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GENERAL ELECTRIC
REVIEW

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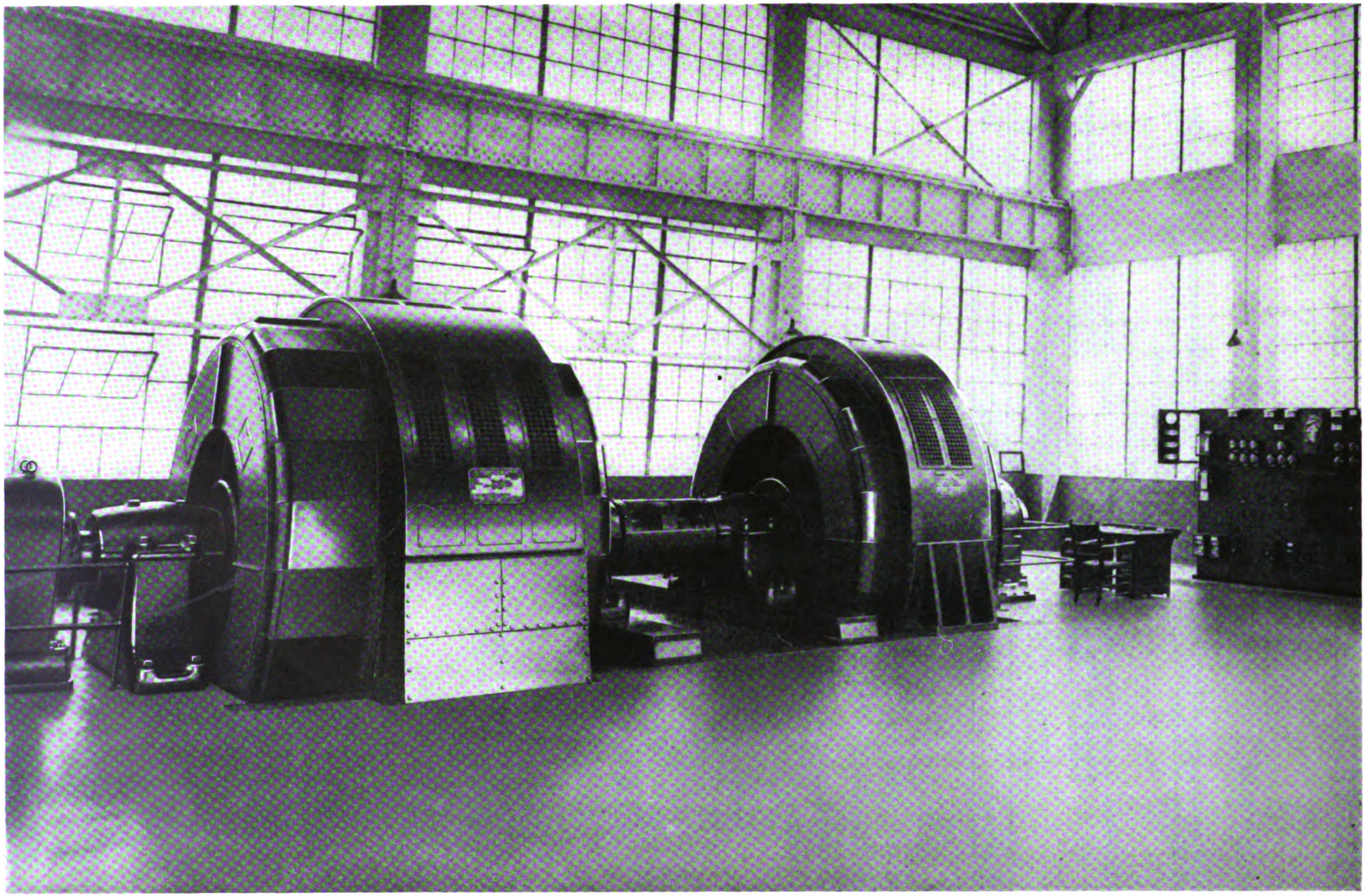

AT SKYKOMISH SUBSTATION

This frequency-changer set, consisting of an 8000-kv-a. 13,200-volt three-phase 60-cycle synchronous motor and a 7500-kv-a. 13,200-volt single-phase 25-cycle generator with their respective direct-connected exciters, provides power for the recently electrified Cascade Division of the Great Northern Railway. The 25-cycle output is stepped up to 44,000 volts at the substation for distribution, and later made available at the locomotives by the use of auto-transformers placed along the right-of-way

GENERAL ELECTRIC

REVIEW

Vol. 31 January, 1928 No. 1

THE VALLEY OF THE GIANTS

This is the Valley where giants are born—where masses of metal become the producers of power, where many-spoked wheels turn slowly to achieve their final balance, and muscles of steel are fashioned into the sinews of huge machines that go to support life and light in all parts of the world. Here a thousand men appear and disappear in the dim shadows, crawling in and out of the great openings where steam will some day rush forth, perching high in the bosom of a casting, calmly presiding over the keen tooth of a lathe as it slides smoothly beneath the iron skin.

Here is the Valley of the Giants. The men swarm out at the tall doors; a few scattered arc lamps high up are left to guard the place. Robed in their purple shadows, the rulers of this new monarchy in the making stand alone in their dignity, gathering in the silence the spirit of their mighty activities to come.

One man lingers there amid the benevolent giants of steel and steam and electricity, and down the narrow aisle between them he sees a vision. The cities and the countrysides of the world stretch out before him in miniature ranks and rows. People move and struggle beneath their burdens. Wars, revolutions; the smoke of industry; ships of commerce headed in and out of ports; sorrow and happiness—but always the struggle to lift a burden. This vision fades and another takes its place—this time a vision of a smaller group of men and women, working quietly over laboratory desks with tubes and wires; immersed in books, with instruments at hand; men with tools—men standing at the fiery doors of furnaces—men who with a single finger control the jaws and gears of great machines.

Then a third vision appears, there among the giants. A pair of balance scales—a symbol of the world. Humanity stands crowded in the one pan, reaching up to greater heights with the right hand and lifting a greater burden with the left—and that side swings slowly down. In the other pan the the men of science and of industry are building—adding constantly to their giants of power, so that the scales swing gradually back and then beyond the balance point, more than restoring human comfort and well-being. And so the swings continue—symbols of the world and progress—cause and effect—struggle and victory.

With these three visions the man sums up the picture of modern progress. First, the struggle for better things, and the people voluntarily accepting heavier and heavier burdens. Then the answering effort of those who, by their professional obligation, have accepted the duty of helping to lift those burdens.

As the lone man leaves, he realizes that the massive shapes are but symbols of something greater than themselves. For there are other giants than those of mere size. In this Valley are but a few of a vast army of other giants of strength and speed—the atoms and electrons which in some strange way have gone into the construction of the world, and so lie behind everything we do and are. The form of these greater giants we have so far but dimly perceived, but light is coming and we are preparing ourselves constantly for what we may find.

This is our age—this age of motion and the use of force. We are not passive. It is not merely according to our pleasure that we enter the stream of progress or stand aside while it flows by. Born into a world that flies, that talks with voices heard half around the globe, that controls energy from a source far greater than itself, we cannot repudiate our birthright nor avoid the responsibilities which are the only payment nature demands of us for these benefits.

The sun has marked off a period—a cycle so small in the total of his travels that he scarcely heeds it as he passes. We call that period a year; and we pause, looking back to see what we have done. However small, that short interval has placed another tier of brick beneath our feet, set down with care to make a firmer foundation for us as we lift our larger burdens with increasing strength.

In the Valley of Giants we are their lively souls. They cannot crush us if they would, for they are dead without our inspiration. Our living energy we have reproduced in steel and copper and motion a thousandfold. This infinitesimal period called a year has made finite return on our investment of brains and strength. New power is ours for the asking. Who shall be idle now?

D. O. WOODBURY

Some Developments in the Electrical Industry During 1927

By JOHN LISTON
General Electric Company

THERE were, during the year, numerous improvements in design, in process work, and in manufacturing methods which resulted not only in improved efficiencies but, in certain classes of electrical equipment, combined this result with lower costs. The economies thus secured were shared with the consumer through the medium of reduced prices in the classes of apparatus affected.

New records in the unit capacity of steam turbine generators were achieved in both the single-cylinder and tandem-compound types, with single generators rated 75,000 kw. and 160,000 kw. respectively. The maximum capacity of the cross-compound type, represented by the 208,000-kw. set under construction, was not exceeded. There was an increased use of high pressures, many of the large turbines utilizing steam at pressures up to 600 lb. There was also a pronounced tendency to utilize higher steam pressure for the industrial type of turbine than had been previously considered practicable.

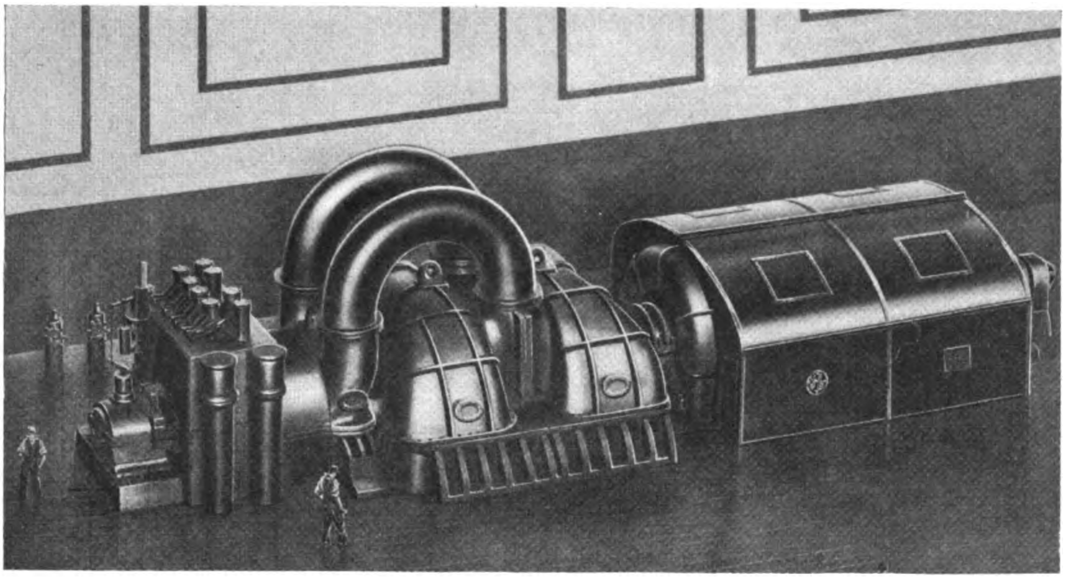
Fig. 1. Drawing of 160,000-kw. Tandem-compound Turbine-generator for the New York Edison Company. The single generator of this set is the largest ever constructed

In the fabrication of large generators for both turbine and waterwheel drive, synchronous condensers, and similar apparatus, there was an increased use of structural-steel and steel-plate welded construction, this method having practically superseded the use of large castings in the building of machines of this class. Another feature of great importance was the cooling of these machines by an enclosed system of circulating gas which not only permits a more accurate control of the temperature than does air cooling, but enables the machines to operate with a higher output than was possible with former methods of cooling.

There were two events of outstanding importance in the marine equipment field, the first being the placing in commission of the airplane carrier *U.S.S. Saratoga*, the most powerful craft afloat. With her electric propelling equipment of 180,000 s.h.p., this ship utilizes approximately as much energy for propulsion as six battleships. The second event was the completion of the turbine-electric passenger liner *California* which has the highest-rated electric propelling equipment in the merchant marine service. She is also the largest craft of her type so far constructed in an American shipyard.

Four motor-generator-type locomotives each of 260 tons were built for the Great Northern Railway. These receive alternating current at 11,000 volts and convert it to direct current for the driving motors. The Chicago, Milwaukee and St. Paul Railway completed the electrification of its entrance into the city of Seattle by the equipment of a 10-mile double-track line.

The system of automatic control of machinery in power stations and substations was utilized for plants of greater capacity in hydro-electric power stations, and in railway, mining, and industrial service. Railway substations were to an increasing extent equipped with mercury-arc power rectifiers which have now been in service for a sufficient length of time to demonstrate their dependability.

Many ingenious applications were made of vacuum-tube control for switching, protective, and indicating devices in power stations and on transmission lines. These were developments supplementing the use of carrier-current communication, which was also further extended during the year, and indicate the possibilities of a great variety of uses for vacuum tubes for purposes other than radio broadcasting and receiving.

The successful operation of the 132-kv. oil-filled type of conductor cable greatly enlarged the scope of the economical underground transmission of large blocks of power. This method is of maximum value where it is desirable to operate high-voltage conductors in cities or other congested areas.

There was a well-defined tendency toward the further adoption of complete, self-contained, switching systems in which the circuit breakers, switches, busbars, and auxiliary equipment are all metal-enclosed and connected to oil reservoirs which maintain the oil at a suitable pressure throughout the switching group. Modifications of this system utilized fluid or semi-fluid compounds in place of oil. This same method of complete metal enclosure of the equipment was also adopted for subway-type transformers.

In presenting a review of these and the many other developments which occurred during the year, all the apparatus referred to are products of the General Electric Company. These cover such a wide range that the references will serve as an indication of the tendencies in design and construction as well as the general progress in the electrical manufacturing industry as a whole.

TURBINES

High-pressure

During the year, eleven turbine-generator sets ranging in capacity from 40,000 to 91,500-kw. were operating at 550 to 600 lb. pressure. In addition to these, two sets were operating at 1200 lb. pressure. No difficulty developed in any of these machines which could be attributed to the use of high-pressure steam.

The third turbine designed for operation on 1200 lb. pressure (Fig. 2) was placed in commercial service. This is a 10,000-kw. 12,500-kv-a. 3600-r.p.m. unit located in the Edgar Station of the Boston Edison Company. It exhausts through a separate superheater into the 375-lb. mains.

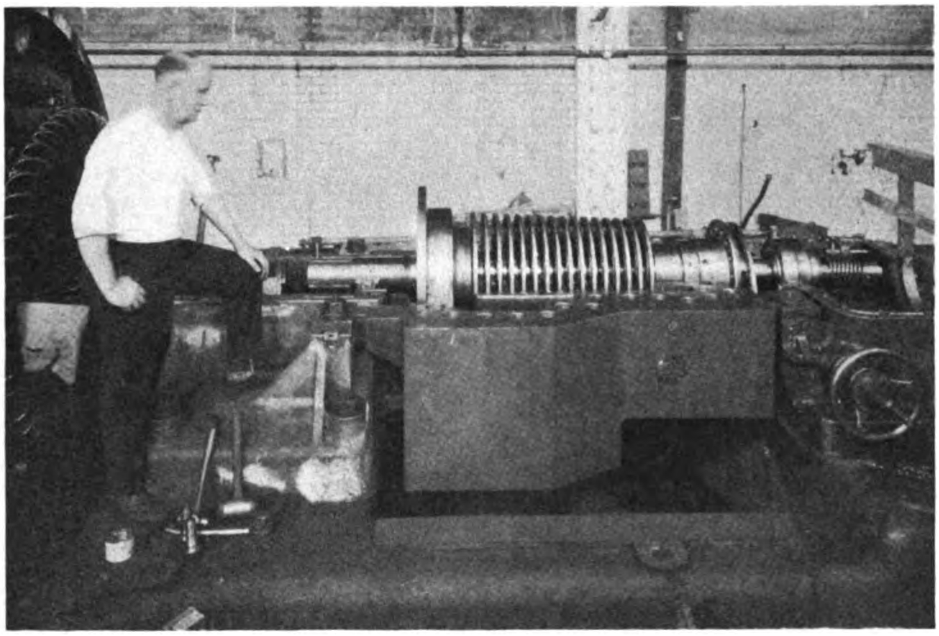

Fig. 2. 10,000-kw. High-pressure Turbine Showing Unusual Form and Massive Construction of the Casing

There are now installed or under construction a total of twenty-one G-E turbines designed for steam pressures of 550 lb. and higher with an aggregate capacity of 1,148,500 kw.

Single-cylinder

The 65,000-kw. unit referred to in last year's review is now in service in the Edgar Station of the Boston Edison Company, and is the largest single-cylinder unit in operation.

A still larger machine is now being built. It is a single-cylinder single-generator unit for the Buffalo General Electric Company and is rated 75,000 kw. at 1500 r.p.m., which is a new record size for this type of machine.

In smaller capacities, new designs of 15,000-kw. 1800-r.p.m. and 10,000-kw. 3600-r.p.m. units were developed to embody in their construction all the new features which would tend to increase the efficiency of machines of this capacity.

Tandem-compound

One 94,000-kw. unit of this type for the Southern California Edison Company was completed and tested.

As the result of recent developments, it was possible to design for the New York Edison Company the largest single-generator unit ever constructed. This machine (Fig. 1) which will be installed in the East River Station is rated at 160,000 kw. with a single 1500-r.p.m. 25-cycle generator. The steam end is tandem-compound with a single-flow high-pressure element and a double-flow low-pressure element.

An unusual feature in the construction of this generator is the fact that the stator winding is composed of two separate windings which are identical

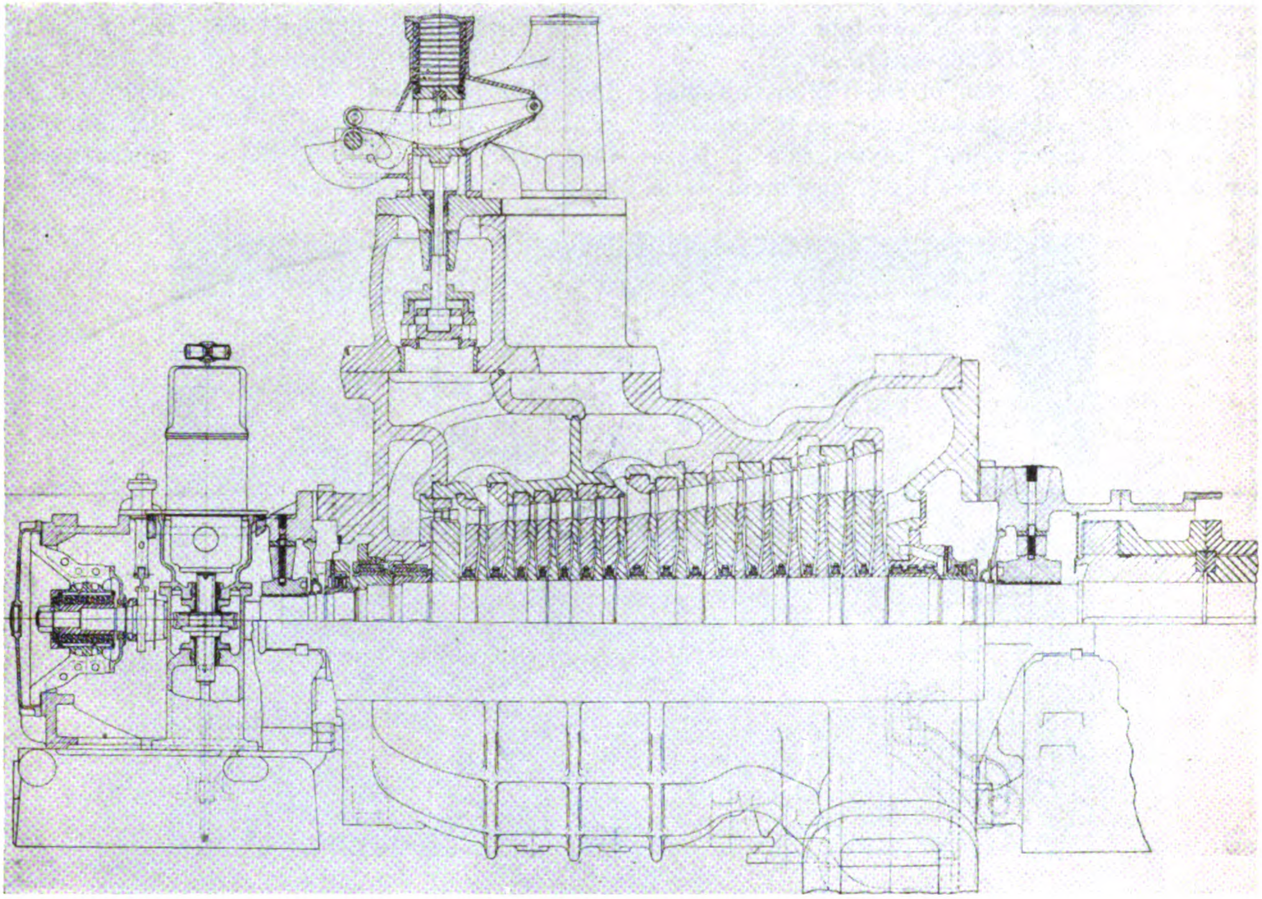

Fig. 3. High-pressure Element of 208,000-kw. Triple-compound Turbine

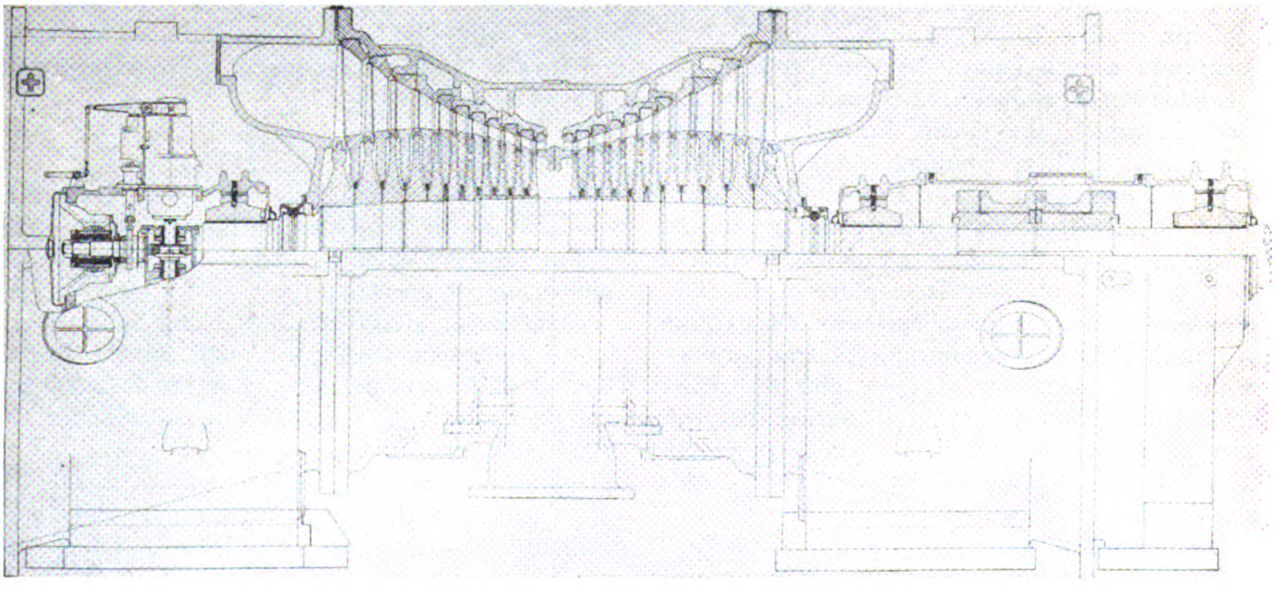

Fig. 4. Low-pressure Element of 208,000-kw. Triple-compound Turbine

in all respects but are electrically separate so that the capacity of each is 80,000 kw. The two windings are connected through two generator breakers to adjacent bus sections.

The advantages afforded by this construction are: (1) the reduction in generator breaker capacity caused by the use of two generator breakers, (2) the elimination of bus reactors made possible by the transfer of power taking place through the coupling effect of the two separate windings, and (3) the increased stability obtained by omitting the bus reactors.

Cross-compound

The 91,500-kw. cross-compound turbine now in service in the Crawford Avenue Station of the Commonwealth Edison Company is the first unit to operate with steam reheated, after partial expansion, by steam at boiler pressure and temperature in a heat exchanger. Previously the steam has been reheated in special reheat boilers. This unit was the first to develop a capacity of over 100,000-kw.

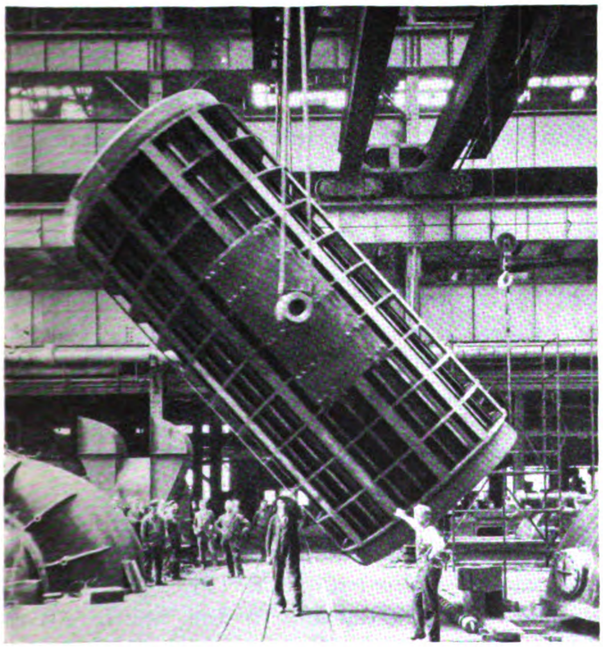

Fig. 5. Stator Frame with Stacked Laminations for 100,000-kv-a., 16,500-volt Generator Showing Latest Form of Welded Construction

Construction of the 165,000-kw., triple, 60-cycle turbine-generator set (Fig 6) for the Philo Station of the Ohio Power Company was well under way. This set is designed for an initial pressure of 600 lb. and a steam temperature of 725 deg. The steam will be reheated to the original temperature between the high-pressure and the two low-pressure elements. All three elements operate at 1800 r.p.m. and each main generator has a capacity of 53,000 kw. at 0.85 power-factor. Each low-pressure turbine will also drive a 3000-kw. service generator. Each main and service generator will have its own direct-connected exciter. The generators are of special construction so as to be suitable for the possible future use of hydrogen as the cooling medium.

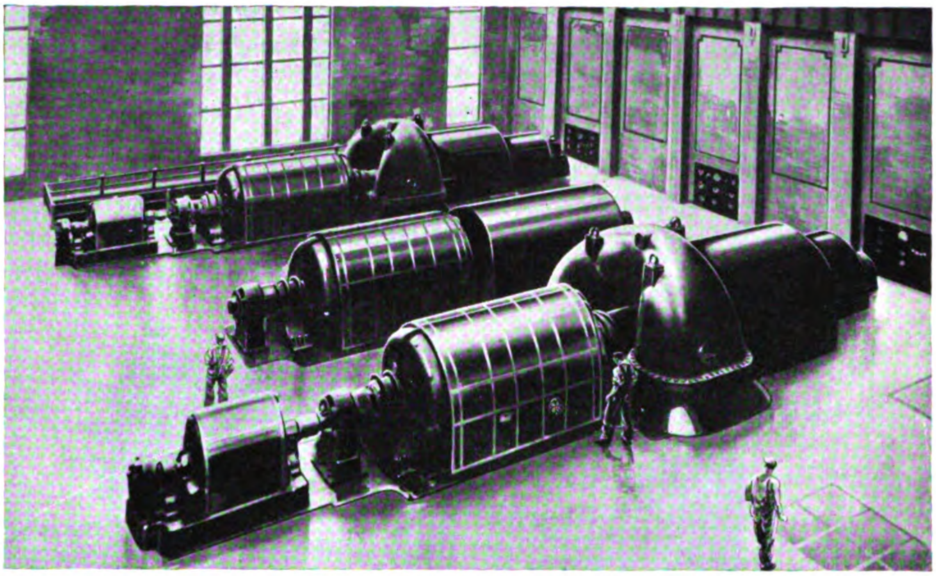

Fig. 6. Drawing of 165,000-kw. Triple-compound Turbine-generator for the Philo Station of the Ohio Power Company

Work on the 208,000-kw. triple unit for the State Line Generating Company (Figs. 3 and 4) was progressing satisfactorily.

Turbine-generators

The use of a fabricated structure for armature frames on all large turbine-generators has the advantage of minimizing foundation requirements and also makes it possible to ship completely assembled and wound machines of much greater capacities than would be possible with cast frames. An excellent example of this construction is found in a 100,000-kv-a. generator for the Southern California Edison Company (Fig. 5), the stator of which, despite its large size, will be shipped as a unit with punchings and windings assembled.

The generators of the 208,000-kw. set for the State Line Generating Company will develop 22,000 volts, and, in order to study the action of this potential on the insulation, sample coils were made up which were repeatedly tested at 75,000 volts with no indication of distress. Similar coils are now undergoing a laboratory life test at 33,000 volts, which is approximately 2.6 times the "Y" voltage of the system on which they will operate.

The 6250-kv-a. generator, which was designed for operation in an atmosphere of hydrogen, was tested during the year and the results substantially checked the estimates that had been made from theoretical considerations.

Briefly, the tests so far have shown that a generator built primarily for hydrogen cooling can be made with 25 to 30 per cent less magnetic material than a machine operating in an atmosphere of air. This is not all clear gain, however, as the construction to retain hydrogen, according to our present knowledge, is very expensive. However, it is possible that as the art progresses this cost might be materially reduced, and would show a substantial saving in the price per kilowatt of such machines over those operating in air.

Apart from the consideration of first cost of the machine, there is a distinct advantage in favor of the hydrogen-cooled generator in the matter of reduced losses, and owing to the absence of an oxidizing atmosphere it is probable that the life of the windings will be materially increased.

INDUSTRIAL TURBINES

There was a notable increase in demand for higher steam pressures and temperatures for industrial type turbines, and a considerable number of units were constructed for operating conditions between 300 and 400 lb. initial steam pressure and temperatures up to 750 deg. F.

An example of this class of machine is found in a 3000-kw. 12-stage non-condensing unit (Fig. 7) built for the International Paper Company. This will operate under steam conditions of 400 lb. initial pressure, 160 deg. F. super-heat, and 125 lb. back pressure.

There was a corresponding increase in extraction pressures and a number of units were placed in service operating with extraction pressures between 50 and 165 lb.; the extraction flow being automatically controlled by internal grid valves.

There were a number of applications of turbines of exceptional capacity to the driving of blast-furnace blowers, and for Bessemer converter service. Some of these turbines were rated as high as 7000 h.p. with blower capacities up to 60,000 cu. ft. per min. For this service also, increased use was made of higher steam pressures and super-heat.

Two turbine-driven blowers for the Central Furnace Company at Massillon (Ohio) are typical of this service. The turbines are 8-stage and are rated 5300 h.p. at 4150 r.p.m., while the compressors are 3-stage and rated 50,000 cu. ft. per min. at 30 lb. pressure. These turbines are operated condensing and the speed of the sets is controlled by constant volume governors.

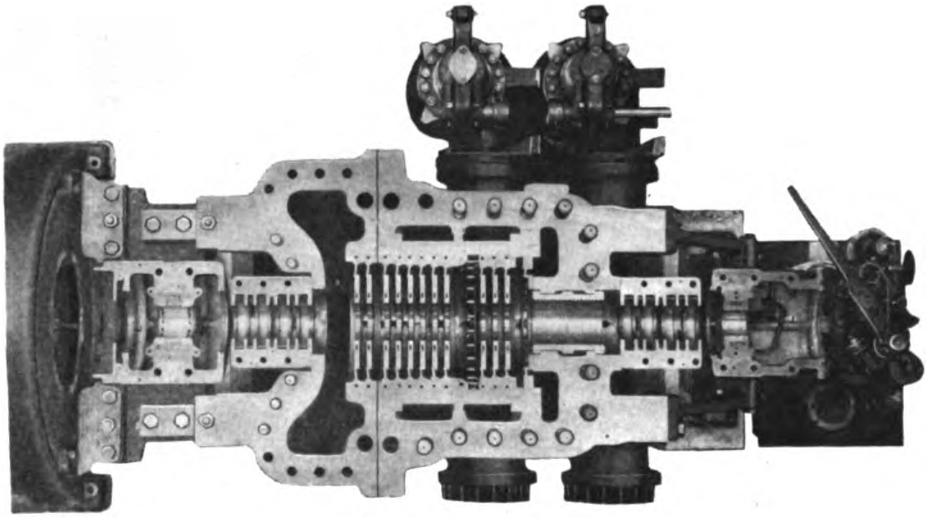

Fig. 7. Lower Half of Casing of a 3000-kw. High-pressure Industrial Type Turbine

EMMET MERCURY VAPOR PROCESS

The 1800-kw. 3-stage mercury turbine equipment installed at the Dutch Point Station of the Hartford Electric Light Company during the summer of 1925 continued to operate for long periods in a manner to warrant the building of a large size equipment for installation over a modern, commercial-size furnace. The results obtained during operation have confirmed the economic value of the process and demonstrated its practicability. Pulverized coal has been successfully burned at high ratings although this furnace was originally designed for burning oil.

Early in 1927, construction was begun on a large commercial mercury boiler and turbine unit for the same company. The equipment (Fig. 8) is now being installed in an extension to the South Meadow Station at Hartford (Conn.). The new mercury turbine-generator is expected to deliver 10,000 kw. when burning about 14,500 lb. of coal per hour in the furnace. In addition, the condensing mercury will generate some 125,000 lb. of steam per hour which will be superheated to 700 deg. F.

Several mercury boiler units were completed and installed early in 1927 in an oil refinery; the mercury vapor generated by them being used for refining oil. They have now been in continuous satisfactory operation for several months.

Pulverized fuel will be used at Hartford, three coal mills discharging directly through the burners into the furnace. The flue gases pass upward along the mercury boiler tubes and between the cylindrical headers. The gases next pass over the mercury liquid heater and steam superheater, and then downward over the water economizer and air preheater, through the induced draft fan, and up the stack. The flue gas temperature at the stack is expected to be reduced to 280 deg. F. After leaving the preheater the air passes over the furnace walls, cooling the refractory surface.

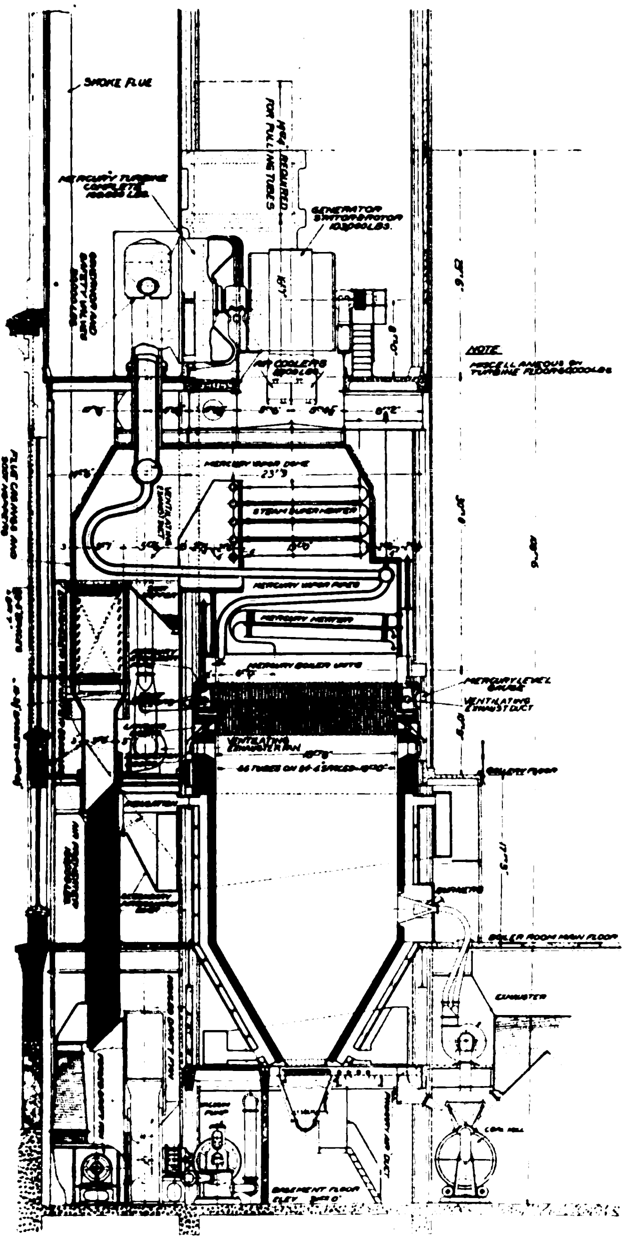

Fig. 8. Side Elevation of Mercury-vapor Power-plant Equipment

The equipment is designed for a mercury pressure at the turbine of 70 lb. gauge and a steam pressure of 350 lb. gauge. The mercury turbine is a 5-stage 720-r.p.m. machine, having an overhung rotor; the complete weight of the revolving parts of the turbine and generator being carried by the two generator bearings. The mercury vapor flows through the turbine toward the generator. The shaft packing, therefore, is at the low-pressure end of the machine. After passing through the turbine, the mercury vapor exhausts into two condensers placed at each side of the turbine. The vapor delivers its heat of condensation to vertical dead-ended tubes containing water, the temperature drop across the tubes being about 20 deg. F. Thus any steam pressure up to the designed value can be generated. Upon condensing, the mercury passes through a cleaning sump and returns by gravity flow to the boiler.

SUPERCHARGERS

Appreciable progress was made in superchargers although no new types were produced.

Exhaust turbine-driven side-type superchargers, equipped with ball bearings and improved turbine-bucket material, have now been in service for nearly a year and have given no trouble whatever due to bearing failures, stretched turbine buckets, or other mechanical difficulties in the supercharger itself.

Superchargers of this type are being used to some extent for high-altitude bombing planes, for towing anti-aircraft gun targets at high altitude, and for pursuit and observation planes. Some of the more recent tests show that pursuit planes so equipped have a speed at high altitudes materially in excess of the speed at sea-level. Such a performance has not been possible heretofore.

Gear-driven superchargers, which are built directly into the engine, have now been adopted by all of the builders of radial air-cooled engines in this country, and have reached a quantity production basis. These superchargers were originally intended to mix and vaporize the fuel and overcome the pressure drop through the carburetor only, but tests have shown that an appreciable gain in the performance of the engine and plane result if supercharging is provided beyond that necessary to overcome the carburetor drop.

MARINE EQUIPMENT

Since the first commercial application of electric propulsion in 1908, when the Chicago fireboat *Graeme Stewart* was placed in commission, there has been a consistent growth in the number and size of electrically-propelled craft.

At the close of the year 1927 there had been placed in commission or were under construction a total of 118 electrically-propelled vessels of various types, ranging from river towboats and small yachts to the largest types of seagoing ships. The equipment of these craft aggregated more than 700,000 s.h.p., about 92 per cent of the primary power being supplied by turbines and 8 per cent by Diesel engines.

The General Electric propelling equipments constitute about 85 per cent of the total shaft horsepower involved and consist of 29 turbine-electric drives totalling 566,250 s.h.p. and 38 Diesel-electric drives totaling 33,800 s.h.p.

Turbine-electric

The outstanding event of the year was the completion and installation of turbine-electric propelling equipment for the Panama Pacific liner *California* (Fig. 9), the largest passenger ship ever built in the United States and the largest electrically-driven ship of her class in the world. The *California* is a twin-screw ship with a displacement of 30,250 tons at load draft.

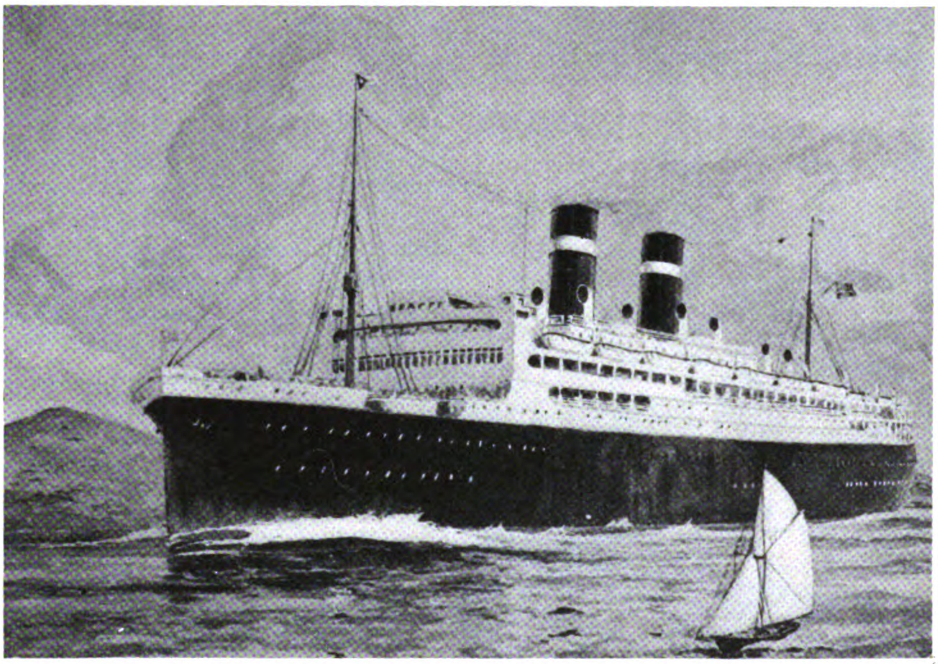

Fig. 9. S. S. *California*, the Largest Turbine-electric Passenger Ship

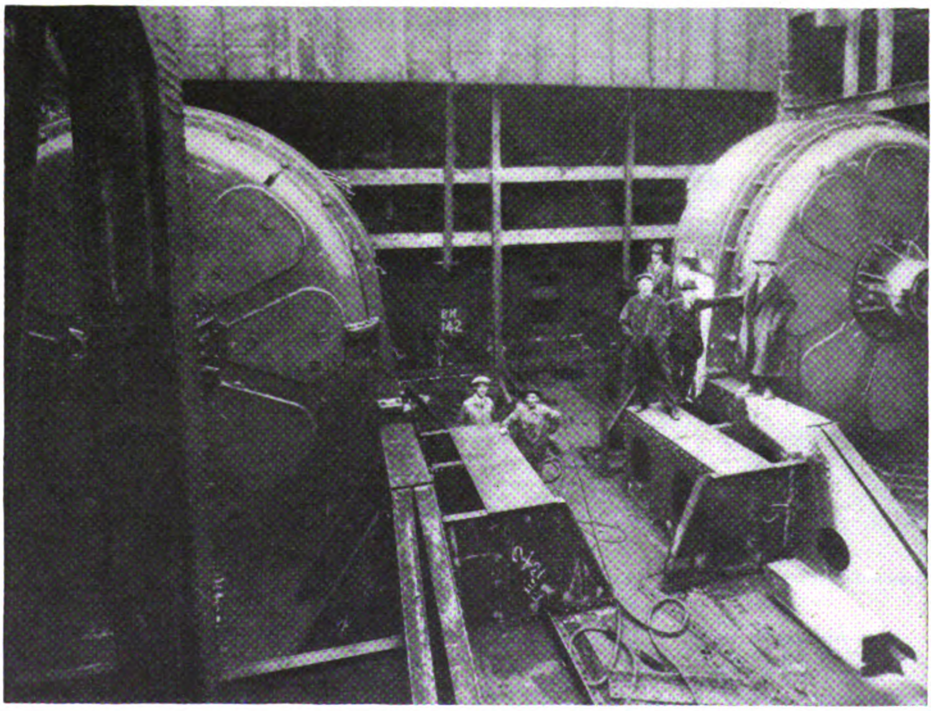

Fig. 10. Main Propulsion Motors of S. S. *California*, Each Rated 8500 s.h.p. at 120 r.p.m.

The maximum energy delivered to her propeller shafts is 17,000 s.h.p. and a speed of 18 knots can be maintained with this power input. At the cruising speed of 16.5 knots, the output of the turbine-generator is 13,500 s.h.p., and at this speed the ship has a cruising radius of 15,400 miles.

The propelling equipment comprises two 16-stage, steam-turbine generators, each having a maximum capacity of 6600 kw. at 2880 r.p.m. This power is transmitted to the propeller shafts by means of two synchronous-induction type motors having a continuous maximum rating of 8500 s.h.p. at 120 r.p.m. These motors (Fig. 10) are direct connected to the propeller shafts and are reversible so that no reversing turbines are required. Both propelling motors can be operated at about three-quarters of their rated output by the current supplied from one of the two turbine-generators, thus insuring economical cruising at reduced speed.

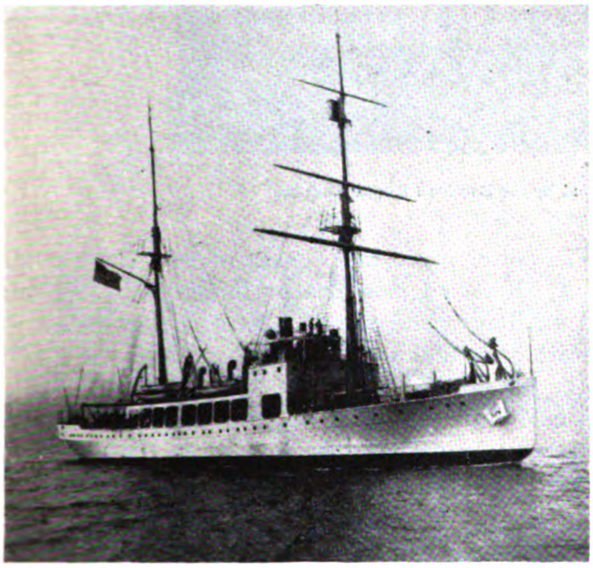

Fig. 11. U. S. Coast Guard Cutter *Northland* Equipped with Diesel-electric Drive

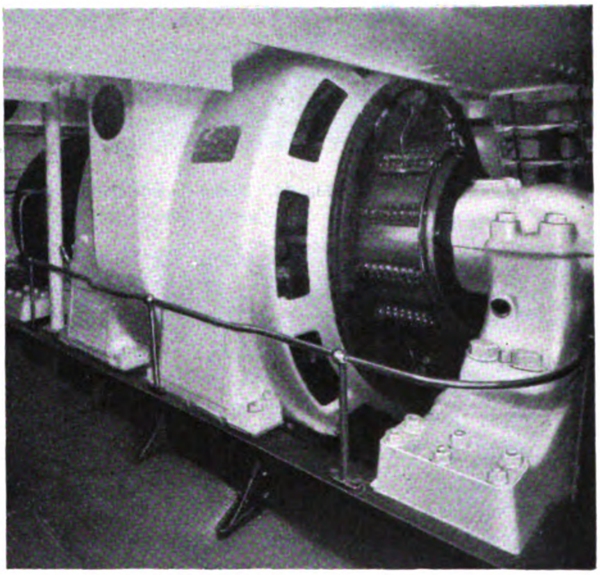

Fig. 12. 1000-s.h.p. Double-armature Propeller Motor, Looking Aft. The dark disk aft of the motor is the magnetic clutch

The *California* was launched October 1, 1927, and work on a second ship of the same size, to be provided with electric propelling equipment of the same power, is already under way.

The second of the turbine-electric self-unloading limestone carriers for the Bradley Transportation Company went into service in July, 1927, and, in addition to being the largest bulk freight carrier on the Great Lakes, is of exceptional interest due to the unusual features of her equipment.

The power developed is normally about 4200 s.h.p., but the turbine-generators are capable of a maximum continuous output of 4800 s.h.p. The steam conditions represent a marked advance over ordinary marine practice in that the turbines are supplied at 300 lb. pressure and a total temperature of 700 deg. F., and will operate on a 28½-in. vacuum.

The electric auxiliaries for use when under way are driven from the main unit directly or from a motor-generator set which operates from the main unit, while the electric equipment for unloading cargo in port is driven directly from the main unit. When under way, a variation in the revolutions per minute of the propeller is accomplished by varying the speed of the turbine. In port, the main turbine is run at constant speed and, in case maneuvering is required simultaneously with the unloading operation, variation in the propeller speed is accomplished by means of resistance inserted in the secondary circuit of the propelling motor.

Diesel-electric

Early in the year the new Diesel-electric coast-guard cutter, *Northland* (Fig. 11) was launched, and is now serving on patrol in the Alaskan waters as the successor of the famous patrol cutter *Bear* which was built in 1874.

The main engine room equipment consists of two Diesel-engine-driven generators, each rated 410 kw., 250 volts at 200 r.p.m. These supply current to a double-unit type, shunt-wound propeller motor. Each section of this motor (Fig. 12) is rated 500 h.p. at 120 r.p.m. The ship will develop a speed of 12 knots with her single propeller operating at 120 r.p.m. and the motor developing 1000 s.h.p. The control is located in the engine room and is of the variable-voltage type.

There are some unusual features in the propelling equipment, one of them being the use of a magnetic clutch between the motor and the propeller shaft. Under normal conditions the motor and shaft are rigidly bolted together, but when cruising at reduced speed in the ice fields the bolts will be removed and the power transmitted through the magnetic clutch. This will transmit a torque equivalent to 500 s.h.p. at 95 r.p.m., but any increase of load beyond this rating will cause the clutch to slip and will thus prevent dangerous stresses.

The propeller-shaft thrust bearing is of the spring-thrust type, and electro-hydraulic steering equipment is used.

In addition to the main generators, there are two auxiliary generators mounted on shaft extensions of the main units and three independently-driven auxiliary generating sets. They supply current for the operation of 22 motor-driven auxiliaries, and for lighting.

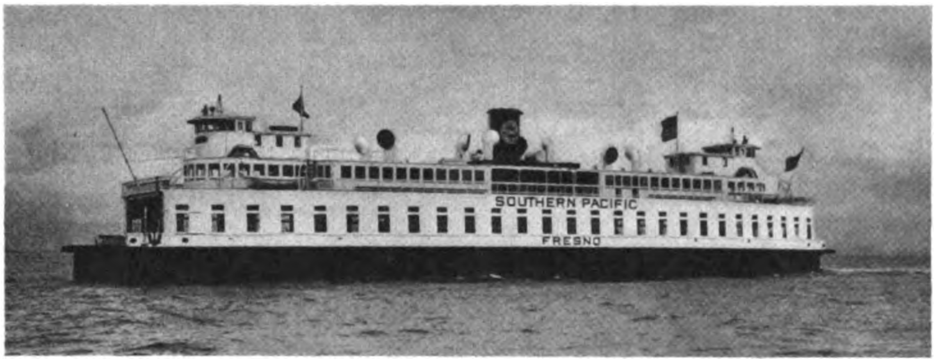

Fig. 13. Diesel-electric Ferryboat *Fresno* Operated by the Southern Pacific Railroad

Four large double-end Diesel-electric ferry-boats were completed for the Southern Pacific Company for service in San Francisco Bay.

The main power plant in each of these boats (Fig. 13) consists of four 450-h.p. Diesel engines, each direct connected to a 275-kw. 250-volt shunt-wound separately-excited direct-current generator. Each set has a 40-kw. 125-volt compound-wound auxiliary generator mounted on the main engine-generator shaft. These generating sets operate at 230 r.p.m. and the generators are connected in series so that they supply current at 1000 volts to the propeller motors, which are of double-unit construction, shunt-wound, and separately excited.

The continuous-duty rating of these motors (Fig. 14) is 1250 s.h.p. at 100 to 130 r.p.m. with 500 volts on each armature. Speed control of the propellers is effected through voltage control of the driving motor.

The two propeller motors are connected in series with each other and in series with the main generators. In accelerating the vessel, full torque is applied to each propeller for a short period, after which the stern propeller comes up to speed while the bow propeller is rotated at just sufficient speed to overcome its own resistance and compensate for friction.

Complete control of propulsion can be maintained from either of the pilot houses or the engine room, the control equipment being similar in appearance and as simple in operation as the conventional engine-room telegraph pedestal.

Equipment was under construction for an electrically-propelled packet boat designed to carry 250 tons of package freight at a speed of 12 m.p.hr. between New Brunswick (N. J.) and points in and adjacent to the harbor of New York.

In this case, electric propulsion was selected primarily because the height of the engines could be kept down and thus allow a clear deck for package freight and on account of the ease with which this packet boat (Fig. 15) could be controlled in maneuvering in congested waterways.

The power plant will consist of two 250-h.p. 300-r.p.m. Diesel engines driving direct-connected main and auxiliary generators. The single propeller will operate at speeds up to 175 r.p.m., and the driving motor will develop 400 s.h.p.

A field hitherto untouched by Diesel-electric propulsion was opened by the decision of the U. S. Shipping Board to authorize the conversion of cargo boats with this type of drive. The use of Diesel-electric equipment for these craft will permit the operation of the propellers at their most efficient speed and this, combined with certain contemplated changes in the hull, should give an increased speed of about one and one-half knots over what would be obtained by direct Diesel-engine drive.

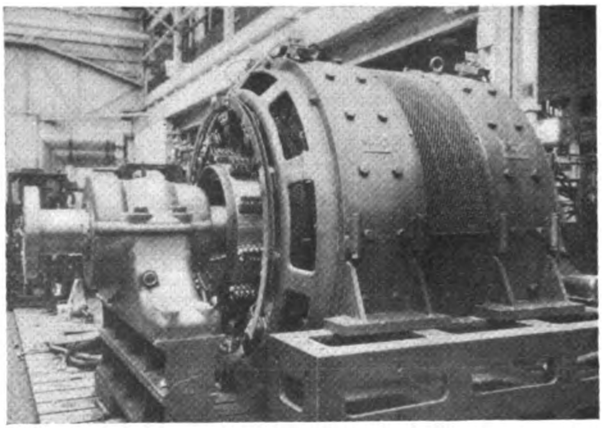

Fig. 14. 1250-s.h.p. Main Propulsion Motor for the Diesel-electric Ferryboat *Fresno*

The engine-room equipment will comprise four direct-current Diesel engine-driven generators, each provided with an auxiliary generator mounted on the same shaft as the main generator. The main propelling motor will be rated 4000 s.h.p., at 70 r.p.m. The control will be of the variable-voltage type, utilizing dead-front panels, and all of the auxiliaries will be electrically driven.

Another novel application of Diesel-electric drive was inaugurated by the Bureau of Light Houses (of the U.S. Department of Commerce) by its decision to adopt this method of propulsion for three new light ships. The equipment for each ship will consist of four 75-kw. main generators and two 7½-kw. separately-driven auxiliary generators, the single propelling motor for each ship being rated 350 s.h.p. at 300 r.p.m.

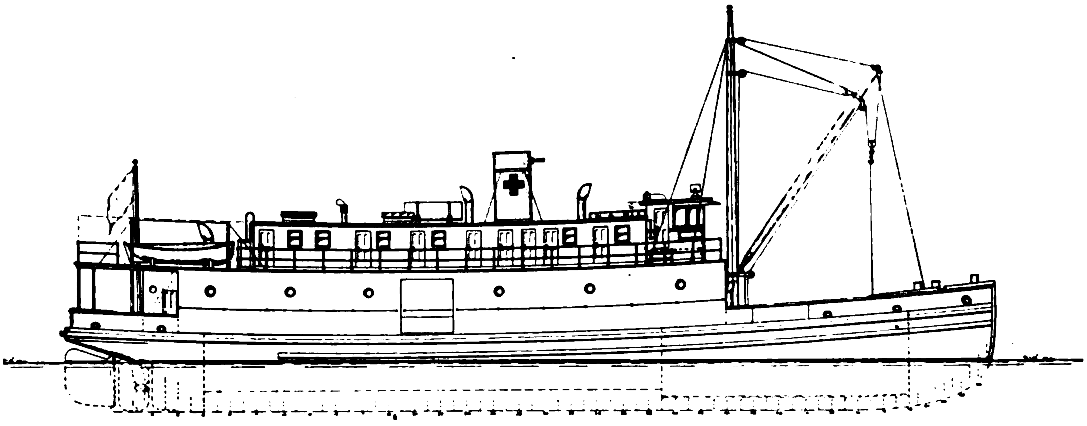

Fig. 15. A New Type of Packet Boat Designed for Diesel-electric Propulsion

In this case, the propelling equipment will be utilized intermittently as the light ships when in service are at anchor most of the time. In severe weather, the propulsion equipment will be used to take the strain off the anchor chains and will, of course, be utilized for maintaining position in the event of the breaking of the moorings.

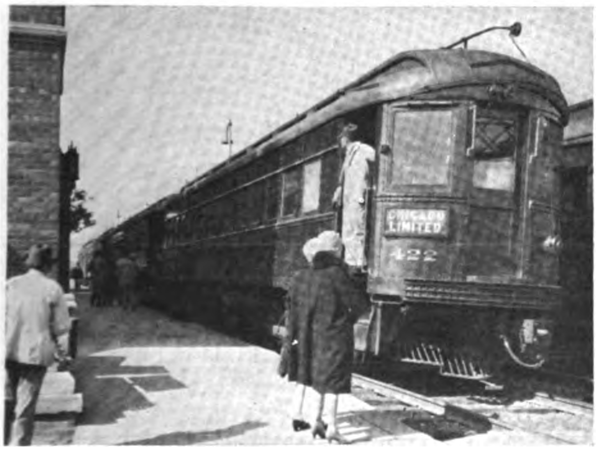

Fig. 16. Train of Four Cars, each Driven by Four 140-h.p. Motors, on the Chicago, Aurora & Elgin Railroad

Auxiliaries

The new single-geared type of motor-operated deck winch, in which the driving motor is designed so that a minimum accelerating torque is required on average loads, was installed on the new International Mercantile Marine liner. This is the first extensive application of these winches and is the result of their ability to give high tonnage rates with low power consumption.

The new 10,000-ton light cruisers for the United States Navy impose severe limitations on the weight of both the main and auxiliary machinery in order to keep the tonnage of these ships within the limitations of the Washington Treaty and at the same time permit them to carry the desired armament.

Work on twenty-four 250-kw. 10,000-r.p.m. turbine-geared direct-current generating sets is now under way, and the adoption of geared connection between the high-speed turbine and the generator has effected a marked reduction in weight as compared with standard equipment. These sets are for the operation of motor-driven auxiliaries on the cruisers.

ELECTRIC RAILWAYS

Activities in the electric railway equipment field were concentrated mainly upon the modernization of rolling stock. Existing cars were replaced with equipment better suited to modern transportation requirements, including motors and control of the latest design.

In the heavy electric railway field, fifteen large new cars each using four 140-h.p. motors and multiple-unit control (Fig. 16) were placed in service by the Chicago, Aurora & Elgin third-rail line. The Milwaukee Electric Railway & Light Company replaced older motors on its suburban line with 36 modern motors each of 140-h.p. rating.

The New York Central Railroad equipped eleven additional motor cars (Fig. 17) for its suburban service with air control and 200-h.p. motors and started rebuilding operations on existing motor cars: lengthening the bodies to increase seating capacity, without changing the electrical equipment. For use on the Hudson and Manhattan lines, the Pennsylvania

Railroad Company added sixteen cars each equipped with multiple-unit control and two 200-h.p. motors.

A number of street railway systems adopted extensive programs involving the use of modern lightweight cars. The Eastern Massachusetts Street Railway and the Worcester Consolidated Street Railway each placed in service 50 new double-truck cars each having four 35-h.p. motors and platform control. Forty similar equipments were placed in service by the Georgia Railway & Power Company on the Atlanta lines and twelve on the Macon lines. The Detroit Street Railway added sixty-two, and the Twin City Rapid Transit Company added twenty-five light-weight four-motor cars.

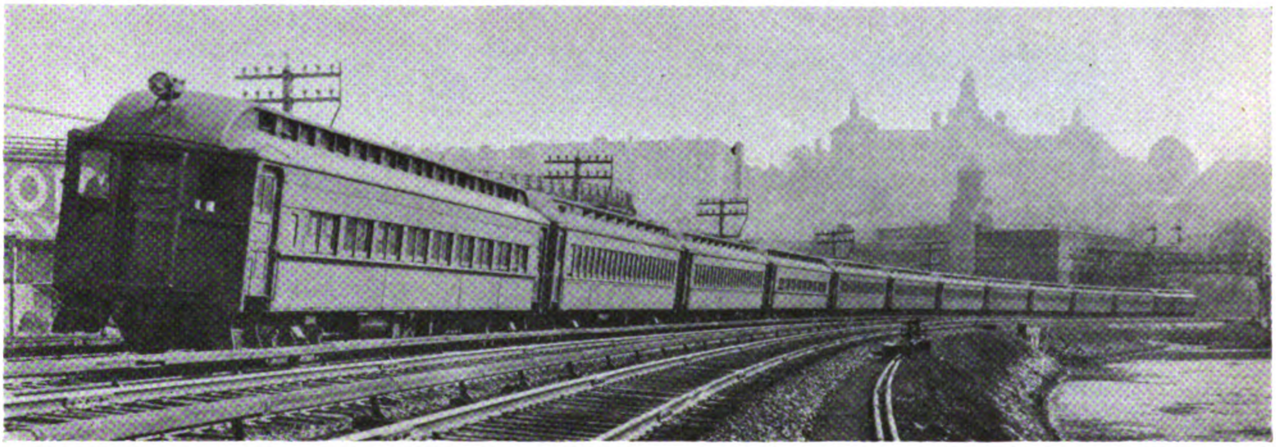

Fig. 17. New York Central Railroad Train Showing Improved Motor Cars Equipped with 200-h.p. Motors

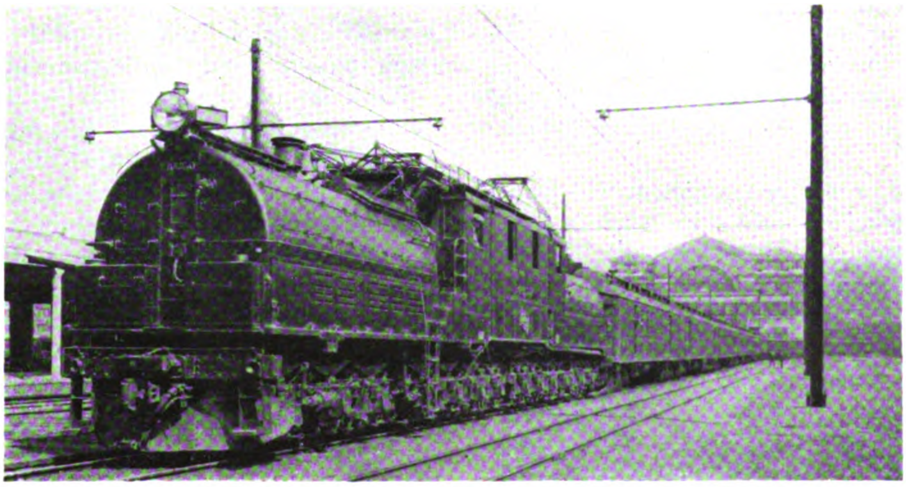

Fig. 18. Transcontinental Train "Olympian" Leaving Seattle Hauled by 260-ton, 3000-volt Gearless Locomotive

Other lines in the mid-west adding this type of equipment include the Gary Street Railway, the Rockford Interurban, the Milwaukee Electric Railway & Light, and the Chicago, North Shore & Milwaukee; in the south, the Northern Texas Traction Company and the New Orleans Public Service Company.

Trial installations of a new type of car using automotive propeller shaft drive were operated on the Cleveland Railways, East St. Louis & Suburban United Railways of St. Louis, Boston Elevated Railway, Detroit Street Railway, and Montreal Tramways. They are equipped with four motors each of 35-h.p. rating.

Gas-electric Buses

Many of the street railway companies which are supplementing rail transportation with motor buses adopted electric drive. The principal additions to gas-electric bus fleets include 50 double-deck and 174 single-deck buses placed in service by the Mitten Management Corporation in Philadelphia and Buffalo, 107 by the Public Service Transportation Company in New Jersey, and 20 by the Capitol District Transportation Company in Albany. During the coming year this latter company will add to its fleet several new single-deck buses each seating 42 passengers. This is said to be the largest seating capacity of any single-deck bus yet built.

Experiments were conducted on an electric transmission suitable for use on taxicabs. This development progressed sufficiently to place a cab in service in Philadelphia to obtain actual operating experience.

Steam Railroad Electrification

Practically no new projects were put forward during the year but a number of the existing electrifications were extended. The Chicago, Milwaukee & St. Paul Railway which had twice previously increased the length of its electric zone completed the electrification of its entrance into Seattle by a 10-mile double-track line from Black River Junction. This new line (Fig. 18) was placed in service without the addition of substations or locomotive equipment, the only expense being for the installation of overhead distribution and rail bonding.

The Mexican Railway Company completed its second addition to the original line and is now operating electrically between Esperanza and Paso del Macho, a distance of 62 miles. In this case it was necessary to install a second substation using two 1500-kw. motor-generator sets to supplement the two 3000-kw. units originally installed in the Maltrata Substation.

The Paulista Railway in Brazil made a third extension of its electric zone (Fig. 19) bringing the total up to 177 miles, thereby completing the electrification of its broad-gauge lines between Jundiahy and Rincao. Any further electrification by this company will probably be accompanied by a widening of the present narrow-gauge lines. Reports by the officials of this system continue to testify to the savings resulting from electrification. (1)

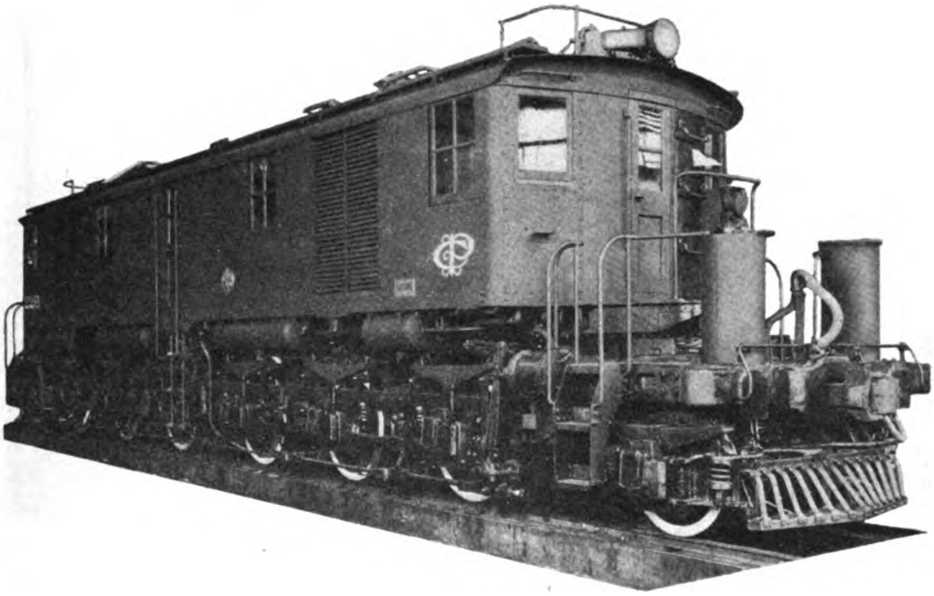

Fig. 19. 150-ton Passenger Locomotive, Paulista Railway (Brazil)

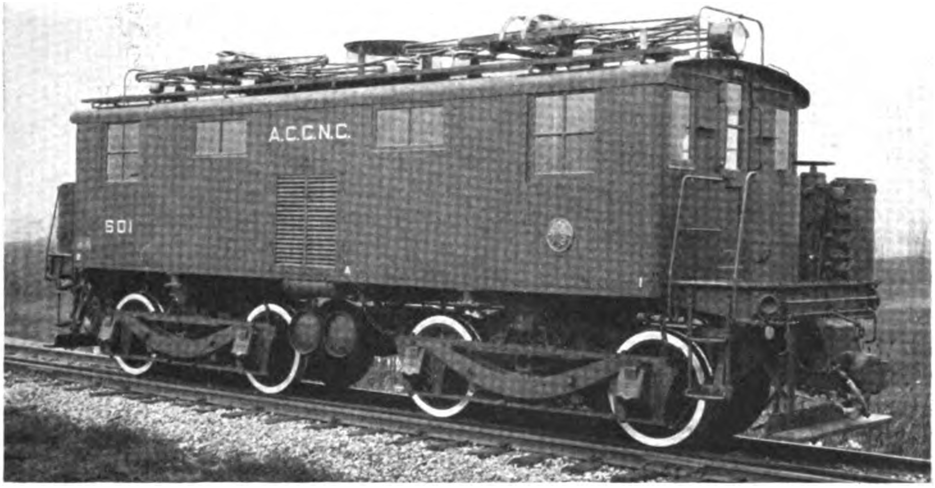

Fig. 20. 66-ton Locomotive for the Anglo-Chilean Consolidated Nitrate Corporation

Equipment for the Anglo-Chilean Consolidated Nitrate Corporation including automatic substations, overhead distribution and locomotives (Fig. 20) went into service in September. In anticipation of a gradual increase in traffic requirements with electric operation, two additional locomotives are now under construction.

In the United States perhaps the most interesting work on railroad electrification is that being done by the Great Northern Railway in the state of Washington. Electric operation between the Cascade Tunnel and Skykomish is to be extended through the new tunnel to Wenatchee before the end of 1928. Two motor-generator-type locomotives, which are the largest single-phase units so far built (Fig. 21), began operation in August, 1927, and two more units of the same type are now under construction together with equipment for a second frequency-changer substation located at Wenatchee.

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(1) Specific data on this subject are contained in the article, "Economies Effected by the Electrification of the Paulista Railway," by S. B. Fortenbaugh, GENERAL ELECTRIC REVIEW, Dec., 1927, p. 595.

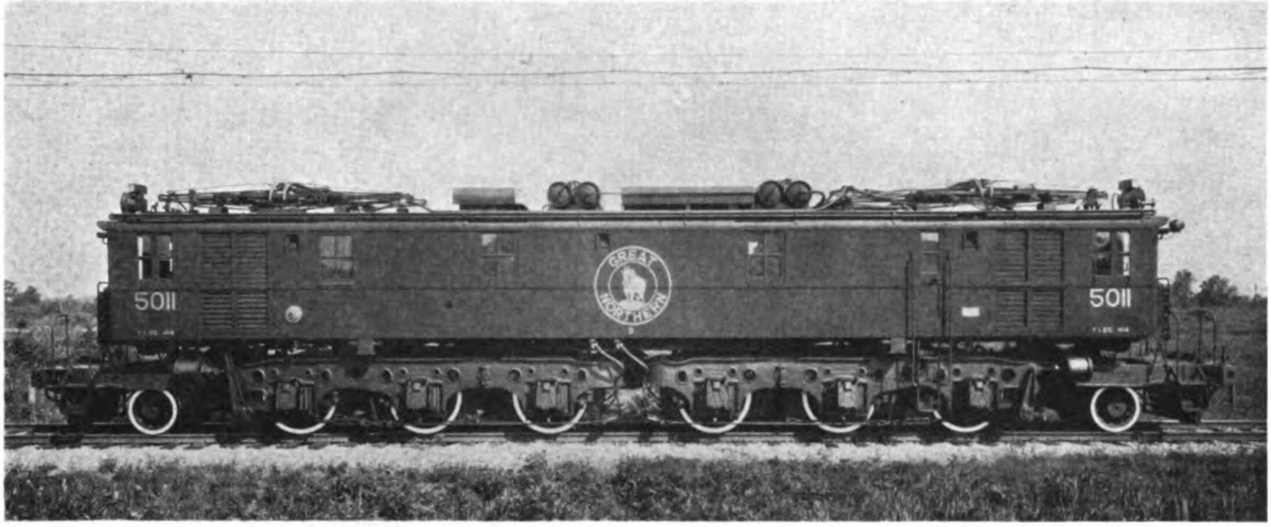

Fig. 21. 260-ton, 11,000-volt A-c.–D-c. Locomotive Built for the Great Northern Railway Company

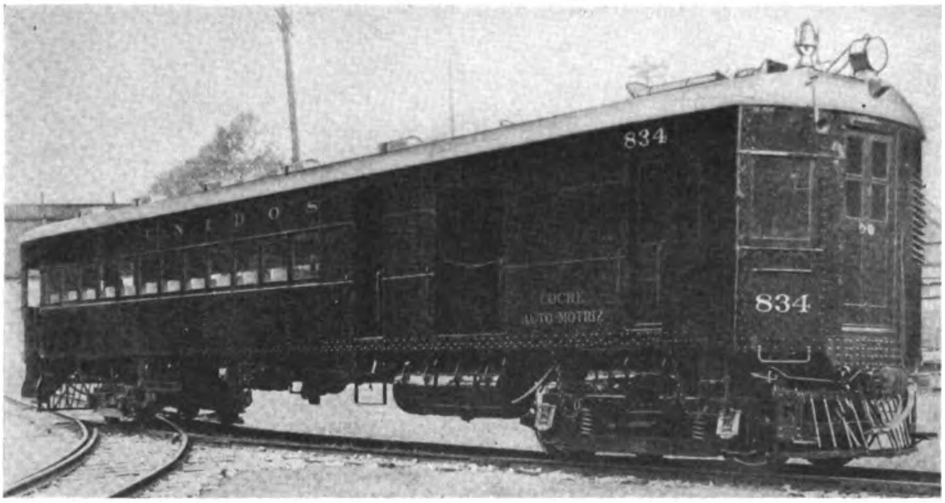

Fig. 22. Gas-electric Motor Car for United Railways of Havana, Equipped with One 275-h.p. Engine-driven Generator and Driven by Two Motors

The New York Central Railroad is actively working on the electrification of its west side yards which will be operated from an overhead trolley north of 60th Street. Below this point self-propelled oil-electric and storage battery units will be used, one of which is already in service. This unit is a 120-ton locomotive equipped with a storage battery and a 300-h.p. oil-engine generating set for battery charging. It will also operate from the third rail.

Equipment for the Wilmington Division of the Pennsylvania Railroad comprising 30 single-phase motor-car equipments and substation switching apparatus includes a new type of air circuit breaker with high-speed characteristics similar to those for direct-current installations. These breakers have a normal rating of 1500 amp. and will be used on 11,000-volt 25-cycle circuits. Oil circuit breakers involving the same operating principle are also to be used.

Electrical equipments for four 200-ton locomotives which will be used on the New York Division utilize motors which are suitable for operation on either alternating or direct current.

Oil-electric Locomotives

Interest continued in the performance of the oil-electric locomotive, a number of which have now been in service for a sufficient period to demonstrate the economies and other operating advantages of this type. Two 100-ton units are now in heavy switching service on the Erie Railroad and 60-ton units were completed for the Chicago & Northwestern Railway, the Union Carbide Company, and the American Rolling Mills Company.

Work is proceeding on the oil-electric freight and passenger locomotives for the Putnam Division of the New York Central Railroad and a 300-h.p. oil-electric motor car is ready for service.

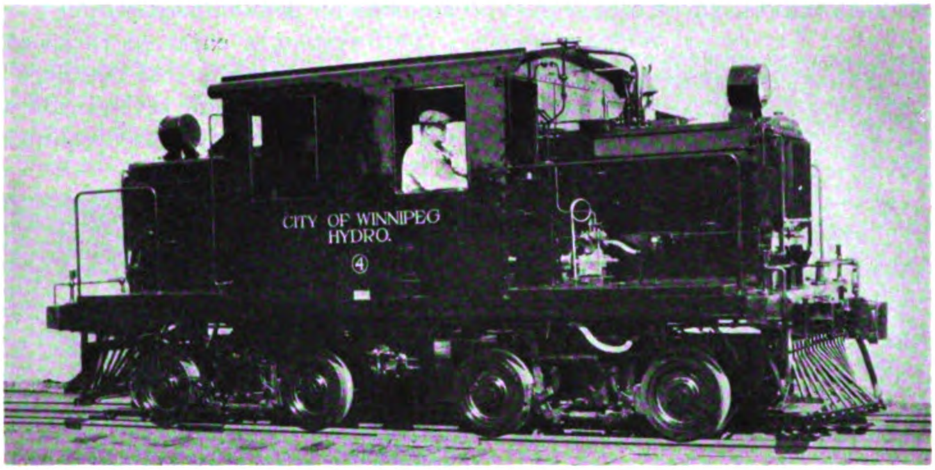

Fig. 23. Typical 20-ton Industrial-type Gas-electric Locomotive

Train Communication

Equipment for radio communication between locomotive and caboose or between train and station was perfected and is giving very satisfactory service on a locomotive operating on the main-line fast-freight service of the New York Central Railroad. A set has also been developed for use in classification yards for directing the operations of the various engines from the yard director's office.

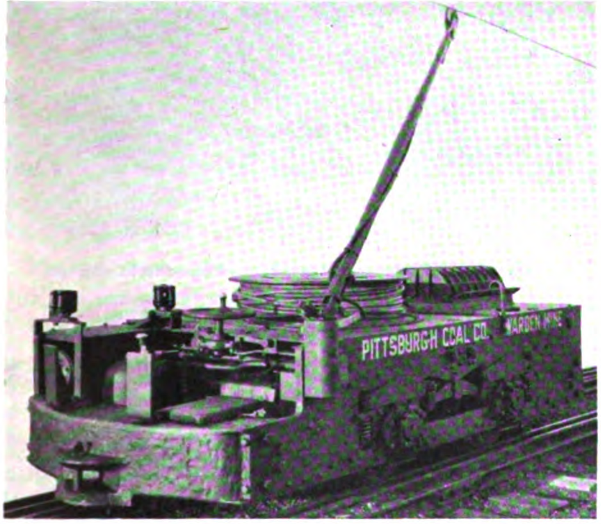

Fig. 24. Cable-reel Gathering Locomotive Provided with "Sealed Equipment"

Gas-electric Cars

As a natural result of its profit-making possibilities, the gas-electric car for branch-line and local main-line service continued to show an increased use. Among the more important additions to existing equipment are sixteen cars to be placed in service by the Chicago, Burlington & Quincy Railroad, one single-unit and two double-power-plant equipments by the Mobile & Ohio Railroad, ten single-unit power plants by the Seaboard Air Line, and three dual-power-plant cars by the New York Central Railroad.

The Chicago, Milwaukee & St. Paul Railway is supplementing its fleet of gas-electric cars with ten 275-h.p. modern cars.

A single-unit-power-plant car was shipped to the Victorian Railways of Australia for trial on the State Lines around Melbourne. Another foreign shipment includes eight cars (Fig. 22) for the United Railways of Havana. In all, about 60 of these cars were placed in service during the year.

Special Type Locomotives

An interesting application of electric transmission is being made on industrial-type locomotives, weighing from 10 to 30 tons (Fig. 23), by using automotive-type engine and generating sets to secure the flexibility of operation inherent in this type of drive.

A new combination trolley and storage battery type of locomotive is to be used by the Chicago, North Shore & Milwaukee Railroad for yard switching. Two units each weighing 70 tons are under construction and will use motor-generator sets for charging the battery when running from the trolley.

MINE LOCOMOTIVES

A new type "sealed equipment" cable reel gathering locomotive (Fig. 24) is provided with two 30-h.p. motors, contactor control of the progressive series-parallel type, and a motor-driven cable reel with 450 ft. of double-conductor, concentric, rubber-covered cable.

The traction motors, the cable-reel motor, cable-reel collector rings, control, headlights, and all parts of the equipment with the exception of the trolley pole and the reel cable are completely enclosed in strong cases which are designed to prevent any gas explosions that may occur within these cases being transmitted to the surrounding atmosphere. With the exception of the short sections of cable leading to the motor, where flexibility is necessary, all wiring (Fig. 25) is enclosed in rigid pipe conduit securely anchored and provided with sealed fittings where it enters the various compartments. The flexible motor cables are enclosed in heavy rubber hose, the ends of which are also provided with sealed fittings.

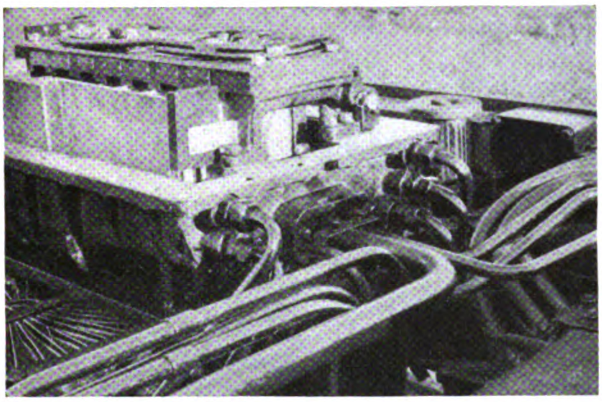

Fig. 25. Arrangement of Rigid Conduit and Gas-tight Contactor Box with Cover Removed, for "Sealed Equipment" Locomotives

The progressive series-parallel control minimizes the necessity of running, to any appreciable extent, on resistance points as the motorman is forced to operate with the motors in series for all low-speed movements. Consequently, there is very little running on resistance points, an important consideration in gathering work where much of the movement is at low speed. The locomotive has a drawbar pull of 4000 lb. at five miles per hour.

QUARRY LOCOMOTIVE CARS

An interesting system of remote control for locomotive cars operating on quarry tracks was developed. With this system a single operator, located in a tower control station from which he can see all the cars, is enabled to start, stop, and even to switch cars on different tracks.

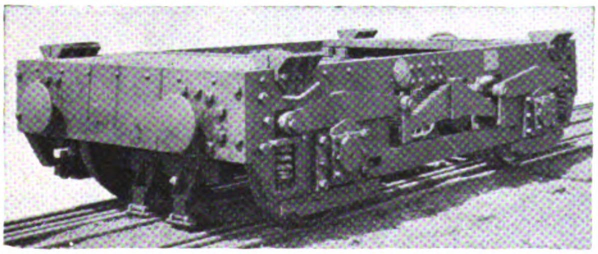

Fig. 26. Body of Locomotive Car for Quarry Service Designed for Operation from a Stationary Control Tower

The electrical equipment of the cars (Fig. 26) consists of high-torque squirrel-cage motors provided with solenoid brakes and controlled through a pair of reversing contactors which throw the motors on the line at full voltage. Two current-collecting shoes are used making contact with two "third rails" located between the traction rails. The separate tracks on which these cars operate have isolated circuits or the cars can be maneuvered on a loop track with isolated sections.

The control operator handles the cars from a pilot switch which actuates the reversing contactors, and when the current is shut off the solenoid brakes set automatically and decelerate the car; the torque being adjusted for smooth deceleration.

This system is expected to speed up production and reduce operating cost inasmuch as one operator can in this way handle several cars whereas with the old system an operator was required for each car.

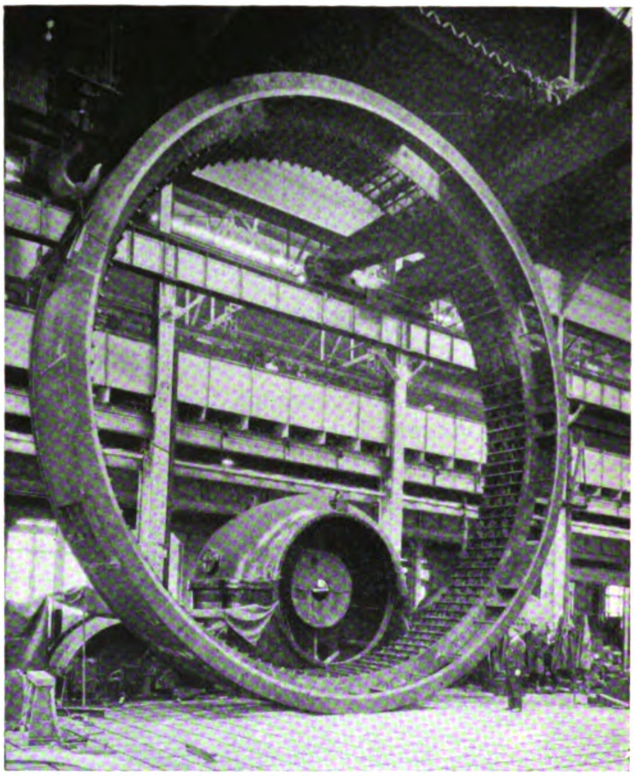

Fig. 27. Stator Frame for 40,000-kv-a. Waterwheel Generator for Conowingo

WATERWHEEL GENERATORS

The first two of the four 40,000-kv-a. 13,800-volt 81.8-r.p.m. 3-phase 60-cycle vertical-shaft waterwheel generators for the Conowingo Development of the Susquehanna Power Company were completed. While these generators are exceeded in capacity by the 65,000-kv-a. machines at Niagara Falls, they have greater physical dimensions owing to their lower speed.

The maximum diameter of the stator frame is approximately 38 ft. and the weight of each generator is in excess of 500 tons. The spring-thrust type of suspension bearing is used and, in operation, will sustain a load of about 640 tons due to the combined weight imposed by the rotor and the hydraulic thrust.

The stator frames for these machines are excellent examples (Fig. 27) of the new method of welded structural steel construction, the weight of the frame, as shown, in spite of its exceptional size, being only about 50 tons; the addition of the windings and laminations bringing the total weight of the stator up to 115 tons.

Among the large machines completed were two horizontal-shaft units rated 25,000 kv-a., 11,000 volts, 450 r.p.m., for the Buck's Creek Power Station of the Feather River Power Company. These machines will operate under the exceptional hydraulic head of 2562 ft.

For the first time the enclosed ventilating system whereby the heated air is passed through water-cooled radiators and recirculated was applied to large waterwheel generators. Two of these machines are rated 30,000 kv-a., 11,000 volts, 300 r.p.m., 3 phase, 60 cycles and will be installed in the Lake Chelan Development of the Washington Water Power Company. The atmospheric temperatures at Lake Chelan are frequently high and at the same time an ample supply of low-temperature cooling water is available, so that, by the use of the enclosed ventilating system, the maximum generator output may be maintained throughout the period of summer operation.

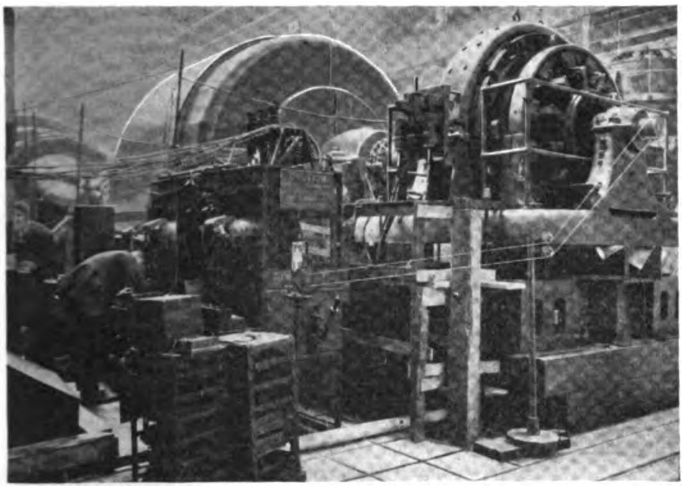

Fig. 28. Testing the Largest Synchronous Condenser, Rated 50,000 kv-a.

This system of cooling was also utilized for eight 12,550-kv-a. 14,000-volt 100-r.p.m. 3-phase 60-cycle units for the Louisville, Kentucky, Gas and Electric Company. In this case, the hydraulic machines are to be operated in conjunction with a steam plant and the use of the enclosed ventilating system provides complete protection to the windings of the generators from dust or fumes, as well as permitting a positive control of the operating temperature.

The construction work for foreign installations included ten waterwheel generators for Japan with an aggregate capacity in excess of 120,000 kv-a. These consisted of two 20,000-kv-a. 11,000-volt units; two rated 13,000 kv-a., 12,000 volts; and six rated 9400 kv-a., 11,000 volts—all being of the vertical-shaft type.

SYNCHRONOUS CONDENSERS

Three record-size synchronous condensers (Fig. 28) were completed for use in regulating the voltage on the 220,000-volt transmission lines of the Southern California Edison Company. They are rated 50,000 kv-a., 3 phase, 50 cycles, 13,200 volts, 60 r.p.m. The total weight of each condenser with its exciter is 375,000 lb. and the overall dimensions are 26½ ft. long, 16 ft. wide, and 12 ft. high above the bottom of the base.

Each machine is designed for operation with surface air coolers located in the pit beneath the base; the blower effect of the rotor being ample to circulate the amount of cooling air required.

The great size of these machines permitted, in the stator windings, the economical use of the transposed-bar conductor that has been employed extensively in turbine alternators. Such a winding has practically no circulating current, and can be installed or removed without distorting the ends. Each bar consists of a single conductor made up of a large number of asbestos-covered strands. Two such bars when installed make up a one-turn coil similar in appearance to the conventional barrel-type coil. The eddy- or circulating-current loss within the conductor is reduced to a minimum by the fact that each strand in traversing the length of the core occupies every possible position in the depth of the bar.

Since there is only one turn on a coil, a failure between turns is impossible unless the external insulation is destroyed.

The field spider consists of four sections shrunk onto a heavy forged-steel shaft; each section being made up from one-piece steel plates. A rotor made in this way can have no hidden flaws and is therefore well adapted to carry the centrifugal force of 3,000,000 lb. which each pole exerts when running at normal speed.

A smaller condenser for the New England Power Company and another for the Appalachian Electric Power Company are of unusual interest because they are designed for outdoor operation and for the use of hydrogen gas as a cooling medium.

The New England Power Company's machine has the equivalent capacity of 10,000 kv-a. when operated in air and an expected capacity of 12,500 kv-a. when operated in hydrogen. It is calculated that the losses when operating at 12,500 kv-a. in hydrogen will be practically the same as when carrying 10,000 kv-a. in air. This saving in losses is caused primarily by the reduced windage when operating in hydrogen. The machine is shaped like a gigantic capsule, the capsule being the stator frame with heavy steel end-bells. The stator core and rotor are on the inside of the frame as are also the bearings. This construction requires only static joints to seal against the escape of hydrogen.

For cooling the hydrogen, semi-circular fin-type surface coolers are placed inside the frame with the headers coming to the outside for attaching the water connections. Thus the machine is entirely self-contained and well adapted for outdoor installation.

SYNCHRONOUS CONVERTERS

Four synchronous converters, each rated 4000/5800 kw., 167 r.p.m., 400/580 volts, direct current, 10,000 amp., 25 cycles, shunt-wound, were designed for service in an electrolytic zinc plant. They are provided with individual transformers with load-ratio control equipment, and a single synchronous motor-generator set is used for supplying the starting current.

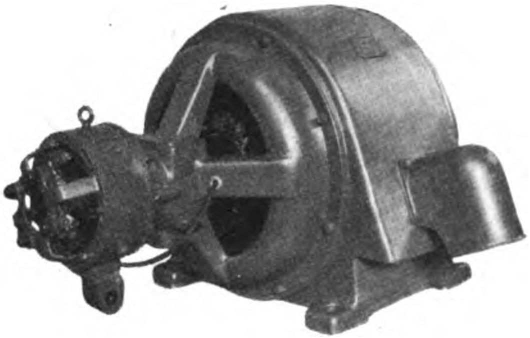

Fig. 29. 150-h.p., 1800-r.p.m., Synchronous Motor, Showing New Standardized Construction

The unusual direct-current-voltage range (400 to 580 volts) is obtained under load by a combination of tap changing in the main transformers and shunt-field control of the synchronous converters. These are the largest machines of this type designed for this method of control.

MOTORS

Due to the increased use of general-purpose synchronous motors, there was considerable progress made in the standardization of this type. To a large extent, induction-motor frames and parts were adopted and this type of motor (Fig. 29) is now nearly as well standardized as the corresponding induction motor. It uses the same compensator and has the same starting torque and current, but has better efficiency and power-factor.

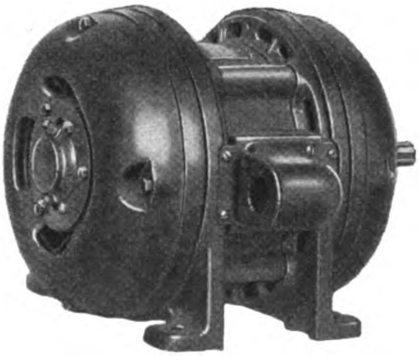

Fig. 30. Three-phase, Totally-enclosed, Fan-cooled Motor

The demand for low-speed synchronous motors to drive compressors for air conditioning in theaters and other public buildings showed a notable increase, and about 12,000 h.p. of this type of motor was so utilized in 1927. A synchronous motor was designed for across-the-line starting. It is rated 350 h.p. and is typical of the modern synchronous motor now available for heavy starting duty with simple control.

Supplementing the existing line of totally-enclosed, fan-cooled, direct-current motors, these features were applied to a similar line of induction motors. These motors (Fig. 30) are intended for operation on 2-phase or 3-phase, 60-cycle circuits and are available in ratings from 2 to 15 h.p., 220, 440, and 550 volts. They are cooled by means of copper-enclosing disks, located at each end of the motor, which absorb heat as generated directly from the stator windings. The heat radiation is effectively assisted by the circulation of the internal air at both ends by the motor fans. Either ball bearings or sleeve bearings can be used and the dimensions of the motors are such that either type of bearing may be used interchangeably.

Improvements were also made in larger induction motors of the totally-enclosed fan-cooled type which permitted their construction in smaller frames, for a given capacity, than had previously been possible. By means of a longer end-shield on the end opposite the pulley, the slip-ring type of motor can be adapted for this construction.

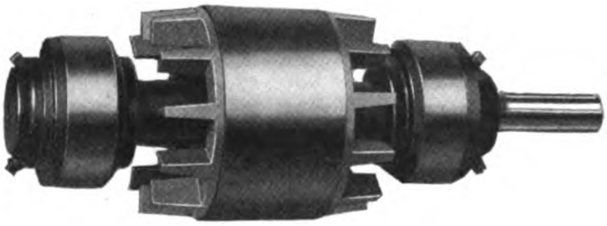

Fig. 31. Rotor of Induction Motor, Showing Location and Appearance of Cartridge-type Housings for Ball Bearings

A new cartridge-type of ball-bearing housing was provided which is so constructed that by taking out the screws locking the housings to the end-shields the rotor can be removed (Fig. 31) with the housings still around the ball bearings. This new type of ball-bearing housing has been so far adopted only for the smaller sizes of general-purpose motors.

A new line of high-reactance normal-torque squirrel-cage induction motors was designed for full-voltage starting by means of a small magnetic line switch. It has as good and in some cases slightly better starting torque with the same starting current as the ordinary squirrel-cage type of motor started with a compensator. It is now available in ratings from 7½ to 100 h.p.; and up to 30 h.p. these machines will meet the accepted limits of starting current even when started on full voltage.

In order to meet the requirements of permissible equipment in gaseous mines, 1-h.p. 3-h.p. and 5-h.p., direct-current, 250- or 500-volt, 1200-r.p.m., explosion-proof motors with self-contained starting control were developed, and a number have already been placed in service.

These motors (Fig. 32) have ball bearings and special insulation and will carry their rated load continuously with a temperature rise not exceeding 75 deg. C. The protection against internal explosions reaching the surrounding atmosphere is by total enclosures and not by the use of baffles.

The application of high-speed motors is increasing, both as to the number of motors and the speeds employed. Motors directly applied to grinding spindles are a part of the regular production equipment in the ball-bearing industry today. The small diameter wheels required for the work necessitate high rotating speeds. Squirrel-cage motors to operate at speeds up to 32,400 r.p.m. and generators to supply these motors with 540-cycle 3-phase power are now an essential part of the plant equipment.

It is interesting to note that drawn-shell motors, standard in every respect, were applied although the operating speeds are extremely high. These applications were made after two years' experience with hundreds of motors operating at 18,000 and 25,200 r.p.m. involving frequencies of 300 and 420 cycles, respectively.

All single-phase motors have an inherent double-frequency torque pulsation which gives rise to some noise and vibration. To absorb these disturbances, some fractional horse power motors were mounted in rubber. An example is a 1/30th-h.p. split-phase motor of the usual construction but with unusual mounting.

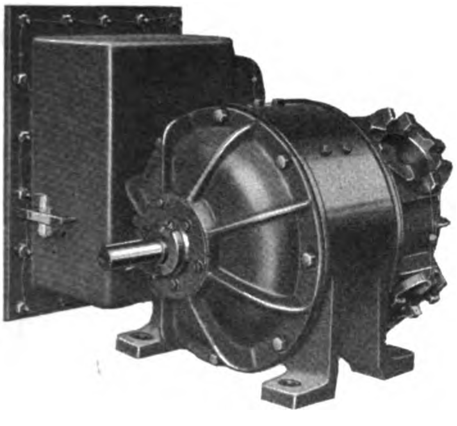

Fig. 32. 3-h.p., 1200-r.p.m., 250-volt, Compound-wound Explosion-proof Motor for Coal-mine Service

The base of the mounting has vertical supports going up through the ventilating holes in the bottom of the end-flanges of the motor and offset around the shaft (Fig. 33) so that the top of the support is directly over the shaft and some distance above it. There are two holes in each support, one above and one below the shaft with rubber bushings in these holes. Two pins extending inward from the end-flange engage these rubber bushed holes and support the motor. The motor is thus supported entirely from these pins in the end-flange and there are no metal-to-metal contacts between the motor and its mounting.

This rubber mounting has two advantages over a rigid mounting in that it eliminates a large part of the vibration normally transmitted to the apparatus on which the motor is mounted and it also reduces the usual motor noise to a minimum, as the motor can produce noise only by vibrations in the relatively small surface of its own frame.

Until recently, fractional horse power motors were used principally on applications which require only intermittent duty, but with the development of electric refrigeration for the home there arose a demand for a motor which has high values of starting torque, efficiency, and power-factor because economy in current consumption is important in the operation of refrigerators.

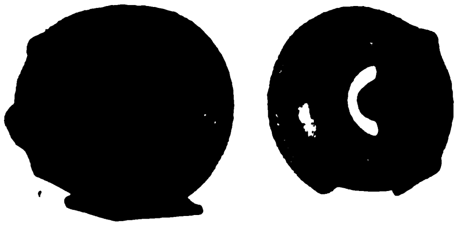

Fig. 33. 1/30-h.p., Split-phase Motor, Showing Arrangement of Rubber-bushed Supports

For this service, a capacitor motor was designed which is essentially a two-phase motor with a squirrel-cage rotor, one phase of the motor being connected directly across the source of supply and the other phase connected in parallel with the first phase through a capacitor. In order to secure best operation, it has been found desirable to make a change in connections on the capacitor as the motor comes up to speed. This change is made by the use of a simple centrifugal device which operates a single-pole double-throw switch.

The motor develops high starting torque for a low value of starting current and has high efficiency and high power-factor. Due to the fact that it has no commutator or brushes, it is extremely quiet and causes no radio interference except one slight click when the centrifugal switch operates.

Magnetic noise has been almost eliminated by careful design, and because the motor operates as a polyphase motor the characteristic hum of a true single-phase motor is absent.

MOTOR CONTROL

Motor Control

A number of blast-furnace skip-hoist equipments were designed for generator-voltage control instead of the rheostatic control which has been used almost exclusively for this service in the past. One such equipment for the Bethlehem Steel Company uses a two-motor drive operated from a single generator, the two-motor drive permitting operation at reduced capacity in case of the failure of either motor. The load on this skip varies widely in handling coke or ore, and generator-voltage control is particularly suitable because of the accurate low speeds which can be obtained for dumping the skip regardless of the load.

This method of control was also applied in the operation of a 100-ton freight-car elevator. The car is started by means of a push button and is automatically slowed down and stopped at the track level by limit switches.

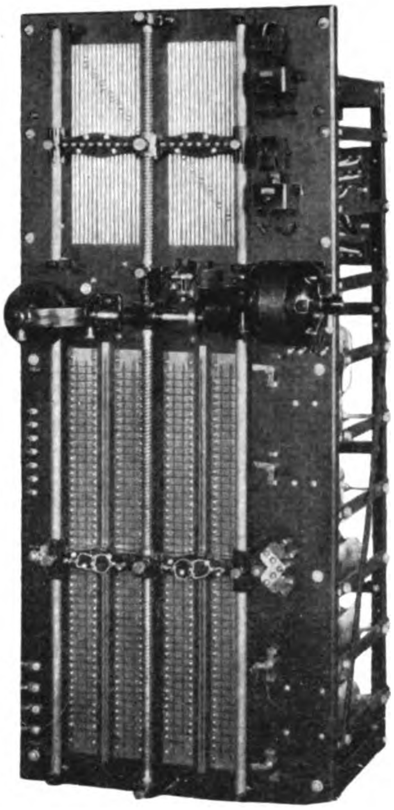

Fig. 34. Motor-operated Field Rheostat for Propeller-shaft Motor Control

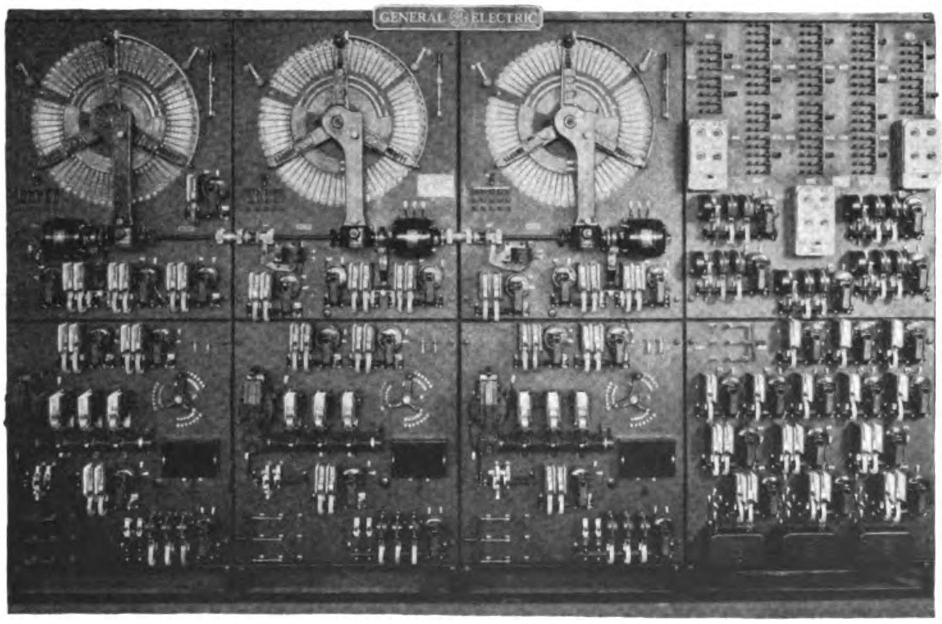

Fig. 35. Six-motor Automatic Printing-press Controller

A motor-operated field rheostat of unusual construction (Fig. 34) was produced for controlling the operation of two motors connected in series and mounted on the propeller shaft of a Diesel-electric ferryboat. There are three 600-kw. engine-driven generators which have their armatures in series and their fields in parallel. They are separately excited through the reversible and adjustable motor-operated field rheostat. With this generator-voltage system, the direction of rotation and speed of the propeller are governed by the direction and strength of the current through the fields of the three generators.

The motor-operated rheostat is controlled from the pilot house by a master switch, the handle of which can be set for the desired speed in either direction. At the top of the rheostat is a limit switch which moves with the field rheostat and automatically positions it to correspond with the position of the master switch.

A control panel for three 100-h.p. double-motor newspaper-press drives (Fig. 35) permits the operation of numerous presses in various combinations. It also permits adjacent drives to be operated in parallel for straight-run newspaper production, and, by means of the equipment, the control stations on the various presses may be transferred from one drive to the other. Provision is also made for warning signals to be given automatically before starting the presses.

Heretofore, the control of dynamometers used in testing internal-combustion engines has been effected by manual operation, but a full-automatic control for this service has now been developed. The new equipment (Fig. 36) permits mounting the master panel on the dynamometer base, thus providing complete automatic control at the point where the testing is being done. The motor-operated field and armature rheostats and the absorption resistor can be installed at any convenient place.

The master panel has a field-reversing switch, and several push buttons for starting and stopping the apparatus, as well as for controlling the motor-operated rheostats on the control panel. A vernier rheostat is mounted on the master panel for fine adjustment of the dynamometer field and a set of instruments gives complete indication of the voltage and current under all operating conditions.

Push-button operated switches for throwing small squirrel-cage induction motors across the line have been widely used in the last few years in all industries, and three new forms were produced which embodied some radical changes. They have bases made of moulded compound, and the shafts of the magnetic contactors which were formerly made of steel with an insulating casing are now made of moulded textolite. This insures permanent insulation and non-corrodible bearings.

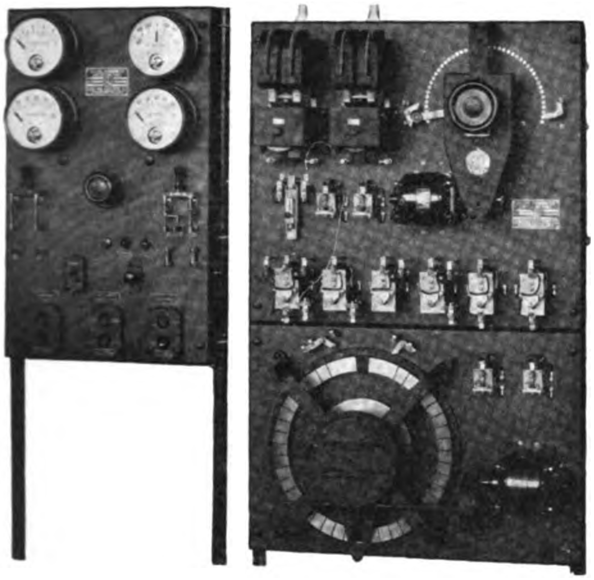

Fig. 36. Automatic Dynamometer Control and Master Panels

The contactor for the 5 and 7½-h.p. size has a magnet of the solenoid type which requires less power for operation than the common clapper type and has the advantage of materially increasing the speed of the contactor on opening because its core drops vertically rather than falling over or away from a vertical position. This increased speed of opening combined with the use of double-break silver contacts, which are unusual in magnetic-switch construction, materially improves the contactor's arc-rupturing ability, particularly on 550-volt circuits.

A small pressure switch for use with motors up to 3 h.p., driving air-compressors or house pumps, has a double-pole double-break switch mechanism with silver to copper contacts. Operation is controlled over a range of 0 to 200 lb., and a range adjuster is provided for changing the differential between the cutting-in and cutting-out pressures.

A new, small, geared-type, limit switch was developed for use on small motor-driven machines to stop the motor at each end of travel. One familiar application of this switch is in connection with the control of the opening and closing of motor-operated factory windows.

A similar switch of heavier design can be applied to the control of larger motors. It is arranged for several circuits, some of which can be utilized for slowing down the motor before the final stopping point is reached. Switches of this type have already been used for steel-mill motor control.

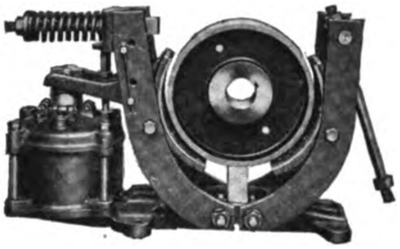

Fig. 37. Solenoid Brake, Showing Method of Removing Wheel

An improved line of solenoids has mechanical parts which are the same for both alternating and direct current, and the coils for both forms of current are also interchangeable mechanically. These solenoids are unusually efficient and require only about one-third as much material for doing a given amount of work as compared with previous designs.

In an improved line of solenoid brakes (Fig. 37), simplicity of design and strength of construction are emphasized. The solenoids are mechanically interchangeable for both alternating-current and direct-current brakes.

The larger sizes are most frequently used in heavy-duty service, such as is found in steel mills. Since quick removal of mill-motor armatures is sometimes necessary, the solenoid brakes for these motors were designed for rapidly and easily removing the wheel from the brake.

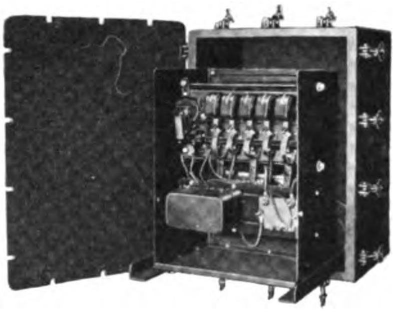

Fig. 38. 50-h.p. 550-volt Compensator Installed in Dust-tight Boiler-plate Case

Industrial users are giving more attention to the protection of electrical control apparatus from dirt, dust, corrosive fumes, etc., because delays in production are becoming increasingly expensive. In some industries, a number of controllers can be grouped in a dust-tight fire-proof control room but where it is desirable to have the controller located near the operator or the machine, dust-tight cast-iron and boiler-plate cases have been adopted. The smaller cases are made of cast iron, but in the larger sizes (eight cubic feet capacity or more) boiler-plate cases (Fig. 38) are used.

STEEL MILLS

The decreased activity in the iron and steel industry resulted in a considerable reduction, as compared with the year 1926, in the number and total horse power of electric motors (Figs. 39 and 40) applied to main roll drives. In spite of this, however, the year's additions brought the total installed capacity of G-E rolling mill motors, of 300 h.p. and above, to more than 1,000,000 h.p.

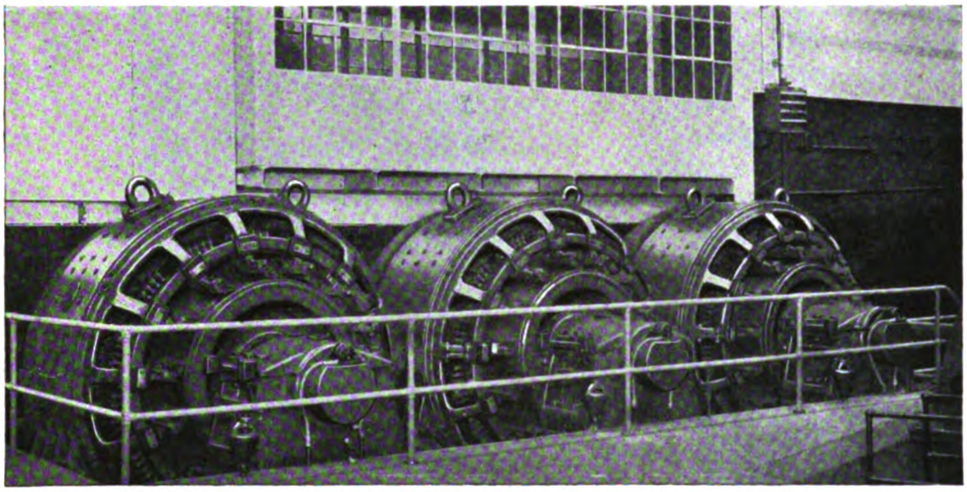

Fig. 39. Three 2000-h.p. 600-volt Direct-current Motors Driving Finishing Stands of 21-in. Continuous Sheet, Bar, and Skelp Mill

A large equipment of unusual interest was placed in operation at the Lackawanna Plant of the Bethlehem Steel Company. The mill to which the motors were applied is used to produce wide flange beams. It consists of three units: a 54-in reversing blooming mill, a 48-in. intermediate mill, and a 48-in. finishing mill. The blooming mill, which is one of the two largest reversing blooming mills in this country, is driven by a single-unit direct-current reversing motor rated 7000 h.p. continuously at 50 deg. C. rise and 40/80 r.p.m. This motor is capable of exerting a maximum torque of 2,400,000 lb. at one foot radius. Direct current is supplied from a flywheel motor-generator consisting of a 5000-h.p. 375-r.p.m. 6600-volt induction motor, a 50-ton flywheel, and two 3000-kw. 750-volt generators which are operated in parallel.

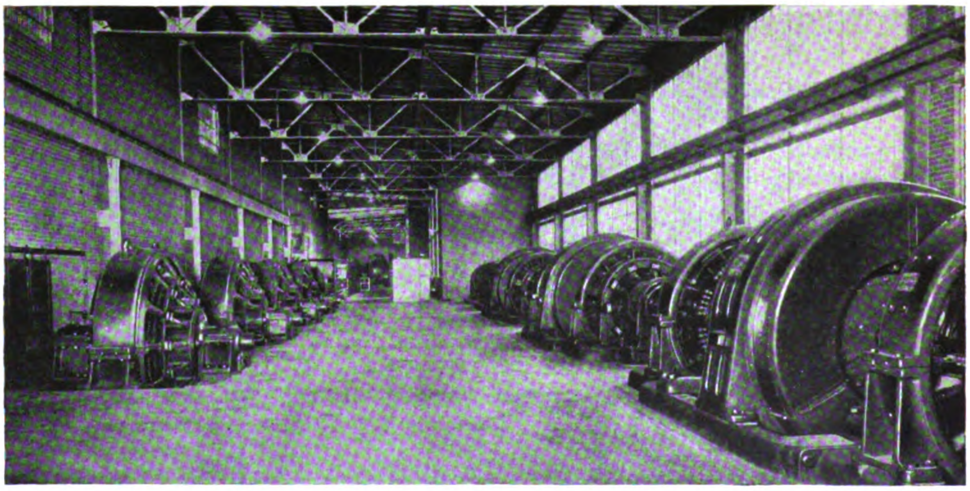

Fig. 40. Motor Room of Continuous Strip Sheet Mill, Showing Four 2000-h.p. Direct-current Motors and Three Motor-generator Sets for Current Supply to the Motors

The 48-in. intermediate mill consists of two separately-driven stands of rolls. The main stand has two horizontal and two vertical rolls, and is driven by a reversing motor rated 7000 h.p. continuously at 50 deg. C. rise and 65/125 r.p.m. The second stand, known as the supplementary mill, is driven by a direct-current reversing motor rated 1500 h.p. continuously at 50 deg. C. rise and 65/225 r.p.m. The flywheel motor-generator which furnishes power to both these reversing motors consists of a 5000-h.p. 375-r.p.m. 6600-volt induction motor (duplicate of that on the blooming mill set), two 3000-kw. generators for the main motor, a 3000-kw. generator for the supplementary motor, and a 50-ton flywheel.

The 48-in. finishing mill equipment is exactly the same as that used to drive the intermediate mill. The total capacity of the main equipment for the blooming, intermediate, and finishing mills is about 71,000 h.p.

The layout of the rail mill at the Lackawanna Plant of the Bethlehem Steel Company will be changed. The 44-in. reversing blooming mill will have a driving motor rated 7000 h.p. continuously at 50 deg. C. rise and 50/120 r.p.m. Blooms from this mill will go to a 36-in. reversing roughing mill which will be driven by a direct-current motor rated 5000 h.p. continuously at 50 deg. C. rise and 50/120 r.p.m. Power for these two reversing motors will be obtained from a single, flywheel motor-generator consisting of an 8000-h.p. 375-r.p.m. 6600-volt induction motor, two 3000-kw. 375-r.p.m. 750-volt generators operating in parallel to supply power to the 7000-h.p. blooming mill motor, two 2200-kw. 450-volt generators operating in series to supply power to the 5000-h.p. roughing mill motor, and a 75-ton flywheel.

The increase in the use of synchronous motors for main roll drives was made noteworthy by seven such applications totalling 12,600 h.p. The largest of these was rated 5000 h.p. continuously at 40 deg. C. rise and 100 r.p.m., 2200 volts, and will be used to drive a 19-in. continuous sheet bar mill at the Kokomo Plant (Ind.) of the Continental Steel Corporation. A motor of the same horse power rating but at 240 r.p.m. will be used to drive a 51-in. piercing mill installed by the Standard Seamless Tube Company of Economy (Penn.). The remaining five motors, two of which are 400 h.p. and three 600 h.p., will drive copper and brass rolling mills.

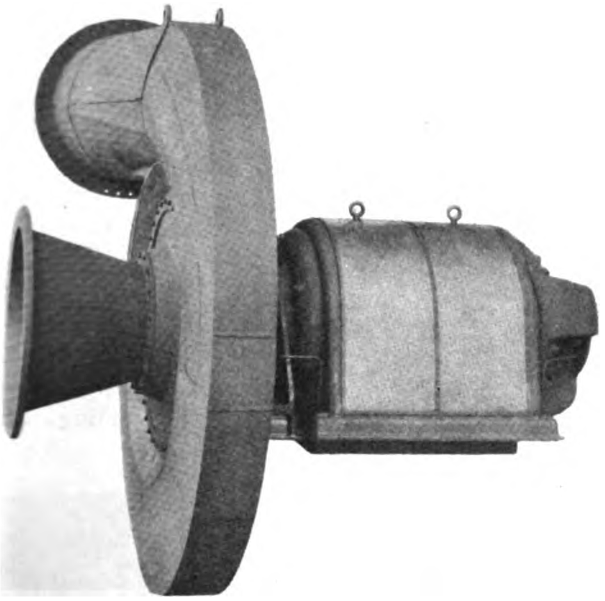

Fig. 41. 60,000-cu. ft. per min. 3.4-lb. Pressure 3600-r.p.m. Centrifugal Air Compressor Driven by Direct-connected Synchronous Motor

Equipments placed in operation during the year for continuous rolling of wide hot strip included eight large motors totalling 16,000 h.p. at the plant of the Weirton Steel Company at Weirton (W. Va.). Three of the motors are each rated 1500 h.p. and take their power direct from the 6600-volt 60-cycle power supply. Of the five motors on the finishing stands, two are of 2000 h.p., 200/400 r.p.m., 600 volts, direct current, and three are of 2500 h.p., 200/400 r.p.m., 600 volts direct current.

The hot-strip mill of the Trumbull Steel Company at Warren (O.) was rearranged for continuous rolling. Scherbius-controlled induction motors are used, the last three stands being individually driven. The main roll drives on this mill total 12,500 h.p.

There was placed on the market during the early part of the year the new line of direct-current mill-type motors which were designed in collaboration with the Standards Committee of the Association of Iron and Steel Electrical Engineers, and which fully meets the standard specifications set up by this Committee.

CENTRIFUGAL COMPRESSORS

The floatation process for the recovery of copper from very low grade ore with the Callow cell was completely revised so that whereas air at 5-lb. pressure was formerly required satisfactory results are now obtained with 1.5 lb. Compressors for this service are built in large volumetric capacities, and because of the steady load conditions synchronous-motor drives are used. Two units of this type (Fig. 41) rated 60,000 cu. ft. per min. at 2.5 lb. or 95,000 cu. ft. per min. at 1.5 lb. capacity were under construction for the Nevada Consolidated Copper Company.

Compressors of low-volumetric capacities but with comparatively high pressures were made possible by employing speed increasing gears and single-stage compressors with small diameter impellers operating at fairly high speeds. Many of these require constant-pressure characteristics and use the radial-blade-type impeller, but in others a drooping-pressure characteristic is necessary and this is obtained with the so-called "backward-flow type" of impeller in which the blades are curved backward.

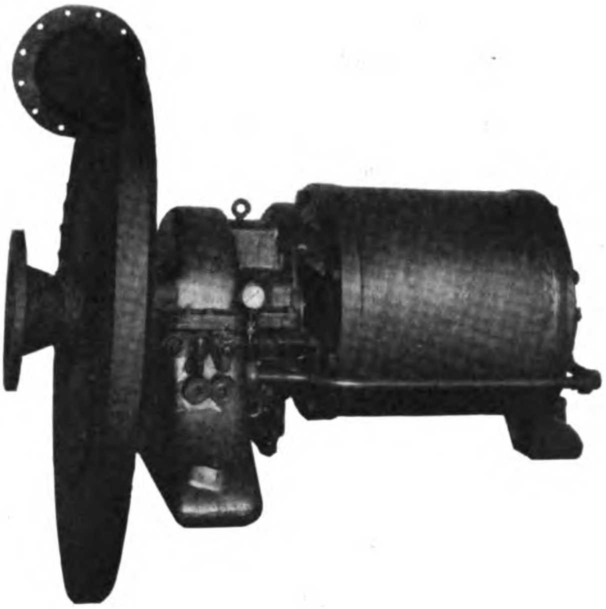

Fig. 42. 3000-cu. ft. per min. 8.8-lb. Pressure 13,650-r.p.m. Centrifugal Compressor Driven Through Step-up Gear by Induction Motor

In the activated-sludge system of sewage disposal, compressors with a drooping characteristic are desirable as this tends to hold constant volume, *i.e.*, as the porous stones through which the air is blown gradually become clogged during a period of years there is a tendency for the air flow to diminish, but this diminution is accompanied by a rise in pressure which in turn tends to keep the flow constant. Compressors of 2500- to 3500-cu. ft. per min. capacity at 7-lb. pressure (Fig. 42) were built for the sewage disposal plants of the cities of Decatur (Ill.), Waco (Texas), Charlotte and Gastonia (N. C.).

Modern blast-furnace practice (Fig. 43) is demanding higher plant efficiency and the latest move in this direction was recently made by the Youngstown

Sheet and Tube Company in adopting three 70,000-cu. ft. per min. blast-furnace compressor units with turbine drivers designed to operate with steam at 400 lb. gauge, and provided with three-stage extraction openings for heating the condensate to boiler-feed temperature. Steam extraction has been applied to many large constant-speed turbine-generators for feed-water heating but this is the first instance where this principle has been extended to a variable-speed blast-furnace turbine compressor.

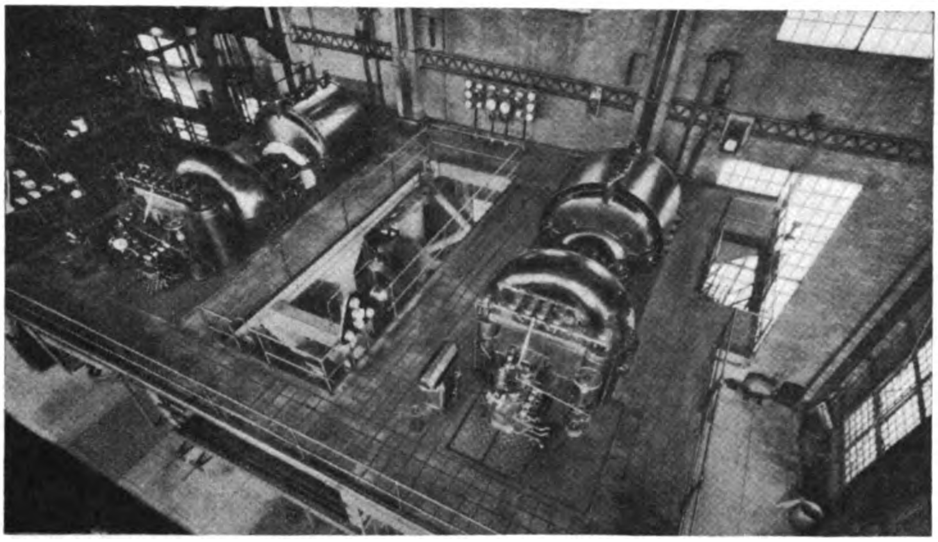

Fig. 43. Two Centrifugal Converter Blowers Direct Driven by Steam Turbines. This is a typical arrangement for modern blast-furnace plants

ELECTRIC MELTING FURNACES

For a number of years the wrought-brass industry, concentrated in a comparatively few large plants, has used electric arc and induction furnaces almost exclusively for melting copper-zinc alloys. The cast-brass industry, consisting of many small plants widely scattered, has been slower in the adoption of electric heat for melting but there was an increased use of electric furnaces for melting brass for castings. The brass industry as a whole uses comparatively small melting units; 75 to 200 kv-a., all single phase. There appears to be less objection than formerly on the part of public-utility systems to single-phase loads of this character and units as large as 300 kv-a. have been installed.

In the steel industry the use of the three-phase arc furnace (Fig. 44) followed the standard practice of the past two or three years and "electric steel" has now become a trademark to indicate a superior quality of steel.

There was a decided growth of interest in the possibilities of cast iron of uniform analysis and of definite physical properties. This is one of the most promising fields for the electric melting furnace.

The coreless-type induction furnace, generally referred to as the "high-frequency furnace," entered the ferrous field for the production of high-grade alloy steels. Completed equipment of this character included a 150-kv-a. 900-volt 2000-cycle unit for melting steel, a 60-kv-a. 900-volt 960-cycle unit installed for laboratory service in a large steel plant, and a 187-kv-a. 900-volt 800-cycle unit shipped to Sweden. All of these units consist of a motor-generator set using a three-phase motor of standard frequency and voltage to drive a single-phase high-frequency generator, together with the corresponding high-frequency capacitor units.

Fig. 44. 1½-ton 3-phase Arc Furnace of the Type Now Used to Produce "Electric Steel"

ELECTRIC WELDING

The developmental work on atomic-hydrogen arc welding was completed and a commercial equipment was produced for use on 60-cycle circuits.

With this method of arc welding, an alternating current is maintained between adjustable tungsten-wire electrodes (Fig. 45), and hydrogen is fed to the arc around the electrodes. The hydrogen molecules are broken up into atoms by the intense heat and, in recombining outside of the arc and in contact with the work, heat is liberated far in excess of that obtainable by any gas flame alone. This heat is used to fuse the metals to be joined and, where additional metal is required, a filler rod may be fused into the work.

The hydrogen, being an active reducing agent, prevents the formation of oxides and hence produces a uniformly strong, ductile, and smooth weld. The metal being welded is not in the electric circuit and need not be grounded or insulated. The arc, constantly maintained, but broadly adjustable in size and intensity, lends itself to a wide range of work. Welds are executed at rates of speed comparable with those attained with a single oxy-acetylene torch.

A transformer furnishes the required voltage, and the current at the arc is controlled by an automatic reactor. In this process, high voltage and low currents are required in the arc, which is the reverse of the conditions in ordinary arc welding. The arc is struck at 300 volts; but while welding, the arc voltage is varied from 90 volts for light work to 60 for heavier, and the current correspondingly varied from 20 to 70 amp.

By this means (Fig. 46) homogeneous ductile welds can be made on very thin metals or on some metals and alloys hitherto considered unweldable.

The line of single-operator variable-voltage arc-welding sets was extended by putting into production a new machine (Fig. 47) having a rating of 300 amp. for one hour, 50 deg. C. rise. It has a self-excited generator with current adjustment obtained by shifting the brushes, and is driven by a standard 15-h.p. motor.

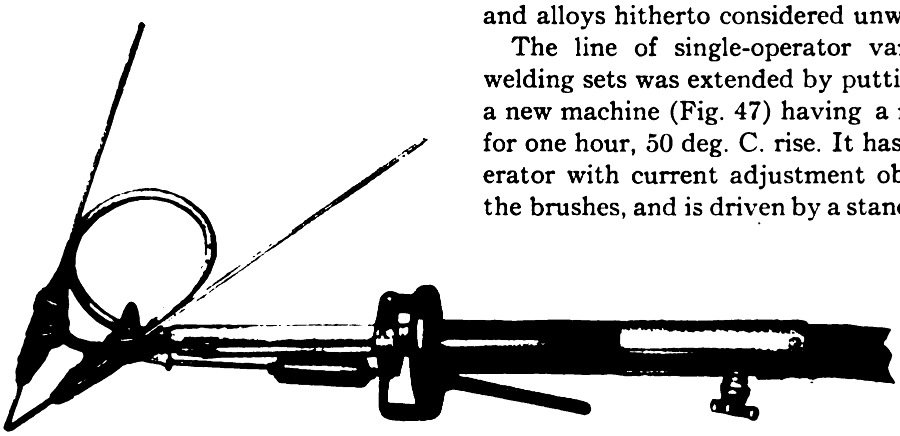

Fig. 45. Electrode Holder for Atomic-hydrogen Arc-welding

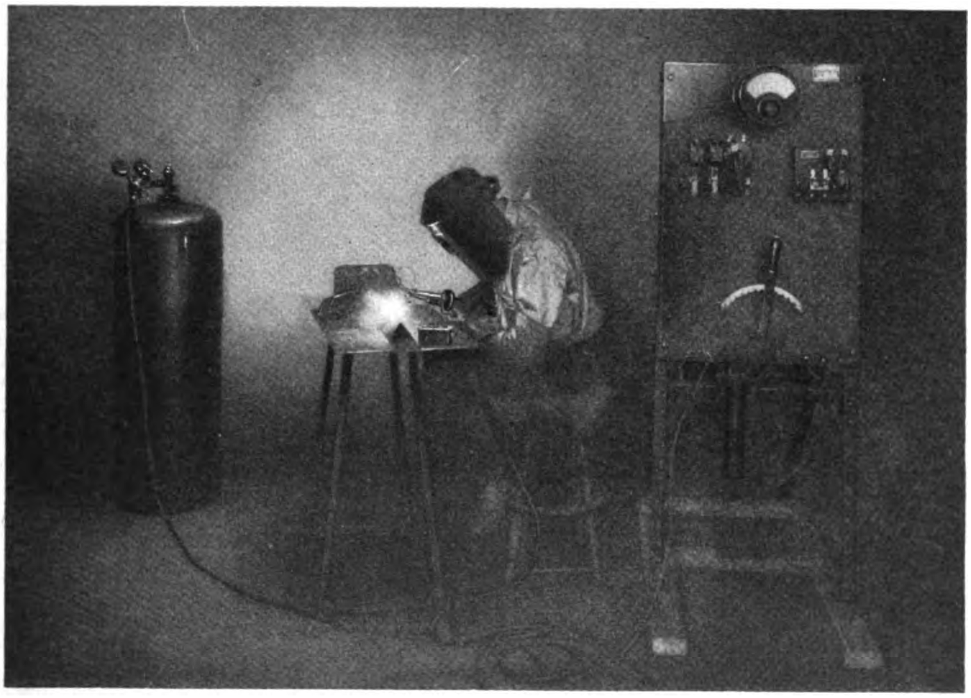

Fig. 46. Atomic-hydrogen Arc-welding Equipment in Operation

The use of automatic welding continued to increase. One important field of application which showed considerable expansion was the manufacture of arc-welded steel pipe. The largest automatic machine produced during the year will weld the longitudinal seam of a steel pipe 36 ft. long.

Automatic welding equipment for the manufacture of steel railroad ties constructed from scrap rails (Fig. 48) was put in service at the Colonie Plant of the Delaware and Hudson Railroad Company. The tie consists of two pieces of worn or scrap rail cut to length and held together at both ends by angle bars which are welded to the pieces of rail. These sections of rail are further held together by two tie plates which are welded to the top or head of the rails at the point where the track work is clamped to the tie.

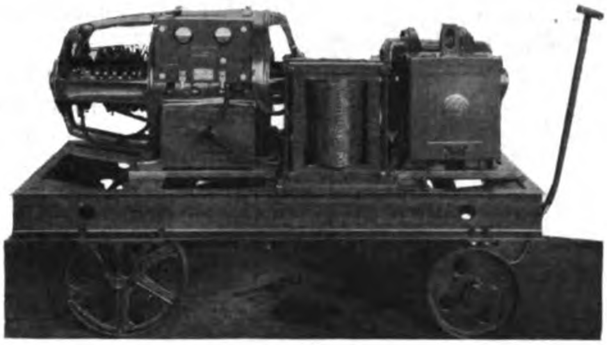

Fig. 47. Improved Type of 300-amp. Portable Arc Welder with A-c. Motor Drive

INDUSTRIAL HEATING

A series of comparative tests on a so-called "jitney" lehr (Fig. 49) used for annealing bottles gave accurate cost data for both fuel-fired and electrically-heated operation.

The test lehr is 3 ft. in active width, approximately 65 ft. long, and the conveyor is of woven wire. The lehr is designed to take the output of one blowing machine. The electric heating equipment is divided into four zones covering a total length of about 19 ft. All of the heaters are located under the conveyor and consist of ribbon units mounted in slotted insulators and supported in a frame.

The first tests were run with natural gas and the lehr was then modified, the electrical equipment added, and further tests were run. The results were such as to make tests with oil unnecessary.

Notwithstanding the fact that this test lehr was not designed primarily for electric heating, it gave an output of 15½ tons with a current consumption of 240-kw-hr. or 129 lb. per kw-hr., the total connected load for the heating units being 71 kw.

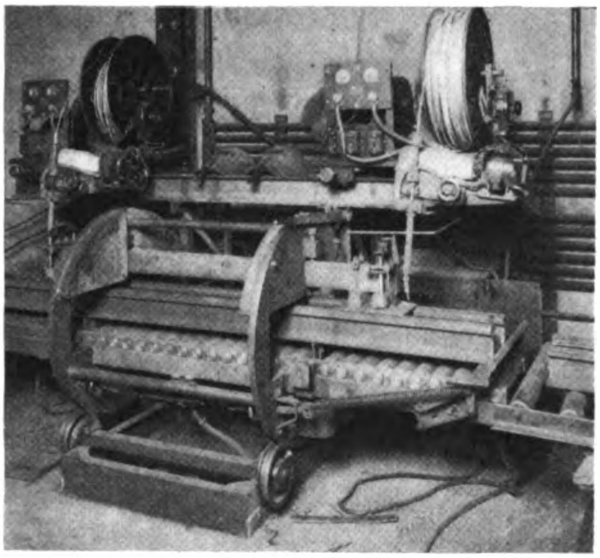

Fig. 48. Automatic Arc-welding Machine for Fabricating Metal Railroad Ties

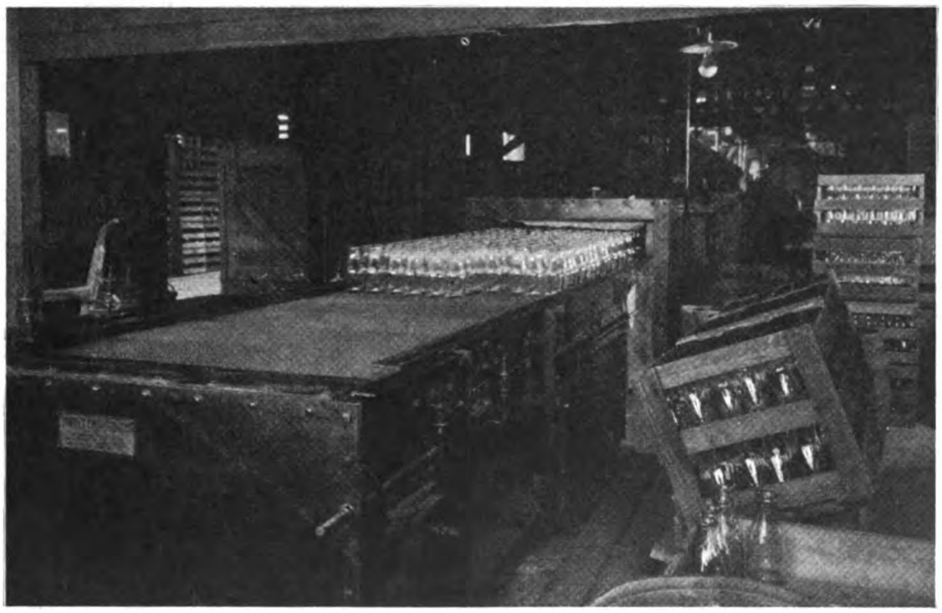

Fig. 49. "Jitney" Annealing Lehr on which Tests Were Run to Determine the Comparative Costs of Fuel Heating and Electric Heating

Electric heating was adapted for the first time to the cloth singeing machines which are used in the finishing rooms of textile mills. The singeing was previously done by gas or oil heat.

Most of the singeing machines in operation at present are of the horizontal type with burners or hot contact plates mounted in horizontal rows so that the cloth is in a horizontal position in its travel over them.

By the adoption of electric heating units, it was possible to arrange them vertically (Fig. 50) over each other in two rows, so that the cloth would be in a vertical position when traveling over them.

This arrangement not only makes available the close control of the temperature which is a characteristic of electric heating, but it also reduces the floor space required for the machine, which occupies an area approximately 10 ft. long by 6 ft. wide. The heating elements come up to operating temperature in 1½ minutes, and, with five heating elements in operation, the connected load is 80 kw. The particular singeing machine on which this installation work was done was built for cloth 40 in. wide, traveling at 140 yd. per minute or 8400 yd. per hour. The power cost of the singeing operation under these conditions was about 9.5 cents per 1000 yd.

By the use of metal-immersion cast-in type of heating units, two electrotype melting pots, originally designed for gas heating, were converted to electric heating. A similar conversion of stereotype melting pots was successfully accomplished in 1926.

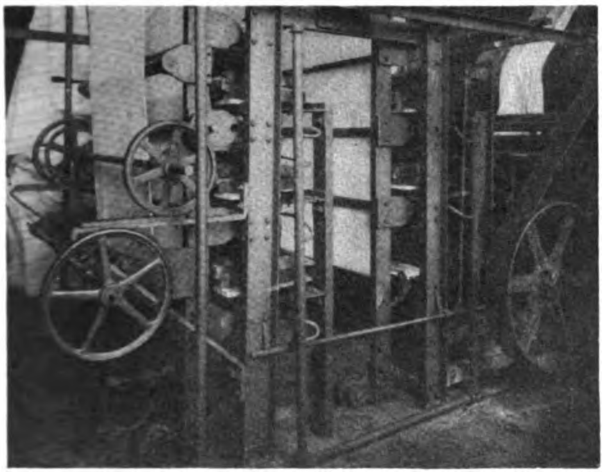

Fig. 50. Electrically-heated Contact Cloth Singer, Showing Cloth in Contact with the Bars and in Running Position

The inside dimensions of the electrotype pots are 36 by 28 by 6 in. deep, and they have a capacity of approximately 2400 lb. of metal. The heating equipment applied for each pot consists of six 5-kw. 220-volt heating elements which were placed directly in the pot, arranged along one 36-in. edge with the units in two groups of three (Fig. 51) near the pot ends. An automatic power-control panel was also installed.

ELECTRIC REFRIGERATION

As the result of experience combined with intensified development, important improvements were made in the construction of household refrigerating units.

In the new unit (Fig. 52) the operating mechanism is totally enclosed in a sheet-steel copper-brazed case and is hermetically sealed. Where the wires enter the case, a special type of metal-glass lead has been used to prevent leaks, and the joint around the bottom of the case is provided with a tongue-and-groove lead seal.

As the motor is located inside the casing, no stuffing box is required and the lubrication problem is simplified by providing a supply of bearing oil which is practically inexhaustible. A force-feed oiling system is used.

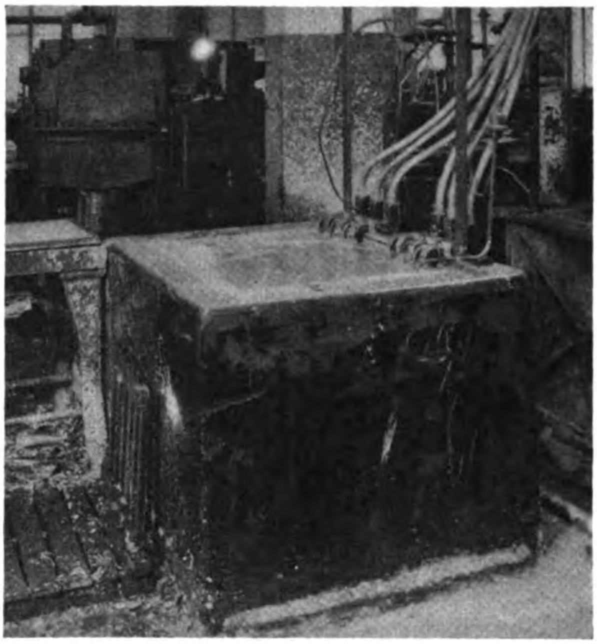

Fig. 51. Electrically-heated Electrotype Metal Melting Pot, Capacity 2400 lb.

The mechanism is essentially free from noise and vibration inasmuch as it is mounted on springs and insulated not only by the casing but by the layer of gas which separates the mechanism from the casing.

A simple float valve is connected in the high-pressure side of the circulating system and functions automatically to return the refrigerant to the evaporator as soon as it becomes a liquid.

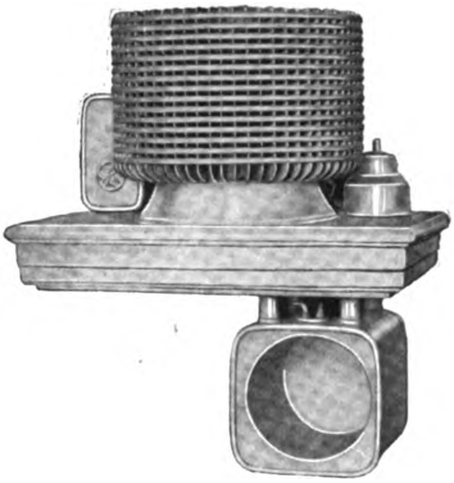

Fig. 52. Improved Type of Refrigerating Unit

Before these units are installed in refrigerators, they are given what is known as the "hot-room" test, where they must maintain continuous temperatures below 50 deg. F. with a room temperature of 100 deg. F.

RESEARCH

The large cathode-ray tube with metallic "window" permitting the discharge of the cathode ray outside of the tube had a maximum operating potential in 1926 of 400,000 volts. During the past year this voltage was increased to 900,000 by a cascade connection of three of these tubes. In this way the velocity and range of the cathode-ray discharge were considerably increased.

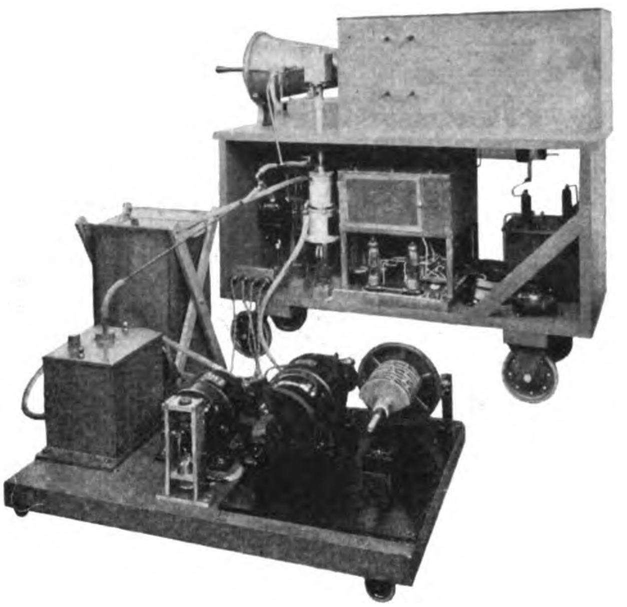
Fig. 53. Complete Portable Cathode-ray Oscillograph Equipment

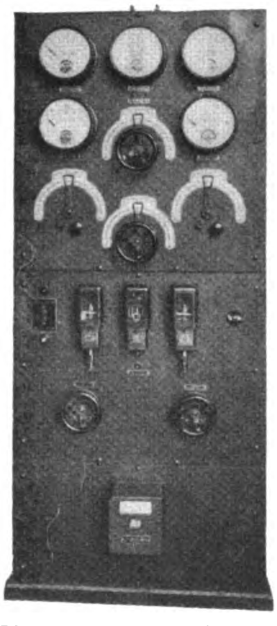
Fig. 54. 20-kw. Power Amplifier, Front View

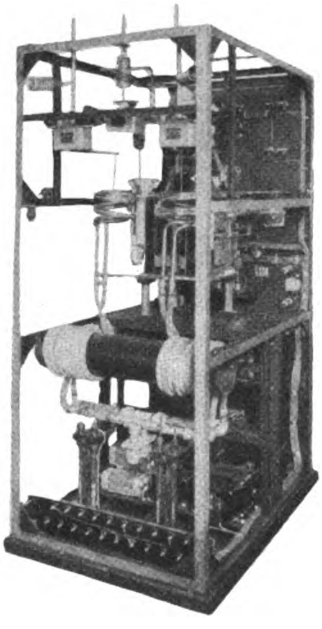
Fig. 55. 20-kw. Power Amplifier, Rear View

In collaboration with the Albany Medical College, the effects of the cathode ray in the production of vitamines in yeast and other foods was studied. The results have been sufficiently promising to warrant further investigation.

The progress in photo-electric cell development included improving the sensitive surface and the cell construction.

Cells were produced with an active surface consisting of a thin layer of caesium instead of the potassium or potassium-hydride formerly used. The caesium surface is very stable and it is not sensitive to quite large variations in temperature. Its color sensitivity is more nearly like that of the human eye, so that the caesium cell "sees" colors practically the same as does the human eye, and needs very little if any correction for work in photometry.

A seven-inch cell which gives very large photo currents was constructed and smaller types of cells provided with metal cathodes which permit a much cheaper construction.

The field of usefulness of the Dufour cathode-ray oscillograph was greatly extended by the construction of an instrument of the high-voltage cold-cathode type designed for both laboratory and field use. This new instrument is completely self-contained and portable so that it may be taken to the work and operated in daylight, either in the factory or in the field.

The oscillograph with all auxiliary equipment (Fig. 53) is mounted on two portable trucks. The cathode tube is mounted in a shielded dark-room and emerges therefrom in the form of a conical metal bell containing the photographic film holder. Other equipment on this truck includes an indicating vacuum gauge, exciting transformer, oscillator for timing wave, and control panel.

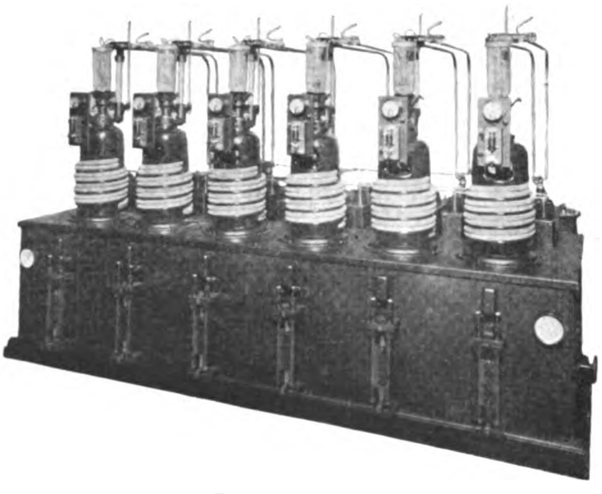
Fig. 56. Kenotron Rectifier, 15 Amp., 4000 to 12,000 Volts, Direct Current

On the second truck there are installed the motor-driven vacuum and water-circulating pumps and synchronous switch. The two units weigh 700 lb. and require floor space about 6 ft. by 10 ft. The maximum power supply is 4 kw. at 110 volts, 60 cycles.

RADIO TRANSMISSION

An improved short-wave transmitter (Figs. 54 and 55) utilizes water-cooled tubes in the output circuit and a rectifier to supply high voltage to the anodes of these tubes. An output of 20 kw. is obtained through the wavelength range from 14 to 40 meters. Wavelengths in this range are proving reliable and useful for communication over great distances. In general, a given signal intensity is produced with a fraction of the power required to produce the same signal by longer waves.

A Piezo-electric quartz crystal is used in the low-power stage to insure accuracy and constancy of the frequency. This crystal is located in a compartment, the temperature of which is held constant by thermostatic control. The output of the crystal, which is in the order of a fraction of one watt, is amplified and the frequency multiplied through several succeeding stages before reaching the output circuit. Four-element tubes are used in several of the low-power stages. These tubes make it possible to obtain highly satisfactory operation with fewer controls than would otherwise be required.

A 50-kw. broadcasting transmitter was placed in operation at the Bellmore Station (Long Island) of the National Broadcasting Company. It is composed of three main parts: the rectifier (Fig. 56), radio-frequency generating and amplifying apparatus (Fig. 57), and modulator. Water-cooled tubes are used in each part. A Piezo-electric crystal is used to insure frequency stability of the radio signal.

The development and production of an improved microphone and amplifier has made it possible for broadcasting stations to obtain more faithful reproduction of sound. The effect of sound on the diaphragm of the condenser microphone is to change its electrostatic capacity. When this effect is amplified through suitable tubes and circuits, a much more faithful reproduction of the sound is obtained than with devices used previously.

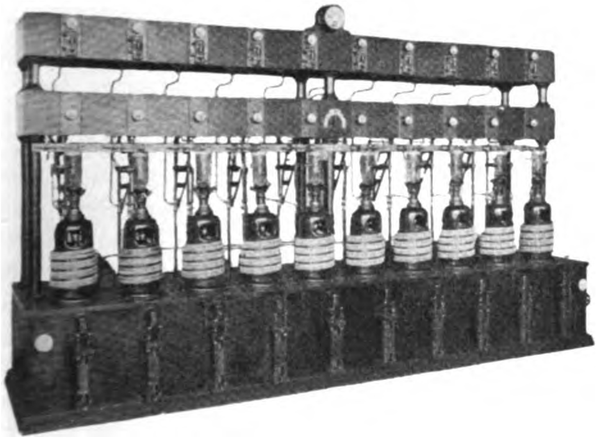

Fig. 57. Power Amplifier for Kenotron Rectifier Shown in Fig. 56

The development of apparatus for radio communication between the front and rear ends of moving freight trains resulted in equipment which is proving very useful. It comprises a radio transmitter (Fig. 58), having an output of approximately 50 watts, and a receiver. This equipment is located on the locomotive and similar equipment in the caboose.

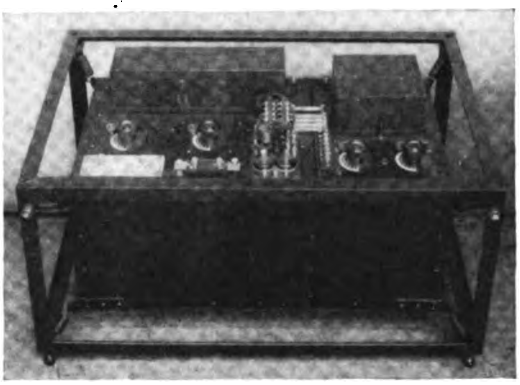

Fig. 58. Train Communication Transmitter-receiver Unit

Developmental Station

In order to ascertain the possibilities of improving service to broadcast listeners by the use of higher power, a new transmitter utilized two 100-kw. water-cooled tubes in the output stage and operated conservatively at an output of 100 kw. into the antenna. Comparisons of the results obtained by using 100 kw. and a relatively low power were made. These show that high power is very effective in overcoming the effects of disturbances at considerable distances from the transmitter and that a power of 100 kw. covers approximately twice the area covered by 30 kw. and gives equally satisfactory reception over the increased range.

Continued efforts were made to improve the rebroadcasting of American programs in foreign countries and satisfactory results were obtained. In this connection, transmitter 2XAF on 32.77 meters and transmitter 2XAD on 21.96 meters were used at various powers up to 35 kw.

CARRIER CURRENT TELEPHONE

The outstanding improvement in the main station equipment was in the form of apparatus to permit the further application of carrier communication to wire-line extensions. Provision was made for the connection of as many as ten line extensions to any one equipment, the apparatus chosen being that used for standard private automatic exchange operation and the service received being somewhat similar to that obtained in such operation.

A secondary station equipment was developed having a nominal output rating of 7½ watts. In general, it has all of the operating features of the primary station equipment except that its range is more limited and it is not provided with the more elaborate extension apparatus.

A still smaller and lower powered equipment is now available and is known as the "Booth Set." This equipment is designed for installation at points of lesser importance along the transmission line and is completely enclosed in a weather-proof metal case so that it may be entirely mounted out of doors. Where remote from other power supply it may be operated entirely from dry cells.

VACUUM-TUBE APPLICATIONS

The vacuum tube usually brings to mind some form of radio device. The year 1927, however, brought forth vacuum-tube equipment other than radio for central-station applications. The natural adaptation of the vacuum tube for particular applications has sponsored these developments rather than a desire to use the vacuum tube where other methods in common use have proved satisfactory.

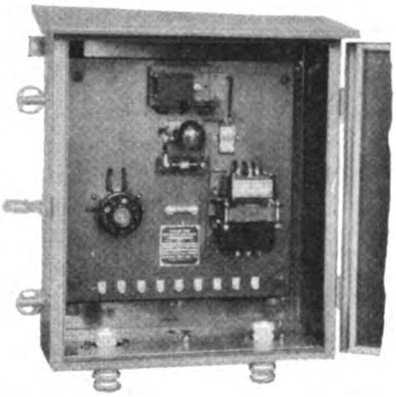

Fig. 59. Biased Detector with Associated Filament Transformer and Throw-over Switch

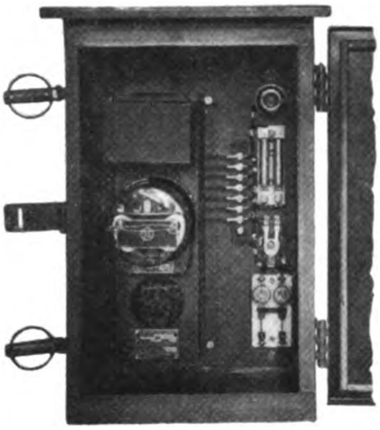

Fig. 60. Carrier-current Receiver for Control of Street Lights

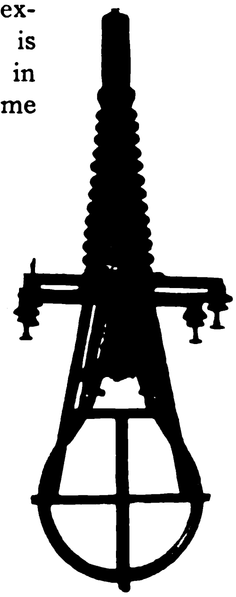

Fig. 61. 132,000-volt Cable-type Coupling Capacitor

Capacitance Transformer

For the higher voltage transmission lines, more particularly of 110 kv. and higher, vacuum-tube apparatus was developed in which the actuating potential is obtained from a capacitance transformer, mounted in an oil-filled bushing, rather than from a potential transformer. A potential of approximately 35 volts from the capacitance transformer is used to excite the grids of vacuum tubes from which various types of apparatus may be operated.

This form of equipment may be installed at a material saving over the potential transformer method, particularly in the case of the higher voltage lines or where an increasingly large number of lines are under consideration.

Standard oil-filled bushings may be replaced by bushings of the same rating equipped with capacitance transformers.

For isolated cases, where bushings having this feature are not installed on any high-voltage apparatus in the vicinity, a separate-tank mounting has been made available.

High-voltage Voltmeter

This application of the capacitance transformer permits the measurements of line potential. The potential obtained from the capacitance transformer is applied to the grid-filament circuit of a biased detector tube equipment (Fig. 59). A milli-ammeter in the plate circuit of the vacuum tube gives an indication of the voltage impressed on the tube.

In order that this indication may be translated into terms of line potential, a calibrating device (which supplies a variable grid excitation from a local source) is adjusted until the instrument in the plate circuit gives the same indication as when connected to the capacitance transformer. The calibrated voltmeter then reads directly the potential of the transmission line, taking into account the ratio calibration of the capacitance transformer.

Pilot-protective System

The usual system of pilot protection of feeders and transmission lines requires pilot wires between the two ends of the line. By the application of carrier current to the feeder or line to be protected, a protective system was developed which eliminates the necessity of pilot conductors.

The equipment consists of a carrier-current transmitter and receiver, installed at each end of the line to be protected. In case of a fault on the protected line, both ends of the line are automatically disconnected from the system through oil circuit breakers. A short-circuit beyond the protected line places the carrier-current equipment in operation to prevent the protected line from being removed from service.

This system gives practically the same protection as would be obtained by the use of pilot wires, but makes use of the transmission line itself, thus giving additional assurance of reliability.

Control of Street Lights

Equipment for the control of street lights by carrier-current consists of a transmitter panel, the output circuit of which is coupled to the feeder by means of coupling capacitors. Only one carrier-current transmitter per station is required, the various feeders being taken care of by transferring the output circuit from one feeder to the other by means of electrically-operated transfer devices. A carrier-current signal of definite time period is sent out over the feeder for turning on the lights, and a signal of a longer duration of the same frequency for turning off the lights.

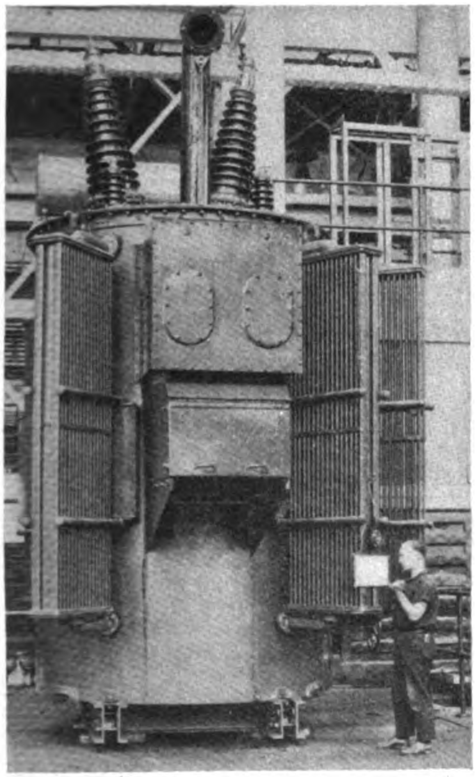

Fig. 62. Transformer with Self-contained Control Equipment

The receiving relays (Fig. 60) are mounted in weatherproof steel boxes arranged for pole mounting. They are located on the distribution feeders adjacent to the transformers which are to be controlled. As in the case of the transmitter, the receiver is coupled to the feeder by means of coupling capacitors.

Cable-type Coupling Capacitor

A new type of carrier-current coupling capacitor was produced in which the capacitance element is a length of high-voltage oil-filled cable (Fig. 61) which is bent into a loop and the free ends brought up through wiping sleeves, case, and a standard bushing shell, to a common terminal. The capacitor is given a vacuum treatment, filled with oil, and hermetically sealed by a positive-pressure expanding reservoir.

The capacitors have been built in 0.001 m.f. sizes for line potentials of 110,000 and 132,000 volts. They may be mounted on the transmission line towers or on a structural steel framework, or on an integral base. Six units of this construction having a capacity of 0.0078 m.f. mounted on herkolite cylinders will be used by the Leland Stanford University Laboratory in connection with a 1,500,000-volt impulse generator.

TRANSFORMERS

The maximum unit capacity of both self-cooled and artificially-cooled transformers was not carried beyond the record ratings of previous years but the total production of units exceeding 10,000 kv-a. in capacity showed a considerable increase; there being more than 100 units of this rating, and above, completed or under construction at the close of the year.

TABLE I

AVERAGE UNIT SIZE OF POWER TRANSFORMERS IN KILOVOLT-AMPERES

Year	Self-cooled	Artificially-cooled	All Types
1919	1175	4,325	2150
1920	1325	3,175	2175
1921	1575	4,150	2750
1922	1700	6,000	3750
1923	2350	5,600	3450
1924	2500	5,325	3525
1925	3550	5,275	4175
1926	2806	11,645	5052
1927	2677	11,354	4316

The use of the oil conservator was standardized for all power transformers in excess of 500-kv-a. rating, with the minor exception of a few special high-current units. The total number of transformers so equipped now exceeds 5000 with an aggregate capacity of more than 27,000,000 kv-a.

The use of the oil air-pressure method of cooling transformers which was first applied commercially about two years ago, showed a remarkable increase in the number and capacity of transformers to which this scheme was applied during the year. The transformers so equipped, installed, or under construction, already total more than 1,000,000 kv-a. in capacity, the unit capacity ranging from 10,000 to 30,000 kv-a.

A corresponding increase in the use of load ratio control was indicated by the fact that the total capacity of the transformers so equipped for the year 1927 was in excess of 900,000 kv-a. This figure is greater than that of all preceding installations since the first unit was placed in service early in 1923.

An interesting feature in connection with the use of this system is the tendency to mount the control equipment directly on the transformer (Fig. 62). This makes the transformer equipment completely self-contained and effects a saving in the floor space required. It also avoids the necessity of bringing cables outside of the transformer to connect with the control apparatus.

The development of higher voltage cables, the desire of operators to avoid exposed connections, and especially the difficulty of making satisfactory end bells, led to the adoption of transformers with cable entrances—the cable either terminating directly inside the transformer tank, or terminating in oil or compound-filled "pot heads" (Fig. 63) which are connected to the transformer by means of bushings, both ends of which are immersed in oil.

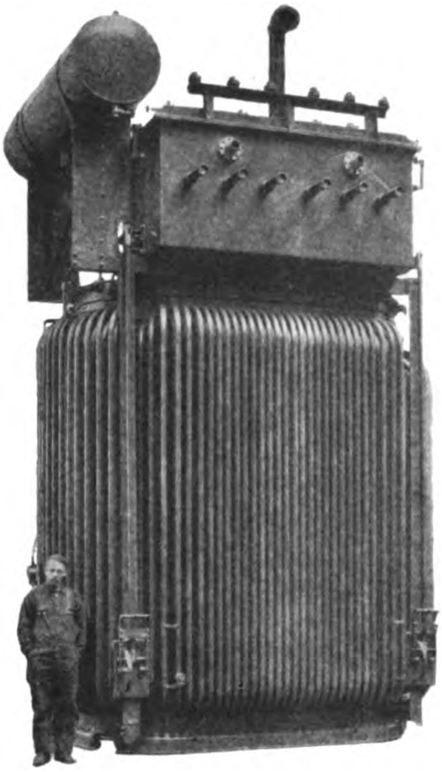

Fig. 63. Outdoor-type Transformer, Showing "Pot Heads" for Cable Terminals

Frequently, the necessity for routine testing of the cable requires means of readily disconnecting the cable from the transformer. To an increasing extent, disconnecting switches are fitted in the pot heads with special testing bushings so that the cable can be switched from the transformer to the testing connection or to ground.

A notable example of the adoption of this means of protection is found in the group of 20,000-kv-a. water-cooled transformers under construction for the State Line Generating Company. These consist of seventeen single-phase and four three-phase units all of which receive current from the generator at 22,000 volts. The high-voltage ratings of seven of the single-phase units is 132,000Y, the remaining ten being 66,000Y, and three-phase units 33,000Y. All of these transformers are provided with cable-entrance bushings on the low-voltage side and all but the 132,000-volt units are equipped with cable-entrance bushings and self-contained disconnected switches on the high-voltage side. All of these transformers are also provided with load ratio control.

A group of transformers of unusual interest is under construction for the Commonwealth Edison Company. They are of the water-cooled type, rated 20,000 kv-a. 66,000Y/12,000/4000Y, and are provided with auxiliary series and regulating transformers to give regulation plus or minus equivalent to 10 per cent of the main transformer voltage. The main transformers in addition to the provision for water cooling are provided with sufficient radiating surface to dissipate the core-loss heat in case of failure of the water supply. They are also provided with extra radiator flanges so that the oil air-pressure system of cooling for full kv-a. rating can be applied, and under these conditions each unit will be equipped with a motor-driven blower. Then too, each tank is fitted with valves for the application of the forced-oil method of cooling. This unusual equipment will permit the adoption of whatever cooling system is best suited to local conditions.

The motor-generator type locomotives for the Cascade Electrification of the Great Northern Railway each includes as an essential part of its equipment an air-blast transformer of unusual design. This unit (Fig. 64) furnishes the electrical tie between the 25-cycle 11,000-volt single-phase overhead catenary system and the locomotive's 25-cycle synchronous motor-generator set which in turn converts the output of the transformer from 2300 volts alternating current to 1500 volts direct current for application to the traction motors. Each of these air-blast transformers is rated 25 cycles, 3285 kv-a.

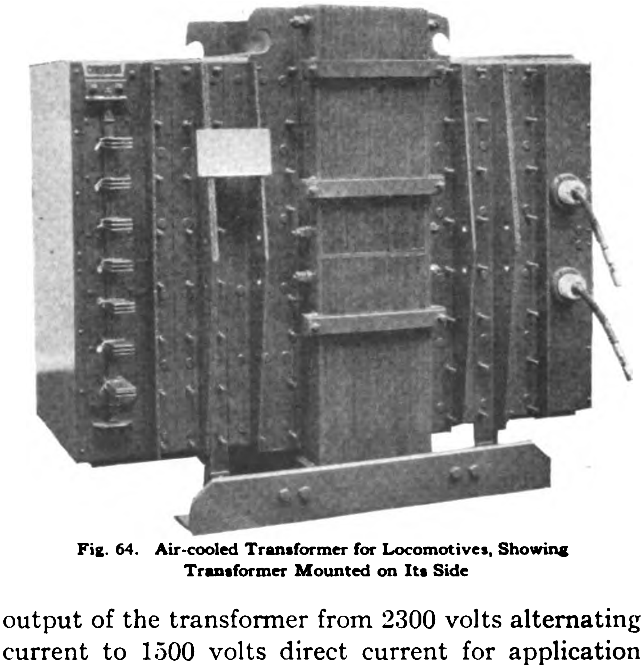

Fig. 64. Air-cooled Transformer for Locomotives, Showing Transformer Mounted on Its Side

The electrical design of the transformers is complicated by the varied operating requirements and the space limitation involved in their installation on the locomotives. In order to meet these conditions, the transformer was not set upright in the usual manner but was laid on its side. A standard steel-housed type of air-blast transformer construction was employed with modifications and the unit is characterized by a low core and an abnormally high coil arrangement.

In order to ventilate these transformers, the air intake is in the bottom and the stream of air is deflected at an angle of 90 deg. to pass longitudinally through the coils. It is again deflected 90 deg. and discharged from the other end of the transformer at the bottom.

A transformer for similar service for use by the Pennsylvania Railroad is rated 450 kv-a. 11,000/1045 volts. It is similar in general construction to those just described, but is modified mechanically because it is intended for mounting underneath the platform and for this reason the construction was made weatherproof throughout.

Network Transformers

The rapid introduction of alternating-current networks in city areas imposed certain modifications on the customary subway transformer designs to meet the requirements of network operation. There is a definite tendency toward the use of three-phase transformers instead of single-phase where new vaults or manholes can be provided with openings of sufficient size to take the larger units. Individual units of as much as 500 kv-a. are being used on certain networks, whereas the limitations of manhole openings and the density of the load have in the past kept the size of subway units down to about 100 kv-a. single phase.

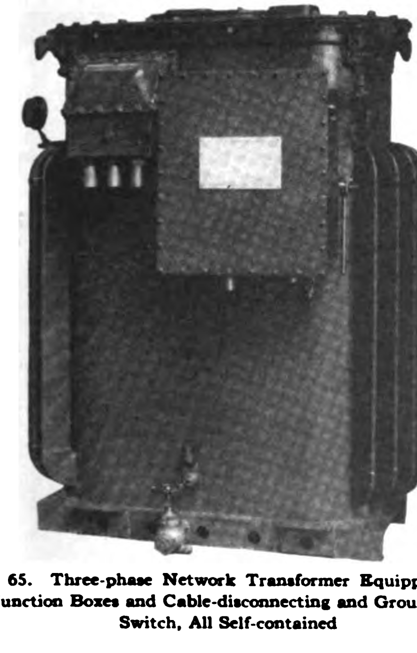

Fig. 65. Three-phase Network Transformer Equipped with Junction Boxes and Cable-disconnecting and Grounding Switch, All Self-contained

The increased voltage of feeders, which is coincident with the use of networks, has introduced certain problems in the handling of cable connections and the testing of cables with the transformers attached or disconnected in accordance with the system used by individual operators. The general requirement is for a disconnecting switch by means of which the cable can be connected to the transformer for service; disconnected from the transformer for phasing-out or routine high-potential tests; and grounded for the protection of operators working on the transformer or cable system.

To meet these requirements a three-position switch (Fig. 65) is mounted either in the transformer tank or preferably in a pothead attached to the side of the transformer.

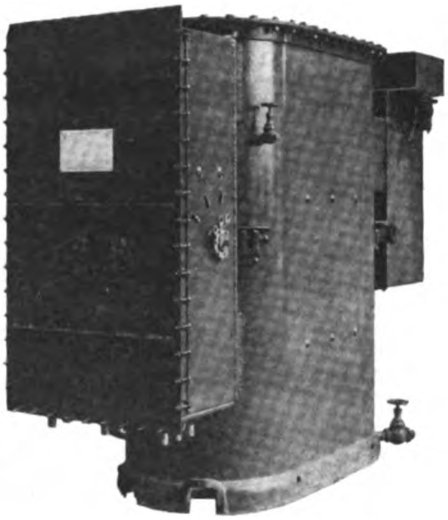

Fig. 66. Low-voltage End of Three-phase Network Transformer Showing Arrangement of Mounting Auxiliaries on Transformer Casing

In at least one large network the advantages of having all equipment in the manhole as a single unit resulted in a design (Fig. 66) which has, in addition to a high-voltage pothead with a switch for grounding purposes only (the transformer in this case being tested with the cable), a low-voltage switch casing in which the automatic network switch is housed. This arrangement greatly simplifies the cable connections, but because of the large size of the unit it is likely that separate housings for the automatic switches in the transformer will generally be used.

Instrument Transformers

The maximum potential of instrument current transformers was increased by the construction of 155-kv. units (Fig. 67) for the Commonwealth Power Corporation.

Four 100/200-amp. current transformers built for the Detroit Edison Company were designed to carry an abnormal short-circuit current of 12,500 amp. and were insulated for a 120-kv. circuit.

There were also constructed six instrument potential transformers for the Philadelphia Electric Company to operate in connection with 220-kv. circuits. These are the highest voltage instrument potential transformers so far constructed, the previous maximum being 154 kv.

Laboratory testing facilities for instrument transformers (Fig. 68) were improved so that phase-angle and ratio tests by the direct-reading method for all voltages up to and including 135 kv. can now be made. The previous limit was 66 kv.

Oil-filled Bushings

The enviable service record of the oil-filled bushing has brought about its adoption for lower voltage ratings than heretofore, it having recently been standardized for all transformers in the 66,000-volt class, superseding the solid-type bushing with this voltage rating.

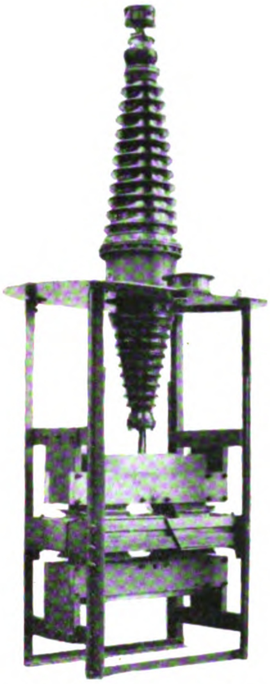

Fig. 67. 155-kv. Instrument Current Transformer with Two Independent Cores and Secondaries

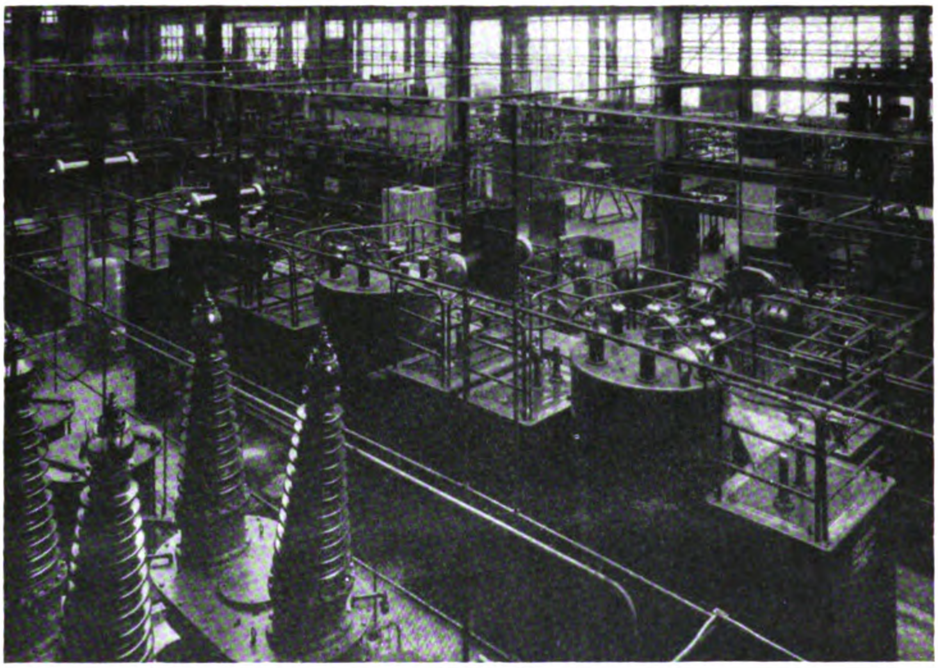

Fig. 68. 135-kv. Instrument Potential-transformer Testing Equipment, Showing Resistance Units and Shielding Transformers

REACTORS

A current-limiting reactor having probably the largest physical dimensions of any so far built (Fig. 69) was constructed for the Consolidated Gas, Electric Light and Power Company at Baltimore, for use in grounding its 13,200-volt 25-cycle system. It has a four-minute rating of 80,000 kv-a., 8000 volts, 10,000 amp. The gross weight is eight tons.

Three reactors for use with the 137,500-kv-a. turbine-generator at the Hudson Avenue Station of the Brooklyn Edison Company will connect the 27,600-volt bus of this generator to the station synchronizing bus. They introduce 10 per cent reactance when carrying the full-load generator current, and are each rated 4583 kv-a., 1593 volts, 2880 amp. They are the largest current-limiting units ever supplied for such high voltage.

There was a remarkable increase in the demand for the oil-immersed air-core type of current-limiting reactor enclosed in a metal tank, which was first used in 1922. During the year more than 50 of these units were completed or under construction.

One group of 19 of these reactors each rated 2750 kv-a., 1170 volts, 2364 amp. will be utilized in sectionalizing the 22,000-volt bus of the State Line Power Station at Hammond (Ind.). The ultimate generating capacity connected to this bus is expected to be over 1,000,000 kv-a.

These reactors (Fig. 70) will be the largest units of the metal-enclosed type and will be connected to the bus by means of lead-covered cable, the lead covering making a wiped joint with a brass casting bolted to the tank.

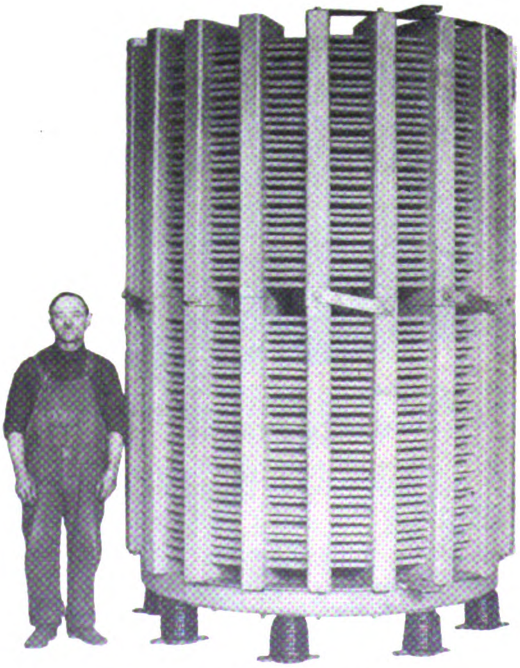

Fig. 69. 80,000-kv-a. 8,000-volt 10,000-amp. Current-limiting Reactor

There was a growing demand for reactors to be used for drawing lagging current through a long transmission line to neutralize the charging current of the line, so as to relieve the generating apparatus from supplying this charging current. What is probably the largest reactor ever built for this purpose (Fig. 71) is rated 4500 kv-a., 13,800 volts, 108.6 amp. and is used in a 24,000-volt line. Since it is not a series reactor but a shunt reactor, with iron core and air gaps in the core, the full voltage of the system is applied across it continuously.

CAPACITORS

A series capacitor (Fig. 72) was built with a rating of 1245 kv-a., and insulated for installation in the 33,000-volt 7500-kv-a. 60-cycle transmission line between the Amsterdam Steam Station and the Ballston Spa Substation of the New York Power and Light Corporation.

The theory involved in this application is quite simple. A capacitance placed in series with a reactance can be made to compensate for the reactance if $\frac{1}{2\pi fC}$ is made equal to $2\pi fL$. For such a circuit the potential drop across the capacitor for all values of current is equal and opposite in phase to the potential drop across the reactance. If this circuit is a transmission line, only the drop due to the ohmic resistance of the line affects the regulation of the circuit. Therefore, by using such a compensated circuit the characteristics of direct-current transmission are approximately obtained, yet with the simplicity and advantages of alternating-current transmission retained.

In the practical operation of the series capacitor, the capacitor element must be insulated from ground for line potential and the dielectric between capacitor plates so proportioned as to stand the potential drop across the capacitor due to normal full-load line current. Under short-circuit conditions, the current increases through the capacitor and abnormal

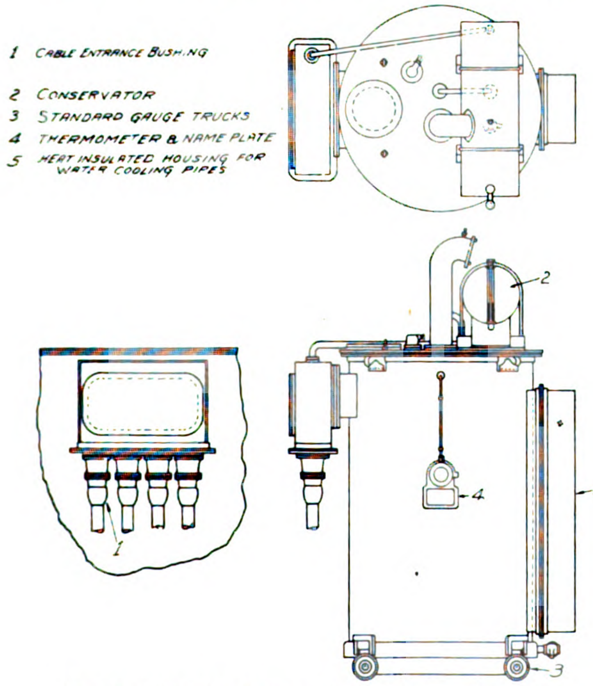
1 CABLE ENTRANCE BUSHING
2 CONSERVATOR
3 STANDARD GAUGE TRUCKS
4 THERMOMETER & NAME PLATE
5 HEAT INSULATED HOUSING FOR WATER COOLING PIPES
1
2
3
4
5

Fig. 70. Oil-insulated Current-limiting Bus-sectionalizing Reactor Having No Live Parts Exposed

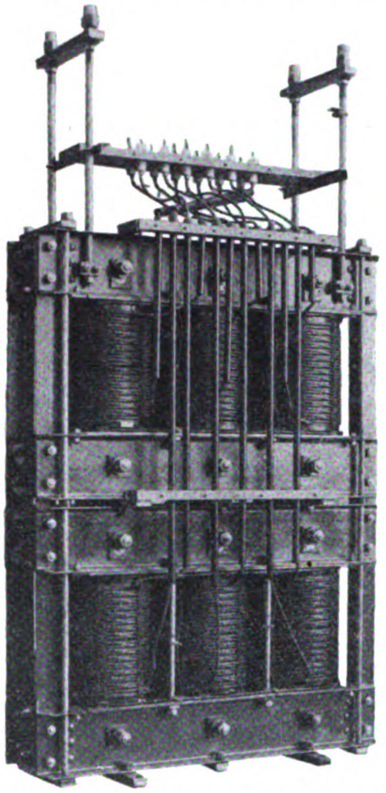

Fig. 71. 4500-kv-a. 13,800-volt 108.6-amp. Current-limiting, Oil-cooled Reactor Assembled and Ready for Installation in Tank

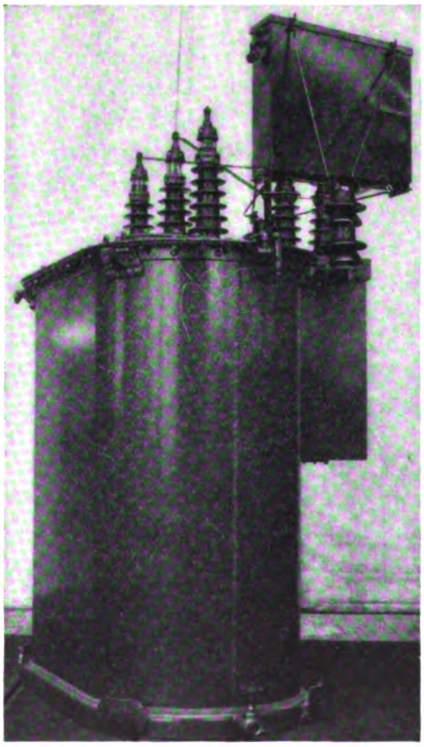

Fig 72. One Phase of a Three-phase Outdoor-type Series Capacitor for 33,000-volt Transmission Line Regulation

drop across the capacitor develops. If the normal drop were 10,000 volts, then ten times normal current on short-circuit gives 100,000 volts across the capacitor. The capacitor is protected from this over-voltage by cutting it out of circuit on the rise of short-circuit current by means of a sphere gap and high-speed contactor.

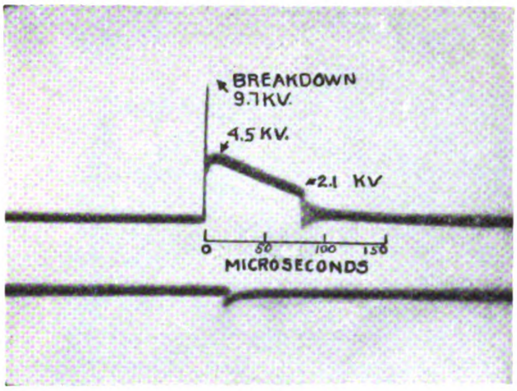

Fig. 73. Cathode-ray Oscillogram, Showing Volt–Time Characteristics of Pellet-type Lightning Arrester

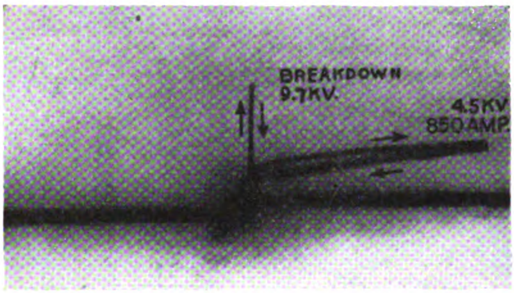

Fig. 74. Cathode-ray Oscillogram, Showing Volt–Ampere Characteristics of Pellet-type Lightning Arrester

Short-circuit tests on the capacitor demonstrated the practicability of the protective equipment. A short-circuit current of 15,000 amp. (r.m.s. value) was easily handled by the sphere gap and high-speed contactor with practically no signs of burning.

It is probable that the series capacitor will be utilized for automatic regulation of transmission-line voltages; increasing the power limit of long transmission lines; and compensation for part of the reactance in a line so that it will share its proper division of load when paralleled with another transmission line of widely different characteristics.

LIGHTNING ARRESTERS

In September, 1925, the Lightning Arrester Laboratories at Pittsfield took the first cathode-ray oscillograms showing the performance characteristics of lightning arresters under impulse conditions. Since that time both in this country and abroad other research workers have taken similar oscillograms. This method of test is being used to determine the volt-time, ampere-time, and volt-ampere characteristics of arresters under the action of discharges which may exist for but a few millionths of a second.

A typical volt-time and volt-ampere curve of a pellet lightning arrester (Figs. 73 and 74) shows how the oscillograph makes possible the accurate determination of the operating characteristics of arresters.

The cathode-ray oscillograph method in these two years has become so important in this field that its use has become standard in the testing of such devices. With this oscillograph an accurate measuring tool is now available, which makes the standardization of arrester testing a possibility.

The effect of time on the operating characteristics of a lightning arrester has always been an important factor and it is of particular interest to note that tests made recently with the cathode-ray oscillograph on service cells, from oxide-film arresters show their operating characteristics to be unimpaired after years of service. The oxide-film arrester has been in service since 1915 and the cells tested were taken from early installations in many sections of the country.

With the growing interest in the accurate determination of the operating characteristics of lightning arresters and the increased use of high-voltage arresters, it is important to have test data to show the protection expected from these higher voltage arresters. During the year, impulse data on 110,000-volt oxide-film arresters were obtained using impulses having a crest potential of 1,400,000 volts.

Test data to be of maximum benefit must be properly correlated with field data. This information is now available through the use of the klydonograph and shows that potential gradients of approximately 50,000 volts per foot can occur on transmission lines during lightning storms. Such high potentials indicate the necessity of protecting terminal equipment from the effects of such impulses.

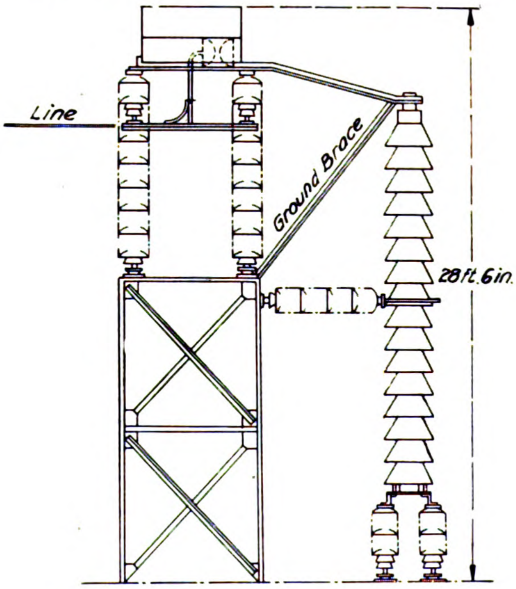

Fig. 75. Side Elevation, Showing Arrangement of 220-kv. Oxide-film Lightning Arrester

With data now available from both laboratory and field, the insulation of transmission lines and of station apparatus is being studied with a view to so co-ordinating the various pieces of apparatus that no one part will be unduly stressed. The arrester is being shown to have a definite function to perform at even the highest transmission voltages.

The highest voltage arresters yet designed are for installation at the Conowingo Station of the Susquehanna Power Co. They are oxide-film arresters (Fig. 75) and are rated 220,000 volts.

CONDUCTOR CABLE

The most notable advance in commercial cable practice was the putting in service of a six-mile line (18 cable miles) of 132-kv. single-conductor oil-filled cable (Fig. 76) by the Commonwealth Edison Company of Chicago, on June 2nd, and a twelve-mile (36 cable-miles) circuit by the New York Edison-United Companies on August 9th.

Both lines were tested with 300,000 volts, using 400,000-volt direct-current kenotron sets. No failures occurred under this electrical test (Fig. 77) in either installation. The lines have each a capacity of 91,000 kv-a., thus bringing the capacity of a single cable line up to that of the largest generating units in service.

Another development in the use of single-conductor cables in connection with three-phase high-tension circuits was the installation by the New York Edison Company of three single conductors in one conduit on a 45-kv. circuit; showing favorable economic results due to the use of a single conduit over the former practice of a separate conduit for each conductor of the circuit.

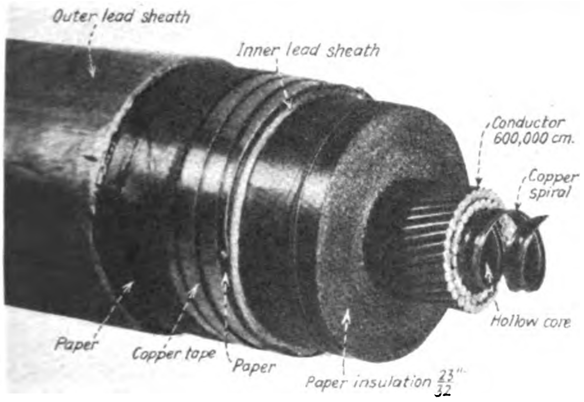

Fig. 76. Construction Details of 132-kv. Single-conductor Oil-filled Cable

There were further large installations during the year of single-conductor cable for 66-kv. circuits, notably in Philadelphia and Chicago.

Practically all circuits from 75 to 33 kv. inclusive are now using oil-filled joints and auxiliary reservoirs to take care of the expansion and contraction of the oil due to variations in temperature and to keep the joint and cable adjacent to the joint under oil pressure. Tests have shown that such joints when properly designed have a dielectric strength equalling or exceeding that of the cable itself.

An improvement in connection with the use of single-conductor cables, particularly those of large size used on alternating-current circuits, was the insertion of insulating joints in the sheaths of the cable, and cross bonding of the sheaths in such a way as to neutralize the induced voltages in the lead sheaths. This results in a reduction of the losses due to such circulating currents and increases the current-carrying capacity of the line by about 15 or 20 per cent.

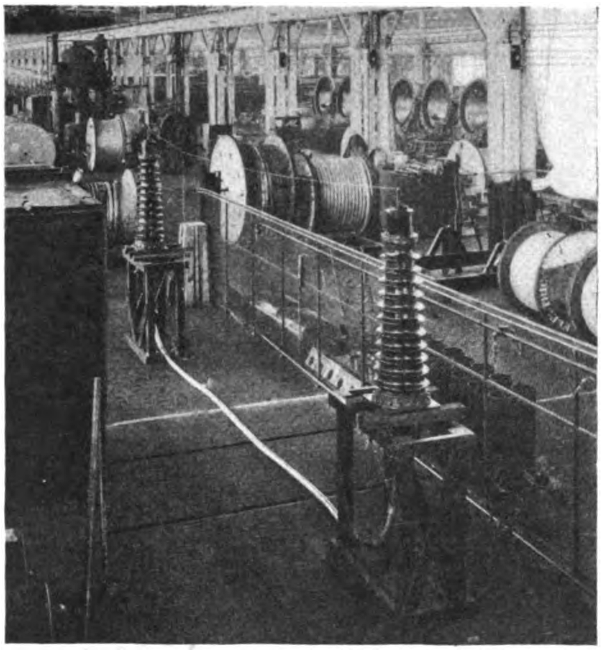

Fig. 77. 300-kv. Test of 132-kv. Single-conductor Oil-filled Cable

AUTOMATIC STATIONS

The demand for automatic control continued to increase. There were no radical changes in the general design, but some very marked improvements were made in construction details. Device functions were combined and some devices were eliminated, thereby tending to simplify the control and reduce the cost.

Hydro-electric

The largest hydro-electric station with automatic control is being equipped by the Louisville Hydro-Electric Company, at its Shipping Port Station. The combined capacity of the eight generators of this station (Fig. 78) is 100,400 kv-a. and space is reserved for two more units which will bring the ultimate capacity to 125,500 kv-a. Power will be generated at 14,000 volts and stepped up through two transformer banks to 66,000 volts for transmission. There will be a main and an auxiliary bus in the station to which each generator will be connected through large oil circuit breakers

A unique feature of this station is the self-supported steel containers for the control equipment. The control devices for each machine are arranged on panels which are mounted within the steel container (Fig. 79), commonly termed a "cubicle". Double doors at front and back permit ready access to the devices and their wiring. The front doors are fitted with glass so that all devices and instruments may be seen without opening the doors. Connecting leads are brought up through small pits in the floor directly under the cubicle and are plugged into the proper terminals by means of multi-plug car couplers. Pull-button switches are built into the front doors so that normal operation may be accomplished without opening the cubicle.

In addition to the eight cubicles for the machine-control equipment there are four which house the control for the transformer-bank switching, one for the station service feeder, and one for the cable-type supervisory-control devices.

A second feature of this station is the compact instrument and supervisory-control cabinet which will enable the chief station attendant to supervise the switching throughout the entire station and to check all meter readings without leaving his desk. The cable-type supervisory system permits him to select and supervise the operation of breakers connecting each machine to the buses and to regulate the load and gate opening of each unit.

The units are started manually by a floor attendant upon the receipt of a signal sent from the supervisory-control cabinet and upon reaching speed are automatically synchronized and connected to the bus. Full-automatic protection is provided at all times. By throwing a switch in each generator cubicle, the entire station may be made manually controlled throughout.

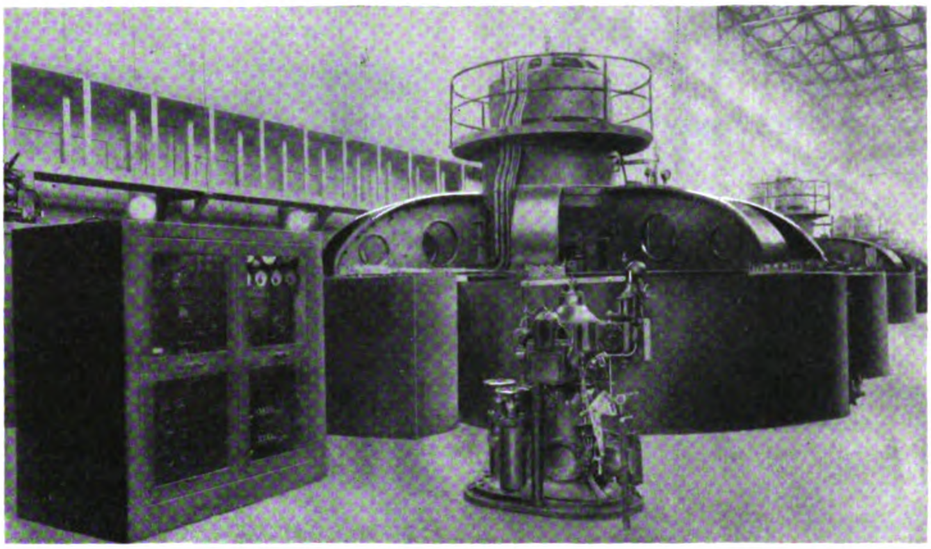

Fig. 78. View in Generator Room of Louisville Hydro-electric Company, Louisville (Ky.), Showing Location of Control Cubicle for Automatic Switching Equipment

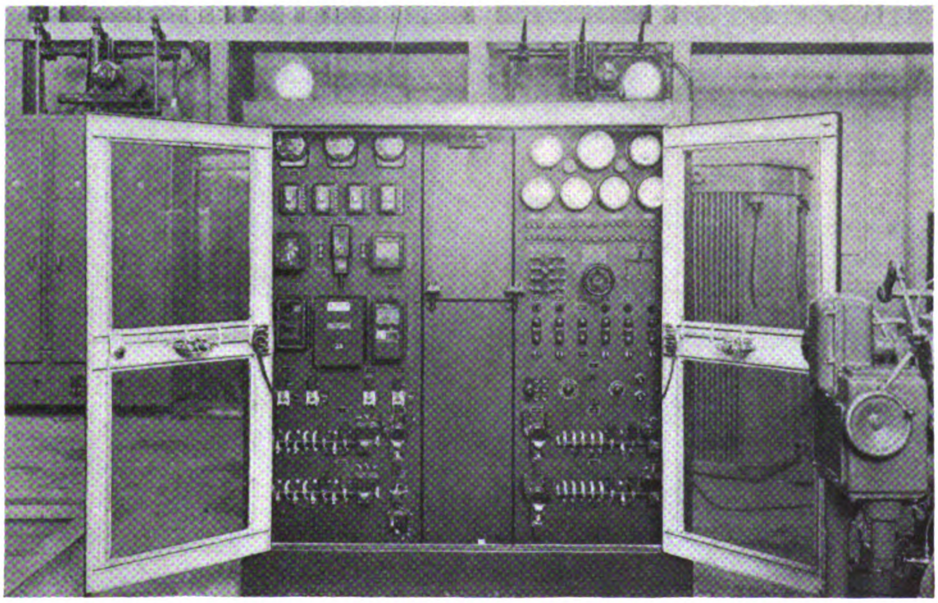

Fig. 79. Details of Control Cubicle for 12,550-kv-a. 14,000-volt Waterwheel Generator, Louisville Hydro-electric Company

The steel cubicles, housing the control equipment and the small compact control and indicating cabinet, are radical departures from the older methods of hydro-electric station layout. The new arrangement permits one man to have supervision of the entire station all within arms' length and gives unit control for each machine and equipment, housed separately in a dust-tight compartment. All control is readily accessible and is easy to maintain, repair, and replace; and each cubicle may be disconnected and moved and a new one substituted if major repair work or replacement is required.

The Bibb Manufacturing Company of Macon (Ga.) installed a two-unit hydro-electric generating equipment with complete automatic control in its Porterdale Station. These machines are started and stopped in accordance with the water level. The control is provided with a regulator having both a voltage and current element so as to allow the machine to deliver full rated leading current under normal conditions. The voltage element is used only to limit the machine voltage in case of insufficient demand.

The majority of the hydro-electric equipments built during the year were arranged to pull into synchronism with the line at 95 per cent synchronous speed, however, one small double-unit station was installed by the Southern Power Company which included a complete automatic synchronizing equipment. The amount of water in the forebay determines whether one or both units will be put in service. A speed-matching relay controls the speed of the unit to equalize as nearly as possible the frequency of the machine with that of the line. The synchronizing equipment then functions to connect the machine to the line when exact synchronism has been reached.

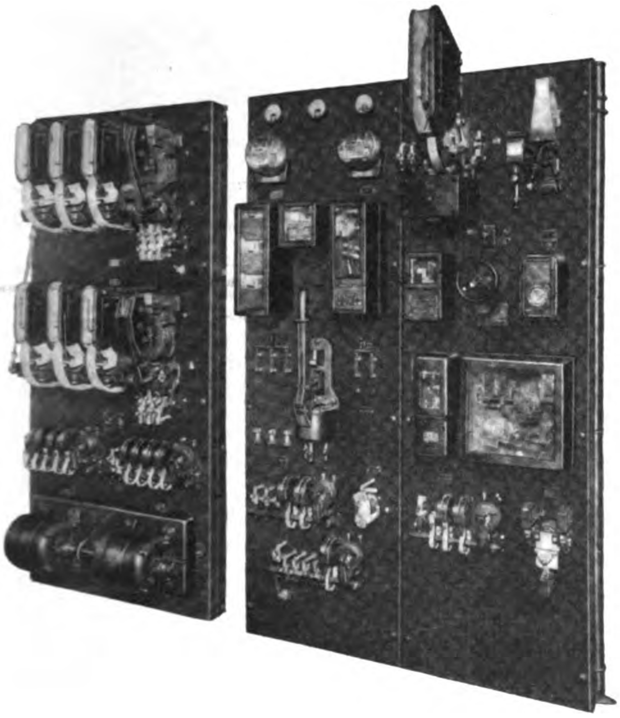

Fig. 80. Automatic Switching Equipment for 200-kw. Mining Synchronous Converter

Mining and Industrial

The control for mining converters and motor-generator sets was further simplified by combining into one device the functions previously performed by several devices. In the case of the synchronous-converter equipment (Fig. 80), one simple contactor now gives the synchronous-speed indication and applies separate excitation to the converter field. Another single relay combines both direct-current reverse-power and direct-current voltage-equalizing functions.

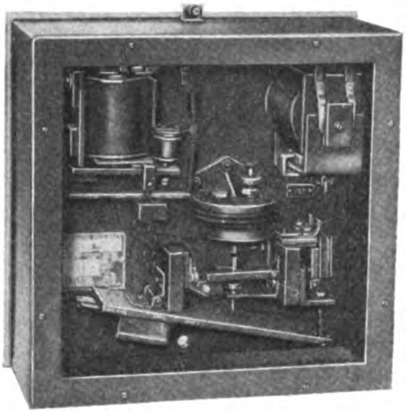

Fig. 81. Automatic Recloser for Stub-end Multiple-feed Direct-current Circuits

A new device known as the direct-current recloser (Fig. 81), and consisting of three elements under one cover, performs the functions which previously required five separate devices. It is entirely self contained with connection terminals and resistors complete, and its range of adjustment is the same as that of the design which it supersedes. It is available for 275-volt direct-current circuits in 300, 600, 900, 1200, 1500, and 1640-amp. capacities. One mining company installed seven single-unit 150-kw. converter stations with this new simplified control, including the direct-current recloser.

The mine sectionalizing and reclosing unit was modified in design and is now available for 550-volt as well as for 275-volt, direct-current circuits.

The American Steel and Wire Company installed at its South Works Station in Worcester (Mass.), control for three 3-unit 600-volt direct-current 1500-kw. motor-generator sets, and three 3-unit 250-volt direct-current 1000-kw. motor-generator sets, all controlled automatically; the latter three being equipped with counter e.m.f. voltage regulation.

All control panels for this installation were arranged on separate self-supported welded frames, each frame being numbered in accordance with its location in the switchboard. These features greatly facilitated the installation work.

Railway Substations

Several important programs were initiated by railway companies for the purpose of modernizing and adding to their existing power supply. A study of information available on the work now being done indicates that, while both manual and automatic substation equipment is being used, a somewhat larger percentage of the projects now in hand are using automatic switching equipment. Many of these are also being provided with supervisory control.

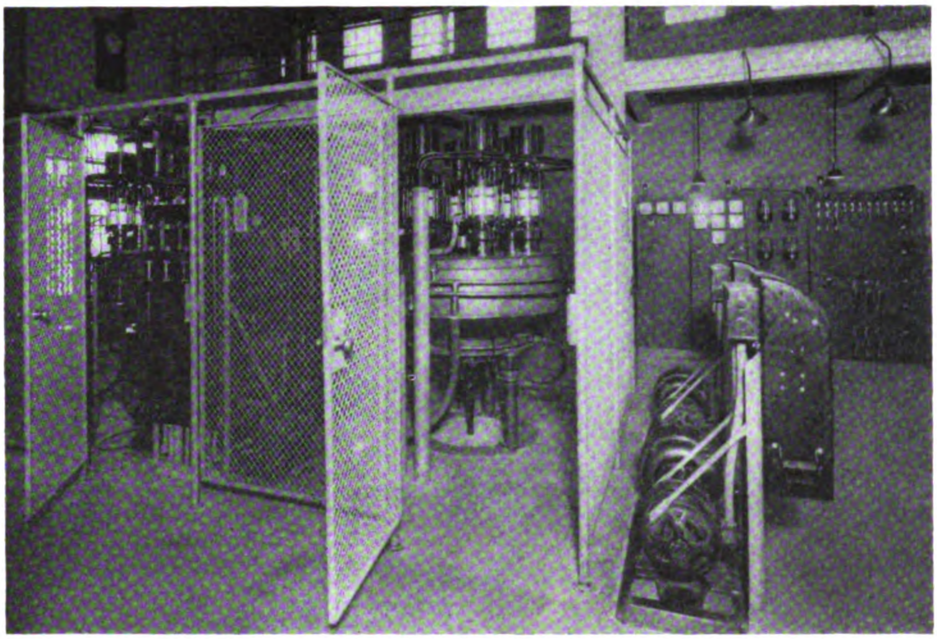

Fig. 82. Laflin Street Substation of Illinois Central Railroad, Showing Arrangement of 1500-volt Mercury-arc Power Rectifiers

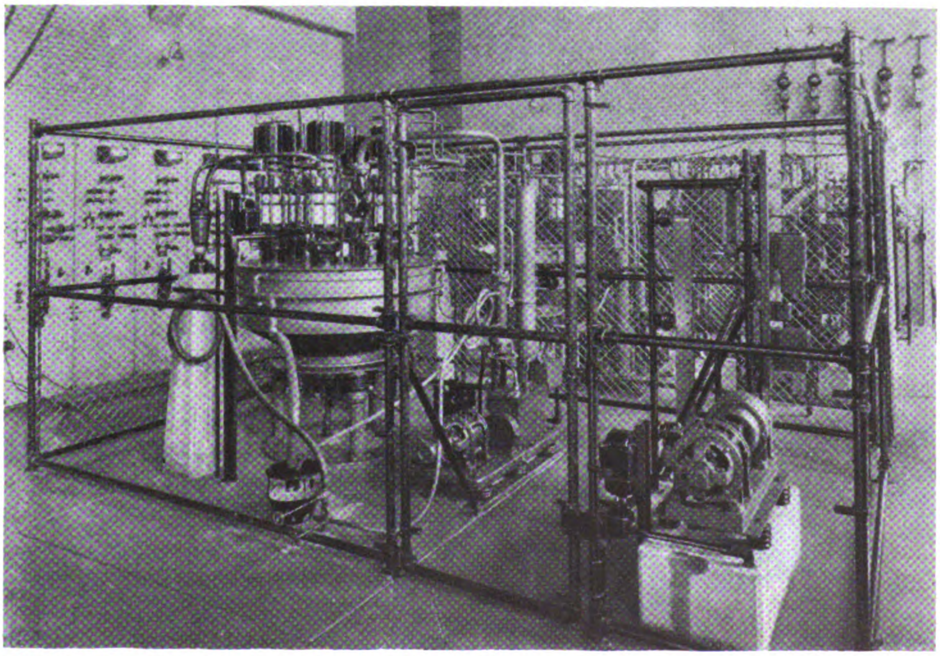

Fig. 83. Jefferson Street Substation of Portland Electric Power Company, Showing Arrangement of Two 1400-volt Mercury-arc Power Rectifiers

One of the largest installations ever undertaken is that of the Cincinnati Street Railway Company which is completely overhauling its substation supply in the city of Cincinnati. Nineteen substations are involved, the larger part of these being completely new. A few are using the synchronous converters in existing substations with the addition of new automatic-control and supervisory equipment.

The supervisory equipment is designed to permit either full-automatic operation or direct control by the load dispatcher. The units controlled are of 1000- and 1500-kw. capacity each. When completed, this installation will represent one of the most extensive applications of automatic control in the world.

The Sacramento Northern Railway after several years' experience with two automatic substations is now providing automatic control for ten additional units in existing stations. These units are all induction motor-generator sets, each of 400-kw. capacity.

The Pacific Gas and Electric Company is equipping a single station with two 1500-kw. units, both automatically controlled. The largest unit to be provided with automatic control is the 4000-kw. synchronous converter shortly to be placed in service by the New York Rapid Transit Company to supply power to the subway and elevated systems. A second converter of the same size is being provided with manual control.

The Boston Elevated Railway, working in conjunction with the City of Boston, is installing a second station to be equipped with two 3000-kw. synchronous converters having automatic and supervisory control. These units duplicate similar equipment placed in service during the year.

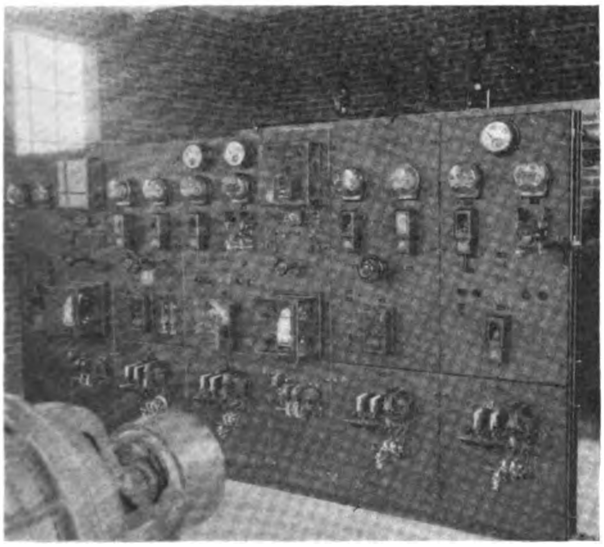

Fig. 84. Switchboard in Automatic Substation at Piqua (O.) for Signalling and Train Control, Pennsylvania Railroad Company

An example of modern substation equipment for a heavy interurban railway is provided by the Chicago, Aurora and Elgin Railroad, which is purchasing power from stations to be equipped by the Public Service Company of Northern Illinois. Three 1500-kw. 60-cycle synchronous converters are being used, two with automatic and one with manual control.

The Shreveport Railways, which installed an automatic substation about three years ago, is now completing the automatic distribution for its entire system by putting in three additional 500-kw. stations, thus permitting the abandonment of the existing power plant.

A number of mercury-arc power-rectifier sets installed during the last two years have now been in use for a sufficient length of time to give assurance of their suitability for railway service. Several 1500-volt units on the Illinois Central Railroad (Fig. 82) and the Chicago, South Shore and South Bend Railway have been supplying power to heavy multiple-unit trains for nearly two years.

During the year the following companies installed rectifiers for rapid-transit service:

The Los Angeles Railway Corp.	Two 500-kw. units
The Long Island R.R. Co.	Three 1000-kw. units

The Philadelphia Rapid Transit Company, which obtained considerable experience in 1926 with two 500-kw. units, increased its equipment by the installation of three 1000-kw. rectifiers. The Long Island and Philadelphia units will use twelve-anode tanks connected in twelve-phase relation.

Rectifiers were also adopted by the Portland Electric Power Company (Fig. 83) for supplying the Oregon Electric Railway; by the Columbus Railway Power and Light Company; and by the Duquesne Light Company for supplying power to the Pittsburgh Railways. Practically all of the rectifiers now being installed are provided with full-automatic control.

Railway Signals and Train Control

Automatic control was used in connection with frequency converters and feeders to supply 100-cycle power for automatic train control and railway signals. The frequency converters, which are usually of the induction-synchronous type, are of moderate capacity and are provided with voltage regulators for maintaining constant voltage.

Systems employing this type of equipment usually maintain a liberal number of stations and generating sets and power from either end of the line insures extreme reliability of service under all conditions. The 100-cycle control power has been adopted by the railroads using the continuous type of automatic train control, because it prevents interference from lower frequency power lines paralleling the right-of-way. As a matter of convenience the signals are therefore operated on the same power supply.

The Pennsylvania, the Reading, and the New York, New Haven and Hartford Railroads are a few of the more prominent roads using this equipment. The Pennsylvania System installed six stations (Fig. 84) on its lines between Columbus (O.) and Indianapolis (Ind.) and three on its Camden-Atlantic City Line. The Reading placed three of these equipments in service on its Bethlehem (Pa.) Branch at the Saucon Creek Power House, Orvilla and Jenkintown.

The New York, New Haven and Hartford Railroad will utilize sixteen equipments on its Shore Line between New Haven (Conn.) and Providence (R. I.). Each of these stations will contain a 5-kv-a. induction frequency converter for supplying the 100-cycle power.

Direct-current Lighting

Although the motor-generator set is considered best adapted to direct-current lighting service, due to the range over which the voltage can be regulated, still a number of automatic control equipments for large booster-type converters were adopted. Load-limiting resistors of the water-cooled type are utilized to provide low voltage for service restoration. This results in an installation which fulfills the essential operating conditions and has a very high efficiency under normal operation.

The Cleveland Electric Illuminating Co. installed a 3900-kw. booster converter with control similar to that just described. This is in addition to two other identical equipments put into operation a few years ago.

Synchronous Condensers

The Alabama Power Company placed in operation a 10,000-kv-a. 11,000-volt synchronous condenser with full-automatic control equipment at Mobile, (Ala.). A voltage regulator maintains constant voltage and the machine is given full-automatic protection at all times.

The Anglo-Chilean Consolidated Nitrate Corporation is installing a synchronous condenser of moderate capacity with automatic control at its nitrate fields in Chile (S. A.).

Alternating-current Feeders and Incoming Lines

Alternating-current automatic-reclosing feeder equipments constituted a large percentage of the automatic switching applications and this type is now handling over 2,500,000 kv-a. of power, mostly in 2300/4000-volt service.

Alternating-current line equipments were the subject of much investigation during the year and as a result a total of eight thoroughly workable control schemes are now available. A total of 250,000 kv-a. is now being handled automatically by such equipments and extensions of the present systems should make duplicate sources readily available and result in greater continuity of service.

Supervisory Equipment

A new type of automatic supervisory control known as the Synchronous Selector System was perfected. With this system the dispatcher may control, and receive indications from, a larger number of equipments located at a remote station. The system requires four line wires between the dispatcher's station and each remote station.

The dispatcher operates the system from a key and lamp cabinet somewhat similar to those used for other types of supervisory equipment. This cabinet contains, for each outlying equipment to be operated, a standard two-position turn key for control, a red light for indicating the closed position, a green light for indicating the open position, and a white light for indicating an automatic operation of the outlying equipment. In addition, each combination has a two-position push-pull holding key to stop the selectors in each station at a point corresponding to the equipment it is desired to operate.

To operate a remote equipment, the dispatcher must first pull the holding key associated with this equipment. By then pressing the starting key, the selectors at each station step in synchronism to the selected position and stop. A control and an indicating circuit for that equipment are thereby connected to the control and indicating wires, and the dispatcher can perform the desired operation by turning the individual control key and pressing the master operating key. The changed position of the supervised equipment will be indicated immediately by the red and green capped lights. Upon completing the remote operation the holding key is pressed, which upsets the setting of the selectors and returns them to the zero or off position.

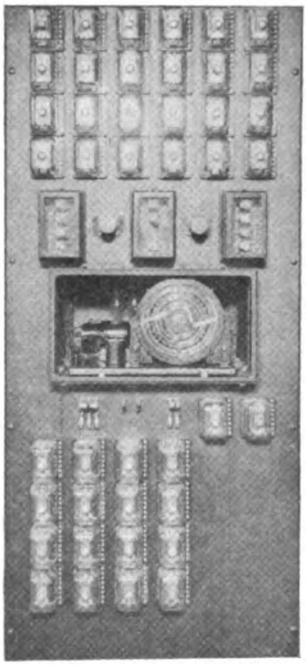

Fig. 85. New Method of Mounting Equipment on Panel for Distributor Type of Supervisory Control

In a similar manner, auxiliary switches and interlocks on the remote equipments set up the selectors to return indications to the dispatcher in case of automatic operations in the remote station.

The distributor type of supervisory system (Fig. 85) was changed from cabinet to panel mounting. All the equipment formerly mounted in the distributor and relay cabinets is now mounted on a 90-in. one-piece compound panel 40 in. wide and 1½ in. thick. The panel in turn is mounted on 4-in. channel supports and the wiring is complete to the terminal blocks. A check-back feature was added which permits the dispatcher to select a station.

OUTDOOR STATIONS

The general application of outdoor equipment to power systems in all parts of the country continued to grow, resulting in improvements in the design of both the mechanical layout and the switching, transforming, and protective apparatus applicable to this service. The increased demand for such equipment was particularly noticeable in the super-power field.

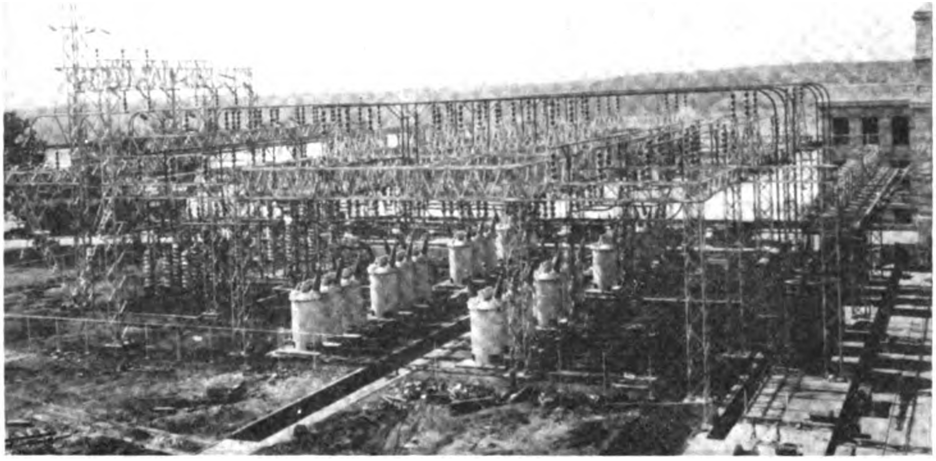

Fig. 86. One of Two Outdoor Stations with a Total Capacity of 260,000 kv-a., Showing Typical Modern Arrangement of Apparatus and Supporting Structure

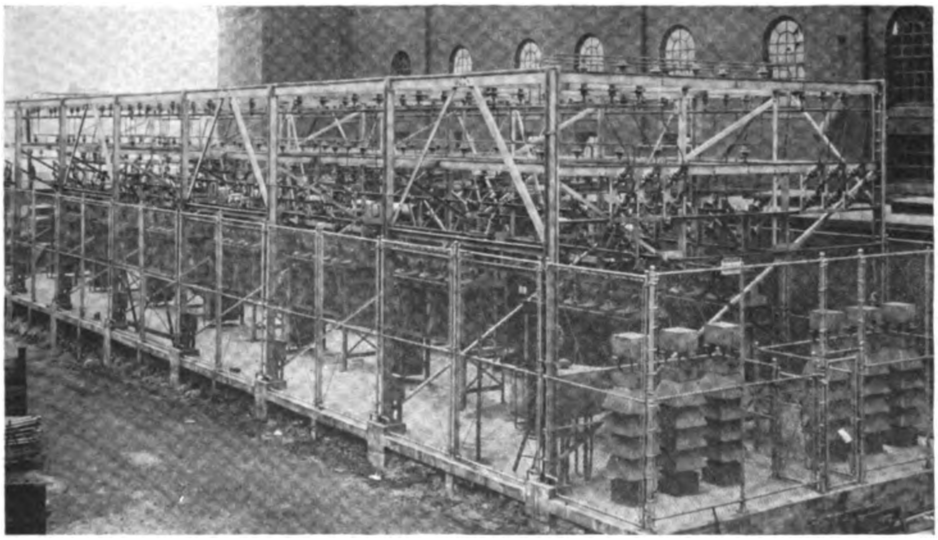

Fig. 87. Complete 20,000-kv-a. 24,000-volt 60-cycle Outdoor Substation

A number of the outdoor substations equipped during the year (Figs. 86 and 87) illustrate the new tendencies by a homogeneous design of the station as a whole. The details of the equipment, including the steel structural work as well as the bus bars and switching apparatus, are now designed as carefully in regard to layout and appearance as the generating station, in an effort to secure maximum operating efficiency combined with low maintenance costs and interchangeability of parts.

A new line of standard outdoor-station equipment for capacities up to 3000 kv-a. and voltages of 15 to 73 kv. was designed. These equipments are especially adapted for the step-down stations of industrial plants and small towns. The standardized structures (Fig. 88) are available in two sizes from which 93 different combinations can be made.

The horn-gap type of switch, utilized for sectionalizing transmission lines (Fig. 89), was improved by omitting the insulator formerly located at the hinge point and providing special clamping devices for attaching the pantograph to the strain clamp used to fasten the line to strain insulators. This results in a short base which is suitable for cross-arm mounting and reduces both the weight of the switch and the space required for mounting it. The ratings of this switch range from 400 to 600 amp. for operation on 15- to 73-kv. lines.

The increased use of group-operated disconnecting and isolating switches for outdoor-station service resulted in the production of a high-speed heavy-duty motor-operating mechanism to replace the manual arrangement previously used.

The salient feature of this mechanism (Fig. 90) is an internal cam which transmits the torque from a worm-gear reduction to the crank. This cam is designed for high torque and relatively low speed at the beginning of the stroke; the speed increasing up to the mid position from which point it again decreases. The blades of the switch are thus put into motion and stopped without shock. Emergency manual operation is provided for by means of a removable wooden operating handle; the motor drive being automatically disconnected when operating by hand.

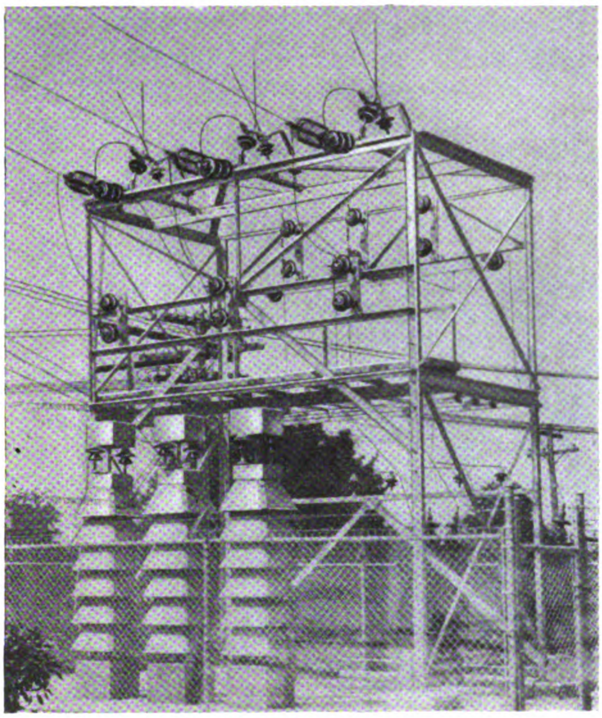

Fig. 88. Standardized Outdoor-station Equipment for 750-kv-a. 33000/2300-volt Service

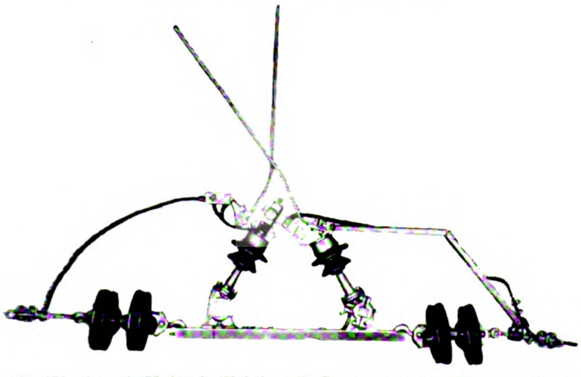

Fig. 89. Single-pole Unit of a Triple-pole Group-operated Horn-gap Switch, Showing Arrangement of High-tension Cables and Strain Insulators

A detail improvement in one form of horn-gap switch (Fig. 91) is the use of torsion springs to balance the weight of the moving blade, thereby facilitating operation. Provision was also made for the lubrication of joints and bearings and a special bearing was provided for the rotating-insulator stack.

Outdoor Switch Houses

Three standard sizes of ground-mounted outdoor switch houses were designed. The smallest size will accommodate the more common types of panel-mounted oil circuit breakers up to and including 6600 volts. The medium size will house similar breakers for 13,200 volts, and the largest size is intended for panel-mounted breakers of the highest interrupting capacity.

Small utility and structural-mounted houses, which embody practically the same general design features, were also perfected.

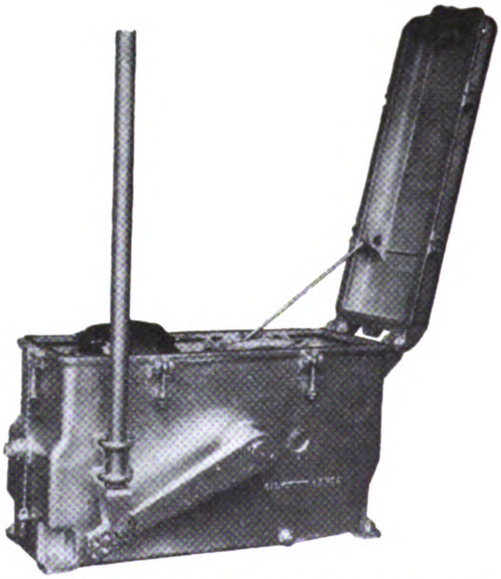

Fig. 90. Motor Mechanism for Operating Disconnecting and Isolating Switches

SWITCHING APPARATUS

The general construction and appearance of the switchboards of today are in keeping with the many recent advancements made in other fields of the electrical industry. As demands were created for larger and more delicately controlled electrical machinery, the switchboard, through necessity, became larger and more complicated, thus making it more difficult to build and especially to install and place in service.

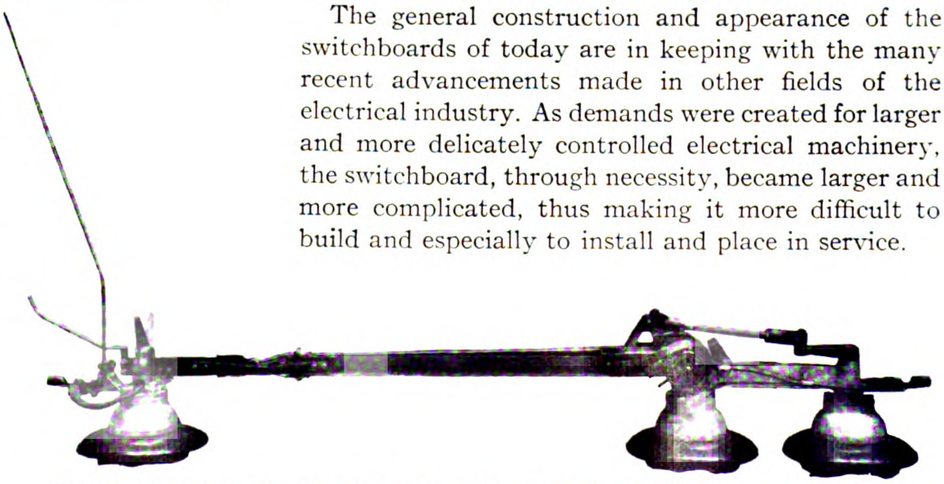

Fig. 91. Horn-gap Switch, Showing Use of Springs to Balance the Weight of the Moving Blade

For some time, engineers have been designing and building many forms of enclosed and self-contained switching equipments to meet present and future demands, and this activity has helped to eliminate many of the installation problems. These equipments need only to be moved into place and a few control and power connections made to be ready for service. Some new and improved designs were brought out for automatic and manual stations both for indoor and outdoor application.

There was an increased use of the unit type or "Factory Built" switchboard (Fig. 92) for manually-controlled stations. The instruments, relays, and oil circuit breakers are mounted and wired on a panel which is supported on a welded, steel and angle-iron frame, the disconnecting switches and bus bars being attached to the superstructure.

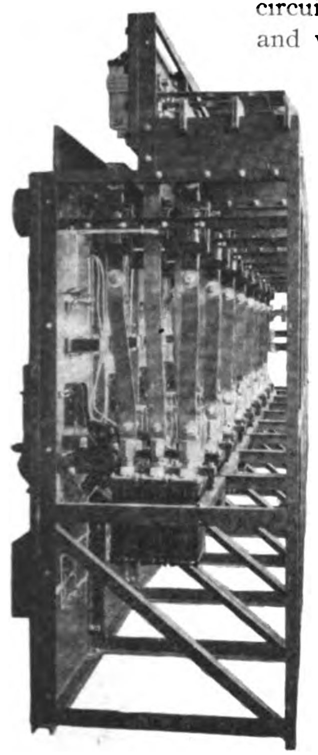

Fig. 92. Typical Factory-built Switchboard, Showing Arrangement of Frame Structure

There were no radical departures in the design of standard safety enclosed-unit removable-truck panel switchboards during the year; the minor changes incorporated tending to simplify the equipment.

Metal-clad Switching Equipment

There was a marked tendency toward the adoption of switching equipments having parts more completely enclosed, even for the largest generating stations, in order to insure maximum protection to operators and maintenance men, the prevention of interruptions in service due to short-circuits between bus bars, connection bars, etc., and at the same time minimize the time and cost of installation. To meet these requirements, metal-clad switching equipment for both indoor and outdoor installation were developed.

In the indoor type (Fig. 93), the oil circuit breaker is carried on a yoke or carriage to insure its position and alignment. It can be raised into its operating position, lowered from it or removed from the frame for inspection or adjustment by means of an elevating truck (Fig. 94), only one truck being required for a number of units.

The movable members of the main disconnecting devices are located on the connection studs of the oil circuit breaker while the stationary members are located in the supporting framework. From the stationary contacts, connections are made to the buses and through current transformers to the outgoing or incoming cables.

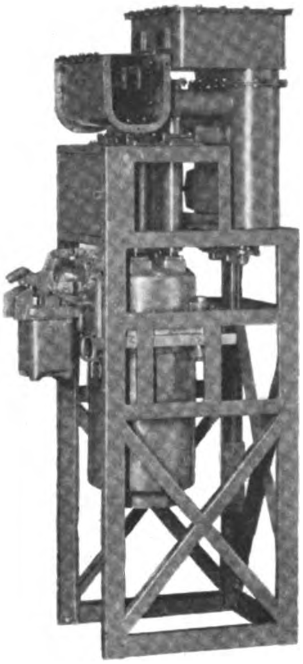

Fig. 93. Metal-clad Switching Equipment in Operating Position

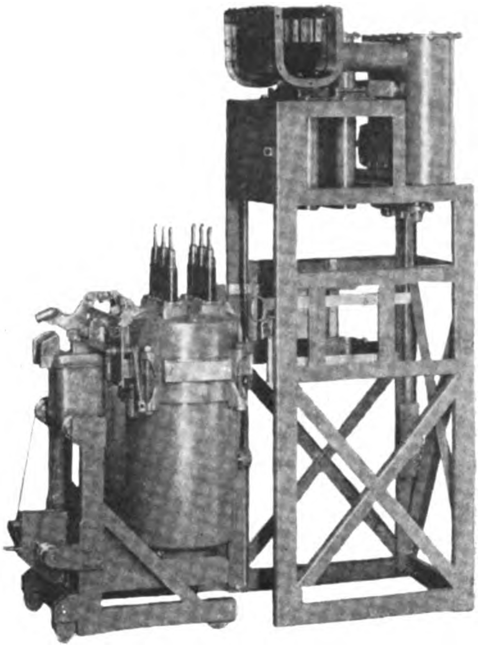

Fig. 94. Metal-clad Switching Equipment, Showing Method of Removing Oil Circuit Breaker

The bus and transformer chambers are so designed that they can be filled with oil or fluid, semi-fluid, or solid insulating compound, and for voltages of 2300 or less may be used unfilled. The secondary wiring is run in rigid conduit and terminal boxes are provided for connections. Incoming or outgoing cables either with or without lead covering may be used.

Positive interlocks are provided so that the breaker cannot be lowered from the operating position until it is open, and it cannot be raised to the operating position unless it is open. In any intermediate position it is locked against operation.

The operating solenoid for the oil circuit breaker is mounted on the circuit breaker itself and is removed with the breaker. This makes it possible to adjust the solenoid and oil circuit breaker with respect to one another. The present designs permit of the units being used on circuits up to 15,000 volts in 600, 800, 1200, 1600, and 2000-amp. capacities.

All buses, connection bars, and main disconnecting devices are insulated for the full-line voltage exclusive of the insulation value of the oil or compound with which the compartments containing them are filled.

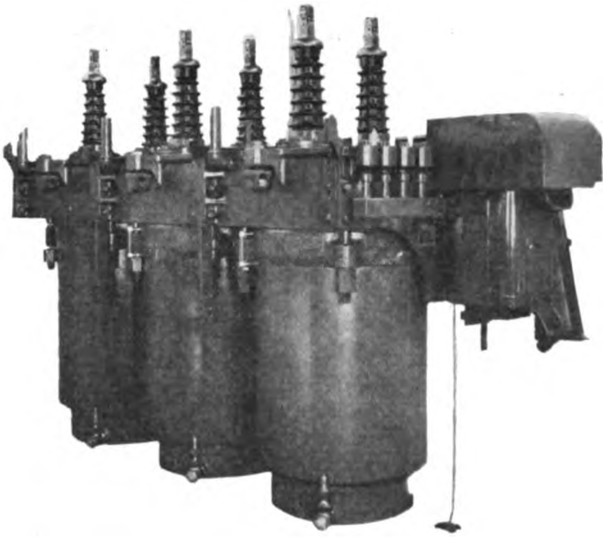

Fig. 95. Oil Circuit Breaker for Metal-clad Switching Equipment Having an Interrupting Capacity of 1,000,000 Kv-a.

One equipment of the outdoor type was designed for a substation which will operate at 33,000 volts, 600 amp. The main and auxiliary bus bars are completely enclosed in oil-filled pipes with connections provided so that either the incoming or outgoing cables can be connected to either bus by the operation of a pair of oil circuit breakers in each circuit. A similar equipment was also designed for a generating station with switching units of 2,000,000-kv-a. interrupting capacity. This equipment required special consideration in the design of enclosing pipes and boxes for the various current-carrying parts which were required to carry 3000 amp. at 22,000 volts continuously.

The oil circuit breakers (Fig. 95) have a rated interrupting capacity of 1,000,000 kv-a., and are so mounted that they can be easily and quickly removed by lowering them from the bus structure by means of a traveling carriage and an elevating platform. When the breaker is removed, shutters automatically close the openings through which the bushings of the breakers normally extend. All operations are mechanically or electrically interlocked.

The enclosing oil-filled pipes for the buses are provided with barriers, thus sectionalizing the equipment. All live parts are insulated for full voltage so that if an oil leak should develop in any section, and even if that leak should not be detected until all of the oil in that section had been lost, the apparatus could be kept in service until the leak was plugged and the section refilled with oil.

Potential transformers and all of the necessary auxiliary devices are provided as part of the complete equipment.

Oil Circuit Breakers

The year was one of exceptional activity in testing and developmental work on circuit-breaker designs. The interrupting capacity tests made—exclusive of insulation and operation tests, heat runs, etc.—numbered more than 3000.

A new interrupting capacity testing station was placed in operation with a specially-designed 100,000-kv-a. generating unit which will furnish short-circuit energy of far greater magnitude than could heretofore be obtained in factory tests. The machine (Fig. 96) will sustain a short-circuit of over 500,000 kv-a.

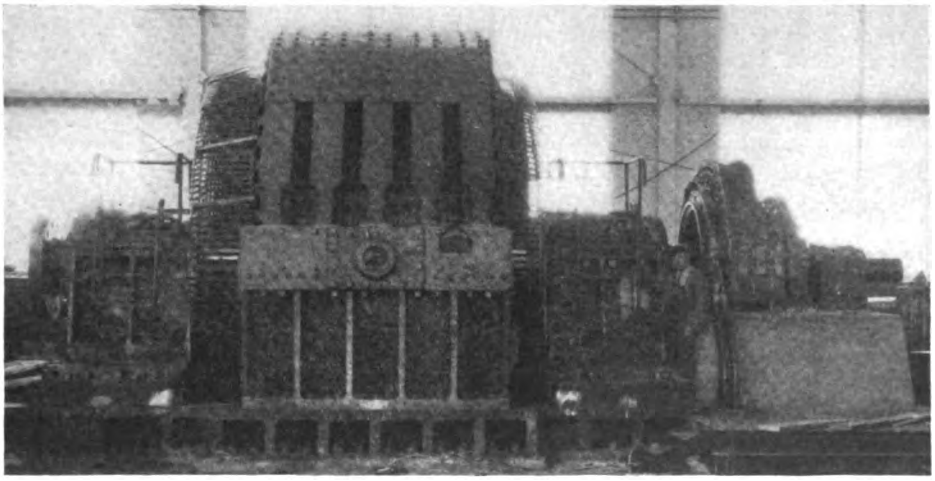

Fig. 96. 100,000-kv-a. Generator Designed Especially for Oil Circuit Breaker Testing

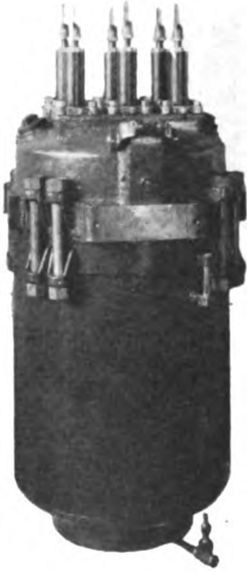

Fig. 97. Indoor-type, Three-pole Single-throw Oil Circuit Breaker Rated 15,000 Volts, 600 Amp.

A new indoor breaker of moderately high interrupting capacity (Fig. 97) includes many constructional principles heretofore found only in breakers of extremely high capacity. It utilizes the short-bolt method of tank suspension, an "internal" mechanism which assures against reclosure of the contacts because of unbalanced pressures, a round tank with domed bottom and domed cover or frame, and a baffle chamber to vent the gases which may be formed.

These features together with conservative ratings give to this breaker the maximum of safety and reliability, yet its construction is such that the space requirement is even less than for some earlier breakers of lower rating. A single, boiler-plate, round tank is used for all three poles of the breaker (Fig. 98); Herkolite linings and fiber phase-barriers are provided in the tanks; short, heavy bolts secure the tank to the top frame; and a gasket insures an oil-tight joint.

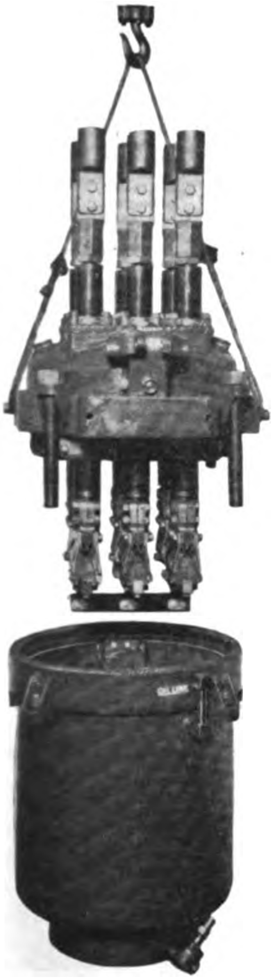

Fig. 98. Three-pole Oil Circuit Breaker Showing Tank Removed

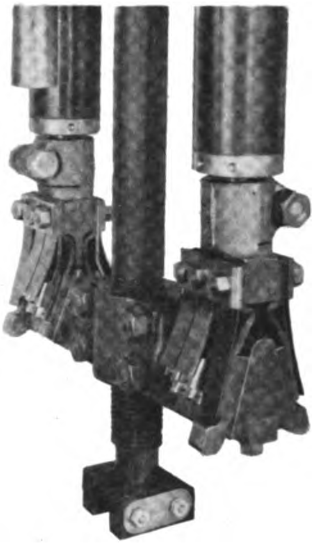

Fig. 99. Self-aligning "Wide-angle" Contacts in Closed Position

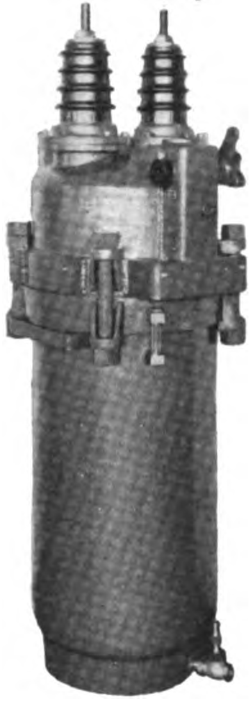

Fig. 100. One-phase of High-capacity Oil Circuit Breaker Rated 15,000 Volts, 600 Amp.

The plain-break contacts employed in this breaker differ in many respects from the finger-and-wedge type thus far employed, in that the main or current-carrying fingers are assembled at a greater angle than is usual with respect to a vertical line through the center of the stud. The fingers of these so-called "wide-angle" contacts (Fig. 99) remain at practically the same angle irrespective of the position of the movable wedge-shaped blade, thus assuring the maximum contact at all times, since the contact surfaces move only through successive parallel planes. Sufficient wiping motion is provided to make these contacts self-cleaning and they are self-aligning.

The extended arcing fingers and the detachable blade tips are provided with masses of metal to take the brunt of the burning. Connections to the stationary contacts are made through studs of solid Herkolite bushings which are individually removable from the frame.

Breakers of this type were being developed in 16, 20, 24, and 28-in. tanks and all are rated 7500 and 15,000 volts except the breaker in the 16-in. tank, which is limited to 7500 volts. The interrupting capacities range from 90,000 to 250,000 kv-a.

A new high-capacity breaker (Fig. 100), utilizing heavy round tanks, with a separate tank for each phase, supersedes a somewhat similar type and includes a number of improvements. It employs boiler-plate top frames and thus obtains a lighter but stronger structure: it is available in both the plain-break and explosion-chamber-break varieties, and utilizes the short-bolt principle of tank suspension, the internal mechanism, and other accepted forms of construction which make possible the attainment of high interrupting capacity. The plain-break breakers of this type are equipped with the new wide-angle finger and wedge contacts.

In the explosion-chamber type, the explosion chambers are threaded to screw onto their adapters instead of being bolted on. This construction provides greater strength, decreases the space requirement, and facilitates removal of the chamber for inspection of the stationary arcing contacts. Where the rated interrupting capacities warrant, the contact retainers and adapters are vented to allow some of the gas generated to escape upward, through the oil, away from the live contacts. Built for rated voltages of 15,000, 25,000, and 37,000 volts and with normal current ratings of 600 to 4000 amp. these breakers use tanks which range in size from 16 to 36 in. diameter by increments of four inches. The interrupting capacities range from 350,000 to 2,000,000 kv-a., the latter value being obtained by use of the 36-in. tank which was not previously available.

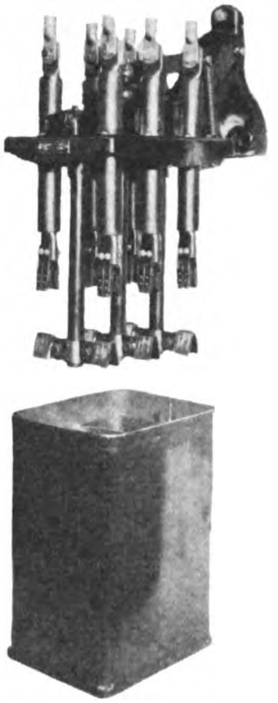

Fig. 101. Improved Triple-pole Single-throw Oil Circuit Breaker Rated 4500 Volts, 200 Amp.

Another new indoor oil circuit breaker (Fig. 101), for application where the interrupting capacity requirement is relatively small, has normal ratings of 200 amp. at 4500 volts and 400 amp. at 2500 volts, and has interrupting capacities of approximately 2100 amp. at the higher or 4250 amp. at the lower of those voltages.

Pressed-steel construction is employed for the frame. The stud is a copper tube, cut to the proper length, over which a Herkolite insulating tube is slipped. The projecting ends of the tube are then pressed flat and drilled, thus providing the necessary terminals for the stationary finger contacts and for the external connections. A supporting ring is then clamped tightly around the Herkolite tube, and the bushing is complete. Thus not only is the construction simplified but the current efficiency is increased because of the elimination of unnecessary joints.

Breakers of this type are adapted for mounting on a panel, five inches back of a panel, or on a remote framework. They are designed for manual operation but can be operated electrically.

Supplementing an existing type of breakers designed for use in manhole or subway installations (Fig. 102), new breakers of an unusual type were developed, rated 15,000 volts, 400 amp. They have an interrupting capacity that is relatively high for this class of service and are mounted on framework of a height sufficient to permit removal of the tanks, thus greatly facilitating inspection or repair of the contacts. All joints are water-tight, allowing these breakers to remain in operation even though totally submerged.

For outdoor installations where the interrupting duty is severe, an entirely new line of high-voltage high-interrupting-capacity breakers was designed. These breakers are the culmination of a very extensive testing program including field tests as well as tests with the high-interrupting-capacity testing station previously mentioned (page 48). These breakers (Fig. 103) make use of the new boiler-plate tank covers, eliminating the heavy cover castings formerly employed and presenting a more attractive appearance. The cover is dome shaped as is also the tank bottom, thus affording maximum strength with minimum possible distortion. The short-bolt principle of tank suspension is employed, decreasing the weight of the supporting members but at the same time materially increasing their strength and effectiveness in holding each tank securely against its supporting cover or frame. The mechanisms which operate the contacts of each unit have also been improved.

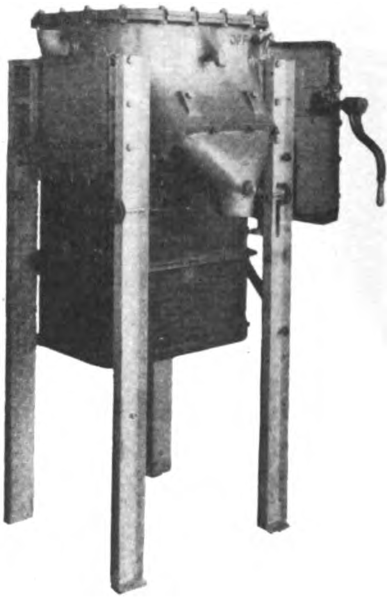

Fig. 102. Oil Circuit Breaker Designed for Use in Manhole or Subway Installations, Rated 15,000 Volts, 400 Amp.

They are made with both plain-break and explosion-chamber-break contacts. The plain-break breakers are built for voltages from 15,000 to 154,000 volts, inclusive; the corresponding interrupting capacities ranging from 350,000 to 750,000 kv-a. Explosion-chamber breakers are available for all standard voltages from 15,000 to 220,000 volts, inclusive; the corresponding interrupting capacities ranging from 750,000 to 2,500,000 kv-a. These ratings approximately parallel those of the earlier breakers. In the breakers equipped with explosion chambers, the explosion-chamber insulation has been materially improved and the outside overall diameter decreased.

These breakers can be mounted on structural steel framework for voltages up to and including 88,000 volts and floor-mounted for higher voltages. Convenience of inspection and facilities for operation and maintenance are provided for by oil drain and fill valves, multiple-ratio bushing current transformers, and manholes for convenience in inspecting the interior parts of the larger units. To these breakers as well as to other outdoor oil circuit breakers, oil-filled bushings have been added for 50,000 and for 73,000-volt service, replacing the earlier solid type bushings.

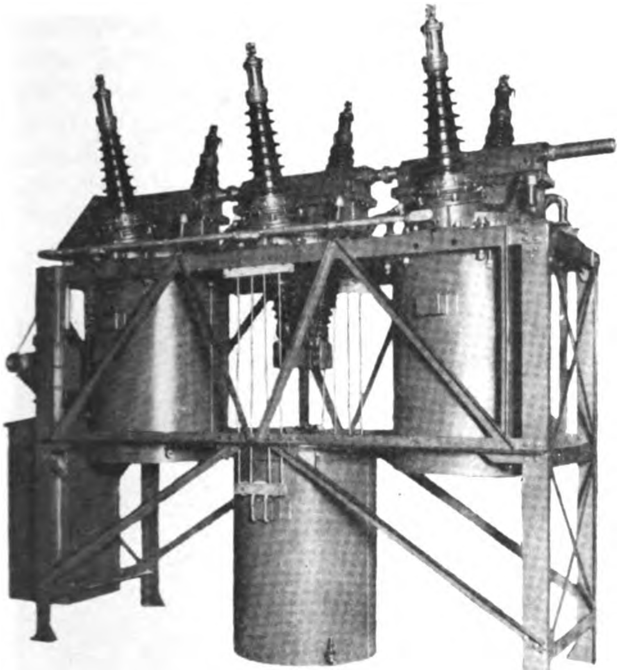

Fig. 103. 73,000-volt, 600-amp. Triple-pole single-throw Motor-operated Oil Circuit Breaker Showing Method of Mounting on Framework and How a Tank Can be Lowered for Inspection

The operating mechanisms and miscellaneous related devices which were produced included a new motor mechanism, a new trip-free solenoid mechanism, a greatly improved motor-operated centrifugal mechanism, and several new time-delay undervoltage devices.

The motor-operated mechanism (Fig. 104) is spring-driven, is trip-free, and is designed primarily for use in isolated-phase installations; being especially advantageous when automatic operation of disconnecting switches is desired. It differs principally from a somewhat similar mechanism which it supersedes in the method of mounting and in the construction and general arrangement of its various parts. It utilizes a motor to charge two sets of springs which, when released by tripping mechanisms, close and open the breaker successively. The mechanism connecting the springs to the oil circuit breaker operating pipes is non-reversible, *i.e.*, it will not allow any magnetic or other forces arising in the oil circuit breaker units to cause rebound of the mechanism from the closed position. The disconnecting switches may be operated automatically or manually, at the discretion of the operator, and emergency manual operation of the entire mechanism is possible. This mechanism may be mounted vertically, thus making it possible to mount it on the back wall of any unit cell in an isolated-phase arrangement, eliminating the necessity for a separate mechanism floor in the station. The fractional horsepower motor will operate from a 125-volt or a 250-volt direct-current source.

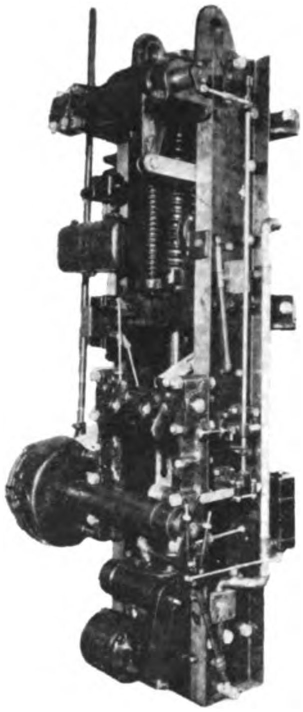

Fig. 104. Motor-operated Mechanism Designed for Use in Isolated-phase Installations

The trip-free solenoid mechanisms are built in two sizes, differing greatly in closing power and in appearance. Both are mechanically trip-free at any point during the closing stroke and are electrically trip-free at any time when an abnormal electrical condition occurs on the line. The oil circuit breakers can therefore be tripped, if necessary, even while closing power is being applied to the solenoid.

The smaller mechanism (Fig. 105) is used principally for the operation of indoor oil circuit breakers of limited size, such as the switchboard and industrial types. In addition to its trip-free mechanism, a self-contained control contactor is mounted with the auxiliary switches in a housing which has provision for conduit connections. Not only is adjustment of the auxiliary switches provided, but the use of a double shaft makes possible an independent adjustment of the circuit-closing and circuit-opening switches. Provision is also made for emergency manual operation.

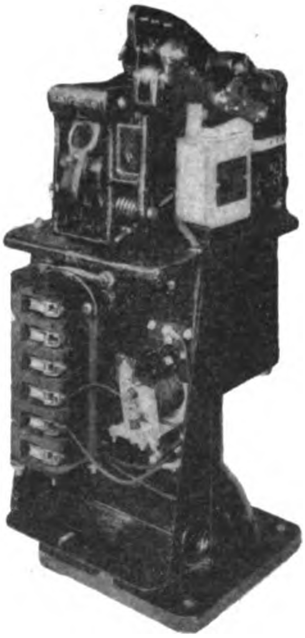

Fig. 105. Trip-free Solenoid Mechanism for the Operation of Indoor Oil Circuit Breakers of Limited Size

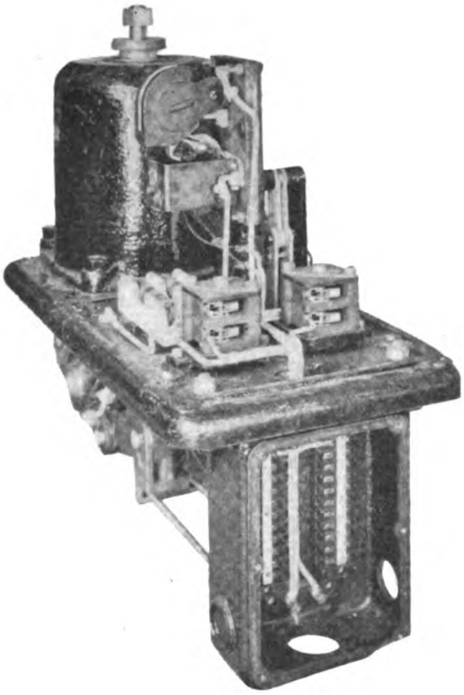

Fig. 106. Trip-free Solenoid Mechanism for the Operation of Large Outdoor Oil Circuit Breakers

The larger mechanism (Fig. 106) has a six-inch plunger which permits the attainment of a closing force sufficient to operate most of the larger breakers, and it is self-contained within a weatherproof housing. Space is provided in the solenoid housing for the self-contained control contactor, the auxiliary switches, and for additional relays if required. All of the operating links, pins, and bearings are made of non-corrodible material, and independent adjustment of the circuit-closing and circuit-opening switches is provided. All control leads are brought to a terminal box mounted at the front of the mechanism and provided with conduit entrances. Easily removable cover plates facilitate inspection of the mechanism and control devices. The mechanism can also be manually operated.

The motor-operated centrifugal mechanism which was improved is used for the operation of the largest oil circuit breakers. To this mechanism a new type of latch was applied, especially designed to give positive reliable operation and at the same time be sensitive to trip. A new self-contained control device (Fig. 107) was also added, replacing both the control contactor and the special auxiliary switch formerly used to control the motor current.

This control device is closed by completing the coil circuit of an electromagnet through the control switch for the mechanism. The contacts then complete the motor circuit and are held closed by a holding coil, thus insuring positive operation of the mechanism even though the control switch may be released before the completion of the closing stroke. Upon completion of the closing operation, the contacts of the control device are forced apart, mechanically, thus preventing any tendency to stick in the closed position. Adjustment is provided for regulating the time of opening of the contacts.

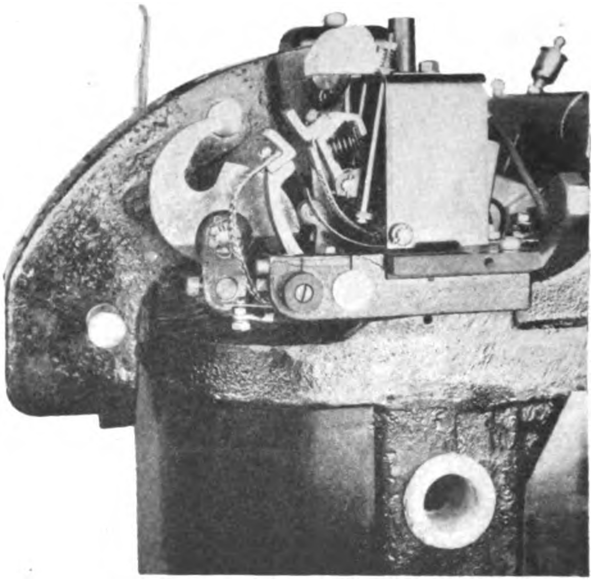

Fig. 107. Motor-control Device for Centrifugal Motor Mechanism with Cover and Arc Chute Removed

A new manual closing device was designed for use with this mechanism when applied to the larger oil circuit breakers, where manual operation by means of a bar inserted in the counterweight ceases to be practical. This device will allow side operation if desired.

The two new undervoltage devices are both designed to give delayed tripping operation upon occurrence of undervoltage, thus preventing the oil circuit breakers from tripping out on momentary fluctuations of voltage but yet affording full protection against serious conditions of undervoltage or loss of voltage. They are respectively applicable to centrifugal motor-operated mechanisms and to manual operating levers.

Those applicable to the motor mechanisms consist essentially of a small motor, a helical spring, and a train of gears. When the device is energized, the motor (Fig. 108) operates through the gear train to store energy in the helical spring (Fig. 109). When the predetermined limiting position is attained, the motor stalls and the spring torque balances the motor torque until such time as the line voltage may fall to approximately 50 per cent of normal, or less. Should such a condition occur, the spring would overcome whatever torque the motor might have at the reduced voltage and would consequently operate the gear train in the reverse direction, finally tripping the latch of the motor mechanism by means of a trip lever. The time delay thus obtained may be varied through a considerable range by adjusting the limiting position of the spring. Should the line voltage be re-established to a value of 80 per cent of normal or more, the motor would again serve to wind the spring, storing energy and completely resetting the device for another operation.

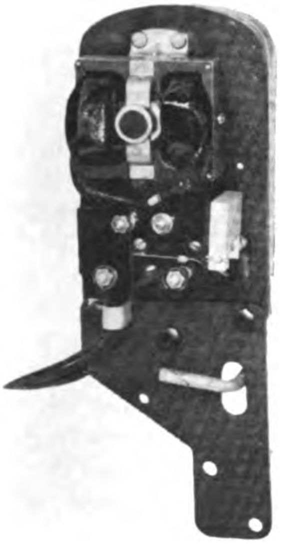

Fig. 108. Undervoltage Device with Cover Removed, Showing Arrangement of Motor

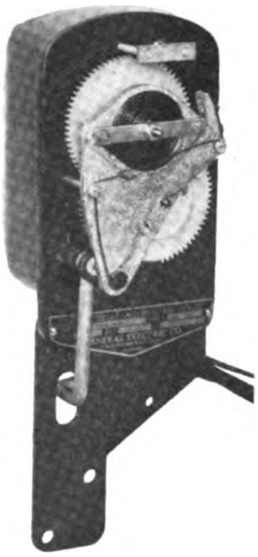

Fig. 109. Undervoltage Device with Cover Removed, Showing Arrangement of Helical Spring and Gearing

The device for the manual operating levers obtains its time delay from an air bellows (Fig. 110) which retards the action of a coil-operated plunger and a tripping mechanism.

So long as the voltage remains at its normal value, the coil is energized and the plunger is held up. Should the voltage fall to approximately 40 per cent of normal the plunger will fall, but it will be retarded in its downward motion by the air bellows. It will finally operate through the tripping mechanism to trip the latch of the manual-operating lever with which it is used. Time can be adjusted by the usual needle valve arrangement.

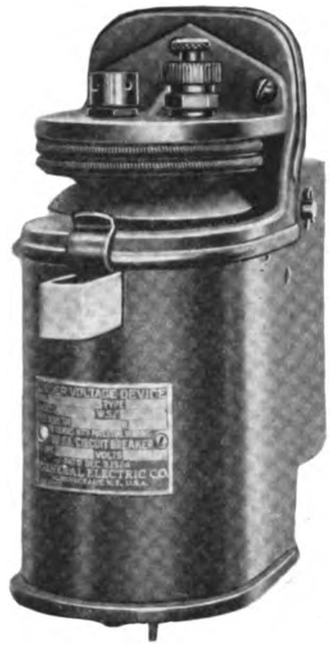

Fig. 110. Undervoltage Device for Use in Connection with Manual-operating Levers

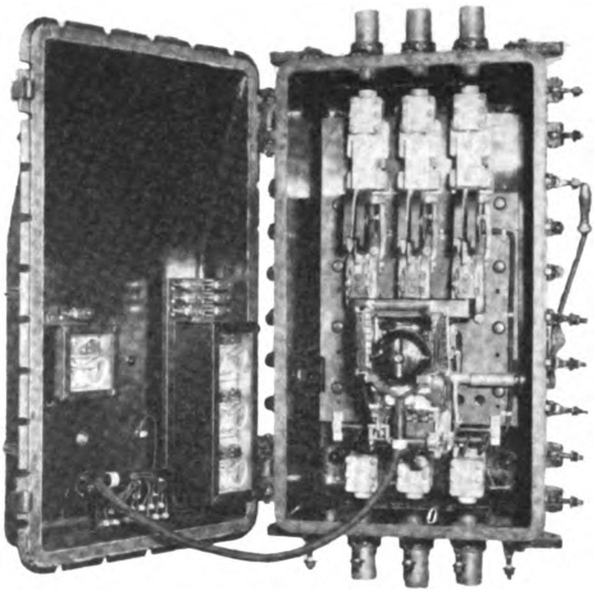

Fig. 111. Submersible Type Triple-pole Network Protector

Network Protectors

Triple-pole network protectors for submersible and non-submersible service were developed (Fig. 111), in which a motor-operated centrifugal mechanism closes the protector when conditions are normal as determined by the operation of a phasing relay and polyphase reverse relay now used as standard equipment. These relays cause the opening of the protector to prevent power flow from the network toward the feeder.

The magnetic blowout circuit breaker has main contacts of improved design, the movable element being a solid piece of copper in place of the laminated copper brush formerly used.

All the faces of the main contacts have silver plates $\frac{3}{32}$ in. thick where contact is made between stationary and movable elements. This construction allows greater currents to be carried without damage to the protector as compared with laminated-type contacts. An improved form of motor cut-off switch prevents "pumping" of the breaker. Provision is made for the use of fuses where conditions require them, and terminals are provided for rope-core cables.

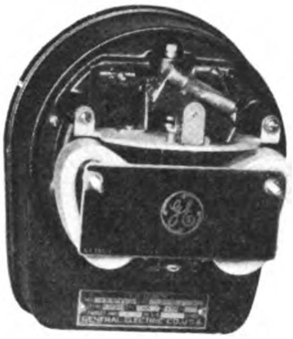

Fig. 112. Time-selector Relay for the Remote Control of Street Lights

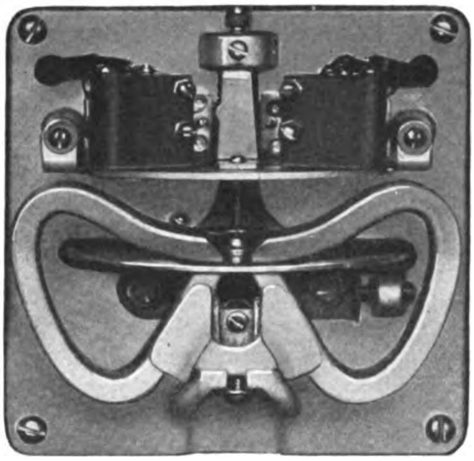

Fig. 113. Temperature Relay for the Protection of Machines from Overheated Windings

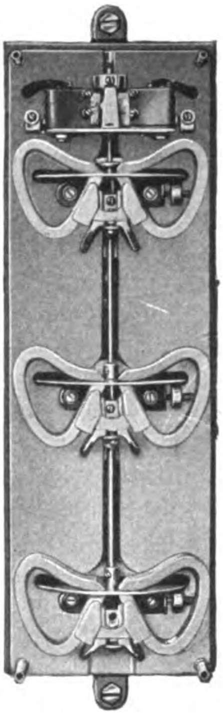

Fig. 114. Three-phase Relay for Network Protection

Relays

A time-selector relay for use in the remote control of street lights by carrier current has a U-magnet, energized from the 110-volt mains, which drives a disk that is damped by a drag magnet. The disk shaft is geared to a mechanism (Fig. 112) which tilts a mercury-tube connector to close and open the lighting circuit.

The mechanism is arranged so that when the disk has completed half its full travel (*i.e.*, after five seconds) the mercury tube will be tilted to the "on" position where it will remain until tilted back to the "off" position when the disk has completed its travel (*i.e.*, after ten seconds). If the signal ends shortly after five seconds, the disk returns to its initial position leaving the tube in the "on" position. The disk is then ready to measure out a ten-second signal to turn the lights off.

The tube is always left in the "on" position following a five-second signal and in the "off" position following a ten-second signal no matter which position it was in before the signal was sent.

An induction-type relay was produced which will close its contacts and energize an auxiliary device if the power-factor of the protected circuit goes beyond a predetermined angle of lag or lead. This relay is of the same construction as a watthour meter except that the recording mechanism has been replaced by contacts.

An induction relay was designed for the protection of machines in automatic substations from over-heated windings. It is of watthour-meter construction (Fig. 113) and is connected in a "Wheatstone Bridge" circuit, where its current coil takes the position of the galvanometer. Two resistance-temperature detectors embedded in the machine winding form two opposite arms of the bridge, while the other two arms are resistors of a value to balance the bridge slightly below the desired operating temperature. The potential coil is connected across the control voltage.

At temperatures below the balance point of the bridge, current flows through the current winding in one direction, holding the contacts open; but as the temperature rises above this point the current, and therefore the torque, reverses its direction and closes the contacts. This principle was made use of to obtain a sensitive and accurate device.

Conditions on certain low-voltage alternating-current networks are such that reliable operation cannot be obtained from relays protecting each phase separately. A polyphase network relay is now available which will disconnect the supply transformer from the network when the net flow of power is from the network into the transformer and will reconnect the transformer when its voltage is of a value which will insure that it take up its share of the load without disturbance to the system.

The new relay consists of three watthour-type elements whose disks are mounted on a single shaft (Fig. 114) and operate a single set of double-throw contacts. Instead of connection studs, the new relay has plugs which fit into spring receptacles on the panel. The relays are held to the panel by two jack screws which free the relay from the panel when turned back and, at the same time, force the plugs out of the socket; similarly turning the jack screws into the panel forces the plugs into the receptacles. This arrangement enables defective relays to be quickly and easily replaced, so that testing and repairing can be done outside the man-hole.

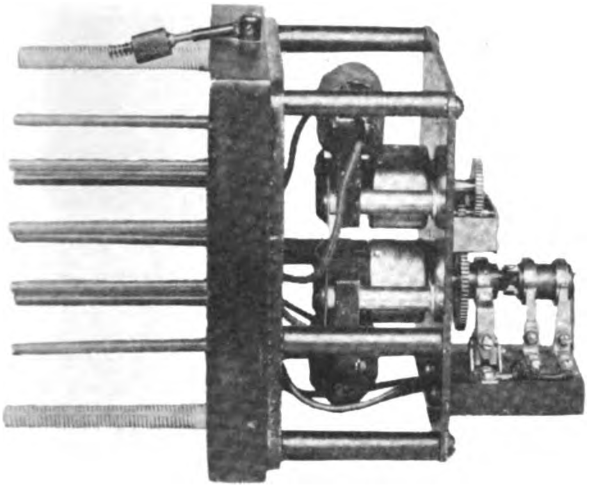

Fig. 115. Speed-control Relay for Synchronizing Turbine-driven Generators

In addition to the polyphase relay, a phasing relay is used to prevent the switch member from reclosing if the incoming transformer voltage should lag the network voltage (due to a faulty regulator or too heavy current fed through the network). Such a condition would tend to cause pumping because of the circulating current set up. This relay is similar to one element of the polyphase relay but has closing contacts and no tripping contacts.

A speed-control relay, which is now used to facilitate the synchronizing of turbine-driven generators (Fig. 115), has double-throw contacts operated by Warren synchronous motors through a differential gear in such a way that if one motor is faster than the other, one contact is closed, and, if slower, the other contact is closed. One motor is connected to the bus and the other to the machine being brought up to speed. The contacts control the governor and increase or decrease the machine speed until it corresponds practically to the bus frequency, at which point the machine may be synchronized either by another relay or by the operator.

Many improvements were incorporated in some of the plunger-type relays. The target is larger and it is reset by a push button in front of the relay cover instead of a rotating knob. The contacts are now silver to silver, which gives them a greater interrupting capacity than silver to brass contacts.

Relays with contacts for opening or closing more than one circuit (Fig. 116) are equipped with the universal type of contact head with which a great range of contact settings is possible. The contacts will close one, two, or three circuits and open one, two, or three others. By using a split-contact bar, two electrically separate circuits can be obtained instead of three circuits, with a common contact. The contacts may be either hand or electrically reset.

Relays with instantaneous pick-up and time drop-out have a new quick-exhaust valve which gives an especially rapid pick-up, and an inverted toggle which gives the contacts a snap action during time drop-out.

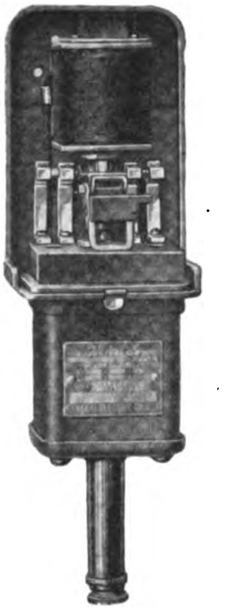

Fig. 116. Instantaneous Sensitive Overcurrent Relay with Electrically Reset Contacts for Opening Three Circuits

Selsyn Devices

A novel application of Selsyn motors to a compound vector indicator shows the combined effect of two induction-type voltage regulators connected in series in the same line. This type of device is also applicable to many other problems where the combined effect of two variables is required.

The Selsyn motors in the indicator (Fig. 117) are each connected to a Selsyn generator driven by the operating mechanism of a voltage regulator. At the indicator, one Selsyn motor is directly connected to the arm which carries the planetary gearing and one of the pointers. The other Selsyn motor is connected, through gearing, to the pointer which rotates about the end of the arm. The drive to this pointer is through a gear train, one gear of which floats on the shaft of the upper Selsyn motor. By means of a polar co-ordinate scale, etched on glass (Fig. 118), the resulting values are read directly.

A combined Selsyn water-level indicator and recorder was designed to give a graphic record of variations in water level at a distant point. As the water level varies, a float rotates the Selsyn transmitter and the results are indicated on the dial (Fig. 119) and recorded through gearing on a rotating chart which may be operated by a spring clockwork or a Warren motor.

Recent developments in Selsyn devices have tended to bring them in line with switchboard and other types of meters and instruments in regard to both appearance and compactness of arrangement. Typical of the advances made along these lines is an indicator (Fig. 120) in which the entire operating mechanism is installed in a horizontal, edgewise, instrument case.

METERS

A new single-phase watthour meter (Fig. 121) is one of the outstanding achievements of the year in the meter field. Owing to the marked increase in the use of electricity for domestic purposes, loads on house meters have been becoming greater, bringing with them the need of a wider working range over which the accuracy of the meter will remain unaffected. This problem has been successfully met in the new meter.

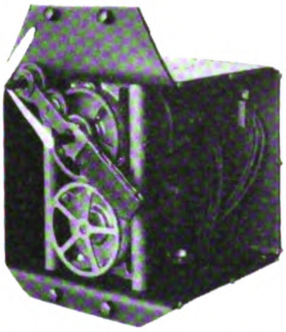
Fig. 117. Motor Elements and Gearing for Compound-vector Indicator

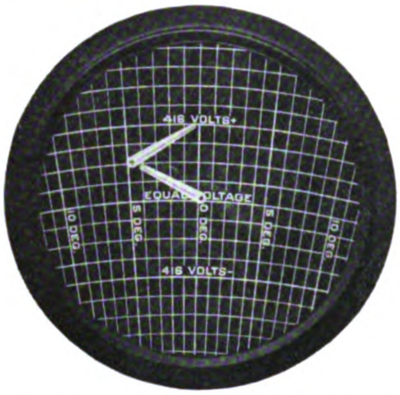

Fig. 118. Co-ordinate Scale and Pointers of Compound-vector Indicator

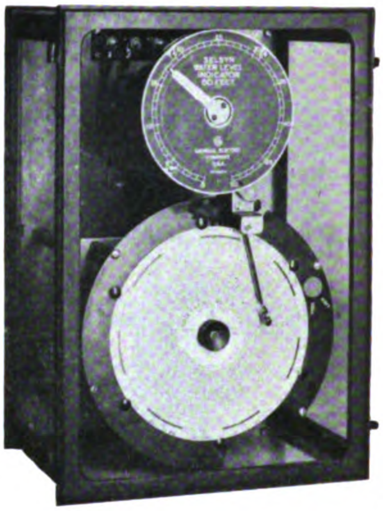
Fig. 119. Remote Water-level Indicator and Recorder

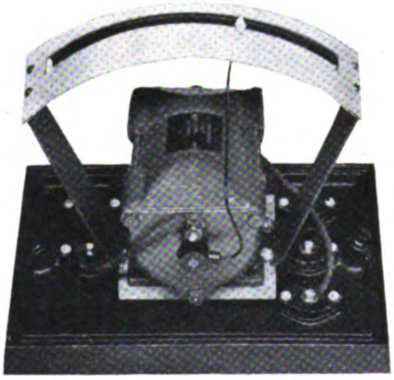
Fig. 120. Selsyn Indicator Designed for Enclosure in a Horizontal Edgewise Instrument Case

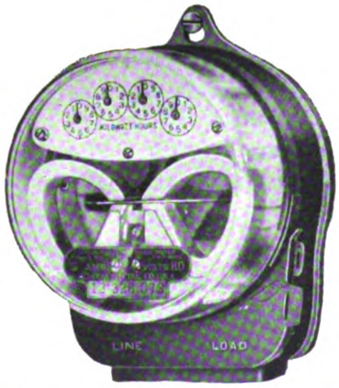

Fig. 121. Improved House-type Single-phase Watthour Meter

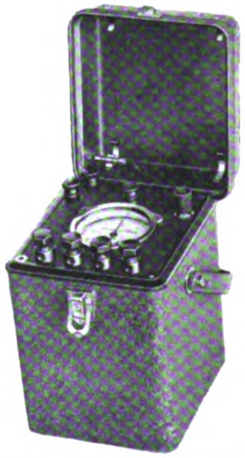
Fig. 122. Improved Type of Portable Induction Test Meter, Capacity 1/5/10 Amp., 110 Volts, 60 Cycles

The inherent characteristic of all induction watthour meters is to tend to run slow on overloads. It has been possible to so reduce this tendency that on loads of 200 per cent the new meter experiences scarcely any error and at loads as great as 400 per cent of normal it is only 2.7 per cent slow. Moreover, the new five-ampere meter will safely carry loads as great as 40 amp. for short periods without detrimental heating.

Improved overload performance is only one of the results of this work and this has been obtained without sacrificing any of the other electrical or mechanical features found in the preceding design such as torque, weight of moving element, influence of varying voltage, temperature, power-factor, frequency, etc. In fact, some of these characteristics were actually improved.

An important factor in securing these results is the improved permanent magnet. The full-load speed of the disk has been reduced to one-half the previous value, requiring twice the damping effect. This is obtained by a more efficient use of the damping flux, making unnecessary a proportionate increase in the size of the permanent magnet.

A new line of portable test meters was also developed (Fig. 122) in which temperature errors have been practically eliminated, giving them load characteristics closely approaching a straight line throughout their working range. They are inherently free from the influence of variation in voltage, frequency, and wave form to a degree which has not been achieved in earlier designs.

It is frequently necessary in electrical metering to obtain the total simultaneous demand on several individually metered circuits. In order to make this possible, an instrument called a totalizing relay was produced, which receives demand impulses from contacts on the various watt-hour meters, totalizes these impulses, and actuates a single demand meter.

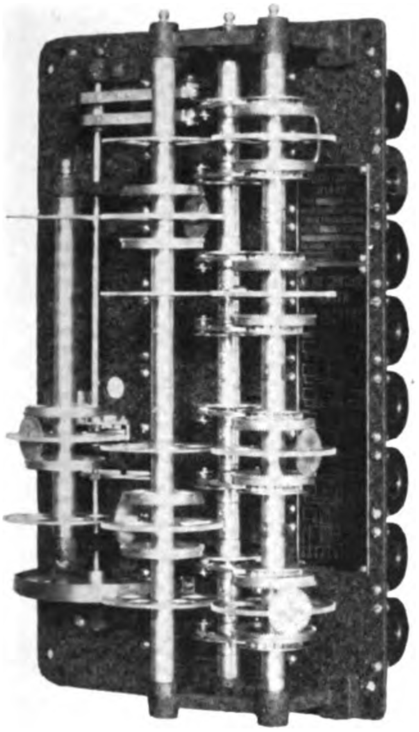

Fig. 123. Eight-element Totalizing Relay, Showing Arrangement of Differential Gearing

The relay can also be used to obtain the total kilowatt-hour consumption as it contains an indicating counter which shows the total number of impulses or energy units. This counter also makes it possible to check the accuracy of the relay and its associated contact devices.

The totalizing relay is operated by solenoids, one pair being used for each circuit totalized. The solenoids operate pawls and ratchets, and the individual impulses are totalized (Fig. 123) through the use of differential gearing.

LIGHTING

A preliminary estimate of the sales of incandescent lamps in the United States during 1927 indicates 320,000,000 large and 218,000,000 miniature lamps. The former is an increase of about 8,000,000 (about 2½ per cent) and the latter about 16,000,000 (about 8 per cent) over the previous year. These sales are the largest in the history of the industry. The former is 75 per cent greater and the latter triple the sales (Fig. 124) of ten years ago.

The industrial lighting campaign of the National Electric Light Association, referred to a year ago, proved so successful in spreading improved lighting practice that a supplementary effort was carried on under the same auspices during the early months of 1927. A number of electric service companies are continuing as a more or less permanent feature the educational and promotional efforts organized for these campaigns.

Ten-kilowatt Mazda lamps were used in increasing quantities, principally for aviation field lighting, although some were also used in motion-picture studios. Great improvements in these lamps have come with increased production, and the indications are that they are now a thoroughly reliable product.

The 5-kw. size was applied to an increasing extent to motion-picture studio lighting and the 2½-kw. size also found considerable favor in the same field, particularly where general illumination relatively free from shadow was desired. This was obtained by installing a large number of 2½-kw. lamps instead of a few of the 10-kw. units.

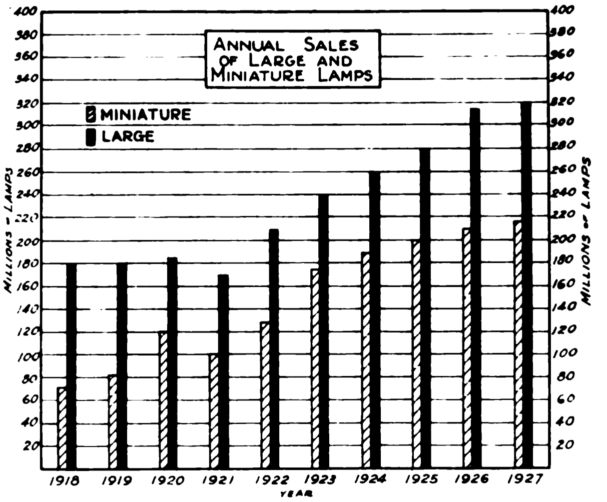

Fig. 124. Number of Incandescent Lamps Sold in the United States 1918–1927 (1927 estimated)

In the past, it has been found difficult to prevent the bases of high-wattage lamps becoming loose, as the temperature at the base is so high that the basing cement deteriorates during the life of the lamp. To prevent this action and insure a permanent secure base, a mogul clamp base was produced which is mechanically fastened to the lamp and not affected by temperature.

A new lamp base, known as the intermediate screw base, was standardized for multiple Christmas-tree lamps and also for small lamps used for sign and decorative use. It was developed because the medium screw base was too bulky for this type of lamp and the candelabra screw base was too small.

The smallest standard lamp for multiple service in the past was the 10-watt sign lamp. A standard 10-watt lamp of smaller size was designed, which utilizes the new intermediate base and is intended to be used principally for sign and ornamental lighting.

A new 36-watt inside-frosted street-railway lamp was standardized. It was especially designed to withstand the severe service incident to street railway operation.

In the ordinary 100-watt Mazda projection lamp, the coiling of the wire reduces its effective length to about 15 per cent of the length of the straight wire. A further improvement reduced the size of the light source by taking the coiled filament and recoiling it. In lamps made with the recoiled coil filaments (Fig. 125), the over-all length of the wire is but 3 per cent of the length of the straight wire, the area of the light source is less than one-half of that occupied by the coiled filament, and a material gain in screen illumination is secured.

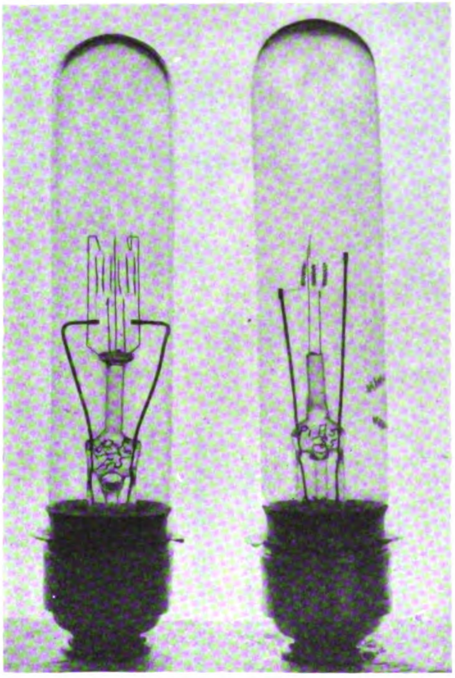

Fig. 125. 100-kw. Projection Lamps, Showing Comparison of Coiled Filament and Recoiled Coiled Filament

The difficulty of aligning Edison-base projection lamps accurately in optical equipment was eliminated by the introduction of a new type of base and socket. The base consists of two parts: an inner shell which is cemented to the lamp bulb in the usual manner; and a flanged cylindrical outer shell which provides freedom for tilting and moving the inner shell to align the filament to a definite position. As a factory operation, the filament is located optically in jigs and the two shells are then fastened together.

The base and socket can be assembled in only one position which insures the accurate replacement of lamps in projection equipment. While these bases have found numerous applications, the largest demand is for use with portable motion-picture machines.

A new type quartz-neon gas-filled lamp was produced which resembles a ball of reddish-orange fire when in use and emits practically all its visible radiation in the long wavelengths, which tests have shown to have great fog-penetrating possibilities.

Another feature of the new lamp is that it operates on low voltage, practically the same as for the lamps used in household illumination, whereas previous types of neon lamps such as those used in signs require a very high potential, often as much as 15,000 volts.

The low voltage is made possible by use of a hot cathode which supplies sufficient electron emission in the lamp to supply the current for the luminous discharge and also tends to lengthen the life of the lamp as compared with previous types. There is also less tendency for the neon gas to disappear and blacken the lamp when low voltages are used.

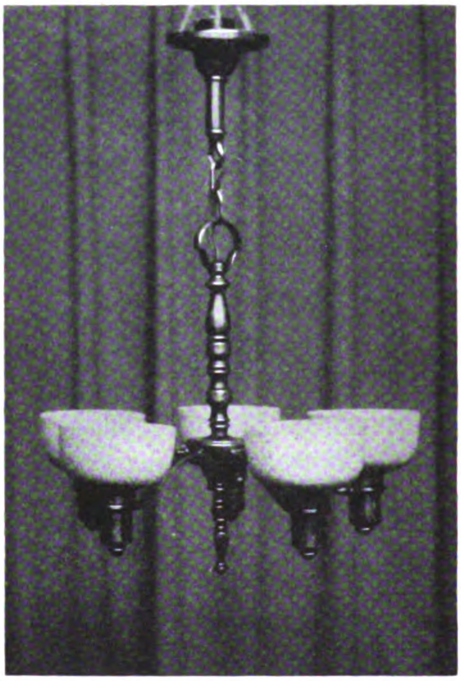

Fig. 126. New Design of Semi-indirect Home Lighting Fixture

The lamp consists of a small quartz tube wound in a close spiral 1½-in. in diameter. This is done to produce a greater concentrated brilliancy so that, when the lamp is placed in one of the standard airport beacons or searchlight projectors, it will produce a narrower and more penetrating beam of light than if the light source were larger.

Quartz was used for the lamp because of the heat generated in its operation. The temperature is about 1200 deg. F., which is above the fusion point of glass whereas quartz will safely withstand temperatures up to 2000 deg.

Home Lighting

In the field of home lighting the falling off in the number of new wiring installations led to an analysis which indicates that the electrical industry has nearly completed the undertaking of putting wiring into homes where electric service is available. It is estimated that 16,000,000 houses in the United States are now wired for electric light, and that at the present time there are only about 2,000,000 unwired houses within reach of electric power lines.

The realization of this condition is having a marked effect upon the commercial elements of the industry and inducing them to give heed to the advice of the illuminating engineering group which has been pointing out the inadequacy of present wiring and luminaire installations to meet the needs and desires of American families.

The coöperating committees of the Association of Edison Illuminating Companies and the Illuminating Engineering Society prepared preliminary specifications for improved equipment, covering illuminating performance, electrical and mechanical construction, and appearance both lighted and unlighted.

An interesting feature of these specifications is the glare measurement, which is made in terms of the wattage of the incandescent lamp, enclosed in a 6-in. inside-frosted ball globe which is found upon observation to be equal in eye comfort to the luminaire under test. Ratings and limits were established and a considerable number of observations were made under this system, with surprising uniformity in the results. These specifications should have a salutary effect upon the practice of equipment design and leading fixture manufacturers have already developed lines of luminaires in accordance with them.

Some of the new designs incorporate the semi-indirect method of lighting in new forms (Fig. 126) which are hardly recognizable as being related in principle to the old-style inverted bowls. They have demonstrated that effectiveness and artistic appearance are not necessarily opposing requirements.

The home lighting committee of the N. E. L. A. adopted the Edison I. E. S. Specifications and induced a number of electric service companies to undertake refixturing campaigns. The electrical group in Chicago made some slight changes in these Specifications to meet local conditions and issued them under the name of "Franklin Specifications."

The use of luminous ornaments (Fig. 127) has for a long time been a small factor in home lighting, but a new note was sounded which bids fair to encourage a widespread conscious application of the practice. A new Mazda lamp was especially designed for this purpose.

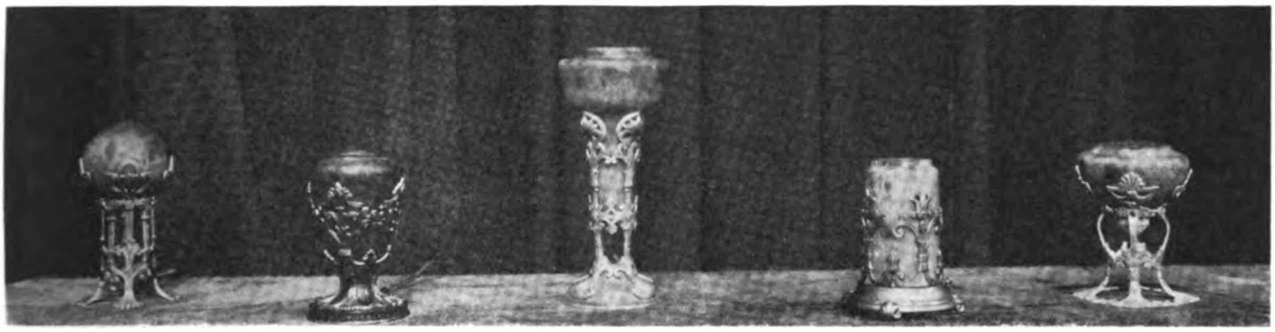

Fig. 127. Luminous Ornaments for which a Special Type of Incandescent Lamp was Designed

Street Lighting

There were two commercial applications of carrier current street lighting control; one on the lines of the New York State Power Corporation at Glens Falls (N.Y.); and the other on the lines of the Pacific Gas and Electric Company at San Francisco. These are among the first commercial applications of this system of control for street lighting.

A new control switch (Fig. 128) for multiple street-lighting service for use in each lamp standard is a single-pole single-throw contact unit rated 7 amp. at 125 volts. In the design of this switch it was necessary to take care of the high-current in-rush when the circuit is closed on a cold incandescent lamp. Therefore, the circuit is made and broken through a pair of silver-to-carbon contacts.

Occasional open circuits in series street lighting systems are unavoidable, especially on aerial circuits, and it is desirable from the standpoint of safety that when an open-circuit does occur the primary of the main transformer be deënergized. A protector was developed to operate in conjunction with a controller and its function is to open the switch in the controller immediately upon the occurrence of an open-circuit in the secondary of the main transformer.

The mechanism of the protector (Fig. 129) consists of two solenoids working in opposition, with a common armature. One solenoid is energized by the controlling circuit, the other by the load circuit to be protected. Under normal conditions, both coils are energized and the common armature remains in its lower position. When an open-circuit occurs in the load circuit, the lower solenoid is deënergized and the armature arises instantly due to the overbalancing of the upper solenoid. This movement of the armature actuates a contact which deënergizes the operating coil of the controller.

The protector can be reset by means of a push rod located at the top of the protector, reconnecting the transformer to the main feeder system. It may also be reset by flashing the control circuit.

The mechanism is housed in a weatherproof casing drawn from copper-bearing steel and the working parts are submerged in oil for insulation and damping.

A new form of dome refractor was designed which would direct the light from the building side of a street-lighting system and redirect it to the road surface. It is a modification of the original dome refractor which gave symmetrical distribution and redirected to the ground the light that would normally go into the upper hemisphere.

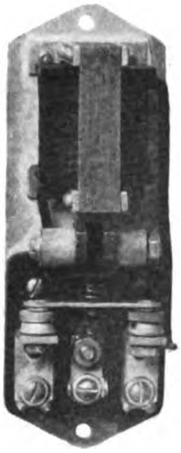

Fig. 128. Individual-lamp Control Switch for Multiple Street-lighting Service

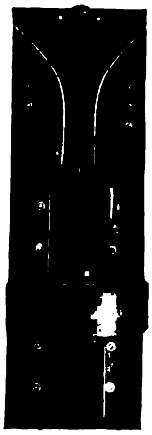

Fig. 129. Protector for Open-circuit Conditions on Series Street-lighting Systems

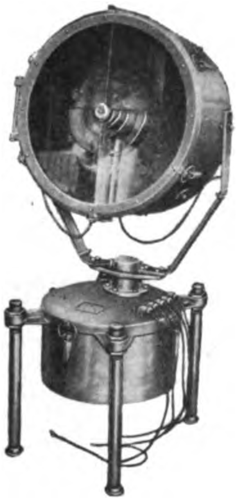

Fig. 131. Standard 24-in. Rotating Airway Beacon

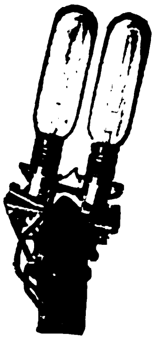

Fig. 132. Automatic Lamp Changer for Airway Beacon

A number of new ornamental type lanterns were designed for street lighting systems; one of these utilizes aluminum for all castings. One-half of the lantern can be swung open and the canopy is hinged so that it can be swung back, this arrangement making the lighting unit readily accessible for cleaning or inspection.

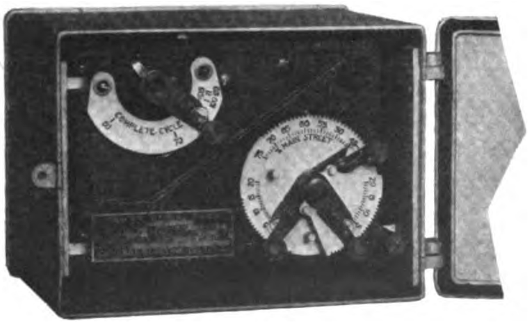

Fig. 130. Improved Type of Traffic-signal Timer

Motor Vehicle Lighting

The policy of the Eastern Conference of Motor Vehicle Administrators in approving only complete headlamp units resulted in a marked improvement in the quality of headlamps. Tentative specifications were adopted covering the depressible beam type headlamps, pending the formulation by the Illuminating Engineering Society of complete test specifications. This type of headlight equipment is now used on approximately 30 per cent of our standard cars.

The use of auxiliary driving lights increased. In some cases, they supplement the fixed-beam type of lighting, so that by switching from the main headlamps to the auxiliary units a depressed beam is obtained. They are also used to throw more light on the right-hand portion of the road to give better illumination of the ditch or edge of the road surface. One form of auxiliary driving light is designed to move with the wheels so as to furnish more light on the right or left-hand side of the road when the vehicle is turning a curve or sharp corner.

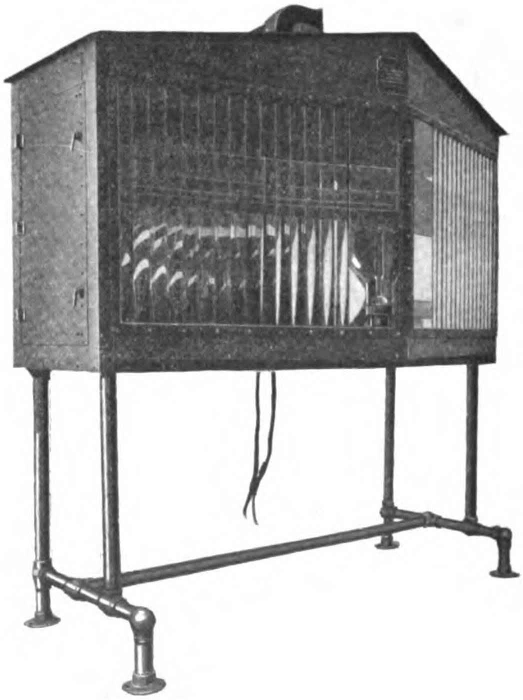

Fig. 133. Airport Twin Floodlight Using Two 5-kw. or 10-kw. Lamps

Traffic Control

To keep pace with the increased demand for automatic traffic-control equipment, a timer (Fig. 130) having increased flexibility of adjustment and refinement of construction was produced. With this device all time adjustments, including variation of the amber signals, are easily made and certain special traffic cycles can be obtained, such as the use of an overlapping amber following the green only.

Aviation Lighting

Under the present program of the Department of Commerce for the establishment of lighted airways as required by the Air Commerce Act of 1926, the United States will have by June 30, 1928, 8121 miles of lighted airways. They will have revolving beacons located at 10-mile intervals and intermediate fields with boundary light marking at every third beacon, or 30 miles apart.

The 24-in. rotating beacon now standard on American Airways (Fig. 131) has many advantages over those formerly used by the Airmail Service. By removing the sheet metal cover from the under side of the base, all the working parts are exposed to view and are readily accessible for oiling or adjustment. The barrel of the searchlight is made an integral casting of aluminum and very light shields have been added to the inside of the front door to prevent stray light striking the ground near the beacon.

An automatic lamp changer (Fig. 132), upon the failure of the operating lamp in the beacon, automatically swings a second lamp into place at the focal point of the mirror, at the same time making connections to bring this lamp into the circuit. The mechanism of this lamp changer is positive, and being operated electromagnetically, there is no time delay such as occurs with thermostatically-operated changers. The total weight of the beacon was also reduced to facilitate its erection on the 51-ft. steel towers used on the airway.

A new type of airport floodlight (Fig. 133) uses large current lamps such as the 5-kw. and 10-kw. It consists of two 24-in. searchlights contained in one housing, the front-door glass of the searchlight being composed of diverging glass strips which spread the light over a large area, at the same time the vertical spread being confined to a narrow limit by louvres. Automatic control of the floodlight from a distance is provided.

The lighting of the planes themselves consists largely of red, green, and white lights; usually 12-volt 21-c.p. lamps carried on the wing tips, and a pair of 12-volt 35-amp. landing lights for use in emergency and landing on unlighted fields. In the large cabin-type planes, the interiors of the cabins are also lighted.

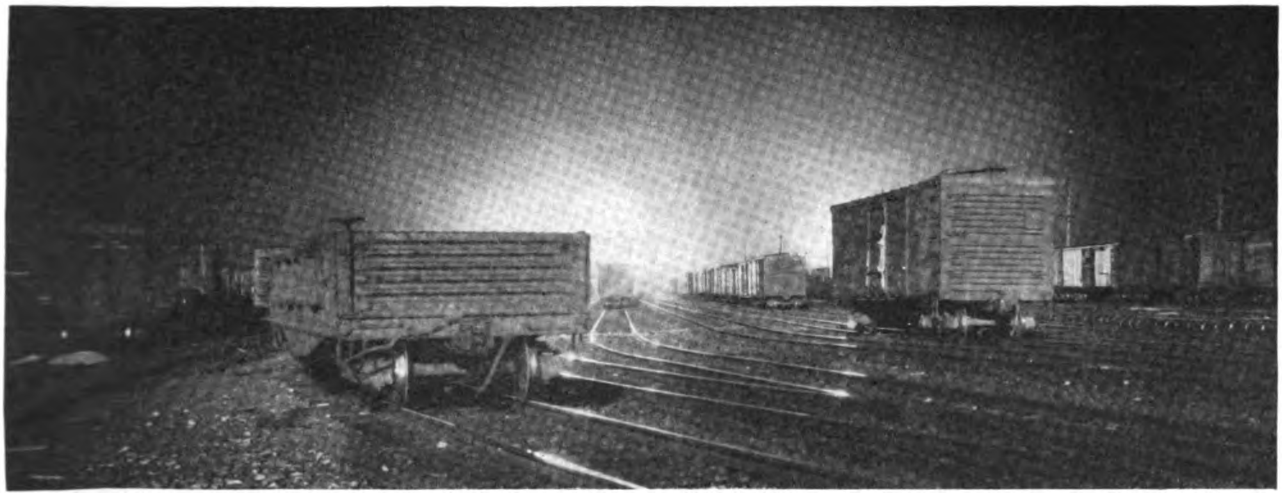

Fig. 134. Boston & Maine Railroad Yard at Mechanicville (N. Y.), Showing Illumination Obtained by Floodlighting

Floodlighting

The Association of Railway Electrical Engineers prepared a specification for the testing of floodlights, such as are used in classification yards, for the purpose of insuring a true and comparable method of evaluating the performance of such equipment.

There were numerous detailed improvements in the floodlight projectors for railroad yards and a number of new installations were made as the result of the benefits derived from the pioneer equipments. An example of modern practice in this field (Fig. 134) is found in the Mechanicville Yard of the Boston and Maine Railroad. Other railroad yards similarly equipped include the Terminal Railway Yard at Louisville (Ky.), the Coxton Yard, Tufts Farm Yard, and Suspension Bridge Yard of the Lehigh Valley Railroad Company.

The submersible fountain-type projector was applied to a number of panchromatic electric fountains and was also for the first time utilized for the under-water illumination of a swimming pool. This form of illumination gives better visibility than can be obtained by locating the floodlights above the surface of the water.

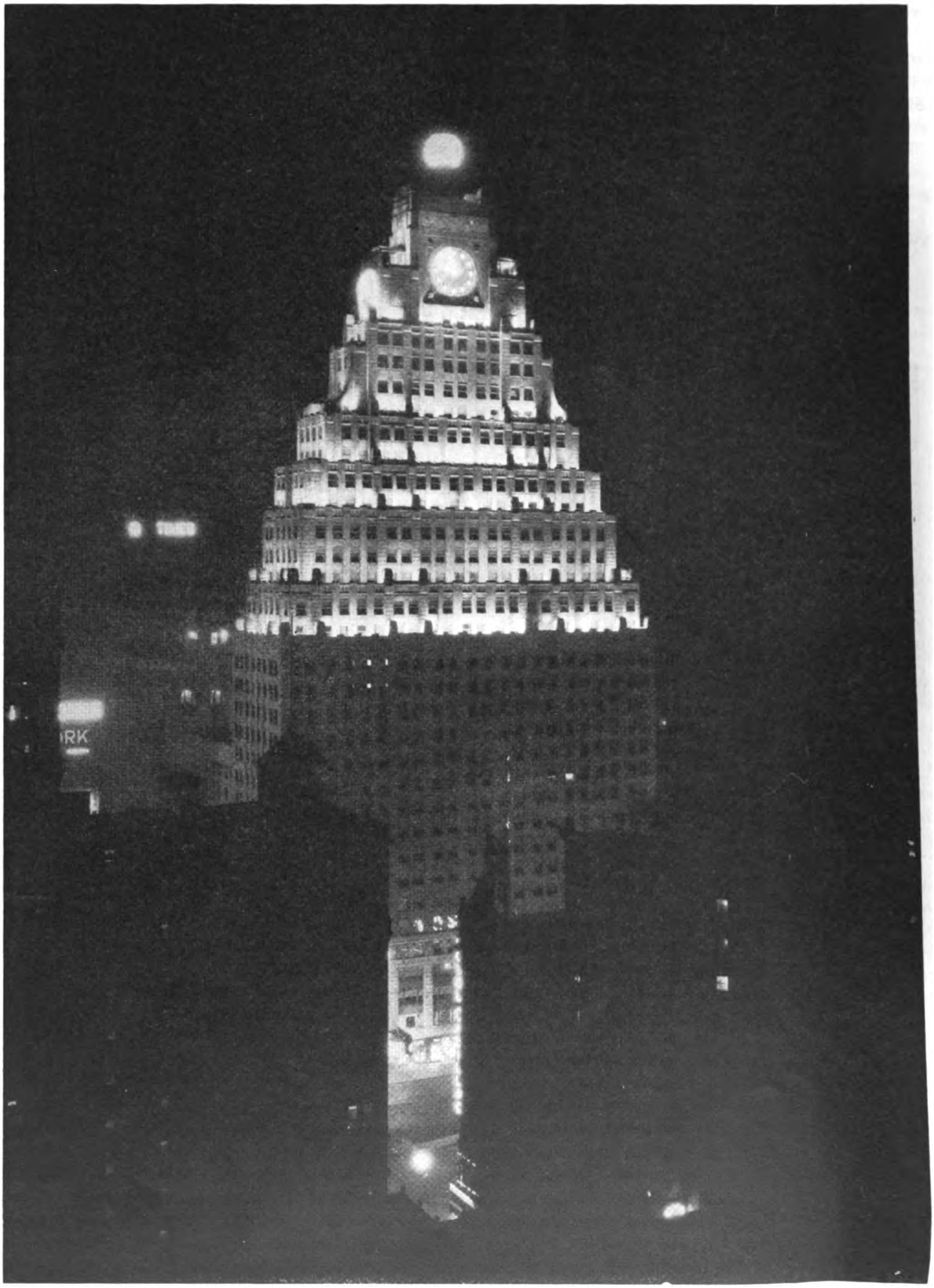

Fig. 135. Paramount Theater and Office Building in New York City Illuminated by Floodlight Projectors

There were numerous applications of floodlighting for the illumination of buildings, and detailed studies in regard to the lighting effects produced by different classes of building materials were made by illuminating engineers in collaboration with architects and building material associations. The object of these studies was to secure more effective use of floodlighting both from the standpoint of appearance and economy. Outstanding examples of building floodlighting during the year were the Paramount Theater and Office Building, New York City (Fig. 135), and the Pennsylvania Power and Lighting Building in Allentown (Pa.).

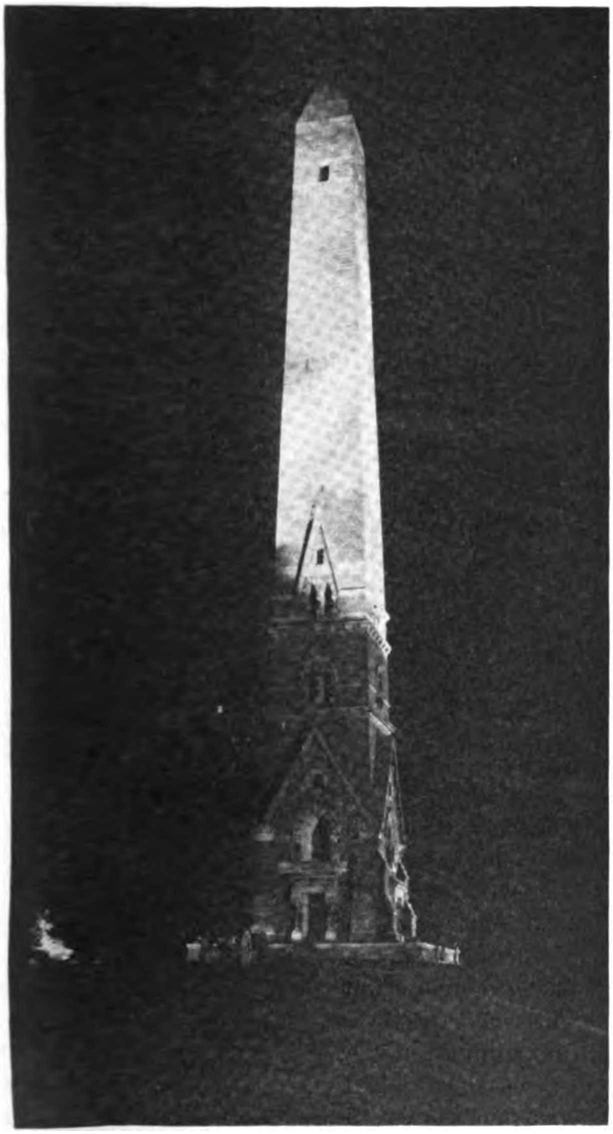

Fig. 136. Saratoga Battlefield Monument at Schuylerville (N. Y.), Floodlighted

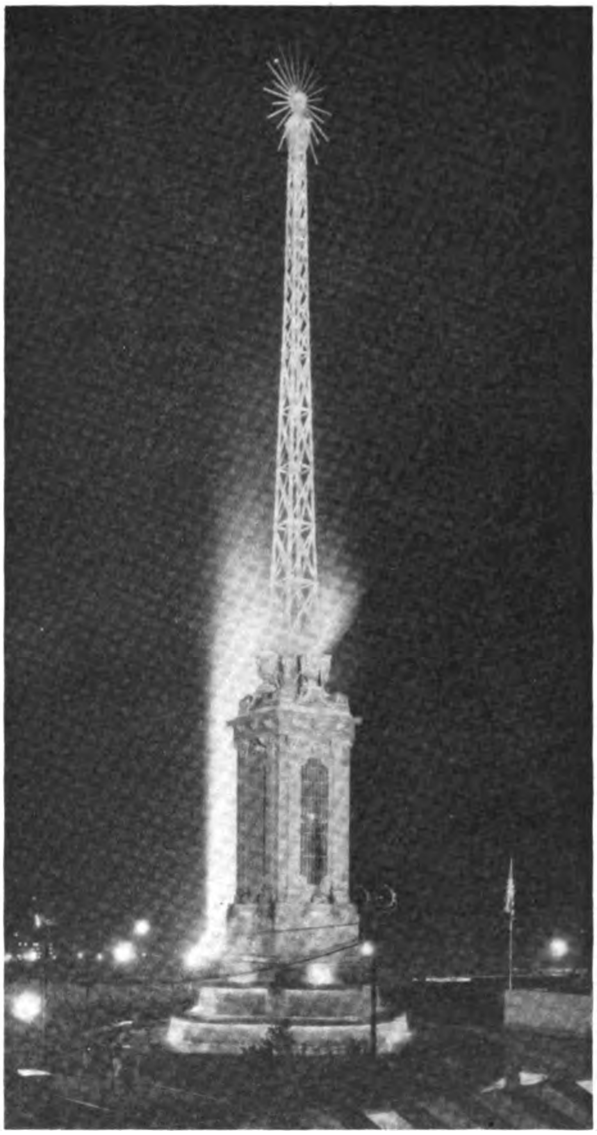

Fig. 137. The Jewelled Tower of Music at the Cleveland Industrial Exposition

In October, the 150th anniversary of the Battle of Saratoga was celebrated by historical pageants supplemented by the illumination (Fig. 136) of the Schuylerville (N. Y.) battlefield monument. This was floodlighted by means of sixteen 18-in. 1200-watt incandescent floodlighting projectors, each unit having a beam capacity of 5,000,000 candlepower. All four sides of the structure were lighted to an intensity of 20 foot-candles.

In August, an industrial exposition was held in the city of Cleveland (O.), the central feature being a 220-ft. illuminated tower (Fig. 137) known as the jewelled tower of music.

The principal structure consisted of a 200-ft. steel radio tower surmounted by a 20-ft. jewelled sunburst. The lower 80 ft. of the structure formed an ornamental housing for six super-panatropes, a new type of auditorium loudspeaker. The base of the tower was surrounded by a 90-ft. basin fed by cascades issuing from the base of the tower.

The total illumination from the searchlights which were utilized for this display was approximately 300,000,000 candlepower, and each searchlight was equipped with color screens so that practically an unlimited variety of color combinations was available for both the tower and the cascades.

Progress in Radio Receiving During 1927

By **DR. ALFRED N. GOLDSMITH**
Chief Broadcast Engineer, Radio Corporation of America

THERE are some years in the development of radio which mark the end of one era and the beginning of another. Among these years has been 1927, for during this year certain basic tendencies have demonstrated their importance in radio development, have been definitely accepted by the engineers and the public, and may now be expected to lead to further continued development along proved lines.

It is now recognized that the era of unregulated or only partially regulated radio transmission has come to an end. The passage of the Federal Radio Act of 1927, and the appointment of the Federal Radio Commission under the provisions of the Law, have for the first time enabled the Governmental regulating powers to be exercised systematically in the direction of minimizing interference between broadcasting stations and the allocation of frequencies to such stations based on a nationally applicable plan and yet flexible toward local conditions. The excellent preliminary work of the Department of Commerce along these lines prior to the passage of the 1927 Radio Law has been utilized as a foundation by the Federal Radio Commission. One great contribution of the Commission has been its consistent unwillingness to permit the erection and operation of unnecessary additional broadcasting stations in regions already adequately served.

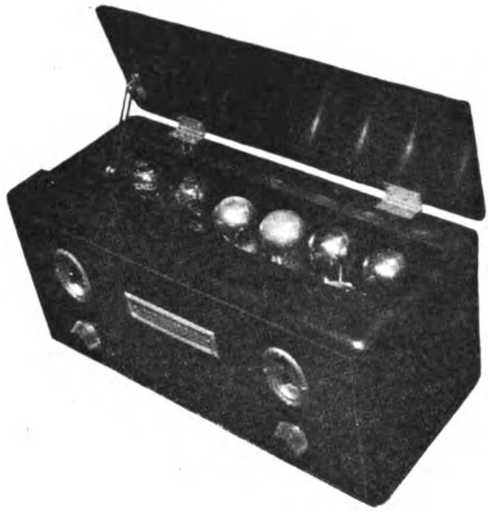

Fig. 1. A Single-selector Battery-operated Receiver, Radiola 16

The circumstances just mentioned have had a considerable influence on radio receiver design. Since it has become evident that an indefinite multiplication of stations will not be permitted, and that the congestion of stations in a given district will be greatly reduced and the field strengths corresponding to the various program signals will be systematically increased, the designers of receiving sets have been enabled to produce generally useful but simple and compact receivers with greater assurance than heretofore. It is now reasonably certain that stations of considerable power having frequency allocations less than 50 kc. apart will not be erected within a given district; that the building of powerful stations in regions of high population density will be discouraged; and that stations giving a demonstrably valuable service to the public will be afforded channels for clear reception up to the reasonable range of the station.

The end of the era of low-power broadcasting transmitters and freak long-distance reception is also at hand. Transmitters of 5 to 50-kw. capacity have become increasingly common, and several stations of 50 kw. have either been established during the year or are planned for early erection. Experimentally, a power of 100 kw. has been successfully used by Station *WGY* at Schenectady. These station powers are in marked contrast to the half-kilowatt power of the average station only a few years ago, which was at that time regarded with entirely undue optimism as capable of spanning tremendous distances.

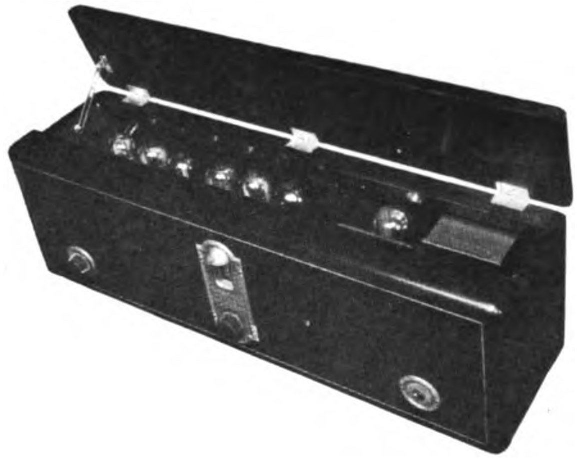

Fig. 2. A Socket-power-operated Single-selector Receiver, Radiola 17

Concurrent with the use of transmitters having a power of several tens of kilowatts has been the inevitable improvement of radio reception in millions of homes. Signal field strengths capable of riding above local man-made and atmospheric electrical disturbances have become so common in the more fortunate districts of the United States that the listeners have been definitely educated to regard such powerful and clear signals as normal. At the same time, the average listener naturally is dissatisfied with the feeble and mangled signal from distant stations and, as a result, only those listeners who are compelled by their location to listen exclusively to distant stations constitute any considerable percentage of "long-distance searchers." This change in listener psychology, corresponding to a demand for loud and clear signals and a refusal to listen to weak or mutilated signals, together with a considerable loss of interest in distant reception, has brought to the fore the matter of quality or fidelity of tone reproduction. The influence of this demand by the listeners has led to the widespread use of higher quality loud speakers, capable of giving considerable sound intensities and fed from so-called "power tubes" in the last audio-frequency stage of the receiving set. Where a few hundredths of a watt was at one time regarded as adequate for the final practically undistorted audio-frequency output of a receiver, today a power in excess of a watt is becoming common.

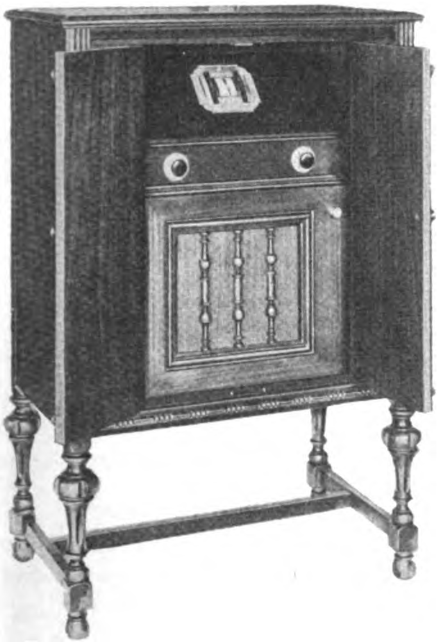

Fig. 3. A Superheterodyne Receiver, Radiola 30-A, Consisting of Radiola 28 and Model 100-A Loud Speaker

As a result of this evolution in the acoustic system of receiving sets, listeners are now enabled to secure a reasonably faithful and natural reproduction of the original studio performance, and an increasingly critical musical faculty is being developed among listeners. Persons who several years ago hardly knew one note from another are today enjoying well-reproduced symphony concerts and are developing a real understanding of the higher types of musical entertainment.

The listeners are concentrating their attention on local stations which give high-quality programs and produce loud and clear signals in their vicinity, and this audience demands receiving sets having excellent tonal quality of reproduction with adequate volume of sound. As is but natural, such listeners insist also upon simplicity of operation.

We have therefore also come to the end of the era of radio receivers which are complicated to operate. Today, the single-selector (uni-control) type of receiving set is practically standard. A minimum of receiver adjustments is regarded with favor by the listeners, and even more marked is the tendency toward socket power operation of receiving sets. The replacement or charging of batteries is on the decline. Simple and reliable receivers utilizing lighting circuit power exclusively for their operation have now become widely available, and the public response to them has been so enthusiastic that there can be no question that the socket-power-operated receiver is the type to be most generally used during the next radio era.

A number of receiving sets have been made available during 1927 which clearly belong to the new era in radio reception.

Typical of the simplicity of the modern receiver is Radiola 16, a six-tube antenna set having only two controls, one for station selection and one for volume. This receiver, Fig. 1, is adapted for use in circumstances or neighborhoods where batteries are regarded as necessary or desirable. The receiver has three tuned radio-frequency circuits, is inexpensive, and utilizes standard UX-201-A Radiotrons in all except the final output stage which utilizes the power-amplifier Radiotron UX-112.

Another receiver, Fig. 2. clearly illustrating the simplicity of the modern operation of such devices is Radiola 17. This also is a six-tube receiver, utilizing an antenna and having three tuned radio-frequency circuits. Adequate output volume is given by the use of the power-amplifier Radiotron UX-171 for the last stage of audio-frequency amplification. This receiver is specifically adapted for socket-power operation and requires no batteries or external device of any sort. It will operate from alternating current of either the 50- to 60-cycle range, or (in an alternative form) the 25- to 40-cycle range. This receiver employs new alternating-current radiotrons, which are described later in this article. This receiver, as well as the preceding, has been produced in the "popular-price range" and, when utilized with a corresponding high-quality loud speaker, is capable of furnishing musically satisfactory service.

A receiver of greater selectivity and more elegance throughout, and with a design based on the same modern ideals of radio reception, is shown in Fig. 3 and known as Radiola 30-A. This is a combination in a single cabinet of Radiola 28 (superheterodyne) and Model 100-A Loud Speaker. The receiver employs a small antenna, and is equipped for complete socket-power operation on either 40- to 60-cycle alternating current or (in an alternative form) on direct current. The output tube is of the Radiotron UX-171 type.

The Model 104 Loud Speaker has proved extremely successful as a device capable of giving fidelity of tone reproduction and sound volume which have astonished many music lovers previously unconvinced of the musical capabilities of high-grade radio reception. This device has been embodied in Radiola 32, views of which are shown in Figs. 4 and 5.

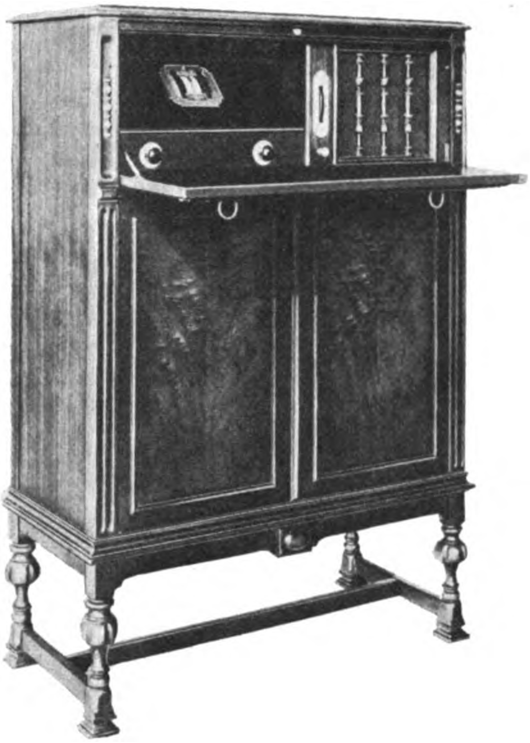

Fig. 4. A Socket-power-operated Superheterodyne Receiver, Radiola 32, Consisting of Radiola 28 and Model 104 Loud Speaker

Unusually difficult electrical and acoustical problems are presented when placing in a single cabinet an extremely sensitive loop receiver, a power-output tube, and a loud speaker of wide tonal range and capable of producing sounds of great intensity, particularly as it is necessary to maintain high quality of reproduction despite the effect of the cabinet enclosure. The enormous amplification which exists in a highly sensitive radio receiver further complicates the production of such an effective system. There have been developed certain electrical and acoustical expedients which are embodied in the Radiola 32 and which lead to the same performance as that of its component parts, Radiola 28 (superheterodyne) and Model 104 Loud Speaker. The receiver is operated on a loop or antenna as desired, and can be used with socket power operation on 40- to 60-cycle alternating current or (in another form) on direct current. In the alternating current form, a UX-210 Radiotron is utilized as the output tube.

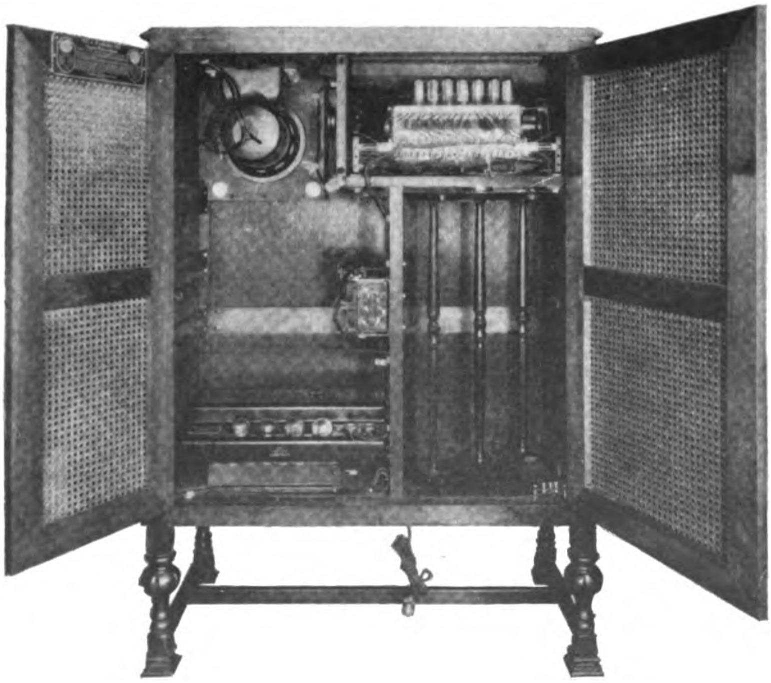

Fig. 5. Interior of the Receiver in Fig. 4 (viewed from the rear), Showing the Loud Speaker, Receiver Catacomb, Loop Antenna, and Socket-power Device which is of the Direct-current Type in the Set Illustrated

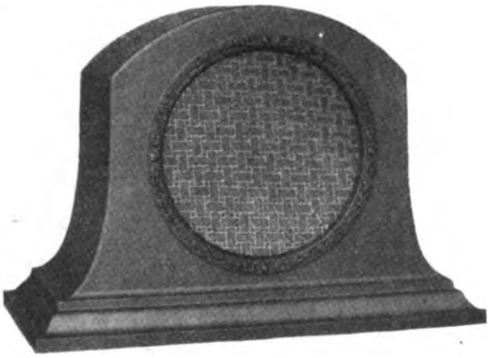

Fig. 6. Radiola Loud Speaker Model 100-A

Loud Speakers

The public is becoming increasingly critical of loud speakers that are incapable of delivering considerable sound outputs without rattling or other disturbing effects. It has also become increasingly insistent on the proper reproduction of the lower audio-frequency tones (which give body and mellowness to music, and naturalness to speech) as well as the higher frequencies. In Figs. 6 and 7 is shown a new seven-inch cone loud speaker, Model 100-A, which has been produced and which is essentially a considerably improved Model 100 type in a newly designed case. Greater volume can be secured with this loud speaker without any rattling, the construction of the driving motor has been made more substantial, and the acoustic response curve has been improved.

A curious result of the excellence of performance of the loud speaker Model 104 has been the insistence of persons residing in districts served by *direct current* that a device of the same capabilities be developed for them as well. Accordingly, there has been produced a Model 104 direct-current loud speaker, Fig. 8, for operation on 110-volt circuits. This device is provided with a new socket power unit incorporating a group of four UX-171-A Radiotrons operating on lower plate voltages. These four output tubes are arranged as a balanced single-stage amplifier (push-pull).

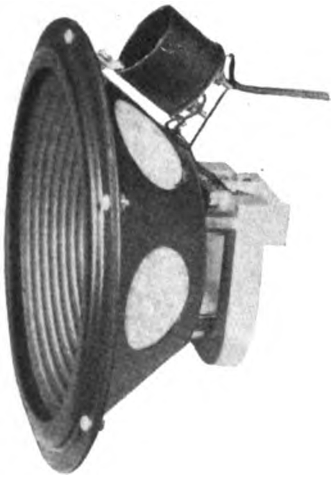

Fig. 7. Cone, Mounting, Driving Motor, and Filter of the Loud Speaker Shown in Fig. 6

Vacuum Tubes

As a necessary part of the rapid development of the socket-power-operated receiver, there have been produced a number of alternating-current tubes during the year. Some other important types of tubes have also been made available commercially.

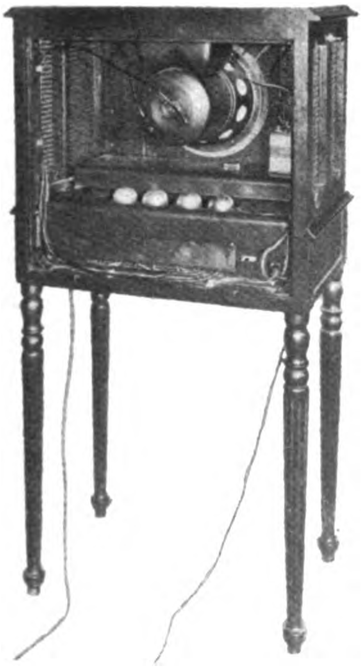

Fig. 8. Rear View of Direct-current Form of Radiola Loud Speaker Model 104, Showing Socket-power Device and Power Amplifier

Alternating-current Tubes

There are two main groups of alternating-current tubes. The first employs alternating current directly as a source of filament supply. This "raw a-c." type of tube, Fig. 9, is known as Radiotron UX-226 and has a special low-voltage high-current filament which permits direct excitation of the filament by alternating current. The average characteristics of this tube at proper voltages are as follows: plate voltage 135, negative grid voltage 9, filament voltage 1.5, filament current 1.05 amp., plate current 5.2 milliamperes, voltage amplification 8.2, and plate resistance 8000 ohms.

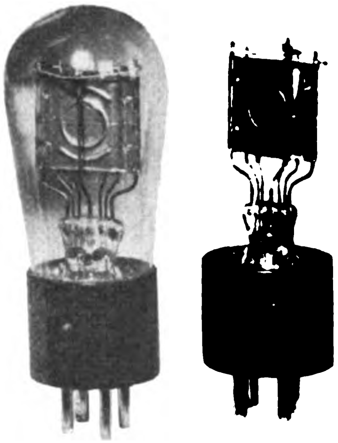

Fig. 9. Alternating-current Radiotron UX-226 ("Raw A-c." Filament Type)

The second type of alternating-current tube employs an indirectly-heated cathode, *i.e.*, an equipotential cathode which is heated by a low-voltage alternating-current heater element but is not electrically connected to the heater. It is known as Radiotron UY-227 and is illustrated in Fig. 10. The output of such a tube is markedly free from "hum" and such tubes are therefore especially suitable for use as detectors. The tube is inherently more elaborately constructed than the UX-226 type. Its average characteristics at proper voltages are: plate voltage 90, negative grid voltage 6, filament voltage 2.5, filament current 1.75 amp., plate current 2.8 milliamperes, voltage amplification 9, plate resistance 10,000 ohms, and mutual conductance 900 micromhos. When used as a detector, the preceding values are changed by

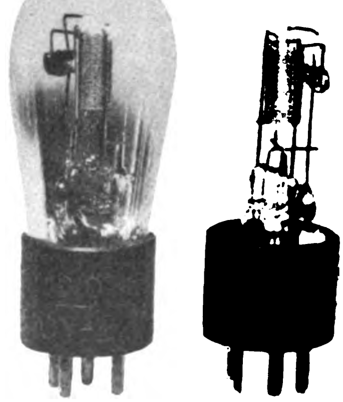

Fig. 10. Alternating-current Radiotron UY-227 (Equipotential-cathode or Heater Type)

using zero grid voltage and a reduction of plate voltage to 45 volts.

Rectifiers

A new heavy-duty oxide-coated-filament type of full-wave rectifier has been produced as the Radiotron UX-280. This tube, which is shown at the left in Fig. 11, is of sturdy construction and the plates have been specially treated to insure its having a long life. Its normal characteristics are as follows: maximum applied alternating-current voltage (r.m.s.) 300 volts per anode, filament voltage 5, filament current 2 amp., and maximum direct-current output 125 milliamperes.

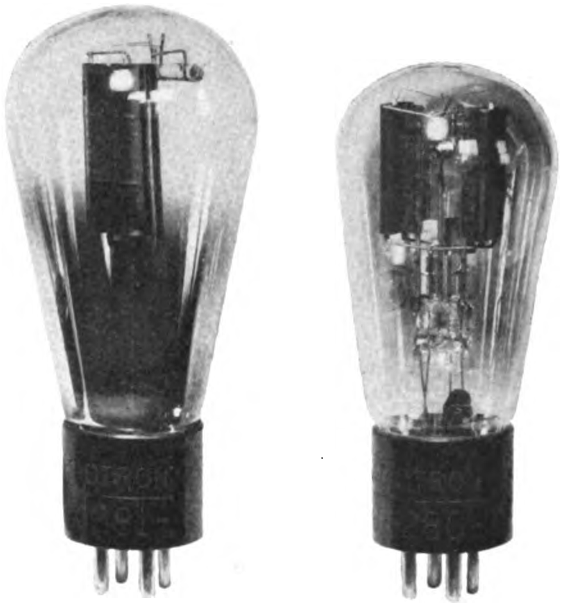

Fig. 11. Full-wave, Heavy-duty, Oxide-coated-filament Rectifier Tube, Radiotron UX-280; and Half-wave, Heavy-duty, Oxide-coated-filament Rectifier Tube, Radiotron UX-281

Another heavy-duty oxide-coated-filament type of rectifier, but for half-wave rectification, is known as the Radiotron UX-281 and appears at the right in Fig. 11. It is generally similar to the UX-280 tube and is useful where higher voltages are desired as indicated by the following characteristics: rated applied alternating-current voltage (r.m.s.) 650 to 750, filament voltage 7.5, filament current 1.25 amp., and rated direct-current output 65 to 110 milliamperes.

Four-electrode Tube

There has recently been developed a shielded-grid amplifier tube known as Radiotron UX-222. This tube, Fig. 12, has a very small effective plate-to-grid capacitance (approximately 0.025 micromicrofarads). This characteristic, together with its high-voltage amplification factor, makes it especially suitable for radio-frequency amplification. Tubes of this type may also be used as two-grid tubes in resistance-coupled audio-frequency amplifiers.

The approximate characteristics of such a tube when used as a screen-grid amplifier are as follows: filament voltage 3.3, negative bias of first grid 1.5 volts, positive voltage of screen grid 45, filament current 125 milliamperes, plate voltage 135, plate current 1.7 milliamperes, plate resistance 850,000 ohms, mutual conductance 340 micromhos, and, most unusual of all, a voltage amplification factor of 300.

The possibilities of tubes of this type in the field of radio frequency amplification are being rapidly developed and it is already evident that they afford a powerful tool to the designer of radio receivers.

Electric Phonograph

The refining of circuit design and construction of radio receivers has been paralleled during 1927 by that of electric phonograph equipment. These devices consist essentially of a "pick-up," which is generally an electro-magnetic device producing an output voltage of wave form controlled by the movement of the needle which rests in the phonograph record groove; a suitable volume control and filter circuit; an audio-frequency amplifier having a power output tube; and a high-quality loud speaker. The fidelity of music reproduction obtainable from the electric phonograph is so high that the public interest in these devices has steadily grown. An effective and economical arrangement consists in the combination of an electric phonograph with a radio receiver, the electric phonograph utilizing the same audio-frequency amplifier system and loud speaker as is alternatively employed for radio reception.

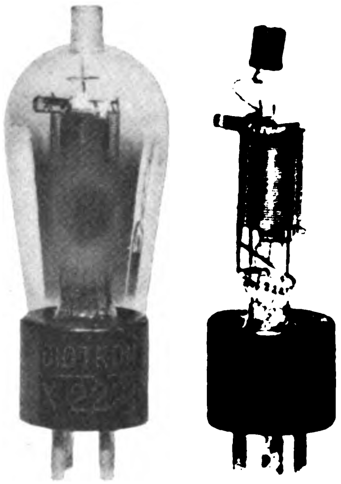

Fig. 12. Four-electrode Tube Having a Voltage Amplification Factor of 300, Radiotron UX-222

Carefully developed forms of this combination instrument have been produced in a number of different types. Those utilizing the Radiola 28 (superheterodyne) receiver, Model 104 Loud Speaker, and associated electric phonograph mechanism in one cabinet are typical of the furthest development of such devices yet reached commercially.

One of these is the Electrola-Radiola Model 9-55, a Victor Talking Machine Company product, Figs. 13 and 14. This electric phonograph is provided with an automatic record-changing mechanism whereby a sequence of records comprising an entire program can be heard in succession without any manipulation of the machine by the listener.

Such devices have a wide range of musical capabilities and combine in themselves all of the possibilities of modern radio reception and the production of music from the most recent records made by electrical recording processes. The two methods of musical reproduction, combined in one instrument, have shown themselves to be mutually complementary in many respects, and give new and valuable musical service and entertainment.

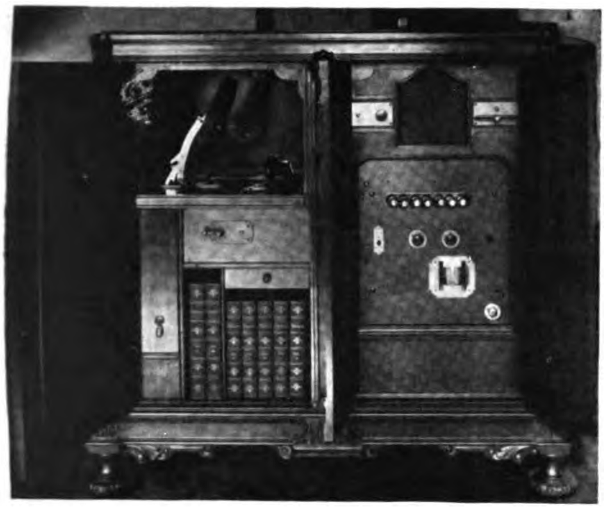

Fig. 13. Victor Electrola-Radiola Showing Automatic Electric Phonograph Mechanism and Radiola 28

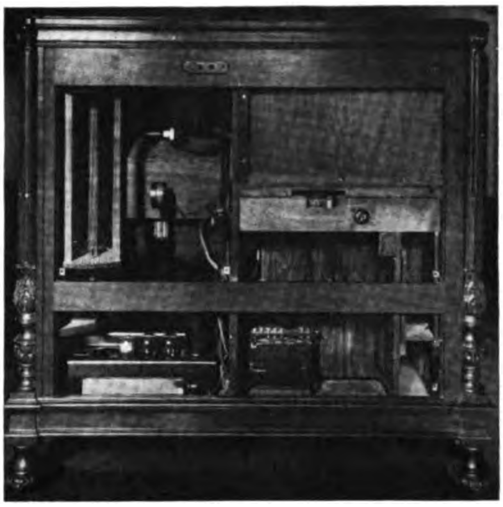

Fig. 14. Rear View of the Combined Radio Receiver and Electric Phonograph Shown in Fig. 13

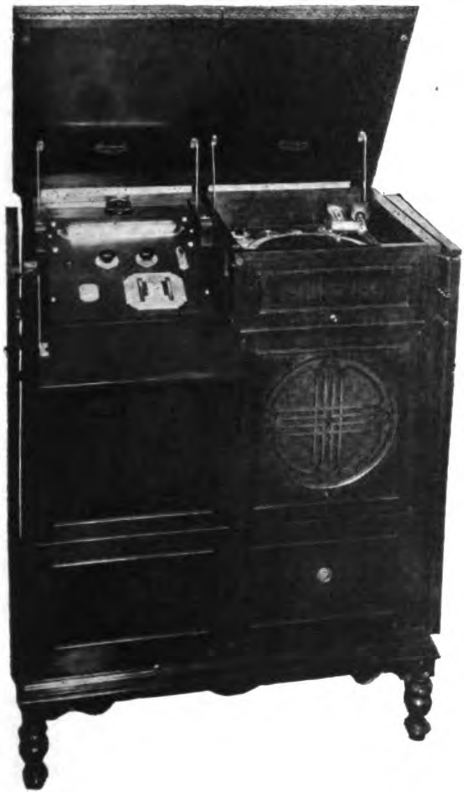

Fig. 15. Brunswick Panatrope-Radiola, Showing Radiola 28, Loop Control, and Electric Phonograph

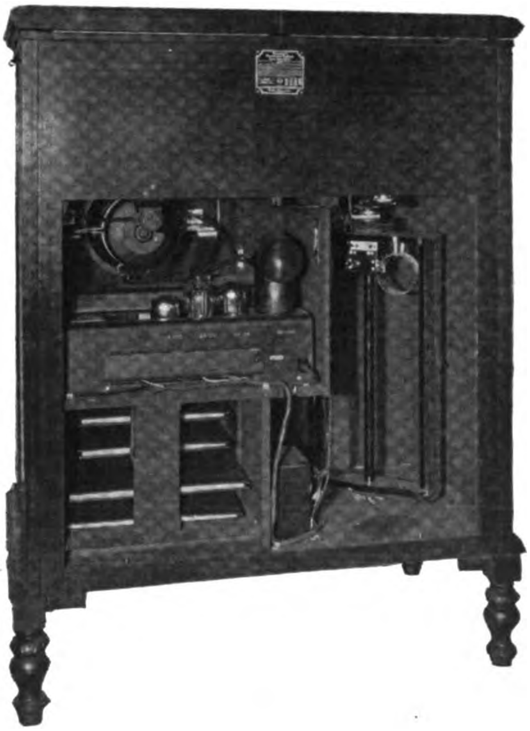

Fig. 16. Rear View of the Combined Radio Receiver and Electric Phonograph Shown in Fig. 15

Another arrangement of the electric phonograph, known as the "Panatrope," and Radiola 28 (superheterodyne), is the Model PR-138-C, a product of the Brunswick Balke Collender Company, Figs. 15 and 16.

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Power-Factor Correction. Effective Power and Reactive Power. W. H. Feldmann.
Iron & St. Engr., Sept., 1927; v. 4, pp. 389-395.

Three-phase Exciters for Improving the Power-Factor of Asynchronous Motor-Generators. W. Weiler.
AEG Progress, Sept., 1927; v. 3, pp. 270-273.

Protective Apparatus

Carrier-Current Pilot System of Transmission Line Protection. A. S. Fitzgerald.
A. I. E. E. Jour., Oct., 1927; v. 46, pp. 1015-1021.
(Describes the equipment and principles involved.)

Radio Communication

Radio Inter-Communicating System for Railroad Train Service. Henry C. Forbes.
Inst. Radio Engrs. Proc., Oct., 1927; v. 15, pp. 869-878.
(An account of tests on an experimental installation.)

Radio Engineering—Detection

Note on Detection by Grid Condenser and Leak. W. Van B. Roberts.
Inst. Radio Engrs. Proc., Sept., 1927; v. 15 pp. 793-796.

Radio Engineering—Loud Speakers

Loud-Speaker Diaphragms; Air Pressure and Energy Distribution in the Space Surrounding the Diaphragm. N. W. McLachlan.
Wireless Wld., Sept. 21, 1927; v. 21, pp. 357-361.

Radio Engineering—Measurements and Testing

Testing of Audio-Frequency Transformer-Coupled Amplifiers. H. Diamond and J. S. Webb.
Inst. Radio Engrs. Proc., Sept., 1927; v. 15, pp. 767-791.
(Includes seven bibliographic references, p. 791.)

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Correlation of Radio Reception with Solar Activity and Terrestrial Magnetism. II. Greenleaf W. Pickard.
Inst. Radio Engrs. Proc., Sept., 1927; v. 15, pp. 749-766.
(A continuation of a paper in the issue of Feb., 1927.)

Radio Atmospheric Disturbances and Solar Activity. L. W. Austin.
Inst. Radio Engrs. Proc., Oct., 1927; v. 15, pp. 837-842.

Two Contrasting Examples Wherein Radio Reception was Affected by a Meteorological Condition. E. H. Kincaid.
Inst. Radio Engrs. Proc., Oct., 1927; v. 15, pp. 843-868.

Relays

Ground Relay Protection for Transmission Systems. B. M. Jones and G. B. Dodds.
A.I.E.E. Jour., Oct., 1927; v. 46, pp. 1089-1094.

Short Circuits

Short Circuit Currents. Raymond C. R. Schulze.
Elec. Lt. & Pr., Oct., 1927; v. 5, pp. 21-25.
(Their nature and calculation. Serial.)

Spark Discharges

Studies on Sparking in Air. A. Pen-Tung Sah.
A. I. E. E. Jour., Oct., 1927; v. 46, pp. 1073-1081.
(Abridgment.)

Speed Regulation

How Problems in Variable-speed Operation Have Been Solved.
Ind. Engng., Oct., 1927; v. 85, pp. 466-470.

Steam Flow

Supersaturation and the Flow of Wet Steam. G. A. Goodenough.
Power, Sept. 27, 1927; v. 66, pp. 466-469.
(Serial.)

Steam Plants

Historical References to the Progress in the Use of High-Pressure Steam. James Mollison.
Inst. Engrs. & Shipbuilders' Trans., Sept., 1927; v. 70, pp. 783-803.

Steam Turbines

Small Steam Turbines. George A. Orrok.
Engrs. Soc. of W. Pa. Proc., July, 1927; v. 43, pp. 267-278.
(A short article summarizing the essential characteristics of different makes of small turbines for auxiliary drives.)

Steam Turbines, Radial Flow

10,000-14,000-kw. Ljungström Steam Turbine.
Engng., Sept. 30, 1927; v. 124, pp. 411-413.
(Illustrated description of the construction of a turbine built by Svenska Turbinfabriks Aktiebolaget Ljungström, Sweden. Serial.)

Substations, Automatic

Automatic Substations. T. W. Ross.
Met.-Vick. Gaz., Sept., 1927; v. 10, pp. 202-207.
(On the necessary equipment and its application. Serial.)

Switches and Switchgear

Armored Switchgear for Main and Auxiliary Station Buses.
Elec. Wld., Sept. 17, 1927; v. 90, pp. 549-553.
(A description with illustrations of equipment in the Waukegan plant of the Public Service Co., of Northern Illinois.)

Switchgear for Alternating Current. H. W. Clothier.
Engng., Oct. 7, 1927; v. 124, pp. 467-470.
(Abstract of a paper read before the British Assoc.)

Switches and Switchgear, Automatic

Automatic Switching Equipments for Converter Plants. G. Meiners.
AEG Progress, Sept., 1927; v. 3, pp. 257-261.

Telemeters and Telemetry

Load Indication by the Current Balance Method. Bernard E. Lenehan.
Power Pl. Engng., Sept. 15, 1927; v. 31, pp. 989-991.
(Describes apparatus for transmitting indications of load, voltage, water level, etc., over distances of 30 miles.)

Transient Phenomena

Transients Due to Short Circuits; A Study of Tests Made on the Southern California Edison 220-Kv. System. R. J. C. Wood and others.
A. I. E. E. Jour., Oct., 1927; v. 46, pp. 985-994.
(Abridgment.)

Water Power

Hydro-Electric Resources of the Empire Overseas. W. R. Cox.
I. E. E. Jour., Oct., 1927; v. 65, pp. 944-956.
(A summary of the water power resources of a number of British colonies, including Canada, Australia, New Zealand, and South Africa.)

NEW BOOKS

(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW)

Berechnung von Drehstrom-Kraftübertragungen. Oswald Burger. 115 pp., 1927, Berlin, Julius Springer.

(A brief text on the electrical design of polyphase transmission lines. Since it was written primarily to satisfy the needs of the designing and operating engineer, it presents the solution of several problems of the type which are met with in practice. In connection with these problems and their solutions the author offers tables of data, graphs and diagrams which should be useful to the designer in solving problems of similar nature.)

Electric Power Equipment. J. G. Tarboux. 455 pp., 1927, N. Y., McGraw-Hill Book Co., Inc.

Electrical Engineering Practice. Ed. 4, rewritten and enl. Vol. 2. J. W. Meares and R. E. Neale. 532 pp., 1927; N. Y., John Wiley & Sons, Inc.

(According to the subtitle of this volume it is "A Practical Treatise for Electrical, Civil, and Mechanical Engineers, With Many Tables and Illustrations." The complete work, in this fourth edition, has grown to three volumes. The one at hand, volume two, treats of transformation, conversion and storage of energy; wiring and control of distribution systems; and lighting, heating, welding and cutting by electrical methods. From this it will be evident that we must look to volumes one and three for such topics as fundamentals of electrical engineering, prime movers and power generation, sale of energy, specifications and testing, and construction, control and applications of motors. We are told that in this book "an endeavor has been made to fill the gap between the many excellent pocket-books of bare data and the highly technical works written for specialists in various branches of electrical engineering." Although written from the British viewpoint, American practice has not been overlooked. Bibliographic lists are appended to many of the chapters, and an unusually extensive index is provided. When the three volumes are all available as a unit they should provide a useful set of books for the student of electrical engineering and for those engaged in the operation of electrical equipment.)

New Reformation; From Physical to Spiritual Realities. Michael Pupin. 272 pp., 1927, N.Y., Charles Scribner's Sons.

(In the dual rôle of scientist-philosopher, Professor Pupin, of Columbia University, takes us away from our more usual contacts with science and engineering to review with him the early beginnings of scientific knowledge, and to study the relations between things scientific and things spiritual. Through the medium of a series of seven "narratives" he sketches for us the chain of evolutionary events which have gone into the building of our present knowledge of scientific facts. His aim is to reveal how completely science has changed our concept of the universe. He tells us that the realities which he describes are those of human experience, and that they are not the result of mere abstract philosophical introspection. In his "Prologue" he says: "These narratives are addressed primarily to persons who have not an elaborate scientific training; hence they speak in terms of a simple and untechnical language. It is hoped that by strengthening our understanding of the physical realities the narratives will reform our mental attitude and make it better prepared for the recognition of the truth that physical and spiritual realities are the fruit of the same tree of knowledge, which was nurtured by the soil of human experience.")

Principles of Radio Communication. Ed. 2, rev. John H. Morecroft. 1001 pp., 1927, N. Y., John Wiley & Sons, Inc.

(In the years intervening since the publication of the first edition of this book, in 1921, much progress has been made in the radio art and much has been written on it. As a result the present edition is much more thorough and complete than was the first. Even with the omission of much that is now out of date, the pages now number 1001 as compared with the former 988. "As in the previous edition, however, no pretense is made that the book is a treatise on radio practice; in general only the principles involved in the operation of radio apparatus have received attention". Those familiar with the first edition will be interested to know that some of the more important additions to the present edition are: new data on coils and condensers at radio frequencies; new material on field strength measurements, reflection and absorption, fading, short-wave transmission and voice analysis; and discussions of loud-speaking telephones, frequency control by crystals, distortionless amplification, radio-frequency amplification, balanced circuits, push-pull arrangements, etc. A portion of the old chapter on spark telegraphy has been omitted and the chapter on radio measurements and the one on experiments have been deleted. We are promised that if the demand warrants it the material of these two latter chapters will be published as a separate volume. Numerous bibliographic foot-notes and an adequate index round out a useful volume.)

Rotary Converters; Their Principles, Construction and Operation. E. P. Hill. 329 pp., 1927, N.Y., D. Van Nostrand Co.

("This book deals with the principles and operating characteristics of the Rotary Converter, and is written from an operation and commercial standpoint. Sufficient material has been incorporated, it is hoped, however, to render the subject matter clear to an engineering student; but primarily the book will be of assistance to engineers responsible for purchasing, operating, or testing rotary converter machinery, the actual detail design of rotary converters not being dealt with in the present volume." The subject-matter is divided into eight chapters as follows: "Fundamental Principles," "Constructional Features," "Transformers and Auxiliary Apparatus," "Methods of Starting Polyphase Rotary Converters and Notes on Switch-Gear," "Service Conditions of Operation and Control," "Operation Characteristics," "Specifications and Testing," and "Comparison with Other Forms of Conversion Systems." Numerous illustrations of typical machines, and many diagrams and graphs are interspersed throughout the text. A classified bibliography is appended.)

Theory of Machines. Louis Toft and A. T. J. Kersey. 408 pp., 1927, N. Y., Isaac Pitman & Sons.

De Wissenschaftlichen Grundlagen des Rundunkempfangs. K. W. Wagner (Editor). 418 pp., 1927, Berlin, Julius Springer.

(This volume on the scientific fundamentals of broadcast reception is not a systematic treatise, but a series of separate papers on the subject. These papers are contributed by prominent German engineers and scientists and are edited by Dr. K. W. Wagner, an authority on the subject. The introductory chapter is written by the editor, and discusses the cultural problems of radio broadcasting, its organization and technic. This is followed by a paper by Prof. F. Aigner on those oscillations within the range of speech and of music, as well as on the causes of distortion. The problems of sound propagation and acoustics are discussed in the next two chapters, one of which is by W. Hahnemann and Dr. H. Hecht and the other by Prof. W. Schottky. The physical foundations of radio reception are reviewed by Dr. H. Salinger, while Dr. R. Rudenberg discusses the problems of radiation, propagation and reception of radio waves. Dr. A. Esau contributes a chapter on radio interference. Next follow several chapters on the vacuum tube by Messrs. Rukop, Möller, Barkhausen and Pohlmann. The final three chapters by Leithäuser, Eppen, and Harbich, are devoted to special radio circuits and to radio devices and apparatus. The volume can scarcely be classed as a text, for the detached papers included do not cover the entire field of radio reception. For this same reason, and also due to the lack of any index, its usefulness as a reference work is somewhat limited. However, for the specialist who cares for authoritative presentations of the topics covered, it no doubt has its field.)

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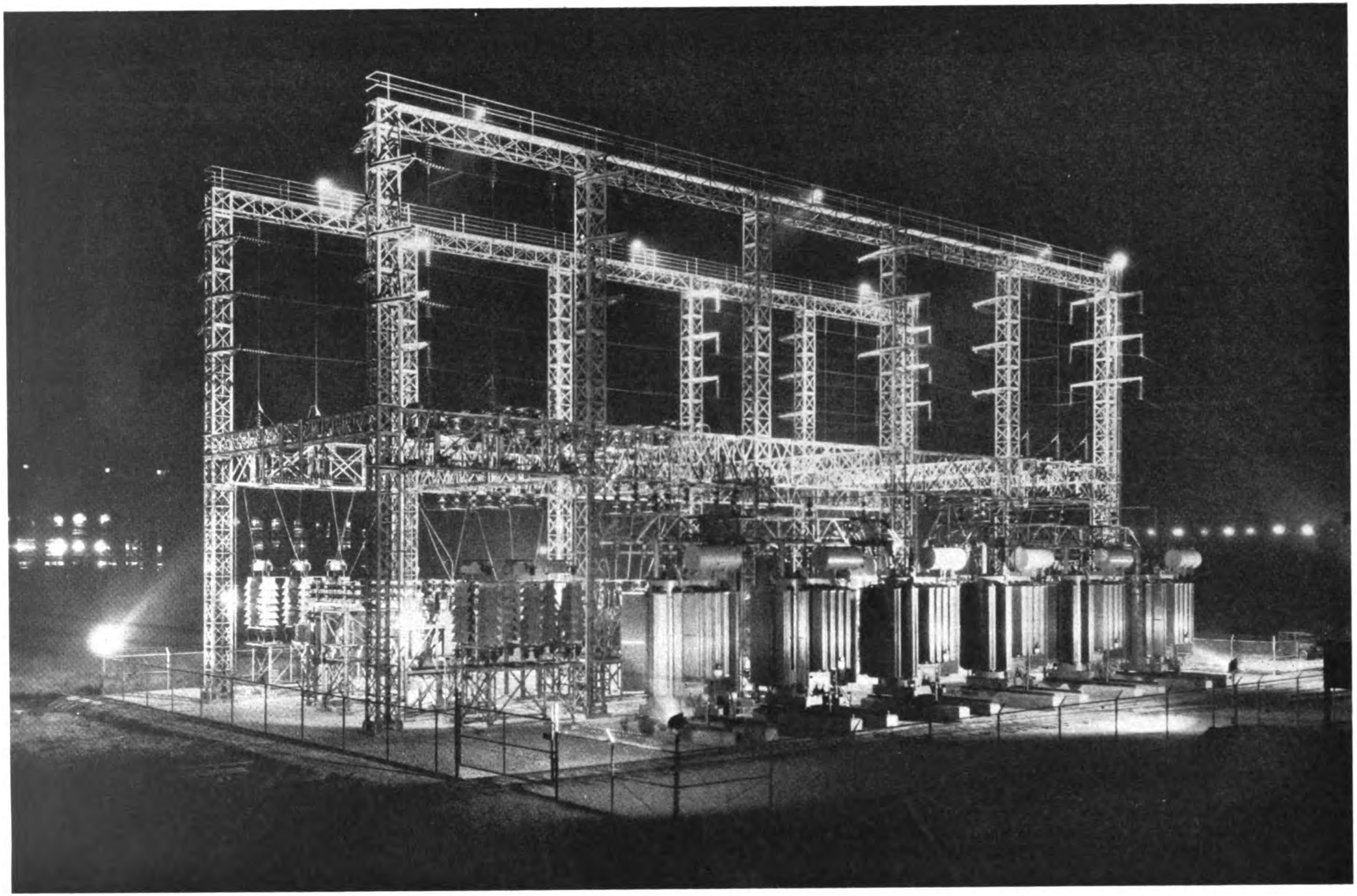

A STAGE WHERE MOTIONLESS ACTORS PERFORM

A night view of the new 100,000-kv-a. Cahokia Substation showing the transformers, lightning arresters, and some of the switching equipment. The transformers are rated 16,667 kv-a. single-phase 69,000/13,800 volts. Four outgoing power lines are shown at the right, and the power plant in the left background. (*See p. 91*)

GENERAL ELECTRIC REVIEW

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SPLITTING UP KILOWATTS

Waterwheel generators of 65,000-kv-a. rating are in service; a turbine-generator capable of developing 100,000 kw. is in operation; single transformers are handling 66,667 kv-a. At the time they were installed they represented the last word in construction and afforded their designers just cause for pride. Also, because of their great size, they gained the admiration of the public; but now that they are no longer new they have become more or less forgotten by these same people. Soon will come the thrill of greater achievements.

Transmission lines run from most of these sources of power, and not long ago we gave attentive ear to the reports of successful line operation at 220,000 volts. Again, the fascination induced by that which is of large measure. These transmission lines, with others of lower potential, carry great bulks of energy hundreds of miles—to where?—to points appreciably short of your home and mine, of your office or factory and mine. What happens in this gap? The delivery to you and to me of all the enjoyments that even small amounts of electricity afford.

If there appears to be nothing to fascinate us in the operation of this local delivery, it is because we have not given it adequate consideration. Even though a transmission line should be run to our own back yard for our exclusive use we could not avail ourselves of it as an individual consumer, unless we were engaged in some activity comparable in magnitude to that of a steel plant. Yet, with no such energy supply within miles of us, there is brought to our property line enough to satisfy our needs at any hour of the day or night; and we can move next door, to the next block, or to the outskirts and still find the same reliable service available at a cost so low that any comparable substitute would be expensive.

There is then something spectacular in this taking of one or a few supplies of energy and splitting it up in exact accord with the individual demands of perhaps several million people, spread over an area that is square miles in extent. In magnitude, this operation may call for an investment practically equal to that required for generating-station apparatus; and, because the electrical losses are naturally greater in the distribution system, exceptional skill must be exercised in its design. Also, the reliability of service to any consumer is as dependent upon the work of the distribution engineer as on that of the power-station engineer. These conditions have brought about the development of distributing systems that are specially designed for this important service.

One of these is the Edison three-wire direct-current network and the other is the a-c. network. The Edison system came into being first, is still extensively used, and probably will continue to be used at least in districts which have grown up with it. Of late years, serious attention has been given to the possibilities of a-c. networks; and in the last five years some twenty such networks have been placed in service and at the present time the installation of half as many more is being contemplated.

In the days when commercial electric service was first gaining a foothold in public estimation, many direct-current engineers claimed that alternating current was of no use and alternating-current engineers claimed that they would make the use of direct current obsolete. We are wiser today, and acknowledge that both kinds of electricity have merits. For this reason we do not predict that a-c. networks will displace d-c. networks, but as the latter are newcomers in the field it behooves us to study their possibilities that we may take full advantage of them where available.

In this issue of the REVIEW is he opening installment of a serial on the subject. As in practice, consideration is given first to the "where and why" of such networks with respect to load density and territory served. When general determinations have been made in these particulars, there comes the problem of choosing one of the several types of a-c. network that combine light and power service.

The prevention of lamp flicker requires special attention because the distribution transformers and circuits possess considerable reactance and the induction motors draw their starting current in one or two abrupt steps.

Then there are the questions of how to effect a proper division of load over the interconnected circuits and how to maintain such a division while automatically regulating the voltage at different points to hold constant potential on the lamps nearby.

Interruptions to service must be prevented, but distribution transformers must not be permitted to suffer because of overload conditions. A-c. network protectors furnish the necessary protection if due thought is given to their proper application in the system.

These are several phases of the subject that must be thoroughly studied prior to connecting up a-c. network.

Ground Detection

General Considerations—Two-wire Low-voltage D-c. and A-c. Circuits—Edison Three-wire Systems—Higher Voltage Requirements—Two-phase Three- and Four-wire Systems—Three-phase Three-wire Systems

By JOHN AUCHINCLOSS
Switchboard Department, Philadelphia Works, General Electric Company

> *The ground is a most useful foundation both for material structures and electrical potentials. In one case the physical elevation is secured by structural members; in the other, the electrical elevation is made possible by insulating materials. If the strength of either type of support becomes weakened in service, a collapse may follow. This article describes various means of detecting faults of this character in electric circuits.*
>
> —Editor

ON grounded circuits, such as railway feeders with grounded negative, Edison three-wire systems, and three-phase systems with grounded neutral, an accidental ground on the isolated pole or poles of the system usually permits the passage of sufficient fault current to operate the nearest over-current protective device, thereby isolating the disturbance from the rest of the system. On such circuits ground detecting equipment is not necessary. On ungrounded systems, however, the occurrence of one ground does not constitute a short circuit, but if the system be permitted to continue in operation under this condition a subsequent ground of different polarity will cause an interruption of service. It is consequently very desirable on such systems to have some provision for indicating immediately the occurrence of the first ground, so that means may be taken for its removal as soon as it can be located and before the occurrence of a second ground which would occasion a short circuit.

Furthermore, on high-voltage circuits having considerable electrostatic capacity, the first ground alone, especially if an arcing one, may endanger the general insulation of the system. On station control circuits accidental grounds may give rise to considerable trouble by causing electrically-operated devices to function under more or less inopportune circumstances. For this reason the battery providing the source of supply for the control circuits is usually equipped with ground-detecting devices.

The various types of commercial circuits call for different schemes of ground detection and each will now be described in connection with the individual type of circuit for which it is adapted.

125-volt D-c. Two-wire Ungrounded Circuit

Ground detectors for this type of circuit are usually in the form of two ordinary incandescent lamps rated for the normal voltage of the control circuit, generally 125 volts. Connected to the bus as in Fig. 1 these lamps will burn at half normal voltage so long as neither the positive nor negative pole of the circuit has any connection to ground.

Obviously, however, a dead ground, say on the negative at *C*, will short out lamp *A* and cause lamp *B* to glow at full brilliancy. Other values of the ground resistance at *C* will be reflected in the relative brilliancy at which lamps *A* and *B* glow. This is further illustrated by the arrows showing the direction of current flow in the detector circuit. Upon the occurrence of a dead ground at *C*, points *C* and *D* are at the same potential, *viz.*, 125 volts below the positive line. Since they are at the same potential, no voltage exists across lamp *A* and consequently no current flows through it. On the other hand, since point *D* is 125 volts below the positive line, lamp *B* will have full line voltage impressed upon it and will therefore burn at normal brilliancy.

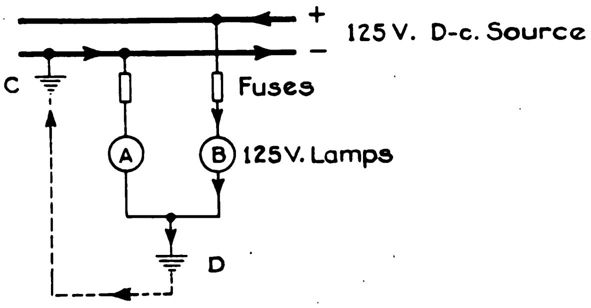

Fig. 1. A Simple Means of Detecting Grounds on Low-voltage Two-wire Circuits by the Use of Incandescent Lamps

Any resistance at point *C*, as in the case of a partial ground, will introduce a difference of potential between points *C* and *D* which will be impressed upon lamp *A*. Let this potential be 50 volts. Then the total voltage across the lamps will be distributed in such a way that lamp *A* will receive 50 volts and lamp *B* 75 volts. Lamp *B* will consequently burn more brightly than lamp *A*.

While the foregoing method does not give very accurate readings of the values of partial grounds, it is usually considered sufficient for circuits of this kind where high-resistance grounds are relatively innocuous. It should be noted that lamps utilized as ground detectors afford a continuous indication of the condition of the system.

If a more accurate measurement of the value of the ground is desired, a differential voltmeter may be substituted for the lamps and connected as shown in Fig. 2. This instrument is provided with a zero-center scale, of the type shown on the switchboard instrument in Fig. 3. Obviously, so long as no ground exists, the two coils will be in series across the full line voltage and since each will receive one-half of this voltage their torques will be balanced and the pointer will stand at zero in the center of the scale. On the occurrence of a ground, however, this balance will be disturbed and the pointer will deflect to one side or the other depending upon the polarity and magnitude of the ground. Each half of the scale should be graduated in full line volts, for with a dead ground on the negative line the right-hand coil of the instrument will have full line voltage impressed upon it.

In those cases where even closer indications are desired, a voltmeter may be used as shown in Fig. 4. The instrument must be suitable for use on the full voltage of the circuit since with a dead ground at *C* and the plug inserted in the right-hand points of the receptacle, the full 125 volts will be impressed on the coil, just as under the same condition in Fig. 1 the full voltage is impressed across lamp *B*.

Direct-current voltmeters are usually of the polarized type and for this reason a definite direction of current flow through the coil is essential to give a positive deflection of the needle. This direction, in the meter shown in Fig. 4, is from left to right. Then, with a dead ground at *C* and the plug inserted in the right-hand points of the receptacle, the current flow through the instrument is in the correct direction. When inserted in the left-hand points there would be no deflection, since the voltmeter is then connected between points *C* and *D* across which there is no potential difference.

On the other hand, were the ground on the positive instead of the negative side of the line, the current instead of flowing from positive to ground, as in the preceding case, would flow from ground to negative. For this reason it will be observed that the receptacle is so wired that when the plug is inserted in the left-hand points to read a ground on the positive line, the ground point *D* connects to the left-hand stud of the voltmeter instead of to the right, when reading negative ground. In other words, since the ground assumes the polarity of the line which is grounded, it may be either positive or negative and since the polarity of the instrument is fixed, it becomes necessary to reverse the connections to the voltmeter when reading positive grounds. This is accomplished, as will be noted in the diagram, by the reversed connections when the plug is inserted in the left-hand points of the receptacle.

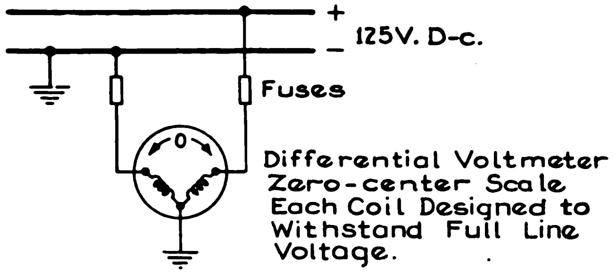

Fig. 2. A Ground-detector Circuit Wherein a Voltmeter is Substituted for Lamps

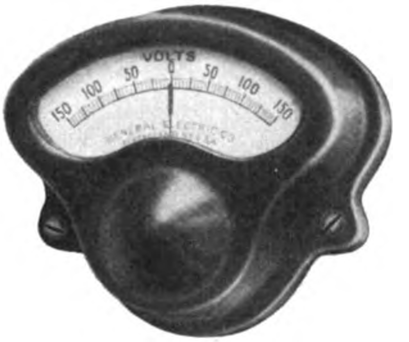

Fig. 3. A Zero-center Scale Instrument of a Type Suitable for Ground Indication

It should be noted that by means of a voltmeter bus and additional receptacles the voltmeter may be used for reading the line-to-line voltages of other machines or circuits when not required for reading grounds. Not more than one plug, however, should be in use with any one voltmeter, for if more than one were used, failure to remove the plug from the ground receptacle before placing another in a line-to-line receptacle would automatically ground one side or the other of the line; or if a ground already existed on one side, a ground on the other side arising from misuse of plugs would result in short circuit.

A method of reading line-to-line voltage and positive and negative grounds on the same receptacle is shown in Fig. 5. Here it will be noted that when the plug is inserted in the two middle points the meter reads the line-to-line voltage, while insertion in the left- and right-hand points will read positive and negative grounds respectively.

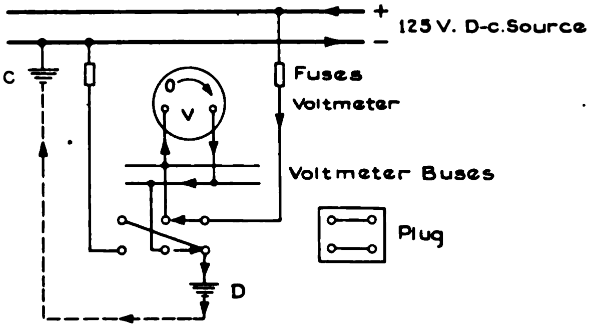

Fig. 4. Connections for Using an Ordinary Voltmeter and Plug Switch to Provide for Accurate Quantitative Indication of Grounds

Another method of reading grounds on such circuits, but by means of a special voltmeter, is illustrated in Fig. 6. In this case the voltmeter is equipped with a zero-center scale, thereby eliminating the question of polarity entirely. The needle will simply deflect in one direction for positive grounds and in the opposite direction for negative grounds, the position of the plug in the receptacle determining which is which. While simple, this method is at a slight disadvantage in that the scale is necessarily short and consequently the readings are not so close as those obtainable from the standard instrument. This renders the meter less suitable for general use as a line-to-line voltmeter. Ground-detecting devices are usually connected to the main bus and will indicate grounds on any circuit having a connection with them. The grounded circuit may be located by opening and closing the various feeder switches, fed from the bus, the opening of the switch on the faulty circuit causing the ground indication to disappear.

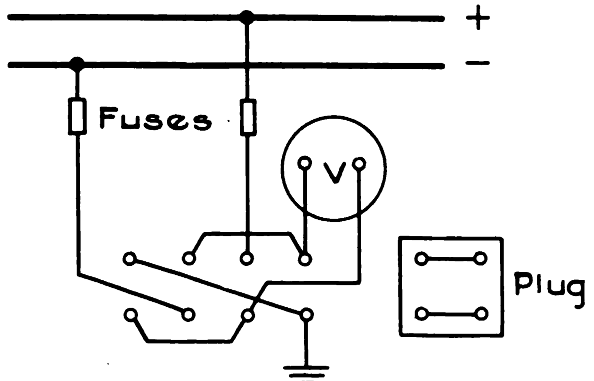

Fig. 5. An Extension of the Plug Switch Connection to Include the Reading of Line-to-line Voltage with the Ground Indicator

Edison Three-wire (Ungrounded Neutral) Systems

The simplest method of detecting grounds on an Edison system is by means of two lamps and a normally-closed push-button switch as shown in Fig. 7. The push-button switch is normally in the closed position; thus it is possible in a convenient manner to temporarily open the circuit. The voltage rating of the lamps should correspond to the potential between the positive and negative lines, or, if such lamps are not obtainable, suitable resistors may be used in series with standard lamps.

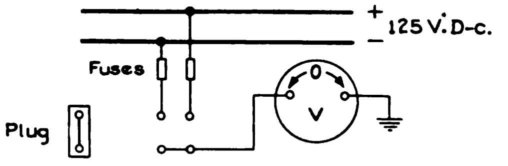

Fig. 6. A Connection Diagram for Ground Reading by Use of a Special Zero-center Scale Voltmeter and a Plug Switch

When no ground exists both lamps will glow at half-normal brilliancy in consequence of the magnitude of the impressed voltage, but they will also glow in the same degree with a ground on the neutral. To test for neutral grounds it is therefore necessary to push the switch to the open position. If, when in this position, the lamp *A* continues to glow, it indicates that a ground exists somewhere on the neutral. On the other hand if lamp *A* ceases to glow the system is free from grounds. Grounds on the positive and negative lines are indicated by a difference in the brilliancy of lamps *A* and *B*, as already described under two-wire ungrounded circuits (Fig 1). The purpose of the push-button switch, therefore, is merely to provide for the detection of grounds on the neutral wire.

Several methods are available for obtaining closer readings by means of a voltmeter. One of these is shown in Fig. 8 and employs an instrument equipped with a zero-center scale. Suppose a dead ground exists on the negative line. If the plug is inserted first in the positive receptacle, a deflection will be obtained to the right of the zero mark on the scale. This indicates a ground, either on the neutral or negative lines. Removing the plug and inserting it in the negative receptacle, the pointer remains at zero, indicating that both studs of the instrument are at the same potential and that, therefore, the ground is on the negative line. If, however, instead of remaining at zero, the pointer swings over to the left of the zero mark an indication would be given that the ground is on the neutral line and not on the negative. A ground on the positive line may be detected in a similar manner, the pointer deflection, of course, being in the opposite direction.

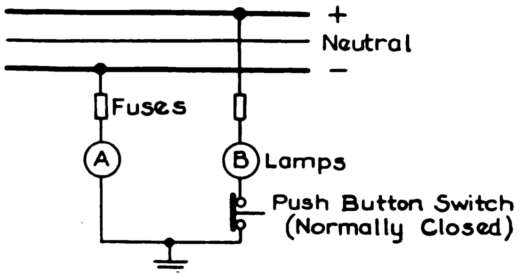

Fig. 7. The Simplest Form of Circuit for Detecting Grounds on an Edison Three-wire System

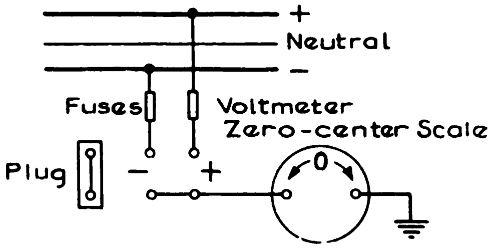

Fig. 8. The Zero-scale Voltmeter Applied to the Three-wire System for the Purpose of Ground Detection

Another similar method, using a so-called differential voltmeter, is shown in Fig. 9. With this arrangement the two coils of the instrument are normally connected in series across the positive and negative lines. The instrument therefore indicates positive or negative ground, just as do the lamps in Fig. 1. To test for ground on the neutral the push-button switch is opened, when, if a ground exists, a deflection in proportion to its value will appear on the scale.

Two-wire A-c. Circuits

On low-voltage (110 volts) single-phase circuits the lamp connection already shown in Fig. 1 may be used. On higher voltages (220–240), resistors must be included to permit the use of standard lamps. It is advisable on these voltages that the equipment be placed beyond the possibility of accidental human contact and that a normally-open test switch be located in some convenient point in the ground connection as shown in Fig. 10. On still higher voltages (480 and above) an electrostatic ground detector may be used. Such an instrument is shown in Fig. 11.

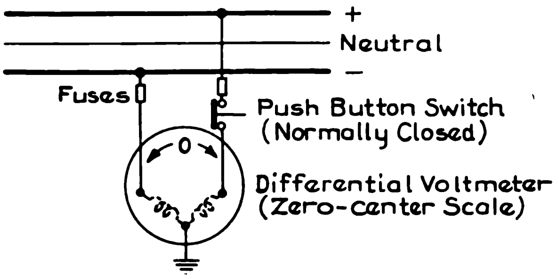

Fig. 9. Ground Detection in an Edison System by Means of a Differential Voltmeter

This simply consists of a light aluminum vane attached to a pointer superposed upon a zero-center scale. The vane is influenced by the electrostatic fields of the two lines. When these are balanced, the pointer attached to the vane remains at zero on the scale thereby indicating that no ground exists upon the system. If, however, a partial ground should occur on one line, its electrostatic field will decrease while that of the other line will increase. The vane, therefore, cannot remain in its former position, but must take up a steady position in which the forces due to the unbalanced fields are again balanced upon it. The movement to this new position carries the pointer over that half of the scale corresponding to the grounded line to a point proportional to the voltage to ground of the affected line. The instrument, therefore, not only indicates which side of the system is grounded, but also gives an approximate indication of the value of the ground resistance. The connections for the device are shown in Fig. 12.

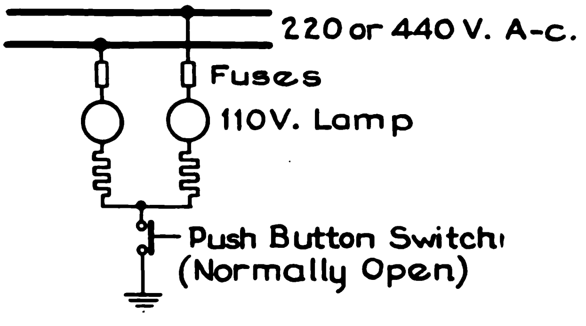

Fig. 10. Diagram Showing the Use of Ordinary Incandescent Lamps with Resistors for Ground Detection on Circuits of 220 to 440 Volts

Two-phase Systems

A two-phase four-wire system with no interconnection between phases is essentially two electrically separate single-phase systems and can be treated as such so far as detection of grounds is concerned. The connections are consequently the same as Figs. 10 and 12, except that of course two separate equipments are required.

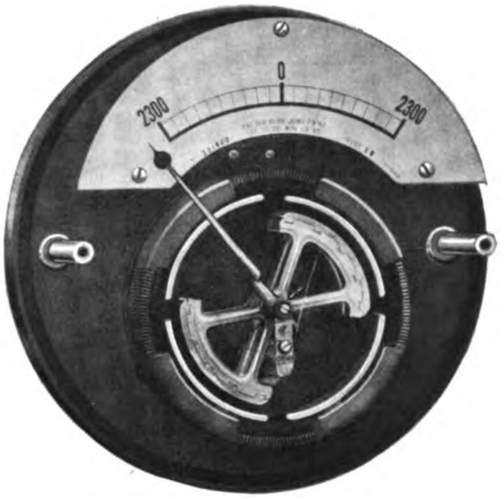

Fig. 11. An Electrostatic Ground Detector for Use in High-voltage Circuits

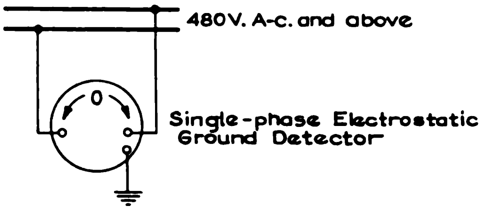

Fig. 12. The Connection Diagram for a Single-phase Electrostatic Ground Detector

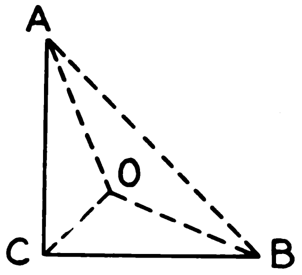

Fig. 13. Vector Diagram of Voltages in a Two-phase Three-wire Circuit to which Ground Detectors Are Not Applicable in the Ordinary Manner

In a two-phase three-wire circuit the end of one phase is electrically connected with the end of the other phase as shown by vectors in Fig. 13. Since the geometrical center of such a system is at *O* and not *C*, ground detectors connected across *A-C* and *C-B* would be unbalanced under normal conditions and would consequently indicate grounds when no grounds exist, and give false indications also when they do exist. It is therefore recommended that on such circuits of potentials of 480 volts and above, three electrostatic ground detectors be used connected from points *A*, *C*, and *B* to ground, as shown in Fig. 14. The stud which is not connected to the line is connected to the ground stud; and the devices function as electrostatic voltmeters indicating the voltage of each line from ground. From Fig. 13 it is apparent that the instrument connected to line *C* will normally indicate about half the voltage of those connected to lines *A* and *B*.

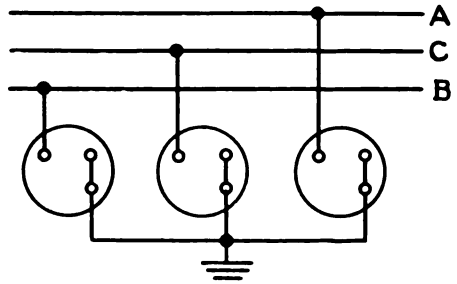

Fig. 14. The Connection of Three Electrostatic Ground Detectors for Proper Indication of Grounds in Two-phase Three-wire Systems

Three-phase Three-wire Circuits

On low-voltage three-phase circuits, two methods of detecting grounds by means of lamps are in common use. In the zone method two lamps only are used, connected as shown in Fig. 15, the switch in the ground connection being optional depending upon the voltage of the circuit and whether it is desirable to have the system permanently grounded through the lamps for the sake of a continuous indication or whether a regular test by means of the switch will prove satisfactory.

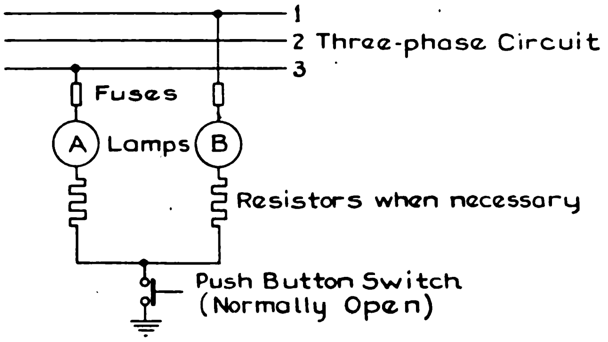

Fig. 15. The Zone Method of Ground Indication by the Use of Two Incandescent Lamps in Low-voltage Three-phase Three-wire Systems

The operation of this equipment is as follows: With the push-button switch closed and the system free of grounds both lamps will glow at half voltage. If, for example, a ground exists on lines *1* or *3*, lamp *A* will be brighter than lamp *B* or vice versa. Should the ground be on line *2*, however, the brilliancy of *both* lamps will increase beyond the brilliancy at which they glow before the push-button switch is closed. This method is not so direct as the three-lamp method shown in Fig. 16.

Here again the push-button switch is optional. When it is closed and the system free of grounds all three lamps glow at approximately half-normal brilliancy. A ground on any line is indicated by the decrease in brilliancy of the lamp connected to the faulty line, with respect to the brilliancy of those connected to the other lines on which no ground exists. Obviously a ground on more than one line, if of considerable magnitude, will constitute a short circuit and automatically disconnect the circuit by means of its over-current protective device.

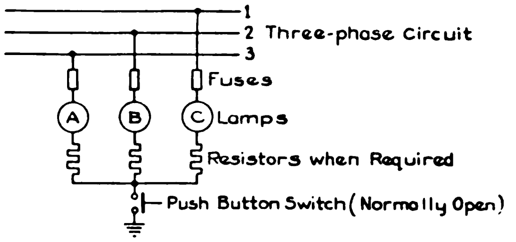

Fig. 16. Diagram of a Direct Method of Ground Indication in Three-phase Three-wire Systems

When more accurate readings are required, a simple method of reading not only the voltage of each phase to ground but also the voltage across the three phases is shown in Fig. 17. For reasons previously stated one plug only should be furnished with each equipment. This method may be extended to read the three-phase voltages on several machines or circuits by means of an eight-point receptacle on each in addition to one six-point and one four-point receptacle for reading grounds at the bus. In this case a voltmeter bus extending across the various circuits would be required. The voltmeter could be conveniently mounted on a bracket at the end of the switchboard.

On higher voltages (480 volts and above) the three-phase electrostatic instrument is usually employed. This device contains three vanes and three scales with three pointers superposed upon them. When a ground occurs on one of the three lines the two pointers adjacent to the segment connected to the grounded line are deflected toward that segment. The connections for this device are shown in Fig. 18. High resistances, usually carbon rods, are connected between each pole of the detector and the high-tension lines. As these are frequently mounted internally they are not shown upon the diagrams.

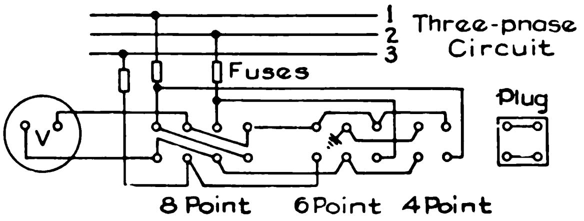

Fig. 17. A Voltmeter and Plug Switch Connection Providing Readings of Phase Voltage as Well as the Voltage to Ground

On extremely high voltages, for which electrostatic detectors are not available, grounds may be detected by means of three potential transformers connected in Y-Y as shown in Fig. 19. The transformers should be rated for the full line-to-line voltage, even though ordinarily they operate at only 57 per cent of this voltage, for the occurrence of a dead ground on any one line will shift the neutral point to that line, thereby impressing the full line-to-line voltage on the remaining two. For example, a dead ground on line *2*, shown in Fig. 20, will short out transformer *B* but at the same time will cause the neutral to shift to line *2*, thus placing transformers *A* and *C* (Fig. 19) across lines *1-2* and *3-2* respectively. Under this condition lamp *B* will become dark while lamps *A* and *C* will glow at full brilliancy. Under normal conditions all three lamps will glow at 57 per cent of their rated voltage (110 volts).

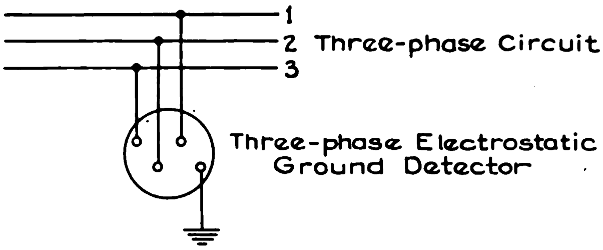

Fig. 18. A Three-element Electrostatic Instrument Connected in this Manner Gives Direct Indication of a Ground Occurring in Any One of the Three Phases

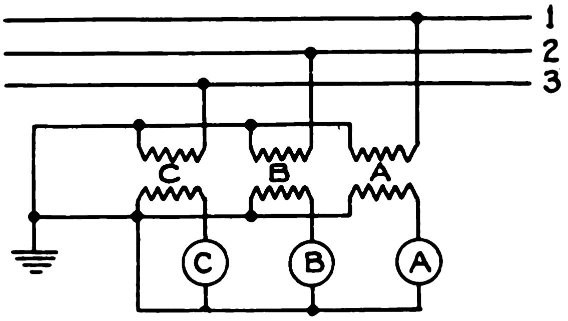

Fig. 19. A Diagram of Three Potential Transformers as Used for Ground Indication on Circuits of Very High Voltages if Proper Care is Used in the Selection of Ratings

A more accurate reading may be obtained by substituting for the lamps a so-called triplex voltmeter having the three scales arranged in three parallel rows one above the other as shown in Fig. 21. With this scale arrangement any difference in the three voltages may be very readily detected. Normally the three pointers will stand in a vertical row. In the event of a ground occurring on one phase, the pointer of the voltmeter element connected to that phase will show a decreased reading while the other two will deflect in the opposite direction.

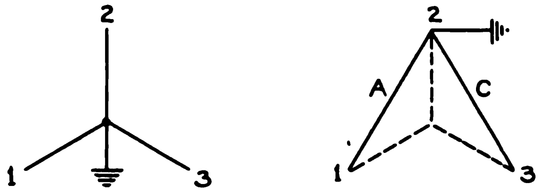

Fig. 20. Vector Diagram Showing the Electrical Effect of a Ground Upon the System of Transformers in Fig. 19.

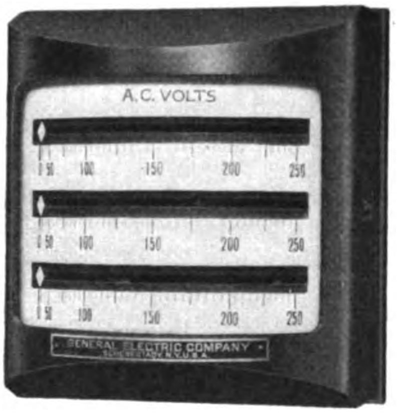

Fig. 21. This Triplex Voltmeter Gives Direct Indication of Grounds in Three-phase Circuits, and Differences in Voltages in Any of the Phases May be Readily Detected

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Modern Fisherman Uses Volts

The most modern method of fishing, based on economic, utilitarian, and humanitarian grounds, is by electricity. In Germany, experiments have recently been conducted on a small scale in one of the trout streams running into the Rhine. The process consists of electrifying the water in a given stretch so that all the fish are stunned and rise to the surface. They are then collected and sorted. Pike and other enemies to the more edible and more profitable fish are dispatched, undersized fish are set aside and later returned to the water. The marketable fish are retained. Those of the catch that are judged eligible for a further lease on life, being only temporarily stunned, appear not to have been harmed by their experience, when again in their native element.

Low-voltage A-c. Networks

PART I

APPLICATION

By D. K. BLAKE
Central Station Engineering Dept., General Electric Company

The Edison three-wire d-c. system of distribution has established an excellent record of performance, but modern conditions of energy supply in metropolitan districts have reached a point where serious thought is being given to the service possibilities of low-voltage a-c. networks. The introductory considerations are outlined below; the specific problems encountered in the design of a-c. networks will be treated in later installments of this serial.—EDITOR

WHEN studying the supply of electrical energy to large American cities it is usually convenient to distinguish three typical areas having diverse load characteristics. These areas may be classed as residential, industrial, and commercial. The residential and industrial areas are supplied with alternating current. The commercial area or downtown business section is usually supplied by the well-known Edison three-wire direct-current system. This area demands the highest order of electric service obtainable; and the Edison system with large standby storage batteries has successfully rendered throughout the years the high order of service required.

The Edison system has appreciable losses and requires a considerable investment in feeder copper, substation equipment, and real estate. Many engineers have for a long time desired to supply this commercial load with alternating current as this does not require converting machinery and can be distributed at high voltage thereby reducing the number of substations in the area of congestion and avoiding the purchase of high-cost real estate. It has been recognized, however, that alternating current cannot provide such reserve capacity as the storage battery and consequently cannot render the same high order of continuous service.

Within the past ten years several factors have caused engineers to further study the application of alternating current to these commercial areas. These factors are: (1) increase of load in the commercial area, (2) increase in extent of the commercial area, (3) new commercial areas in outlying sections of very large cities, and (4) greatly increased reliability of supply to a-c. substations due to interconnection of two or more generating stations. The first factor has greatly increased the difficulty of obtaining suitable locations for additional Edison substations. The first two factors have greatly increased the expense of storage-battery capacity, while the fourth has reduced the value of such reserve capacity. The fourth factor is the reason why virtually all engineers believe the less expensive and more efficient a-c. system should be used for the new commercial areas mentioned as the third factor. The difference in the order of continuous service is not considered sufficient to warrant the additional cost of the Edison system with the storage-battery reserve for these areas.

There is a diversity of opinion as to the use of the a-c. system for (1) increase of load in the commercial area and (2) increase in extent of the commercial area. Some companies do not permit the present area supplied by the Edison system to increase but reinforce the Edison system for the increasing load in its present area and use alternating current to supply the increasing extent of the commercial area. Other companies restrict the growth of the Edison system in kilowatt capacity and supply all new load in the present area as well as the extending area with alternating current. This involves a certain amount of changeover of existing d-c. service to a-c. service to prevent the load on the Edison system increasing beyond the capacity of the equipment. Still other companies, particularly in the smaller cities, intend to eventually replace all of the Edison system with alternating-current service. This of course involves the changeover of d-c. supply to a-c. in order to remove Edison equipment from service as it becomes obsolete.

The extent to which alternating current is supplied to the commercial area, and the Edison system is curtailed, depends to an appreciable extent on the emphasis placed on the decreasing value of the storage battery as a reserve supply. The more reliable the a-c. supply to substations the less is the value of a storage-battery reserve. There are some prominent engineers who believe the difference in reliability between the a-c. system and the Edison system with the storage battery is still sufficient to warrant a larger investment in the latter. In other words, it will cost more to give equal reliability with the a-c. system than with the Edison system and storage-battery reserve. However, investigations tend to indicate that an a-c. system can be devised to give at a lower cost as reliable service as the Edison system without the storage-battery reserve.

There are factors of importance other than reliability and comparative cost which have to be weighed carefully when considering the introduction of an a-c. system into an existing commercial area supplied from an Edison system. These are such things as the investment in present equipment, the cost of changing consumers' equipment, and in designing a combined light and power a-c. secondary system. There are numerous problems such as voltage standards, lamp flicker, metering, and protection. Some engineers believe, after considering these factors, that it is more practical and economical to continue with the Edison system.

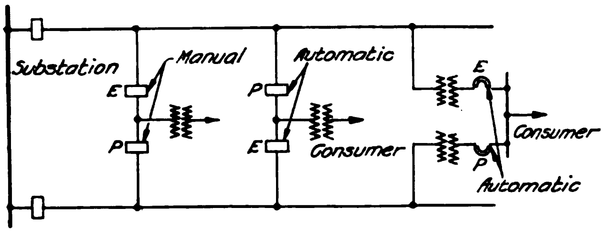


Fig. 1. Diagram of Various Combinations of Preferred and Emergency Feeder Arrangements for A-c. Distribution

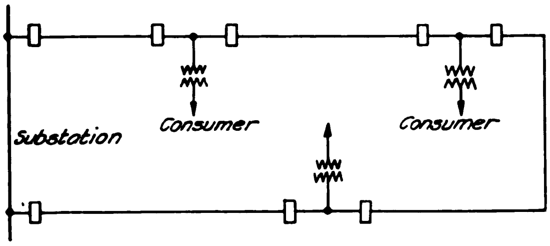


Fig. 2. Loop-feeder Arrangement of A-c. Distribution to Supply Large Consumers

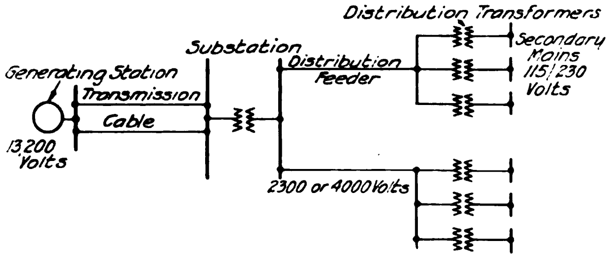


Fig. 3. Section of a Typical Radial A-c. Distribution System

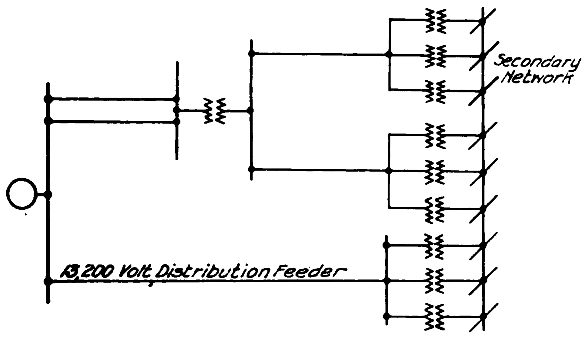


Fig. 4. Connection of the Radial Circuit in Fig. 3 into an A-c. Network to which is Added a 13,200-volt Distribution Feeder

It is well to mention in this connection some of L. L. Elden's remarks on the subject of relative costs of distribution systems: (1)

> "Many of those present will be interested to note the many elements which it has been necessary to consider where attempts have been made to compare the costs of distribution systems. Cost of land for substation locations varies widely in different cities; $100,000 not being an unusual price in one locality, while equivalent facilities may be secured elsewhere for $3000.
>
> "Cost of buildings varies widely, both as to type, period of construction, and local conditions governing construction methods.
>
> "Substation equipment varies widely as to design and efficiency, both of these factors materially affecting costs.
>
> "Certain systems omit regulating equipment, being satisfied to limit regulation to, say, five per cent, while other important companies take steps to limit voltage variations to two per cent.
>
> "Reserve capacity in transmission lines is nil in some instances, and in other cases insurance is provided reaching 100 per cent where the desire for service of the highest quality is paramount.
>
> "For low-tension service, practice varies from where it provides no facilities to meet emergencies as compared with others where large sums are expended in this direction.
>
> "Cost of capital, taxes, and labor and material for maintenance varies widely throughout the country owing to local conditions.
>
> "The degree of maintenance authorized by a company is a substantial factor in system costs.
>
> "Cost of energy from which to compute losses varies 100 per cent or more according to the source and locality where power costs are incurred.
>
> "Storage-battery reserves in connection with d-c. systems are an important element in comparisons of this character.
>
> "It will, therefore, be seen that comparisons of costs of d-c. and a-c. systems are of no value unless reduced to a comparable basis. It is easily possible to compare two d-c. systems and show that one is better than the other although both are of identical characteristics as to equipment and output.
>
> "One system with a higher load-factor may be highly efficient and may be favored with low-cost energy as well as lower labor and maintenance costs, and thus show more favorable results than another less favorably situated.
>
> "The same applies to a-c. systems, and in fact it may be shown that a favorably situated d-c.

---

(1) From the Proceedings of the Eighteenth Annual Convention of the New England Division of the National Electric Light Association, 1926.

system may even outclass well-designed a-c. systems operating under favorable conditions."

* * *

"In dealing with present and prospective costs it should be noted that the figures which are quoted from time to time, purporting to represent the savings which can be effected in the future, if a definite system is installed, are merely the figures on paper which are never proved. Advances in the art over a period of years generally modify plans for expansion with attending variations and changes in costs from those which were originally developed.

"In considering the projects outlined at this meeting, it may well be that any and all of them should be used in future developments of various systems, but it may fairly be suggested that each and every location should be studied on its merits to determine what system of distribution is best suited to each case, regardless of whether it is best to utilize networks or some other form of distribution."

Where it is decided to use alternating current in the commercial area, some form of duplicate supply or a-c. secondary network system must be used. The duplicate supply system consists of some form of preferred and emergency feeder arrangement or the well-known loop-feeder arrangement for important and large consumers. Fig. 1 illustrates various types of the former and Fig. 2 the latter.

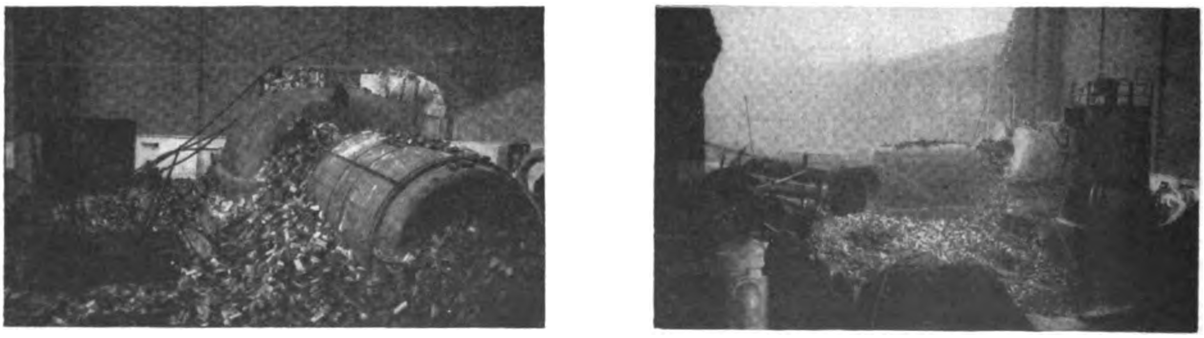
The low-voltage a-c. network has been applied to the commercial areas in about fifteen cities during the past three years. It is believed by the companies' engineers that some form of secondary network is necessary to approach the continuity of service given by the Edison system. It is also believed that the secondary network can give, at a better economy, a reliability equal to the Edison system without a storage-battery reserve.

Some cities that receive their supply from distant hydro-electric stations over high-voltage transmission lines are subject to occasional interruptions of service due to transmission-line faults. In such cases it seems inadvisable to discontinue the use of the Edison system with storage-battery reserve. The portion of the commercial area which is supplied from the a-c. system may not benefit from a low-voltage network because the interruptions of service due to distribution-system faults are a very small percentage of those due to transmission-system faults. A transmission-line fault affects all or a large part of the city while a distribution-circuit fault affects only the consumers connected to it. Some form of preferred and emergency feeder arrangement may be sufficient until the transmission system reliability is increased.

Sometimes the low-voltage a-c. network may be utilized to give not only a more reliable service but with a better economy than continuing with the present radial system. Such a case is illustrated in Figs. 3 and 4. The usual radial distribution method (Fig. 3) is utilized until the load density increases to such a point as to justify forming a low-voltage network from the existing secondary mains, transformers, and feeders, and supplying the increase in capacity required direct from the 13,200-volt or 26,400-volt system without the necessity of building additional transformer substations as shown in Fig. 4. As the distribution transformer size and spacing increases with an increase in load density, it becomes more favorable to form a network and to utilize economically the higher-voltage transformer. These cases are more likely to occur in those areas which contain a large number of apartment houses and apartment hotels.

(*To be continued*)

Turbines Survive Tornado



Eight Hours After a Tornado Had Literally Brought Down the House at the Venice (Ill.) Plant of the Union Electric Light and Power Company, One of the Station's Smaller Units Had Been Put Back Into Operation

Daylight Recording by Means of the Photo-electric Cell

By **DR. L. R. KOLLER**

Research Laboratory, General Electric Company

> *The recording of daylight intensity is another one of the applications of the versatile photo-electric cell. This device, coupled to suitable automatic recording devices in an altogether simple arrangement, provides a means that has fascinating possibilities for investigating daylight conditions. Limitations of the visual photometer in this service are pointed out by Mr. Frank Benford in his article on page 87.*—EDITOR

THE variations in the intensity of daylight are of interest and importance to every one; perhaps especially so to the illuminating engineer, the power company, the meteorologist, and the farmer. A convenient method of observing and recording these variations automatically has been made possible by the development of large, sensitive, photo-electric cells.

The photo-electric cell is that member of the vacuum-tube family which has the property of allowing a current to flow through it when light falls upon it, the current being very nearly proportional to the intensity of the light. As the light varies in intensity, the current through the cell varies accordingly and so can be used as a measure of the light. One method of doing this is to pass the photo-electric cell current through a small electrolytic cell and collect in a graduated tube the hydrogen which is generated. This method integrates the light, the total quantity of hydrogen collected being a measure of the total quantity of light which has fallen upon the cell over a given period of time. If this current is passed through a recording meter, the variations in the daylight intensity are recorded automatically and accurately in permanent form on a strip of paper.

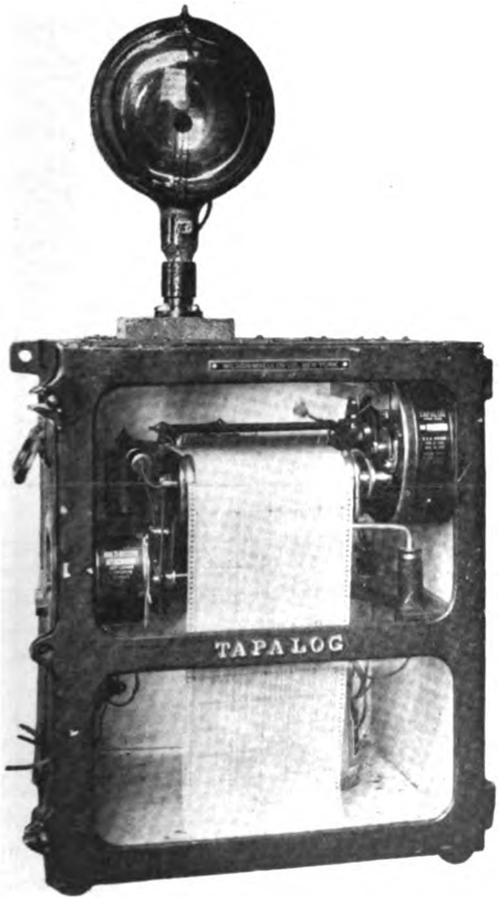


Fig 1. Photo-electric Cell and Current-recording Device which Was Used to Produce the Daylight-intensity Records Illustrated in This Article

Fig. 1 shows the apparatus used for this purpose in the laboratory, consisting simply of a large caesium photo-electric cell, a radio "B" battery, and a standard form of recording meter having a full-scale deflection of one-half a milliampere. The simplicity of this apparatus, which operates without the use of any amplifiers, is now made possible by the availability of high-sensitivity cells which give large currents for small light intensities.

The electrical connections are shown in Fig. 2. The negative terminal of the photo-electric cell is connected in series with a recording meter to the negative side of the "B" battery; the positive side is connected to the other cell terminal. Thus the current which passes through the photo-electric cell also passes through the meter. The variations in this current are recorded on the moving strip of paper and are faithful reproductions of the variations in light intensity in that location.

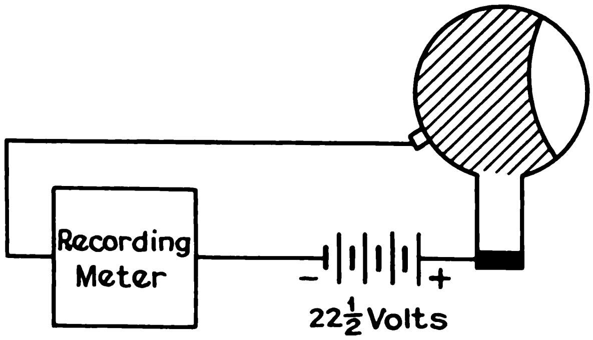


Fig. 2. Elementary Wiring Diagram of the Photo-electric Cell Photometer

Typical records are shown in Figs. 3, 4, 5, and 6. The cell was set up indoors facing an east window, but in such a position that direct sunlight never reached it. The portion of the records shown in these illustrations covers a period of a little more than six hours, the time being indicated by vertical lines at

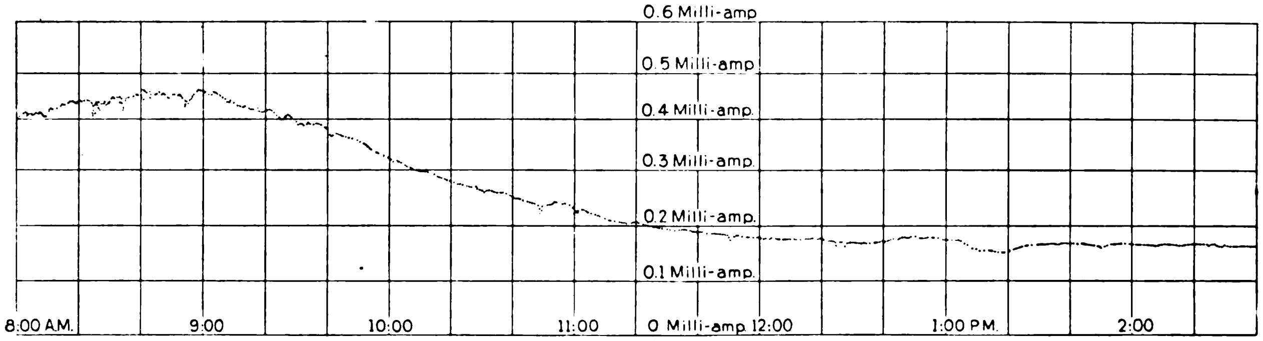


Fig 3. Record Made on a Day During which a Uniform Light Haze Prevailed. The vertical displacements are in terms of the milliamperes passed by the photo-electric cell. These currents are very closely proportional to the intensity of light falling on the cell

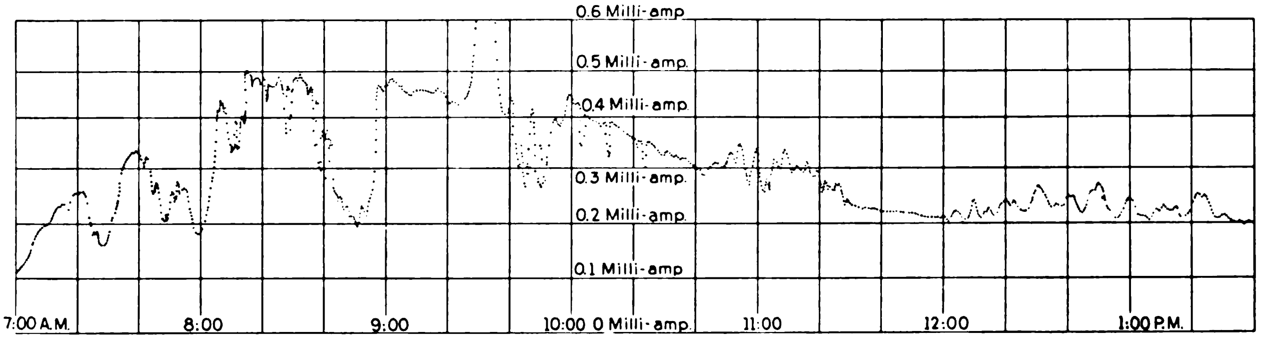


Fig. 4. Record Made on a Day During which Light Clouds Passed Before the Sun

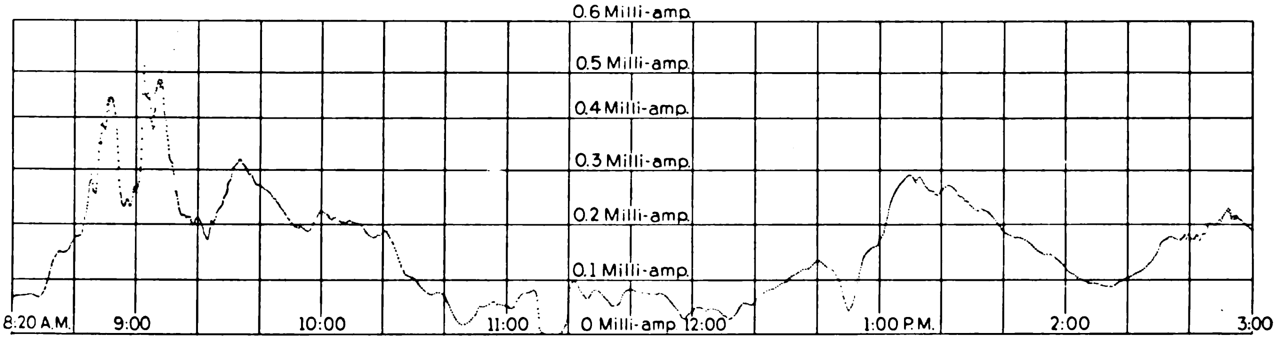


Fig. 5. Record Made on a Day in which a Thunderstorm Took Place in the Late Morning and Was Followed by the Return of a Dense Cloud About 2 p.m.

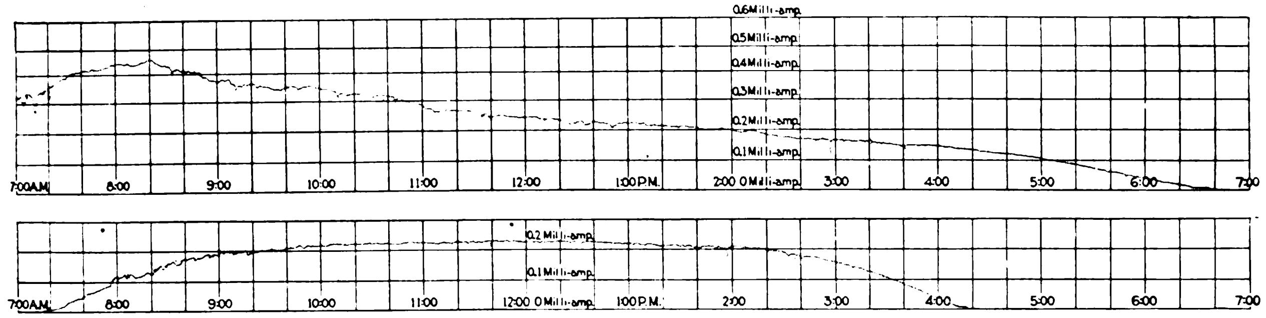


Fig. 6. A Comparison of a Day in July with a Day in December as Shown by the Photo-electric Cell Daylight Recorder

20-min. intervals. All four records indicate a general decrease in light brilliancy from morning to afternoon which is to be expected in the case of an eastern exposure. Aside from this same general trend, the curves show a number of interesting differences. Fig. 3 is a record made on a day of uniform light haze when there were few minor fluctuations and is, in consequence, comparatively smooth.

Fig. 4, on the other hand, was for a day of small light clouds and shows clearly the fluctuations due to the passage of those clouds across the sun. Fig. 5 shows the interesting phenomenon of a thunder-shower which is indicated by the rather sudden darkening of the sky followed by clearing after the storm, and then another shorter period of darkness due to the passage of a large thunderhead.

Daylight Measurement by Means of the Visual Photometer

By FRANK BENFORD
Physicist, Illuminating Engineering Laboratory, General Electric Company

Where the source of illumination is under control, the ordinary or visual photometer is a thoroughly practical device for measuring the intensity of the light. The investigation of daylight, however, introduces difficulties that lead the author to make favorable reference to the possibilities of the photo-electric cell photometer, p. 85.—EDITOR

A RECENT investigation, to gain information about the present daylight conditions in a seven-story factory building, and to get data that could be used to compute the effect of another building that it is proposed to erect beside the present one, made very apparent some of the more trying limitations of the customary visual photometer. The nature of the photometric problem called for data from which all effects due to variations in daylight had been eliminated, and a study of the data of the tests suggests that the visual photometer (combined with the usual generous "equation" of the operator) is not wholly adequate for such requirements.

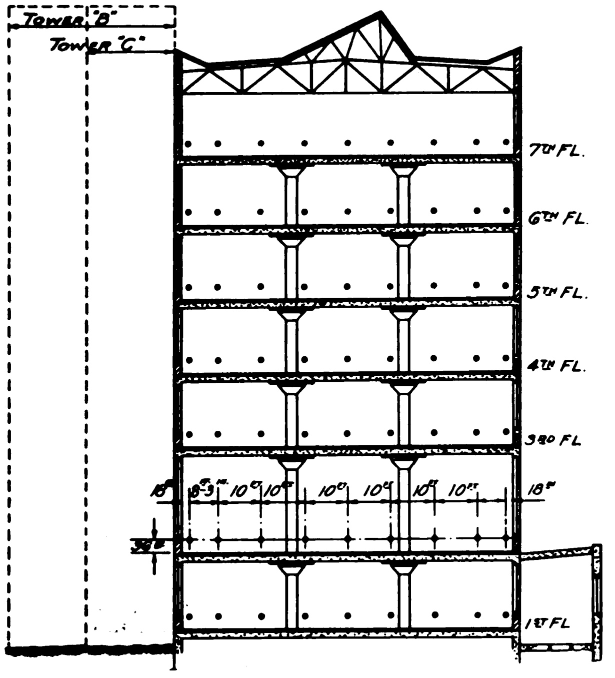


Fig. 1. Vertical Section, Between Towers *B* and *C*, of a Factory Building, the Daylight Conditions of which Were Investigated Along Its Length by a Visual Photometer Transferred from Station to Station (represented by small circles in this section)

The building under investigation is 600 ft. long, 80 ft. wide, and has seven floors. Even if the photometrist could go to all stations without interruption, and spend only two minutes at each arranging the photometer and reading it, over two hours would be required to make a traverse of all floors. During this period, daylight may readily change by a factor of two or three—as is illustrated by the curves in Figs. 2 and 3. Fig. 2 is the plotted data from readings made at three-minute intervals on a stationary photometer located on the fifth floor. A record of time was kept along with the photometric readings, and the same procedure was followed by the photometrist making the floor traverses, the plan being to correct the floor readings for daylight variations as measured at the stationary photometer.

An examination of the curves suggests that a variation of one minute between the recorded and

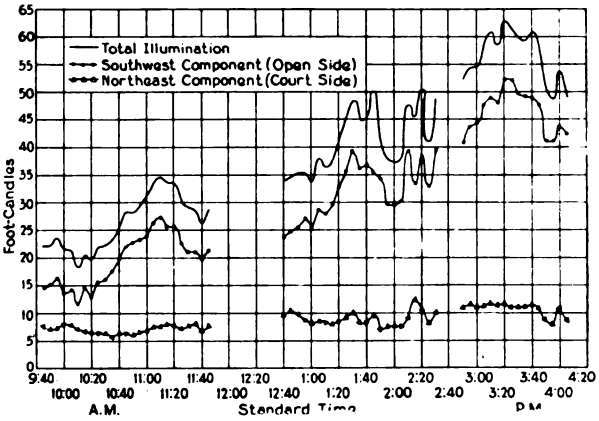


Fig. 2. Curves Plotted from Data Taken at Three-minute Intervals, by the Visual Photometer Method, at a Station on the Centerline of the Fifth Floor of the Building Shown in Fig. 1 and Midway Between Towers *B* and *C*

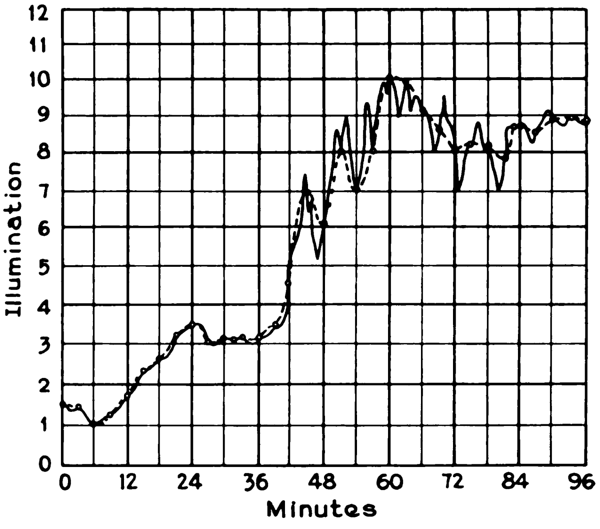


Fig. 3. The Full-line Curve is Reproduced from a Photo-electric Cell Record of Daylight Intensity. The original record consisted of a series of dots automatically made at 15-sec. intervals, therefore practically continuous. The circle points were afterward located at three-minute interval spacing and the smooth dotted curve drawn through them for comparison

the actual time of reading might make a difference of five to ten per cent in the recorded value. The same also applies to the record of the traveling photometer, and in using one record to correct the other the probable errors are of too great a size to permit of fair photometric accuracy.

The solid curve in Fig. 3 is a daylight record taken at another time in another city, and it is here included to illustrate how the use of a recording photometer of the photo-electric cell type eliminates the errors of visual photometry. The instrument made a record of the illumination every fifteen seconds, or twelve times as frequently as has been found practicable for a photometric exploration that takes nearly three hours. These points on the original record sheet form practically a continuous line and are therefore to be regarded as a faithful continuous record of daylight intensities. The circles were "spotted" on the curve at three-minute intervals to represent instantaneous readings taken by a visual photometer at the same time spacing as used in the photometric test.

The broken curve, drawn through the three-minute points, is notably smoother than the fifteen-second record; and there are at least eleven points that might be read ten per cent in error due to a one-minute error in recording the time of actual balance.

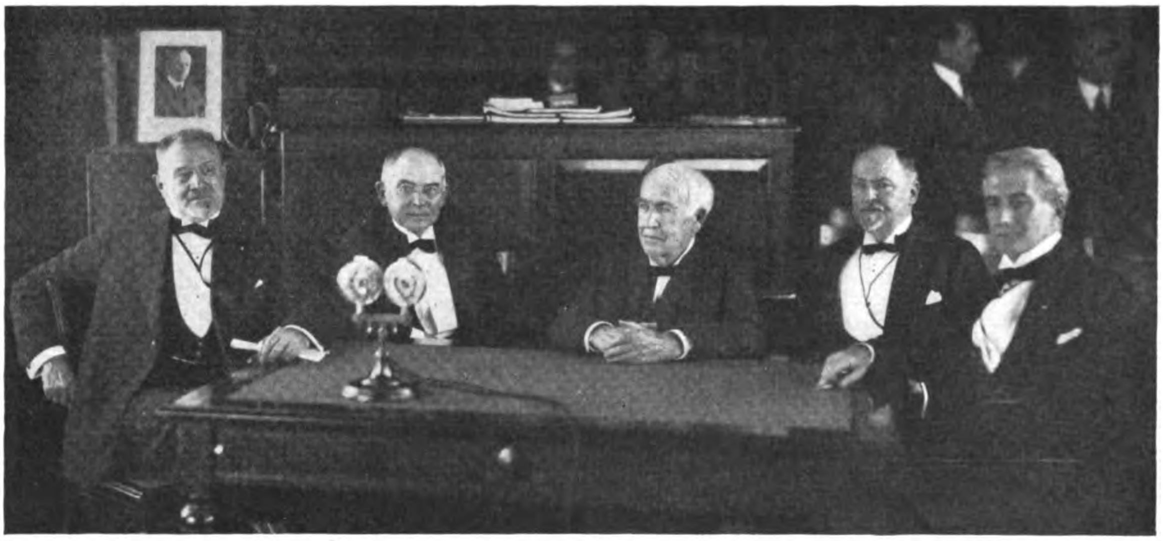
The experience gained in these tests makes it evident that a photo-electric photometer, and particularly a portable one, would be a valuable instrument for daylight measurements; and some recent developments in the photo-electric cell and its auxiliary parts lead to the hope that this instrument will appear in practice in the near future.

Edison's Voice and Photograph Recorded by the Kinegraphone

ON the latest anniversary of the invention of the incandescent lamp by Thomas A. Edison, in 1879, the ceremonies took a unique form, in that for the first time the inventor's voice was broadcast over radio. This was accomplished through a network of forty-three stations of the National Broadcasting Company. Among these were the two General Electric short-wave stations at Schenectady, 2XAF and 2XAD, by means of which the program was heard on several continents.

In addition to the broadcasting of the voice of Mr. Edison, motion-picture and voice records were made simultaneously; the former at his laboratory in West Orange (N. J.), and the latter in the laboratory at Schenectady, nearly two hundred miles away. Later, the two films were combined into a "talking movie" or Kinegraphone film.

The film is of standard size. Along the left margin, in a space about one-eighth of an inch wide, the voice is recorded as a small saw-toothed line. When the record is being reproduced, a beam of light passing through this margin becomes a broken ray and falls upon a photo-electric cell, which translates the light into electrical impulses that are changed into sound again by a radio receiver and loud speaker. The projector used in showing such a film is of the standard theatre type with the addition of the device for reproducing the sound.



John W. Lieb, E. W. Rice, Jr., George F. Morrison, and Gerard Swope with Mr. Edison in the Library of His Laboratory on Edison Night

The record of Edison Night was made possible through the maintenance of synchronism between the power supply at Schenectady and at West Orange. How this was accomplished is described in the article by Mr. Lawson which appears on page 89 of this issue.

Holding Two Isolated Power Systems at the Same Frequency

An Undertaking That Contributed to the Successful Recording of a Talking-Motion Picture of Mr. Edison and Others—Joint Operation of the Two Systems and the Telephone Company Involved

By JAMES T. LAWSON
Assistant to General Superintendent of Generation,
Public Service Electric & Gas Company of New Jersey

In the laboratory of Thomas A. Edison at West Orange (N. J.), a celebration was held on October 21, 1927, to commemorate the 48th anniversary of the birth of the incandescent lamp; and this meeting was broadcast over a great chain of radio stations throughout the United States and Canada.

One of the unusual features of the evening was the making of records from which a "talking-motion picture" of the event could later be produced. In connection with the making of these records was a detail of noteworthy interest to power system operators. This was the part played by the Adirondack Power & Light Company of New York, and the Public Service Electric & Gas Company of New Jersey. The two companies had been requested to hold their system frequencies exactly at 60 cycles from 10 to 11 p.m. on that evening to enable the synchronous taking of motion pictures of the meeting at West Orange (N. J.) and the recording of the speeches, music, etc., on a film at Schenectady (N. Y.); the motion-picture film and the voice-record film of the broadcasting to be combined later to make the "talking-motion picture."

The apparatus for taking the motion picture at West Orange was to be driven by synchronous motors from service supplied by the local Public Service Electric & Gas Company of New Jersey, and that for recording the speeches, etc., at Schenectady was to be driven by synchronous motors from service supplied by the Adirondack Power & Light Company. To make the undertaking a success it was necessary that the motors run at exactly the same speed. The engineers of the local company promised to aid in every way possible to make the endeavor a success but were initially at a loss as to how their system and the distant one, *which were not interconnected*, could be operated without some slight frequency variation, even with every precaution to keep each system individually at exactly 60 cycles and with no knowledge of what variation might occur in the other company's system.

From 10 to 11 p.m. there is usually a reduction in load on the Public Service system, which would also tend to vary the frequency. While no great amount of frequency variation is experienced at any time, operation without any variation at all had been requested. As the two systems are not interconnected, the local power engineers thought it would be a great help in keeping the systems at the same frequency if it were possible to have two frequency meters, one connected to each system, placed side by side, so that their joint readings might serve as a guide to the operation of the systems of the two companies.

A consequent study of the two companies' transmission lines by the Public Service engineers showed that the Adirondack Power & Light Company was interconnected through the Central Hudson Company of New York to the New Jersey Power & Light Company, which in turn was connected with the Jersey Central Power & Light Company. The Jersey Central Power & Light Company feeds the territory in New Jersey west of and adjacent to that served by the Public Service Electric & Gas Company. It was also learned that these interconnections were operated in multiple so that the frequency of the Adirondack system was the same as that of the Jersey Central Power & Light Company. This information was obtained and confirmed at about 3 p.m., October 21st, the meeting in West Orange being set for 10 p.m. the same night.

Several schemes to make use of this knowledge were conceived, one of them being to find two residences which were close enough together to permit setting up frequency meters connected to the two systems. The thought then occurred to the Public Service engineers that if it were possible for the Telephone Company to select a telephone pair from its Millburn (N. J.) Exchange, which is served by the Jersey Central Power & Light Company, this pair could be connected to a lamp socket at Millburn and the Adirondack frequency brought in over it to the load dispatcher's office of the Public Service Electric & Gas Company in Newark (N. J.) which is approximately 12 miles from Millburn. This plan was taken up immediately with the Chief Engineer of the New Jersey Bell Telephone Company, who, when the situation had been explained to him, replied that if it were at all possible he would take care of assigning a telephone pair and connecting it to a lamp socket in the Millburn Exchange at once. The Telephone Company immediately set to work and by 4 p.m. the telephone connection was made available.

Two synchronous timers were set up, one connected to the Jersey Central Power & Light Company—Adirondack Power & Light Company through the

Millburn connection as described and one connected to the Public Service Electric & Gas Company. Each of the synchronous timers had two indicating pointers, one of which traveled around the dial at one-second intervals and the other at twelve-second intervals; the twelve-second pointer giving an audible click as it passed the twelve-second mark. By setting the hands together and listening for the click of the two instruments and by watching the hand movement, it was possible to see and to adjust the frequency of the two systems. The object was to keep the two system frequencies exactly alike.

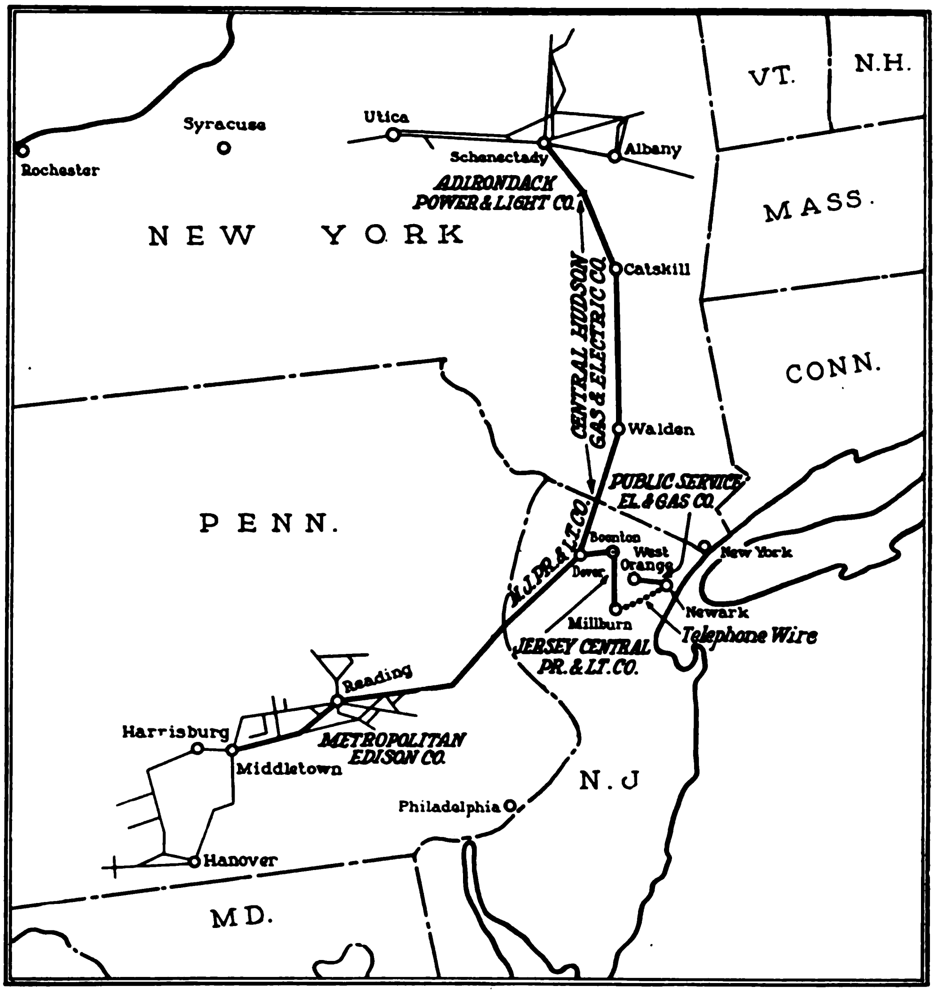


Fig. 1. Map Showing the Shortest Run of the Transmission and Telephone Lines Used in Securing the Frequency Indications

After an interval of observation it was evident that an indication of the frequency of the two systems had been made available, and the telephone connection was temporarily discontinued. There remained yet to arrange for the joint operation of the two systems. For this purpose, telephone communication was established by way of Dover (N. J.) to Reading (Pa.), where the system operator of the New Jersey Power & Light Company and other interconnected companies is located. This Reading operator was in control of the frequency of the Adirondack Power & Light Company at Schenectady. At 9:40 p.m. the frequency-indicating telephone pair was again connected between the Millburn Exchange and the office of the load dispatcher of the Public Service Electric & Gas Company in Newark and the two synchronous timers were used to bring the two systems together at exactly 60 cycles for the prearranged hour of 10 p.m.

The frequency indications over the telephone pair were maintained for the entire period from 9:40 to 11:15 p.m., the load dispatchers of the two systems holding the frequencies together as closely as possible during this time.

The readings of the speed variations were tabulated. These indicate that the precision in observing the gain of one system over the other was probably two cycles where the deviation was small, and about three cycles for the larger deviations. The maximum gain that took place during the entire period was 17 cycles; but for a great portion of the time the two systems were held exactly together without any variation. The greatest difficulty in holding them together occurred from 10:55 to 11 p.m., when apparently load was dropping off both systems very rapidly. This made it extremely difficult to hold the frequencies together without the fraction of a cycle variation as requested.

The General Electric Company's engineers say the undertaking was a success and they have combined the picture and voice film records. The contribution of the Adirondack Power & Light Company and the Public Service Electric & Gas Company was made possible largely through the aid of the New Jersey Bell Telephone Company in making the rapid connection and allowing use of its telephone wires for frequency-indicating purposes. Without these wires and the two system indicators, a wider range of variation would have undoubtedly occurred.

Quite apart from the success of the General Electric Company in combining the voice records with the motion-picture records, the success attained in the attempt to hold two large generating systems at exactly the same frequency without their being connected is considered of sufficient importance to bring it to the attention of operating men at large.

The Cahokia Substation

Service Performed by New Substation—Illumination of Structure—Connection of Power Lines at Substation—Layout of Present and Future Circuits—Substation Equipment

By STANLEY STOKES

Power Supervisor, Union Electric Light and Power Company, St. Louis, Mo.

A NEW 66-kv. substation known as the Cahokia substation was put into operation recently by the Union Electric Light and Power Company of Illinois. The construction of this substation was made necessary after entering into a contract with the Illinois Power and Light Corporation to furnish a supply of power for most of the latter's Southern Illinois requirements. Part of this load was to be supplied at Venice (Ill.), five miles north of the Cahokia Plant, and the Belleville (Ill.) load was to be supplied at Cahokia over a new two-circuit line. Moreover, it was very desirable to have a sizeable connection between the newly acquired Venice Plant and the Cahokia Plant. This puts the Venice Plant on a reserve basis and at the same time gives the Illinois Power and Light Corporation more flexibility of supply.

The Alton load of the East St. Louis Light and Power Company is also being supplied out of the Venice station over a new two-circuit line. Previous to the acquisition of the Venice Plant, the Alton load had been supplied from Cahokia by way of East St. Louis (Ill.); but the increasing demand for the 60-cycle service for the industrial district in the Alton territory made it imperative to supply additional capacity. This fitted in very well with the Cahokia-Venice tie, since Alton also is located on the Mississippi River, seventeen miles north of Venice. With all of these requirements in mind, it was decided to build the 66-kv. substation with an installed capacity of 100,000 kv-a., and an ultimate capacity of 150,000 kv-a.

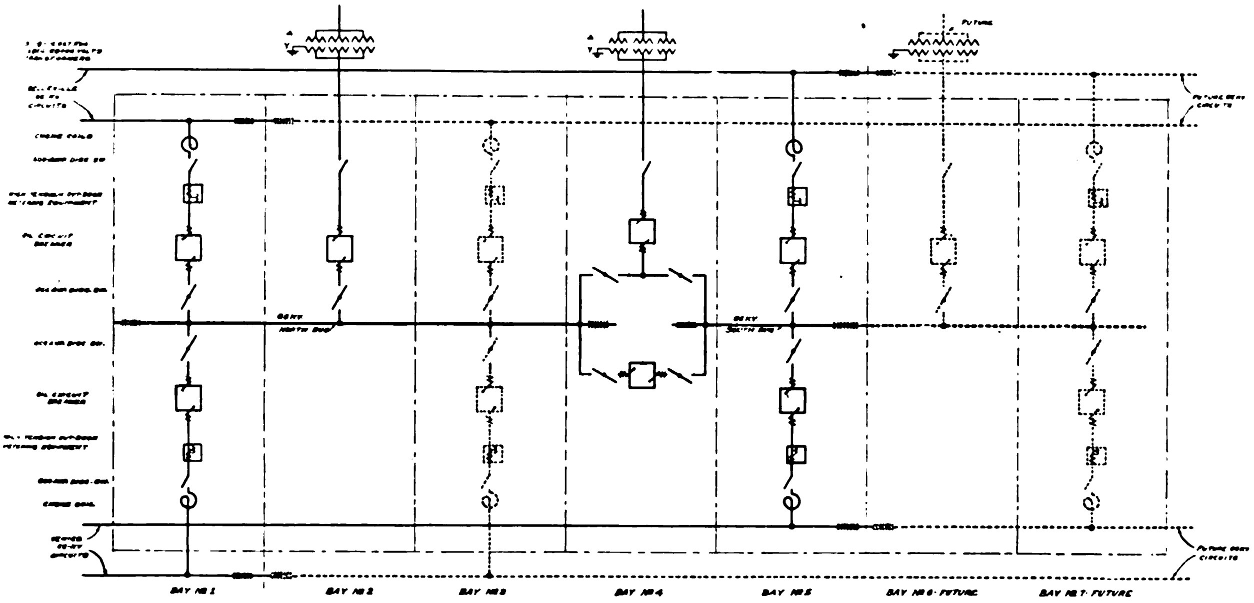


Fig. 1. Schematic One-line Diagram of Connections of the Cahokia Substation

Flood Lighting

The night view of the Cahokia substation, shown in the frontispiece of this issue, indicates the effects which may be created by adequate and well-directed floodlighting. On account of the fact that this substation is of such great importance to the various companies served, in respect to flexibility and continuity of service, it was necessary to supply such a degree of illumination that any switching to be done would not be hindered by poor lighting.

Nineteen floodlighting units are arranged on the structure in such a way that the beams are concentrated on the switching equipment. The spread of the light, however, overlaps sufficiently to produce a uniform intensity of illumination throughout the entire substation, eliminating to a surprising degree any disagreeable shadows from the larger equipment. Eleven of the units are equipped with 500-watt lamps and eight with 200-watt lamps. The larger ones are placed along the top of the structure and fence and the smaller units are located at other advantageous points. Some of the units can be seen readily in the frontispiece; others were blocked out or shaded while making the photographic exposure.

Outgoing Lines

At present there are four outgoing 66-kv. circuits, leaving the structure from the north on two steel tower lines. The circuits in all cases are dead-ended on the structure; and the last tower on each line is an angle tower. One line connects the Cahokia substation to a new 66-kv. substation on the Venice property. This line consists of two circuits of 350,000-cir. mil copper on steel towers built along the river front. The other line extends to the Illinois Power and Light Corporation's new substation in Belleville, a distance of approximately fourteen miles. The two circuits on this line are 4/0 A.C.S.R., also on steel towers. An ultimate installation of four more circuits can be accommodated from the south by addition of two more bays and switching equipment.

Structure

The steel structure as now completed is 75 ft. high, 150 ft. long and 50 ft. wide, consisting of five bays, each 30 ft. by 50 ft. At present there are two bays for feeders, two for transformers, and one for future feeder requirements. The latter bay, with an additional one for transformers and one for feeders, will take care of the four future circuits mentioned. The general layout is evident from the one-line diagram of the substation shown in Fig. 1. The structure was designed mechanically to act as a dead-end for the outgoing lines. Both north and south lines dead-end on the same bracket. After future requirements have been provided for, the substation will have four identical bays for outgoing circuits, two for transformers; and the seventh or center bay will contain the bus junction as well as the transformer connections of the reserve set of transformers. The switching for this transformer bank is such that it can be switched to either the north or south bus by means of air break switches.

Equipment and Feed

The transformers now installed are six 16,667-kv-a. units, single-phase, self-cooled and wound for 13,800/69,000 volts. They are connected delta on the low and star on the high side. The feed to the transformers from the Cahokia Plant, dimly outlined to the left in the frontispiece illustration, is supplied through eighteen 1,500,000-cir. mil single-conductor underground cables. These cables are of stranded conductor over a spiral core, and are insulated to avoid sheath currents and consequent electrolysis. They are terminated directly on the low-voltage buses, three for each phase, and brought up from the conduit run through concrete pillars containing fibre conduit.

Of the seven oil circuit breakers installed, six are for the line and transformers and have a 600-amp. rating; and the seventh is a bus junction breaker having a rating of 1200 amp., all of them of 73,000 volts. The air break switches are of 600 amp. rating and are gang operated. The lightning arresters are of 66 kv.

Relays and Metering

All of the circuit breakers are equipped with two bushing-type current transformers for relay operation. The relays are all of the induction type and perform the usual functions of overload protection, current balance, and directional and overcurrent protection.

In addition, metering equipment has been incorporated on all outgoing feeders. The metering has been designed to give totalizing demand and kilowatt-hour data, these being of prime importance due to the contracts with other companies, all service being supplied on both demand and energy basis.

The Municipal Pier End of the New White Way at St. Petersburg (Fla.)

Heaviside's Operational Calculus as Applied to Engineering and Physics

PART II: THE NATURE AND PRACTICAL VALUE OF THE OPERATIONAL SOLUTION

PART III: THE EXPANSION THEOREM

By DR. ERNST JULIUS BERG
Consulting Engineer, General Electric Company
Professor of Electrical Engineering, Union College

HEAVISIDE evidently concluded that the use of differentials and integrals was undesirable —if it were possible to do without them. He also believed that even the most primitive electrical engineer could apply Ohm's law to the permanent condition in a direct-current network involving only e.m.f.'s and resistances. In short, he adopted the premise that in the circuit illustrated in Fig. 2 any electrical engineer could show that the total resistance is

$$R_0=\frac{RR_1}{R+R_1}$$

Thus

$$i=\frac{E}{R_0}=E\,\frac{(R+R_1)}{RR_1}$$

the steady current after the system has become stable.

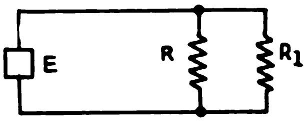


Fig. 2

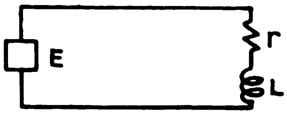


Fig. 4

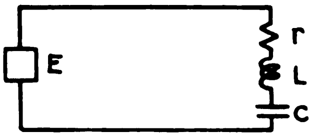


Fig. 6

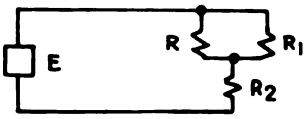


Fig. 3

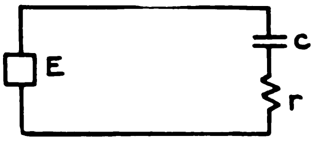


Fig. 5

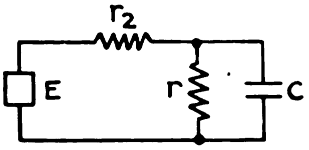


Fig. 7

Again, in the circuit shown in Fig. 3 he assumed it as evident that the total resistance was

$$R_0=R_2+\frac{RR_1}{R+R_1}$$

and that the current in any branch can easily be found as a function of the steady impressed e.m.f.

He then considered more complicated circuits as, for instance, that shown in Fig. 4. Here a resistance is connected in series with an inductance. The instantaneous relation

$$e=ir+L\,\frac{di}{dt}$$

he assumed as well known. Thus under his conditions we write

$$E\mathbf{1}=ir+pLi=i\,(r+pL)$$

In the case shown in Fig. 5, where the circuit consists of a condenser in series with a resistance, the instantaneous relation is

$$e=ir+\frac{1}{C}\int idt$$

Thus under his conditions,

$$E\mathbf{1}=ir+\frac{i}{pC}=i\left(r+\frac{1}{pC}\right)$$

In the case shown in Fig. 6

$$e=ir+L\,\frac{di}{dt}+\frac{1}{C}\int idt$$

Thus under his conditions,

$$E\mathbf{1}=ir+pLi+\frac{i}{Cp}=i\left(r+pL+\frac{1}{pC}\right)$$

His so-called "resistance operator" for a circuit of resistance and inductance in series is $z=r+pL$. Note that this does not mean that the impedance is $r+pL$. It merely means that if $\frac{1}{z}$ operates on the unit function of E, the current under Heaviside's condition can be found. The resistance operator for a circuit of resistance and capacity in series is $z=r+\frac{1}{pC}$.

The operator in the case of resistance, inductance, and capacity in series is $z=r+pL+\frac{1}{pC}$.

Consider next the circuit shown in Fig. 7, which might represent a leaky condenser in series with a resistance. We may proceed to set up the equation of the steady current as if the various branches contained resistances only, *i.e.*, from Fig. 3 we get

$$R_0 = R_2 + \frac{RR_1}{R+R_1}$$

Substituting for the R's the corresponding resistance operators we get:

$$z_0 = r_2 + \frac{r\,\frac{1}{pC}}{r+\frac{1}{pC}} = r_2 + \frac{r}{1+rpC} = \frac{r_2+rr_2pC+r}{1+rpC}$$

Therefore

$$i = \frac{E\,\mathbf{1}}{z_0} = E\,\frac{(1+rpC)}{r+r_2+rr_2pC}\,\mathbf{1} \qquad (8)$$

This relation he calls the *operational solution* and writes it:

$$i = E\,\frac{Y}{Z}\,\mathbf{1} \text{ or } E\,\frac{Y_{(p)}}{Z_{(p)}}\,\mathbf{1} \qquad (9)$$

Thus in this case

$$Y = 1+pCr$$
$$Z = r+r_2+rr_2pC \qquad (10)$$

It is evident that the operational solution is not limited to finding the current caused by the application of a steady voltage E at $t=0$. Another problem might be to find, for instance, the operational solution of the e.m.f. consumed by the condenser at any time after the application of E at the terminal of the network. To get this solution, it is at present better for the student to refer to the equivalent d-c. network illustrated in Fig. 3 and to solve for the voltage consumed by R_1 which in that diagram takes the place of the condenser in Fig. 7.

This voltage is readily seen to be

$$e = E\,\frac{RR_1}{RR_2+RR_1+R_1R_2}$$

Substituting the resistance operators we get

$$e = E\,\frac{\frac{r}{pC}}{rr_2+\frac{r}{pC}+\frac{r_2}{pC}}\,\mathbf{1} = E\,\frac{r}{rr_2pC+r+r_2}\,\mathbf{1}$$

Let us return for a moment to Equations (8) and (9).

The question may well be asked why not write the operational solution

$$i = E\,\frac{\frac{1+pCr}{r+r_2+rr_2pC}}{1}\,\mathbf{1} \qquad (11)$$

in which case Y would be $\frac{1+pCr}{r+r_2+rr_2pC}$ and $Z=1$, or why not write

$$i = E\,\frac{1}{\frac{r+r_2+rr_2pC}{1+pCr}}\,\mathbf{1} \qquad (12)$$

in which case Y would be unity and

$$Z = \frac{r+r_2+rr_2pC}{1+pCr}$$

It will be clear from what follows in the proof of the expansion theorem that Equations (8) and (12) are equally applicable, but that Equation (11) would not do.

The rule is that the operational solution must be of such form that when Z, the denominator, is equated to zero we obtain the largest number of roots of p. In this case $Z=0$ gives only one root, $p=-\frac{r+r_2}{rr_2C}$ whether we use Equations (8) or (12); but Equation (11) gives no roots at all. In most problems Z contains higher powers of p than the first when, of course, several roots are obtained.

From this consideration it should be evident how the operational solution is obtained at least in networks of concentrated resistance, inductance, and capacity.

A number of examples will be given later which involve not only resistance, inductance, and capacity, but also mutual inductances. These will help to clear up any doubts that may exist at this time.

PART *III*: THE EXPANSION THEOREM

In Part II has been shown the method of obtaining the operational solution. It must be remembered that the operational solution gives the answer only in case a steady force is suddenly applied to the system at $t=0$. This is, of course, not always the case in actual problems where the forces frequently are varying or indeed where no external forces are applied, as in the case of the discharge of a condenser. Such cases are easily solved, however, as will be shown in Parts VI, VIII, and XIV. As a rule, the solution then depends upon that obtained by first assuming a steady force applied at $t=0$. Briefly, whatever the nature of the problem, it is usually essential to know the solution in the case of a steady force applied at $t=0$, that is, under the so-called Heaviside condition.

In Equation (9) in Part II, the operational solution for the line current was given as $i=E\,\frac{Y_{(p)}}{Z_{(p)}}\,\mathbf{1}$ where $Y_{(p)}$ and $Z_{(p)}$ were certain functions of p.

If $Z_{(p)}$ is of higher power in p than $Y_{(p)}$, then $\frac{Y_{(p)}}{Z_{(p)}}$ is a fraction. If on the other hand $Y_{(p)}$ is of higher power in p than Z^p, it is always possible by division to reduce $\frac{Y_{(p)}}{Z_{(p)}}$ to a sum of terms some of which are integral functions of p and one of them a fraction. So the resultant expression becomes, for instance, $Ap\,\mathbf{1}+B\mathbf{1}+C\,\frac{Y_{(p)}}{Z_{(p)}}\,\mathbf{1}$. In the last case the expression

$\frac{Y_{(p)}}{Z_{(p)}}$ is obviously not the same as that in the first case. The solution of the problem is then the sum of the solutions of $Ap\,\mathit{1}$, $B\mathit{1}$ and $C\frac{Y_{(p)}}{Z_{(p)}}\mathit{1}$.

From what has been said about $p\mathit{1}$ it is evident that this term $Ap\mathit{1}$ contributes something which is infinite at $t=0$ and zero afterward. The solution of the third term, the fraction in p, is obtained by the Expansion Theorem.

Z always contains p in various powers. Thus, in general, $Z_{(p)}$ can be written

$$Z_{(p)}=p^n+ap^{n-1}+\cdots\cdots$$

Let the roots corresponding to $Z_{(p)}=0$ be p_1, p_2, p_3, etc. Therefore

$$Z_{(p)}=(p-p_1)\,(p-p_2)\,(p-p_3)\cdots$$

and

$$\frac{Y_{(p)}}{Z_{(p)}}=\frac{Y_{(p)}}{(p-p_1)\,(p-p_2)\,(p-p_3)\cdots} \qquad (13)$$

When this expression is broken into partial fractions and all roots are different and none is zero, we get

$$\frac{Y_{(p)}}{Z_{(p)}}=\frac{A}{p-p_1}+\frac{B}{p-p_2}+\frac{C}{p-p_3}+\cdots \qquad (14)$$

in which

$$A=\frac{Y_{(p_1)}}{Z'_{(p_1)}},\ B=\frac{Y_{(p_2)}}{Z'_{(p_2)}},\ \text{etc.} \qquad (15)$$

where Z' is $\frac{d}{dp}Z$ and $Z'_{(p_1)}$ is the particular value of Z' when p_1 is substituted for p. For the explanation of this relation see, for instance, Todhunter's "Integral Calculus" (second chapter) or Pierce's "Table of Integrals" (p. 14).

We get also

$$\frac{Y_{(p)}}{Z_{(p)}}\mathit{1}=A\frac{1}{p-p_1}\mathit{1}+B\frac{1}{p-p_2}\mathit{1}+\cdots \qquad (16)$$

Substituting the value for $\frac{1}{p-p_1}\mathit{1}$ from Equation (4), Part I, it is easily seen that Equation (14) becomes

$$\left[-\frac{A}{p_1}-\frac{B}{p_2}-\cdots\right]\mathit{1}+\left[\frac{A\epsilon^{p_1t}}{p_1}+\frac{B\epsilon^{p_2t}}{p_2}+\cdots\right]\mathit{1} \qquad (17)$$

From Equation (14) it is evident that the first parenthesis is $\frac{Y_{(p)}}{Z_{(p)}}\mathit{1}$ for $p=0$ or for the sake of brevity $\frac{Y_{(o)}}{Z_{(o)}}\mathit{1}$. The second parenthesis after substituting the values of $A\ B\cdots$ from Equation (15) is

$$\left[\frac{Y_{(p)}\,\epsilon^{pt}}{p\frac{dz}{dp}}\text{ for }p=p_1+\frac{Y_{(p)}\,\epsilon^{pt}}{p\frac{dz}{dp}}\text{ for }p=p_2+\cdots\right]\mathit{1}$$

Therefore

$$i=E\left[\frac{Y_{(o)}}{Z_{(o)}}+\sum_{p_1p_2p_3\cdots}\frac{Y_{(p)}\epsilon^{pt}}{p\frac{dz}{dp}}\right]\mathit{1} \qquad (18)$$

which is the same equation as Heaviside gives on page 135 (Vol. ii). Note, however, that in this demonstration it has been assumed that all roots of $Z_{(p)}=0$ are unequal and not zero. The method to be used when several roots are equal, or when one root is zero, will be given in Part VII.

Heaviside always omitted writing the unit function $\mathit{1}$ in terms involving the independent variable t, but usually, though not always, wrote it when the solution is given in some function of p. There is a good practical reason for this—we would otherwise have too many $\mathit{1}$'s in the equations. For this reason it seems wise to follow his style and from now on the unit function will be omitted when the solution is given in t. It should be remembered, however, that it belongs there.

It is an interesting fact that while the expansion theorem is deduced under the assumption that $\frac{Y_{(p)}}{Z_{(p)}}$ is a fraction in powers of p, it is not a necessary limitation under the Heaviside condition. The expansion theorem can be used as given in Equation (18) even though the highest power in p is the same in $Y_{(p)}$ and $Z_{(p)}$. Even if the power in p is higher in $Y_{(p)}$ than in $Z_{(p)}$, it is generally not necessary to divide through. Instead, we can proceed in the usual way with the mental reservation that to the result obtained there must be added a term which is zero before and after $t=0$ but is infinite at $t=0$.

Consider the case where the highest power in p is the same in the two expressions.

$$Y_{(p)}=p^n+\alpha p^{n-1}+\beta p^{n-2}+\cdots\cdots\cdots\gamma$$
$$Z_{(p)}=p^n+a\,p^{n-1}+bp^{n-2}+\cdots\cdots\cdots c$$

By division we get:

$$\frac{Y_{(p)}}{Z_{(p)}}=1+\frac{(\alpha-a)\,p^{n-1}+(\beta-b)\,p^{n-2}\cdots\gamma-c}{p^n+\alpha p^{n-1}+\beta p^{n-2}+\cdots c}=1+\frac{Y_{1(p)}}{Z_{(p)}}$$

Here the second term is a fraction in p and we can without hesitation proceed with the expansion theorem.

The solution of the second term generally contains a constant term $\frac{Y_{1(o)}}{Z_{(o)}}$ and a series of terms involving ϵ^{pt}. The solution of the problem is then

$$1+\frac{Y_{1(o)}}{Z_{(o)}}+\Sigma\,k\,\epsilon^{p}$$

$\frac{Y_{1(o)}}{Z_{(o)}}$ would of course be $\frac{\gamma-c}{c}$ so that the total value of the constant term would be $1+\frac{\gamma-c}{c}=\frac{\gamma}{c}$ which is also the value obtained if we had proceeded without the division. It is also evident that since $Z_{(p)}$ is the same whether we divide or not, the number of roots and the values of the roots corresponding to $Z_{(p)}=0$ are the same.

In the first case, when we do not divide through, a number of exponential terms result

$$m_1\,\epsilon^{p_1 t}+m_2\,\epsilon^{p_2 t}+\cdots\cdots$$

In the second case, the same number of terms result

$$k_1\,\epsilon^{p_1 t}+k_2\,\epsilon^{p_2 t}+\cdots\cdots$$

and it happens that under the Heaviside condition $k_1=m_1$, $k_2=m_2$, etc.

Since $Y_{1(p)}=Y_{(p)}-p^n-[Z_{(p)}-p^n]=Y_{(p)}-Z_{(p)}$

Therefore $k_1=\dfrac{Y_{(p)}-Z_{(p)}}{p\dfrac{dZ}{dp}}$ for $p=p_1$. But this is $\dfrac{Y_{(p)}}{p\dfrac{dZ}{dp}}$ since $Z_{(p)}$ for $p=p_1$ must be zero. Thus $k_1=m_1$ and similarly $k_2=m_2$, etc.

Experience has indicated that the student is little likely to go astray in the use of the Expansion Theorem if he introduces a definite order of procedure.

(1) Write down the value of $Z_{(p)}$.

(2) Write $Z_{(p)}=0$ and solve for p which gives values p_1, p_2, p_3, etc.

If $Z_{(p)}$ contains the first power of p, only one root p_1 results.

If $Z_{(p)}$ is quadratic in p, two roots p_1 and p_2 result. A cubic equation gives three, etc.

(3) Differentiate $Z_{(p)}$ in respect to p, *i.e.*, find $\dfrac{dZ_{(p)}}{dp}$.

(4) Write out $p\dfrac{dZ_{(p)}}{dp}$.

(5) Substitute in (4) $p=p_1$ $p=p_2\cdots\cdots$etc.

(6) Write out $Y_{(p)}$ and substitute $p=p_1$, $p=p_2$, etc.

(7) Write $\dfrac{Y_{(p)}}{Z_{(p)}}$ and substitute in this equation $p=0$ to find $\dfrac{Y_{(o)}}{Z_{(o)}}$.

In general, the solution then contains one constant term $\dfrac{Y_{(o)}}{Z_{(o)}}$ and a number of terms

$$C_1\,\epsilon^{p_1 t}+C_2\,\epsilon^{p_2 t}+\cdots\cdots$$

where C_1 is $\dfrac{Y_{(p)}}{p\dfrac{dZ}{dp}}$ for $p=p_1$, $C_2=\dfrac{Y_{(p)}}{p\dfrac{dZ}{dp}}$ for $p=p_2$, etc. At this point the mathematician may say that this is nothing but the usual solution of a linear differential equation where the right-hand number is a constant; and this is of course the truth. The beauty of Heaviside's treatment lies in the wonderfully simple method used to determine the integration constants C_1, C_2, etc. They are obtained by the simplest kind of operation involving only the knowledge of differentiation in respect to p.

The simplicity of his method is especially appreciated when dealing with differential equations of infinite order, such as are met in systems of "distributed" constants where the number of integration constants are infinite and often hopelessly difficult to evaluate by conventional methods.

Heaviside really uses two rather distinct methods in solving his problems: one, and the safest one, the Expansion Theorem; the other, the "algebrizing" method in which case he expands the operator in a power series.

The disadvantage of the first method is that it is often very laborious. $Z_{(p)}=0$ frequently gives a number of complex roots, difficult to locate.

The disadvantage of the second method is that it leads frequently to series expansions which are very difficult to interpret, which are valid over certain regions only, and which even in those regions give only a fair approximation.

Unfortunately, his operational calculus which depends upon the expansion of functions in a power series is poorly fitted to certain problems as will be shown in Part XVI. But there are shorter ways as will be discussed in that part and elsewhere. The use of Duhamel's integral (Part XIV) or the vector equations when dealing with permanent conditions due to periodic forces is strongly to be recommended. If the "algebrizing" method gives a series which is not convergent or is convergent only over a certain region, do not expect it to tell the whole story; it seldom does. Try then another method. Heaviside says (page 492, Vol. ii) in connection with expansions involving divergent series: "There is more to be said on this subject and I have no doubt a good deal more will be said when proper mathematicians will thoroughly explore divergent series for physical purposes."

It looks as if Carson were well on the way.

(*To be continued*)

Vacuum Tubes as Oscillation Generators

PART III: DESIGN OF THE SIMPLER VACUUM TUBE CIRCUITS

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IT has already been shown that a vacuum tube oscillating circuit must contain a certain amount of stored energy in order to maintain oscillations. The energy leaves the circuit as output at a different instantaneous rate than it enters as input, and, consequently, some flywheel effect is necessary.

To see how this energy storage effect may be calculated, consider a circuit having an inductance L, a capacity C, and oscillating at a frequency f and voltage V with W watts loss. At a point in the cycle when the current is zero, the instantaneous voltage will be $\sqrt{2}\ V$; all the energy is stored in the condenser; and its amount is CV^2. The r.m.s. current in this circuit is $2\pi fCV$, and the joules lost per cycle are W/f. Dividing energy stored by joules lost per cycle gives:

$$\frac{\textit{Energy stored}}{\textit{Energy lost per cycle}}=\frac{CV^2}{W/f}=\frac{fCV^2}{W}=\frac{2\pi fCV^2}{2\pi W}=\frac{VI}{2\pi W}$$

which shows that the ratio of energy stored to energy dissipated per cycle is $\frac{1}{2\pi}$ times the ratio of reactive power to watts. Circuits having less than twice as much energy stored in them as they dissipate per cycle have been found by actual test to have a tendency toward erratic operation. Circuits having a greater relative amount of stored energy give excellent operation but require more condenser capacity and more expensive inductance coils, and the useless power losses in the oscillating circuit are higher. Unless, therefore, there is some other determining factor, circulating volt-amperes should approximate 4π times the total power output in watts as a desirable minimum value. Another way of saying this is that the power-factor should not be over 8 per cent or, in the older radio nomenclature, the decrement should not exceed 0.25. This circulating energy has the same effect as the flywheel of a single-acting engine and must be coupled closely to the plate, *i.e.*, the circuit must be so arranged that the sinusoidal voltage wave of the oscillating circuit is available at the tube terminals without distortion due to the current passing between the two. As a mechanical analogue, consider the value of a flywheel driven through a spring—an oscillating circuit connected to a vacuum tube through a loose coupling of any sort would be equally ineffective.

After assurance is obtained that the circulating energy is adequate, the design of an oscillating circuit becomes a matter of obtaining the proper voltages and phase relations for the leads to the tube. Two general types of circuit are in common use, and, in either of these, the production of the desired values is quite simple.

Fig. 22 shows a form of the Hartley circuit. In this circuit, the oscillating current is made to flow through two inductances or one inductance with a tap. The voltage drops across the two windings are used as the alternating components of the plate and grid voltages. Energy is supplied at constant potential from the blocking condenser C_2. This condenser supplies the pulses of current drawn through the tube at the frequency of the oscillating circuit. The charge is replaced at a more uniform rate from the direct current source G through the choke L_c. The oscillating circuit consists of the plate inductance L_p, the grid inductance L_g, the condenser C_1, and the load resistance r_L. It will be noticed that the pliotron, the oscillating circuit, and the immediate source of energy C_2, are connected in series in a manner similar to that already described in Part II. The impedance of the load resistance will be quite small compared with the other impedances in the oscillating circuit, and its effect on the voltage distribution may be neglected in an approximate calculation. The function of the grid leak resistance r_g and bias condenser C_3 is to maintain a bias for the grid voltage. The condenser has sufficient capacity to pass the current pulses through to the grid without distorting the voltage wave, but the average current thus passed is discharged at a uniform rate through the resistance, thereby providing a steady bias voltage.

In designing a circuit of the type just described, the first step is to determine the volt-amperes in the oscillating circuit, which value should be at least 4π times the watts output. The voltages across L_p and L_g are obtained from the curves giving optimum operating conditions for the tube, and this fixes the current and impedance of the various parts of the oscillating circuit. The value of the grid leak resistance is also obtained from the optimum operating conditions curves. This leaves only the plate choke L_c, blocking condenser C_2, and grid leak condenser C_3 to be determined. In general, if the choke has enough inductance, so that it passes only a very small current, and the condensers have sufficient capacity to present no appreciable impedance to the currents which they carry, the circuit will operate satisfactorily. There are certain refinements to be considered if the best design practice is to be followed, but consideration of these will be deferred.

Fig. 23 shows a form of the Colpitts circuit which differs from the Hartley circuit in that the plate and grid alternating voltages are obtained by taking the voltage drop across two condensers instead of across two inductances. The oscillating circuit, therefore, consists of the plate condenser C_p, the grid condenser C_g, the inductance L_1 and effective resistance r_L. A plate choke L_c and blocking condenser C_B are used as in the Hartley circuit. The capacity C_g serves also as the grid leak condenser. In order to prevent a loss in the grid leak resistance, r_g, due to the alternating voltage across the condenser, the former has in series with it a substantial choke coil, L_2. The Colpitts circuit offers practically the same design problems as the Hartley circuit and will therefore not be considered separately.

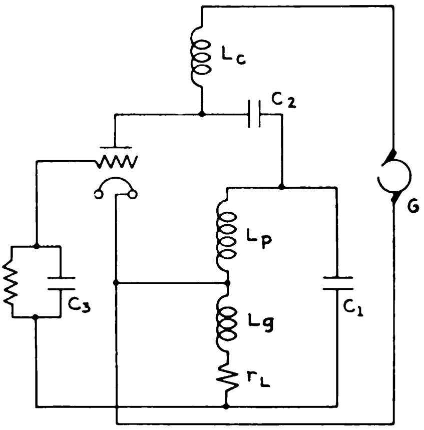

Fig. 22. The Hartley Circuit

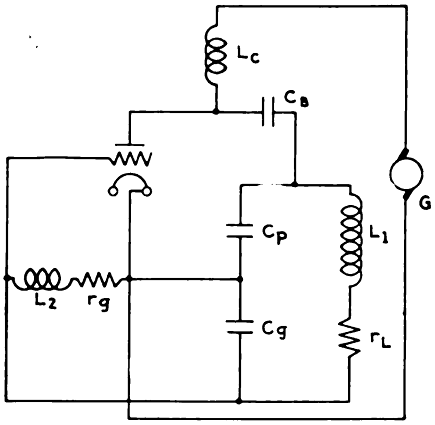

Fig. 23. The Colpitts Circuit

An example will serve to illustrate the methods of calculation just described. Assume a Hartley circuit as in Fig. 22 driven by a one-kilowatt pliotron operating at 15,000 volts direct potential. The curves of optimum operating conditions are given in Fig. 21. The output is to be one kilowatt at 10^6 cycles.

For an output of one kilowatt, the circulating volt-amperes immediately available in the oscillating circuit will be 4 π, or about 12.5 kv-a.

From Fig. 21, the minimum instantaneous plate volts should be 630. Deducting this from 15,000 volts leaves 14,370 for the peak value of the plate alternating voltage, which corresponds to 10,160 volts r.m.s. Fig. 21 gives also the grid alternating voltage, which is 2690 volts in this case. Adding the plate and grid voltages together gives 12,850 volts as the total a-c. drop across the oscillating circuit.

In order to be conservative, it will be assumed that only that part of the energy stored in the oscillating circuit which corresponds to the volt-amperes in the plate coil is coupled closely enough with the vacuum tube to be useful as flywheel effect. This means that the plate coil must carry a current corresponding to the 12,500 volt-amperes at 10,160 volts, or 1.23 amp., and this is the circulating current of the oscillating circuit. Hence, there results the accumulation of the data in Table IV.

The grid leak resistance should be about 165,000 ohms, from Fig. 21, and a condenser of a few thousandths of a microfarad capacity representing an impedance of less than a hundred ohms should operate quite nicely as a grid leak condenser, C_3. A condenser of similar capacity, but insulated for a higher voltage, should be satisfactory as a plate-blocking condenser C_2. It is very desirable to keep the high frequency current out of the supply source, because of the damage it can do there. As far as the radio frequency is concerned, the choke coil, L_c, is in parallel with the plate coil, L_p, for there will be enough capacity in the leads and windings of the generator G, to make sure that it will present very little impedance to high frequency. Hence, if the choke coil has an inductance of several hundred or a thousand times that of the plate coil, the operation will probably be altogether satisfactory. As a practical precaution, however, it is advisable to shunt G with a condenser to by-pass any high-frequency current reaching its terminals.

A point of the greatest importance, which is sometimes overlooked, is the fact that the effective resistance of the oscillating circuit is absolutely fixed. Since the voltages in the circuit permit only slight variation without greatly disturbing the efficiency, it is necessary that the load absorb the correct power at the current fixed by this condition. In the case in hand, the current is 1.23 amperes and the output one kilowatt, which means the effective resistance of the oscillating circuit must be 660 ohms. If the load itself does not have this actual resistance, it must be coupled into the oscillating circuit in such a way that it presents that effective resistance. Considerations which follow, dealing with more complicated circuits, will illustrate some of the ways in which this can be accomplished.

TABLE IV

Element of Circuit	Volts	Amperes	Impedance	Value at 10^6 Cycles
Plate coil, L_p	10,160	1.23	8,270	1.32 milli-henries
Grid coil, L_g	2,690	1.23	2,185	0.348 " "
Tuning condenser, C_1	12,850	1.23	10,450	0.000,0152 mfd.

(*To be continued*)

Graphical Determination of Magnetic Fields

PART I

THEORETICAL CONSIDERATIONS

By A. R. STEVENSON, JR., and R. H. PARK
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IN the design of electrical apparatus, the accurate calculation of the characteristics of a proposed design depends on the knowledge of the distribution of magnetic flux in the machine.

> *The refinements of modern electrical design have made more than ever necessary an accurate knowledge of flux distribution, particularly within the current-carrying region. In this two-part article the authors present in mathematical form and in sets of rules for graphical constructions the disclosures of American and European research. This material was presented also at the 1927 Winter Convention of the A.I.E.E.*—EDITOR

A few shapes assumed by the lines of magnetic flux lend themselves to rigorous mathematical analysis. One of the most famous examples of this is the calculation of the fringing coefficient due to the slots along the periphery of an induction motor air-gap, by F. W. Carter (1) who used the theory of functions of a complex variable to obtain a solution of Laplace's equation.

The mathematical complications encountered in analyzing the magnetic fields surrounding many of the irregularly shaped iron boundaries commonly found in electrical machinery, however, have led many authors to advocate and describe graphical and experimental methods. One of the best bibliographies on this subject is given by J. F. H. Douglas in an article (2) in which he reviews the many methods suggested by other authors, but especially emphasizes the use of high-resistance templets for experimentally plotting a field.

There are many articles (3) that describe the graphical method of plotting magnetic fields in air, but almost none of them give any suggestion of a method for plotting fields within current-carrying conductors. One of the best articles on the subject is "The Free-Hand Graphic Method of Determining Stream Lines and Equipotentials," by L. F. Richardson (4). This author makes the statement that in a region occupied by current, the difference of successive chequer ratios in a direction perpendicular to the lines of force, divided by the mean chequer area, is equal to a constant times the current density in the region (5). But he goes on to say: .

"To drawn chequers free-hand, so as to satisfy a difference relation of this sort between chequer ratios, is likely to be toilsome, and we will here consider only the case where $\Delta^2 v = 0$."

The remainder of Richardson's paper is given up to the sketching of various fields, in none of which does he take into account the effect of current-carrying conductors in the space occupied by the flux.

The most complete treatise on the free-hand plotting of fields of magnetic flux will be found in a series of French articles by Lehmann (6). The 1909 article contains many valuable suggestions for plotting fields in air, and the 1923 articles give an elaborate set of rules for plotting magnetic fields in regions where the curl of the field is not zero, *i.e.*, in space occupied by current-carrying conductors.

It has always been said that there are no lines of magnetic equipotential in current-carrying space. This is true, but Lehmann pointed out that a system of lines orthogonal to the lines of force can be drawn. He called these lines gradients, and directed attention to the fact that they all converge to one or more points in the copper which he called points of indifference.

Consider the field in and around a cylindrical conductor. The lines of force are circles. The gradients, or lines of no work, as the authors prefer to call them, are radial lines; and the point of indifference, or kernel, is the center of the wire. It is also seen that the lines of no work on leaving the copper become lines of equipotential.

This conception of lines of no work radiating from one or more kernels greatly facilitates the plotting of fields in copper. A very brief set of rules for plotting the flux in the current-carrying regions has been compiled by Lloyd P. Shildneck, and applied as an illustration to the distribution of flux between the field poles of an alternator. These are included in a later installment of this article.

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(1) "Air Gap Induction," by F. W. Carter, *Electrical World and Engineering*, Vol. 28, No. 22, November 30, 1901, p. 884.

(2) "The Reluctance of Some Irregular Magnetic Fields," by John F. H. Douglas, *Proceedings A.I.E.E.*, Vol. 34, No. 2, 1915, p. 867.

(3) "The Air-Gap Magnetic Field in Non-Salient Pole Machinery," by B. Hague, *World Power*, Vol. 4, October, November, December, 1925.

"Mapping Magnetic and Electrostatic Fields," by Arthur D. Moore, *The Electric Journal*, Vol. 23, No. 7, July, 1926.

(4) "The Free-Hand Graphic Method of Determining Stream Lines and Equipotentials," by L. F. Richardson, *Philosophical Magazine*, Vol. 15, No. 96, February, 1908, p. 237.

(5) A proof of this statement will be found in a footnote in the ensuing installment of this article.

(6) "Graphical Method for Determining the Tracings of Lines of Force in Air," by Th. Lehmann, (Three Installments), *La Lumiere Electrique*, Vol. 8, October 23, October 30, and November 6, 1909.

"The Graphical Determination of Two-Dimensional Magnetic Fields in Regions Where Laplace's Equation is Satisfied, and also in Regions Where the Curl of the Field is Not Zero," (Two Installments), by Th. Lehmann, *La Revuew Generale de l'Electricite*, Vol. 14. September 15 and September 22.

"Sketches of Magnetic Fields in Iron" (Two Installments), by Th. Lehmann, *La Revuew Generale de l'Electricite*, January 9 and January 16, 1926.

In June, 1918, R. E. Doherty and O. E. Shirley published a paper(7) which made practical use of many sketches of magnetic fields in air. Since that time, considerable use has been made of free-hand sketching in the predetermination of the air-gap fields of salient-pole machinery. Except in a few special cases, however, our attempts to sketch magnetic fields in regions occupied by current-carrying conductors were rather unsatisfactory.

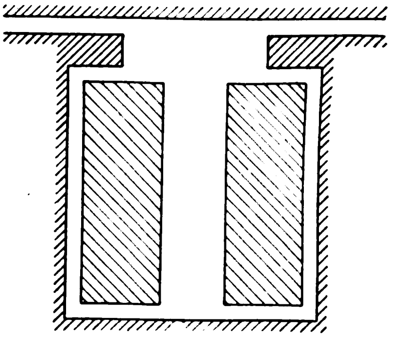
Fig. 1

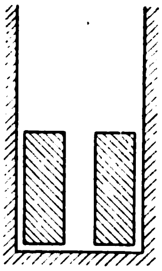
Fig. 2

In the fall of 1924, not yet having noticed Lehmann's 1923 article,(6) we started to investigate this problem seriously. The task of sketching is greatly simplified when one knows in advance the approximate shape which the sketch should have. Not yet having gained the conception of a kernel from which lines of no work radiate, only vague and incorrect ideas prevailed as to the detail distribution of flux within the copper coils surrounding alternator field poles, which was the problem then demanding a solution.

After many attempts to make a reasonable looking sketch, it appeared that it would be worth while to attempt a mathematical analysis of this field, in the hope of obtaining a picture which would at least suggest the general arrangement of the lines of force in the copper. The purpose of this article is to describe this mathematical method, the route by which it was derived, and the additional suggestions for free-hand sketching which resulted from this work.

Professor V. Karapetoff called attention to a most valuable article by Rogowski(8) which was immediately adopted as a basis for our work. Unfortunately, this is out of print in German and had not appeared in English until recently, when it was abstracted and translated by one of the present authors.(9) Rogowski mathematically plotted the leakage field in and around transformer coils. The illustrations in Rogowski's article suggested the kernels or points of indifference.

The two fundamental ideas obtained from Rogowski's work were:

(1) The simultaneous solution of four relatively simple differential equations set up to satisfy the conditions in four adjacent regions into which the field was divided. This simplified the problem greatly because each region, taken by itself, could be analyzed easily; whereas the alternative course, a consideration of all four regions at once, would have been a very difficult task.

(2) This division into regions made it possible to represent the current distribution by a Fourier series of only one of the space coördinates.

The contribution of the present article consists in the extension of Rogowski's method to more complicated magnetic shapes. Rogowski was followed by dividing the field of a salient-pole alternator into several regions the equations for which could be solved simultaneously. The current-carrying region was represented by an infinite series of reflected and re-reflected images, while the mathematical expression for the boundary region between the pole tips was obtained from a free-hand sketch. This combination of a strictly mathematical method with the theory of images and with free-hand sketching, in the parts which can easily be sketched free-hand, into a single consistent system of equations which gives mathematical expressions for the lines of force in the part of the field which cannot be easily sketched, is, we believe, a valuable engineering tool which can be applied to many other problems. Illustrations are given of its application to a problem of rectangular boundaries and also to a problem involving circular boundaries. The method can probably be extended to an even wider range of problems by expressing the equations of Poisson and Laplace in more complicated systems of coördinates.

Experience has shown that, in certain cases, so much sketching and re-sketching is required before the kernel can be found by the free-hand method that this mathematical method could actually be worked out, by the use of comptometers, more quickly than the field could be sketched. This theoretical work led also to additional rules for guiding the construction and checking the accuracy of free-hand sketches.

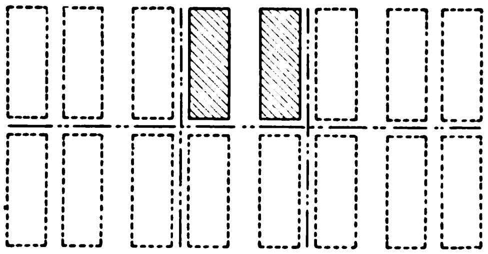
Fig. 3

In presenting the considerations relating to the determination of magnetic fields, the first part of the present article is arranged in three sections, *viz:*

(1) Calculation of the flux leakage between poles in an alternator with approximately parallel pole sides.

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(7) "Reactance of Synchronous Machines and Its Applications," by R. E. Doherty and O. E. Shirley, *A.I.E.E.*, Vol. 37, Part 2, 1918, p. 1209.

(8) Dr. W. Rogowski: *Mitteilungen uber Forschungsarbeiten* No. 71, 1909.

(9) "Fundamental Theory of Flux Plotting," by A. R. Stevenson, Jr., GENERAL ELECTRIC REVIEW, Vol. 29, No. 11, November, 1926, pp. 797-804.

(2) Point conditions in coplanar magnetic fields. In this section are given new rules for constructing and checking free-hand sketches.

(3) Application of the method to polar coördinate systems.

Part of the second and concluding installment is merely in the interest of completeness, as for instance the presentation of rules for plotting flux in air, and a method of plotting two-dimensional magnetic fields in space occupied by current-carrying conductors. The remainder is devoted to valuable original material pertaining to vector potential in coplanar magnetic fields, the relation of the scalar potential function to the vector potential function in coplanar magnetic fields, and the calculation of inductance from a knowledge of vector potential.

Calculation of the Flux Leakage Between Poles in an Alternator with Approximately Parallel Pole Sides

It is desired to calculate the flux distribution in the two-dimensional magnetic circuit of Fig. 1, which is supposed to represent a cross-section of an actual pair of field poles. The shaded rectangles are intended to represent current-carrying copper. It is assumed that the current density is constant over the cross-section of the conductors and that the iron is of infinite permeability.

One step in the solution of the problem, in which the actual magnetic circuit was replaced by an infinitely deep slot, as in Fig. 2, was suggested by one of the writers' associates, C. J. Koch. Under this assumption, the complete magnetic circuit may be replaced by a double row of images extending to infinity, as shown in Fig. 3. It was found that the solution used by Rogowski[10] for the flux in a transformer with pancake coils could be applied to this case, by using a Fourier series for the infinite string of images.

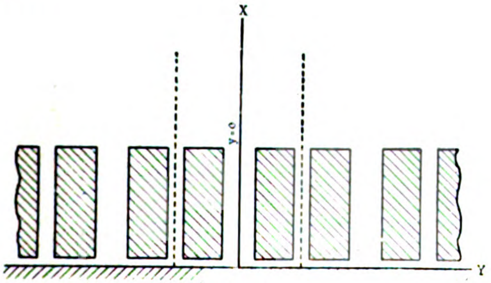

Fig. 4

The problem may be reduced, in fact, to a consideration of an infinite series of rectangular conductors parallel to an infinite plane of iron, as in Fig. 4, and the solution is the same as Rogowski's for the case $\mu = \infty$ except that, since all the currents are of the same sign, there will be a constant term, α_0, in the Fourier series for the current density, which requires an additional term, $2\pi \alpha_0 x^2$, in the equation for vector potential R, in current-carrying space; and it will also be found that the constant terms and coefficients of x will not drop out as they did in the transformer problem.

The solution for dimensions of coils and slots given in Fig. 5, is as follows:

Region I:

$$R = \sum_{1}^{\infty} \frac{2\pi\alpha_n}{(k\,n)^2}[1 - \epsilon^{-kn(b-a)}][\epsilon^{-kn(a-x)} + \epsilon^{-kn(a+x)}] \cos n\theta$$

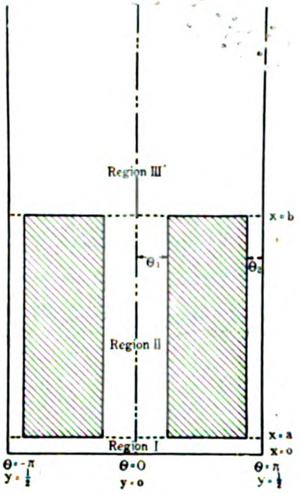

Fig. 5

Region II:

$$R = 2\pi\alpha_0 (x-a)^2 + \sum_{1}^{\infty} \frac{2\pi\alpha_n}{(k\,n)^2}[2 - \epsilon^{-kn(b-x)} - \epsilon^{-kn(x-a)}(1 - \epsilon^{-2kna} + \epsilon^{-kn(b+a)})] \cos n\theta$$

Region III:

$$R = 2\pi\alpha_0 (b-a)[2x - (b+a)] + \sum_{1}^{\infty} \frac{2\pi\alpha_n}{(k\,n)^2}[1 - \epsilon^{-kn(b-a)} - \epsilon^{-2knb} + \epsilon^{-kn(b+a)}]\,\epsilon^{-kn(x-b)} \cos n\theta$$

wherein, for convenience, the y coördinate has been replaced by θ which varies from $-\pi$ to π across the total width of the slot. Also,

$$k = \frac{2\pi}{l} \text{ where } l \text{ is the width of the slot.}$$

$$\alpha_0 = \frac{\pi - (\theta_1 + \theta_2)}{\pi}\, i$$

$$\alpha_n = \frac{2\,i}{n\,\pi}[\sin(n\,\theta_1) + (-1)^n \sin(n\,\theta_2)]$$

where i is the current density in the copper and θ_1 and θ_2 are indicated in Fig. 5. The field distribution for this case is given in Fig. 6.

In order to extend the solution to include the effect of the pole tips and the finite depth of slot, it was suggested that the field be calculated by replacing the actual magnetic circuit with a closed rectangular magnetic circuit of the same dimensions as though the pole faces came together, leaving no air-gap. The effect of the air-gap was to be simulated by a narrow strip of copper along the surface of the iron, assumed to carry a total current equal to the sum of the currents in the two field coil sides (see Fig. 7).

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[10] For translation, see reference given in Footnote (9).

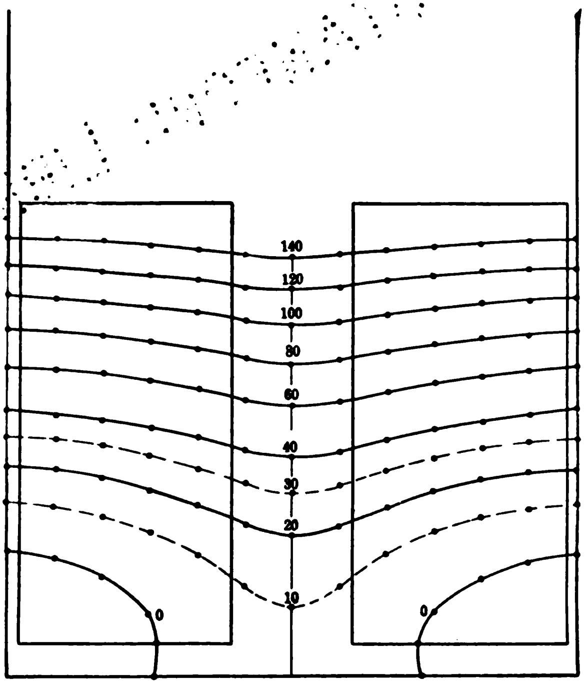


Fig. 6

A better method of solution is provided by first estimating one component of the field in the air-gap between the pole faces and using this estimated field component as a boundary condition. The procedure is as follows:

The field at the air-gap is first calculated or plotted free-hand on the basis of a definite potential between the field poles.

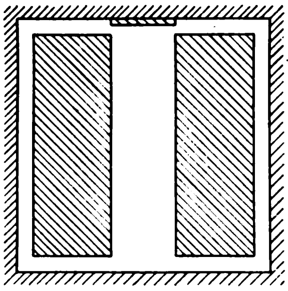


Fig. 7

The Y component of the field along the line $d\ d'$ (Fig. 8) is zero from d to e. It is approximately inversely proportional to the vertical distance S between two lines of force in the neighborhood of the line $d\ d'$ from e to e' and is again zero from e' to d'. The arrangement is symmetrical with respect to the x axis.

The variation of Y is calculated in this way from d to d', or from $\theta = -\pi$ to $\theta = \pi$, and is plotted to any convenient scale. The general character of the curve is shown in Fig. 9.

This curve of Y along the upper boundary of Region III can be expressed as a Fourier series(11) of the form:

$$Y = \beta_0 + \sum_1^\infty k\,n\,\beta_n \cos n\,\theta$$

Unfortunately, it seems necessary to consider a dozen or so harmonics in using this equation as a boundary condition. The solution for the vector potential R at any point in the three regions is then given by three equations:

Region I:

$$R = \sum_1^\infty \left[a_n\,\epsilon^{-kn(a-x)} + a_n'\,\epsilon^{-knx}\right] \cos n\theta$$

Region II:

$$R = 2\,\pi\,\alpha_0\,(x-a)^2 + \sum_1^\infty \left[b_n\,\epsilon^{-kn(b-x)} + b_n'\,\epsilon^{-kn(x-a)} + 2\,K_n\right] \cos n\theta$$

Region III:

$$R = 2\,\pi\,\alpha_0\,(b-a)\,[2\,x - (b+a)] + \sum_1^\infty \left[c_n\,\epsilon^{-kn(c-x)} + c_n'\,\epsilon^{-kn(x-b)}\right] \cos n\theta$$

where the six constants of integration determined from the original Fourier series for current distribution and the six boundary conditions, one of which is another Fourier series, as are follows:

$$a_n = \frac{\beta_n\,\epsilon^{-kn(c-a)} + K_n\left[1 - \epsilon^{-kn(b-a)} + \epsilon^{-kn(2c-b+a)} - \epsilon^{-2kn(c-a)}\right]}{1 - \epsilon^{-2knc}}$$

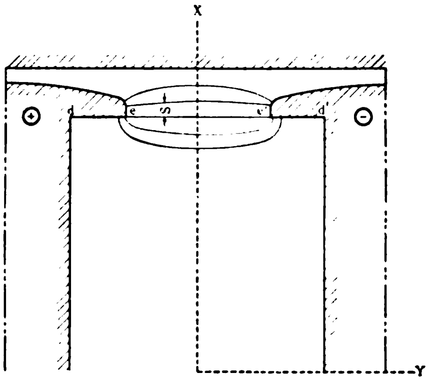


Fig. 8

$$a_n' = a_n\,\epsilon^{-kna}$$

$$b_n = \frac{\beta_n\,\epsilon^{-kn(c-b)} - K_n\left[1 + \epsilon^{-kn[2c-(a+b)]} - \epsilon^{-2kn(c-b)} - \epsilon^{-kn(2c+a-b)}\right]}{1 - \epsilon^{-2knc}}$$

---

(11) The determination of the Fourier series will be simplified if the curve (Fig. 9) between $-\theta_0$ and $+\theta_0$ is assumed of the form: $Y = f + g\,\theta^2 + h\,\theta^4$.

$$b_n' = \frac{\beta_n\, \epsilon^{-kn(c+a)} - K_n \left[1 - \epsilon^{-2kna} + \epsilon^{-kn(b+a)} - \epsilon^{-kn(2c+a-b)}\right]}{1 - \epsilon^{-2knc}}$$

$$c_n = \frac{\beta_n + K_n\, \epsilon^{-kn(c-b)} \left[1 - \epsilon^{-kn(b-a)} + \epsilon^{-kn(b+a)} - \epsilon^{-2knb}\right]}{1 - \epsilon^{-2knc}}$$

$$c_n' = \frac{\beta_n\, \epsilon^{-kn(c+b)} + K_n \left[1 - \epsilon^{-kn(b-a)} - \epsilon^{-2knb} + \epsilon^{-kn(b+a)}\right]}{1 - \epsilon^{-2knc}}$$

wherein the letters a, b, and c refer to Fig. 10, and

$$K_n = \frac{2\,\pi\,\alpha_n}{(k\,n)^2}$$

By assuming that the function of Fig. 9 between $-\theta_0$ and $+\theta_0$ may be represented in the form,

$$Y = f + g\,\theta^2 + h\,\theta^4$$

the values of β_n become

$$\beta_n = \frac{\left[\begin{array}{c}\left[\dfrac{1+h'\,\theta_0^4}{n} + \left(g' - \dfrac{12\,h'}{n^2}\right)\left(\dfrac{\theta_0^2}{n} - \dfrac{2}{n^3}\right)\right] \sin n\theta_0 \\ + \left[\dfrac{4\,\theta_0^3\,h'}{n^2} + \left(g' \dfrac{12\,h'}{n^2}\right)\dfrac{2\,\theta_0}{n^2}\right] \cos n\theta_0\end{array}\right]}{\dfrac{\theta_0}{2} + \dfrac{g'\,\theta_0^3}{6} + \dfrac{h'\,\theta_0^5}{10}} \bullet \left[\frac{2\,\pi\,\alpha_0\,(b-a)}{(k\,n)}\right]$$

where:

$$g' = \frac{g}{f}$$

$$h' = \frac{h}{f}$$

Any inaccuracy in the determination of g' and h' will result in inaccuracy in the field plot only in the immediate vicinity of the pole tips.

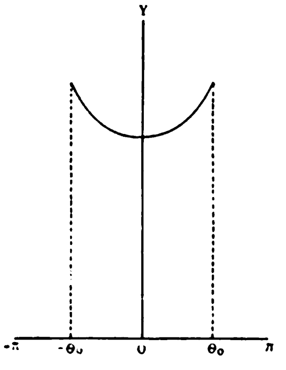


Fig. 9

In the foregoing equation, θ_0 is the value of θ at the pole face edge, as indicated in Figs. 9 and 10, and k and α_n are the same as in the solution for the infinite slot.

The solution, assuming the current density such that $\alpha_0 = \frac{1}{2\,\pi}$, is shown in Fig. 11. The dotted lines in the figure indicate that portion of the flux distribution which was sketched in free-hand.

Although the problem has been worked out for the particular case of the flux distribution between two field poles, it is evident that the same method may be applied to any problem with rectangular boundaries. In case the magnetic circuit or the current distribution is not symmetrical about a central axis, it will be necessary to express R as a series of sines and cosines.

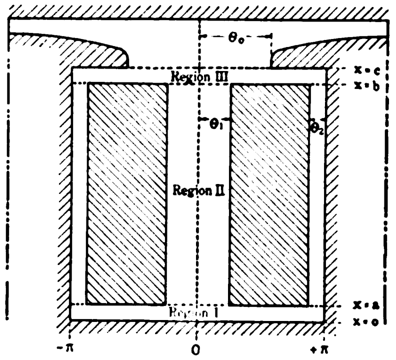


Fig. 10

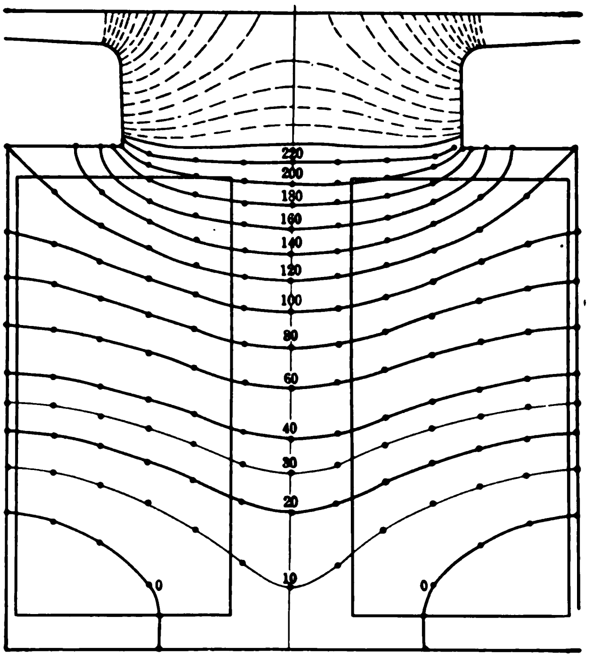


Fig. 11

Point Conditions in Coplanar Magnetic Fields

The theory used in the preceding calculation can be made to answer various questions which arise in the free-hand sketching of magnetic fields; and it also offers various criteria for judging the accuracy of the sketch.

The question is sometimes asked whether there is any refraction of the lines of force at a copper-air boundary. As a preliminary theorem it will be proved that there is no such refraction, or discontinuity in slope, but that the radius of curvature of a line of force may change suddenly when the line crosses the boundary of a current-carrying region. This change in the curvature of the field at the boundary of current-carrying copper is readily detected on a correctly drawn field plot, especially where the lines emerge from the copper perpendicularly and the field intensity is low.

Theorem I: There is no refraction at a copper-air boundary.

In studying the possibility of refraction at copper-air boundaries where the permeability is continuous, the classical problem of refraction at a boundary where the permeability is discontinuous will be considered, taking into account a flow of current on one side of the boundary.

Fig. 12 indicates the boundary of two regions, I and II, of permeabilities μ_1 and μ_2. The normal and tangential components of the magnetic field on either side, but immediately adjacent to the boundary, are H_{n1} and H_{n2}, and H_{t1} and H_{t2}, respectively.

In order that the total flux emanating from the space included by the dotted area shall vanish, it is necessary that the integral of the outward component of flux around the area must vanish. If the flux density is everywhere finite, in the limiting case where the area is made vanishingly thin the flux emanating from the ends of the area may be neglected. Therefore

$$\int_a^b \mu_1 H_{n1}\, dt = \int_a^b \mu_2 H_{n2}\, dt$$

and

$$\mu_1 H_{n1} = \mu_2 H_{n2} \qquad (1)$$

From the application of the condition that the line integral of the field around the dotted area must equal 4 π times the current included, and the assumption that the current density is finite, it follows in the limiting case, in which the area is made vanishingly thin, that the current included, and therefore the complete line integral around the boundary, vanishes, *i.e.:*

$$\int_a^b H_{t1}\, dt + \int_b^a H_{t2}\, dt = 0$$

Therefore,

$$H_{t1} = H_{t2} \qquad (2)$$

From Equations (1) and (2), it is evident that the well-known rule, stating that at any point of a magnetic field, the normal component of flux density μH_n and the tangential component of field intensity H_t are continuous across any boundary, holds even when there is current flowing on one side of the boundary, provided that the current density is not infinite at the point under consideration.

Since the permeability of copper and air are approximately equal, it therefore follows that there can be no discontinuity in the normal and tangential components, or in the direction of the field at the boundary of the copper, even though it be carrying current, provided that the current density at the boundary is not infinite.*

Theorem II: There will be a change in the normal gradient of tangential field intensity at a boundary of current-carrying copper.

At a copper-air boundary the relation, curl $H = 4\,\mu\, i$, may be written in the form,

$$\frac{\partial H_t}{\partial n} - \frac{\partial H_n}{\partial t} = 4\,\pi\, i \qquad (3)$$

where the vectors n, t, i form a right-handed system of unit vectors, as in Fig. 13, and t and n are respectively tangent and normal to the copper-air boundary. Since H_n is continuous across the boundary, it follows that $\partial H_n / \partial s$ is continuous across the boundary, where the term s refers to distance measured along the boundary. But, at any point on the curve, neglecting infinitesimals of higher order, the arc and the tangent coincide. Therefore, $\partial H_n / \partial s = \partial H_n / \partial t$ and $\partial H_n / \partial t$ is also continuous across the boundary, from which it follows that

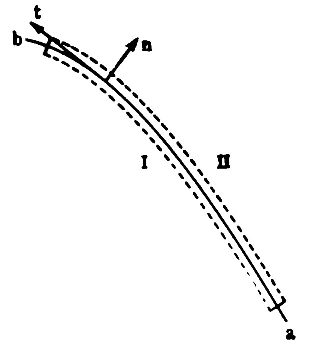


Fig. 12

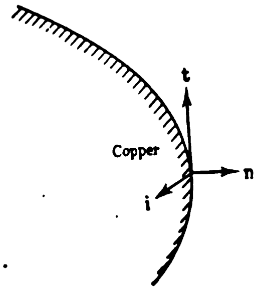


Fig. 13

$$\Delta\left(\frac{\partial H_t}{\partial n}\right) = 4\,\pi\,\Delta\,(i) \qquad (4)$$

The discontinuity in $\partial H_t / \partial n$ across the boundary at any point, therefore, is equal to 4 π times the discontinuity in current density at that point.

Theorem III: At any point in the field, the quotient of the magnetic density divided by the radius of

---

* In the analysis of practical electrical problems involving coplanar fields, it is sometimes convenient to idealize to the extent of assuming the exciting currents to be concentrated in a ribbon of negligible thickness, *i.e.*, to assume that a finite amount of current flows into the plane of the field through a line drawn in that plane.

In this case, the current density is infinite for points on the line. It will be readily verified that, at a boundary along which a linear distribution of current density exists, Equation (2), Theorem I, becomes:

$$H_{t2} - H_{t1} = 4\,\pi\, i'$$

where i' is the current density at the point in question.

In particular, for a linear distribution of current density along an iron surface, we have:

$$H_{t1} = 0$$

and thus:

$$H_{t2} = 4\,\pi\, i'$$

i.e., the tangential field intensity along an iron surface faced with a distribution of linear current density depends only on the value of the linear density at the point in question. This relation, which is obvious as soon as appreciated, may be seen to necessitate the generally understood rule that equipotential lines shall enter such a surface at points separated so as to mark off sections including equal amounts of current. From the point of view that such equipotential lines when continued into the copper to form lines of no work shall enclose equal amounts of copper, this conclusion might appear to be open to question. A further obvious point, however, that is sometimes neglected, is that equipotential lines and lines of force must, in general, enter such linear distributions of current at a slant.

curvature of the lines of force, minus the rate of change of the magnetic density along a line normal to the lines of force, equals 4 π times the current density in the region under consideration.

Equations (3) and (4) may be used to check the accuracy of a field plot sketch, but a somewhat more convenient relation, and one which involves the curvature of the field directly, is available.

Let the curve in Fig. 14 be a line of force within a field carrying current. It is desired to investigate its curvature at the point 0. Shift the axes of coördinates so that the origin coincides with the point in question, and rotate them until the x axis is tangent to the curve as shown.

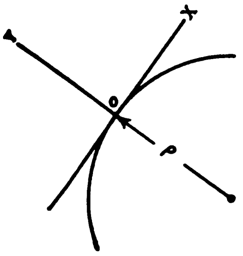


Fig. 14

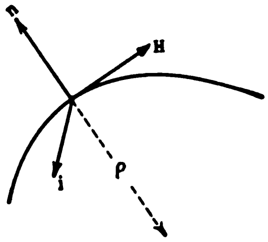


Fig. 15

Using these axes, the curve can be expressed by an equation,

$$y=f\ (x)$$

The relation between the curl of the field and the current density is

$$\frac{\partial Y}{\partial x}-\frac{\partial X}{\partial y}=4\,\pi\,i \tag{5}$$

And since X and Y are the components of the field intensity, the slope of the line of force can be expressed:

$$\frac{d\,y}{d\,x}=f'\ (x)=\frac{Y}{X} \tag{6}$$

or

$$Y=X\frac{d\,y}{d\,x}$$

Differentiate this(12) with respect to x;

$$\frac{\partial Y}{\partial x}=\frac{d\,Y}{d\,x}=\frac{d\,X}{d\,x}\frac{d\,y}{d\,x}+X\frac{d^2\,y}{d\,x^2}$$

But, since the x axis is tangent to the field at the point in question,

$$X=H \text{ and } \frac{d\,y}{d\,x}=0$$

Therefore,

$$\frac{\partial Y}{\partial x}=H\frac{d^2\,y}{d\,x^2}$$

Substituting in Equation (5),

$$H\frac{d^2\,y}{d\,x^2}-\frac{\partial H}{\partial y}=4\,\pi\,i \tag{7}$$

but the radius of curvature of any curve $y=f\ (x)$ is

$$\rho=\frac{[1+(dy/dx)^2]^{3/2}}{d^2\,y/d\,x^2}$$

and, since $dy/dx=0$, at the point in question,

$$\frac{d^2\,y}{d\,x^2}=\frac{1}{\rho}$$

Making this substitution, Equation (7) becomes

$$\frac{H}{\rho}-\frac{\partial H}{\partial y}=4\,\pi\,i$$

Or, since y is the coördinate normal to the curve, a more general form would be

$$\frac{H}{\rho}-\frac{\partial H}{\partial n}=4\,\pi\,i \tag{8}$$

where the quantities H, n, i form a right-handed system of vectors, as in Fig. 15, and the radius of curvature ρ is to be measured to the position of the center of curvature on the vector n.(13) It should be noted that, in Equation (8), n is a unit vector normal to the field rather than a unit vector normal to the copper-air intersurface, as in Equations (3) and (4).

Equation (8) provides a method of checking up with tolerable accuracy the shape of lines of force which have been sketched in free-hand.

Numerical Application of Theorem III. In order to test its probable accuracy, this method was used to check a field plot at a point inside a square conductor in the bottom of an infinitely deep slot as in Fig. 16. The lines of force had already been plotted from an exact formula for this case. Since the current is flowing downward (perpendicular to the paper), the positive directions of H and of the normal are as indicated by the arrow heads.

Assume that the cross-section lines of the figure are spaced a centimeter apart. The tube (22½–12½) has a width of 1.11 cm. and contains 10 lines of force. Therefore, the flux density is 9.0 lines per sq. cm. The tube (12½–2½) has a width of 1.86 cm., and contains 10 lines. Therefore, its flux density is 5.38. The flux density at the point P is assumed to be the average of these two.

Therefore, $H=7.19$ lines per sq. cm.

The distance between the center lines of tube (22½–12½) and tube (12½–2½) is 1.5 cm. Since the difference in density is -3.62, the rate of change of density with respect to distance measured along the normal is

$$\frac{\partial H}{\partial n}=\frac{-3.62}{1.5}=-2.41$$

The radius of curvature of the 12½ line at the point P determined with a pair of dividers is

$$\rho=+3.95$$

---

(12) At the origin, the slope of the line $y=f\ (x)$ is zero and the partial derivative of Y with respect to x becomes equal to the total derivative with respect to x at that point.

(13) The value of ρ shown in Fig. 15 is negative because it is measured in the negative direction.

Substituting these in the left portion of Equation (8),

$$\frac{H}{\rho}-\frac{\partial H}{\partial n}=\frac{7.19}{3.95}+2.41=4.23$$

The plot was made with an assumed current density in the coil of $1/\pi$ abamperes per sq. cm. (14). Therefore,

$$i=\frac{1}{\pi}$$

and

$$4\ \pi\ i=4$$

Substituting these values in Equation (8),

$$4.23=4.00$$

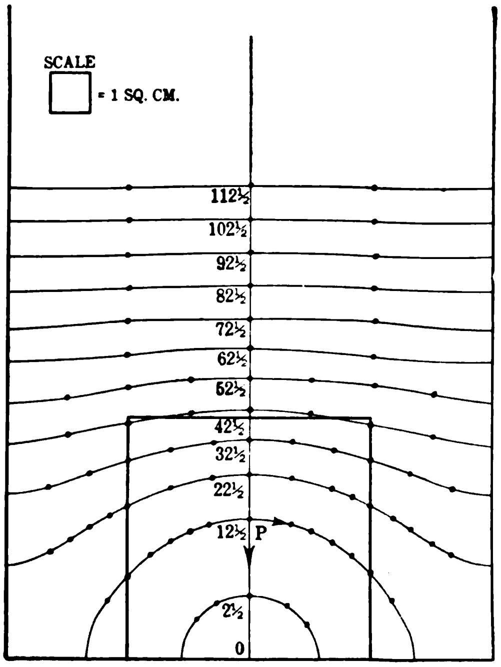


Fig. 16

This inequality would indicate an error of about 5.75 per cent.

Special Case Where the Field Crosses a Copper-Air Boundary Perpendicularly. In the case that the field crosses a copper-air boundary perpendicularly, as in Fig. 17, the term $\partial H/\partial n$ is identical with the quantity $\partial H/\partial s$, where s refers to distance measured along the copper-air boundary, and $\partial H/\partial n$ is, therefore, continuous across the boundary. The discontinuity in H/ρ across the boundary, therefore, is equal to $4\ \pi$ times the discontinuity in current density across the boundary; *i.e.*,

$$\Delta\left(\frac{H}{\rho}\right)=4\ \pi\ \Delta\ i$$

Or, since H is not discontinuous,

$$\Delta\left(\frac{1}{\rho}\right)=\frac{4\ \pi}{H}\Delta\ i$$

Hence, it follows that near the nucleus, where H is small, a discontinuity in current density at a given point may easily produce a large discontinuity in the curvature of the field at that point.

Special Case Where the Field is Tangent to the Copper-air Intersurface. If the field is tangent to the copper-air intersurface, this surface coincides with the x axis in Fig. 14. It should be noted that

$$\frac{H}{\rho}=\frac{\partial Y}{\partial x}$$

It is equal, therefore, to the rate of change of the normal component along the surface of the copper which is continuous across the boundary.

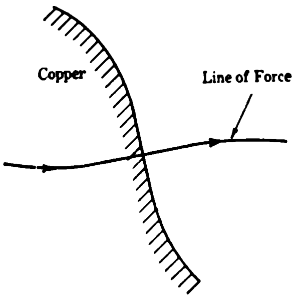


Fig. 17

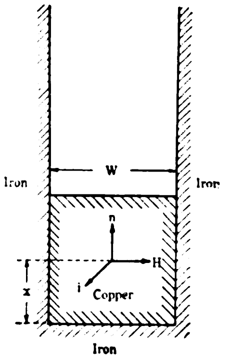


Fig. 18

Therefore, in this case, any discontinuity in Equation (8) must be written

$$\Delta\ (\partial H/\partial n)=-4\ \pi\ \Delta\ i$$

Thus, in the case of an isolated circular wire carrying a longitudinal current, the field inside the wire is given by

$$H=-2\ \pi\ i\ r$$

and differentiating:

$$\frac{\partial H}{\partial n}=\frac{\partial H}{\partial r}=-2\ \pi\ i$$

If r_0 is the radius of the wire, the field outside the copper is given by

$$H=\frac{-2\ (\pi\ r_0^2)\ i}{r}$$

and, differentiating,

$$\frac{\partial H}{\partial n}=\frac{\partial H}{\partial r}=\frac{+2\ \pi\ r_0^2\ i}{r^2}$$

Therefore, at $r=r_0$,

$$\frac{\partial H}{\partial n}=-2\ \pi\ i \text{ inside the copper}$$

$$=+2\ \pi\ i \text{ outside the copper}$$

$$\Delta\ \frac{\partial H}{\partial n}=4\ \pi\ i=4\ \pi\ \Delta\ (i)$$

---

(14) This value of current density can be checked as follows:
The area of the conductor is 36 sq. cm. Therefore, the total current is

$$I=\frac{36}{\pi}$$

$$\text{m. m. f.}=4\ \pi\ I=144$$

The width of the slot is

$$l=12 \text{ cm.}$$

$$H=\frac{\text{m. m. f.}}{l}=\frac{144}{12}=12 \text{ lines per sq. cm.}$$

Therefore, in the region where the lines of force are straight parallel lines, the width of the tubes containing 10 lines should be 10/12 = 0.835 cm.

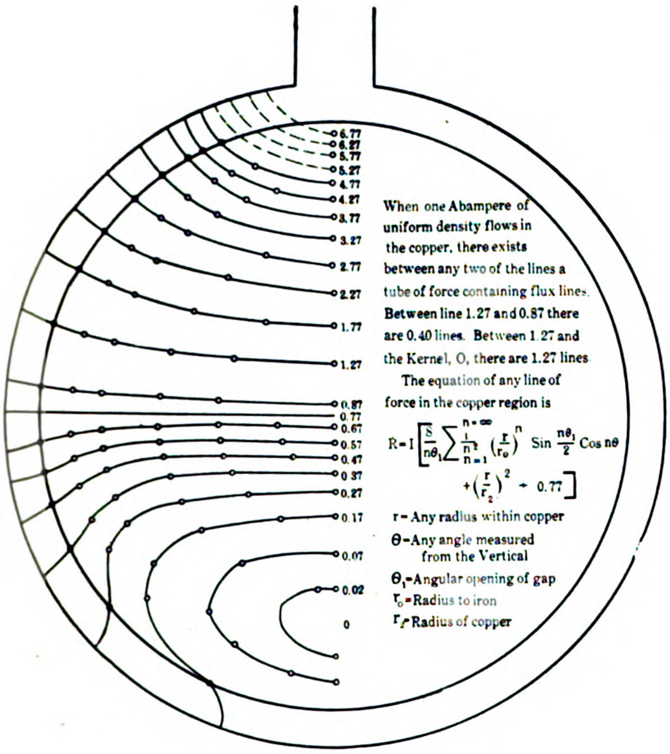
as would be expected since the field at the boundary is tangent to the copper.

In the case of an infinitely deep slot partly filled with copper, as in Fig. 18, the curvature is zero; and we have

$$H = -4\,\pi\,i\,x$$

and, differentiating,

$$-\frac{\partial H}{\partial n} = -\frac{d\,H}{d\,x} = 4\,\pi\,i$$



When one Abampere of uniform density flows in the copper, there exists between any two of the lines a tube of force containing flux lines. Between line 1.27 and 0.87 there are 0.40 lines. Between 1.27 and the Kernel, O, there are 1.27 lines.

The equation of any line of force in the copper region is

$$R = I\left[\frac{S}{n\theta_1}\sum_{n=1}^{n=\infty}\frac{1}{n^2}\left(\frac{r}{r_0}\right)^n \operatorname{Sin}\frac{n\theta_1}{2}\operatorname{Cos} n\theta + \left(\frac{r}{r_2}\right)^2 + 0.77\right]$$

r – Any radius within copper

θ – Any angle measured from the Vertical

θ_1 – Angular opening of gap

r_0 – Radius to iron

r_2 – Radius of copper

Fig. 19

while outside the copper the field is constant, so that $\partial\,H/\partial\,n$ vanishes. Hence, at this type of copper-air intersurface also,

$$\Delta\,\frac{\partial\,H}{\partial\,n} = -4\,\pi\,\Delta\,(i)$$

as would be expected.

Application of the Method to Polar Coördinate Systems

One of the writers' associates, T. R. Rhea, has plotted the flux in and about a round bar[15] in a round slot, as shown in Fig. 19. The tangential component of field intensity H_θ will be zero around the iron surface $\mu = \infty$ except at the air-gap. Here the value of H_θ will rise abruptly and follow some sort of curve to the other side of the air-gap. If H_θ, obtained from a free-hand sketch, is plotted against θ around the circumference (at $r = r_0$), a curve something of the shape shown by A in Fig. 20 would be obtained.

The sharp peaks on the curve are caused by the corners where the slot cuts into the circumference. To obtain a boundary condition that would exactly represent the boundary conditions of our problem, the Fourier series for the foregoing curve should be obtained, after this curve had been determined from an accurate and quantitative field sketch at the air-gap.

It is assumed, however, that these peaks may be neglected and the average value of H_θ across the air-gap used. This will greatly simplify the Fourier series representing the boundary conditions, and will not greatly affect the calculations farther down in the regions which are to be investigated.

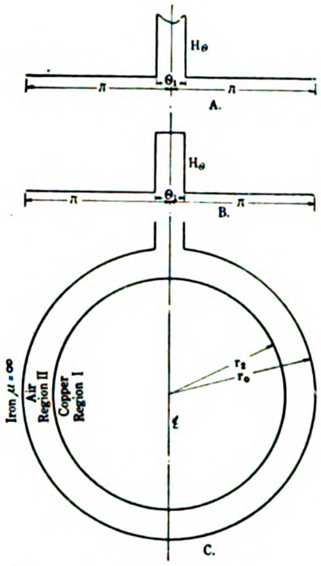


Fig. 20

With this assumption the boundary condition curve for H_θ will be of the form shown by B in Fig. 20.

Here θ_1 is the angular opening of the slot.

The total m.m.f. across the slot is $4\,\pi\,I$, where I is the total current in abamperes. As the medium within the slot is air, the field intensity, assumed uniform across the gap, will be

$$H_{\theta_1} = \frac{4\,\pi\,I}{r_0\,\theta_1}$$

The Fourier series expressing such a curve will be of the form,

$$H_\theta = C_0 + \sum_{n=1}^{n=\infty} C_n \cos n\,\theta \tag{9}$$

The constant C_0 is the average value of the curve, and has the value

$$C_0 = \frac{\dfrac{4\,\pi\,I}{r_0\,\theta_1}\times\theta_1}{2\,\pi} = \frac{2\,I}{r_0} \tag{10}$$

---

[15] With alternating-current flowing in the bar, the current distribution would not be uniform, but the problem of skin effect is outside the scope of this article and, for purposes of illustration, it is here assumed that the current density is uniform.

The value of the general constant C_n may be found from the expression

$$C_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} H_\theta \cos n\,\theta\, d\theta$$
$$= \frac{1}{\pi}\int_{-\theta_1/2}^{+\theta_1/2} \frac{4\,\pi\, I}{r_0\,\theta_1} \cos n\,\theta\, d\theta$$
$$= \frac{8\,I}{r_0\, n\,\theta_1}\sin n\,\frac{\theta_1}{2} \qquad (11)$$

Having obtained a Fourier series for the boundary condition, it is now necessary to set up expressions for the vector potential that will satisfy all the conditions.

The four conditions are:

Condition I: Poisson's equation for the vector potential R in Region I, expressed in polar coördinates,[16] as

$$\frac{1}{r}\,\frac{\partial R}{\partial r} + \frac{\partial^2 R}{\partial r^2} + \frac{1}{r^2}\,\frac{\partial^2 R}{\partial \theta^2} = +\,4\,\pi\, i \qquad (12)$$

where i is the current density (assumed uniform).

Condition II: At the boundary between Regions I and II, both H_θ and H_r are continuous functions because there is no change in permeability.

Condition III: Laplace's equation for the vector potential in Region II, expressed in polar coördinates, as

$$\frac{1}{r}\,\frac{\partial R}{\partial r} + \frac{\partial^2 R}{\partial r^2} + \frac{1}{r^2}\,\frac{\partial^2 R}{\partial \theta^2} = 0 \qquad (13)$$

Condition IV: The outer boundary condition at $r = r_0$ which has already been given in Equation (9).

A solution which will satisfy these four conditions is provided by the form:

$$R = R_1 + R_2$$

where R_1 is the vector potential of an isolated wire in space and R_2 is a function satisfying Equation (13) and so adjusted that Equation (9) is satisfied. That this solution satisfies the conditions imposed in both regions may easily be verified.

Choosing $R_1 = 0$, at $r = 0$, the function R_1 is, in Regions I and II, respectively,

$$R_1{}^{\text{I}} = I\left(\frac{r}{r_2}\right)^2 \qquad (14)$$

$$R_1{}^{\text{II}} = 2\,I\left[\log_\epsilon\left(\frac{r}{r_2}\right) + \frac{1}{2}\right] \qquad (15)$$

A sufficient form for the function R_2 in both Regions I and II will be found to be:

$$R_2 = b_0 + \sum_{n=1}^{n=\infty}\left(\frac{r}{r_0}\right)^n (b_n \cos n\theta + a_n \sin n\theta) \qquad (16)$$

In Region I, the total vector potential is

$$R^{\text{I}} = R_2 + R_1{}^{\text{I}}$$

where the expression for $R_1{}^{\text{I}}$ and R_2 would be taken from Equations (14) and (16).

In Region II, the total vector potential is

$$R^{\text{II}} = R_2 + R_1{}^{\text{II}}$$

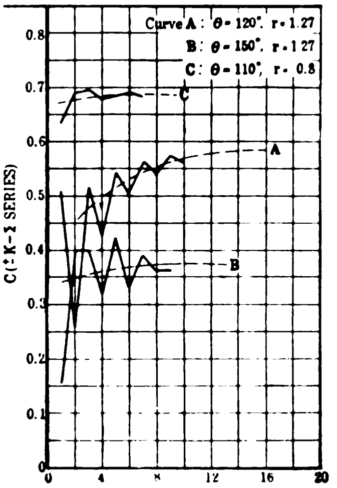


Fig. 21

From Equations (15) and (16),

$$R^{\text{II}} = b_0 + \sum_{n=1}^{n=\infty}\left(\frac{r}{r_0}\right)^n (b_n \cos n\theta + a_n \sin n\theta).$$
$$+2\,I\left[\log_\epsilon\left(\frac{r}{r_2}\right) + \frac{1}{2}\right]$$

Maxwell has shown that

$$H_\theta = \frac{\partial R}{\partial r} \qquad (17)$$

$$H_r = -\frac{1}{r}\,\frac{\partial R}{\partial \theta} \qquad (18)$$

Taking the partial derivative of R^{II} with respect to r, we therefore obtain

$$H_\theta = \frac{\partial R^{\text{II}}}{\partial r} = \frac{1}{r_0}\left[\sum_{n=1}^{n=\infty} n\left(\frac{r}{r_0}\right)^{n-1} (b_n \cos n\theta + a_n \sin n\theta)\right] + \frac{2\,I}{r} \qquad (19)$$

This must satisfy the terminal conditions imposed by Equation (9), when $r = r_0$ Hence, Equation (19) must equal Equation (9) when $r = r_0$. Therefore,

$$\frac{1}{r_0}\sum_{n=1}^{n=\infty} (b_n \cos n\theta + a_n \sin n\theta) + \frac{2\,I}{r_0} = C_0 + \sum_{n=1}^{n=\infty} C_n \cos n\theta$$

---

[16] In order to make the sign of R positive it has been found convenient to define it by the relations:

$$Y = \frac{\partial R}{\partial x}, \qquad X = -\frac{\partial R}{\partial y}$$

instead of

$$Y = -\frac{\partial R}{\partial x}, \qquad X = \frac{\partial R}{\partial y}$$

as in Rogowski's work.

A comparison of these series, term by term, gives

$$a_n = 0$$

$$b_n = \frac{r_0 C_n}{n}$$

And from Equation (10), we already know that

$$C_0 = \frac{2I}{r_0}.$$

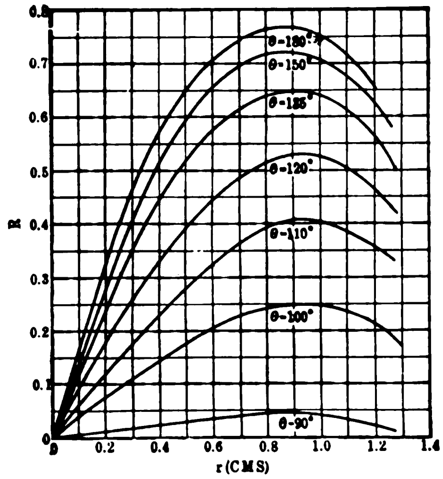


Fig. 22

All the constants except b_0 are now known, and the equations for vector potential in Regions I and II are

$$R^{\mathrm{I}} = b_0 + \sum_{n=1}^{n=\infty} \frac{8I}{n^2 r_0 \theta_1} \left(\frac{r}{r_0}\right)^n \left[\sin \frac{n\theta_1}{2}\right] \cos n\theta + I\left(\frac{r}{r_2}\right)^2 \qquad (20)$$

$$R^{\mathrm{II}} = b_0 + \sum_{n=1}^{n=\infty} \frac{8I}{n^2 r_0 \theta_1} \left(\frac{r}{r_0}\right)^n \left[\sin \frac{n\theta_1}{2}\right] \cos n\theta + 2I\left[\log_\epsilon\left(\frac{r}{r_2}\right) + \frac{1}{2}\right] \qquad (21)$$

where

$r_0 = 1.43$ cm.
$I = 1$ abampere = total current in conductor
$\theta_1 = 0.222$ radians
$r_2 = 1.27$ cm.

It is not necessary to know b_0 in order to plot the field. Make the calculation on the assumption that $b_0 = 0$. This will make $R_1 = 0$ when $r = 0$. Later, b_0 will be given the proper value to cause R_1 to be zero at the kernel.

It is well known that curves of constant R are lines of force and that unit difference of R is one Maxwell.(9) The field can be plotted, therefore, from the expressions for R^{I} and R^{II} in Equations (20) and (21).

The arithmetical evaluation of R is interesting, and a part of this work is therefore included. A number of terms of the series are calculated for particular values of θ and r. Unfortunately, for the larger values of r, the series does not converge rapidly. Plotting the sum of the series against the number of terms n was found to be a convenient method of determining the value to which the series converged. Several of these plots are given in Fig. 21. It is most important that the convergence value of the series be determined accurately, because it is subtracted from the value $(r/r_2)^2$, which, when r approaches r_2, is of the same order of magnitude. Thus it is a case where small differences between the relatively large numbers have to be determined accurately.

After having found the value of R for a sufficient number of points, r and θ, a set of curves was drawn of R against r, with θ as a parameter. Also, a set of curves was drawn of R against θ, with r as a parameter. Examples of these curves are given in Fig. 22. With the help of these curves and parameters of r and θ, curves for constant values of R could easily be plotted as lines of force in the magnetic field.

Neglect of the constant b_0 made the flux at the center of the figure zero and the flux at the kernel +0.77. As it was desired to have the flux at the kernel zero, the sign of Equations (20) and (21) was changed and a value of +0.77 assigned to the constant b_0, in order to make the entire flux plot positive.

The field plot in Fig. 19 is believed to be very accurate within the copper, but due to the slow convergence of the series it is not so accurate outside the copper in Region II.

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In connection with this article, reference may profitably be made to the paper by Th. Lehmann, appearing in the December, 1927, issue of the *Journal of the A.I.E.E.*

(*To be continued*)

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Engng., Nov. 4, 1927; v. 124, pp. 579-580.
(Short account of test results obtained by Prof. Stodola on a turbine supplied by Escher, Wyss & Co., for the Wehrden power station.)

Substations

Portable Distribution Substation. Herndon C. Jones.
Elec. Wld., Nov. 12, 1927; v. 90, pp. 991-993.

Serving Large Power Blocks with 24-Kv. Feeders. Wellen H. Colburn.
Elec. Wld., Oct. 22, 1927; v. 90, pp. 845-847.
(A description of recent substation enlargements of the Edison Elec. Ill. Co. of Boston.)

Substations, Automatic

Another Automatic Substation on the Pacific Electric.
Elec. Rwy. Jour., Nov. 5, 1927; v. 70, pp. 863-864.
(Describes the San Gabriel automatic substation of the Pacific Electric Railway.)

Switches and Switchgear, Automatic

Development of Automatic Switching Equipments in the United States and Europe. A. H. de Goede.
A.I.E.E. Jour., Nov., 1927; v. 46, pp. 1209-1214.

Television

Radio Vision. C. Francis Jenkins.
Inst. Radio Engrs. Proc., Nov., 1927; v. 15, pp. 958-964.

Television. F. K. D'Alton.
Bul. of Hyd. Pr. Comm. of Ont., Oct., 1927; v. 14, pp. 393-400.
(On the principles involved in its accomplishment.)

Turbine-Generators

Efficiency Guarantees and Determination of Efficiency of Oerlikon Turbo-Generators.
Bul. Oerlikon, Oct., 1927; pp. 313-315.

High-Speed Turbo-Generators. William Sharp.
Elec'n., Oct. 28, 1927; v. 99, pp. 524-525.
(Short article on recent design features.)

Vacuum Tubes

New Uses for Valves. A. Dinsdale.
Wireless Wld., Nov. 9, 1927; v. 21, pp. 643-645.
("Now employed in power stations to control a-c. and d-c. generators.")

Screened Valve in L. F. Circuits. N. W. McLachlan.
Wireless Wld., Oct. 19, 1927; v. 21, pp. 536-538.
(Serial.)

Voltage Regulators

Thermionic-Valve Type Close Voltage Regulator. F. C. Turner.
Engng., Oct. 21, 1927; v. 124, pp. 537-538.
(A paper read before the British Assoc.)

Water Turbines—Testing

Hydro Turbine Tests—Pitot and Venturi Tubes.
Power Pl. Engng., Oct. 15, 1927; v. 31, pp. 1086-1088.

NEW BOOKS

(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW.*)*

Alternating-Current Machinery; a Text-Book on the Theory and Performance of Generators and Motors. Richard E. Brown. 247 pp., 1927, N. Y., John Wiley & Sons, Inc.
("The purpose of this book is to provide a text-book on the principles of alternating-current machinery which thoroughly treats the subject and which may at the same time be adapted to almost any type of course. It is best suited for students who already have a good training on the principles of alternating-current circuits. Throughout the book emphasis has been placed on the analysis of the performance of the machines. Lengthy descriptions of the construction of machines have been avoided." General alternating-current theory, transformers, alternators, synchronous motors, synchronous converters, mercury vapor and other types of rectifiers, single-phase and polyphase motors, parallel operation, unbalanced polyphase loads, and measuring instruments are all treated from the theoretical and operating standpoints. Problems are provided at the end of nearly every chapter, and a special chapter of machinery laboratory problems is included.)

Boiler Feed Water Purification. Sheppard T. Powell. 363 pp., 1927, N. Y., McGraw-Hill Book Co., Inc.

Elements of Telephone Transmission. H. H. Harrison. 147 pp., 1927, N. Y., Longmans, Green & Co.

Engineering Problems Manual. Forest C. Dana and Elmer H. Willmarth. 187 pp., 1927, N. Y., McGraw-Hill Book Co., Inc.

Machine Design Drawing Room Problems. Ed. 2. C. D. Albert. 355 pp., 1927, N. Y., John Wiley & Sons.

Mechanics of Machinery. C. W. Ham and E. J. Crane. 504 pp., 1927, N. Y., McGraw-Hill Book Co., Inc.

Pocket-Book of Marine Engineering Rules and Tables. Ed. 17. A. E. Seaton and R. M. Rounthwaite. 770 pp., 1927, Lond., Chas. Griffin & Co.

Propagation of Electric Currents in Telephone and Telegraph Conductors. Ed. 4, rev. & enl. J. A. Fleming. 422 pp., 1927, N. Y., D. Van Nostrand Co.
(We are informed by the author that "this discussion is intended to provide telegraph and telephone engineers with some necessary information to enable them to follow the original writings of leading investigators, and also with the means of solving for themselves practical problems in connection with the subject." To this end he has provided a mathematical introduction the purpose of which is to help the reader acquire a working knowledge of the mathematical processes involved. He has simplified the theory as much as possible, and has presented illustrative examples of the more essential calculations. After his "Mathematical Introduction," Prof. Fleming continues with chapters on propagation of electromagnetic waves along wires, telephone and submarine cables and the propagation of currents along them, transmission of high-frequency and low-frequency currents, electrical measurements and determination of constants of cables, cable calculations, loaded cables, and recent advances in telephone and telegraph line construction.)

Theory of Thermionic Vacuum Tube Circuits. Leo James Peters. 256 pp., 1927, N. Y., McGraw-Hill Book Co., Inc.

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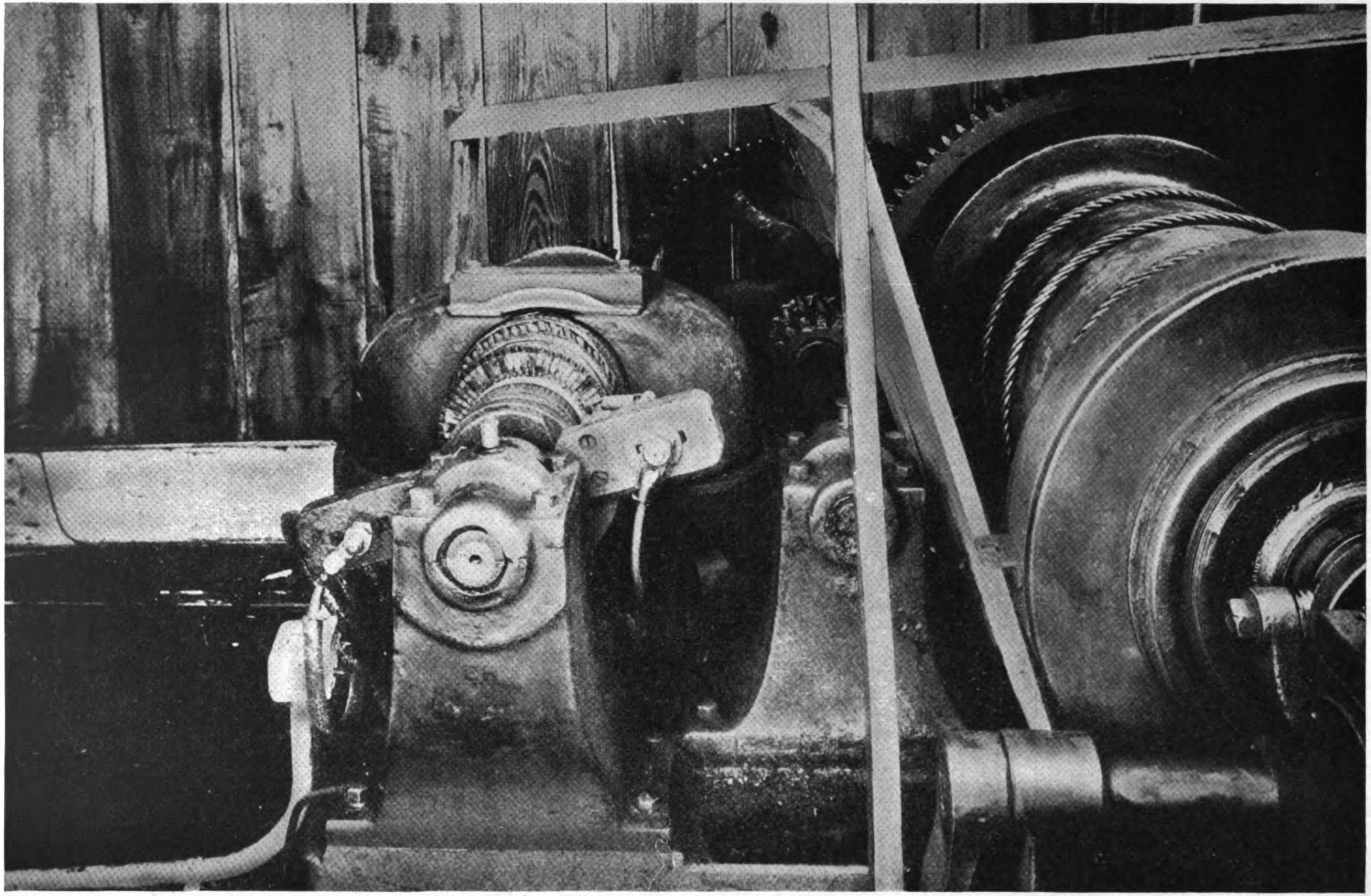

STILL IN ACTIVE SERVICE AFTER THIRTY-EIGHT YEARS

This Thomson-Houston motor was built in 1889 and was exhibited at the Maritime Exposition in Boston in the fall of that year, following which it entered upon its long career in hoisting operations in various localities. For many years it has been at work in its present location on a dock at Lynn (Mass.)

(An article on the development of d-c. motor design appears on p. 116 of this issue)

GENERAL ELECTRIC REVIEW

Vol. 31 March, 1928 No. 3

THE WHY, WHERE, AND HOW OF DIRECT-CURRENT UNITS

Mark Twain once said: "Everybody talks about the weather but nobody does anything about it." The reverse is very nearly true of direct-current applications for they are seldom discussed but are very much used.

Prof. Elihu Thomson reveals in this issue how great has been the advance in direct-current design; and we are prompted to look into present-day applications to learn why, where, and how direct-current units are being used.

Disregarding their almost exclusive use in traction work, we find that direct-current motors deliver 31 per cent of the electric power in industry and by number constitute 36 per cent of all motors in service. The limited areas where direct-current distribution prevails account for only a small fraction of these applications. Most of them are found in the presence of alternating-current power supply; and the extra fixed charges and operating costs involved in making the transition, or in first-hand generation, indicate that there must be some very good reasons for the practice. There are; and "adjustable speed" is by no means the principal one, although it is the one most often mentioned.

Direct current is required for battery charging, for electrolytic work, and for the excitation of the fields of many a-c. and d-c. machines. These uses account for great numbers of direct-current generators in the smaller sizes. That wonderfully flexible system of control known as "generator-voltage control" or "Ward-Leonard control" inherently requires a generator for each motor, or for each group of motors that operates as a fleet. One big motor, that comes to mind, is waited upon by a retinue of two major generators, a lesser generator, three exciters, and two little pages or messenger generators. These generator-voltage control applications employ many generators some of which run into large sizes.

Process steam requirements, at one or more reduced pressures, have caused the introduction of the steam turbine from which reduced-pressure steam is taken from the stages or from the exhaust, or both. The turbine drives a generator which furnishes power to the motors driving the process machinery. In many cases this is a d-c. system within an industry which is mainly operated on an a-c. power supply.

Direct-current motors are preferred for some applications and are practically indispensable for others, because of certain characteristics peculiar to them. Base speeds are not limited to synchronous zones and they may be made constant, adjustable, varying, or adjustable-varying by choice. The control devices handle only small currents for speed adjustments or reversals. Stable speeds from zero to maximum in either rotation are easily obtained and controlled by the generator-voltage control system.

Dynamic braking is effective even at very low speeds, thus reducing friction braking to practically a holding function only. Starting and accelerating torques of high values are available with comparatively small energy demands because speeds may be inherently tempered to compensate for the high torques.

Commutation, the one-time bugbear of direct-current design, has been so thoroughly mastered that it is no longer an important factor in the equations leading to a choice between a-c. and d-c. power.

Direct current is used extensively in the iron and steel industry for both major and minor operations. It is indispensable in the making of paper. Machine tools which enter into hundreds of industries are largely equipped with direct-current motors. High-speed passenger elevators as well as the slowest heavy-duty freight elevators are usually operated by direct current though the primary supply is usually alternating current. Power shovels of the large sizes rely on the "iron hand in the velvet glove" qualities of generator-voltage control to perform operations that would otherwise tear them to pieces. Mining requires direct current for cutting, hauling, and hoisting. Material handling operations throughout the whole industrial field consistently employ this flexible power medium. Feed mechanisms for all kinds of materials take advantage of the adjustable-speed direct-current motor.

How d-c. motors are used would require pages to tell and only a few examples can be cited here. A car dumper that hoists and upsets 120-ton carloads of coal every 40 sec. uses generator-voltage-controlled motors on the cradle hoist, on the "mule" that delivers the cars, and on the apron that guides the coal into the ship; yet no excessive load peaks can break through to the a-c. supply lines. A 1000-hp. mine-hoist motor, subject to 1800-hp. peaks, works on a fast cycle yet the a-c. motor driving the generator takes 600 kw. from the line as steadily as though it were driving a fan.

The paper machine, a line of ten disconnected units, makes "news" 25 ft. wide at the rate of 1200 ft. per min. Each section is driven by a d-c. motor. The ten motors are speed-adjustable as a fleet by a single rheostat. Each can be dropped to a stable low speed for the servicing of its section. Each section runs at an independently micro-adjustable speed slightly faster than the preceding section. The paper must not break.

The giant reversing blooming-mill motor with its retinue previously mentioned is regularly fed with white-hot steel ingots the size of office tables. Its speed from zero to 80 turns per minute, its reversals as rapid as one every three seconds, and its torque up to 2,400,000 lb. are controlled by the manipulation of comparatively microscopic currents. The bumps incident to reducing ingots of steel to long slender blooms hardly cause a ripple in the a-c. supply system.

The one-time general impression that a-c. supply required a-c. motors has faded away and most of the fading has taken place in very recent years.

R. H. Rogers

A Review of the Development of Direct-current Motor Design

By **PROF. ELIHU THOMSON**
Director, Thomson Research Laboratory, General Electric Company

WHEN reviewing the developments that have brought direct-current motors into existence as we know them, all those crude early efforts to obtain power from the currents yielded by voltaic batteries may be neglected. We may also exclude motors whose power depended on the intermittent pulls of iron armatures toward magnet poles, the magnetic drawing of cores into solenoids, and the successive exertions of force on iron plates or armatures mounted on wheels for rotation by electromagnets whose circuits were closed and opened by a breakpiece or cam on the shaft.

ELIHU THOMSON

Not indeed until the development of machines of the Gramme dynamo type, and its modification, the Siemens type—both inherently foreshadowed in the middle sixties of the past century by Paccinotti—did we have the basis for a real electric motive power development. In the early seventies it was not unusual to find references to the reversal of dynamos, such as the Gramme, and their use as motors. It began to be recognized that a well-organized dynamo-electric machine would make a good motor if supplied with such a current as it would produce when driven as a generator. This dual characteristic was exemplified at the 1876 Philadelphia Centennial Exhibition where a Gramme dynamo driven by power was connected to a similar machine receiving current and running as a motor. This dynamo motor was first connected to a centrifugal pump which raised water a few feet. The water then flowing over a dam gave the appearance of a small waterfall. No other exhibit in Machinery Hall was more impressive. This application took place long before the appearance of the first incandescent lamp, and was contemporaneous with the first announcement and exhibition of the then new, speaking telephone of Alexander Graham Bell.

In those early days there was much to learn regarding the proper proportioning and relationship of windings in the construction of dynamos, and necessarily much more as to the factors which must enter into the construction of a satisfactory direct-current motor; *e.g.*, the endeavor to reduce "idle wire." This attempt led to elongated cores and small armature diameters in both dynamos and motors. The bugbear of idle wire as found in the Gramme and similar machines was corrected through improvements in design made by the writer and others. Another superstition of that time influenced existent design; *viz.*, that long rather than short magnet cores should be used. Long magnet cores were supposed to project the lines of force through the armature, which the shorter magnet cores would fail to do. We early recognized that after all it was a question of ampere-turns, and that if these turns were allowed to exert their full effect on the armature core there would be no need for any special length in a field magnet core.

In the early years, or those around 1878-80, there was very little demand for electric motors. There were no lines to which they could be connected. Consequently, the electrical engineer of the time devoted his efforts to dynamo-generators, feeding arc lamps as the sole system of distribution. Not until 1880 was there any promise of extension in incandescent lighting. This broadening of the field came through the invention by Edison of his high-resistance carbon-filament lamp in October, 1879. With the gradual development of electric stations, generating direct current in large centers, a field was opened for direct-current motors to replace small steam or gas engines economically.

When the factory of the American Electric Company was established in New Britain, Conn., in 1880, this plant furnished one of the earliest examples of transmission to and operation of an electric motor of several horsepower. The available space for a pattern shop was too far away from the main engine to allow power transmission by shafting and belting. In this case, a T-H dynamo was placed near the main shaft and belted to it; two wires from the dynamo were led to the pattern room where they were connected to a second dynamo which, however, took current as a motor and was belted to the woodworking lathes, saws, etc., of the pattern room. Novel appliances were added for changing power and speed. This drive proved to be very satisfactory for the woodworkers and was continued in use while the plant was operated, or until the Works were removed late in 1883 to Factory A of the West Lynn Plant of the Thomson-Houston Electric Co.

Outside of a few small motors for use on series lines operating arc lamps, there was still little demand for electric motors until about 1885. A few appeared as exhibits in the Franklin Institute Electrical Exhibition in Philadelphia in the Fall of 1884. The great convenience and effectiveness of direct-current motors, especially for isolated printing-plant machines, woodworking, pumping, and similar drives, resulted in a considerable and increasing demand for central generating stations to furnish the driving current.

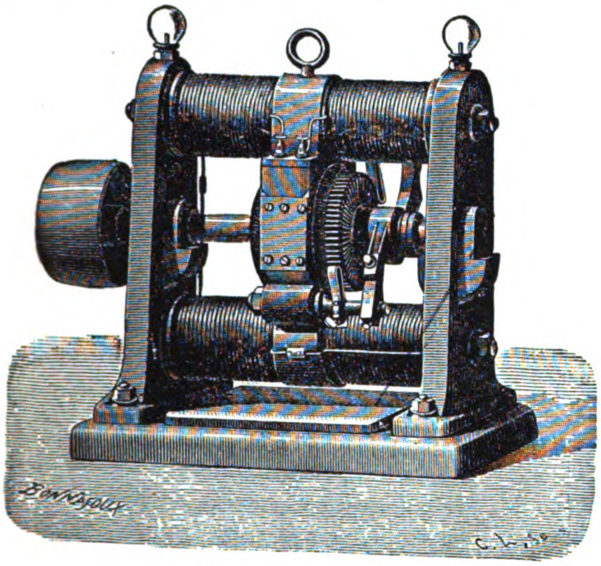

Fig. 1. Gramme Dynamo, "A" Pattern

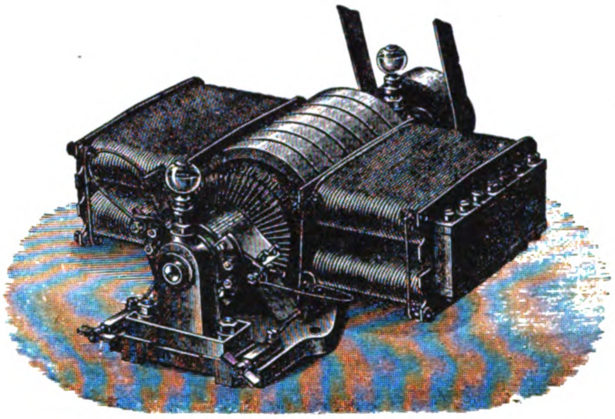

Fig. 2. Siemens Dynamo, Horizontal Pattern

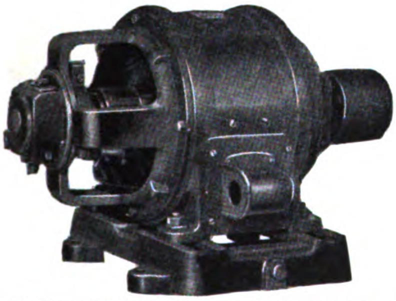

Fig. 3. Typical Two-pole Riveted-frame General-purpose Direct-current Motor of Size Ranging from ½ to 3 hp.

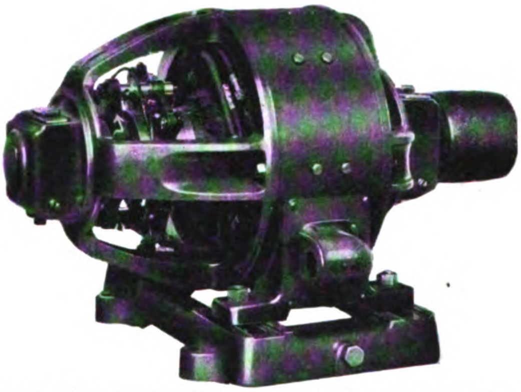

Fig. 4. Typical Four-pole Cast-magnet-frame General-purpose Direct-current Motor of Size Ranging from 3 to 200 hp.

Improvements in design took place rapidly; composite carbon brushes replaced copper wire or gauze, thereby reducing commutator and brush wear to a minimum. Fortunately, the problem of securing good commutation was more easily solved for motors than for generators, because the reactions which in generators are additive, are differential or compensatory in motors. As a later development, the so-called commutating field pole was made available in dynamo construction, including, of course, in such designation, direct-current motors.

Beginning about 1887, sturdily constructed direct-current motors were first applied to electric traction, diverting the skill of the best designing engineers to machinery demanding mechanical perfection, robustness, reliability in electrical qualities, good insulation, and other properties to meet the harsh conditions of use under street cars.

The wonderful flexibility of a well-built direct-current motor is probably not equalled by any other device producing power. As its running speed depends on the development of a counter e.m.f., speed variations within fairly wide limits can be obtained by simple modification of the effective magnetizing ampere-turns of the field magnets. The load can be widely varied under any given speed, for the speed is self-regulating within a small per cent even though the load vary from nil up to full or the reverse. Also, starting under normal or heavy load is accomplished easily and promptly by using resistance in the armature circuit, this resistance being cut out when the motor reaches full speed. The efficiency factor in a well-designed direct-current motor, even of relatively small output, leaves practically nothing to be desired; while reliability in operation depends on the excellencies of mechanical and electrical design coupled with good workmanship. Change of direction of rotation can, in like manner, be attained by the simple reversal of current in either the armature or the field circuit, practically no other adjustment being required.

This ease of reversal and very close regulation of speed control assures an indefinite continuance of demand for direct-current motors and, in fact, compels their adoption and use for driving many of the automatic and special machines now so largely required in mass and repetition manufacture.

As an indication of the satisfaction that has attended the use of direct-current motors, there are today great numbers of such machines in operation which were placed in service 20 or 30 years ago, and which may reasonably be expected to continue in satisfactory operation for years to come.

Briefly traced, we have seen the direct-current motor, progenitor of all the commercially practicable types, first appear as a crude conversion of the Gramme or Siemens dynamos.

Slowly improved to secure the proper proportioning and relationship of windings, these "trial and error" initial methods employed were the necessary precursors of a technique which, step by step, has made dynamo-electric design and construction an exact science. We now reach culmination in the modern direct-current industrial motor with its detailed wealth of mechanical and electrical features.

Carefully proportioned main and commutating poles and stabilizing windings permit close speed regulation. Refinements of commutator, brush rigging, and other essentials insure practically complete freedom from commutation troubles.

Bearings, for years a source of recurrent trouble, have as a result of refined accuracy secured through modern design, manufacture, and assembly, become available in sleeve, ball, and roller types with a degree of trustworthiness beyond cavil.

In summary, the arrangement and distribution of all active and inactive materials embodied in the latest direct-current motors provide a combination of strength, compactness, attractive appearance, stability, and efficiency guaranteeing permanent satisfaction under stress of hard and continuous service.

Professor Thomson Honored

Prof. Elihu Thomson, director of the Thomson Research Laboratory of the General Electric Company, has been named a member of the American Committee of the World Congress of Engineers to be held in Tokyo (Japan) in November, 1929. The appointment was made by Herbert Hoover, Secretary of Commence.

This is the first congress of its kind ever held and according to Baron K. Furuichi, President of the Engineering Society of Japan, is for the purpose of promoting international coöperation in the study of engineering in all its branches and in stimulating a sense of brotherhood among the engineers of the world.

Among the other well-known members of the American Committee appointed by Secretary Hoover are Thomas A. Edison, John Hays Hammond, Charles M. Schwab, and Orville Wright.

Electric Heaters Prevent Freezing Troubles in Compressed-air Application in Cascade Tunnel

In connection with the construction of the new Cascade Tunnel, seven and three-fourths miles long, and two shorter tunnels for the Great Northern Railway Company, A. Guthrie & Company, of St. Paul (Minn.), has demonstrated the versatility of electric heating equipment in a unique manner. This consists in applying water immersion heaters to the preheating of air employed to operate an air-driven shovel.

The heating units, which were of General Electric manufacture, are used to preheat the compressed air that operates a No. 40 Marion shovel employed in tunnel excavation work. This shovel is of the ordinary steam engine-driven type, from which the boilers have been removed and replaced by a large air receiver.

Much engine trouble was experienced as a result of the freezing of the moisture in the air on expansion, to avoid which it was found necessary to heat the air.

The receiver on the shovel is four feet in diameter by seven and one-half feet high. Water immersion heaters of helical type were chosen because such units are easily installed. To prevent the units from burning out in the air receiver it was necessary to use two 220-volt units in series, on a 220-volt circuit. Nine of these groups were placed in multiple, 18 units thus being employed.

When the shovel is operating, sufficient air passes through the receiver to keep the heating units from burning out. To prevent overheating the air, and to guard against damaging the units, an indicating thermometer is placed where the shovel operator can readily note the temperature of the air. When this temperature rises above 300 deg. F., the operator shuts off the heaters. Although a higher temperature might not burn them out, it might occasion other difficulties, as, for example, those arising from a breakdown of the lubricating oil.

A large share of the heat energy placed in the air is regained through the expansion cycle of the engine on the shovel.

Load Ratio Control

PART I

Method Employed in Voltage Control—Principle of Operation of Various Types of Load Ratio Control—Possible Reduction in Reactor Capacity and Losses—Symmetrical, Unsymmetrical, and Bridging Connections—Special Considerations

By L. F. BLUME
Assistant Engineer, General Transformer Engineering Department
Pittsfield Works, General Electric Company

At a joint meeting of the Electrical Section of the Western Society of Engineers and the Chicago Section of the A.I.E.E. the author presented the material contained in this article, which will be published in the Review *in two parts. The present installment deals with the general principles of load ratio control; a later one will treat of its fields of application.—*Editor

It frequently happens in the progress of applied science or engineering that an invention does not find a positive field of application until many years after its original development. This long delay may not be due so much to the undeveloped state of the invention, as to the fact that the field in which it eventually will be used successfully has not yet been established. Of this the present subject is a very good illustration. Many years ago, the inherent weakness of constant-potential electric lines in general, and transformers in particular, to the effect that voltage could not be maintained constant at all points if the load varied through a wide range, was clearly recognized, and various steps were taken to overcome or neutralize this inherent defect.

The problem was approached by three different methods. First, serious attempts were made to overcome the regulation drop inherent not only in the transformer, but also in the line by designing the transformer so that the secondary voltage would actually increase in value with increasing loads.

Some very ingenious schemes were suggested, such as connecting two transformers having different ratios in series in both primary and secondary sides. By properly choosing the magnetizing characteristics of the two transformers, the combination can be made to give inherently rising voltage characteristics with increasing loads.

Ingenious as this scheme was, it had many practical limitations which caused it to be abandoned. As it had rising voltage characteristics only at high power-factors, it could not be made to compensate correctly for variations in applied voltages; and it had other serious limitations.

The second method attempted was to change the ratio of transformation by mechanical means, by providing the transformer with a number of taps and a suitable switching device so that, at the will of the operator, the voltage ratio could be varied. Many years ago, a considerable number of these outfits were built and put into successful operation, and the principle upon which they were based is quite similar to that embodied in what are now being built and known as transformer ratio control or tap-changing equipments (Figs. 1 and 2).

However, the induction regulator was being developed at the same time, as the third method of obtaining voltage control; and for many years, the induction regulator has very successfully dominated the field. It is, therefore, a pertinent question to ask for the reasons that have caused the present rapid development of transformer ratio control equipments. The following two facts, if taken together, seem quite sufficient to account for the growing demand for them.

In the first place, the kv-a. capacity concentrated in central stations, and the kv-a. of individual transformer banks has been steadily increasing from year to year, and these stations are being more and more connected into networks. With this naturally has come about an increasing demand for voltage control devices in larger units than heretofore, and at higher voltages. The second fact is found in the comparison of the cost curves of induction regulators and ratio control equipments, plotted as a function of the kv-a. ratings, which shows that the induction regulator is the less expensive for the smaller ratings and for moderate voltages; whereas for the larger transformer ratings, and especially for higher voltages, ratio control equipments are proving to be considerably less expensive. Thus, the present return of the tap-changing method has been largely brought about by the development of a field of application in which it is proving to be the most economical, and at the same time the most efficient, solution for voltage control.

The principle of operation underlying transformer ratio control should first be examined, so that the various types can be compared as to electrical performance. It is quite possible, by employing two transformers operating in parallel on a given line, to shift from tap to tap without dropping the load, if the operating handles of the ratio adjusters are brought out of the tanks, and so located that they may be operated with excitation on the transformers. The diagram in Fig. 3 represents two transformers connected in parallel on both high- and low-voltage sides; and each transformer is provided with ratio adjusters. A tap change can be made without dropping the load by first opening circuit breaker *A*, which shifts the load from transformer *A* but does not remove the excitation since the high-voltage side is still connected to the line. The ratio adjuster in transformer *A* can now be moved to the next position, after which circuit breaker *A* is again closed. The two transformers are now operating, with unequal ratios, in parallel; and a circulating current as indicated by the arrows flows between them. Next, circuit breaker *B* is opened so as to permit the ratio adjuster in transformer *B* to be shifted to the adjacent position. Circuit breaker *B* is then closed and the two transformers are again operating under normal conditions, but with a changed ratio.

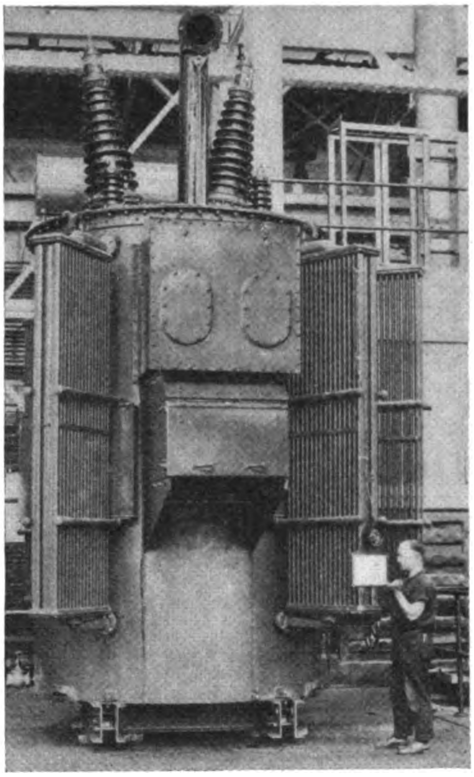

Fig. 1. External View of a Load Ratio Control Transformer

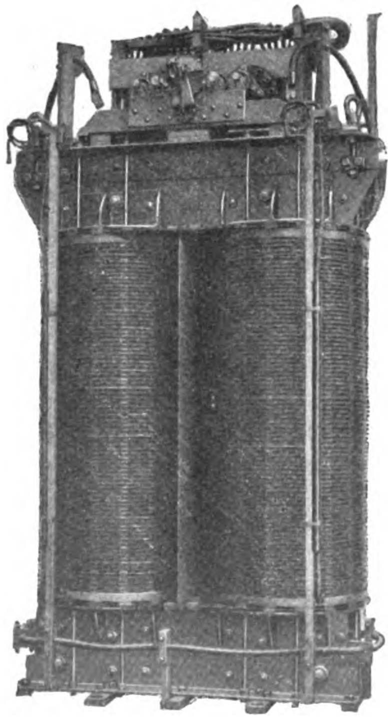

Fig. 2. Load Ratio Control Transformer Removed from Tank

Examining more closely what happens during this process, it is important to note that when one of the circuit breakers is open, the voltage which exists across the open breaker is not the line voltage, but a very small fraction of it, owing to the fact that the disconnected transformer receives excitation from the high-voltage side. Thus the voltage which the breakers are obliged to open is equal to that arising from the difference in the ratio of transformation of the two transformers, plus the impedance drop which exists in the loaded one.

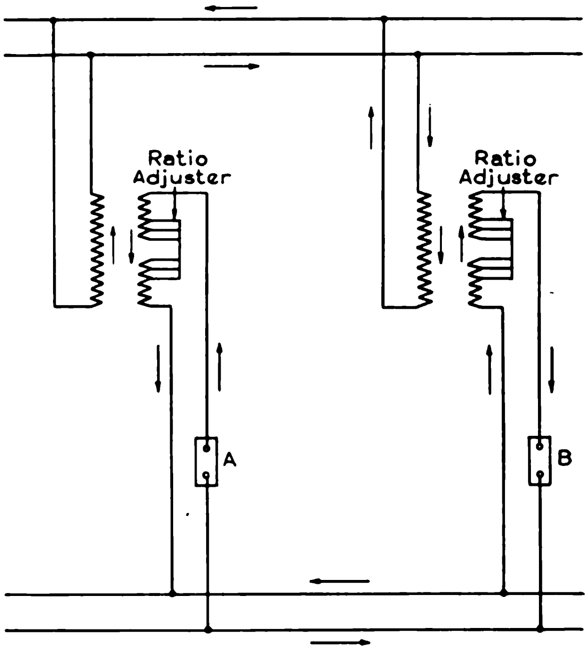

Fig. 3. Elementary Scheme of Connections for Voltage Adjustment by the Use of Two Parallel Transformers Having Suitable Means for Changing Taps

Aside from the inconvenience of operation in this manner, and the liability of the operator to make serious mistakes, this method of tap changing is undesirable, because the doubling of the load on one winding doubles the impedance drop, so that, during the switching period, the increase in impedance results in an undesirable fluctuation in secondary voltage. Thus, with 10 per cent transformer reactance, at full load the reactance drop is increased to 20 per cent which is almost five times the voltage between adjacent taps.

By designing one transformer with a multiple circuit, and connecting it as shown in Fig. 7, these various difficulties are readily overcome. The reactance between the multiple paths can now be controlled independently of the main transformer reactance, and it can be made small enough to prevent objectionable fluctuations in voltage during the process of switching, and large enough to prevent excessive circulating current.

When transformer ratings are such that a multiple circuit can be provided without additional expense, this arrangement is very desirable, since it does not involve additional kv-a. capacity of apparatus. This scheme, however, possesses two inherent objections: First, during the process of switching, momentary full load is thrown on one-half of the transformer winding and, in the very remote possibility of the operating motor failing to complete its cycle, this load may be present for a sufficient time to seriously overheat the windings. To provide against this contingency, means either to design the windings with an appreciably greater copper cross-section (and this cannot be done without increase in size and cost) or, resorting to a less expensive method, to provide circuits which warn the operator of the stopping of the motor in a non-operating position. The second limitation of the circuit is that in some ratings, the transformer design does not lend itself so readily to a multiple circuit without appreciable increase in cost.

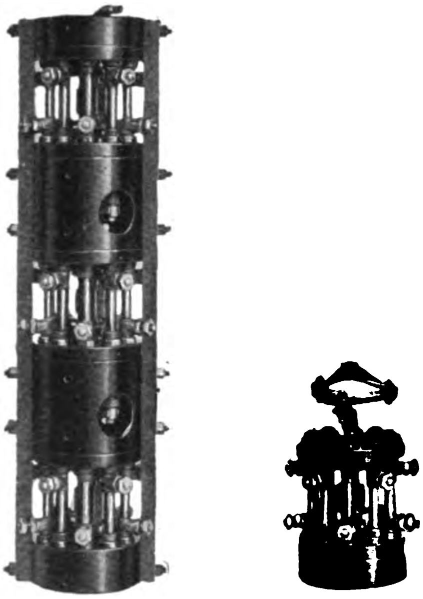

Fig. 4. Ratio Adjusters Used on Load Ratio Control Transformers. Three-phase at left; single-phase at right

These practical considerations make it desirable to depart in many cases from the simple arrangement shown in Fig. 7, by placing only a small portion of the circuit in multiple, as shown in Fig. 8. Here, the multiple circuit consists of only the ratio adjusters and circuit breakers, and an auxiliary reactance is employed to limit the circulating current. Here, also, nine taps are used to derive nine operating voltages. Whether this or the circuit shown in the Fig. 7 should be used depends largely upon design considerations; and the choice should, for the most part, be determined by whether the multiple circuit with extra taps and leads costs more than the current-limiting reactor.

It will be well at this point to examine in greater detail the condition that the current-limiting reactor must meet during the tap change. In the operating position, both sides of the reactor being connected symmetrically to the same tap, the only voltage which appears across it is due to the leakage reactance between the reactor halves, and this may be made negligible by interlacing. The load current divides equally between the two halves.

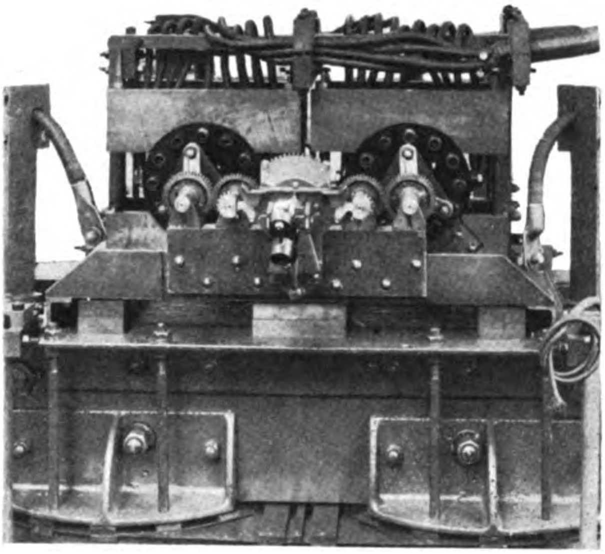

Fig. 5. Close-up View of Intermittent Gear Used to Drive Load Ratio Control Equipment. Single-phase application

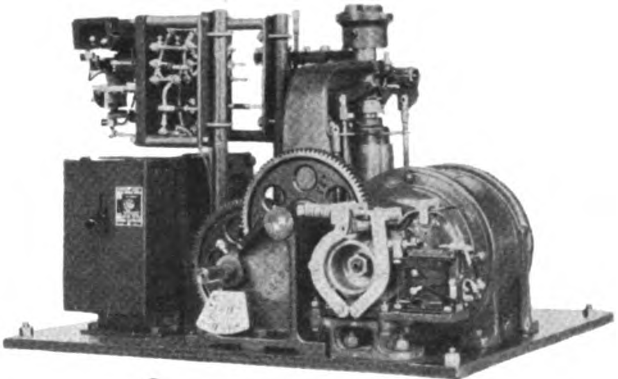

Fig. 6. Driving Mechanism for a Transformer Load Ratio Control. Direct-current Drive

In the process of switching from one tap to the next, there are two unsymmetrical positions: the first, in which one ratio adjuster only is connected; the second, in which both ratio adjusters are connected, but on adjacent taps. In Fig. 9, a close-up of all three positions are shown; diagram (a) being the operating position with the load current dividing equally in the two reactor halves; diagram (b) with connection through reactor to one tap only, in which case the full load current flows through one half of the reactor; diagram (c) where the reactor is bridging adjacent taps, and therefore tap voltage is impressed on the reactor. To aid in considering these diagrams, let us call (a) the symmetrical, (b) the unsymmetrical, and (c), the bridging positions. In the bridging position, two currents are flowing in the reactor, *viz.*, the load current, which divides in two equal halves as in (a), and a magnetizing current or circulating current, which flows as a result of the reactor bridging across adjacent taps. The conditions imposed by the bridging and unsymmetrical positions are conflicting; since to design with relatively high reactance, so as to have a small circulating current, will result in a relatively large reactance drop in the unsymmetrical position and, consequently, an undesirably large fluctuation in line voltage. If the reactor is of the iron type, gaps are necessary in the magnetic circuit. A magnetizing current of not less than 50 per cent full-load current is considered necessary to give good results.

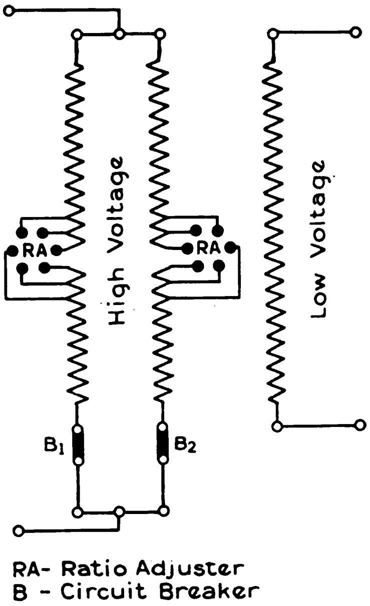

Fig. 7. Diagram of a Transformer Designed with a Multiple Circuit, by Means of which Many Difficulties which Accompany Tap-changing Under Load Are Overcome

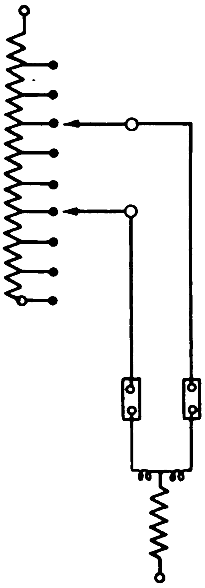

Fig. 8. A Connection Diagram for Load Ratio Control in which Nine Taps Are Provided for Deriving Nine Operating Voltages

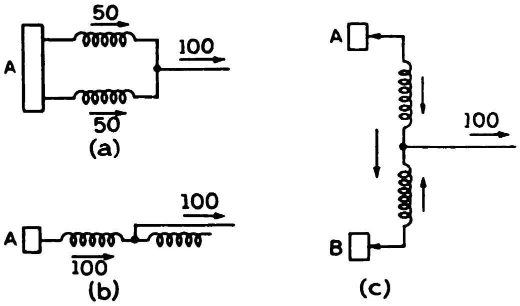

Fig. 9. Current-limiting Reactor Connections in Load Ratio Control

(a) Symmetrical (b) Unsymmetrical
(c) Bridging

It would be logical to ask, at this point, why better use were not made of all of these positions by making them permanent operating positions. This is possible; and it has been done. For example, the connection shown in Fig. 8, in which nine taps are used to derive nine operating voltages, could be replaced by the connection shown in Fig. 10, if all positions are made operating positions. Here, nine voltages are derived by using five taps instead of nine, and three-point instead of nine-point ratio adjusters. In this, the symmetrical position is not used, the sequence of operation being alternately the unsymmetrical and bridging positions.

Occasionally, the arrangement of windings is such as to make it desirable to reduce the number of taps to a minimum, and there are other cases where a particularly large number of taps are desired. For example, in one instance when eighteen operating voltages were wanted, and where, on account of the design of the windings, it was not desirable to have a multiplicity of taps, the design selected had only five tap sections; and by using all positions for permanent operation, the eighteen steps were obtained.

In the majority of cases, however, it has been found desirable to make the unsymmetrical and bridging connections only transitional or switching positions and not operating positions, in order to make the size of the reactor smaller and, at the same time, to save in losses. Permanent operation on only the symmetrical position incurs no losses due to circulating current, or to core loss in the reactor. Furthermore, since twice the number of taps must be used for permanent operation on only the symmetrical position, the voltage between taps is only one-half, and therefore the voltage impressed on the reactor during switching is only one-half of what it would be if all possible positions were made operating positions. The net result is that the kilovolt-ampere rating of the reactor is about one-third, and the energy losses in the reactor only one tenth. In the larger ratings, this is by no means negligible. For example, in a 60,000-kv-a. bank having ratio control in 2 per cent steps, *the reactor rating can be reduced from 3000 to 1000 kv-a.* by doubling the number of taps and operating only on the symmetrical position; and at the same time, *the reactor losses can be reduced from 10 to a single kilowatt.* These considerations alone have made it very desirable to use a larger number of taps in equipments involving the larger ratings.

Another advantage of operating permanently on only the symmetrical positions is that the voltage steps will always be uniform. For the purpose of comparing the voltages derived, the diagrams in Fig. 11 are useful, (a) giving the voltages when operating only on symmetrical positions and (b) the voltages when operating alternately on unsymmetrical and bridging positions. The diagrams are drawn to the same scale, and both show the voltages for two steps, the desired voltage of each step being 100. In Fig. 11 (a) the three operating tap positions are *A*, *B*, and *C*, the voltages derived on the switching positions being *1*, *2*, *3* in going from *A* to *B*, and *3*, *4*, *5* in going from step *B* to *C*. Both diagrams are plotted for full load at 80 per cent power-factor, and it is assumed that the reactor has been designed for a circulating current of 60 per cent. In (b), Fig. 11, the voltage between adjacent taps *A* and *B* is 200, but the tap voltage derived on the operating positions *1*, *2* and 3 are alternately 50 and 150. The reactance drops introduced by the reactor during operation on the unsymmetrical position are *A-1* and *B-3*, whereas in the bridging position, no drop, or relatively little drop, is introduced by the reactor. The resulting non-uniformity in tap voltage may be improved by designing the reactor for greater circulating current, and of course, the conditions are better for lighter loads and higher power-factor; but on the other hand, with over-loads, it is quite possible for one of the steps to become reversed so that when the equipment is operated to gain in the voltage, actually a lower voltage is obtained.

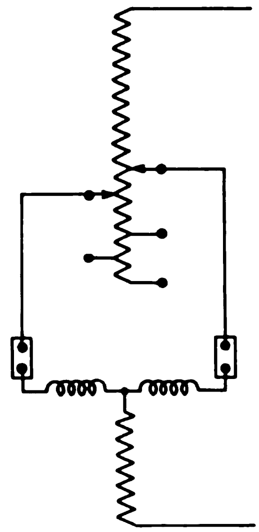

Fig. 10. A Connection Diagram for Load Ratio Control in which All Positions Are Made Operating Positions, Thus Making it Possible to Obtain Nine Voltages by Using Only Five Taps

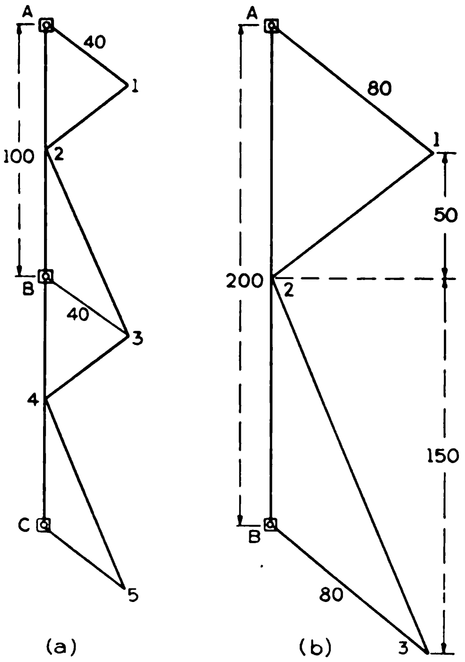

Fig. 11. Diagram Showing the Voltages Derived in Tap-changing with a Circulating Current of 60 Per Cent Full Load at 80 Per Cent Power-factor

(a) Voltages derived from symmetrical position shown in (a), Fig. 9.

(b) Voltages derived from unsymmetrical and bridging positions (b) and (c). Fig. 9. Note the inequality of voltage steps produced.

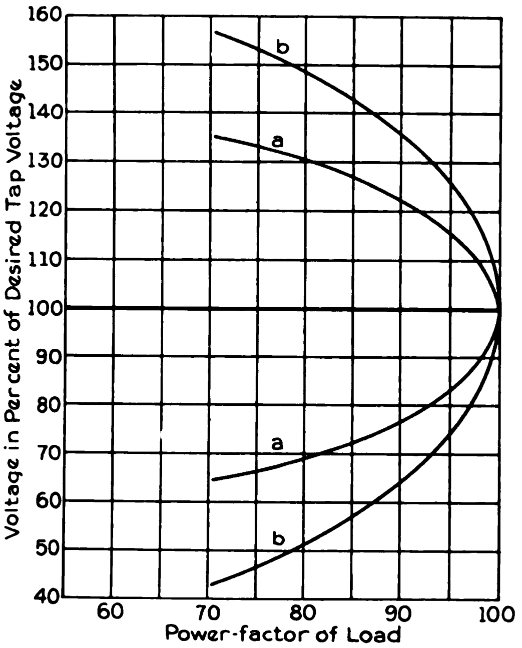

Fig. 12. Unequal Voltage Steps, Caused by Operating Alternately with One and Two Taps Closed, Plotted as Functions of Power-factor

Curve *aa*—Reactor designed for 100 per cent circulating current.

Curve *bb*—Reactor designed for 60 per cent circulating current.

In Fig. 12, the relations between unequal voltage steps are plotted as a function of power-factor. Curve *aa* is obtained when the reactor is designed for 100 per cent circulating current and curve *bb* when the reactor is designed for 60 per cent circulating current. The curves show that, even at the higher power-factors, a considerable inequality in adjacent voltage steps is obtained; for example, at 90 per cent power-factor and full load, we obtain from curve *bb* adjacent voltage steps equal to 65 per cent and 135 per cent, where two 100-per cent steps are desired.

Another objection to operating with connections on only one tap is that short-circuit currents are concentrated on one circuit; consequently, unless the current-carrying parts are designed much more liberally, the safety-factor, under short-circuit conditions, will be very greatly reduced.

Furthermore, during short circuits, the voltage drop across the reactor will rise, so that a relatively high voltage appears across the open breaker. For example, a short-circuit current of ten times normal, flowing through one side of the reactor, may increase the voltage across the open circuit breaker in the neighborhood of 50 per cent of line voltage. With symmetrical operation, both breakers being closed, this voltage rise cannot occur.

The circuits which have just been described show what can be done with the combination of ratio adjusters and circuit breakers, to obtain voltage control. When a group of circuit breakers or contactors are used alone, they are connected in a manner shown in Fig. 13. This method (1) is one of the oldest forms of ratio control arrangements; and it was used in connection with relatively small units many years ago.

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(1) It was fully described in the paper, "Tap Changing Transformer Under Load," by L. H. Hill, at the A.I.E.E. Convention, Pittsfield, Mass., May, 1927.

The arrangement just described involves two kinds of operating positions, *viz.*, one in which only one contactor is closed, and another in which two adjacent contactors are closed and bridged by the current-limiting reactor. The voltage diagram at the right-hand side of Fig. 13, showing the voltages obtained in three successive positions at full load, marked 1, 2, and 3, indicates that the steps are not uniform as was previously described. However, by adding (Fig. 14) another contactor, 6, which is connected so as to short-circuit the reactor in alternate steps, uniform voltages may be obtained. This arrangement involves the operating voltages 1, 2, and 3, shown in the voltage diagram at the right-hand side. Thus, a considerable improvement in operating characteristics is obtained in that all steps are now uniform, and two contactors are always closed in an operating position. In alternate positions the reactor bridges adjacent taps, and therefore, the reactor is under excitation and circulating current is flowing. As both the core losses and the copper losses are independent of load because of the circulating or magnetizing current in the bridging positions, they should be included in the no-load losses of the equipment, although allowance should be made for the fact that they are present only on alternate operating positions. Of course, by avoiding the use of the bridging positions as operating positions these losses can be eliminated and, at the same time, the reactor size can be reduced to about one-third, as previously explained. However, to the writer's knowledge, this has not been attempted, since it involves doubling the number of contactors.

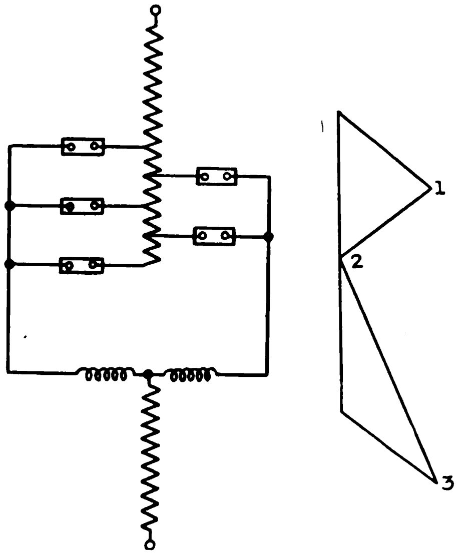

Fig. 13. Connection Diagram Showing How Load Ratio Control May be Accomplished by the Use of Circuit Interrupting Equipment Alone. The voltage diagram appears on the right

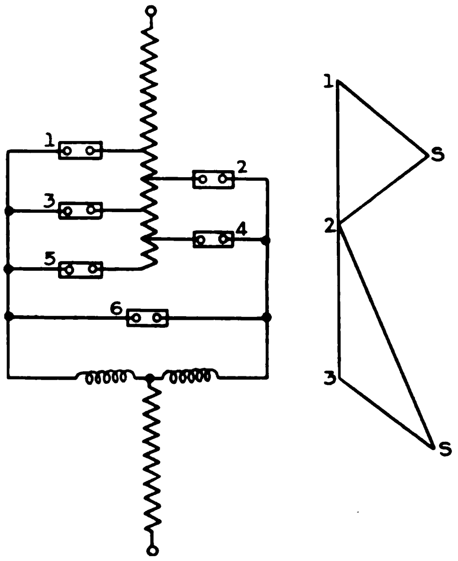

Fig. 14. The Switching Equipment of Fig. 13, Supplemented by the Use of an Additional Unit, 6, Permits Tap-changing in Uniform Voltage Steps. In the voltage diagram, on the right, *1, 2, 3* are operating positions; *S, S* are transition positions

The foregoing characteristics have been considered in rather minute detail, in order to show that although various ratio control equipments differ very greatly from each other in mechanical and electrical design, they are all nevertheless reducible to equivalent electrical circuits which are similar to each other; and the major difference in electrical characteristics depends mostly upon the positions in a cycle of operation which are selected by the designer to be the permanent operating positions. By using all possible positions, the number of mechanical parts is reduced; but when this is done it is necessary to design the electrical circuits more liberally and even then, in some cases, the electrical performance is not particularly good.

While having the two types of equipments in mind, it is worth noting that the segregation of the switching duty from the tap-changing duty, which is accomplished by the combination of circuit breakers and ratio adjusters, materially reduces the voltage between parts in the switch tank. This is because, in the one case, the full tap range must be brought out into the switch tank; whereas when ratio adjusters are used, only the voltage of one tap appears across the circuit breaker. Take for example the case of ratio control inserted in the grounded neutral end of a 132,000-volt line. Without ratio adjusters, it is necessary to bring out of the transformer tank the full tap range, *i.e.*, from 7000 to 20,000 volts; whereas with ratio adjusters the voltage of only one tap is brought out which is one-tenth as much. These values are also the voltages to ground. Considering that the oil in a switching compartment is very quickly carbonized, and also contains particles of copper, it is certainly advantageous to keep the voltage in these tanks as low as possible.

(*To be continued*)

Industrial Electric Heating

Part IV

MELTING NON-FERROUS METALS (Cont'd)

By N. R. STANSEL
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THE second division of the non-ferrous metals in the melting point scale, Group II, Table VIII, begins with silver and is extended to include all alloys containing a metal from that group as one of the major constituents of the alloy. Copper and nickel are the two metals in Group II which are of greatest utility in the industrial arts. The copper alloys, brass, bronze, nickel-silver, and some of the bearing metals which contain copper are first in importance in this group of alloys. Of these, brass is most widely used; the term "brass" including all the copper-zine alloys which contain 55 per cent or more of copper.

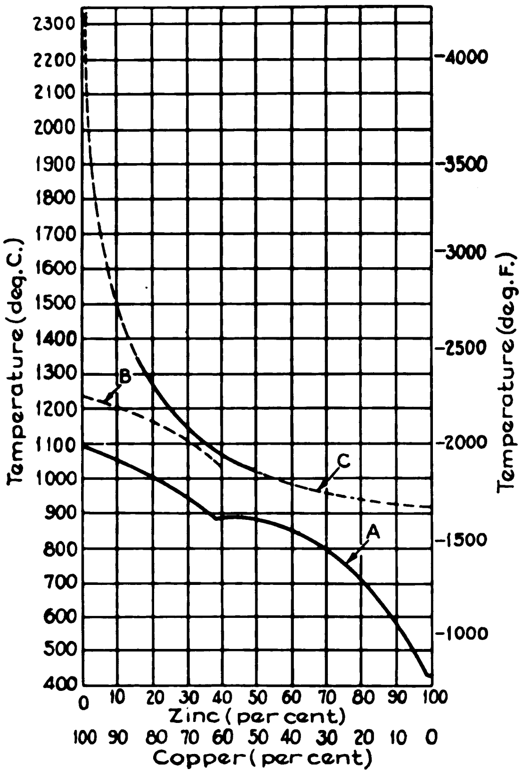

Fig. 58. Temperature Chart for Molten Copper-zinc Alloys (*Gillett*)

(*A*) Melting temperatures.
(*B*) Pouring temperatures, assumed 150 deg. C. above melting point.
(*C*) Approximate boiling points.

The extent of the present use of electric heat for melting in this branch of the non-ferrous metal industry is shown in Table X.[16] While the resistor-type electric furnace has been used to some extent for this class of melting, by far the larger part of electric heat for this service is now obtained from induction furnaces and arc furnaces.

TABLE X

Electric Brass and Bronze Furnaces in Active Use in United States and Canada, Feb., 1927. Excludes idle or spare furnaces. Includes those melting copper, nickel brass, and aluminum bronze. Includes only arc and induction furnaces

Number of Furnaces	Connected Load Kw.	Tons of Melt for 1926 (Estimated)	Kilowatt-hours Used (Estimated)
Wrought Brass Industry			
321	27,100	*450,000	113,000,000
Brass Foundry Industry			
303	29,200	†225,000	67,500,000
Total 624	66,300	‡675,000	180,500,000

* Approximately 90 per cent of the total output of mills.
† Approximately 30 per cent of the total output of foundries.
‡ Approximately 50 per cent of the grand total.

The figures of Table X show that while the numbers of electric furnaces in active use for melting in the two divisions of industry noted are about the same, the electric energy used for melting alloys for the production of bars, sheets, tubes, etc., is about twice that used in foundry practice. This wrought-metal production is concentrated into a few large plants which use electric heat almost exclusively for melting. The total cast-metal output comes from a large number of comparatively small, widely-scattered foundry plants. In these plants the growth of the adoption of electric heat for melting has been much slower.

Pouring Temperatures

The pouring temperatures of non-ferrous metals of high melting temperatures and their alloys vary with the character of the metal or alloy, the purpose for which the melt is to be used, and with the type of mold. In general, the pouring temperatures range from 150 to 350 deg. F. above the melting point. The chart in Fig. 58, by Gillett and Mack[17], gives

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[16] Gillett: "Twenty-five Years of Non-ferrous Electrothermics," *Trans. Amer. Electrochem. Soc.* Apr., 1927.

[17] Gillett and Mack: "Electric Brass Furnace Practice," *Bul. No. 202*, U.S. Dept. of the Interior.

References made to material hitherto published in this series of articles may be readily located from the following table:

	Issue	*Figs.*	*Equations*	*Examples*	*Tables*	*Footnotes*
Part I	Oct. 1927, p. 488	1 to 23	(1) to (8b)	1 to 3	I to III	(1) and (2)
Part II	Nov. 1927, p. 551	24 to 43	(9) and (10)	4 to 6	IV to VII	(3) to (13)
Part III	Dec. 1927, p. 600	44 to 57	(11) and (12)	8 to 11	VIII to IX	(14) and (15)

approximate boiling points, melting points and assumed pouring temperatures for the entire range of commercial copper-zinc alloys.

Heat Content

Only a comparatively small amount of work in the determination of the heat content of molten non-ferrous alloys has been done. The values in Table XI are given by Gillett and Mack(17) who state: "According to the probable specific heat of molten copper alloys it will probably take from 10 to 15 kw-hr. per ton of brass or bronze extra for each 100 deg. C. above the melting point to which the metal is raised;" also, "It seems likely that 150 kw-hr. per ton for average yellow brass poured at about 1050 deg. C. (1922 deg. F.) and 170 kw-hr. per ton for average red brass poured at 1200 deg. C. (2192 deg. F.) are better figures for the theoretical power requirements." The heat content curves of Fig. 59 for two copper-zinc alloys and of Fig. 60 for two nickel silver alloys are given by Tama.(18)

TABLE XI

Alloy	TEMPERATURE		Heat Content Kw-hr. per Ton
	Deg. F.	Deg. C.	
61 Cu 36 Zn 3 Pb	1832	1000	146 (Gillett and Mack)
65 Cu 35 Zn	1832	1000	136 (Richards)
66⅔ Cu 33⅓ Zn	1832	1000	138 (Clamer and Hering)
80 Cu 20 Zn	2012	1100	161 (Hansen)
85 Cu 15 Sn	1922	1050	136 (Richards)

The heat content curves of non-ferrous metals and their alloys, for example, Figs. 46, 59 and 60, consist of a line broken into three sections:

(*a*) A section showing the heat absorption up to the melting point.

(*b*) An intermediate section representing the transition from the solid to the liquid state.

(*c*) A section showing the heat absorption above the melting point.

TABLE XII

Temperature, Deg. C.	Microhms Per Centimeter Cube
20	1.7347
1000	9.42
1082.6	10.30 solid state
1082.6	21.30 liquid state
1100	21.43
1340	23.39
1450	24.22

The change from the solid to the liquid state may be sudden or gradual, depending upon the character of the metal or alloy. If this change is gradual, we have the pasty stage noted with reference to the equilibrium diagrams, Figs. 44 and 45. The change from the solid to the liquid state is accompanied by a marked increase in the specific resistance of the metal. For example, Northrup(19) obtained the values given in Table XII for copper (99.39 per cent conductivity).

In each case the value given for the heat content represents only the heat absorbed by the metal or metals. If the charge of metals to be melted contains impurities, such as oxides, dirt, moisture, etc., these also absorb heat. Usually the most important impurity, from the standpoint of heat absorption, is water. Moisture in a charge may account for an appreciable quantity of heat in the melting operation. The evaporation of one pound of water requires approximately 1200 B.t.u. (0.35 kw-hr.) From the figures just given the heat content of one pound of a molten non-ferrous alloy runs from 250 to 300 B.t.u. If the charge contains, say, three per cent moisture the heat absorption of the total charge will be from 10 to 15 per cent greater than the theoretical value for the metal alone.

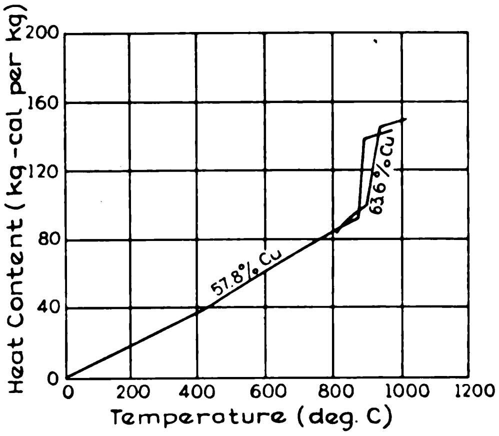

Fig. 59. Heat Content Curves of Two Copper-zinc Alloys (*Tama*)

Factors in Melting

In non-ferrous melting practice the more important factors which guide the melter are:

(*a*) The low vapor pressures of some metals, notably zinc, which cause volatilization at comparatively low temperatures;

(*b*) The readiness with which oxidation of these metals occurs at high temperatures;

(*c*) The tendency in case of some alloys towards segregation while in the molten state;

(*d*) The effects of underheating, overheating, and of unequal distribution of temperature within the mass of the charge;

(*e*) The effects that impurities have upon the metal or alloy.;

(*f*) The analysis of the charge and the nature of the different parts of the charge; and

(*g*) The kind and size of the castings to be poured.

In brass melting it is easy to lose zinc by vaporization. Values of the vapor pressure of zinc at different temperatures, together with the corresponding values for some other metals for comparison, are given in

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(18) Tama: *Zeitschrift fur Metalkunde*, Vol. 18, p. 7.

(19) Northrup: *Jour. Franklin Institute*, Vol. 177, p. 21.

Table XIII. It will be noted that zinc has an appreciable vapor pressure not far above its melting point. Also by reference to Fig. 58 it is seen that the pouring temperatures of the copper-zinc alloys near the 60-40 proportion (which constitute a large part of the brass used) are close to the boiling points of these alloys.

If brass is melted in an open crucible the rate of loss of zinc from the charge depends upon:

(*a*) The percentage of zinc in the charge;
(*b*) The temperature of the metal;
(*c*) The rate of diffusion of zinc through the mass to the surface; and
(*d*) The velocity of gases over the surface of the metal.

The total loss of zinc from a given weight of charge, as determined by these factors, will depend upon the area of the exposed surface and upon the length of time that the mass of metal is held at the stated temperature. With a sealed melting chamber the loss by vaporization during melting would of course be zero.

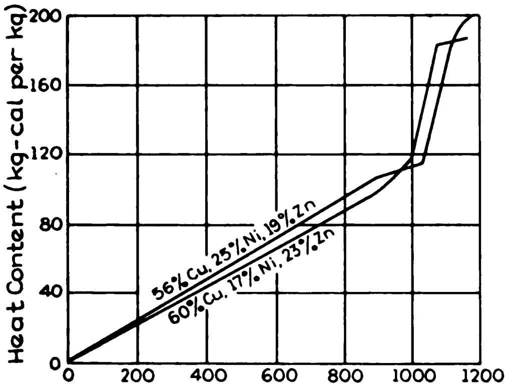

Fig. 60. Heat Content Curves of Two Nickel Silver Alloys (*Tama*)

A loss of a part of one or more of the constituents of a charge of metals during melting is undesirable in itself as a metal loss. A more important consideration, in the case of alloys, is the consequent loss of the control of the analysis of the alloy. In the process of melting steel a check analysis of each charge is usually made before pouring. This checking of the melt before pouring is not practicable, under average conditions, in melting non-ferrous alloys.

TABLE XIII

Metal	Melting Point Deg. C.	Vapor Pressure in mm. of Mercury 10^{-3}	10^{-2}	10^{-1}	1	10	50	100	760
		Temperature (Deg. C.)							
Zinc.....	419	290	350	420	500	610	700	750	920
Copper...	1083	1080	1200	1340	1520	1740	1930	2030	2350
Lead.....	327	620	710	820	960	1130	1290	1360	1640
Tin......	232	1010	1130	1270	1440	1660	1850	1940	2260

The poisonous character of zinc fumes is another factor in melting alloys which contain that metal. Before the advent of the electric melting furnace in the brass industry the malady known as "brass shakes" was common among workmen employed around open crucible melting furnaces. While zinc has been singled out as the metal requiring much care in melting to prevent loss by vaporization, it should be kept in mind that other metals (see Table XIII) have vapor pressures at elevated temperatures that must be considered in melting operations.

As the constituents of non-ferrous alloys are readily oxidizable at the pouring temperatures of these alloys, the atmosphere of the melting chamber should be neutral.

Stirring of an alloy while it is in the molten state is often essential to prevent segregation of the metals. This is particularly the case with alloys containing lead, a metal quite common in brass mixtures. The degree of the stirring necessary in any particular case depends upon the character of the alloy, *i.e.*, upon its tendency towards segregation and also upon the nature and the quantity of the impurities which may be present. The stirring of the molten mass should be a uniform procedure. This requires that the action be automatic. Stirring also brings about equalization of temperatures within the mass and prevents overheating of any portion of the metal.

The Ideal Melting Furnace

A convenient method of considering equipment for a given service is to first consider a specification that represents ideal conditions. This method, however, will lead to wrong conclusions unless the degree of importance attached to each item of the specification is based upon the conditions under which the equipment will operate. The ideal specification for a furnace with which to melt non-ferrous metals and their alloys would include many items. However, what is thought to be a reasonably complete catalogue of requirements follows:

(*a*) A closed melting chamber to prevent loss of metal by volatilization and for the control of the atmosphere of the chamber.
(*b*) A continuous and uniform stirring of the molten metal the degree of which can be controlled to suit the particular alloy being melted.
(*c*) A close control over the flow of energy into the metal. This is necessary in order to obtain accuracy in pouring temperature of the individual melt and uniformity in the pouring temperatures of successive melts.

(*d*) A temperature gradient from the metal to the outer boundary of the furnace walls. This prevents a flow of heat from the furnace structure into the metal when the supply of heat to the furnace is cut off, as in holding a charge before pouring.
(*e*) There should be no contamination of the metal or alloy by impurities or gases during the melting process.
(*f*) The furnace should be suitable for melting all classes of charges, *i.e.*, virgin metal, scrap of all kinds, and in fact any mixture of metals.
(*g*) The design of the furnace as regards size should be such that the type can be fitted to operations both large and small.
(*h*) The furnace should be adaptable to both intermittent and continuous operations and to the melting of alloys of different compositions in intermittent service.
(*i*) The furnace should be suitable for location at any point in the line of manufacturing operations.
(*j*) The conversion efficiency must be high and the design such that a high operating efficiency can be obtained easily under average operating conditions.
(*k*) The nature of the ambient atmosphere due to the operation of the furnace should affect neither the efficiency nor the health of the operators.
(*l*) The refractory lining of the melting chamber should have a long life.
(*m*) Simplicity and ruggedness of design and construction are essential.
(*n*) The furnace should be simple in operation.
(*o*) Safety in operation is essential.
(*p*) The labor cost of operation should be a small item.
(*q*) The maintenance charge should be low.
(*r*) Adaptability of design to any desired method of charging and pouring should be possible.

In addition, for the electric melting furnace, the following features relating to the power system which supplies the furnace load should be considered:

(*s*) The utilization of standard voltages and frequencies;
(*t*) A unity power-factor load;
(*u*) A balanced polyphase load;
(*v*) An even demand upon the power supply; and, with all these requirements, the furnace must meet the economic phase of the service.

Obviously no one type of furnace complies wholly with this specification. Hence in each case the class of service required and the local conditions should be considered to determine the features which should be given priority. There is a wide variety in melting service. In some cases two or more types of furnaces will serve the purpose. In others a furnace of a particular type is essential for satisfactory results.

Electric Melting Furnaces

Both the induction furnace and the arc furnace meet in a large measure the ideal specification. The wide use of both of these types shows that electric heat for melting meets the economic demands of melting service besides being of particular value in the production of high-grade metals and alloys of uniform quality; the latter often being the only reason assigned to the selection of electric heat for melting service.

The induction furnace and the arc furnace differ fundamentally in the methods of conversion of electric energy into heat and in the methods of applying the heat to the metal to be melted. In induction heating, the energy is transferred to the metal by electromagnetic induction and the electric currents thus induced in the metal develop heat in the mass as expressed by Joule's law, given as Equation (1). In the arc furnace, the electric energy is converted into heat in the arc and the heat is transferred to the metal by heat radiation, in accordance with Equation (8). These fundamental differences necessitate separate considerations of the two types of furnaces, which will be taken up in turn.

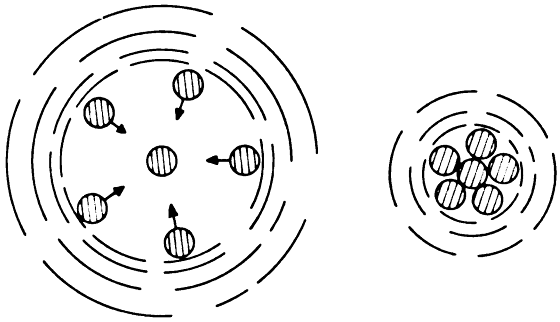

Fig. 61. The Constricting Effect of Electromagnetic Forces Between Parallel Conductors Carrying Currents in the Same Direction

The Induction Furnace

The induction furnace is a combination of an air-cooled transformer and a direct-resistance furnace; a transformer in that the energy supplied to a primary coil is transferred by induction to a secondary coil, and a furnace in that this energy is converted into heat within this secondary coil. The secondary coil is a closed loop formed by the charge of metal to be melted. The character of this secondary circuit is the distinguishing feature of the types of induction melting furnaces.

There are certain principles which are peculiar to the use of induction heating for melting metals, *viz.:*

(*a*) The heat is developed within the mass of the metal and the time required to melt the charge does not depend upon the thermal resistance of the surface.

Equation (1) applied to melting by induction heating can be further rearranged thus:

$$t' = \frac{h_a}{I_s^2 \times R_s \times 1000} \qquad (1d)$$

$$= \frac{h_a}{I_s \times E_s \times 1000} \qquad (1e)$$

in which

t' = theoretical time in hours for the metal to reach a given temperature.
h_a = kilowatt hours absorbed by the metal from Equation (2).
I_s = current induced in the secondary circuit.
R_s = resistance of the secondary circuit.
E_s = voltage induced in the secondary circuit.

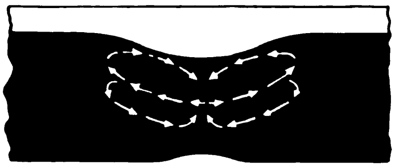

Fig. 62. A Diagram of a Portion of a Molten Metal Conductor, Showing the Constriction of Its Sectional Dimension Occasioned by Electromagnetic Forces

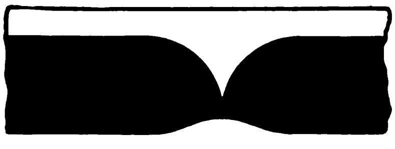

Fig. 63. The So-called "Pinch Effect," which Sometimes Occurs When the Constrictive Effect of Electromagnetic Forces Becomes Pronounced

Equations (1d) and (1e), together with Equation (2), state *first* that the theoretical time required to melt a given charge of metal depends upon the specific heat, the latent heat, and the final temperature; and, *second*, that this time is inversely proportional to the input of energy into the metal. The value h_a in a given case is a definite quantity. Hence the minimum value of t' is only a matter of the rate at which energy can be supplied. The actual time is greater than the theoretical time by an amount depending upon the limitations of the power input imposed by the design of the furnace and upon the rate at which heat can be accumulated in the metal, the last mentioned being a matter of the degree to which the mass of metal is insulated against the flow of heat to the surrounding atmosphere.

(*b*) The secondary coil of an induction furnace becomes, when melted, a liquid conductor at high temperature, *i.e.*, molten metal. A liquid conductor can be considered as an infinite number of current-carrying elements or conductors in parallel. The law of electro-magnetic forces between parallel conductors is:

$$F = \frac{I_1 \times I_2 \times c}{s} \qquad (13)$$

in which

F = force acting per unit length; attractive if the currents flow in the same direction, repulsive if in opposite directions.
I_1, I_2 = currents in the conductors.
c = a constant.
s = the distance between the conductors.

In the case of parallel conductors free to move and in which the currents flow in the same direction this force will tend to bring the conductors together to form a closed bundle of circular cross section as indicated in Fig. 61. If the conductor is a liquid contained in an open channel the radial pressure created tends to force the liquid towards the center and out at the ends thus decreasing the cross section of the conductor. If there is an irregularity in the channel of the liquid conductor, as shown in Fig. 62, the liquid will be constricted and consequently the current density and the pressure will be increased at this point. The result is that the liquid is forced away from the point of higher pressure, a movement being set up as indicated. If the current in the liquid conductor is great enough the liquid will be forced away faster than it can return and the circuit will be ruptured, as in Fig. 63. This phenomenon is known as the "pinch effect." (20) It is a factor which has much bearing upon the design of induction furnaces.

Equation (14) by Hering (21) applies to liquid conductors in open channels whose width is one-half the initial height of the liquid. The value I_c in this equation is the current which will cause rupture of the circuit and is termed the critical current.

$$I_c = 449\sqrt{G \times H^3} \qquad (14)$$

in which

I_c = critical current.
G = specific gravity of the liquid.
H = initial height of the liquid.

From Table XIV it will be noted that the current densities corresponding to the critical currents for various sizes of the channel varies with the size of the channel. In the use of induction furnaces with open channels the current in the liquid conductor (secondary circuit) must be limited to a safe operating value below the critical value for the size and type of channel used. As a general rule, three-fourths of the critical value provides sufficient margin.

(*c*) The high temperature of the liquid conductor makes it necessary to use a refractory container for the secondary circuit of the furnace. This construction requires a comparatively large amount of space in the transformer construction, *i.e.*, between the primary and secondary coils. Thus the reactance is high; and unless the power supply is of low frequency the power-factor of the furnace load will be low.

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(20) Northrup: *Physical Review*, Vol. 24, p. 474.

(21) Hering: *Trans. Amer. Electrochem. Soc.*, Vol. 11, p. 330, and Vol. 15, p. 255.

(*d*) The heating effect of the currents in the secondary circuit being proportional to the electric resistance of the circuit, the specific resistance of the metal to be melted is a determining factor in the use of induction heating for melting. Data on the specific resistances of metals and alloys in the molten state are limited. The values of Table XV were compiled from various sources. In any case the value of specific resistance depends, of course, upon the temperature of the metal. Tama(18) gives the chart of Fig. 64, showing the relative values of specific resistance of copper-zinc alloys in the solid state at 0 deg. C. and in the liquid state at the melting temperature. The chart(22) in Fig. 65 shows the resistivity of a brass of composition 63 Cu, 34.58 Zn, 2.42 Pb. It will be noted that the molten metal has a negative resistance temperature coefficient. According to Northrup this is due to the zinc in the alloy, he having determined previously(23) that the resistance temperature coefficient of zinc in the molten state (up to 750 deg. C.) is negative.

TABLE XIV

CHANNELS		2 IN. BY 1 IN.		4 IN. BY 2 IN.		6 IN. BY 3 IN.		8 IN. BY 4 IN.	
Metal	G	I_c	CD_c	I_c	CD_c	I_c	CD_c	I_c	CD_c
Tin......	6.99	3300	1650	9800	1225	17000	945	26000	810
Lead.....	10.60	4100	2050	11500	1440	21000	1165	33000	1030
Zinc......	6.48	3200	1600	9100	1140	16500	920	25500	800
Aluminum	2.67	2000	1000	5800	725	10500	585	16500	515
Silver....	9.51	3900	1950	11000	1375	20000	1110	31000	970
Copper...	8.20	3600	1800	10000	1250	18500	1030	29000	900
Nickel....	8.80	3700	1850	10500	1310	19500	1080	30000	940
Steel.....	6.88	3300	1650	9500	1185	17000	945	26500	830

G = specific gravity.
I_c = critical current for liquid conductors in open channels whose width is one-half the initial height of liquid.
CD_c = current density corresponding to I_c.

TABLE XV

	MICROHMS PER CENTIMETER CUBE	
	Solid	Liquid
Metal		
Tin..........................	11.3	54.6
Lead.........................	20.8	102.8
Zinc.........................	6.1	36.6
Aluminum.....................	2.94	24.0
Silver.......................	1.65	17.0
Copper.......................	1.78	24.8
Nickel.......................	11.8	...
Steel........................	10.8	1.66
Alloys		
Cu 66⅔ Zn 33⅓..............	4.0	41
Monel Metal..................	42.5	80
Ni 66 Cr 22 Fe 10 Mn 2.......	109	...

(*e*) The forces acting upon the liquid conductor which forms the secondary circuit of the furnace tend to cause motion of the liquid, *i.e.*, the molten metal. In general, the more viscous the molten metal the more sensitive it is to the forces tending to cause motion of the metal. The viscosities of molten metals vary. However there is but little, if any, data on this property.

From the foregoing considerations we can classify the properties of metals with reference to melting by induction heating, as in Table XVI.

The lower the values of specific heat, latent heat and melting temperature, the lower the heat content for any given pouring temperature. None of these values affect the permissible operating current in the secondary circuit. The higher the specific gravity, the higher may be the operating current, as in Equation (14). The higher the specific resistance, the higher will be the rate of heat development by the permissible current. The heat developed by the current in the secondary circuit must supply the heat loss of the furnace as well as raise the temperature of the charge of metal. The limit of this temperature is reached when the rate of heat loss equals the rate at which heat is developed. If in a given furnace the specific gravity and the specific resistance of the metal are low, it may not be possible with the limited current to develop heat at a sufficient rate to raise the temperature of the metal to the desired value.

TABLE XVI

Property	Favorable	Unfavorable
Specific heat..............	low	high
Latent heat................	low	high
Melting point..............	low	high
Specific gravity...........	high	low
Specific resistance........	high	low
Viscosity..................	low	high

Types of Induction Furnaces

There are two types of induction furnaces in use today, *viz.*, the core type and the coreless type. The transformer of the former type has a magnetic core and is designed for use on circuits of a standard frequency—60, 50, 40, 30 or 25 cycles—and in some cases for a special low frequency, below 25 cycles.

The coreless, or high-frequency type, furnace has no magnetic core and requires a frequency considerably higher than those adopted as standard for power service. Here the term high frequency means, say,

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(22) Northrup: *Metallurgical and Chemical Engineering*, Vol. 12, p. 161.
(23) Northrup and Suydam: *Jour. Fran. Inst.*, February, 1913.

500 cycles or higher; although no particular value of frequency is assigned to the term.

For reasons of simplicity of design and construction, induction furnaces are as a rule single-phase. Some three-phase induction furnaces have been built; but since the three-phase design has only the added merit of a balanced load as compared with the single-phase furnace, the three-phase design has made but little progress. With the growth of the size of power supply systems the magnitude of the permissible single-phase load has increased; and it is now not uncommon to find single-phase furnace loads up to 300 kv-a. In non-ferrous melting service large melting furnace units are not usually required. As a rule, two or more small units are better adapted to the operating conditions; and this arrangement lends itself to obtain a balanced load on the power system. For special frequencies a motor-generator set is required which, of course, constitutes a balanced load. The load of an induction furnace is inherently steady, the load chart being practically a straight line. For holding the temperature of a charge of molten metal a low voltage, obtained by a transformer tap, is used. The power-factor depends upon the design of the furnace and upon the metal being melted. This will be referred to again in the later considerations of furnace types.

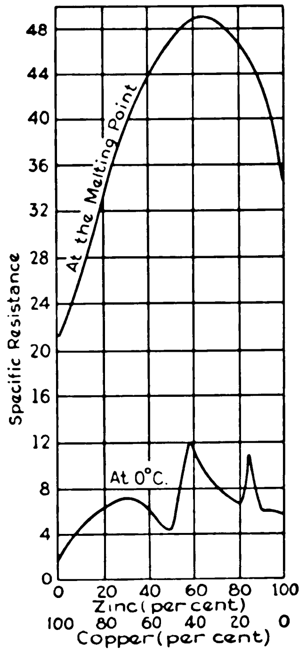

Fig. 64. Relative Values of Specific Resistances for Copper-zinc Alloys in Solid and Molten States (*Tama*)

Two forms of the core-type induction furnace have been developed, *viz.*, the Horizontal Ring furnace and the Vertical Ring furnace. While the Horizontal Ring furnace is designed primarily for melting and refining ferrous alloys and non-ferrous alloys of high specific resistance and is not intended for general application to melting non-ferrous alloys, it is logical to include this form at this point in the treatment of induction furnaces.

The Horizontal Ring Induction Furnace. The name of this furnace itself indicates the arrangement of the secondary circuit of the transformer, *i.e.*, a ring or loop of metal in a horizontal plane. The general design of the furnace is shown in Fig. 66. The entire mass of the charge of metal is contained in a ring crucible, which is an *open* channel in a refractory material. The charge of metal is thus formed into a single-turn loop for the secondary circuit. The magnetic core is of the three-leg type made up of laminated steel in accordance with standard transformer practice. The primary coil, of the circular disk type insulated with mica and asbestos, is located above the secondary coil (the molten metal loop) so that it will not be damaged in the event of a leak in the crucible. A small motor-driven blower supplies air under a low pressure, two to four ounces, for cooling the primary coil. A description of a 6-ton furnace(24) of this type has been previously published. A typical installation is shown in Fig. 67.

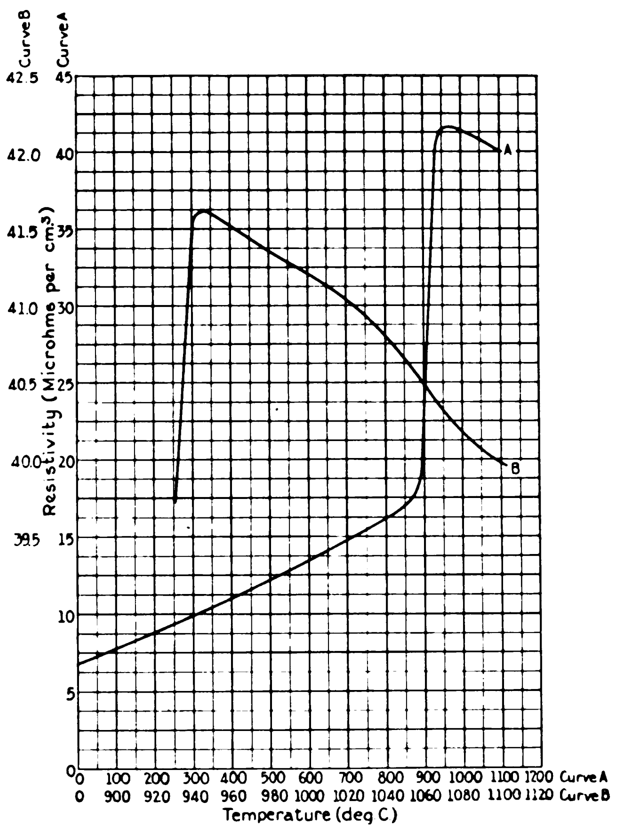

Fig. 65. Resistivity of a Copper-zinc-lead Alloy (*Northrup*)

In this furnace there is an open channel for the liquid conductor, and the forces acting upon the secondary circuit so formed produce the following results:

(*a*) Keeping within the permissible limits of operating current value for the size and proportions of the open channel used and for the metal to be melted (see Table XIV) there is an unbalanced pressure set up within the molten metal, as indicated in

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(24) "A Six-ton Induction Furnace Installation," by M. Unger, General Electric Review, August, 1924, p. 498.

Fig. 62, which causes a more or less gentle motion of the metal along the horizontal axis of the channel. If there are slag or gases—comparatively poor conductors—in the metal these impurities act in the same way as irregularities in the channel, *i.e.*, they reduce the cross-section of the liquid conductor and thereby aid in producing motion of the molten metal.

(*b*) Owing to the annular shape of the secondary circuit, the current density in the loop of metal is greater at the inner face of the ring, *i.e.*, where the path of the current is the shortest and resistance lowest. Hence, the attractive force within the liquid conductor is not uniform, being greatest at the inner face. This inequality of forces tends to set up a circulating motion in the molten metal as indicated in Fig. 68.

(*c*) With the arrangement of primary and secondary coils as shown in Fig. 66, the repulsive force between these two circuits, given by Equation (13), tends to repel the current-carrying elements of the liquid conductor towards the bottom of the channel. This action forces the impurities in the molten metal to the surface.

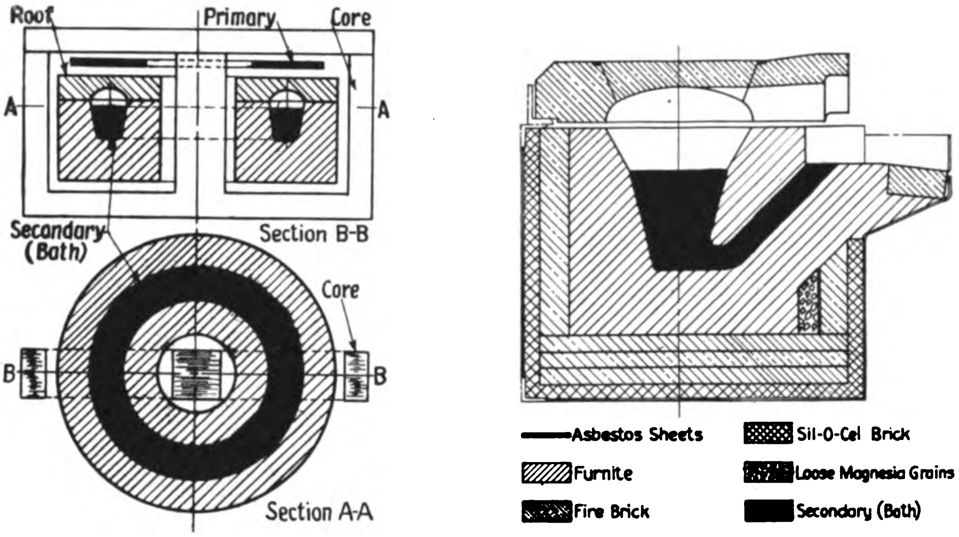

Fig. 66. A Diagram of the Horizontal Ring Induction Furnace, Showing a Section Along the Transverse Horizontal Axis and Another Through the Pouring Spout

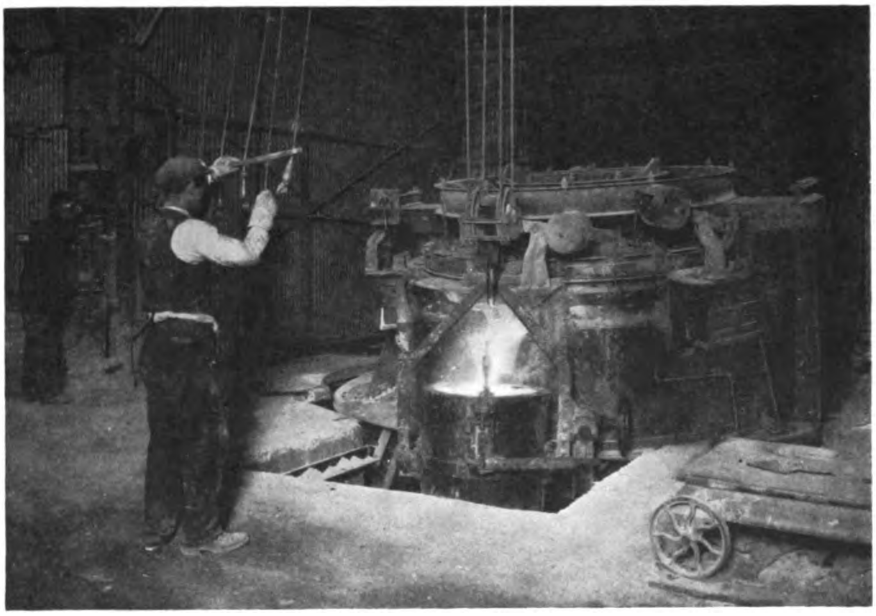

Fig. 67. An Installation of a Horizontal Ring Induction Furnace of 4000-lb. Capacity. The electrical rating is 250 kw., 2200 volts, 15 cycles

The combined effect of these forces in the operation of the Horizontal Ring furnace may be summarized thus: The molten metal is forced towards the center of the channel so that any impurities in the metal are squeezed out. The molten metal is furthermore subjected to circulation, one motion in a vertical plane, perpendicular to the axis of the channel, and another in horizontal planes in the direction of the axis of the channel. This combination of forces provides a stirring action the degree of which is governed by the extent of the unevenness of the channel, the amount of impurities present, and upon the value of the current in the conductor. The current is regulated to give the desired degree of stirring as dictated by the metallurgical conditions of the melting or refining operation.

The starting of the Horizontal Ring furnace is accomplished either by pouring molten metal into channel for the secondary circuit or by the use of solid starting rings made to fit the contour of the channel, one of which is shown in Fig. 69. These rings must of course be of the same metal as the charge. In the production of alloys, instead of using a single starting ring which may cause some trouble due to rupture of the circuit, two rings of different melting points are used. After the starting rings are melted, cold metal is added to complete the charge. When pouring, about one-third of the charge is left in the furnace for starting the subsequent charge.

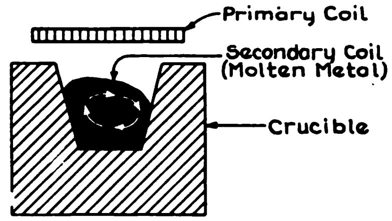

Fig. 68. A Sectional Diagram Showing the Forces Acting Upon the Secondary Circuit of a Horizontal Ring Induction Furnace

The power-factor of this type of furnace is comparatively low owing to the loose coupling between the primary and secondary coils. The higher the specific resistance of the metal of the charge the higher will be the power-factor; moreover, the furnace will operate at a higher power-factor with a portion of a charge than with a full charge. With the larger sizes of furnaces the power-factor at a standard frequency is lower than is desirable; and a motor-generator set is used which both supplies a low frequency for the furnace circuit and balances the load on the power system. Also, if desirable, a synchronous motor can be used for the motor-generator set and capacity added to this machine for power-factor correction. A furnace having a capacity of 500 lb. of steel has a power-factor of approximately 50 per cent on a 60-cycle circuit. This value of power-factor is also obtained with a 1500-lb. furnace on a 25-cycle circuit, and with a 4000-lb. furnace on a 15-cycle circuit, in each case with a full charge of molten metal.

The Vertical Ring Induction Furnace. The distinguishing feature of this form of core type induction furnace is the use of a *closed* channel of a construction and location which eliminate the current density limitation of open channels and adapt induction heating to the melting of metals and alloys which are outside of the range of furnaces with open channels. The Vertical Ring induction furnace is particularly well suited to the melting of copper-zinc alloys and is widely used for that purpose by the brass industry. The construction is shown in Fig. 70.

The secondary circuit of the transformer is a single-turn loop of molten metal in a channel which forms a V in a vertical plane below, and connecting with, the main body of the charge of metal. The mass of metal in the V loop is comparatively small, from 3 to 10 per cent of the total mass of the charge, the amount in each case depending upon the size of the furnace and the properties of the metal or alloy for which the furnace is designed. The heat developed by the current induced in the secondary circuit is transferred to the main body of the charge by liquid convection. The V loop of molten metal is thus an indirect heating unit. The convection movement of molten metal in the channel is effected by the combined actions of the force of gravity and the electrodynamic forces acting on the liquid conductor.

Fig. 69. Starting Rings for a Horizontal Ring Induction Furnace

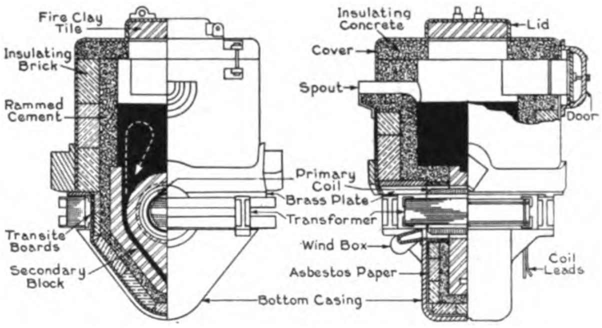

Fig. 70. Sectional Views of a Vertical Ring Induction Furnace

The force of gravity effect is that which follows from the difference in densities of the particles of molten metal in the channel and of the cooler (and hence denser) particles in the main body of the charge. The electrodynamic forces acting are due in part to the geometrical arrangement of the liquid conductor, this element of the forces being a maximum at the apex of the triangle formed by the channel, and in part to the "pinch effect" phenomenon which action is a maximum at the junctions of the channel with the main body of the charge. A complete analysis of the actions of these forces is given elsewhere.⁽²⁵⁾ The channel in the refractory material for the secondary circuit is designed to utilize these forces to produce a movement, or circulation, of the molten metal, *i.e.*, the liquid convection noted in a preceding paragraph and indicated by the arrow in Fig. 70. The channel is designed as a secondary circuit of a transformer with a cross-section and length corresponding to rate of melting desired; and in addition there must be taken into account the viscosity of the molten metal and the frictional resistance of the channel. The primary coil and magnetic core follow in design standard transformer practice. The primary coil is cooled by low-pressure air supplied by a small motor-driven blower.

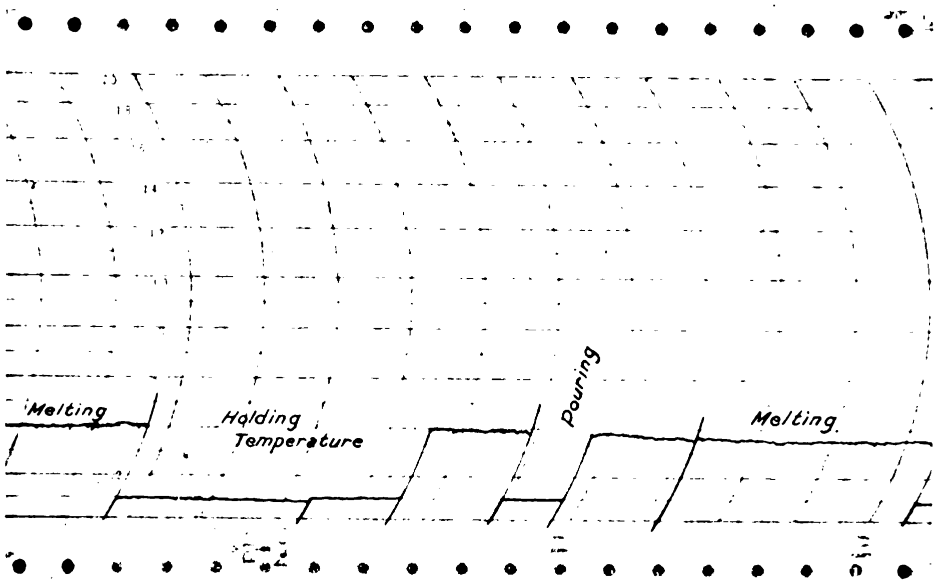

Fig. 71. A Typical Load Chart of a Single-phase 60-kw., 220-volt Vertical Ring Induction Furnace

The rate of melting a given metal or alloy in the Vertical Ring furnace is a matter of the permissible power input into the secondary circuit. There is, of course, a limit to the temperature to which the liquid conductor of the secondary circuit can be carried both on account of the volatilization of the metal and the temperature limit of the refractory material which contains the channel. As in all transformers, the primary coil is limited also to a definite kilowatt input at the voltage for which it is designed. Furnaces of this type operating with pouring temperatures up to 2400 deg. F. are in commercial service.

The power-factor of this type of furnace is comparatively high, being 75 to 80 per cent on 60-cycle circuits and somewhat higher on 25-cycle circuits. In operation, the loop forming the secondary circuit is constantly kept full by the hydrostatic head of the charge above; and the resistance of the circuit is practically constant. A chart showing the uniformity of the load is reproduced in Fig. 71. A regulating transformer with a number of taps is provided for holding the temperature of a charge. A diagram of the furnace connections is shown in Fig. 72.

In starting the Vertical Ring furnace it is first necessary to fill the V channel with molten metal; and this loop must be kept in a molten state as long as the furnace is in service. This requirement makes the furnace inflexible as regards frequent changes of alloy compositions. In general, this type of furnace

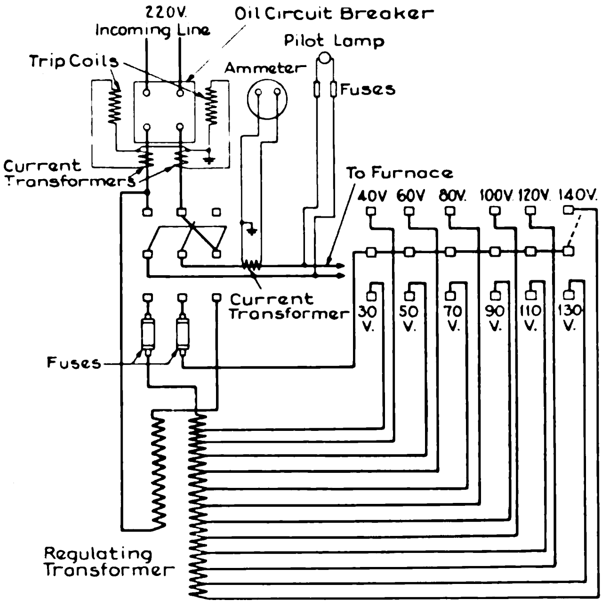

Fig. 72. The Connection Diagram of a Single-phase 60-kw. 220-volt Vertical Ring Induction Furnace

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(25) Northrup: "Nature and Explanation of the 'Motor Effect' in the Ajax-Wyatt Furnace," *Jour. of the Franklin Institute*, Vol. 190, p. 817.

is intended for a continuous melting service although there may be idle periods, as nights, holidays, and Sundays in the schedule of operations. During such periods the metal in the furnace is held molten by a low-voltage tap on the regulating transformer.

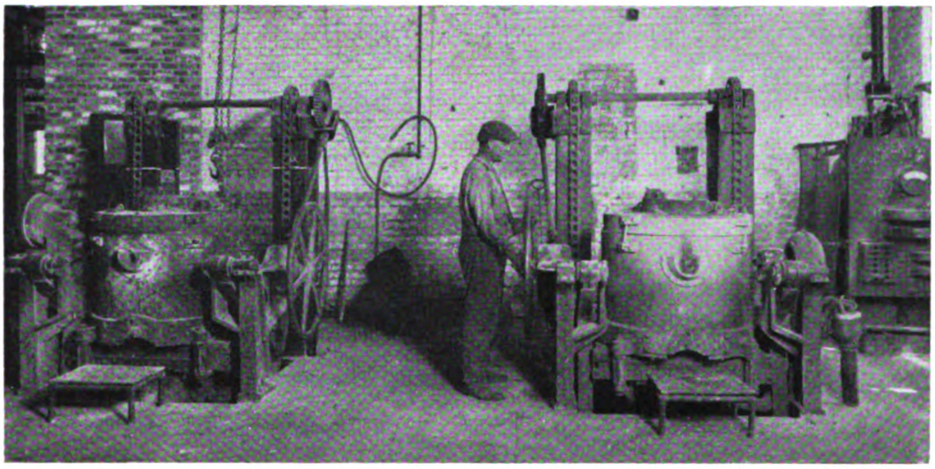
TABLE XVII

	Output, Lb.	Per Cent
Gross Charges	3,352,581	100
Net Castings	3,285,062	97.99
Net recovery on skimmings and splashings	53,552	1.59
Total metal shrinkage including oil and non-metallics in original charge	13,967	0.42

The data of a test (26) of this type of furnace melting brass for metal loss, analysis of output, and energy consumption are given in Tables XVII, XVIII, and XIX respectively.

The operating efficiency, *i.e.*, the kilowatt-hours per ton of castings over an extended period of operation of a furnace, depends of course upon the many factors that enter into a schedule of manufacturing operations. Thus the operating efficiency of a furnace on the same metal will vary as the operating conditions are changed. The highest operating efficiency is obtained when the furnace runs continuously, when the metal is poured as soon as it reaches the pouring temperature, and when the charging and pouring times are a minimum percentage of the total time.

The manufacturers of the Ajax-Wyatt Vertical Ring induction furnace give as representative results with this furnace the data of Table XX.



Courtesy Ajax Metal Co.

Fig. 73. A Typical Installation of Two 60-kw. 220-volt Single-phase Furnaces of the Vertical-ring Induction Type in Operation in a Foundry

TABLE XVIII

ANALYSES OF POURINGS FOR LEAD CONTENT

Heat No.	FIRST POURING Cu	FIRST POURING Pb	LAST POURING Cu	LAST POURING Pb
1	64.44	1.03	64.38	1.05
2	64.38	1.01	64.38	1.08
3	64.27	1.09	64.24	1.17
4	64.49	1.10	64.52	1.02

TABLE XIX

Alloy	Estimated Pouring Temperature	Kw-hr. Per Ton
60 Cu 40 Zn	1850 deg. F.	172
65 Cu 35 Zn	1960 deg. F.	186
66.7 Cu 33.3 Zn	2000 deg. F.	192
70 Cu 30 Zn	2070 deg. F.	200

TABLE XX

Foundry Operation: 9 hr. per day.
Product: brass, 75 Cu, 23 Zn 2 Pb medium weight castings, 7 lb. per mold.
Charge: Clean medium-weight scrap and ingots.
Pour: 500 to 800 lb.
Average melting time per heat (600 lb.): 60 min.
Production (8 heats per day): 2½ to 3 tons per day.

	Melting Cost Per Ton
Energy consumption, 275 kw-hr. per ton (includes stand-by nights and week ends) at 2 cents per kw-hr	$5.50
Lining (life, 500 tons)	0.30
Maintenance	0.18
Metal loss (0.5 per cent) at 11 cents per lb.	1.10
Labor at 65 cents per hr.	1.17
Ladle heating	0.40
Total Cost per ton of brass melted	$8.65

Refractories

The vital part of any melting furnace is the refractory lining. This is a general subject common to all types of furnaces and is reserved as the subject matter of a later article of this series.

(*To be continued*)

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(26) Heuer: "The Ajax-Wyatt Furnace in the Brass Mill Casting Shop," *Jour. Ind. and Eng. Chem.*, Vol. 14, p. 1021.

The Neon-electric Stroboscope

By E. E. STEINERT
Engineering General Department, General Electric Company

ABOUT three years ago there arose an urgent need for a stroboscope that could be used in the study of power system stability. The system involved in the study consisted of an alternator, a synchronous condenser, a synchronous motor, and two "π" sections of line. Among other things it was necessary to measure various displacement angles caused by different load conditions. This need brought about the development of the neon-electric stroboscope. It is the purpose of this article to review some of the previous types of stroboscopes, and to describe the new form developed by C. A. Nickle.

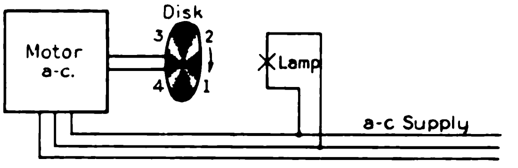

Fig. 1. Diagram of a Simple Form of Stroboscope

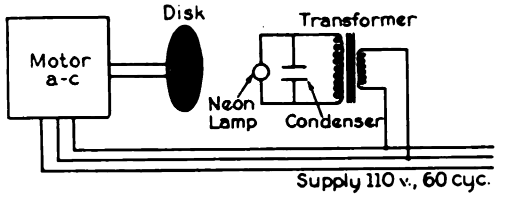

Fig. 2. Diagram of the Neon-electric Stroboscope

General

The word "stroboscope," derived from Greek, means "whirling mark." A stroboscope usually consists of a disk rotating with the shaft of the machine under consideration and is provided with some device which allows it to be seen by the eye only at certain regular intervals. The operation depends on the property of the eye to retain an image. For example, if a moving object attains a certain position twenty times or more a second, and is allowed to be seen by the eye only when in the stated position, it appears to be stationary. There are a variety of stroboscopes designed to meet specific problems. A simple form is shown diagrammatically in Fig. 1.

If the motor in Fig. 1 is a four-pole synchronous machine, there is utilized in the procedure a disk having four black and four white sectors equally and alternately spaced. The disk is attached to the motor shaft and rotates with it. Light is furnished by an electric lamp (usually a carbon arc) connected to the a-c. supply and giving to the disk an illumination which pulsates at a frequency twice that of the a-c. supply.

Action

If in the case under consideration the motor is connected to a 60-cycle supply, it will run at 1800 r.p.m., or 30 r.p.s. The current through the arc will be positive for one-half cycle and negative for the next half cycle, but the light intensity will be the same for each half cycle. Thus the pulsations of light from the arc lamp will have a frequency equal to twice that of the supply, or 120 cycles per second. While the disk is turning through 90 degrees, the intensity of illumination passes through one cycle. During the next light cycle, the disk moves through another 90 degrees and allows a second image to be registered. Suppose the first image shows the white sectors in positions as indicated in Fig. 1; then in the second image, sector two occupies the position previously occupied by sector one, sector three occupies that of sector two, etc. If the intensity of illumination is great for a very small portion of each cycle, the shift in sector position is not detected. Thus the impression transmitted to the observer is that of a stationary disk. Usually, however, the image is blurred because the light furnished by the a-c. arc, instead of giving an instantaneous flash each cycle, has a more or less continuous variation.

Fig. 3. The Neon-electric Stroboscope

In any study which requires precise measurement by the use of a stroboscope, it is necessary that the image recorded by the eye be very sharp. This is particularly the case when it is desired to measure the displacement angle of a synchronous motor.

Types of Stroboscopes

One form of stroboscope (1) makes use of a slotted disk, back of which is located a small neon lamp.

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(1) "A Simple Method of Determining Slip of Induction Motors and Torque Angle of Synchronous Motors by Means of the Neon Lamp," by W. E. Maserve and D. Ramadanoff, *Sibley Journal of Engineering*, June, 1927.

the neon lamp being chosen because of its nominally instantaneous responsiveness to applied voltage and because of its low thermal capacity which prevents its continuing as a light source while the voltage wave is crossing the zero axis. When the disk revolves, it interrupts the passage of light between the neon lamp and the observer. In this connection there is described also an ingenious method of reading high values of induction motor slip by means of two disks and a system of pulleys.

Another form of the stroboscope is that of Dr. D. Robertson(2) in which the light interruption is obtained by means of an electrically-driven tuning fork placed between a segmented disk and the observer's eye. The prongs of the fork have slotted aluminum wings. With the fork vibrating, light reaches the observer's eye when the slots are opposite and is cut off when they are not opposite. Thus the number of interruptions per second depends on the fork frequency.

Still another form is that of Dr. C. V. Drysdale(2) which makes use of a tuning fork, an induction coil, and a neon tube. The tuning fork, which is electrically driven, makes and breaks the primary current of the induction coil. The secondary of the coil is connected to the neon tube. Thus the vibrating fork causes the neon tube to flash, producing the usual stroboscopic results.

Of particular significance is the fact that all previous stroboscopes which give a fair degree of accuracy involve moving parts in addition to the rotating disk.

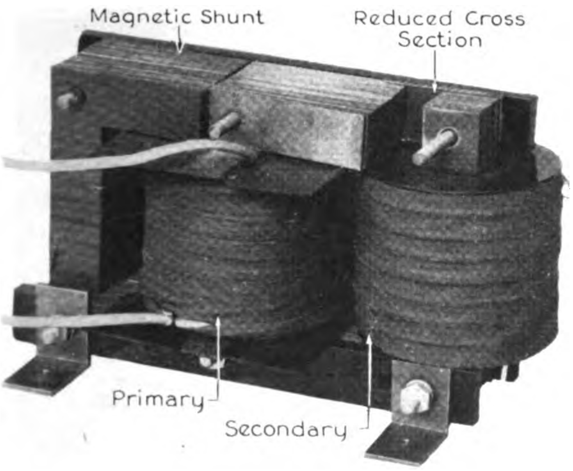

Fig. 4. The Special Transformer

Neon-electric Stroboscope

The new or neon-electric form of stroboscope is unique in that it has no moving parts except the disk. This stroboscope consists of four parts: a special transformer, a condenser, a neon lamp, and a disk. The description of these parts will follow. Fig. 2 shows the general arrangement of the parts. The transformer, condenser, and neon lamp are enclosed in a case (see Fig. 3) for protecting the lamp against breakage and the operator against high voltage. The only electrical connections to the case are those leading to the transformer primary, which operates from a 110-volt 60-cycle supply; and the only technique required in its operation is to place the box where the light can fall upon the disk. No tuning or adjusting is necessary. Moreover, one looks directly at the disk, and not through slots or past vibrating parts. Hence it has the merit of great convenience.

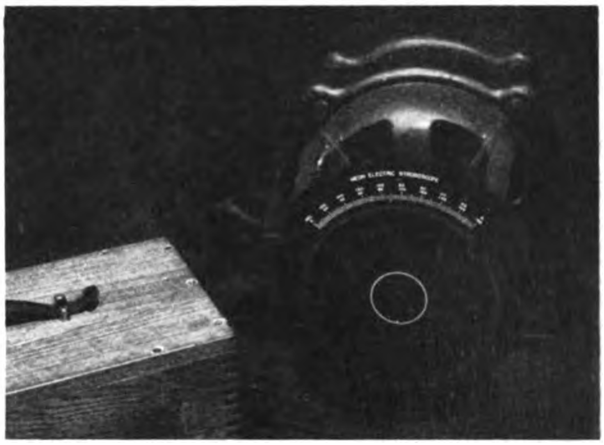

Fig. 5. A Laboratory Set-up with the Neon-electric Stroboscope in Position but Without the Neon Lamp in Operation, for which Reason the Arrows on the Rotating Disk Are Invisible
(See cover illustration)

The element which makes this stroboscope effective is the special transformer shown in Fig. 4. To describe its action it will be necessary to treat the functioning of the various parts separately. It is a well-known fact that the voltage induced in a coil is proportional to the rate of change of flux with respect to time. Thus, if a transformer is supplied with a sine-wave voltage, and saturation does not occur, the secondary voltage will also be a sine wave. With an approximately rectangular flux wave, as shown in Fig. 6; the voltage wave will consist of lines perpendicular to the time axis.

For the purpose in hand, the flux wave of Fig. 6 would be the one to strive for, because it would produce instantaneous flashes in the neon lamp. Even ordinary grades of transformer iron will give a slightly peaked voltage because of saturation; and special grades which saturate readily, and thus permit only a very slight increase of flux after the knee of the curve is passed, might further increase the peaked effect. However, to radically accentuate the peak of the induced voltage wave, a magnetic circuit was constructed which had one part of much smaller cross-section than the remainder (see Fig. 4).

The effect of the reduced section is to increase the rapidity with which the core becomes saturated and thus give a flux wave which approaches rectangular shape. Hence the voltage generated in the secondary is characterized by a sharp, well-defined peak. The nature of this wave shape (Fig. 7) was determined in a test by means of an exploring coil and the oscillograph.

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(1) Miles Walker: "The Diagnosis of Troubles in Electrical Machines."

Still better results were obtained by tuning the secondary circuit with a condenser. With this arrangement, an oscillogram of voltage in the secondary was obtained, the nature of which was determined by test and is shown in Fig. 8.

This voltage wave, when impressed on a suitable neon lamp, produces the sharp flashes of light which are essential for satisfactory operation of the stroboscope. There remained one difficulty to be overcome in the use of the oscillatory voltage wave, *viz.*, that the neon lamp responds not only to the first peak of the cycle, but also to the second and third peaks. However, there was no difficulty in distinguishing the fundamental image from the others, for it is much brighter and is separated from them by a considerable distance.

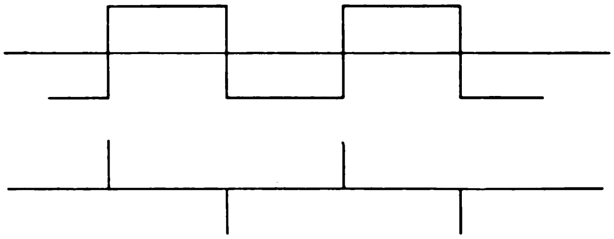


Fig. 6. Rectangular Flux Wave and Induced Voltage

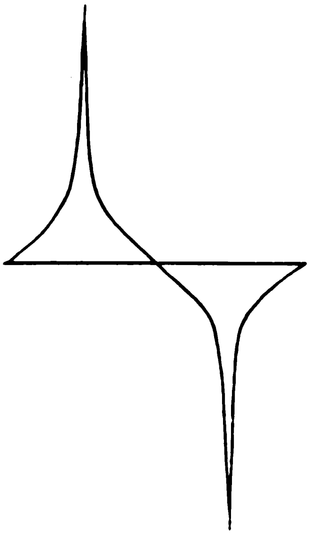


Fig. 7. Secondary Voltage Generated by the Special Transformer Used with the Neon-electric Stroboscope

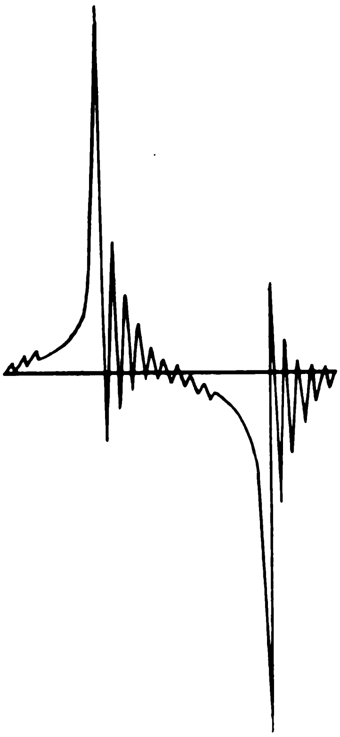


Fig. 8. Tuned Secondary Voltage Obtained Through the Use of a Condenser

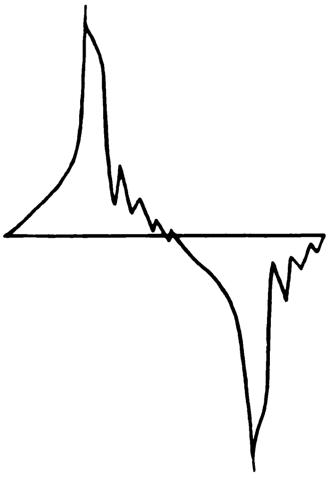


Fig. 9. Tuned[Secondary Voltage with Load

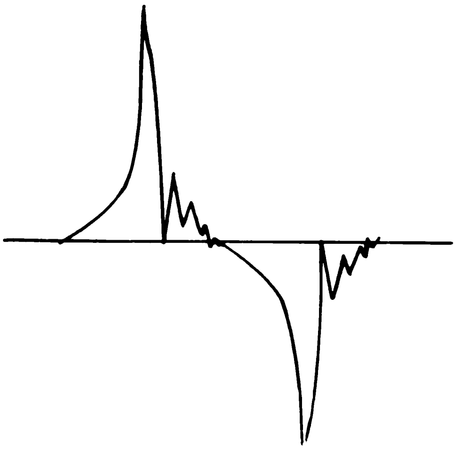


Fig. 10. The Wave Shape for which the Neon Lamps Are Designed to Operate in the New Stroboscope

There is another factor entering to aid the lamp design, and that is the effect of transformer load on the sharpness of tuning. Increasing the secondary load produces not only the detrimental effect of making the peaks flatter, but it produces also the favorable result of decreasing the secondary peak with respect to fundamental peak (see Fig. 9).

The neon lamp was designed to operate on a compromise wave shape (Fig. 10) between that of Fig. 8 and Fig. 9. The result is that a single sharp flash of high intensity is obtained each half cycle.

Another feature of the transformer is the magnetic shunt shown in Fig. 4. By varying the air gap between the shunt and the primary core, the primary current can be adjusted with respect to the resultant variation of light intensity from the neon lamp. The shunt also makes the transformer more compact, since without it an external reactance would be needed. During some tests, a small exploring coil was wound on the shunt and oscillograms taken. The voltage induced in this coil by flux passing through the magnetic shunt is shown in Fig. 11, while the voltage applied to the transformer primary is shown in Fig. 12. The sudden dip in voltage wave shown in Fig. 11 occurs when the restricted magnetic section between the primary and secondary windings becomes non-saturated.

Disk

It is not necessary to have a very large disk to secure quite precise measurements. In the case mentioned earlier in this article, black disks one foot in diameter were used. The marking employed consisted of a $\frac{1}{16}$-in. radial white line. (Enough equally spaced lines must be employed so that the frequency of appearance of the image shall be at least twenty per second. Furthermore, the number of lines must be an integral divisor of the number of poles on the machine to which the disk is attached.) Lines as narrow as 0.02 in. have been used on a one-foot disk rotating at 1800 r.p.m., giving readings which were accurate to within 0.02 in. at the periphery of the disk. Thus the effective duration of illumination of the disk is about 1/50,000 of a second for each flash of the neon lamp.

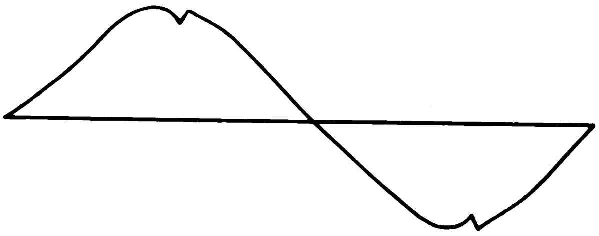


Fig. 11. Voltage Induced in Exploring Coil Used in Making Oscillographic Investigations of Stroboscope Performance

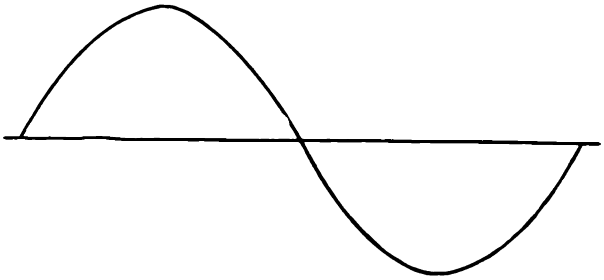


Fig. 12. Primary Voltage Wave Applied to Stroboscope Transformer

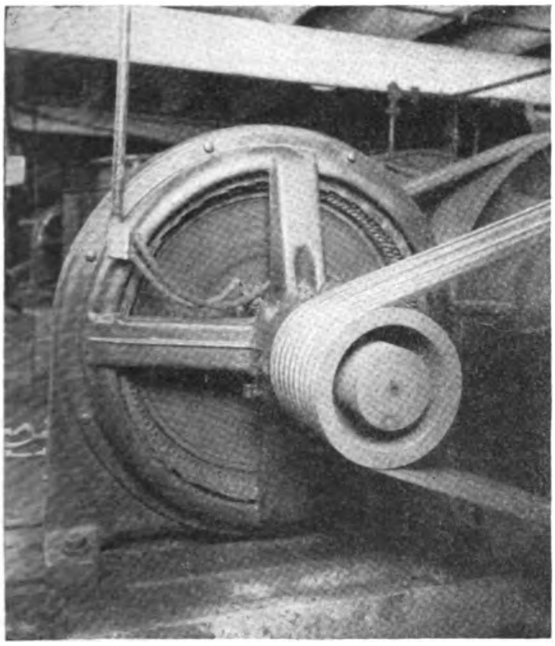
Summary

The success with which the neon-electric stroboscope has been used in recent tests is a direct result of its high degree of accuracy, which is possible only because of the extreme clearness of the image. Other features of this type of stroboscope are that it is compact and quite rugged. Also, it is as portable as an ordinary voltmeter.

Synchronous Motors Replace Steam Engine in Paper Plant

A 250-hp. Corliss engine, belted to four beaters in the Camden (N. J.) plant of the West Jersey Paper Mfg. Co., has been displaced by two 140-hp. 600-r.p.m. 440-volt 60-cycle 0.8-p-f. synchronous motors each driving two beaters.

These motors not only provide the power-factor correction that was needed but successfully accelerate and carry the load. With the beaters filled with pulp under certain conditions, and particularly when working manila rope stock, the partly ground stock has a tendency to pack between the beater rolls and the housing. To obviate this difficulty it was found that by modifying the control so that the motors could be given a jogging start, in the reverse direction, the mass in the beater is loosened. On again reversing to standard rotation, the motors pick up the load without difficulty.



One of the Synchronous Motors Installed

Low-voltage A-c. Networks

PART II

TYPES OF COMBINED LIGHT AND POWER NETWORK SYSTEMS

By D. K. BLAKE

Central Station Engineering Department, General Electric Company

IN residential areas, where the lighting and household appliance loads predominate, the single-phase transformer is used to supply three-wire secondary mains to which the consumers are connected as shown in Fig. 5. The voltage supplied at the socket may be 110/220, 115/230 or 120/240. The 115/230 predominates and is the standard preferred by the N.E.L.A., while the other voltages are recognized departures from the standard. The National Electric Safety Code requires that the circuits supplying lamps and appliances must be grounded so that the nominal voltage to ground does not exceed 150 volts. In order to meet this requirement, the neutral of the three-wire circuits is grounded. Wherever motor loads are encountered the electric service company usually specifies that single-phase motors must be used in applications of 7½ hp. or less. The distribution cost to supply three-phase service is excessive for such small ratings. Three-wire services with three-wire meters are used for the larger loads occasionally encountered.

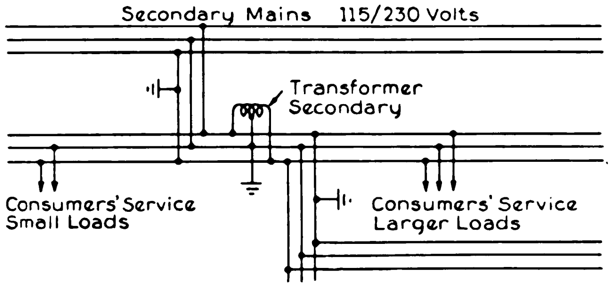

Secondary Mains 115/230 Volts
Transformer Secondary
Consumers' Service Small Loads
Consumers' Service Larger Loads

Fig. 5. A Three-wire Secondary Distribution System for Supplying Power for Lighting and Household Appliances

The motor load predominates in the industrial areas. Where the load consists of 220-volt motors, the delta-connected transformer secondary is used almost universally. The consumer's lighting load, being a small percentage of the total load, is supplied from a separate single-phase transformer in some cases; in others, it is supplied from one phase of the power transformers as illustrated in Fig. 6. The latter method is more economical than the former, but is limited to applications where the motor load characteristic is such that the motor starting currents will not cause objectionable flicker. The transformer on the phase supplying the lighting load is usually greater in capacity than the other two. The unbalance on the primary circuit is as a rule not objectionable because the lighting load is small and different phases are used for the lighting load of different consumers. The combined light and power supply requires three-coil protection on the control equipment because of the ground.

The power consumption in the commercial areas of large American cities is divided approximately into 50 per cent lighting and 50 per cent motor load. Some of the smaller cities may show various ratios approaching 90 per cent lighting and 10 per cent motors. An appreciable percentage of the motor load in the commercial area is made up of numerous small consumers. The d-c. Edison system now supplies both lighting and power loads from a single set of mains. It is very desirable to likewise supply light and power from a common set of mains when an a-c. network is used. The reasons for this are obvious, such as the diversity between light and power loads, the saving in duct and transformer vault space where there is already so much congestion, the use of one set of meters for certain loads where the rate structure permits, and the simplicity. The combined light and power a-c. network, however, presents several problems of a practical nature which are very important and very complex owing to the prevalent voltage standards of the devices constituting the load and existing practices of distribution and metering. There is also the additional technical problem of motor starting currents causing lamp flicker due to the voltage drop in the reactance of the transformers and secondary mains.

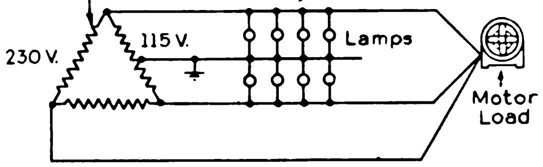


Fig. 6. A Method of Supplying a Lighting Load from One Phase of a Power Transformer that Furnishes Motor Service

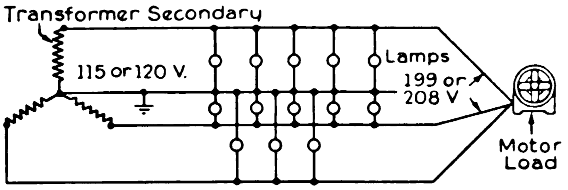


Fig. 7. The Y-secondary System of Distribution for Supplying Combined Lighting and Motor Loads

A combined light and power network should permit the load to be easily balanced over the phases, should permit grounding to limit the voltage on lighting circuits below 150 volts to ground, and should have the same nominal voltage for lamps as the circuits that are not in the network area.

About fifteen of the networks now installed use the Y-connected transformer secondary illustrated in Fig. 7. The lamps are supplied from phase to neutral and the motor load from the phase wires. Most of the networks supply 120 volts for light and 208 volts for motors measured at the customer's meter. A much smaller number of networks supply 115/199 volts. There are two important things to consider when contemplating this system. The first is that the voltage supplied to polyphase motors is appreciably below the standard rating of 220 volts. The second is that the three-wire service for lighting precludes the usual three-wire meter, and requires either two single-phase meters or the two-element polyphase meter because the voltage across the phases is 30 deg. out of phase with the current in the line.

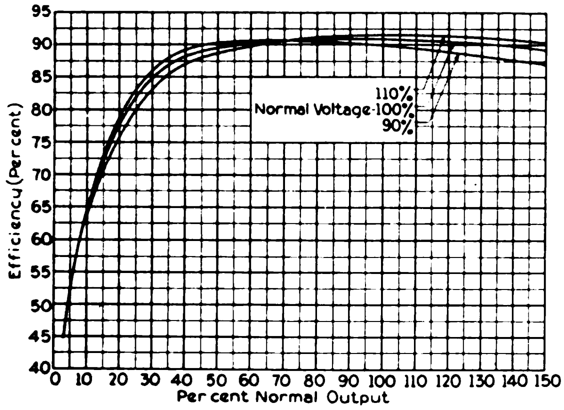


Fig. 8. Approximate Efficiency Curves for a Standard Induction Motor

The various operating companies who have adopted the Y-secondary system have advanced the following reasons for their action:

(*a*) The standard 220-volt induction motor is designed to operate successfully on potentials as low as 198 volts.

(*b*) The great majority of motor applications have larger motors than necessary for successful performance. (Various estimates are given, such as that 75 per cent of the motors are 50 to 60 per cent loaded).

(*c*) The network system will be regulated to give a plus or minus variation of three to four per cent, which is far better than now given on the usual power circuit. (This accounts for the large number of 120/208-volt systems, since a variation from 200 volts to 216 volts is above the lower limit of 198 volts.)

(*d*) The power-factor is slightly improved above 70 per cent output, and the efficiency slightly decreased but it is increased below 70 per cent output which covers the great majority of motor applications. The speed is not appreciably affected. (The approximate characteristics of standard general-purpose 220-volt motors at 90, 100, and 110 per cent voltage are given in Figs. 8, 9, and 10.)

(*e*) In the very few cases where the load is such as to demand full voltage applied to the motor, auto-transformers may be used to step up the voltage. (These cases are estimated to be less than 5 per cent of the total number of installations.)

(*f*) An increase in the lighting voltage to 120 volts makes 208 volts available for motor service, and this is adequate as mentioned in Item (*a*). Standard lamps of 120 volts rating are available; and large operating companies report that, in their experience, no difficulties are encountered with conservatively rated 110-volt appliances even though the voltage variations exceed the manufacturer's limit of 120 volts maximum. Also, that distribution transformers may be excited above 120 volts to maintain 120 volts at the service. (Some companies find it too difficult to raise their system voltage to supply 120 volts.)

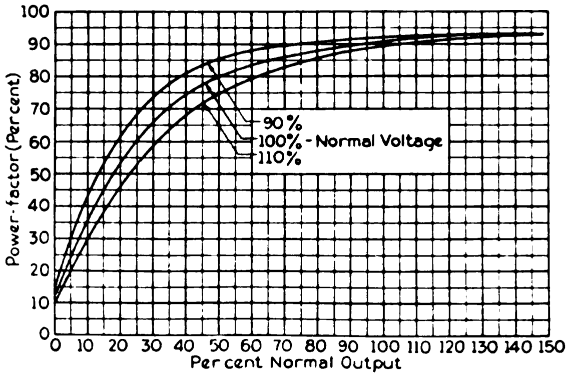


Fig. 9. Typical Power-factor Curves for a Standard Induction Motor

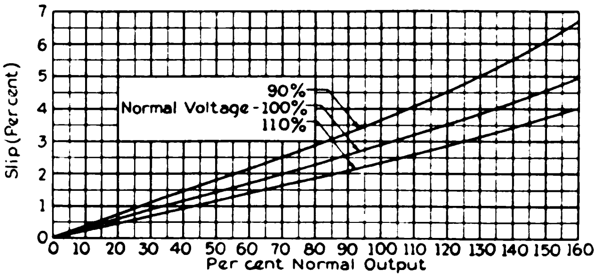


Fig. 10. Approximate Slip Characteristics of a Standard Induction Motor

Two companies already using two-phase for distribution purposes considered the three-phase Y-connected system when planning their network; but in view of the subnormal voltage supplied to motors, the three-wire metering difficulties, and the cost of changeover from two-phase to three-phase, decided to continue two-phase operation with the five-wire connection shown in Fig. 11. These two companies consider their demand for two-phase motors large enough to assure their consumers of no difficulty in readily obtaining two-phase motors. Aside from this consideration this system fits present voltage standards and meter practice perfectly.

Three other companies of moderate size consider it preferable to accept a more expensive system than to adopt a system which supplies subnormal voltages. One of these companies installs two sets of mains with auto transformers stepping up the voltage from the lighting mains to the power mains as illustrated in Fig. 12. Another company uses a similar system except that the transformer secondary winding is extended to 133 volts and a tap is provided at 115 volts as shown in Fig. 13. The 18-volt portion of the winding presents difficulties to the transformer designer when parallel operation of different sizes is required because of the small number of turns and the high voltage per turn. Neither of these systems eliminates the three-wire meter difficulty.

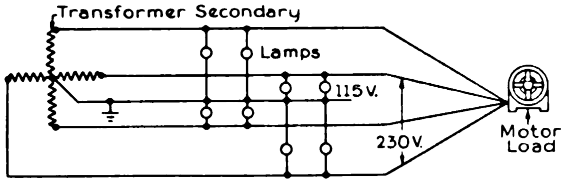


Fig. 11. A Two-phase Five-wire System for Supplying Lighting and Motor Loads

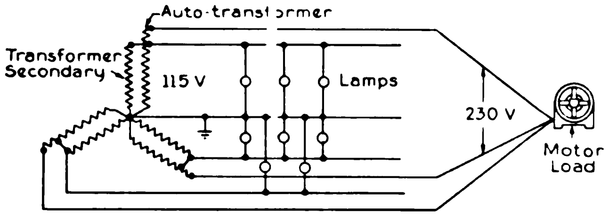


Fig. 12. Auto-transformers Used in Conjunction with a Lighting Supply Circuit to Provide for a Motor Load

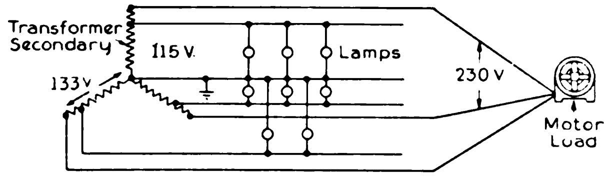


Fig. 13. A System Providing Proper Voltages for Lamps and Motors Through the Employment of Transformer Taps

The third company uses the well-known delta secondary illustrated in Fig. 6. The network is segregated into three separate areas in order to balance the load on the three phases. The installed transformer capacity on the lighting phase of a particular area exceeds that on the other phases. This system, of course, fits standard voltages and permits the usual three-wire meter practice. The necessity of three separate phase areas is the reason why this system is not more generally used. The delta-connected secondary also presents difficulties in the relay characteristic for the automatic network protector now extensively used. This company proposes to try out the so-called translator system, employing devices which are essentially auto-transformers for tying together the secondary mains into one large network with the lighting load on a different phase in different areas.

The translator network may be understood by referring to Fig. 14, 15 and 16. The connection of the translators between two transformers having different phases grounded is shown in Fig. 14. The vector relation of the two transformers is shown in Fig. 15. The translator is simply an iron core with four 115-volt coils as illustrated in Fig. 16. The vector diagram in Fig. 15 shows that there is a difference of potential of 115 volts between A_1 and A_2, B_1 and B_2, C_1 and C_2 due to the grounds on the different phases being at zero potential. Coil *a* connects A_1 to A_2, coil *b* connects B_1 and B_2 and coil *c* connects C_1 to C_2. Therefore, each coil is energized at 115 volts and excites the iron core. Suppose the lighting load on Bank 2 is very heavy and some of this load is supplied from Bank 1. A current will flow from A_1 to A_2 through coil *a* and from B_2 to B_1 through coil *b*. If the lighting load is on Bank 1 then Bank 2 will likewise supply current which will flow from B_2 to B_1 through coil *b* and from C_1 to C_2 through coil *c*. The arrows show clearly that the load currents flow through the coils in opposite directions and therefore pass readily through the translator as they would through a one to one ratio transformer. It is also evident from the arrows that a heavy three-phase load may be readily passed through the translater because the current in one coil is equal and opposite

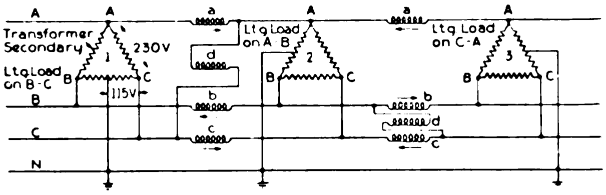


Fig. 14. The Translator Network, Consisting of a Combination of Mid-point Transformer Taps in Separate Phases and Special Devices for Supplying Lighting Loads

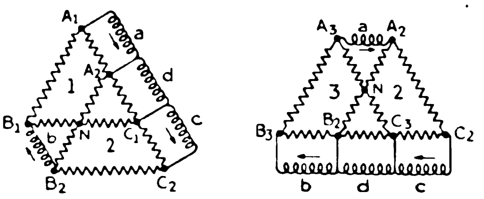


Fig. 15. Diagrams Showing the Relationship of Voltages in the Translator System

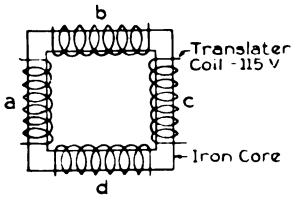


Fig. 16. Elementary Construction of an A-c. Network Translator

to the vector sum of the currents in the two other coils. Fig. 15 also shows the diagram for Banks 2 and 3. The load is always carried by coils *a*, *b*, and *c* unless the lighting load is unbalanced; then coil *d* carries some of the current. The chief function of coil *d* is to maintain the three-phase relation when the transformers of a particular section are removed. Suppose transformer No. 1 is removed. It is then obvious from Fig. 15, that coils *d* and *c* are in series across A_2 and C_2 and that conductors A_1 and B_1 are maintained at the same position as when the transformer is energized. Therefore, coil *d* need only be designed for a small percentage of the current rating of the other coils.

Each of the coils *a*, *b* and *c* should be designed with the same current-carrying capacity as the cables connected to it. The translator is preferably located between transformer vaults; but usually subway conditions demand that they be located in the transformer vault. It is obvious that this system requires an additional piece of apparatus at an increase in cost, losses, space requirement and maintenance. All of these are borne by the electric service company and none by the consumer. The consumer is not called upon to operate his single-phase devices at excessive voltage or his polyphase motors at subnormal voltage. It seems, therefore, that the selection of the translater system must be upon the grounds that the electric service company does not want to run any risk of inconvenience to, or complaint from, the customer but prefers to accept the extra cost and complications of the translater. The electric service company also gains the advantage of being thus enabled to use standard 3-wire meters on three-wire circuits.

(*To be continued*)

Heaviside's Operational Calculus as Applied to Engineering and Physics

Part IV: The Expansion Theorem Applied to Some Definite Problems
Part V: Operations on Unit Function Squared Have No Physical Significance
Part VI: Additional Operators Employed when a Network is Suddenly Connected to an Alternator Instead of to a Battery of Constant Voltage

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To illustrate the application of the Expansion Theorem to the problem cited in Part II when a leaky condenser in series with a resistance is suddenly connected to a storage battery of potential difference E, we have $Z_{(p)}=r+r_2+rr_2pC$. (See Equation (8) and Fig. 7 in Part II.)

Therefore $Z_{(p)}=0$ gives $p=-\dfrac{r+r_2}{rr_2C}$

$$\text{and } p_1=-\frac{r+r_2}{rr_2C}$$

$$\frac{dz}{dp}=rr_2C$$

Therefore $p\dfrac{dz}{dp}=prr_2C$

$$p\frac{dz}{dp} \text{ for } p=p_1=-\frac{r+r_2}{rr_2C}\times rr_2C=-(r+r_2)$$

$$Y_{(p_1)}=1-\frac{r+r_2}{rr_2C}\times rC=1-\frac{r}{r_2}-1=-\frac{r}{r_2}$$

$$\frac{Y_{(o)}}{Z_{(o)}}=\frac{1}{r+r_2}$$

Therefore
$$i=E\left[\frac{1}{r+r_2}+\frac{r}{r_2\,(r+r_2)}\,\epsilon^{-\frac{r+r_2}{rr_2C}t}\right]$$

For $t=0$, $i=\dfrac{E}{r_2}$ and for $t=\infty$, $i=\dfrac{E}{r+r_2}$.

It is of interest to note that the initial and final currents can always be obtained directly from the operational solution. In this case

$$i=E\,\frac{(1+pCr)}{r+r_2+pCrr_2}\,\mathbf{1} \tag{19}$$

When we remember the shape of the unit function, we realize that for $t=0$, $p=\dfrac{d}{dt}=\infty$; for $t=\infty$, $p=0$.

Therefore, for $t=0$ when $p=\infty$ or is very large compared with any finite quantity, we get

$$i=\frac{EpCr}{pCrr_2}\,\mathbf{1}=\frac{E}{r_2}$$

For $t=\infty$, $p=0$, and therefore $i=E\dfrac{1}{r+r_2}$.(2)

---

(2) The operational solution is the solution of the permanent alternating-current condition when vector representation is used if for p is substituted $j\omega$, thus $p^2=-\omega^2$, $p^3=-j\omega^3$, etc.

Heaviside brought out this fact in 1892, or perhaps earlier; it had also been shown by Kennelly in 1892.

The proof is simple when it is recollected that a convenient equation of a revolving vector is:

$$\underset{\cdot}{I}=I\epsilon^{j\omega t}$$

$$\text{Thus } \frac{d}{dt}\underset{\cdot}{I}=p\underset{\cdot}{I}=j\omega I\,\epsilon^{j\omega t}=j\omega\underset{\cdot}{I}$$

Referring then to Equation (19)

$$\underset{\cdot}{I}=\underset{\cdot}{E}\,\frac{(1+j\omega rC)}{r+r_2+j\omega Crr_2}$$

$$=\underset{\cdot}{E}\,\frac{r+r_2+\dfrac{r^2}{x_c^2}r_2+j\dfrac{r^2}{x_c}}{(r+r_2)^2+\dfrac{r^2}{x_c^2}r_2^2}$$

$$\text{where } x_c=\frac{1}{\omega C}.$$

Other examples involving complex roots will be given in Parts VII and VIII.

The operational equation brings out some other interesting features. Since for $t=0, p=\infty$, and for $t=\infty$, $p=0$, it is evident that if the denominator is of a higher power of p than the numerator, the initial current must be zero.

If numerator and denominator are of the same power in p, the initial current is of a definite value, not zero.

If the numerator is of higher power of p than the denominator, then the initial rush of current is infinite; but after an infinitely short time it decreases to a definite value. It might be well to say here that this condition can never be met in any actual problem, whether mechanical or electrical. It would exist in the case, for instance, where a condenser is suddenly connected to a battery with wires of no resistance, which obviously is impossible.

However, even then the method throws some light on the subject. We would obviously expect to find an instantaneous infinite rush of current during an exceedingly short time such that $\int idt$ would be the charge which corresponds to the voltage impressed.

The operational solution obviously is:

$$i=E\frac{1}{\frac{1}{pC}}\mathit{1}=pCE\,\mathit{1}$$

which means an instantaneous rush of current of infinite magnitude and the charge $=\frac{i}{p}=CE$, which is the proper charge.

Problem

A condenser made up with two different kinds of dielectric is suddenly connected to a storage battery of voltage E as in Fig. 8. Find the charge on the middle plate.

Let the leakage conductance and the capacity of the upper portion be g_1 and C_1 and the corresponding values for the lower part be g_2 and C_2. Let e_1 be the instantaneous voltage consumed by the upper part and e_2 that of the lower part, then the current taken by the upper part is obviously $i_1=e_1\,(g_1+pC_1)$ (the resistance operator of the condenser being $\frac{1}{pC_1}$; the corresponding admittance is $\frac{1}{\frac{1}{pC_1}}$). Similarly, the current taken by the lower part is $e_2\,(g_2+p\,C_2)$. These two currents are obviously the same. Thus

$$e_1\,(g_1+p\,C_1)=e_2\,(g_2+p\,C_2)$$

But $e_1+e_2=E\,1$. Therefore

$$E\,1=\frac{i}{g_1+pC_1}+\frac{i}{g_2+pC_2}$$

$$\text{or } i=E\frac{(g_1+pC_1)\;(g_2+pC_2)}{g_1+g_2+p\,(C_1+C_2)}\mathit{1}$$

Therefore

$$e_1=\frac{i}{g_1+pC_1}=E\frac{g_2+pC_2}{g_1+g_2+p\,(C_1+C_2)}\mathit{1}$$

and the charge on the top plate is

$$e_1\,C_1=E\,C_1\frac{g_2+p\,C_2}{g_1+g_2+p\,(C_1+C_2)}\mathit{1}.$$

The corresponding charge on the top side of the middle plate is numerically the same but of course with minus sign.

Consider next the lower part of the condenser. Its upper plate has a charge equal to C_2e_2 which is easily shown to be

$$C_2e_2=E\,C_2\frac{g_1+p\,C_1}{g_1+g_2+p\,(C_1+C_2)}\mathit{1}.$$

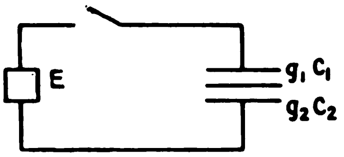

Fig. 8

Thus the total charge on the middle plate is

$$q=E\frac{C_2\,(g_1+p\,C_1)-C_1\,(g_2+p\,C_2)}{g_1+g_2+p\,(C_1+C_2)}\mathit{1}$$

$$=E\frac{g_1\,C_2-g_2\,C_1}{g_1+g_2+p\,(C_1+C_2)}\mathit{1}$$

$$=E\frac{A}{G_0+p\,C_0}\mathit{1}$$

where

$A=g_1\,C_2-g_2\,C_1$

$G_0=g_1+g_2$

$C_0=C_1+C_2$

This can be written

$$q=\frac{A_1}{p+a}\mathit{1}$$

$$\text{where } A_1=\frac{A}{C_0}$$

$$\text{and } a=\frac{G_0}{C_0}$$

The solution can be written down at once by comparing this equation with Equation (4) in Part I. It is

$$q=E\frac{A}{C_0}\,\frac{C_0}{G_0}\left[1-\epsilon^{-\frac{G_0}{C_0}t}\right]$$

$$=E\frac{g_1C_2-g_2C_1}{g_1+g_2}\left[1-\epsilon^{-\frac{g_1+g_2}{C_1+C_2}t}\right]$$

The student should verify this by the use of the Expansion Theorem.

PART V: OPERATIONS ON UNIT FUNCTION SQUARED HAVE NO PHYSICAL SIGNIFICANCE

The student may be tempted to write the operational expression for electric power and expect from this to get by the Expansion Theorem the instantaneous values. He will fail because the operational solution will contain 1^2 and neither Heaviside nor anybody else has shown how to operate on 1^2.

To illustrate, assume that it is desired to find the power supplied to the inductance of an inductive circuit. The power is the instantaneous product of the voltage consumed by the inductance and the current flowing in the inductance.

The operational solution for the current is

$$i = \frac{E}{r+pL} 1$$

and since the voltage consumed by the inductance is

$$e = pLi = E\frac{pL}{r+pL} 1$$

he might write

$$P = E^2 \frac{pL}{(r+pL)^2} 1^2$$

and erroneously proceed to use the Expansion Theorem. It must be remembered that we know only how to operate on 1; we know nothing about rules of operations on 1^2.

The solution is of course the product of the voltage and current when these are given as functions of t, not of p. Thus the power in this case is

$$P = E^2 \frac{\epsilon^{-\frac{r}{L}t}}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$$

Incidentally this difficulty is similar to that encountered when trying to get the power in an alternating-current circuit from the vector expressions of the current and voltage. The student will remember that the answer is obtained by "telescoping" the vectors; ordinary multiplication does not "work."

PART VI: ADDITIONAL OPERATORS EMPLOYED WHEN A NETWORK IS SUDDENLY CONNECTED TO AN ALTERNATOR, INSTEAD OF TO A BATTERY OF CONSTANT VOLTAGE

Referring to Equation (3) in Part I

$$\frac{p}{p-a} 1 = \epsilon^{at}.$$

Thus $\quad \frac{p}{p \pm j\omega} 1 = \epsilon^{\mp j\omega t} \quad$ when $a = j\omega$

$$\sin \omega t = \frac{\epsilon^{j\omega t} - \epsilon^{-j\omega t}}{2j} = \frac{1}{2j}\left[\frac{p}{p-j\omega} 1 - \frac{p}{p+j\omega} 1\right]$$

$$= \frac{p}{2j}\left[\frac{p+j\omega-p+j\omega}{p^2+\omega^2}\right] 1 = \frac{p\omega}{p^2+\omega^2} 1.$$

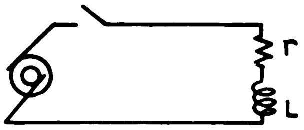

Fig. 9

This shows how a sine wave can be converted to a "Unit function."

Similarly $\quad \cos \omega t = p\frac{\sin \omega t}{\omega} = \frac{p^2}{p_2+\omega^2} 1$ (3)

Therefore $\quad \sin(\omega t \pm \varphi) = \sin \omega t \cos \varphi \pm \cos \omega t \sin \varphi$

$$= \frac{p\omega \cos \varphi \pm p^2 \sin \varphi}{p^2+\omega^2} 1$$

and $\quad \cos(\omega t \pm \varphi) = \frac{p^2 \cos \varphi \mp p\omega \sin \varphi}{p^2+\omega^2} 1$

This relation the writer believes was first shown by Pleijel.

Similarly

$$\epsilon^{-\beta t} \sin(\omega t \pm \varphi) = \frac{p\omega \cos \varphi \pm p(p+\beta) \sin \varphi}{(p+\beta)^2+\omega^2} 1$$

$$\epsilon^{-\beta t} \cos(\omega t \pm \varphi) = \frac{p(p+\beta) \cos \varphi \mp p\omega \sin \varphi}{(p+\beta)^2+\omega^2} 1.$$

In connection with the last two equations it should be noted that, while $\epsilon^{-\beta t} = \frac{p}{p+\beta} 1$ and $\sin \omega t = \frac{p\omega}{p^2+\omega^2} 1$, $\epsilon^{-\beta t} \sin \omega t$ cannot be obtained directly by multiplying the two operators because the resultant operator would involve the unit function squared.

An application of these operators will be given in the case of an alternator of e.m.f. $E \sin(\omega t + \varphi)$ being suddenly (at $t = 0$) connected to an inductive circuit as shown in Fig. 9.

The procedure is to write first the operational solution as if unit e.m.f. were impressed. It is $i = \frac{E}{r+pL} 1$. Then introduce the additional operator which converts the sine wave to a wave of unit function. Thus the operational solution becomes

$$i = \frac{E}{r+pL} \cdot \frac{\omega p \cos \varphi + p^2 \sin \varphi}{\omega^2+p^2} 1 \qquad (20)$$

Therefore $\quad Y_{(p)} = \omega p \cos \varphi + p^2 \sin \varphi$

and $\quad Z_{(p)} = (r+pL)(\omega^2+p^2)$

The solution of Equation (20) gives the instantaneous values of the current. The denominator is of a higher power of p than the numerator, thus the initial current value is zero. But in this case we cannot say that for $t = \infty$, $p = 0$. It is not zero because the final current is obviously not steady. If we desire to know the final value without solving the equation by the Expansion Theorem, we may resort to the vector representation as discussed previously and write

$$I = \frac{E}{r+pL}$$

where $p = j\omega$.

---

(3) For the development of $p^n \sin \omega t$ where n is a fraction, see Part XXVI.

Therefore

$$I = \frac{E}{r+j\omega L}$$

which is the well-known solution.

It is now perfectly simple to solve Equation (20) by the Expansion Theorem. Since $Z_{(p)} = (r+pL)(\omega^2+p^2)$, we get three roots, $p_1 = -\frac{r}{L}$, $p_2 = +j\omega$, $p_3 = -j\omega$. We proceed then to find $d\frac{Z_{(p)}}{dp}$ and then $p\frac{dZ_{(p)}}{dp}$, for $p=p_1$, p_2 and p_3, etc.

The result will be three terms ($\frac{Y_{(o)}}{z_{(o)}}$ is easily seen to be zero in this case), *viz.*,

$$A\epsilon^{-\frac{r}{L}t} + B\epsilon^{j\omega t} + C\epsilon^{-j\omega t}$$

The last two terms will combine to a pure sine wave without decrement and are, therefore, the permanent condition, which usually is well known to engineers.

In this case the permanent alternating-current is, of course,

$$i_p = \frac{E}{Z}\sin(\omega t+\varphi-\alpha)$$

so that it is only necessary to calculate the value of i from the Expansion Theorem for the root $p=p_1=-\frac{r}{L}$, and then to add i_p to the solution.

However, the solution is not always so easily obtained. We will therefore consider a general simplification of work when the denominator $Z_{(p)}$ is in the form of a product.

Let

$$\frac{Y_{(p)}}{Z_{(p)}} = \frac{Y_{(p)}}{h_{1(p)}\cdot h_{2(p)}}$$

Therefore

$$Z'_{(p)} = \frac{dZ_{(p)}}{dp} = h_{1(p)}\,h_2'{}_{(p)} + h_{2(p)}\,h_1'{}_{(p)}$$

and

$$pZ'_{(p)} = p\,[h_{1(p)}\,h_2'{}_{(p)} + h_{2(p)}\,h_1'{}_{(p)}]$$

When in this expression we substitute the root or roots obtained when $h_{1(p)}=0$, we get only the second term left since $h_{1(p)}$ obviously becomes zero. A similar argument applies in regard to the second set of roots, so that the net result is, for instance, in case $h_{1(p)}$ has only one root, p_1, and $h_{2(p)}$ two roots, p_2 and p_3,

$$i = E\Bigg[\frac{Y_{(o)}}{Z_{(o)}} + \frac{Y_{(p)}\,\epsilon^{pt}}{ph_{2(p)}\,h_1'{}_{(p)}} \text{ for } p=p_1 + \frac{Y_{(p)}\,\epsilon^{pt}}{ph_{1(p)}\,h_2'{}_{(p)}} \text{ for } p=p_2 + \frac{Y_{(p)}\,\epsilon^{pt}}{ph_{1(p)}\,h_2'{}_{(p)}} \text{ for } p=p_3\Bigg]$$

This relation the writer believes was first worked out by Pleijel or Herlitz.

In the particular problem under consideration

$h_{1(p)} = r+pL$; therefore $h_1'{}_{(p)} = L$ and $ph_1'{}_{(p)} = pL$

$h_{2(p)} = \omega^2+p^2$; therefore $h_2'{}_{(p)} = 2p$ and $ph_2'{}_{(p)} = 2p^2$

Thus $ph_{2(p)}\,h_1'{}_{(p)} = pL\,(\omega^2+p^2)$

and $ph_{1(p)}\,h'_{2(p)} = 2p^2\,(r+pL)$

$ph_{2(p)}\,h_1'{}_{(p)}$ for $p=p_1$ becomes $-\frac{r}{L^2}z^2$

where $z^2 = r^2+\omega^2L^2 = r^2+x^2$

$Y_{(p)}$ for $p=p_1$ becomes $z\frac{r}{L^2}\sin(\varphi-a)$

where $\tan\alpha = \frac{x}{r}$

Thus the first term becomes

$$-\frac{E}{z}\epsilon^{-\frac{r}{L}t}\sin(\varphi-a)$$

$ph_{1(p)}\,h_2'{}_{(p)}$ for $p=p_2=j\omega$ becomes $-2\omega^2\,(r+j\omega L)$

$Y_{(p)}$ for $p=p_2=\omega^2\,(j\cos\varphi-\sin\varphi)$

Therefore

$$\frac{Y_{(p)}}{ph_{1(p)}\,h_2'{}_{(p)}} \text{ for } p=p_2 = \frac{j\cos\varphi-\sin\varphi}{-2\,(r+j\omega L)}$$

$$= \frac{\cos\varphi+j\sin\varphi}{2j\,(r+j\omega L)} = \frac{\cos(\varphi-a)+j\sin(\varphi-a)}{2jz}.$$

$$\frac{Y_{(p)}}{ph_{1(p)}\,h_2'{}_{(p)}} \text{ for } p=p_3 = \frac{j\cos\varphi+\sin\varphi}{2\,(r-j\omega L)}$$

$$= \frac{-\cos\varphi\,(\varphi-a)+j\sin(\varphi-\alpha)}{2jz}.$$

Thus the last two terms become

$$\frac{E}{z}\left[\cos(\varphi-\alpha)\frac{\epsilon^{j\omega t}-\epsilon^{j\omega t}}{2j}+\sin(\varphi-\alpha)\frac{\epsilon^{j\omega t}+\epsilon^{j\omega t}}{2}\right]$$

$$=\frac{E}{Z}\sin(\omega t+\varphi-\alpha)$$

which is the permanent value and the solution is

$$i = \frac{E}{z}\left[\sin(\omega t+\varphi-\alpha) - \epsilon^{-\frac{r}{L}t}\sin(\varphi-\alpha)\right]$$

The process is somewhat lengthy, but as has been stated, when it is once done, it is known that the roots $\pm j\omega$ give the permanent condition, so that it is really only necessary to calculate the condition for the other root or roots, if the permanent condition is known. If not, it is simplest to obtain it first in vector form by substituting $j\omega$ in the operational solution, $i = \frac{E1}{r+pL}$ in this particular case.

This problem will be treated in an entirely different way in Part XXVI. The object then will be to bring out certain peculiarities of power series developments and to show the application of Heaviside's "shifting."

(To be continued)

Vacuum Tubes as Oscillation Generators

PART IV

SPECIAL CONSIDERATIONS BEARING ON THE DESIGN AND OPERATION OF OSCILLATING CIRCUITS

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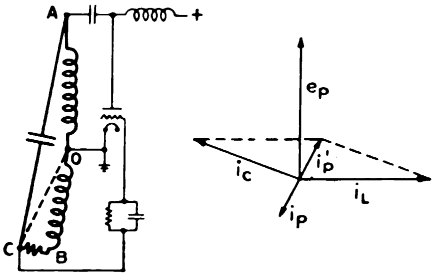
Grid Phase-angle Corrections

In dealing with the design of oscillating circuits the effects of the load resistance, the plate-choke, and the blocking condenser were described as though only their primary functions were fulfilled and, this accomplished, they exerted no other influence on the circuit. All three parts of the circuit can, however, affect its operation; and, though the changes produced are usually small, it is satisfying to know just what may be expected. In general, the effects consist of the introduction of small voltages or currents into the simple scheme of things first described. This results in a small change in magnitude of practically all of the various quantities, combined with a slight change in the phase relations.



(A) (B)

Fig. 24. Effect of Load Resistance in a Hartley Circuit

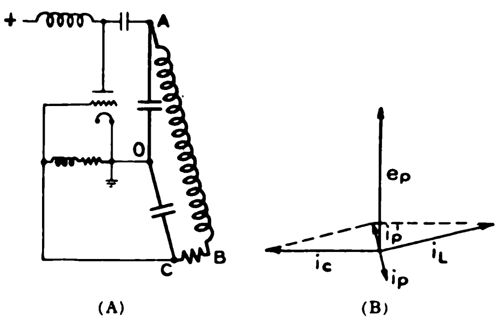
In order to simplify the discussion, only the fundamental frequency component of the current supplied to the oscillating circuit through the blocking condenser will be considered. The harmonics are relatively much less important, and their omission will entail no serious error. The value of the fundamental component can be obtained by dividing the watts input to the oscillating circuit by the voltage between the leads to the plate and filament of the tube.

Fig. 24 illustrates the effect of the load resistance in a Hartley circuit. In (A) the oscillating circuit itself, $AOBC$, is drawn in such a way that the angles between various parts of the circuit correspond to the electrical phase differences of the voltages across them. Thus the grid coil OB and the condenser AC are in geometrically parallel sections of circuit, for they carry the same current, and both are pure reactances. The load resistance is drawn at right angles to them for similar reasons. Part of the load resistance might be located in the plate coil section AO, but this would not affect the diagram, as far as the object in hand is concerned, which is to show the relation between plate and grid voltages. It will be observed that the grid current has been neglected. This is not the case with the plate current, which may be represented by a sinusoidal current flowing between the leads to the circuit at A and O. For this reason the plate coil in Fig. 24 is not drawn parallel with the grid coil but instead there is a phase difference between their voltages. For the present it will be assumed that there is no drop across the blocking condenser and no current passed by the plate choke. Hence, OA will be the alternating component of the plate voltage.

The grid voltage will be represented by the dotted line OC in Fig. 24(A), and the plate current will be in phase with it. Thus, in Fig. 24(B), the plate voltage e_p is drawn parallel with OA in Fig. 24 (A) and i_p, the alternating component of the plate current, is drawn parallel with OC. The currents through the condenser and plate coil are 90 degrees out of phase with the corresponding voltages and are represented by i_c and i_L perpendicular to AC and OA respectively. Adding the two vectorially results in i_p', which must be equal and opposite to the alternating component of the plate current i_p. It will be seen that i_p' must always lag behind e_p or, in other words, the oscillating circuit absorbs power not as a perfect resistance, but as a resistance in connection with an inductance. The circuit itself produces this effect by running slightly below the resonant frequency, thus drawing the lagging component of current by increasing the current through the inductance and decreasing the current through the capacity. The deviation of the frequency from that corresponding to the natural frequency of oscillation can be calculated by applying numerical values to the vectors.

The corresponding phenomena in a Colpitts circuit are shown in Fig. 25, (A) and (B). In this case i_p' leads e_p by some angle which requires that the oscillating circuit operate slightly above its natural frequency in order to produce this power-factor. This self adjustment of the simpler types of circuit is a very valuable property under many conditions, as it automatically insures against serious losses due to the grid excitation being out of phase.

In Fig. 26, (A) and (B) indicate the effect of the plate choke and the blocking condenser upon the operation of the circuit. This is the same Hartley circuit as shown in Fig. 24(A) but the blocking condenser and plate choke are no longer considered to be perfect in operation. The alternating component of the plate voltage is, therefore, no longer represented by *OA* but by a vector *OD* displaced from *OA* by the addition of *AD*, the drop in the blocking condenser. The plate choke may be considered to be grounded on the side next to the high voltage generator as far as the high frequency is concerned, so this choke appears as an inductance connected between *O* and *D*.



(A) (B)

Fig. 25. Effect of Load Resistance in a Colpitts Circuit

By choosing a blocking condenser of the correct impedance, it is possible to bring the plate voltage *OD* exactly 180 degrees out of phase with the grid voltage, the conditions to be desired for efficient operation. This will result in the oscillating circuit plus the blocking condenser drawing a load with a leading current component. This component may then be neutralized by arranging the choke so that it will draw a lagging current of equal magnitude. In the vector diagram 26(B), the current through *AC* is represented by i_c and that through *AO* by i_L. These combine to form a short vertical current vector to which is added the choke current, i_s, to form a total current equal and opposite to i_p supplied by the tube. The voltage across *OA* has been taken to be vertical, and, if the sum of i_c and i_L is vertical, the drop which this current causes in the blocking condenser must be represented by a horizontal line. This, when added to that representing the voltage across *OA*, results in the alternating component of the plate voltage, e_p. This voltage can, by this means, be made opposite to i_p and e_g; *i.e.*, by control of the drop in the blocking condenser.

To make a complete calculation of a circuit, including the points just discussed, the circuit *OBCA* is treated as though the choke and blocking condenser were not present. This circuit may be equivalent either to a pure resistance between *A* and *O* (as shown in the vector diagram) or the equivalent load may contain some reactance. Let it be so proportioned that it will be equivalent to a resistance. The angle *COA*, and its supplement *AOD*, can then be calculated. The equivalent resistance between *A* and *O* is known since it represents a given load at a given voltage. The capacity reactance *AD* can, therefore, be selected so that $AD/OA = \tan(180° - AOC)$.

The alternating voltage is impressed at *D*, and the circuit *DAO* will draw some leading current, the amount being easily ascertainable. It is then only necessary to choose the choke *DO* of such a value that it will draw the same amount of lagging current.

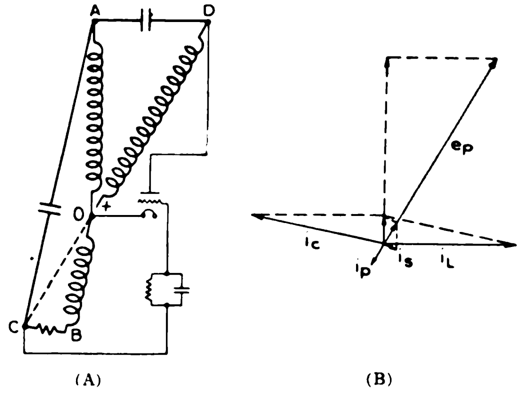
The procedure in arriving at the proper values of blocking condenser and line choke will be clearer if the solution of a numerical example is carried through.

The first item to be determined is the ratio of the inductance on which the alternating components of the plate and grid voltage depend. In Fig. 26, the plate voltage is represented by *OD* and the drop across the inductance by *OA*; and it will be noticed that the two may be considered equal in magnitude unless the circuit is far from normal in design. The effect of the resistance in the grid inductance on the magnitude of the grid voltage can also be neglected. The two inductances will then be in the same ratio as the alternating components of plate and grid potentials, E_p and E_g respectively, as determined from the data on optimum operating conditions. Assume that this gives

$$\frac{E_p}{E_g}=\frac{OA}{OC}=4.$$

Let $OA = 100$ ohms inductance with 5 ohms resistance.
$OB = 25$ ohms inductive reactance.
$BC = 2.5$ ohms resistance.
$CA = 125$ ohms capacity reactance.

Then angle $OAC = \sin^{-1}\frac{2.5}{100} = 1$ deg., 27 min.



(A) (B)

Fig. 26. Effect of Plate Choke and Blocking Condenser

If 100 volts be impressed across *OA*

$i_l = 1$, watts in $OA = (i_l)^2 \times 5 = 5.0$
$i_c = 1$, watts in $BC = i_c^2 \times 2.5 = 2.5$
Total watts $= 7.5$

Equivalent resistance $= \frac{E^2}{W} = \frac{100^2}{7.5} = 1333$ ohms. It will be arranged to have this circuit operate at the resonant point so that its reactance between O and A is zero.

In the triangle AOC

$$\frac{OA}{\text{Sin } ACO}=\frac{OC}{\text{Sin } OAC}=\frac{AC}{\text{Sin } AOC}$$

$$\frac{100}{\text{Sin } ACO}=\frac{25}{0.025}, \text{ Sin } ACO=0.1$$

$$\text{Angle } ACO=5 \text{ deg., } 45 \text{ min.}$$

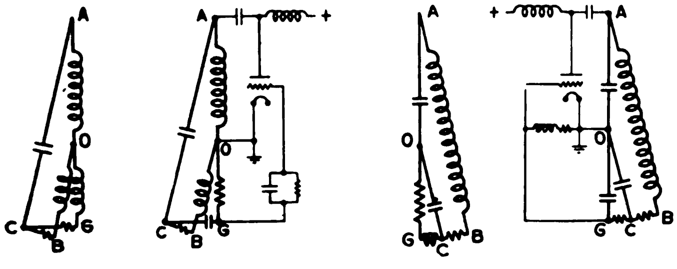
$$\text{Angle } AOD=\text{Angle } OAC+\text{Angle } ACO=7 \text{ deg. } 12 \text{ min.}$$

$$AD=\text{impedance between } A \text{ and } O\times\tan AOD$$

$$=1333\times0.126=168 \text{ ohms capacitance.}$$

The total impedance between O and D through A is 1343 ohms, so that the ratio between the plate and grid voltages suffers no appreciable change due to the presence of the plate-blocking condenser. If 100 volts are impressed between D and O, the current is

$$\frac{100}{1343}=0.0745 \text{ amp.}$$



(A) (B) (C) (D)

Fig. 27. Special Circuits to Compensate for Errors in Phase of Grid Voltage

and the wattless volt-ampere component is

$$0.0745^2\times168=0.933 \text{ volt-amperes.}$$

In order to correct for these leading volt-amperes, the choke should draw the same amount lagging, thus giving

$$DO=\frac{100^2}{0.933}=10{,}700 \text{ ohms}$$

If the impedance of the plate-blocking condenser had been high, the desired ratio between grid and plate voltages would not have been obtained. It would then have been necessary to assume an initial value somewhat larger than desired for the final result, proceeding by a series of approximations. However, for other reasons, it is not likely that a high-impedance blocking condenser would be desirable. A high-impedance blocking condenser corresponds to a low-impedance choke which would allow radio frequency currents to flow in circuits with high effective resistance and thus, possibly, damage power generating apparatus.

Proportioning the plate-blocking condenser and line choke is not the only method of bringing plate voltage and current into the 180-degree phase relation. The angle OAC may be compensated for by de-phasing the grid in such a way as to cause the oscillations to occur at the natural resonant frequency and the plate current and voltage to be properly related.

In Fig. 27, diagrams (A) and (B) show two methods by which a Hartley circuit may be restored to operation at the natural resonant frequency of the circuit with proper phase relations, while (C) and (D) show the two equivalent methods for the Colpitts circuit. In (A) the phasing is accomplished by connecting resistance CG and inductance GO in series. The grid is attached at G. The same result is accomplished in (B) by a resistance from O to G and a condenser from G to C. The elements are reversed in (C) and (D) to produce lag instead of lead.

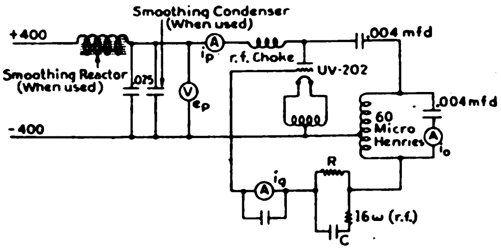


Fig. 28. Circuit Used in Taking Oscillograms Shown in Figs. 30, 31 and 32

Although grid phasing may correct the various angles, it is probable that adjustments of the choke and the blocking condenser are to be preferred since these devices are normally present and so do not constitute added complication.

Grid Bias Condenser

It will be noted that no criteria have yet been developed governing the choice of a grid-blocking condenser. Since the function of this condenser is to pass the alternating-current component of grid excitation without the occasion of serious voltage drop while forcing the direct component to flow through the grid leak or biasing resistance, its value is not critical. Its value should be large enough to make the grid-plate capacity small by comparison. The values of tube capacity are normally so small that this requirement causes no concern. The individual pulses of direct current should cause no

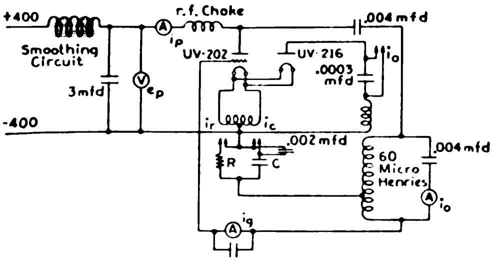


Fig. 29. Circuit Used in Taking Oscillogram Shown in Fig. 33

considerable change in the bias, but this requirement also causes small concern.

Intermittent Oscillation

Troubles may be encountered because of excessive capacity of the grid leak condenser. Since the reason for this is not apparent at first glance, it will be developed in more detail. The evidence of trouble appears as a falling off in input and output, and the radiation of an interrupted wave.

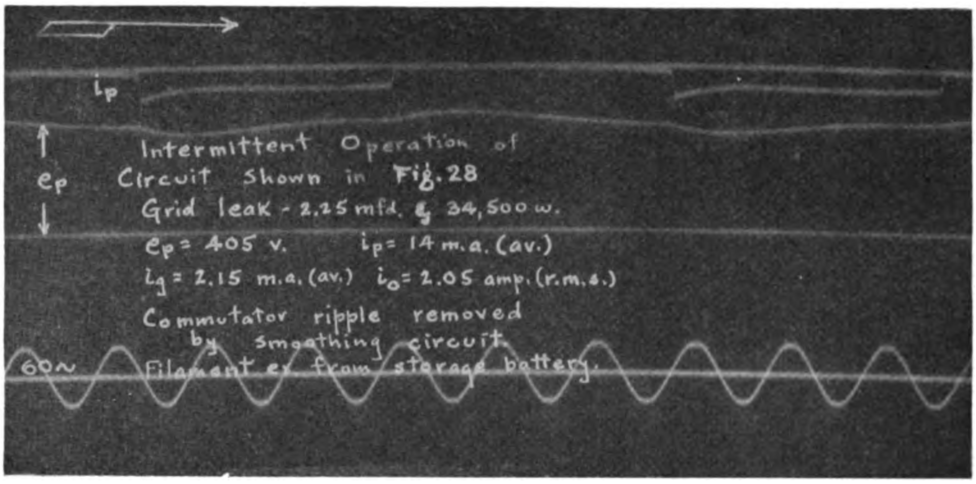


Fig. 30. Intermittent Operation with a Very Large Grid Condenser

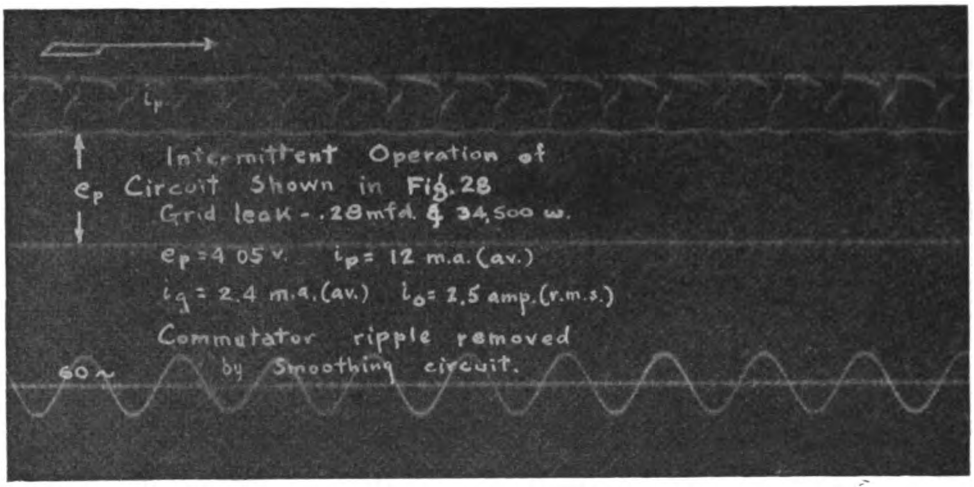


Fig. 31. Intermittent Operation with a Grid Condenser of Smaller Capacity Than is Used for Fig. 30

To show the effects of different grid capacities, the circuits illustrated in Figs. 28 and 29 were subjected to oscillographic study. The result of using a very large grid condenser is shown in Fig. 30. This oscillogram was taken in connection with the circuit shown in Fig. 28. The oscillations alternately build up and die out, the period being quite long. By reducing the capacity of the condenser, the period is changed as shown in Fig. 31. The phenomenon persists even when considerable care is used in smoothing out the impressed voltage by means of a filter, as shown in Fig. 30. That the phenomenon is not due to resonance is indicated by the irregularities shown in Fig. 32. In the foregoing oscillograms, plate current is used as a measure of oscillations. Fig. 33 shows that the actual oscillation amplitude follows the same general form. This was taken with the circuit shown in Fig. 29.

The explanation of the phenomenon just described is arrived at as follows: Referring to Fig. 34, let the ratio of alternating plate and grid volts, and other adjustments be fixed. If a constant bias in volts is then assumed, the tube output for any amplitude of oscillation can be calculated directly by the method developed in the latter half of Part II of this serial. Several such curves are plotted. The resistance loss in the circuit is proportional to the square of the voltage and is also plotted. An intersection between tube output and circuit loss represents a point of equilibrium. If a decrease in oscillation amplitude causes the tube output to become higher than the losses, the point is stable; if the reverse is the case, it is unstable. By determining the grid current for various points on the curves of output at constant bias, the corresponding leak resistances can be determined. These points may then be joined by curves which represent output variation with change in oscillation amplitude at constant leak resistance. It will be observed that there is a value of bias such that the bias curve and the loss curve are tangent, as at *C*. A greater bias than this cannot be used as there are no intersections with the loss curve and hence no points of equilibrium.

For a small grid condenser, the constant resistance line is more nearly the criterion of stability. It is apparent that in the region of *C*, in the direction of point *A*, stable operation is obtained with either constant resistance or constant bias. In the direction *B*, constant bias operation is unstable, while constant resistance operation is still stable.

Best operation, especially for a highly efficient tube, is very likely to fall in the *C*—*B* zone; thus stability requires a grid condenser small enough to cause the slope of the resultant output curve for the probable maximum rate of amplitude variation to be less than the slope of the loss curve. This condition is also frequently observed in regenerative receiver sets.

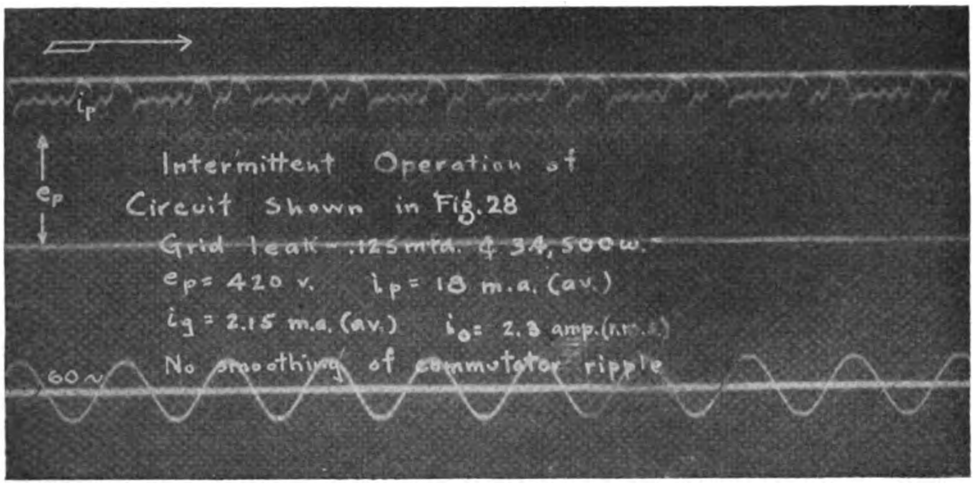


Fig. 32. Irregularities in Intermittent Operation

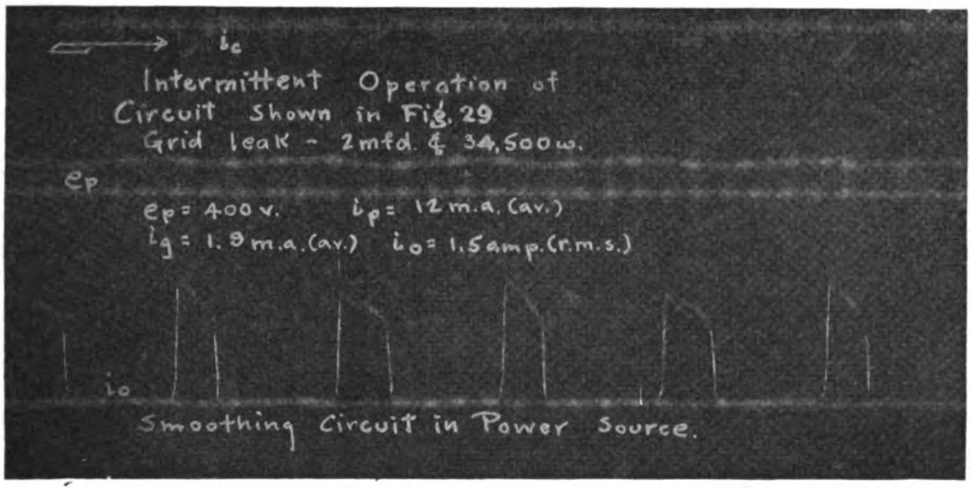


Fig. 33. Intermittent Nature of Oscillating Current

Through any point, as *A* or *B*, on the loss curve two lines can be drawn, one representing the change of output variation at constant bias and the other the change of output variation at constant leak resistance. If a fixed bias is being used, the equilibrium test must be applied to the bias line. If resistance bias is being used, the test is applied with reference to the constant-resistance line. The grid-blocking condenser tends to hold constant bias; therefore, with a large condenser, the diagram of operation, as far as equilibrium is concerned, utilizes a constant bias line.

Having established a condition of either instability or a very narrow margin of stability, equilibrium may be disturbed by a large number of causes, including generator ripple, line regulation, keying, and the like. The cycle followed is somewhat as follows: Assuming the 20,000-ohm value of grid leak, the tube, starting with zero bias, will build up both oscillations and grid bias until equilibrium is reached at 7.3 watts output and approximately 50 volts bias. If the grid condenser is small, the operation will be stable. If the condenser is large, any momentary disturbance will shift the operating point up or down on the 50-volt bias line, the condenser tending to hold the bias constant over a brief interval. If it shifts down, the tube output becomes less than the circuit losses, and the oscillations die out. The grid condenser now discharges through the leak until the bias is low enough for oscillations to start again and the cycle is then repeated.

Operating Difficulties

If a circuit does not operate as expected after making the calculations, the trouble will usually be found to be due to one of two causes. The tube may not correspond to the characteristics, or the circuit constants may not be as they seem. The most frequent variation in tube characteristics is in the emission. If this is low, the tube is very apt to refuse to operate at all. If it should be too high, the operation will be good, but the tube life can be extended by decreasing the filament current. In cases where it is desirable, output and efficiency can both be increased at the expense of tube life by overburning the filaments.

Occasionally the grid current will differ from that given by the characteristic curves. In this case a satisfactory adjustment can usually be obtained by varying the grid leak resistance.

Circuit difficulties are of two sorts: the measured values may be in error, or the circuit unexpectedly may operate at a much higher frequency than desired, utilizing the inductance of condenser leads and the internal capacities of coils rather than the normal reactances of these elements.

The resistance of the oscillating circuit is perhaps the most difficult constant to determine, besides being especially apt to make trouble if it actually has a greater value than supposed. Hence the greatest care should always be used to see that it is correctly determined. Certain objects, such as concrete floors or walls within a few feet of radio-frequency apparatus, are likely to have considerable losses produced in them which means that they increase the effective resistance of the circuits. In general, any object near a high-frequency circuit should be built of either very good insulating materials or good conductors. Very little loss can occur in good insulators; and good conductors shield themselves by the currents induced in them which flow with only a relatively small loss.

It is often difficult to foresee the causes of parasitic oscillations in which an inductance coil will behave as a condenser due to its internal capacity, or a lead whose reactance is negligible at the desired frequency develops a high reactance due to the greatly increased frequency. Such annoyances are always to be expected; sometimes they will require the replacement of some part of the circuit, but often they can be stopped by placing a resistance in some part of the circuit where it will cause very little loss under the normal operating conditions, but form a heavy burden for the parasitic oscillations. A very good place to put such a resistance is in series with the grid leak condenser. In many cases, a few hundred ohms at this point will be all that is required.

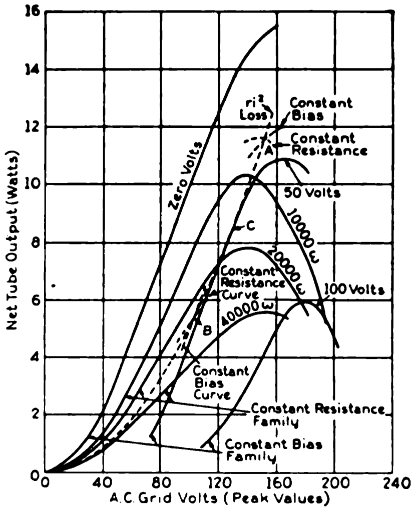


Fig. 34. Curves Showing Grid Circuit Stability of a Small Pliotron Assuming the Alternating Plate Volts as 1.7 Times the Alternating Grid Volts and a Direct Plate Potential of 350 Volts

(To be continued)

Graphical Determination of Magnetic Fields

PART II

RULES FOR CALCULATING AND PLOTTING FIELDS

By A. R. STEVENSON, JR., and R. H. PARK
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ALTHOUGH various authors have given rules for plotting flux in air, it seems well in the interest of completeness to briefly sum up the more important principles, in the group of brief rules which follows.

The method used by Doherty and Shirley can best be described by reference to a particular problem illustrated in Fig. 23 which represents a pair of alternator field poles.

In attacking a problem of this sort, Doherty and Shirley made the following assumptions:

(1) That the magnetomotive force is due to an infinitely thin coil distributed along the pole(17).

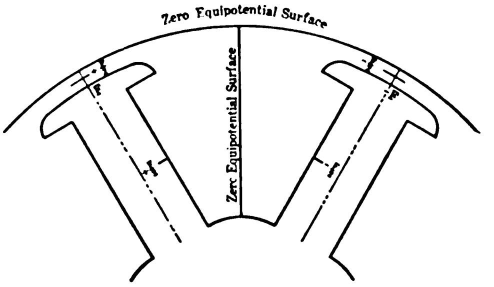

Fig. 23

The armature reaction is neglected while plotting the flux due to the field m.m.f.; and later, after another sketch is made of the field due to the m.m.f. of armature reaction, the two are superposed.

(2) The magnetic potential of one pole due to the m.m.f. of its own winding is + F. The magnetic potential of the other pole due to its own m.m.f. is $-F$.

(3) Assuming infinite permeability, the armature and field pole faces are equipotential surfaces.

(4) By symmetry, a zero equipotential surface is drawn midway between the two poles.

(5) The armature surface is also assumed to be a zero equipotential surface.

(6) At the surface of the left-hand pole, the potential is + F. As the air-gap is crossed, this drops to 0 at the surface of the armature. In re-crossing the gap from the armature surface to the right-hand pole, the magnetic potential drops to $-F$. Therefore, the total drop from one pole face to the other is $2F$.

(7) In any place where the flux lines are known to be straight and parallel, as is approximately true in the air-gap, the potential drop is a linear function of distance. Thus, a line half-way across the gap represents one-half potential.

(8) The equipotential planes $+F/2$ and $-F/2$ will curve from the center of the air-gap to the midpoints of their respective poles. The beginning and ends of the one-quarter potential planes can be similarly located. In sketching the traces of these planes through points known to be at the same potential, there must be discontinuity in the gradual change in shape of adjacent potential lines, *i. e.*, when near the pole they follow its configuration, but those near the zero equipotential line must approach its rectangular shape.

(9) After the equipotential lines have been sketched, the lines of force are drawn perpendicular to them. The whole surface is thus divided into chequers. If enough lines are drawn, these chequers become very small rectangles. The sketch is not correct until all the angles are right angles.

(10) In space through which no current is flowing, all these rectangles must be similar, *i.e.*, the spacings of lines of force at two different places in the field must be proportional to the spacing of equipotential lines at the same respective points. It is recommended as a matter of convenience that the flux density be represented by such a number of lines as will make these rectangles curvilinear squares.

This method of plotting magnetic fields is a cut-and-try method. The first few sketches will obviously be wrong, but there are sufficient conditions to provide that the final picture obtained after several readjustments will be approximately correct.

The actual flux density can be calculated at any point from this picture as follows:

a. The total m.m.f. per pole,

$$F = 4\,\pi\, n\, I \tag{22}$$

where n is the number of turns per pole and I is the field current in abamperes.

b. If there are m equipotential surfaces including one of the boundary surfaces drawn between the pole and the zero equipotential surface, the potential gradient between any two of these surfaces is

$$\frac{d\,P}{d\,s} = \frac{4\,\pi\, n\, I}{m\,\delta} \tag{23}$$

where δ is the perpendicular distance measured in centimeters between adjacent equipotential surfaces at the point in question; *i. e.*, $m\,\delta$ = air-gap.

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(17) This approximation is more accurate than neglecting the distribution of the coil entirely, as has been done by some others, and apparently does not very much affect the accuracy of flux distribution in the air-gap.

c. Since the permeability of air is unity, the density in lines of force per sq. cm. at every point is exactly equal to the potential gradient.

$$B=\frac{dP}{ds}=\frac{4\,\pi\,n\,I}{m\,\delta} \tag{24}$$

d. **But, if the figure has been drawn so that the small chequers are square, the lines of force will have the same spacing, δ, as the equipotential surfaces. Therefore, the flux included in the tube between** consecutive lines of force is

$$\Delta\,\phi=B\,\delta=\frac{4\,\pi\,n\,I}{m} \tag{25}$$

e. **Attention is called to the fact that if δ is the spacing of lines of force, the area of a tube δ wide and 1 cm. thick is δ, and the density at any point is**

$$B=\frac{\Delta\,\phi}{\delta}=\frac{4\,\pi\,n\,I}{m\,\delta} \tag{26}$$

Equation (26) is identical, of course, with Equation (24), and merely represents another viewpoint.

In order to check the accuracy of the free-hand method, 23 different men determined the effective air-gap of a motor by making free-hand sketches of the flux. By Carter's equation, the correct value of the effective gap was 0.575. The results of the free-hand sketches were as follows:

Results Between	Number
0.570–0.605	8
0.605–0.640	6
0.640–0.675	4
Rejected for gross inaccuracy	5
Total	23

The above table indicates that the more accurate sketches gave a minimum reluctance. This agrees with Arnold's[18] statement: "The most nearly correct distribution of the flux will be shown by the sketch which makes the permeance of the flux tubes a maximum."

It is probably obvious that the flux will arrange itself in such a manner as to follow the path of least resistance; but it is interesting to note that a comparison of the reluctances determined from several independent flux plots with the mathematically correct permeance confirms this theory so well.

Method of Plotting Two-dimensional Magnetic Fields in Space Occupied by Current-carrying Conductors

Consider a section of an infinitely long cylindrical conductor, shown in Fig. 24. The work done in making a complete circuit about such a conductor with a unit pole is

$$W=4\,\pi\,I \tag{28}$$

The difference of potential between two points is usually defined as the work done against the field in transporting a unit pole from one point to the other by any path whatever.

Let the potential at *A* be zero. Then the potential at *D* is

$$P_D=2\,\pi\,I \tag{29}$$

The potential, however, continues to increase as the circle is completed. On returning to *A* after a complete circuit, the potential is no longer zero, but

$$P_A=4\,\pi\,I \tag{30}$$

Every time the unit pole is taken around the conductor, the potential increases by this amount.

It might be asked, "How does the law of conservation of energy apply?" The answer is that, in taking the unit pole around the conductor, a voltage has been induced. The product of this induced voltage multiplied by the current in the conductor represents electrical power which, integrated for the elapsed time, gives electrical energy. Thus the mechanical work done in carrying the unit pole about the conductor is converted into electrical energy.

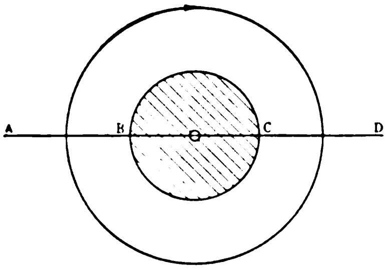

Fig. 24

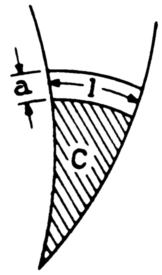

Fig. 25

Again referring to Fig. 24, it should be noticed that no work would be done in going from *A* to *D* by the path *A B O C D*, because this path at all points is perpendicular to the lines of force. Such lines, therefore, can be called lines of no work.

The following are the most important rules to be followed in plotting flux in regions containing current-carrying conductors:

(1) The equipotential lines in the air space, when projected as lines of no work into the copper, must divide the copper into equal areas. It is thus seen that each particular part of the ampere conductors may be regarded as responsible for a particular part of the field.

(2) The work done in carrying a unit pole along any line of force from one point to another is proportional to the current flowing in the area enclosed by the line of force and the lines of no work passing through the two points.

(3) The spacing of lines of force must be inversely proportional to the copper enclosed and proportional

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[18] E. Arnold: *Die Wechselstromtechnik* (Julius Springer, Berlin), p. 78.

to the length of the part of the tube enclosing the copper.(19) Thus

$$H\,l = 4\,\pi\,C$$

If k is the number of lines in a tube, the width is

$$a = \frac{k}{H} = \frac{k\,l}{4\,\pi\,C}$$

The spacing between lines of force must be proportional, therefore, to the ratio l/C, where l is the length of the tube under consideration between the lines of no work and C is the total current included by this length of tube and the boundary lines of no work in Fig. 25.

(4) The line of half potential outside the copper, when extended as a line of no work into the copper, does not necessarily intersect every line of force at a point which would divide into two equal halves the work done in carrying a unit pole around the line. A similar statement can be made, of course, for the other potential lines.

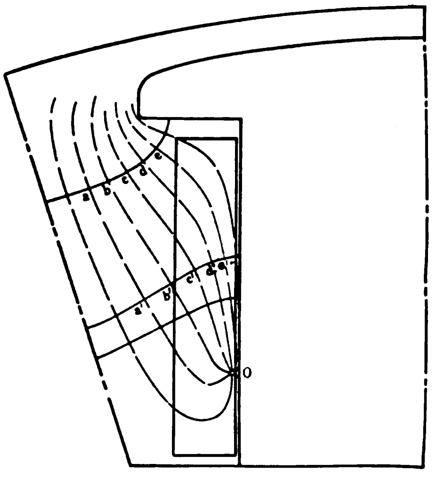

Fig. 26

Brief Rules for Construction of Field Flux Plots Inside the Copper

The following directions as given by L. P. Shildneck have proved very helpful in making flux plots of the fields of salient-pole machines.

Consider the case of a copper and iron distribution as given by a field pole (Fig. 26), with lines of no work *o a*, *o b*, *o c*, *o d*, *o e*, etc., drawn so as to divide the current into equal areas. Then the work done in transporting a unit pole from *a* to *b* is equal to the work done in transporting a unit pole around the path *a b o a*.

Work ($a\,b$) = work ($a\,b\,o\,a$)
= work ($b\,c$) = work ($b\,c\,o\,b$)
= work ($c\,d$) = work ($c\,d\,o\,c$), etc.
for work ($a\,o$) = work ($b\,o$) = work ($c\,o$) = 0.

That is, the work done in going from one equipotential line such as *a* to another such as *b*, is equal to the m.m.f. between these two lines, or 4 π times the current enclosed by the lines *a b*, *b o*, and *o a*. Consequently, work *a b* equals work *a' b'*; but work *c d* is greater than work *c' d'*, for more current is enclosed by *c d*. It is well to remember that the m. m. f. between any two points in the copper, such as *c'* and *d'*, is proportional to the amount of current enclosed by the line of flux *c' d'* and the lines of no work *c' o* and *d'* o. Therefore, if a tube of flux is desired along *a' b' c' d'*, so that it may enclose the same flux at all points of the tube, the reluctance must vary in direct proportion to the m.m.f. And since the m.m.f. varies in direct proportion to the amount of current enclosed, the ratio l/a(20) for the curvilinear rectangles must vary in direct proportion to the amount of current enclosed. This relation makes it possible to extend the plot into the copper.

The subsequent rules will aid materially in shortening the time necessary to obtain an accurate field plot. The reasons for following the directions in the order given will be obvious.

(1) Draw bisectors of the angles at the points *a*, *b*, *c*, *d*, *e*, *f*, as shown in Fig. 27. These are the directions of the lines of flux at these points. Continue them wherever it is possible to do so with any degree of accuracy, as at *a*, *b*, *c*, *d*, entering the opposite side at right angles.

(2) Divide the current-carrying conductor section into eight equal regions, as shown in Fig. 27. The

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(19) L. F. Richardson has given a rule for the plotting of fields in current-carrying copper to the effect that in a region occupied by current "the difference of successive chequer ratios in a direction perpendicular to the lines of force, divided by the mean chequer area is equal to a constant times the current density in the region." A proof of this relation is briefly outlined below.

Let n and t represent distance measured along lines of no work and along lines of force, respectively. Let small quantities of the order x be represented by the symbol $o\,(x)$. Then, (referring to Fig. A), if n and t are small enough increments so that the field intensity and current density change only slightly for points included in their variation, the relations exist:

$$H = o\,(0)$$

$$n,\ t = o\,(1)$$

$$H_1\,t_1 - H_0\,t_0 = 4\,\pi\,i\,n_1\,t_1 + o\,(3) \qquad (30a)$$

$$H_2\,t_2 - H_1\,t_1 = 4\,\pi\,i\,n_2\,t_2 + o\,(3) \qquad (30b)$$

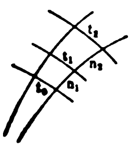

Fig. A

where

H_n = field intensity along t_n.
i = current density at any point near n_1, n_2, t_1, t_2.

If n_1 and n_2 are chosen so that the flux between t_2 and t_1 is the same as the flux between t_1 and t_0, then

$$(H_1 + H_0)\,n_1 = (H_2 + H_1)\,n_2 + o\,(3) = 2\phi + o\,(3) \qquad (30c)$$

where

ϕ = the flux between t_1 and t_0, and between t_2 and t_1.
= o (1).

Also,

$$n_2 - n_1 = o\,(2)$$

$$n_2\,t_2 = n_1\,t_1 + o\,(3) = n\,t + o\,(3)$$

Adding Equations (30a) and (30b), there results:

$$(H_2 + H_1)\,t_2 - H_1\,(t_2 - t_1) - (H_1 + H_0)\,t_1 + H_0\,(t_1 - t_0)$$
$$= 8\,\pi\,i\,n\,t + o\,(3) \qquad (30d)$$

But

$$H_0\,(t_1 - t_0) - H_1\,(t_2 - t_1) = (H_0 - H_1)\,(t_1 - t_0) - H_1\,(t_2 + t_0 - 2t_1) = o\,(3)$$

Therefore, Equation (30d) becomes

$$(H_2 + H_1)\,t_2 - (H_1 + H_0)\,t_1 = 8\,\pi\,i\,n\,t + o\,(3)$$

Substituting in this equation from Equation 30(c), there results

$$\phi\left(\frac{t_2}{n_2} - \frac{t_1}{n_1}\right) = 4\,\pi\,i\,n\,t + o\,(3)$$

$$\frac{t_2}{n_2} - \frac{t_1}{n_1} = \frac{4\,\pi\,i\,n\,t}{\phi} + o\,(2)$$

Thus, omitting second order terms, we obtain a relation involving quantities of the first order on either side:

$$\frac{\dfrac{t_2}{n_2} - \dfrac{t_1}{n_1}}{n\,t} = \frac{4\,\pi}{\phi}\,i$$

which is the rule stated by Richardson.

(20) The reluctance is proportional to the length l of the path and inversely proportional to the area a of the path. Since unit thickness is chosen for the plot, the width of a tube represents its area.

reason for the peculiar division is that, later, the lines of no work will roughly coincide with the construction lines, thereby providing an easy method of sketching the lines of no work as that they will divide the current into equal parts. If it is found later that some of the construction lines are not placed to best advantage, others may be drawn.

(3) Draw a trial set of seven lines of no work as in Fig. 28, crossing the lines of flux *a*, *b*, *c*, *d* at right angles, hugging the sharp projections closely, as at *b* and *c*, keeping away from the inverted corners as at *a* and *d*. This trial set must divide the current region into eight equal portions. Arbitrarily choose some point *P* in the corner closest to the iron, for the kernel. If no iron were present, it would lie in the center of the copper; if iron were touching the copper along one surface, the kernel would be at the copper-iron boundary; if iron were touching the copper along two surfaces, the kernel would be on both copper-iron boundaries, at the corner. Obviously, the kernel, in all practical cases, would lie somewhere between the center of the copper and the lower corner near the iron. Line No. 4 (Fig. 28) must divide the current into two equal regions. The advantage of the straight construction line between Regions 4 and 5 of Fig. 27, dividing the current into two equal portions, is evident. The other construction lines also are located so as to be most useful in enabling an accurate division of the current to be made by judging only small differences in area with the eye.

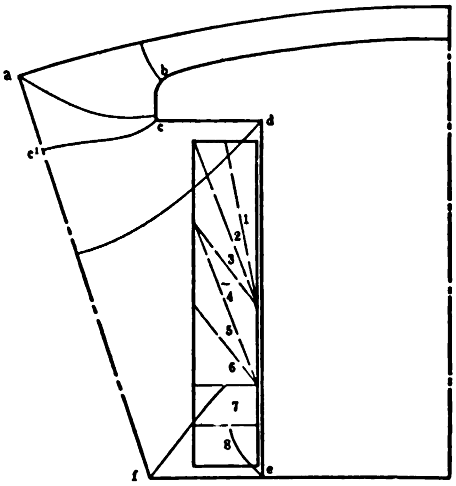

Fig. 27

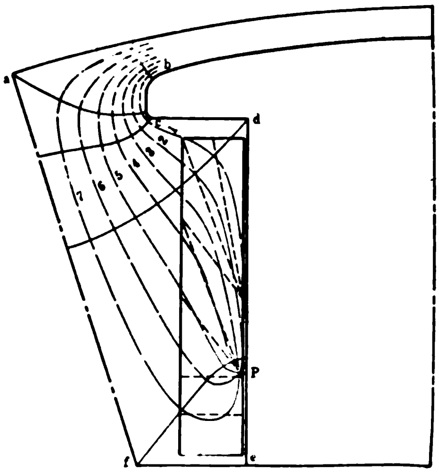

Fig 28

(4) Starting from some line of flux *c c'* (Fig. 29), draw lines of flux, making curvilinear squares along the line *a c' f* in the region where there is no current. Then extend the lines of force into the current region orthogonal to the lines of no work.

(5) The correctness of the plot may now be tested as follows:

a. Lines of flux must cross lines of no work at right angles.

b. Lines of flux must enter the iron at right angles (assuming infinite permeability).

c. All rectangles outside the current must be curvilinear squares (ratio $l/a = $ unity).

d. Within the current region, the rectangles must have a ratio l/a less than unity and equal to the ratio between the current enclosed and the one-eighth portion of the total current. Thus at a point *g* (Fig. 29), if the flux line and the two lines of no work intersected by it enclose one-half of the eighth portion of current, then the length of the rectangle at this point should be one-half of the width.

(6) If the various ratios of l/a are *not* proportional to the current enclosed, then the plot must be redrawn, either changing the position of the kernel or else shifting the position of the lines of no work, and making the correspondingly necessary changes in the lines of flux. It is well to use tracing paper, for then each previous trial may be used for a guide.

The final plot is shown in Fig. 30. Any required accuracy may be attained by continuing the process indefinitely. By this method, however, a large portion of the cut-and-try is eliminated because as many of the required conditions as possible are fulfilled in the first part of the construction, and after three or four trials a surprisingly accurate sketch can be obtained.

Vector Potential in Coplanar Magnetic Fields

Although neither the problems nor the general methods given in this connection are new it is believed that the application of the little-used conception of vector potential to a group of familiar problems will be of interest.

In problems where the flux distribution can be represented by a two-dimensional sketch,(21) the vector potentials are all perpendicular to the plane of the paper and consequently can be added and subtracted exactly like scalar quantities. This makes the construction of field plots by lines of equi-vector potential very much easier than by the more usual method which employs the two components of the vector field intensity.

---

(21)Such fields are said to be coplanar in the plane of the paper.

Let X and Y be the x and y components of the field. Then, integration around any small area in the $x\ y$ plane will show that (22)

$$\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} = 4\pi i \qquad (27)$$

where i is positive when directed upward out of the plane of the paper.

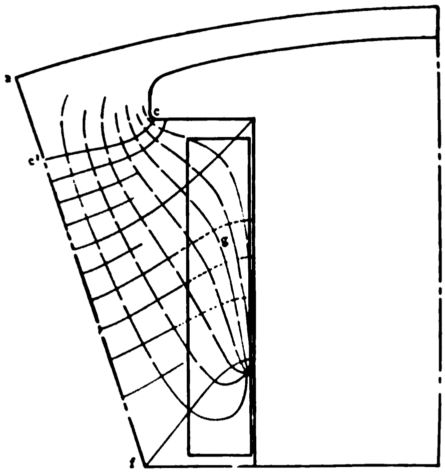

Fig. 29

Let R be a function (23) such that

$$X = -\frac{\partial R}{\partial y} \qquad (28)$$

$$Y = +\frac{\partial R}{\partial x} \qquad (29)$$

Then

$$\frac{\partial^2 R}{\partial x^2} + \frac{\partial^2 R}{\partial y^2} = +4\pi i \qquad (30)$$

The function R is known as the vector potential of the field. In a coplanar magnetic field, a line of force is characterized by the equation $R = \theta =$ a contant, because

$$dR = \frac{\partial R}{\partial x}dx + \frac{\partial R}{\partial y}dy = 0 \qquad (31)$$

and solving for the slope of the line $R =$ a constant:

$$\frac{dy}{dx} = -\frac{\partial R/\partial x}{\partial R/\partial y} = \frac{Y}{X} \qquad (32)$$

which shows that this line has the same direction as the magnetic field.

Also, $\Delta\phi = \phi_1 = \phi_2$ is equal to the flux included between the lines of force $R = \phi_1$ and $R = \phi_2$, for

$$\Delta\phi = \int_1^2 dR = \int_1^2 Y\,dx - X\,dy \qquad (33)$$

may be seen to give the flux included between the points 1 and 2 whatever the path of integration employed.

In any problem in which the only sources of m.m.f., are currents within the regions under consideration, the boundary conditions imposed consist in variations of permeability frum the region under consideration to the adjoining regions and, in general, from the adjoining regions to other regions. This type of boundary condition may be recognized as consisting of a sum of terms containing constants and partial derivatives operating on R.

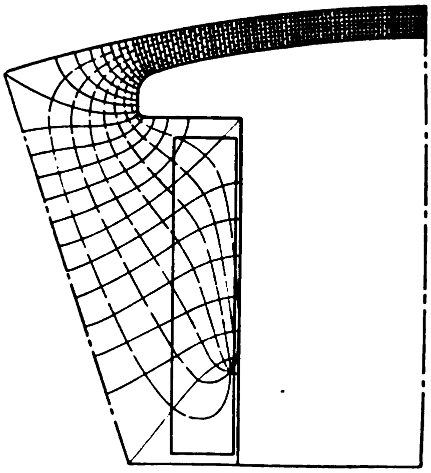

Fig. 30

It is then clear that if two solutions R_1 and R_2 satisfy these boundary conditions and the point Equation (30) for current densities i_1 and i_2 separately, their sum must also satisfy these boundary conditions and the point Equation (30) for a current density $i_1 + i_2$.

It is possible, therefore, to superpose solutions for vector potential due to two distributions of current density and the result may be extended to any number of superpositions; and thus from a knowledge of the vector potential of an element we may by integration determine the vector potential of a complicated distribution of current density.

The Vector Potential of an Isolated Straight Wire of Circular Section. Let the radius of the wire be a and let the total current through the wire be I.

Then inside the wire the field intensity (24) is

$$H = \frac{2Ir}{a^2} \qquad (34)$$

and outside the wire

$$H = \frac{2I}{r} \qquad (35$$

---

(22) "Fundamental Theory of Flux Plotting," by A. R. Stevenson, Jr., GENERAL ELECTRIC REVIEW, Vol. 29, No. 11, Nov., 1926, p. 797-804.

(23) As in Part I, in order to make R positive in sign, it is defined by the relations.

$$Y = \frac{\partial R}{\partial} \qquad X = -\frac{\partial R}{\partial y}$$

instead

$$Y = -\frac{\partial R}{\partial x} \qquad X = \frac{\partial R}{\partial y}$$

as in Rogowski's work

(24) $H = \sqrt{X^2 + Y^2}$.

If we choose $R=0$ at $r=0$, then inside the wire

$$R=\int_{r=0}^{r=r} H\,d\,r=\frac{I\,r^2}{a^2} \tag{36}$$

and outside the wire

$$R=\int_{r=a}^{r=r} H\,d\,r+\frac{I\,a^2}{a^2}=2\,I\left[\log_\epsilon\left(\frac{r}{a}\right)+\frac{1}{2}\right] \tag{37}$$

The Vector Potential of Two Isolated Straight Wires of Circular Section. Let r_1 and r_2 be the radius vectors from wires 1 and 2, respectively, and let a_1 and a_2 be the respective radii of the conductors. Then inside conductor 1

$$R=2\,I_2\left[\log_\epsilon\left(\frac{r_2}{a_1}\right)+\frac{1}{2}\right]+I_1\frac{r_1^2}{a_1^2} \tag{38}$$

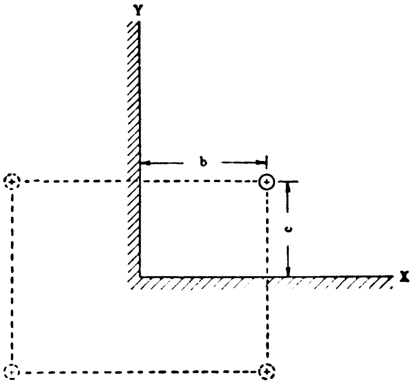

Fig. 31

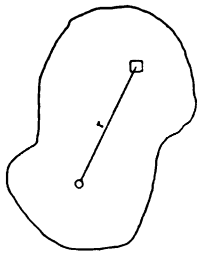

Fig. 32

Inside conductor 2

$$R=2\,I_1\left[\log_\epsilon\left(\frac{r_1}{a_1}\right)+\frac{1}{2}\right]+I_2\frac{r_2^2}{a_2^2} \tag{39}$$

Outside both conductors

$$R=2\,I_1\left[\log_\epsilon\left(\frac{r_1}{a_1}\right)+\frac{1}{2}\right]+2\,I_2\left[\log_\epsilon\left(\frac{r_2}{a_2}\right)+\frac{1}{2}\right] \tag{40}$$

(*a*) In the *special case* where the currents are in opposite directions, $I_2=-I_1=I$, and $a_1=a_2$, the vector potential outside the conductors is

$$R=2\,I\log_\epsilon\frac{r_1}{r_2} \tag{41}$$

and the equation of a line of force, $R=$ a constant, is

$$\frac{r_1^2}{r_2^2}=\epsilon^{R/I} \tag{42}$$

The well-known fact follows that the lines of force outside the conductors are circles.

In particular, if the centers of the wires are located at $x=-b$, and $x=b$, respectively, Equation (42) becomes

$$[x+b\coth(R/2\,I)\,]^2+y^2=\frac{b^2}{\sinh^2(R/2\,I)} \tag{43}$$

(*b*) In the *special case* where the currents are in the same direction, $I_1=I_2=I$, and $a_1=a_2=a$,

$$R=2\,I\left[\log_\epsilon\left(\frac{r_1\,r_2}{a^2}\right)+1\right] \tag{44}$$

The equation of the line of force in this case is of the fourth degree.

The Vector Potential of a Straight Wire Near the Corner of two Infinitely Permeable Planes. The wire and images for this case are shown in Fig. 31. Outside the wire

$$\frac{R}{I}=\log_\epsilon\frac{(x-b)^2+(y-c)^2}{a^2}+\log_\epsilon\frac{(x-b)^2+(y+c)^2}{a^2}$$
$$+\log_\epsilon\frac{(x+b)^2+(y-c)^2}{a^2}+\log_\epsilon\frac{(x+b)^2+(y+c)^2}{a^2}+4 \tag{45}$$

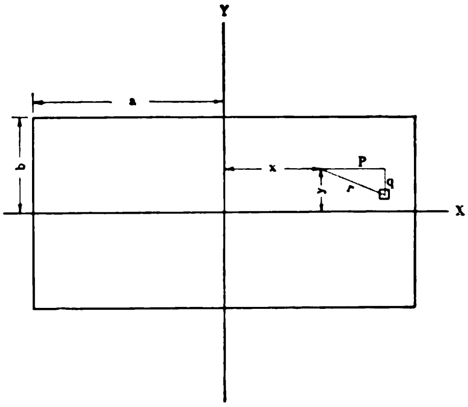

Fig. 33

Equations of the type of (45) may be solved by plotting since only the real roots are desired. In the case under consideration, R/I should be plotted as a function of x with y as a parameter. Points on a line of force are then determined by the intersections of the curves along lines of $R/I=$ a constant.

The Vector Potential of an Isolated Wire of Any Cross-Section in Air. Referring to Fig. 32, the vector potential due to an element of the wire is,

$$d\,R=i\,(\log_\epsilon r^2)\,da \tag{46}$$

where r is the distance from the point at which R is determined to the element under consideration, i is the current density, and da the area of the element. The value of R for the whole wire is, then,

$$R=\int_S i\,(\log_\epsilon r^2)\,da+\text{a constant} \tag{47}$$

the subscript S indicating that the integral is taken over the whole surface of the wire. If the current density over the cross-section is constant, we have

$$\frac{R}{i}=\int_S(\log_\epsilon r^2)\,da+\text{a constant}^{(25)} \tag{48}$$

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(25) This result may be expressed in the form $\frac{R}{i}=\log D^2+$ a constant, where D equals the geometric mean distance of the point at which R is to be computed from the area of the section of the conductor.

(*a*) For the *special case* of an isolated wire of rectangular section we have, referring to Fig. 33.

$$\frac{R}{i} = \int_{q=y-b}^{q=y+b} \int_{p=x-a}^{p=x+a} \log_\epsilon (p^2+q^2)\, dp\, dq \qquad (49)$$

Integrating and adding the constant terms.

$$-4\,a\,b \log_\epsilon (a^2+b^2) - 6\,a\,b - 4\,a^2 \tan^{-1}\frac{b}{a} - 4\,b^2 \tan^{-1}\frac{a}{b}$$

In order to make $\frac{R}{i} = 0$ at the origin, Equation (49)

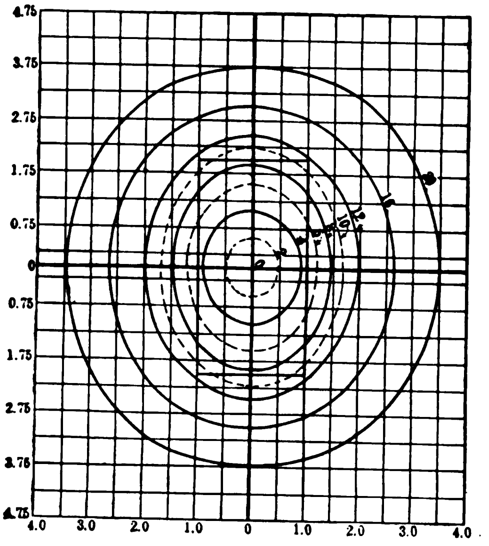

Fig. 34

may be reduced to the symmetrical form: (26)

$$\begin{aligned}
\frac{R}{i} = {} & (x+a)\,(y+b) \log_\epsilon \left[\frac{(x+a)^2+(y+b)^2}{a^2+b^2}\right] \\
& -(x-a)\,(y+b) \log_\epsilon \left[\frac{(x-a)^2+(y+b)^2}{a^2+b^2}\right] \\
& -(x+a)\,(y-b) \log_\epsilon \left[\frac{(x+a)^2+(y-b)^2}{a^2+b^2}\right] \\
& +(x-a)\,(y-b) \log_\epsilon \left[\frac{(x-a)^2+(y-b)^2}{a^2+b^2}\right] \\
& +(x+a)^2 \left[\tan^{-1}\frac{y+b}{x+a} - \tan^{-1}\frac{y-b}{x+a}\right] \\
& -(x-a)^2 \left[\tan^{-1}\frac{y+b}{x-a} - \tan^{-1}\frac{y-b}{x-a}\right] \\
& +(y+b)^2 \left[\tan^{-1}\frac{x+a}{y+b} - \tan^{-1}\frac{x-a}{y+b}\right] \\
& -(y-b)^2 \left[\tan^{-1}\frac{x+a}{y-b} - \tan^{-1}\frac{x-a}{y-b}\right] \\
& -4\,a^2 \tan^{-1}\frac{b}{a} - 4\,b^2 \tan^{-1}\frac{a}{b} \qquad (50)
\end{aligned}$$

The field in and around a rectangular conductor of dimensions two units by four units, carrying a current of 80 amp., *i. e.*, $a=2$, $b=1$, $i=1$, has been plotted from Equation (50) by one of the writers' associates, R. S. Arthur, and is shown in Figs. 34 and 35. Figs. 36 and 37, showing R as a function of x and y, were used in plotting Figs. 34 and 35.

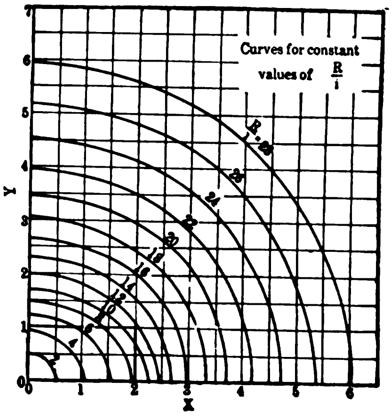

Fig. 35

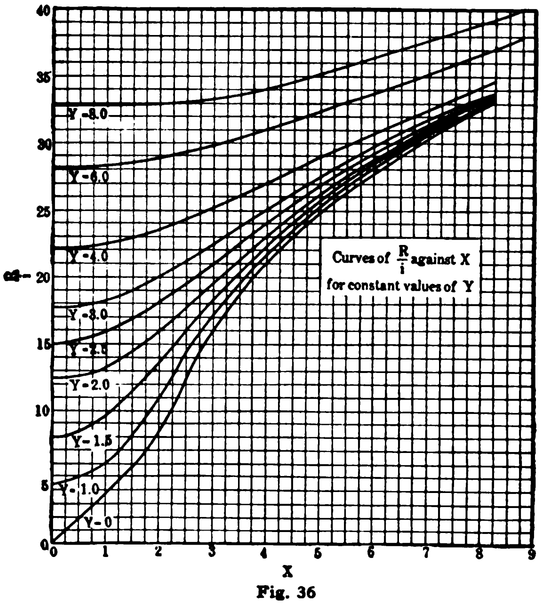

Fig. 36

(*b*) *Special Case.* Infinitely thin ribbon.

For sufficiently small values of b, that is, if the rectangle degenerates into a ribbon, putting $2\,b\,i = i'$ = current per unit length o the ribbon and adding terms such that $\frac{R}{i'} = 0$ at $x = y = 0$, Equation (50) may be written (26)

$$\begin{aligned}
\frac{R}{i'} = {} & (a+x) \log_\epsilon \frac{(a+x)^2+y^2}{a^2} \\
& +(a-x) \log_\epsilon \frac{(a-x)^2+y^2}{a^2} \\
& +2\,y \left[\tan^{-1}\frac{a+x}{y} + \tan^{-1}\frac{a-x}{y}\right] \qquad (51)
\end{aligned}$$

Equation (51) corresponds precisely to the case of a vanishingly thin isolated conductor, or to a narrow strip of conductor on an infinite iron surface.

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(26) The variable terms in Equations (50) and (51) may be checked against those given by Maxwell "Electricity and Magnetism," (paragraph 692) for the mean geometric distance of a point from a rectangle and a straight line, respectively.

In the case of a ribbon conductor two units long, carrying a current of 20 amp., the expression for vector potential is

$$R=(1+x)\log_\epsilon[(1+x)^2+y^2]+(1-x)\log_\epsilon[(1-x)^2+y^2]$$
$$+2y\left[\tan^{-1}\left(\frac{1+x}{y}\right)+\tan^{-1}\left(\frac{1-x}{y}\right)\right] \qquad (52)$$

The field for this case as plotted by one of the writers is shown in Fig. 38. The curves of R as a function of x and y which were used in plotting Fig. 38 are shown in Figs. 39 and 40.

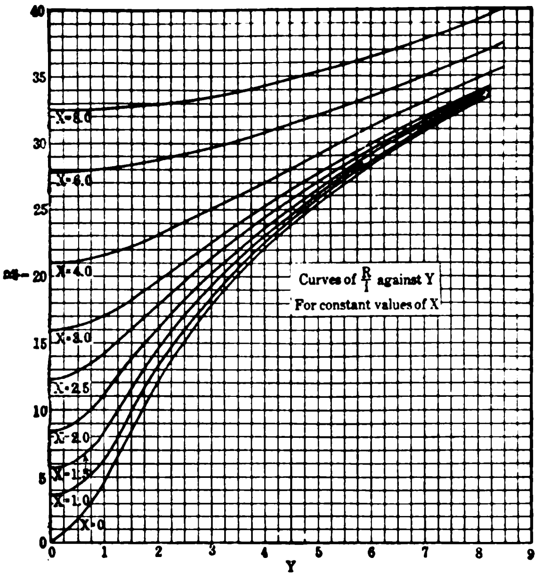


Fig. 37

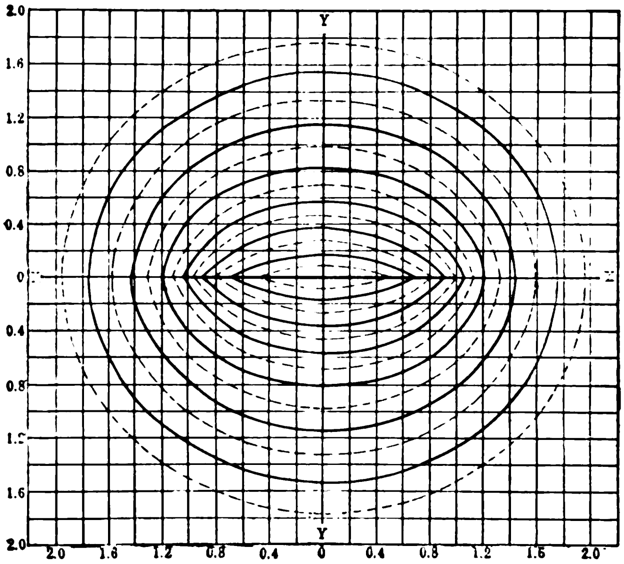


Fig. 38

The Relation of the Scalar Potential Function to the Vector Potential Function in Coplanar Magnetic Fields

For points outside of regions in which current density exists, it is legitimate to construct a potential function V having the property:

$$H_n=\frac{\partial}{\partial n}V \qquad (53)$$

where H_n is the field intensity in a direction n and where n is distance measured in any direction from the point at which H_n exists.

Then, in particular,

$$dV=\frac{\partial V}{\partial x}dx+\frac{\partial V}{\partial y}dy=H_x\,dx+H_y\,dy$$
$$V_2-V_1=\int_1^2 H_x\,dx+H_y\,dy \qquad (54)$$

where x and y form any convenient set of coördinate axes. The path of integration must be arranged so that it does not traverse regions in which current density exists. At the same time, it is necessary to construct arbitrary boundary surfaces, one beginning at some point of each current-carrying region and dividing the field into arbitrary sections such that within any section it is impossible to enclose current by any path of integration outside the zone of current density. Equipotential lines may be carried through these regions by joining lines which differ by an amount 4 π times the current enclosed by any circuit which does not cut through any boundaries, and starting at one side of the boundary and ending at the other side at the same point.

If the vector potential of the field is known, we may use Equation (54) in the form:

$$V_2-V_1=\int_1^2\frac{\partial R}{\partial x}dy-\frac{\partial R}{\partial y}dx \qquad (55)$$

Solutions of scalar potential due to particular distributions of current density will be superposable in the same way and subject to the same limitations as apply to solutions of vector potential.

In the following, the scalar potential functions in the various examples of magnetic fields given in other parts of the article will be worked out briefly.

Field Pole, or Slot. Referring to Fig. 41, it will be convenient to divide Region II into a and b parts, the a part being between the current-carrying zones and the b part between the current-carrying zones and the iron, and to choose the line $x=a$ from $\theta=\pi-\theta_2$ to $\theta=\pi$, and from $\theta=\theta_2-\pi$ to $\theta=-\pi$ as the arbitrary boundary surface previously referred to.

Then, choosing $V=0$ at $x=y=0$, and remembering $y=\frac{\theta}{k}$,

$$V=\int_0^{\theta,x}-k\frac{\partial R}{\partial\theta}dx+\frac{1}{k}\frac{\partial R}{\partial x}d\theta=\int_0^\theta\frac{1}{k}\frac{\partial R}{\partial x}d\theta \qquad (56)$$

Thus, for Region I:

$$\frac{\partial R}{\partial x}=\sum_1^\infty kn\,[a_n\,\epsilon^{-kn(a-x)}-a_n'\,\epsilon^{-knx}]\cos n\theta$$

$$V_I=\sum_1^\infty[a_n\,\epsilon^{-kn(a-x)}-a_n'\,\epsilon^{-knx}]\sin n\theta \qquad (57)$$

For Region II:

$$\frac{\partial R}{\partial x} = 4\,\pi\,\alpha_0\,(x-a) + \sum_1^{\infty} n\,[b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-a)}]\cos n\theta$$

$$V_{IIa} = 4\,\pi\,\alpha_0\,(x-a)\,y + \sum_1^{\infty} [b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-a)}]\sin n\theta \tag{58}$$

In order to obtain the scalar potential for Region IIb, it is convenient to subtract from the known potential at the iron surface the potential difference from the point in question referred to that surface. Thus:

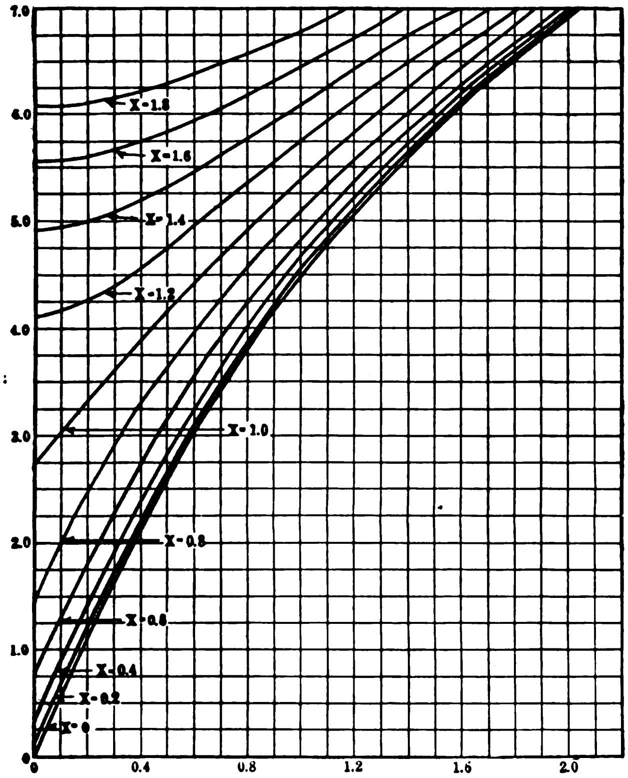


Fig. 39

$$V_{IIb} = 4\,\pi\,\alpha_0\,(b-a)\,\frac{l}{2} - \int_{\theta}^{\pi} \frac{1}{k}\,\frac{\partial R}{\partial x}\,d\theta$$

$$= 4\,\pi\,\alpha_0 \left[(b-a)\,\frac{l}{2} - (x-a)\left(\frac{l}{2} - y\right)\right]$$

$$+ \sum_1^{\infty} [b_n\,\epsilon^{-kn(b-x)} - b_n'\,\epsilon^{-kn(x-a)}]\sin n\theta$$

$$= V_{IIa} + 2\,\pi\,\alpha_0\,l\,(b-x) \tag{59}$$

For Region III:

$$\frac{\partial R}{\partial x} = 4\,\pi\,\alpha_0\,(b-a) + \sum_1^{\infty} [c_n\,\epsilon^{-kn(c-x)} - c_n'\,\epsilon^{-kn(x-b)}]\cos n\theta$$

which gives

$$V_{III} = 4\,\pi\,\alpha_0\,(b-a)\,y + \sum_1^{\infty} [c_n\,\epsilon^{-kn(c-x)} - c_n'\,\epsilon^{-kn(x-b)}]\sin n\theta \tag{60}$$

The solution for an infinitely deep slot may be obtained by inserting $c = \infty$.

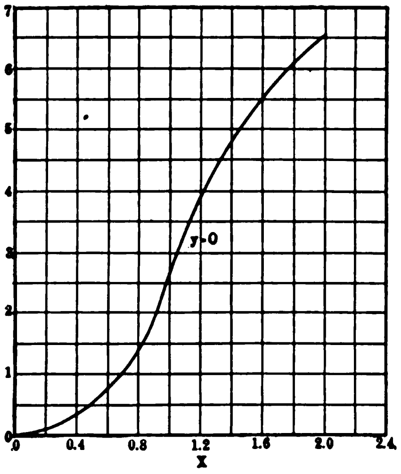


Fig. 40

Circular Slot. In polar coördinates, Equations (54) and (55) become

$$V_2 - V_1 = \int_1^2 H_r\,dr + H_\theta\,r\,d\theta \tag{61}$$

$$= \int_1^2 -\frac{1}{r}\,\frac{\partial R}{\partial \theta}\,dr + \frac{\partial R}{\partial r}\,r\,d\theta \tag{62}$$

Taking $V = 0$ at $\theta = 0$, there results:

$$V = \int_0^{\theta} r\,\frac{\partial R}{\partial r}\,d\theta \tag{63}$$

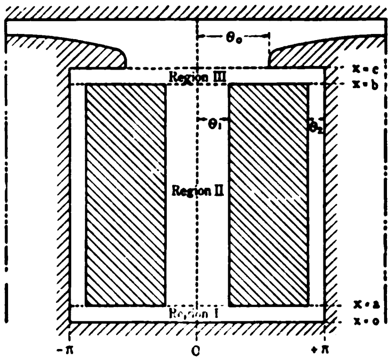


Fig. 41
(Fig. 10 from Part I, repeated)

For Region II:

$$\frac{\partial R}{\partial r} = \sum_1^{\infty} \frac{n}{r_0}\left(\frac{r}{r_0}\right)^{n-1} [b_n \cos n\theta] + \frac{2\,I}{r}$$

which gives:

$$V_{II} = \sum_1^{\infty} \left(\frac{r}{r_0}\right)^{n} b_n \sin n\theta + 2\,I\,\theta \tag{64}$$

Scalar Potential of a Wire of Any Section. Evidently, for an isolated circular wire,

$$V=2\,I\,\theta$$

The solution for several circular wires is obtained by superposition. The solution for a wire of any section is

$$V=\int_S 2\,i\,\theta\,da \qquad (65)$$

the integral to be extended over the whole section in which i exists. It may be found more convenient, however, to obtain V indirectly through Equations (47) and (55).

Calculation of Inductance from a Knowledge of Vector Potential

In general, in order that it shall be permissible to employ the conception of inductance to a conductor of large section, it is necessary that the conductor consist of a bundle of smaller conductors, all of these conductors supposedly connected in series, or in such a way that the same current flows in each. (27) If there is a sufficient number of small conductors, it is permissible as an approximation to calculate the inductance of the system as the average inductance of a continuous distribution of small current filaments, the density of the filaments being in proportion to the density of small conductors. It will further be assumed that the current distribution may be regarded as continuous within the section of the large conductor. (28)

Let ϕ_0 be the total flux outside some particular line of force $R=R_0$ and between that line and the line with respect to which induced voltage is to be computed, where R is the vector potential function of the field.

Then (ϕ_0+R_0-R) will be the flux outside any line R.

Let n = the density of small conductors.

The average flux linkages per conductor are

$$\int_S \frac{(\phi_0+R_0-R)\,n\,da}{N}=(\phi_0+R_0)-\int_S \frac{R\,n\,da}{N}$$

where

$$N=\int_S n\,da$$

= the total number of small conductors in the section.

If I is the total current through the section, the component of average inductance due to flux up to the point that ϕ_0 is computed, is

$$L=\left(\frac{\phi_0+R_0}{I}\right)-\int_S \frac{R\,n\,da}{N\,I}$$

But

$$i=\frac{n}{N}I$$

$$I=\int_S i\,da$$

or

$$L=\left(\frac{\phi_0+R_0}{I}\right)-\int_S \frac{R\,i\,da}{I^2} \qquad (66)$$

Equation (66) may be put in the form,

$$L=4\,\pi\,(P_0-P) \qquad (67)$$

where P_0 and P are permeance factors:

$$P_0=\frac{\phi_0+R_0}{4\,\pi\,I} \qquad (68)$$

$$P=\frac{\int_S R\,i\,da}{4\,\pi\,[\int_S i\,da]^2} \qquad (69)$$

In Equations (68) and (69), the quantity P_0 is the permeance factor that would apply if the flux ϕ_0+R_0 linked all of the conductors, while the factor P takes into account the effect of partial linkages.

It is interesting that in the foregoing equation, the vector potential R need not be computed so that the minimum value of $R=0$. Thus it is not necessary to calculate the value of R at the kernel. If the kernel is known to exist on some line such that one coördinate x or y, or r or θ, for example, is fixed, the calculation of the value of R at the kernel is comparatively easy. If, as in the field-pole problem, however, the value of neither coördinate is known, then on account of the unsatisfactory convergence which usually occurs at the kernel, the calculation of the position and value of R at the kernel is a task of considerable difficulty. It is fortunate, therefore, that this calculation is not required.

Inductance of Two Parallel Cylindrical Conductors. As a simple example of Equation (66), consider the case of two straight conductors of circular section carrying currents I and $-I$, respectively. From Equation (38), the vector potential at a point inside one wire is

$$R=I\frac{r_1^2}{a^2}-2\,I\left[\log\left(\frac{r_2}{a}\right)+1/2\right]$$

where a = the radius of the section.

Let $r_1=r$

l = distance between centers of the conductors.

The component of inductance due to half the flux will be calculated. Thus ϕ_0 will be chosen equal to 0 for $r_1=r_2$. But also, from Equation (41), $R=R_0=0$ at $r_1=r_2$. Thus, Equation (66) gives

$$L=-\int_S \frac{R\,i\,da}{I^2}$$

---

(27) From the restricted point of view that stored magnetic energy equals one-half the product of inductance and current squared, it is permissible to apply the conception of inductance to conductors which are not subdivided.

(28) If desired, this approximation may be corrected. A correction of this type is given by Maxwell, "Electricity and Magnetism," par. 693.

which may be put in the form:

$$L = \frac{1}{\pi a^2}\int_0^a \int_0^{2\pi} \left[1 - \frac{r^2}{a^2} + \log\left(\frac{l^2+r^2-2\,l\,r\cos\theta}{a^2}\right)\right] r\,d\theta\,dr \tag{70}$$

$$= \frac{2}{a^2}\int_0^a \left[1 - \frac{r^2}{a^2} + \log\frac{l^2}{a^2}\right] r\,dr$$

$$= 2\log\frac{l}{a} + \frac{1}{2} \tag{71}$$

which is the usual expression for one-half the inductance of a circuit formed by two parallel wires.

Application to Field-Pole Problem. The use of Equation (69) will now be illustrated by applying it to the field-pole problem. In this case, and considering all the copper as comprising a single coil side, there is

$$P = \frac{2\,i \int\limits_{\theta_1}^{\pi-\theta_2} \int\limits_{x=a}^{x=b} R_{11}\,dx\,dy}{4\,\pi\,[\alpha_0\,(b-a)\,l]^2} \tag{72}$$

This integral reduces to the expression

$$P = \frac{b-a}{6\,l} - P'$$

where

$$P' = \sum_1^\infty \frac{\alpha_n}{16\,\pi^2\,\alpha_0^2\,(b-a)^2\,n}\left[(b_n+b_n')\left\{1-\epsilon^{-kn(b-a)}\right\} + 2\,K_n\,k\,n\,(b-a)\right] \tag{73}$$

Thus, for the field-pole shown in Fig. 11

$$P' = 0.0045$$

For the case of an infinite slot, $c=\infty$, Equation (73) becomes explicitly:

$$P' = \sum_1^\infty \frac{1}{8\,\pi^2}\left(\frac{\alpha_n}{n\,\alpha_0}\right)^2\left(\frac{l}{b-a}\right)\left[1 - \frac{(2-\epsilon^{-2kna}+\epsilon^{-kn(b+a)})(1-\epsilon^{-kn(b-a)})}{k\,n\,(b-a)}\right] \tag{74}$$

Thus, for the "infinite" slot shown in Fig. 6,

$$P' = 0.0037$$

In the case of an infinitely deep slot, it may be verified that the factor P' provides a correction which gives the increase of inductance due to concentration of current above the inductance which would be calculated on the assumption that the lines of force were everywhere perpendicular to the slot sides.

Circular Conductors in a Circular Slot. The expression for P' in the case of a circular conductor is very simple because for b_0 chosen equal to 0, the integral over the area of the copper of the term which involves the effect of the slot is zero; and there remains only the term which would exist were the conductor isolated in space.

Thus, with $b_0=0$:

$$P' = \frac{i}{4\,\pi}\int\limits_S \frac{R\,i\,d\,a}{(\int\limits_S i\,d\,a)^2} = \frac{1}{4\,\pi^2\,r_2^2}\int_0^{2\pi}\int_0^{r_2}\frac{r^2}{r_2^2}\,r\,dr\,da \tag{75}$$

$$= \frac{1}{8\,\pi} \tag{76}$$

Conductors in Air. The calculation of the inductance of conductors in air permits the development of special formulas involving the conception of geometric mean distance. Thus, for a system of positive currents with return currents within a finite distance, the vector potential calculated from Equation (47) will be zero at any point which is an infinite distance from the system in question, if the constant term is taken equal to zero. Thus, if the voltage due to flux between the system of positive currents and infinity is to be calculated, and if the system of currents fulfills the requirements permitting the calculation of inductance, then

$$L_1 = -\int\limits_{S_1}\frac{R\,i\,da}{I^2} \tag{77}$$

where S_1 refers to the area of the sections of conductors carrying positive currents and

$$R = \int\limits_{(S_1+S_2)} i\log r^2\,da$$

Thus

$$L_1 = \frac{-1}{I^2}\int\limits_{S_1}\left[\int\limits_{S_1+S_2} i\log r^2\,da\right] i\,da \tag{78}$$

The voltage due to flux from infinity to the return is

$$L_2 = \frac{-1}{I^2}\int\limits_{S_2}\left[\int\limits_{S_1+S_2} i\log r^2\,da\right] i\,da \tag{79}$$

The total inductance of the circuit is

$$L = L_1 + L_2 = \frac{-1}{I^2}\int\limits_{S_1+S_2}\int\limits_{S_1+S_2} i\log r^2\,da\,i\,da \tag{80}$$

which may be put in the symmetrical form:(29)

$$L = +2\,\frac{\int\limits_{S_1+S_2}\int\limits_{S_1+S_2} i\log r\,da\,i\,da}{[\int\limits_{S_1} i\,da]\,[\int\limits_{S_2} i\,da]} \tag{81}$$

---

(29) It is understood that, subject to the conditions imposed

$$\int\limits_{S_1} i\,da = -\int\limits_{S_2} i\,da$$

For uniform current density, remembering that the current density is negative over the section of the return conductors, Equation (81) becomes:

$$L = -2\,\frac{\left\{\int_{S_1} - \int_{S_2}\right\}\left\{\left[\int_{S_1} - \int_{S_2}\right]\log r\,da\right\} da}{A_1 A_2} \qquad (82)$$

$$= -2\,\frac{\left[\int_{S_1}\int_{S_1}\log r\,da\,da - 2\int_{S_1}\int_{S_2}\log r\,da\,da + \int_{S_2}\int_{S_2}\log r\,da\,da\right]}{A_1 A_2}$$

$$= 2\log\frac{R_{12}^2}{R_{11}\,R_{22}} \qquad (83)$$

where

R_{11} = geometrical mean distance of section 1 from itself.

R_{22} = geometrical mean distance of section 2 from itself.

R_{12} = geometrical mean distance of section 1 from section 2. (30)

A_1 = area of section 1.

A_2 = area of section 2.

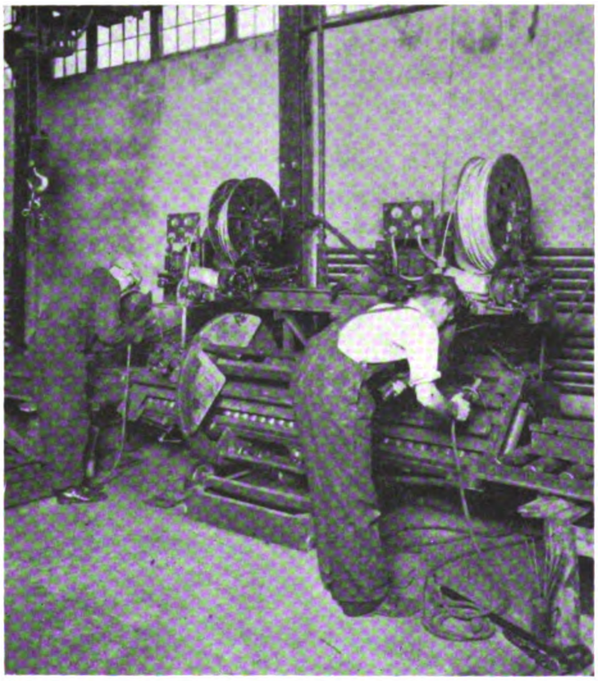
(Concluded)

---

(30) The geometrical mean distance for sections of several types are given in Maxwell, loc. cit, p. 63.

A New Machine for Producing Welded Railroad Ties

Following the development of a new type of railroad tie constructed from scrap rails, as described in the June, 1927, issue of this magazine, there has come the application of automatic welding machinery to the production of such ties.



The Apparatus by Means of which Ties Are Quickly Produced from Scrap Rails by the Simultaneous Application of Automatic and Hand Welding Operations

The component parts of the tie are assembled and clamped rigidly together on a jig which rotates with the work into positions most favorable to the application of the process. While two automatic welding heads are fastening the tie plates to which the rails are keyed when installed, angle bars which further strengthen the construction and provide for tie spacing are welded to the ends of the tie by the operators, both of whom are provided with manual welding equipment.

The current is supplied by a 1500-amp-hr. motor-generator welding set. Separate control panels provide two circuits for the hand welding electrodes and two circuits for the automatic welding heads.

By means of this equipment it is possible for two operators to produce about twelve ties per hour. The attendants work simultaneously upon the same tie, but each independently of the other, since the machine embodies two separate sets of welding appliances.

IN MEMORIAM

ELMER ELLSWORTH FARMER CREIGHTON

A CAREER of much technical achievement was terminated by the untimely death of Elmer Ellsworth Farmer Creighton on the twelfth of last January, at Schenectady, N. Y. He was in his fifty-fifth year, having been born on April 11, 1873, at Vallejo, California.

For nearly twenty-five years, beginning in 1904, Prof. Creighton was a special research engineer with the General Electric Company, devoting most of his time to the subject of protective equipment for electrical circuits and transmission systems. During the greater part of that period he was a technical associate and a personal friend of the late Dr. Charles Proteus Steinmetz. Together they studied lightning in its relation to electric power transmission; but Prof. Creighton also performed much independent engineering work in this field.

He was educated at Leland Stanford University, graduating in 1895, and receiving the degree of Electrical Engineer in 1897. These early years were divided between service as special testing engineer for the Pacific Postal Telegraph Company, teaching at his alma mater, and assisting Dr. David Starr Jordan on the Fur Seal Commission among the Pribilof Islands, Alaska.

From 1898 to 1900, he was assistant to Andre Blondel, in the Sorbonne, Paris, and at the Ecole Superieure d'Electricite. Upon returning to this country he again taught, for a time, at Leland Stanford.

In 1901, Prof. Creighton became the head of the Experimental Department at the Stanley Electric Manufacturing Company's plant at Pittsfield, Massachusetts, removing to Schenectady in 1904 to become assistant professor to Dr. Charles P. Steinmetz, who was at that time the head of the Electrical Engineering Department at Union College.

In 1913, a research laboratory for investigating protective methods, which Prof. Creighton had assisted in conducting at Union College in behalf also of the General Electric Company, was moved to the General Electric Works, and was made part of Dr. Steinmetz's consulting engineering department. A few years ago this laboratory was combined with the Standardizing Laboratory to form the present General Engineering Laboratory, under which Prof. Creighton had since served.

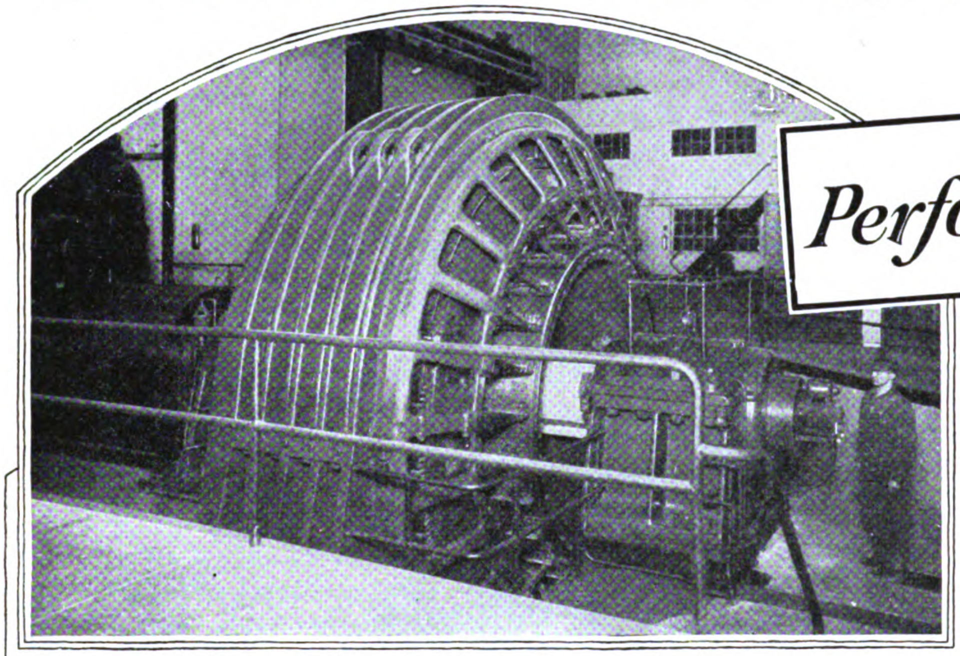
Prof. Creighton was a contributor to the electrical engineering profession both by inventions and by technical papers. He held sixty-six United States patents, almost all in the field of protective devices. These were principally the aluminum arrester, the compression-chamber arrester, the dry-film arrester, concrete reactors, and several types of direct-current and alternating-current arresters.

He served on many technical committees, particularly those on meetings and papers and on protection for the A.I.E.E., as well as on several American engineering standards committees. Since 1906, he had contributed more than seventy-five technical papers and discussions to the transactions of the A.I.E.E.

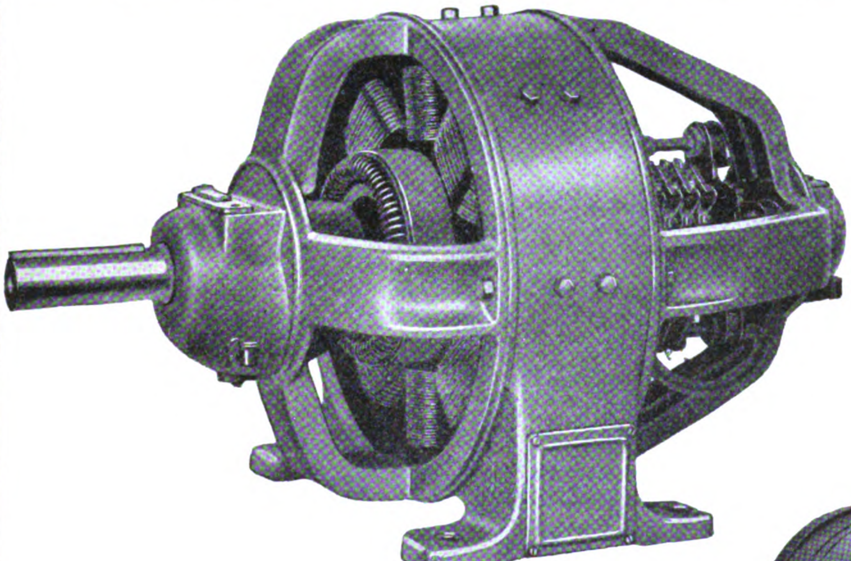
He was a Fellow of the American Institute of Electrical Engineers, a member of the Society for the Promotion of Engineering Education, the American Association for the Advancement of Science, the National Electric Light Association, the American Society of Mechanical Engineers, the American Physical Society, the American Electro-Chemical Society, the American Ceramic Society, and the Society Francaise des Electriciens. He was also a member of the Sigma Xi fraternity.

G-E Direct-Current

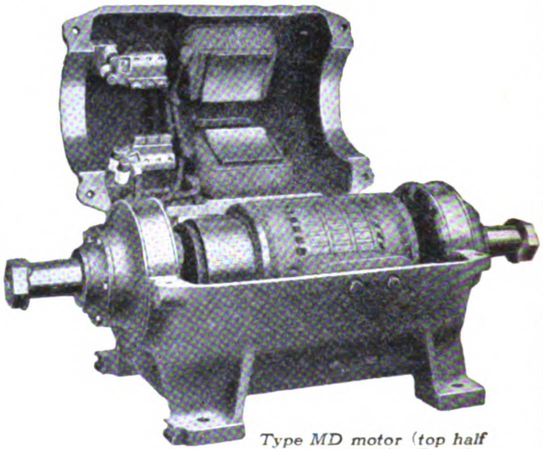
Performance that



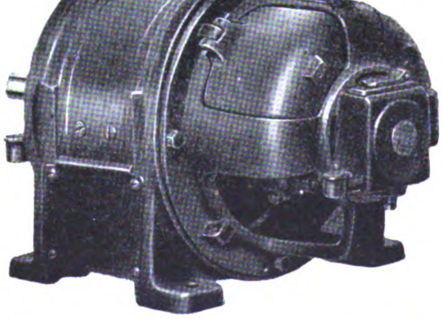
7000-hp., 40/80-r.p.m., direct-current, reversing motor driving a 54-in. blooming mill at the Lackawanna plant of the Bethlehem Steel Corporation.



Type CD motor, 3 to 200 hp. For adjustable-speed, constant-speed, high-speed, or low-speed applications.



Type MD motor (top half of frame lifted). For the severest duty on steel-mill auxiliaries, cranes, shovels, etc. Conforms to A.I.&S.E.E. specifications.



Type BD motor. ½ to 3 hp. A small d-c. motor adaptable to a great variety of applications.

GENERAL

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GENERAL ELECTRIC REVIEW

SCHENECTADY, NEW YORK

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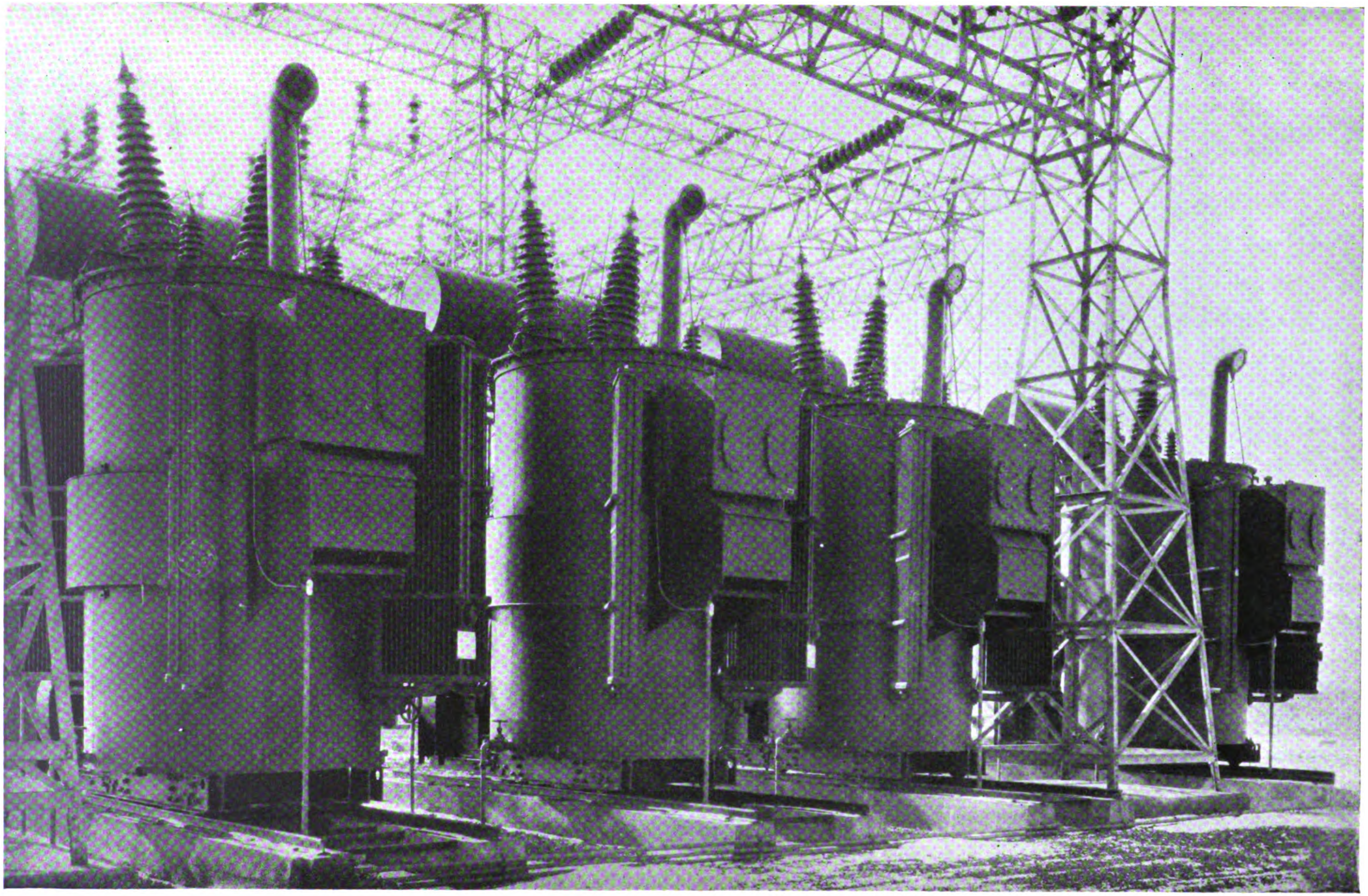

FLEXIBLE RATIO TO DELIVER A CONSTANT VOLTAGE

The Massillon Substation of the Ohio Public Service Company receives its energy at a fixed voltage from this 25,000-kv-a. bank of 132,000Y/24,000-volt step-down transformers (one a spare). The voltage control necessary to compensate for changes in line potential is afforded by the latest type of load ratio control equipment comprising adjusters and intermittent gear inside each transformer tank, and mechanically-operated high-voltage contactors immersed in oil in a separate compartment, with all live parts enclosed

The concluding part of a general article on Load Ratio Control appears on p. 191

GENERAL ELECTRIC REVIEW

Vol. 31 **April, 1928** **No. 4**

HEAT IT ELECTRICALLY

A survey of the literature of electric heating leads to the conclusion that almost everything has been tried at least once. Probably over-enthusiasm led many astray. On looking at present-day electric heating equipment one notes the comparatively few forms of this equipment, its simplicity, and how well it is adapted to the service. The crystallization process has been indeed severe but industry is benefitting mightily thereby.

Few manufacturing processes can be carried on without the use of heat, to change the shape and properties of the materials so that they can be fashioned into the final product. In many of these processes the energy required in the form of heat is by far the largest item. The extent to which electric energy can be used for this service is of interest to both the executive and the engineer of manufacturing organizations. The executive wants to know what is the extent that electric heat will increase production and at the same time reduce costs and improve working conditions. To be in position to render the most favorable answer to this triple question is the business of the engineer.

In this third major use of electric energy, the first consideration of the electrical industry is the value of electric heat to the user of heat. This has required the acquisition of reliable information concerning the utilization of heat in the various branches of manufacturing industry, and also the general problem. Fortunately, however varied may be the uses of electric heat, the engineering principles of its application are comparatively simple within the range of exactness sufficient for practical use. The aim of the serial entitled "Industrial Electric Heating," now running in this magazine, is to discuss these principles and their application with reference to the economical utilization of electric energy for industrial heating service.

The accurate control of electric energy is its valuable characteristic. Control includes both distribution and regulation of flow. When electric energy is converted into heat energy, the control of the electric energy is at the same time the control of the flow of heat.

Uniformity of product is essential in manufacturing. This becomes a paramount consideration in mass production. Uniformity is dependent upon the uniformity in degree of control over the flow of energy applied to the material. Individual quality can be left to the artisan; this, however, is not manufacturing. The trend today is toward both quality and uniformity in manufactured products. This is the result of the machine age in which the power of the individual to create is multiplied many fold by exact control over a flow of energy, and by means of which can be obtained a product that combines the art of the artisan and the uniformity required for mass satisfaction. This ideal is approached as exact control is obtained over the entire flow of energy that is employed. Hence the value of electric heat to supplement the electric motor to that end.

The range of the application of electric heat in manufacturing is wide—perhaps doughnuts to steel ingots is not sufficiently inclusive. This breadth of conception however should be kept within the field of use in which the return justifies the means. Electric heat is not a universal substitute for heat obtained by combustion. Broadly speaking, the field of electric heat in manufacturing is that suggested by the preceding paragraph: *i.e.*, for production where uniformity and quality are the important considerations. As examples to illustrate this classification we may name a few common manufactured products, such as alloy steels, non-ferrous alloys, glassware, vitreous enamel products, automobiles, machine parts, motors, tools, shoes, newspapers, bread, etc. Aside from the elements of uniformity and quality, considerations of convenience, speed, and hygiene are often controlling factors which lead to the selection of electric heat.

Progress in manufacturing is in a large measure the result of the transfer of the exact methods of the laboratory to the shop. This transfer is made a practical procedure in heat applications through the use of electric heat. The properties of materials as affected by heat follow definite laws in which time and temperature are primary factors. Electric heat has in itself no temperature limitation, low or high. Whatever temperature is needed is available, the upper limit in any case being the maximum temperature which the material being heated—including its container—can stand without injury.

In addition to raising the temperature of a material, it is frequently the case that there is need for controlling or utilizing the effects of the contiguous atmosphere on the material being heated. An individual characteristic of electric heat is its independence of the atmosphere. Hence the atmosphere of an electric heating chamber can be made suitable to the need and be made a participating or non-participating agent in the heat process as desired. This feature materially extends the practical possibilities in heat application and gives to every-day procedure in the shop a new freedom in the utilization of natural laws.

The use of electric heat requires a certain readjustment of both ideas and methods. Steinmetz's forecast of the electric motor reads: "The introduction of electric power in place of other forms of power rarely can be a mere substitution but usually requires a change of methods of power application, a reorganization of industry to secure maximum economy." That reorganization was accomplished easily and swiftly. A similar reorganization in the use of heat in manufacturing is now taking place, more slowly than was the case with the electric motor but as certainly for electric heat is demonstrating its value to that industry.

Switching Induction Regulators Into and Out of Service

Voltage Surges Commonly Encountered—Arrangement of Switching Equipment for Feeders—Means of Interrupting Exciting Current—Applications of Small Oil Circuit Breakers—Switching Sequence

By C. L. GRIFFETH and R. C. HARDY
Central Station Engineering Department, General Electric Company

WHEN interrupting the exciting current of a highly inductive circuit, as for example the field of a large motor or generator, a voltage "kick" or surge is often observed. In some types of apparatus this induced voltage is sufficiently high to be a source of danger to the operator as well as to the machine winding. Inductive voltage surges are likewise observed when switching induction feeder regulators.

The object of this article is to treat briefly the cause of the voltage surge at times of switching single-phase regulators and to suggest the proper sequence of operations when regulators are placed in, or removed from, service.

A brief theoretical consideration of the behavior of a self-inductive circuit such as that of an induction regulator will be helpful. The voltage induced in the shunt or excitation winding of the regulator obeys the relation expressed by the equation

$$E=N\frac{d\phi}{dt}$$

That is to say, with a fixed number of turns (N), the induced voltage (E) is proportional to the time rate of change of flux. In steady-state operation this change of flux is sinusoidal with respect to time; the fundamental circuit frequency giving, of course, an induced or counter e.m.f. substantially equal to the impressed voltage.

When a regulator is disconnected from the line by the opening of a switch it is quite possible that the contacts may be made to part when the flux in the magnetic circuit of the regulator is at a maximum. Likewise, the breaking of the switch contacts may be quite rapid and the introduction of resistance into the circuit very fast. If such is the case, the flux will decay at a high rate, and the induced e.m.f. will be high. In fact, it may be many times the normal induced voltage.

This high induced voltage is exhibited in particular when the exciting current of a regulator is interrupted by opening an air break or disconnect-type switch, at which time a tenacious flaming arc may be observed. This initial arc in turn may even result in a flashover and a destructive power arc.

The induced voltage caused by interrupting the exciting current is much more severe in the case of a regulator than with a transformer of comparable kv-a. rating because of the presence of the air gap and the greater stored magnetic energy in the regulator excitation circuit. However, it is not proposed to suggest schemes for reducing the magnitude of these potential surges, since all connected apparatus and buses should be liberally insulated to withstand this strain. This presentation will consequently be limited rather to describing an arrangement and a sequence for switching regulators which is of more importance in avoiding flashover, arcing to ground, and hazard to operators.

Most operating companies prefer to use single-phase regulators in order to obtain individual phase regulation. In such cases the series winding is connected in the conductor of one phase and the shunt winding between this and an adjacent phase or neutral conductor.

The relative arrangement of the substation equipment according to the practice of several large operating companies is shown in the one-line diagrams given in Figs. 1 to 12 inclusive.

The typical arrangement shown in Fig. 1 is used when no provision is made for removing the regulators from service without also de-energizing the feeder. With the addition of by-pass switching equipment, as indicated in Fig. 2, it is possible to disconnect the regulator without interrupting service, but the exciting current must be broken on the blades of air break switches. In a preceding paragraph it was indicated that this practice may result in flashover. Moreover, such flashovers have actually occurred in service. This danger may be avoided by the addition of an oil circuit breaker in the shunt circuit of each regulator as shown in Fig. 3.

Cases have been reported where the breaking of the regulator exciting current by the parting of the primary stab contacts of a truck-type switch has given rise to flashover. The possibility of such failures may be eliminated by inserting an oil circuit breaker in the shunt circuit of each regulator. This arrangement of substation equipment will be seen upon reference to Fig. 4.

When a transfer bus is available, as in Figs. 4 to 11 inclusive, it is always possible to break the excitation current on an oil circuit breaker in the feeder. This current may be interrupted by the feeder selector oil circuit breaker, when such is available as in the cases shown in Figs. 6 and 7. If selector breakers are not installed, and it is not desired to open the exciting current on the feeder breakers, then an oil circuit breaker must be inserted in the shunt circuit of each regulator as shown in Fig. 12.

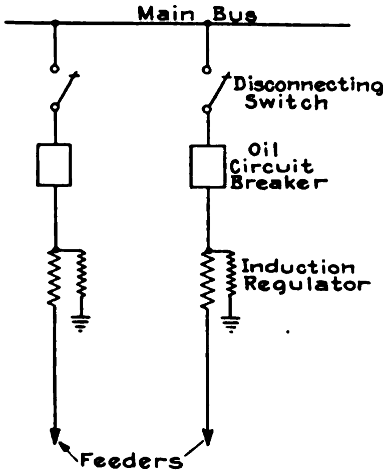

Fig. 1. A Single-bus Radial Feeder Circuit Containing Induction Regulators but Without Special Switching Equipment for Bringing Them Into and Out of Service

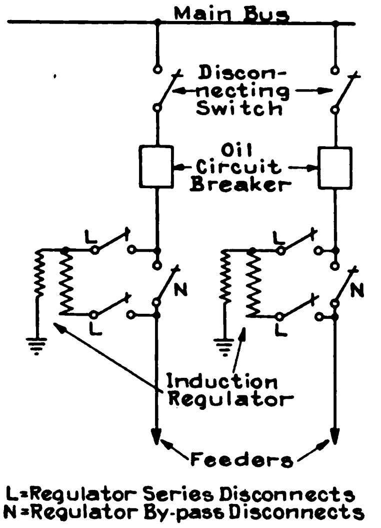

Fig. 2. A Regulated Feeder Circuit Having a By-pass Arrangement for Interrupting the Exciting Current with Disconnecting Switches

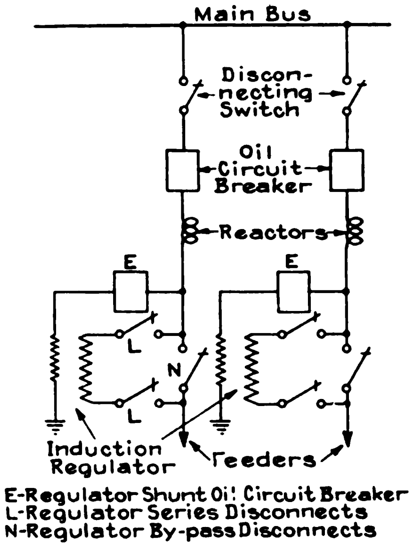
Main Bus
Discon-necting Switch
Oil Circuit Breaker
Reactors
E
E
L
N
L
Induction Regulator
Feeders
E-Regulator Shunt Oil Circuit Breaker
L-Regulator Series Disconnects
N-Regulator By-pass Disconnects

Fig. 3. A Regulated Feeder Circuit Provided with By-pass Switching Equipment in which Exciting Current is Interrupted by an Oil Circuit Breaker

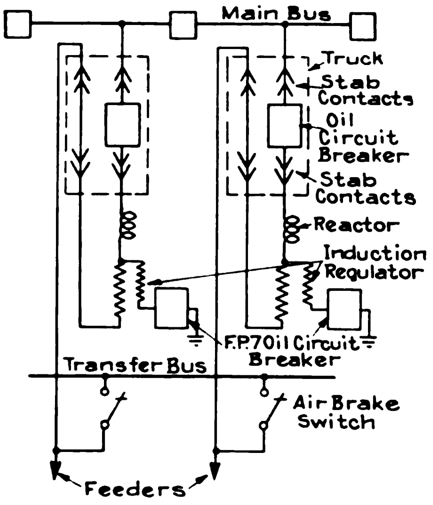

Fig. 4. Main and Transfer Buses of a Circuit Employing Truck-type Breakers with Regulators Having Oil Circuit Breakers in Exciting Circuits

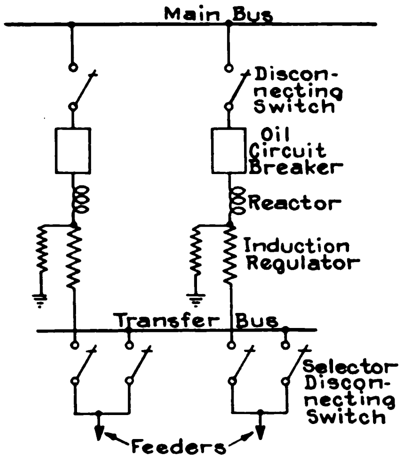

Fig. 5. Main and Transfer Buses of Regulated Circuit Employing Selector Disconnecting Switches in the Feeders

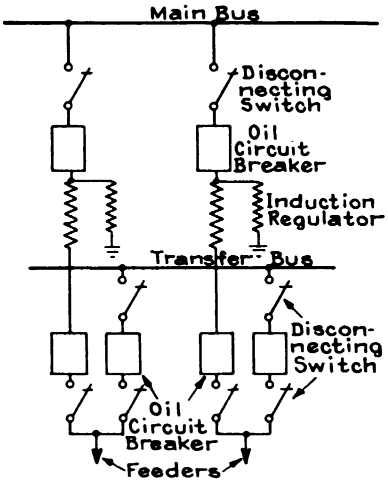

Fig. 6. The Circuit of Fig. 5 Supplemented by Oil Circuit Breakers in the Selector Switching Equipment of the Feeders

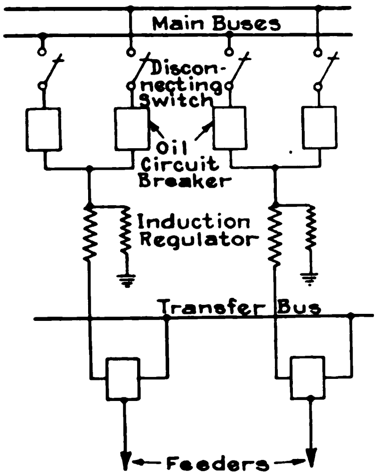

Fig. 7. Feeders Supplied Through Selector Oil Circuit Breakers and Regulators from Two Main Buses

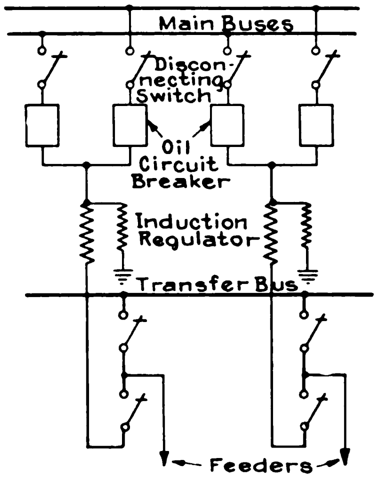

Fig. 8. Feeders Supplied from Two Main Buses Employing Regulators with Selector Equipment Consisting of Disconnecting Switches

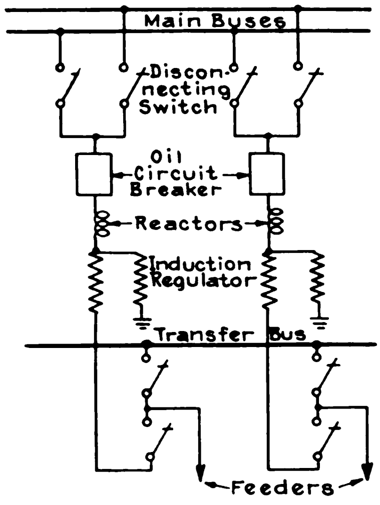

Fig. 9. A Modification of the Selector Switching Equipment for the Regulated Circuit Shown in Fig. 8

The foregoing remarks indicate the desirability of interrupting the exciting current by a switch which will not itself break down under repeated high-voltage discharges and which will not allow the inductive arc to cause flashover. The feeder oil circuit breaker will, of course, take care of this duty satisfactorily. It should always be used as a means of breaking the exciting current wherever possible, as when transferring the feeder to the transfer bus. With some arrangements, however, it is either inconvenient or impossible to interrupt the exciting current by the feeder breaker; and in such cases, as already pointed out, several companies have installed a small manually-operated non-automatic oil circuit breaker in the shunt circuit of the regulator.

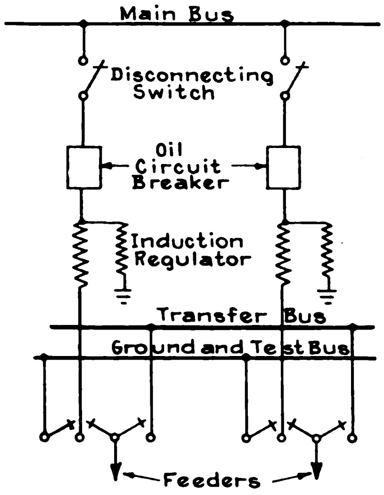

Fig. 10. A Regulated Feeder Circuit Consisting of Main, Transfer, Ground and Test Buses

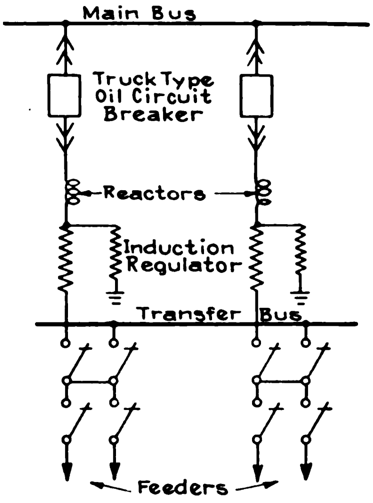

Fig. 11. The Circuit of Fig. 5 Equipped with Truck-type Breakers and Provision for Multiple Feeders

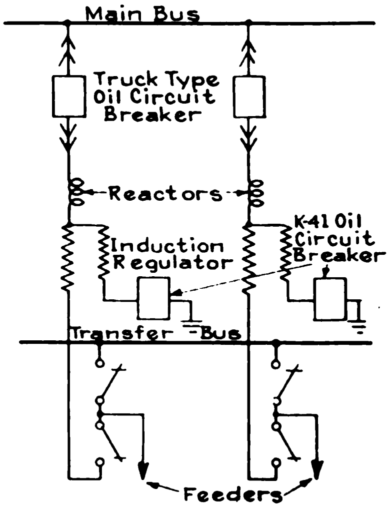

Fig. 12. Main and Transfer Buses with Truck-type Feeder Circuit Breakers and Selector Disconnecting Switches. Oil circuit breakers are employed in the regulator exciter circuit

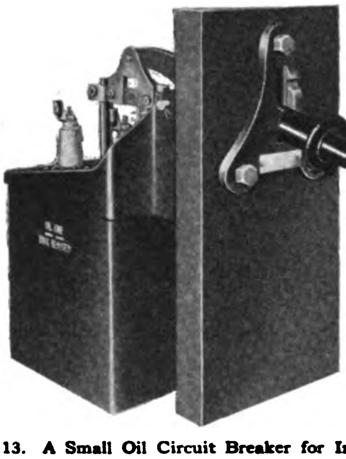

Fig. 13. A Small Oil Circuit Breaker for Indoor Mounting which May be Utilized for Interrupting Regulator Exciting Current as Shown in Fig. 15

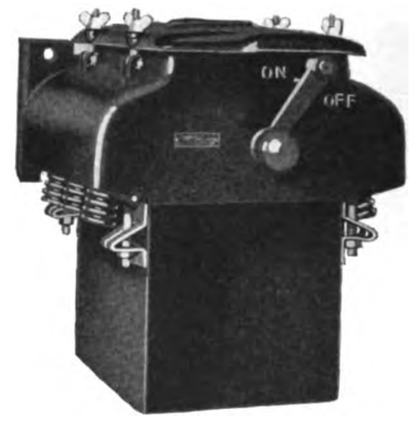

Fig. 14. A Small Outdoor-type Oil Circuit Breaker Suitable for Use in Installations Similar to That Shown in Fig. 16

Either of the oil circuit breakers of the types shown in Figs. 13 and 14 is suitable for this service and both have been satisfactorily applied. Both are designed to withstand such high voltage surges as result from interruption of regulator exciting current. They are of simple, rugged design, the difference between them being mainly in the manner of mounting.

In Fig. 15 is shown one of these breakers in the shunt circuit previously described and which is shown in Fig. 12. Here a breaker is located on the right of each regulator panel. An application of the other type appears in Fig. 16, where the switch is mounted adjacent to the truck-type feeder breaker in accordance with the scheme of connections shown in Fig. 4. The function of this switch is to break the exciting current of the regulator before the truck is removed from the housing and thus avoid the arc which might otherwise follow the parting of the primary stab contacts. The vertical rod shown is connected to an operating handle which is mechanically interlocked with the adjacent truck to prevent rolling out the truck, on which the feeder breaker is mounted, until the small breaker is open and to prevent rolling in the truck until this breaker is closed.

Some operating companies have arranged safety interlock schemes so that the only possible way of interrupting the exciting current is by opening the oil circuit breaker in the exciting circuit. The interlocks also prevent placing the regulator back into service until this breaker is closed. The latter precaution is necessary to avoid accidentally placing the series winding of the regulator in service without excitation on the shunt winding.

The sequence of the operations to be performed when switching induction regulators, connected with buses and switches according to either of the two schemes more commonly followed, as shown in Figs. 3 and 5, will now be outlined.

Switching Manipulations

When a transfer bus is not installed in the substation, as indicated in Fig. 3, the feeder regulator may be removed without interrupting service by means of by-pass switching. With this method of taking the regulator out of service, throw the regulator control from automatic to non-automatic and run the regulators to the neutral position. These settings may be checked by inspecting the mechanical position indicators on the regulators. Close the by-pass disconnecting switches, thus taking practically all load off the regulators. Then open the series disconnecting switches, thereby isolating the series windings. Finally interrupt the regulator exciting current by opening the oil circuit breaker in the shunt winding. An oil circuit breaker, rather than disconnecting switches, should be employed to interrupt the exciting current for the reasons previously enumerated.

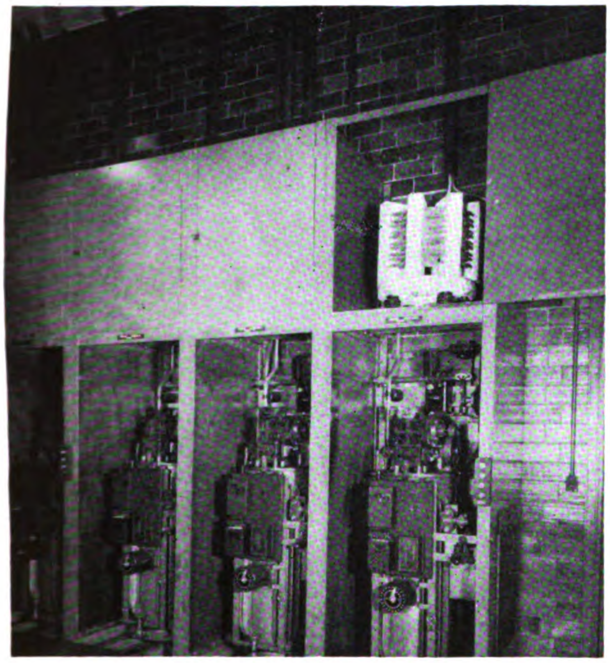

Fig. 15. An Installation of Regulators in Which the Switching is Made Through the Arrangement of Breakers Shown in Fig. 12

Failure to place the regulator in the neutral position causes circulating current to flow when the by-pass disconnecting switches are closed.

Before isolating the regulator with by-pass switching it is important to be sure that the regulator is carrying only normal connected load. An indication of overload might possibly be due to a short-circuit or ground in one of the single-phase regulators, in which case the operator should not of course open the series circuit by disconnecting switches. Where there is any doubt, the safest plan would be to forfeit continuity of service and open the feeder breaker.

In case a transfer bus is provided, as in Fig. 5, to switch a feeder regulator out of service, first energize the transfer bus, then throw the regulator control from automatic to non-automatic operation for this change-over and run the regulators to the same relative position to equalize voltages. These settings also may be checked by the mechanical position indicators on the regulators. Close the transfer selector disconnecting switches, thus dividing the load between the two buses. Open the feeder selector disconnecting switches, which removes all load from the feeder regulator. Finally, interrupt the regulator exciting current by opening the feeder oil circuit breaker. If selector oil circuit breakers are installed in addition to selector disconnecting switches, as shown in Fig. 6, the exciting current to the feeder regulator may be interrupted by the feeder selector breaker.

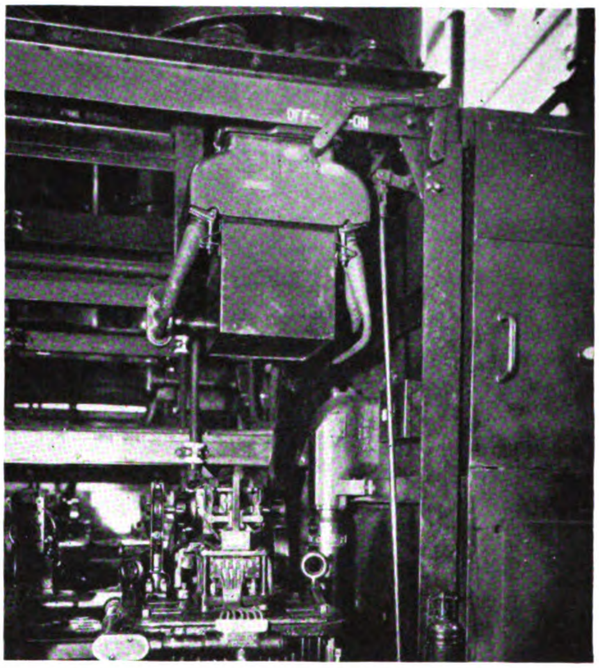

Fig. 16. The Mechanically-interlocked Equipment for Regulator Switching Carried Out by Means of the Breaker Arrangement Shown in Fig. 4

To place an isolated feeder regulator back in service with either of the arrangements described (Figs. 3 and 5) the procedure is the reverse of the operation and sequence of each step in its removal except that the voltages of transfer and feeder regulators must be equalized before paralleling the main and transfer buses. If the by-pass switching scheme is used, care must be taken to close the shunt circuit of each regulator to furnish excitation before placing it in service; otherwise the series windings may act as a high impedance reactor and a high voltage may be induced in the shunt winding.

Liquids as Insulators

Liquid Dielectrics as Protection to Other Insulators—Selection of an Oil Dielectric—Sludging—Physical and Chemical Changes in Oil Under Various Conditions—Electrical Properties—Factors Relating to Dielectric Strength

By **FRANK M. CLARK**
Physicist, Pittsfield Works, General Electric Company

Of the various liquids, oils possess the best combination of those properties which are necessary in an insulator of this type. Considering the great variety of oils, it is rather startling to learn that they all exhibit practically the same dielectric strength when in the pure state. This fact, however, does not make them of equivalent value as insulators, because some retain their original properties better than others when in this service.—EDITOR

LIQUID dielectrics, as might well be expected from a study of their physical properties, bear strong resemblance to gaseous dielectrics whose characteristics have already been described.(1) Thus, for example, as electric stress is applied between conductors properly spaced in oil, luminous discharge will finally be noted at a certain voltage whose value is dependent upon the experimental testing conditions. This luminous discharge is really a spark-over phenomenon in which the oil itself functions as one of the electrodes. Unlike in air, the discharge appears to begin quite suddenly and to extend farther out from the electrode surface. Like in air, however, its formation depends upon the electrode spacing. Under certain conditions of spacing, actual spark-over may occur without preliminary corona formation.

The behavior of gaseous dielectrics was discussed largely from the standpoint of air.(1) In a similar manner, the discussion of liquids will be illustrated by means of data obtained from petroleum oils.

Many classes of electrical apparatus depend for their insulation on liquid and solid materials. The usefulness of the latter varies inversely as the water content. Since most solid insulators are hygroscopic in nature, they must, before use, be permanently freed from water even in minute amounts. Cellulose materials form the backbone of solid dielectrics, as does air in the case of gaseous and petroleum oils in the case of liquid insulators. The use of cellulose products places great emphasis upon the importance of the problem of protection from atmospheric moisture. These materials constitute one of the best drying agents known, showing an affinity for water even greater than that of calcium chloride or sulphuric acid, and being exceeded as dehydrating agents only by phosphorous pentoxide. After having been dried to a constant weight in an air oven at 100 to 110 deg. C., and again brought into contact with atmospheric conditions, cellulose will almost immediately take up sufficient moisture to produce equilibrium with the prevailing humidity. Some protection is therefore demanded for cellulose materials which are to be used as electrical insulators. It is here that liquid dielectrics play an important role. The best liquid dielectric is that one which, other properties being equal, possesses the least affinity for water and water vapor. Petroleum oils, among other liquids, possess such characteristics and occupy a position of extreme importance in relation to high-voltage dielectric phenomena.

Various Types of Insulating Oils

Insulating oils may be of animal, vegetable, or mineral origin. Perhaps the most widely used oils are the mineral oils obtained in the distillation of petroleum. These oils possess, to a marked degree, the water-proofness and all-round desirable characteristics required of liquid dielectrics.

It was formerly true that insulating oils were considered as oils and nothing more. Any cylinder oil of the viscosity demanded was considered good enough for use without special treatment. It took but little practical experience, however, to indicate the wide difference in requirements for a good lubricating oil, as compared to those which are absolutely indispensable for oils to be used for electrical insulating purposes. To-day, the latter are taken from carefully selected stock, scientifically treated to bring out the desirable characteristics, and carefully protected during transportation from the oil refinery to the electrical machine in which they are ultimately used.

Oils other than those of petroleum origin are in more or less common use at the present time. Among these may be mentioned the fish oils, such as sperm oil, cod-liver oil, shark-liver oil, etc.; the vegetable oils, such as cotton-seed oil, linseed oil, and castor oil, the various nut oils, China wood oil, etc. Each has its own advantages and each possesses approximately equal dielectric strength when pure. As shown by Table I the other electrical properties may vary to a great degree.

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(1) "High-voltage Dielectric Characteristics of Gaseous Insulators," by Frank M. Clark, GENERAL ELECTRIC REVIEW, March, 1925, p. 158.

Ideal Liquid Dielectric

The ideal liquid dielectric would possess the following variety of exacting properties:

Chemical Properties

(1) High resistance to chemical oxidation.
(Chemical oxidation produces increased acidity, which results in lowered insulation resistance and increased power-factor.)

(2) Oxidation products when formed must be soluble.
(The formation of insoluble materials as a result of oxidation leads to a precipitation which decreases the surface resistance of the insulation on which the deposit occurs. In addition, the dissipation of heat by radiation is retarded. This action, in electrical machines such as transformers, is of primary importance.)

(3) Chemically inert to metals and organic materials.

(4) No solvent action on or chemical affinity for water vapor.

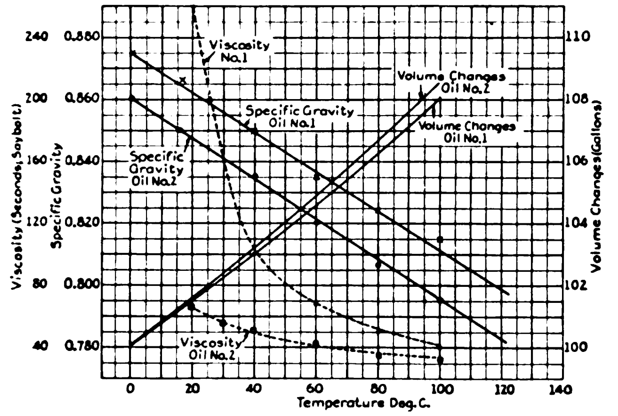

Fig. 1. The Effect of Temperature Upon Volume, Specific Gravity, and Viscosity of Mineral Oils

(5) Absence of mineral matter.
(The presence of mineral matter such as sulphur, although having little if any effect upon the dielectric strength of the liquid dielectric, leads eventually to a corrosive action upon the material of the metallic conductor.)

(6) Absence of inorganic and organic acidity.

(7) Chemically inert to voltage stress.

TABLE I

CHARACTERISTIC PROPERTIES OF TYPICAL INSULATING OILS

Material	Permittivity	Resistivity 500 v. d.c. 100 deg. C. ohms per cc.	Power-factor Bridge Method 100 deg. C.	Dielectric Strength; kv. 25 deg. C. 0.1-in. Gap
Mineral oil.....	2.23	21.0 $\times 10^{12}$	0.0004	30–40
Whale oil......	3.05	0.032 $\times 10^{12}$	0.0015	30–40
Linseed oil.....	3.3	0.61 $\times 10^{12}$	0.0027	30–40
Castor oil......	4.7	0.066 $\times 10^{12}$	0.0070	30–40
Cotton-seed oil..	3.2	0.01 $\times 10^{12}$	0.0005	30–40
China wood oil.	3.2	0.08 $\times 10^{12}$	0.0090	30–40

Physical Properties

(1) Low pour point.
(This is of especial importance with those liquid dielectrics which are subject to low temperatures in their practical application.)

(2) High flash point and burn point.

(3) Non-volatile.

(4) High specific heat.

(5) In general of low viscosity.

(6) Homogeneous and free from suspended matter.

Electrical Properties

(1) High electrical resistance.

(2) Low power-factor.

(3) High dielectric strength.

(4) Low temperature coefficient of electrical properties.

(5) Electrical properties unaffected by voltage application.

While no liquid dielectric possesses all these properties of the ideal case it is nevertheless true that in those oils at present used in transformers, switches, capacitors, cables, and the like a combination of properties has been obtained, as the result of extensive researches, endowing the oil developed with properties highly adapted for the apparatus in which it is to be used.

Temperature Effects

The immediate result of heating liquid dielectrics is observed in volumetric changes. For mineral oils the coefficient of expansion varies with the oil viscosity and specific gravity. For a Saybolt viscosity of 87 sec. at 37.5 deg. C. the coefficient of expansion is 0.00081; for a viscosity of 40 sec., it is 0.00089. The relation between temperature on the one hand and observed changes in specific gravity, viscosity, and volume of transil oil on the other, is shown in Fig. 1.

With the continued application of heat, the flash and burn points of the oil are approached. With the more viscous materials these are of high value. Switch oils having a viscosity at 37.5 deg. C. of 100 sec. (Saybolt), for example, possess a flash point at 152 deg. C., and burn at 170 deg. C. as compared to a lighter oil having a viscosity of 53 sec. and a flash point and burn point at 140 and 155 deg. C. respectively.

With oils of low flash and burn values, the use of "snuffers" has been investigated. The use of only 5 per cent of halogenated hydrocarbons has been found to produce a beneficial effect. The use of such materials, however, is limited because of their highly corrosive characteristics.

On long continued heating a variety of effects is noted. Poppy seed, linseed, castor, and in fact most of the vegetable oils, thicken, become dark and in some instances produce a scum over the surface as a direct result of oxidation, the speed of which is promoted by the presence of the unsaturated bodies present in the oil. The formation of surface scum on such oils as castor and linseed furnishes a protective coating for the lower layers. Oils which thicken without scum formation oxidize rapidly throughout.

China wood oil, although practically colorless when pure, soon darkens and acquires a peculiar, strong odor. At 200 deg. C., an insoluble, jelly-like substance is formed, which formation is accompanied by a marked decrease in the iodine number. The gelation, however, can be retarded by the presence of a scum-forming oil such as linseed.

Mineral oils when subjected to long continued heating at 100 deg. C. exhibit progressive changes in properties. Prominent among these changes is a continual increase in the acid number. The rate of increase in the acid number varies considerably from one type of oil to another, and even shows considerable difference among oils of the same class. Typical changes in the acidity of oils subjected to heat treatment in an air oven at 100 deg. C. are shown in Fig. 2. Here the changes in linseed oil, a typical vegetable oil, are compared with the changes in two typical mineral oils, the one a highly refined oil and the other 10-C transil oil.

The selection of an oil dielectric for use in electric machines must be determined primarily by a complete review of the conditions to which the oil will be subjected in practice. For example, oils operating under high stress and exposed to air must have high resistance to chemical oxidation, since the electrical resistivity is progressively lowered as the acidic oxidation products accumulate.

On the other hand, in practically all electric machines the ready dissipation of heat by conduction, radiation, and convection is absolutely fundamental to the life of the apparatus. In such cases the use of an oil which will precipitate an insoluble material, as a result of oxidation, must be avoided since the dissipation of heat is greatly retarded by the presence of such "sludge" materials.

Sludge may be described as consisting of those insoluble materials which are precipitated from solution when an oil is subjected to conditions favoring oxidation. Despite considerable research, the problem of oil sludging, its detection and prevention, is still unsolved. Various theories have been suggested. The decrease in the iodine and bromine value of oils during the sludging process, accompanied as it is by increased acidity, viscosity, and specific gravity, at once suggests a molecular polymerization and oxidation resulting in the formation of organic acidic materials of high molecular weight. This explanation has been repeatedly attacked. Wolff,(2) for instance, has devoted considerable study to the changes in oil during a heat run and claims that such an explanation is not warranted by the facts observed. When the oil is subjected to oxidation, it is true that a decrease in the iodine and bromine value is observed together with an increase in the viscosity, but Wolff's experiments indicate that such changes are not concurrent. The bromine number may reach a minimum even before any material change in viscosity occurs. On the other hand, it is claimed by Wolff that the resultant viscosity changes are typical of those characteristic of colloidal materials. This suggests that the viscosity increase is due to colloidal phenomena and not to polymerization. In fact, the molecular weight researches of Wolff indicate no possibility of polymerization. The decrease in the bromine value (which all researches reveal) is said to be due to the grouping of the colloidal materials. This occurs in such a way as to effectively block bromine or iodine addition.

The drying of China wood oil lends support to the theories advanced by Wolff. The process is essentially one of oxidation, but the accompanying changes in specific gravity, bromine and iodine numbers, and viscosity are primarily of a colloidal and physical-chemical nature. Thus China wood oil, according to the results of Eibner(3) and others, shows considerable oxidation effects after six months' exposure. Molecular weight determinations, however, show that the oxidation is accompanied by a splitting rather than a polymerization process.

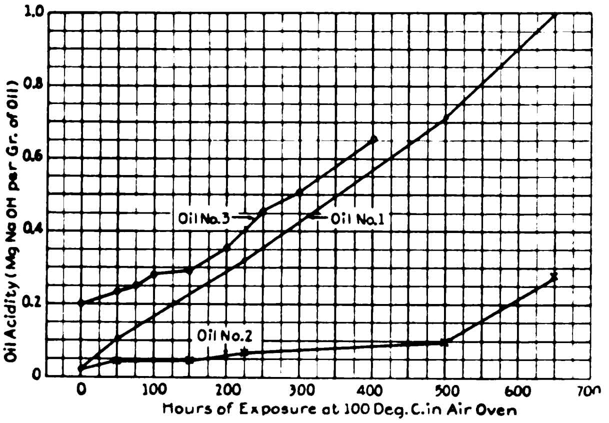

Fig. 2. The Increase in the Acid Number of Oils Subjected to Long Continued Heating

Oil No. 1—A highly refined mineral oil.
Oil No. 2—10-C transil oil.
Oil No. 3—Raw linseed oil.

With linseed oil, a study of the iodine number, density, and molecular weight, shows that the oxidation is accompanied by a polymerization which is probably intra as well as extra molecular. This polymerization is slow below 100 deg. C. but becomes marked above 200 deg. C. Agents such as carbon tetrachloride tend to retard the polymerization process.

TABLE II

CHANGING PROPERTIES OF MINERAL OILS DURING A HEAT RUN

Properties	Before Heat Run	After Heat Run
Color	Colorless	Light red
Acidity	0.02 mg. NaOH per gr.	0.15 mg. NaOH per gr.
Power-factor (100 deg. C.)	0.0004	0.0028
Resistivity (100 deg. C.)	21.0×10^{12}	0.3×10^{12}

It has of course long been known that the fast-drying vegetable oils, such as linseed, possess high iodine values, denoting a considerable degree of unsaturation. It is reasonable to believe that the presence of these unsaturated bonds accounts for the great affinity of these oils for oxygen. Coffey (4) has assumed the complete oxidation of these bonds, and furthermore inclines toward the belief that a selective oxidation reaction occurs.

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(2) Wolff: "Polymerization of Drying Oils." *Zeit. Angew Chem.*, *37* (1924), p. 729.

(3) Eibner *et al.*: "Polymerization and Oxidation of Oils." *Chem. Umschau*, *31* (1924), p. 69. Eibner and Wiblietz: "Linseed and Poppyseed Oils." *Chem. Umschau*, *31* (1924), p. 109.

The fundamental chemical reactions during the oxidation of the fast-drying oils, such as linseed, involve the absorption of oxygen at the point of unsaturation in the molecule. The peroxides so formed are catalysts and further extend the oxidation, resulting in the formation of organic acids and lactones, or the intermediate formation of aldehydes. These, if formed, are easily capable of condensation and polymerization, especially in the presence of basic oxides.

The heat-acidity relationship of oils, especially in the case of the unsaturated organic oils is distinctly of a catalytic nature. Rutt, Watkins and others (5) have shown the effect of catalytic materials in the oxidation of paraffin wax. This material was heated at 208 deg. C. under a vacuum for 509 hours. It was then exposed to air at 100 deg. C. for 48 hours, giving a product containing 9.9 per cent oxygen. The wax so treated was then extracted with methyl acetate and re-crystallized. It was then heated in air at 100 deg. C. for 457 hours and a product containing only 0.7 per cent oxygen was obtained. Other examples could be cited. Thus, Grunback and Holm (6) observed that the oxidation of fats was catalyzed positively by the formation of the unsaturated acids during the process, and negatively by the presence of water and water vapor. The search for an efficient negative catalyzer in the oxidation of electrical insulating oils has been the long-sought goal of the oil chemist and the electrical engineer.

Catalysts to the Oil Oxidation Process

The problem of chemical oxidation has long been a subject of careful research because of its paramount importance both in the field of pure and applied science. Its characteristics, in general, are well known and the influence of various materials upon the rate of the reaction clearly recognized. The presence of traces of water vapor, for example, plays a prominent part in the oxidation of metals, of phosphorous, and the explosive reaction between hydrogen and oxygen. Such reagents which change the rate of a chemical reaction are termed catalysts. Two classes of catalyzers are recognized. Those materials which assist the rate of reaction are called positive catalyzers. Those which retard the rate of reaction are called negative catalyzers.

Positive catalyzers for the oxidation of insulating oils are many. With the unsaturated organic oils, as has already been stated, the products of the oxidation act positively upon the rate of the reaction. The most widely known catalyzers of insulating oils, however, are the metals. Heyden and Typke, (7) for example, have recently confirmed previous experiments showing that copper and copper alloys, such as brass, rheotan, constantan, and the like, increase the rate of oxidation and sludge formation. Tin, nickel, and iron act likewise. Lead is attacked by the oil acids giving a low molecular weight material, insoluble in oil. This material is gradually acted upon, however, and redissolves. Lead, copper, and zinc are all attacked to a more or less extent by the oil acids.

A most interesting question is that of the negative catalyzer. Some metals are claimed as such, *i.e.*, zinc and aluminum by Heyden and Typke. To these may be added the weakly basic oxides such as those of magnesium and copper. The action of phenolic materials has been investigated by Krypsin and others. Apparently no retardation in the rate of the oxidation is caused by the use of pyrogallol, alpha naphthol, or meta cresol, using 0.1 gram per 30 grams of the oil, the amount of resin and asphaltic material being determined by the Kissling method. (8)

Detection and Measurement of the Oil Sludging Characteristics of Insulating Oils

A variety of tests for the detection and estimation of the oil sludging characteristics of insulating oils are now in use. None has as yet been found entirely satisfactory or widely accepted. The prevailing tests may be roughly grouped into two classes: the first consisting of those tests which are chemical in nature; and the second, those tests which might be described as accelerated life tests.

The chemical tests which were formerly in more or less general use are being, at least in this country, slowly superseded by the second type mentioned, the accelerated life tests. Among the more important chemical tests now being used are the Nastjukoff test, (9) the Kissling test, (8) the Michie test, (10) and the Waters carbonization test. (11)

The Nastjukoff (Formalite) test is merely the reaction of oils with formaldehyde in the presence of concentrated sulphuric acid. The insoluble product so formed is filtered, washed with petroleum ether, dried, and weighed, the weight being accepted as an estimate of the oil sludging characteristics.

The Kissling test consists of passing a stream of oxygen through oil at 120 deg. C. for a fixed period with or without the presence of a catalyzer. The acidic insoluble products are extracted with alcoholic sodium hydroxide and precipitated with light naphtha. The procedure varies considerably with different users. For general insulating oils, 30 grams of the material when blown for 60 hours at 120 deg. C. should not give a precipitate of more than 0.6 per cent.

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(4) Coffey: "Drying of Oils." *Jour. of the Oil and Color Chem. Assn.*, June, 1923.

(5) Rutt, Watkins *et al.*: "Velocity of the Oxidation of Paraffin Wax." *Jour. Chem. Soc.*, *125* (1924), p. 38.

(6) Grunbank and Holm: "Auto Oxidation of Fats." *Ind. and Eng. Chem.*, *16* (1924), p. 598.

(7) Heyden and Typke: "The Effect of Metals on Transformer Oils." *Pet. Z.* 1924–2320.

(8) Kissling: "Quantitative Determination of Oil Sludge." *Chem. Zeit.*, *30* (1906), p. 932; *32* (1908), p. 938; *33* (1909), p. 521.

(9) "Nastjukoff Formalite Reaction." *Chem. Zeit. 34* (1909) 893; *Pet.* 4 (1909) 1336.

(10) Nutall: "Sludge Tests for Transformer Oils." *World Power*, *1* (1924), p. 92.

(11) "Waters Carbonization Test." *Bur. Stds. Pub. No. 99* (1920).

The Michie test exposes an oil to air or oxygen at 150 deg. C. in the presence of copper for 45 hr. The oxidized products are precipitated with benzine, dried, and weighed. The asphalt or Michie number is influenced by the test factors which must therefore be rigidly controlled.

In the Waters carbonization test the oil is oxidized in air at 250 deg. C. for 2½ hours. The product is then extracted with petroleum ether, filtered, dried, and weighed. The result gives the so-called carbonization value.

Various other chemical methods have been used to estimate the sludging tendencies of oils. Among these may be mentioned reactions based upon the acid-forming properties of the oils, studies made of the iodine and bromine values, and chemical reactions involving the use of bleaching powder, metallic sodium, etc. None, however, has shown decided advantage and none has been widely accepted.

A method which is coming into prominence as a means of determining the sludging properties of oils is that which has been called an accelerated life run.(12) In this test, samples of oils are placed in a constant-temperature air oven at 120 deg. C. A typical oven devised for this test by the Pittsfield Laboratory of the General Electric Company is shown in Fig. 3. On an internal revolving platform, so arranged as to secure uniform heating throughout all of the samples, are placed beakers, each containing 500 cc. of the oil to be tested. The time at which the samples are placed in the already heated oven is noted and regarded as the start of the run. Each day thereafter the revolving internal platform is stopped and the samples are observed by momentarily lifting out a specimen of the oil in a small pipette. Changes in the color of the oils have been found to be excellent criteria as to the progress of the tests. As soon as any sample is found to be near the sludging condition, 10 cc. of it is removed and later centrifuged, the process being repeated daily thereafter. The time elapsing from the start of the experiment to the appearance of an insoluble deposit is taken as an estimate of the sludging characteristics of the oil under consideration.

Viscosity

Viscosity values play a large part in the selection of an oil for electric machinery. In many cases this problem is closely related to that of heat dissipation. In oil-filled transformers, for example, viscosity specifications are carefully set up and rigidly adhered to. The lowest viscosity compatible with a safe flash and burning point is demanded.

When a spark discharge passes through oil, carbon and other products are formed. The presence of this carbon considerably lowers the electrical properties of the oil. Its elimination is essential for the proper functioning of the dielectric in such apparatus as oil switches and the like. Other factors than viscosity must of course be considered, but it is evident that, other properties being equal, oil of lower viscosity will more easily allow the removal of the carbon by means of precipitation.

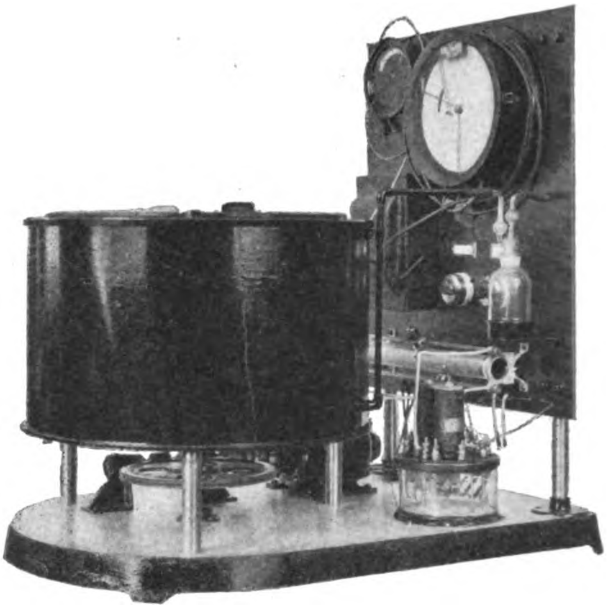

Fig. 3. A Constant-temperature Air Oven Used in Determining Sludging Characteristics of Oil Samples

Of the liquids used to impregnate solid insulation, those of low viscosity are best suited to thoroughly treat the material. Even after a satisfactory treatment, highly viscous materials suffer obvious disadvantages. With machines in operation, vibratory motion, or in the case of cables, bending, may easily cause the formation of voids in the space between solid dielectric laminations. It is the presence of these voids to which many instances of insulation failure can be traced. With the more fluid materials such as mineral oils, the formation of these voids is less likely to occur.

Fortsch and Wilson(13) have made a study of the temperature-viscosity relation for a variety of oils. Contrary to the work of Hershel,(14) their results point to a logarithmic relation only in the case of light paraffin oil. With oils of greater or less viscosity, this relationship does not hold. This is illustrated in Fig. 4.

Conductivity and Resistivity

When an electric potential is applied to a liquid dielectric, the current flowing is made up of two parts: absorption and conduction. The former slowly dies out. Liquid conductivity is greatly affected by the temperature, method of sampling, previous history of the material, incident light, etc. It seems however to be independent of the material of the electrodes. Schrader,(15) for instance, has found no difference in the conductivity of oil tested between electrodes of brass, copper, nickel, silver, or gold. Table III shows the resistivity of transil oil at several temperatures.

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(12) **Snyder:** "Sludging Tests for Transformer Oils." *Proc. A.S.T.M.*, 1924.

(13) Fortsch and Wilson: "Viscosity of Oils at High Temperature." *Ind. and Eng. Chem.*, *16* (1924), p. 789.

(14) Hershel: "Viscosity-Temperature Relation in Oils." *Ind. Eng. Chem.*, *14* (1922), p. 715.

TABLE III

TEMPERATURE-RESISTIVITY RELATION IN MINERAL OIL

Temperature (Deg. C.)	Resistivity (Ohms Per cc.)
30	250×10^{12}
60	100×10^{12}
100	15×10^{12}

Permittivity

The permittivity of organic liquids is dependent upon the presence of a polar carbon within the molecule. A polar carbon is a carbon atom to which is attached two or more different atoms besides carbon, or those to which one or more electro-negative atoms are combined. Liquids containing such carbon atoms are dipolar and possess increasing permittivity and high temperature coefficient. With two or more polar carbons, the molecular permittivity and temperature coefficient are mean values of the individual effects. Many unsuccessful attempts have been made to explain the magnitude of the permittivity of organic liquids upon a basis of the physical constants involved. The recent researches of Grim and Patrick,(16) for example, showed no definite relationship to exist between the permittivities of a large variety of pure organic liquids, and such properties as the surface tension, and latent heat of vaporization.

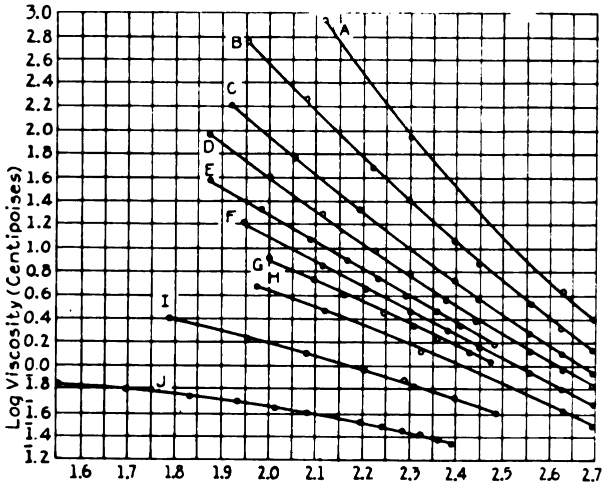

Fig. 4. Logarithmic Curves of the Temperature-viscosity Relation for Various Oils

Curve *A*—145 gravity M.C. residuum (*384 sec. at 210 deg. F.*)
Curve *B*—Heavy motor oil (*121 sec. at 210 deg. F.*)
Curve *C*—Red oil (*485 sec.*)
Curve *D*—Light motor oil (*219 sec.*)
Curve *E*—White oil (*113 sec.*)
Curve *F*—Light paraffin oil (*79 sec.*)
Curve *G*—Pressed distillate (*65 sec.*)
Curve *H*—Mineral seal (*44 sec.*)
Curve *I*—Kerosene
Curve *J*—Octane

Values in italics denote Saybolt Universal Viscosity of the oils taken at 100 deg. F. unless otherwise stated.

(Curves based on data of Fortsch and Wilson.)

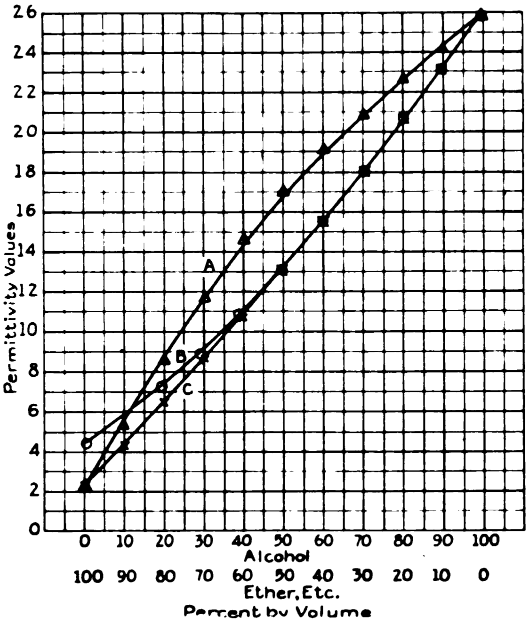

Fig. 5. Permittivity Values in Liquid Solutions at 25 Deg. C. and 1000 Cycles, by Bridge Method

Curve *A*—Carbon tetrachloride—alcohol mixture.
Curve *B*—Ether-alcohol mixture.
Curve *C*—Alcohol-benzene mixture.

(Curves based on data of King and Patrick.)

The permittivity and specific gravity of various liquid dielectrics is given in Table IV. No definite relationship seems evident.

The resultant permittivity of liquid mixtures varies continually and consistently as the composition of the mixture is changed from one extreme to the other. This is illustrated in Fig. 5.

TABLE IV

Liquid	Specific Gravity at 25 deg. C.	Permittivity at 25 deg. C.
Acetone	0.797	21
Ethyl alcohol	0.789	25.8
Aniline	1.022	7.1
Benzene	0.886	2.2
Castor oil	0.956	4.7
Cotton-seed oil	0.960	3.8
Linseed oil	0.930	3.1
Mineral oil	0.850	2.23
Paraffin	0.899	2.29
Shellac	1.173	3.7
Toluene	0.866	2.3
Vaseline	0.863	2.17
Wax, ceresin	0.917	2.4

The effect of temperature change upon the permittivity of liquid dielectrics varies considerably with the material investigated. In general, an increase in temperature results in decreased values. Thus water at 2.3 deg. C. gives a permittivity of 90.7, while at 99.5 deg. C. the value has fallen to 57.4. Aniline at 25 deg. C. has a value of 7.4, at 174.5 deg. C. only 4.19. Nitrobenzene at 18.6 deg. C. shows a permittivity of 35.6, at 203.9 deg. C. it is 15.35.

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(15) Schrader: "Conduction in Liquid Dielectrics." *Elec. Jour.*, *16* (1919), p. 336.

(16) Grim and Patrick: "Dielectric Constants of Organic Liquids." *J.A.C.* *45* (1923), p. 2794.

There is however no linear relation between temperature and the permittivity value. Isnardi(17) has found that with benzene the permittivity rises suddenly just before the melting point. With acetone, a linear increase occurs as the temperature falls. For glycerine, the permittivity increases rapidly as the temperature increases from −32 to −14 deg. C. after which a slow decrease occurs with a continual increase in temperature.

Waible(18) has investigated the relation between applied pressure and permittivity. With pressures varying from 1 to 130 kg. per sq. cm. (14 deg. C.), the effect may be summed up in the expression

$$E = E_0 (1 + aP + bP^2)$$

where E = permittivity at pressure P.
E_0 = permittivity at pressure of 1 kg. per sq. cm.
P = pressure applied.
a and b are constants.

Frequency changes seem to have but a negligible effect upon the permittivity value of good dielectrics. However, commercial dielectrics in many cases show a value which although approximately constant up to 10^4 cycles per second gives decreasing permittivity at higher frequencies.

Dielectric Loss

Dielectric losses in liquids increase with an increase in the applied voltage, though up to the point of breakdown no rapid increase occurs such as would be noticed if a large number of discharges took place. The insulating properties show no great change until the point of breakdown is reached. This agrees with the theory advocated by many physicists that the breakdown of a liquid insulator is a function of the ionization by collision phenomena, as with air, little or no loss resulting up to the point of failure. The conductivity which is always noted to a greater or lesser extent with mineral oils is generally accredited to the presence of water, dust fibers, and the like. Such conductivity, however, shows a somewhat slow and consistent growth with the increase in the applied voltage.

Quite different results are obtained with aerated oils, especially with the more viscous materials. Here, as the voltage is slowly increased the losses at some fairly definite point suddenly show a rapid increase, leading to a decided variation in the shape of the dielectric loss-voltage relationship. It is at this point that the gas bubbles in the material between the electrodes become ionized. Conduction losses therefore depend upon the elimination of water and solid impurities; and ionization losses upon the removal of gas bubbles. Losses resulting from gaseous ionization constitute an ever-present problem in the practical application of oil-treated fibrous insulation, especially in insulation subjected to relatively high voltage stress such as is found in transformers, cables, and capacitors. The complete removal of the last traces of air in such materials is possible only by means of a carefully regulated vacuum-heat treatment.

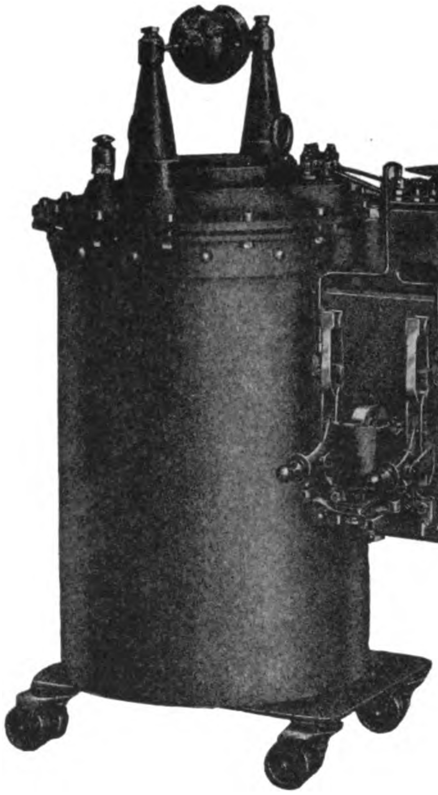

Fig. 6. A Testing Set for Determining the Dielectric Strength of Oils. The set consists of a transformer, a hand-operated regulator with a graduated voltage-indicating disk on the movable element, and a receptacle for the sample which surrounds the electrodes

The effect of frequency upon the dielectric losses in liquids is practicably negligible. This is entirely different from the effect of frequency upon the losses observed in solid dielectrics. It seems probable that the losses in liquids (when pure) are chiefly due to ionic conduction, in solids largely to hysteresis.

Dielectric Strength

Dielectric strength may be defined as that minimum voltage able to produce an arc between electrodes. A portable oil testing set, by means of which samples are readily tested, is shown in Fig. 6. The mechanism of the electrical breakdown of liquid dielectrics is considered by many as similar to that which has been well established for the electrical failure of gases, *i.e.*, an accumulation of ionized particles due to electronic or ionic collision effects, any experimental differences being traced to differences in the physical characteristics of the two materials.

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(17) Isnardi and Gans: "The Dependency of the Dielectric Constant of a Liquid on Temperature." *Zeit. f. Physik, 9* (1922), p. 1953.

(18) Waible: "Dielectric Constant and Pressure." *Ann. d. Physik, 72-3* (1923), p. 161.

Schulze (19) has suggested a somewhat different mechanism. According to this theory, gas formation is the fundamental cause of failure. All liquids contain ions, the kinetic motion of which is accelerated under an applied electric stress. This accelerated motion through the dielectric results in the generation of heat, which in turn generates a "gas trail." Other ions upon hitting these vapor paths cause ionization by collision as in gaseous materials. The accumulation of these ionization effects results in spark propagation. The dielectric strength will therefore depend upon the size of the ions involved, the viscosity of the liquid, the heat of vaporization, density of the vapor, the ionization potential of the vapor, etc. Whatever be the true explanation of the phenomena occurring in liquid dielectric failure, the effects produced by the variously shaped electrodes, by imposed pressure, by increased temperature, etc., must be adequately explained.

Strength-thickness Relation

Oil and other liquid dielectrics possess lowered dielectric strength in bulk. The strength falls off approximately as the square-root of the thickness of the layer tested ($\text{kv.} = AT^{\frac{1}{2}}$), using disk electrodes at 25 deg. C. (Fig. 7).

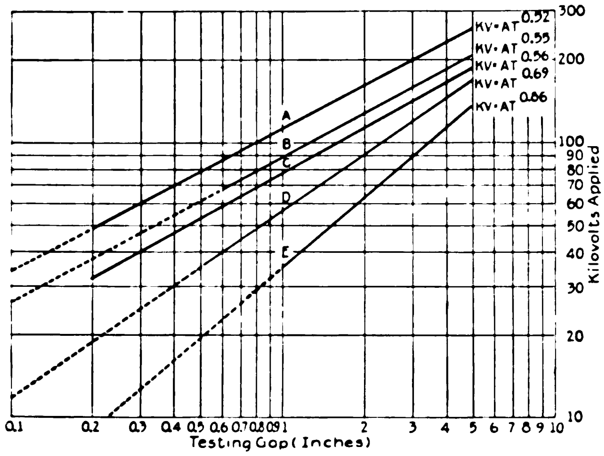

Fig. 7. The Strength-thickness Relation, Showing Its Variation as Electrodes of Different Shape Are Used

Curve *A*—4-in. disks.
Curve *B*—2-in. spheres.
Curve *C*—Blunt conical points.
Curve *D*—Needles.
Curve *E*—4-in. disk and needle.
(Curves based upon data given in Fig. 8.)

Effect of Electrode Shape

As the shape of the testing electrodes is changed from very sharp points to blunt needles, to spheres, and finally to flat disks, a great variation in the thickness-strength relation for liquid materials will be observed. Typical results are given in Fig. 8. The effects are produced by differences in the stress distribution occurring with the various shaped electrodes.

Effect of Electrode Size

W. Spath (20) has observed that with small gap settings (0.05 to 1.0 cm.) the strength of oil is independent of the diameter of the testing electrodes (disks) within the range used, which varied from a minimum diameter of 40 cm. to a maximum of 90 cm. With long gaps, the larger electrodes give the larger breakdown values probably through a better distribution of the electric stress. It has been estimated that the gap distance should not exceed a value greater than 10 per cent of the electrode diameter.

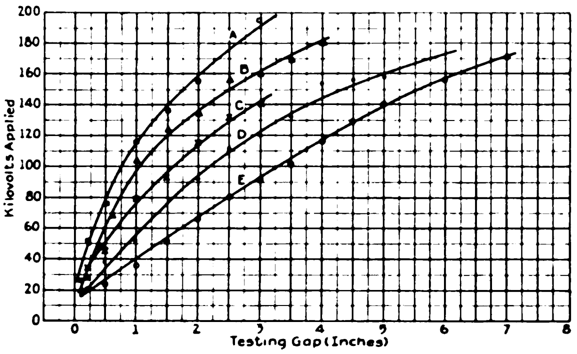

Fig. 8. Effects of Dimensions of Gap and Shape of Electrode Upon Dielectric Strength as Shown in Tests Made at Room Temperature, Using 60-cycle Sine-wave Voltage

Curve *A*—4-in. disks.
Curve *B*—2-in. spheres.
Curve *C*—Blunt conical points.
Curve *D*—Needles.
Curve *E*—4-in. disk and needle.

Effect of Temperature

The dielectric strength of oils is affected by the temperature of the test. As illustrated by Tobey (21) and by Hendricks, (22) the strength decreases with decreasing temperature to a minimum value which is reached at a point just before the actual solidification of the oil. On either side of this value an increase in the dielectric strength occurs. (Fig. 9.)

Dielectric Strength-Resistivity Relation

There seems to be no definite relation between the strength and the resistivity of oils. Those of low resistivity demand in many cases high voltage to puncture.

Dielectric strength and resistivity changes of mineral oil with temperature are given in Fig. 10. From the data in Table V, oxidation at 115 deg. C. under partial vacuum produces a drop in resistivity which is not accompanied by a change dielectric strength.

The Time Factor

The time for spark-over in gases is measured in terms of microseconds.* In liquids, the time factor is of greater importance although in no way comparable to its value in solid insulation. With unlimited time at low frequency, the voltage needed to puncture is of constant value. With transient voltages, however, greater applied stress is necessary. The effect of short-time voltage application on solid insulations seems cumulative.(23) With oil, no such effect is noted.

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(19) Schulze: "Dielectric Strength of Liquids and Solids." *Jahrb. Radioakt Elektronik, 19* (1922), p. 92.

(20) Spath: "Breakdown of Transformer Oils." *Arch. f. Elektrot 12.* (1923), p. 331.

(21) Tobey: "Dielectric Strength of Oils." *Jour. A.I.E.E., 29* (1910), p. 1189.

(22) Hendricks: "Transformer Oils." *Jour. A.I.E.E., 30* (1911), p. 327

*A microsecond is a millionth of a second.—Ed.

With long-time application, the breakdown values of oils at high frequency are considerably less than at 60-cycle voltages.

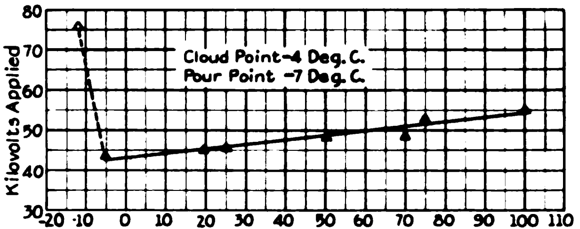

Fig. 9. The Variation of Dielectric Strength of Mineral Oil with Temperature

Specific gravity of the oil, 0.87.
Viscosity of the oil at 37.8 deg. C., 110 sec. Saybolt.

Effect of Pressure

The effect of applied pressure upon the dielectric strength of liquids bears marked resemblance to the effects produced in gaseous breakdown. An increase in pressure leads to increased strength, a 50 per cent increase resulting from a gauge pressure change from 0 to 200 lb. per sq. in. In gases, dielectric strength changes with applied pressure are explained by the changing values of the mean free electronic and ionic paths. The similarity in the effects of pressure application seems to further emphasize the possibility of a common cause of failure for both gases and liquids; *i.e.*, ionization resulting from electronic and ionic impacts.

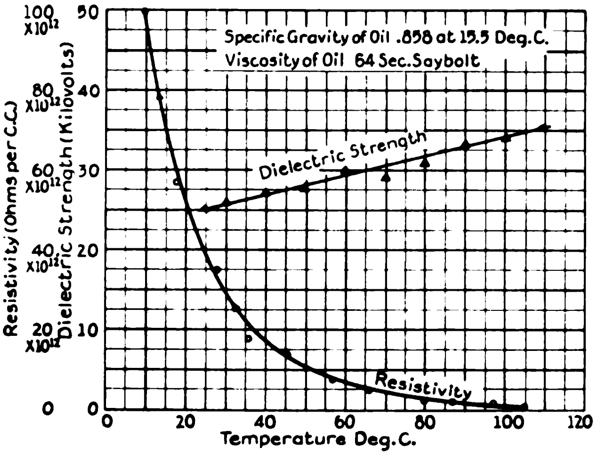

Fig. 10. The Relation of Dielectric Strength and Resistivity of Mineral Oils to Temperature

Dielectric strength tests with 1-in. disks 0.1 in. apart.
Resistivitiy tests between concentric cylinders at 110 volts alternating current.

Frequency Effects

With frequency from 30 to 111 cycles per second, Langsdorf(24) has found that breakdown occurs after a definite number of repetitions of stress. However, under the conditions of operating practice, the breakdown voltage of oil has been found to be independent of frequency.

Effect of Moisture

One of the greatest problems confronting the electrical engineer in the use of liquid insulating materials is the removal of moisture. It has been pointed out by several investigators(25) that the dielectric strength of mineral oil is greatly affected by the presence of only minute traces of water (Fig. 11).

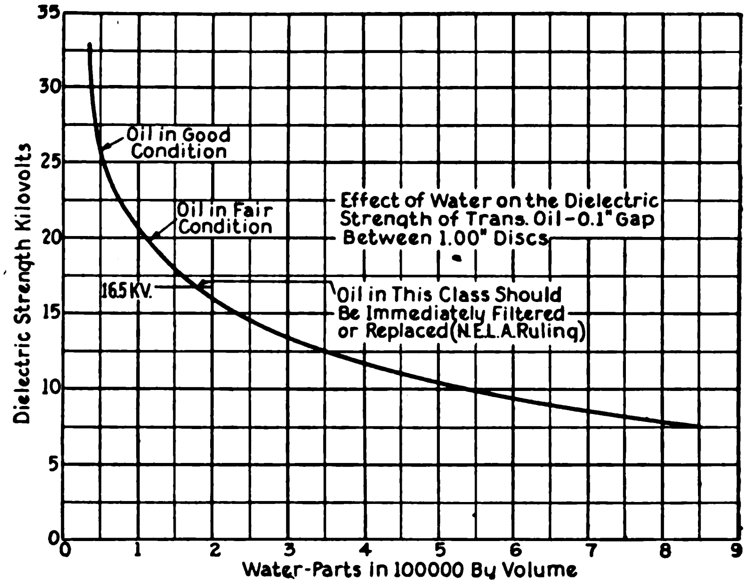

Fig. 11. The Dielectric Strength of Mineral Oil as Affected by Water

The satisfactory removal of water from mineral oils may be accomplished mechanically, or by means of chemical dehydrating agents such as calcium chloride, sodium, and the like. Common methods are by paper or centrifugal filtration by means of which large bodies of oil may be quickly and efficiently dried.

TABLE V

CHANGES IN RESISTIVITY AND DIELECTRIC STRENGTH OF MINERAL OIL

As Effected by a Heat Run of 72 Hours in an Iron Tank at 115 Deg. C. Under Vacuum

Mineral Oil	Power-factor Bridge Method 100 deg. C.	Resistivity 500 v. d.c.; 100 deg. C. Ohms Per cc.	Dielectric Strength. kv. 0.1-in. Gap; 25 deg. C.
Original......	0.0004	21.0×10^{12}	36
After heat run.	0.0028	0.3×10^{12}	36

A method has been suggested by Woog(26) for the detection of water in liquid hydrocarbons when present in amounts as small as 1 part in 60,000 (0.001 per cent). This method has been described as follows:

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(23) Peek: "Impulse Effects." *Jour. A.I.E.E.*, *34* (1915), p. 1695. Clark: "The Dielectric Properties of Fibrous Insulation." *Jour. A.I.E.E.*, *44* (1925), p. 3.

(24) Langsdorf: "Dielectric Strength-Frequency." *Elec. World*, *52* (1908), p. 942.

(25) Hirobe: "The Effect of Moisture on the Dielectric Strength of Oils." *Elec. Chem. Lab. (Japan) No. 25* (1916).

(26) P. Woog: "Surface Alteration in Glass Detected by High Voltage Currents." *Le Verre*, *4* (1924), p. 54.

> "The liquid was placed in a tall cylindrical vessel, standing on a metal plate. The top of the cylinder was closed with an ebonite strip holding through its center a brass rod which terminated in a brass sphere. The brass sphere and rod were centrally located in the liquid. A 25-cycle alternating-current voltage was applied to the metal plate and the brass rod. With sufficient voltage a discharge was obtained, the current flowing from the brass rod to the surface of the glass and over the glass surface to the metal plate. When the carefully dried cylinder was filled with anhydrous mineral oil the resistance was high. The moisture retained on the surface, however, was sufficient to conduct a current and a discharge was obtained. With a rise in temperature of the oil, lower discharge voltages were necessary. If on the other hand a liquid hydrocarbon containing a trace of water were used, the discharge voltage decreased with the temperature on cooling and then rose again on heating the liquid."

The explanation is given by Woog that the surface of the glass effloresces liberating basic materials by the hydrolysis of the silicates. The CO_2 of the air causes a thin film of Na_2CO_3. This salt will crystallize with four or five molecules of water at room temperature but on cooling will form the hydrate with ten molecules of water if water is available. The latter effloresces on heating if its vapor pressure is higher than that of the water vapor contained in the liquid.

Perhaps the largest use of liquid insulators occurs in connection with solid materials. The latter after a thorough drying process, generally under a vacuum at a temperature of 100 to 115 deg. C., are treated with the impregnating liquid. The function of the liquid and the solid have been frequently discussed. The liquid imparts to fibrous materials a greater dielectric strength and permittivity than originally possessed by the untreated solid, in addition to protecting it from atmospheric humidity effects. The solid on the other hand, although throwing greater electric stress upon the liquid present (as a result of the lower permittivity of the liquid) endows the liquid with greater dielectric strength by acting as a barrier to the lining up of fibers and other solid impurities, as well as by serving to limit the energy distance available for the production of ions and electrons by collision.

Impurities

The dielectric strength of oils seems but little affected by the presence of inorganic impurities such as sulphur. It is however very sensitive to fibers. These under electric potential line up in the areas of greatest stress and lead to insulation failure at greatly reduced voltages. The importance of such materials has been emphasized by the work of Hayden and Eddy,(27) Hirobe,(26) Ogawa and Kirbo, and others. The latter investigators have noticed, in the presence of very small amounts of fibrous materials, a reduction of as much as 33 per cent in the dielectric strength. In fact, the effect of dust, fibrous materials, and the like on the dielectric strength of oils was so great that Hirobe suggested that the great lowering which previously had been accredited to the presence of water was not so much due to the water itself as to the dust and other contaminating materials absorbed by the emulsified water droplets.

The absorption by colloids or suspensoids of the emulsified water in mineral oils is of great importance in those oils which are subjected to continual arcing in their practical use, such as for example the switch oils. Here, water and carbon are formed with each arc struck. The carbon must be precipitated. If water be present, it is absorbed by the suspended carbon particles which are thus held in suspension and are attracted to the electric field with disastrous results. They may finally be drawn to the electrodes and precipitated upon the neighboring parts, resulting in greatly decreased surface resistance.

The acidity of oil must be low. Sulphuric acid of the purifying process must be removed. The presence of slight traces of organic acidity is not of great harm, but if present in large amounts leads to a gradual weakening effect upon the mechanical strength of the fibrous insulation present and lowers the demulsibility of the oil.

Voltage Effects

As ordinarily used in electrical apparatus, insulating oils are but little affected by the applied potential. Oils subjected to continual discharge, however, are greatly changed both in their chemical and physical properties. The action of glow discharge upon oils has been investigated by Eichwald.(28) He has found that fatty oils subjected to a glow discharge break down with the formation of hydrogen as a result of the electronic bombardment. This hydrogen is not evolved but reacts chemically with the unsaturated hydrocarbons present. The residual groups from the hydrogen evolution process combine or polymerize to large molecules which are not identical with the ordinary products of polymerization. Thus an oil originally possessing 42 per cent saturated and 58 per cent unsaturated products was found by Eichwald to contain, after subjection to a glow discharge, 69 per cent saturated and only 31 per cent unsaturated materials. This process has been made use of in the production of lubricating oils.

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(27) Hayden and Eddy: "Five Hundred Tests on The Dielectric Strength of Oil." *Jour. A.I.E.E.*, *41* (1922), p. 138.

(28) Eichwald: "Glow Discharge on the Primary Tars." *Zeit. Angew Chem.*, *36* (1923), p. 611.

Some Past Developments and Future Possibilities in Very High Voltage Vacuum Tubes

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Assistant Director, Research Laboratory, General Electric Company

*On the fifteenth of last February, Dr. Coolidge was honored with the Edison Medal for his "contributions to the incandescent electric lighting and the x-ray arts." The presentation took place at the Annual Convention of the A.I.E.E. in New York City, on which occasion he delivered the paper that is here reproduced complete except for the omission of the introductory remarks.—*EDITOR

EARLY in our work on the hot-cathode high-vacuum x-ray tube, we were made conscious of a certain limitation. Such a tube behaved consistently only so long as a certain applied voltage was not exceeded. When this voltage *was* exceeded, current flowed through the tube even when the cathode was not heated. This current increased rapidly as the applied voltage was increased. Depending on the design of the tube, the discharge might pass directly from one electrode to the other, or it might pass from one electrode to the glass wall of the tube and then to the other electrode.

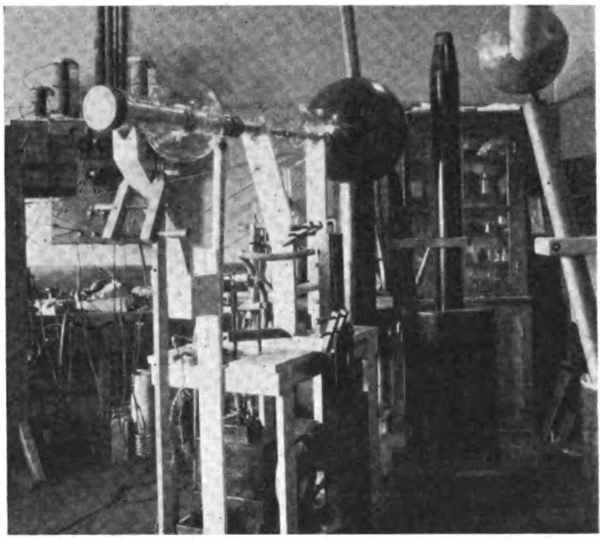

Fig. 1. Two-section Cathode-ray Tube for Operation at 600,000 Volts

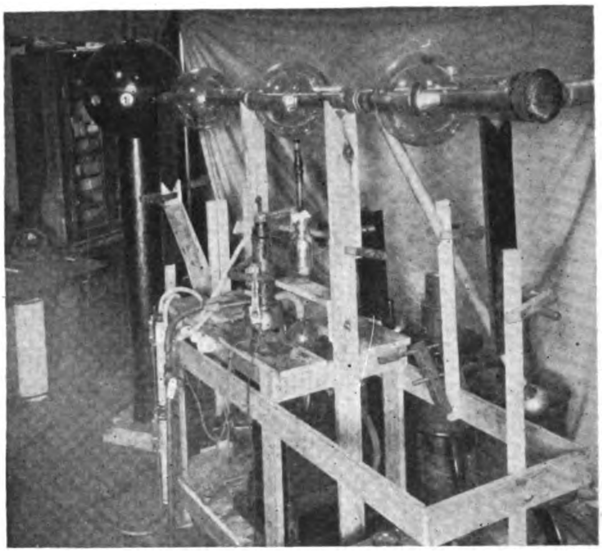

Fig. 2. Three-section Cathode-ray Tube for Operation at 900,000 Volts

The discharge was shown to consist of tiny jets of electrons coming from the cathode. When these jets went directly to the anode, their points of impact could be recognized by the heating, fluorescent, and x-ray effects which they produced. When they impinged upon the glass, they produced the small fluorescent patches familiar to those who have worked with the original gas-filled x-ray tube when the latter had too high a vacuum to function properly. In the x-ray tube the discharge in question took place from the edges of the focussing device and, by means of the x-ray pinhole camera, it was easy to show that it was quite independent of whether the filament was lighted or not.

Experiment showed that the determining factor was not the magnitude of the applied potential difference, but rather the electrical *potential gradient* at the surface of the cathode. The cold cathode discharge was favored by close spacing of electrodes and by the presence of sharp edges and corners on the cathode structure. While our first acquaintance with the effect had been made at voltages of the order of magnitude of a hundred thousand, we found later that by placing the electrodes only a few mils apart and using a cathode consisting of a tiny tungsten wire, say 1 mil in diameter, the cold-cathode effect could be observed with only a couple of thousand volts.

It is as though electrons could be pulled out of the cathode by the simple application of a sufficiently strong electrostatic field.

The cold-cathode effect sets a limit to the voltage which can be used on a given tube; for if one attempts to raise the voltage appreciably in spite of it, he either punctures the tube (through the local heating attending bombardment of the glass) or experiences a runaway arc discharge (through bombardment of the anode).

In our earlier attempts to build x-ray and cathode-ray tubes for voltages appreciably in excess of 250,000, we seemed to be continually contending with and limited by the "cold-cathode" effect.

More recently we have found that we can remove this limitation by subdividing the total potential difference applied to the tube between different pairs of tubular electrodes. The electrons coming from the cathode pass through the tubular intermediate electrodes without touching them, and are given successive accelerations as they pass between successive pairs of electrodes. This in effect divides the tube into sections, each of which may be good for as much as 300,000 volts. We have already successfully operated such a cathode-ray tube, with three sections, on 900,000 volts.

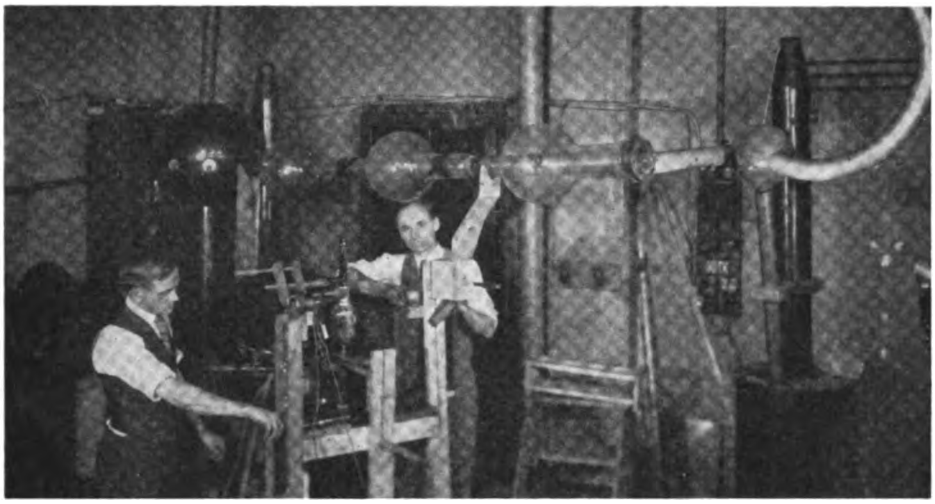

Fig. 3. Another View of the Three-section Cathode-ray Tube Showing (at the Right and Left) the Two Halves of the Vertical Induction Coil That furnishes High-voltage Energy to the Tube

This cascade or multi-sectional system promises to allow us to build vacuum discharge tubes for as high voltages as we can generate. This applies as well to an x-ray tube as to a cathode-ray tube, as the former may be derived from the latter by the addition of a suitable target. It also applies equally well to a high-voltage kenotron.

This opens a vista of alluring scientific possibilities. It has tantalized us for years to think that we couldn't produce in the laboratory just as high-speed electrons as the highest velocity beta rays of radium and just as penetrating radiations as the shortest wavelength gamma rays from radium. According to a statement of Sir Ernest Rutherford we need over two million volts, or a little more than twice the voltage which we have already employed, to produce x-rays as penetrating as the most penetrating gamma rays from radium; and with three million volts we could produce as high-speed beta rays. The capacity or quantity factor would be tremendously in our favor, as with twelve milli-amperes of current we would have as many high-speed electrons coming from the tube as from a ton of radium in equilibrium with its decomposition products. Another factor in our favor would be the easy control which we would have of the output, the radiation ceasing upon the opening of a switch. This would be quite different from our position with respect to radium, in which case the radiation takes place continuously whether it is desired or not, so that we must always be protected from it.

We expect to find that a high-voltage positive-ray tube can be constructed along the general lines of the multi-sectional cathode-ray tube. In this case we should need, according to Rutherford, about eight million volts to produce positive rays having the energy of the highest-velocity alpha rays from radium.

The problem of vacuum-tube operation at very high voltages is two-fold, as it involves not only the design of the tube but also the design of a suitable high-voltage source.

It seems quite possible that it will prove advantageous to proceed along the lines of our present oil-immersed x-ray outfit, in which the x-ray tube is placed in the same container and in the same oil with the high-voltage source. With such an outfit no part of the high-tension circuit is brought out into the air, and the corona problem is therefore greatly simplified.

We now have transformers that will produce several million volts, even with the high-tension circuit brought out into the air, and it should be possible for us to go much higher still, and especially so if the whole high-tension circuit is under oil.

What shall we do with the high-speed particles obtainable from tubes operating at a potential difference of millions of volts? The lure lies in the fact that we can't answer the question, beyond saying that we shall experiment with them. They should eventually help us to further knowledge of the atomic nucleus and to further knowledge of radiation laws.

Low-voltage A-c. Networks

PART III: LAMP FLICKER

By D. K. BLAKE
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WHEN supplying squirrel-cage induction motors from a combined light and power network system it is necessary to make sure that the starting current does not cause a dip in voltage sufficient to produce a noticeable or objectionable change in illumination. The first thing to be determined is the permissible dip in voltage that will not cause objectionable lamp flicker. Results of tests(2) "indicate that two per cent instantaneous variation in voltage cannot be detected and that three per cent is the maximum possible for the highest grade service. Changes of five per cent, however, will not be observed in most conditions found in practice." The permissible voltage dip having been decided upon the maximum motor size depends on the size of transformers and secondary cables, transformer reactance, the starting tap used on the compensator, and the location of the motor relative to the transformers.

In calculating the voltage dip it is important to use both the resistance and reactance of the cable; but the transformer resistance may be neglected. The starting currents and their power-factor should be averaged for all sizes to simplify the problem and permit the compilation of a simple set of tables for use in checking the connection of motors to the network. Standard general-purpose squirrel-cage motors have a starting current between six and seven times normal at full voltage. The current drawn from the network is proportional to the square of the voltage corresponding to the tap used on the compensator as given in Table I.

TABLE I

Motor Starting Currents Drawn from Network at Various Starting Voltages

Compensator Starting (Voltage Per Cent)	Network Current (Times Normal)
100	6.50
80	4.16
65	2.75
50	1.63

Several tables have been prepared for the study of the effect of such variables as motor location, network arrangement, secondary cable sizes, transformer reactance and spacing, and the use of starting taps upon voltage dip and motor size. The starting currents were assumed as given in Table I. A power-factor of 35 per cent was chosen although lower power-factors may be occasionally encountered; but they will not affect the results appreciably. The 300-kv-a. transformer in combination with 350,000-cir. mil and 500,000-cir. mil secondary cables was chosen because this seems to be representative of most networks using the three-phase, four-wire system. The transformer reactances of 5 per cent and 10 per cent cover the extremes of 13,200-volt transformers although there are a large number of 2300-volt network transformers having an approximate reactance of 3.5 per cent. Tables II, III, and IV were first prepared to determine the dip in voltage caused by a 50-hp. motor. For other sizes the voltage dip would be proportional to the horsepower rating. From these tables were prepared Tables V, VI, and VII to show the horsepower rating that will not cause a voltage dip exceeding 3 per cent.

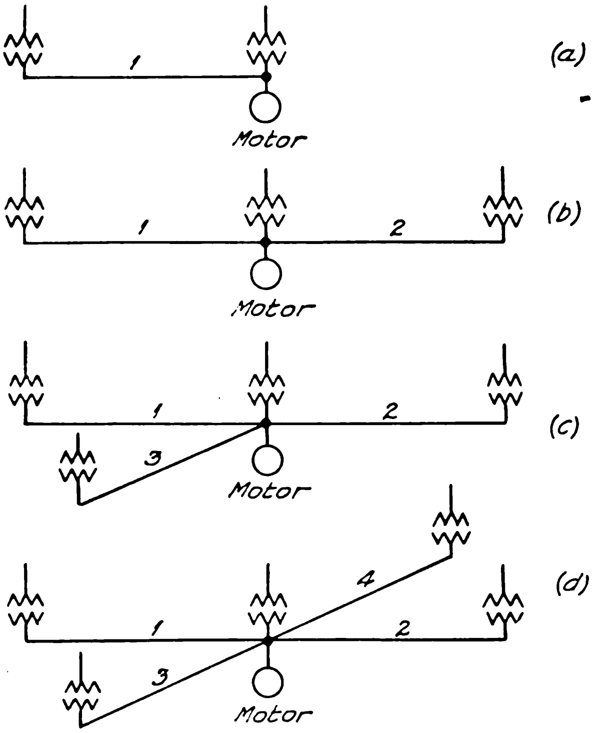

Fig. 17. Single-line Circuit Diagrams Showing Location of Transformers and Arrangement of Cable Sections Tied in at the Motor

Large motors are usually accompanied by other loads of sufficient size to justify a transformer vault located in the building supplied so that it is well to consider the effect of the number of network sections connected to the building transformer. The arrangement of the various sections are shown in Fig. 17, where the cable sizes and transformer ratings are uniform.

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(2)"Underground Alternating-Current Network," by A. H. Kehoe, *A.I.E.E. Journal*, June, 1924.

TABLE II

VOLTAGE DIP (LINE TO NEUTRAL) OF A 120/208-VOLT CIRCUIT

Standard 50-hp. Motor Located at 300-kv-a. Transformer

(Refer to Fig. 17)

Cable Size (Cir. Mils)	Starting Voltage (Per Cent)	Radial	NETWORK							
			SPACING OF TRANSFORMERS							
			500 ft.				1000 ft.			
			NO. OF CABLE SECTIONS TIED IN AT MOTOR LOCATION							
			1	2	3	4	1	2	3	4
			10 Per Cent Reactance Transformer							
350,000	100	9.95	7.08	5.46	4.48	3.80	7.97	6.65	5.71	5.01
500,000	100	9.95	6.80	5.20	4.22	3.53	7.74	6.33	5.35	4.64
350,000	80	6.35	4.52	3.50	2.87	2.43	5.10	4.26	3.66	3.21
500,000	80	6.35	4.36	3.33	2.70	2.26	4.95	4.05	3.43	2.97
350,000	65	4.2	2.98	2.32	1.90	1.60	3.37	2.82	2.42	2.12
500,000	65	4.2	2.88	2.20	1.78	1.50	3.27	2.68	2.27	1.97
350,000	50	2.48	1.76	1.37	1.12	0.95	1.99	1.67	1.43	1.25
500,000	50	2.48	1.70	1.30	1.04	0.83	1.93	1.58	1.34	1.16
			5 Per Cent Reactance Transformer							
350,000	100	4.96	3.98	3.33	2.86	2.50	4.37	3.91	3.53	3.20
500,000	100	4.96	3.84	3.14	2.67	2.31	4.27	3.75	3.33	3.01
350,000	80	3.18	2.55	2.13	1.83	1.60	2.80	2.50	2.26	2.05
500,000	80	3.18	2.48	2.01	1.71	1.48	2.73	2.40	2.13	1.93
350,000	65	2.10	1.69	1.40	1.20	1.05	1.85	1.65	1.49	1.35
500,000	65	2.10	1.63	1.34	1.13	0.98	1.80	1.58	1.40	1.28
350,000	50	1.24	1.00	0.83	0.71	0.63	1.09	0.98	0.88	0.80
500,000	50	1.24	0.97	0.79	0.67	0.58	1.07	0.94	0.83	0.75

TABLE III

VOLTAGE DIP (LINE TO NEUTRAL) OF A 120/208-VOLT Y CIRCUIT

Standard 50-hp. Motor Located at Various Points Between Two 300-kv-a. Transformers

(Refer to Fig. 18)

Cable Size (Cir. Mils)	Starting Voltage (Per Cent)	SPACING OF TRANSFORMERS							
		500 ft.			1000 ft.				
		DISTANCE FROM MOTOR TO NEAREST TRANSFORMER							
		0 ft.	125 ft.	250 ft.	0 ft.	125 ft.	250 ft.	375 ft.	500 ft.
		10 Per Cent Reactance Transformer							
350,000	100	7.08	8.60	9.11	7.97	10.30	12.05	13.00	13.20
500,000	100	6.80	7.97	8.45	7.74	9.54	10.85	11.65	11.70
350,000	80	4.52	5.50	5.83	5.10	6.60	7.70	8.30	8.45
500,000	80	4.36	5.10	5.40	4.95	6.10	6.95	7.45	7.50
350,000	65	2.98	3.60	3.83	3.37	4.35	5.05	5.45	5.55
500,000	65	2.88	3.34	3.55	3.27	3.99	4.57	4.80	4.93
350,000	50	1.76	2.13	2.27	1.99	2.56	3.00	3.23	3.29
500,000	50	1.70	1.96	2.10	1.93	2.36	2.71	2.85	2.92
		5 Per Cent Reactance Transformer							
350,000	100	3.98	5.96	6.68	4.37	7.70	9.30	10.40	10.65
500,000	100	3.84	5.46	5.90	4.27	6.60	8.10	9.10	9.38
350,000	80	2.55	3.82	4.28	2.80	4.60	5.95	6.65	6.83
500,000	80	2.48	3.50	3.78	2.73	4.23	5.18	5.81	6.00
350,000	65	1.69	2.55	2.83	1.85	3.03	3.94	4.38	4.50
500,000	65	1.63	2.32	2.48	1.80	2.79	3.41	3.83	3.97
350,000	50	1.00	1.47	1.67	1.09	1.80	2.33	2.60	2.68
500,000	50	0.97	1.37	1.48	1.07	1.65	2.02	2.27	2.35

TABLE IV

VOLTAGE DIP (LINE TO NEUTRAL) OF A 120/208-VOLT CIRCUIT

Motor Designed for Full Voltage Starting

MOTOR LOCATED AT 300-KV-A. TRANSFORMER

Cable Size (Cir. Mils)	Radial	NETWORK							
		SPACING OF TRANSFORMERS							
		500 ft.				1000 ft.			
		NO. OF CABLE SECTIONS TIED IN AT MOTOR LOCATION							
		1	2	3	4	1	2	3	4
		10 Per Cent Reactance Transformers							
350,000	6.88	4.90	3.80	3.11	2.63	5.53	4.61	3.97	3.48
500,000	6.88	4.73	3.61	2.93	2.45	5.36	4.39	3.73	3.22
		5 Per Cent Reactance Transformers							
350.000	3.44	2.75	2.30	1.98	1.73	3.03	2.70	2.44	2.22
500,000	3.44	2.68	2.17	1.85	1.60	2.95	2.60	2.30	2.09

MOTOR LOCATED AT VARIOUS POINTS BETWEEN TWO 300-KV-A. TRANSFORMERS

		SPACING OF TRANSFORMERS							
		500 ft.			1000 ft.				
		DISTANCE FROM MOTOR TO NEAREST TRANSFORMER							
		0 ft.	125 ft.	250 ft.	0 ft.	125 ft.	250 ft.	375 ft.	500 ft.
		10 Per Cent Reactance Transformers							
350,000		4.95	5.95	6.30	5.53	7.15	8.35	9.00	9.15
500,000		4.73	5.52	5.84	5.36	6.62	7.53	8.08	8.14
		5 Per Cent Reactance Transformers							
350,000		2.75	4.14	4.63	3.03	4.98	6.45	7.20	7.40
500,000		2.68	3.79	4.10	2.95	4.57	5.61	6.30	6.50

TABLE V

MOTOR SIZE (HORSEPOWER) WHICH WILL NOT CAUSE AN EXCESS OF 3 PER CENT VOLTAGE DIP WHEN STARTING FROM A 120/208-VOLT CIRCUIT

MOTOR LOCATED AT 300-KVA. TRANSFORMER

(Refer to Fig. 17)

Cable Size (Cir. Mils)	Starting Voltage (Per Cent)	Radial	NETWORK							
			SPACING OF TRANSFORMERS							
			500 ft.				1000 ft.			
			NO. OF CABLE SECTIONS TIED IN AT MOTOR LOCATION							
			1	2	3	4	1	2	3	4
			10 Per Cent Reactance Transformer							
350,000	100	15	25	30	40	40	20	25	30	35
500,000	100	15	25	30	40	50	20	25	30	35
350,000	80	25	35	50	60	60	35	40	40	50
500,000	80	25	40	50	60	75	35	40	50	60
350,000	65	40	60	75	75	100	50	60	60	75
500,000	65	40	60	75	100	100	50	60	75	75
350,000	50	60	100	125	150	150	75	100	125	125
500,000	50	60	100	125	150	200	75	100	125	150
			5 Per Cent Reactance Transformer							
350,000	100	35	40	50	60	60	40	40	50	50
500,000	100	35	40	50	60	75	40	40	50	50
350,000	80	50	60	75	75	100	60	60	75	75
500,000	80	50	60	75	100	100	60	75	75	75
350,000	65	75	100	125	150	150	75	100	100	125
500,000	65	75	100	125	150	150	100	100	125	125
350,000	50	125	150	200	250	250	150	150	200	200
500,000	50	125	150	200	250	300	150	150	200	200

TABLE VI

MOTOR SIZE (HORSEPOWER) FOR WHICH VOLTAGE DIP WILL NOT EXCEED 3 PER CENT WHEN STARTING FROM A 120/208-VOLT CIRCUIT

MOTOR LOCATED AT VARIOUS POINTS BETWEEN TWO 300-KV-A. TRANSFORMERS

Cable Size (Cir. Mils)	Starting Voltage (Per Cent)	SPACING OF TRANSFORMERS							
		500 ft.			1000 ft.				
		DISTANCE FROM MOTOR TO NEAREST TRANSFORMER							
		0 ft.	125 ft.	250 ft.	0 ft	125 ft.	250 ft.	375 ft.	500 ft
		10 PER CENT REACTANCE TRANSFORMER							
350,000	100	25	20	15	20	15	10	10	10
500,000	100	25	20	20	20	15	15	15	15
350,000	80	35	30	30	35	25	20	20	20
500,000	80	40	35	30	35	25	25	20	20
350,000	65	60	50	40	50	40	35	30	30
500,000	65	60	50	50	50	40	35	35	35
350,000	50	100	75	75	75	60	60	50	50
500,000	50	100	75	75	75	75	60	60	60
		5 PER CENT REACTANCE TRANSFORMER							
350,000	100	40	30	25	40	20	15	15	15
500,000	100	40	30	30	40	25	20	15	15
350,000	80	60	40	40	60	35	30	25	25
500,000	80	60	50	40	60	40	30	30	30
350,000	65	100	60	60	75	50	40	40	40
500,000	65	100	75	60	100	60	50	40	40
350,000	50	150	100	100	150	100	75	60	60
500,000	50	150	125	100	150	100	75	75	75

TABLE VII

MOTOR SIZE (HORSEPOWER) WHICH WILL NOT CAUSE AN EXCESS OF 3 PER CENT VOLTAGE DIP WHEN STARTING FROM A 120/208-VOLT CIRCUIT

Motor Designed for Full-voltage Starting

MOTOR LOCATED AT 300-KV-A. TRANSFORMER

Cable Size (Cir. Mils)	Radial	NETWORK							
		SPACING OF TRANSFORMERS							
		500 ft.				1000 ft.			
		NO. OF CABLE SECTIONS TIED IN AT MOTOR LOCATION							
		1	2	3	4	1	2	3	4
		10 PER CENT REACTANCE TRANSFORMER							
350,000	25	35	40	50	60	30	35	40	50
500,000	25	35	50	60	60	30	35	40	50
		5 PER CENT REACTANCE TRANSFORMER							
350,000	50	60	75	75	100	50	60	60	75
500,000	50	60	75	75	100	60	60	75	75

MOTOR LOCATED AT VARIOUS POINTS BETWEEN TWO 300-KV-A. TRANSFORMERS

		SPACING OF TRANSFORMERS							
		500 ft.			1000 ft.				
		DISTANCE FROM MOTOR TO NEAREST TRANSFORMER							
		0 ft.	125 ft.	250 ft.	0 ft.	125 ft.	250 ft.	375 ft.	500 ft.
		10 PER CENT REACTANCE TRANSFORMER							
350,000		35	30	25	30	25	20	20	15
500,000		35	30	25	30	25	20	20	20
		5 PER CENT REACTANCE TRANSFORMER							
350,000		60	40	35	50	35	25	25	20
500,000		60	40	40	60	35	30	25	25

With the motor located between transformers, a single section with a transformer at each end was considered and the effect of the rest of the network was neglected because of the added complication. It should be remembered, however, that the network does improve the result, although not so much as when the motor is connected to the transformer; for as the distance between the motor and transformer increases, the secondary cable impedance predominates.

It is advisable to consider also the possibilities of full-voltage starting as provided by the large transformers and cables interconnected into a network. Not all standard general-purpose motors are suitable for full-voltage starting but a great many of them are, particularly those rated 30 hp. and below. Above 30 hp. it depends a great deal on the rated speed of the motor. In some cases it is possible to thus use motors of 75 hp., and even up to 200 hp., at low-speed ratings.

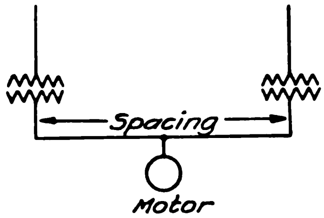

Fig. 18. A Single-line Circuit Diagram Showing Motor Location with Respect to Transformer Spacing

An available standard line of general-purpose motors, designed particularly for full-voltage starting, draws a starting current of about four to five times normal when started in this manner. These motors are considered in Tables IV and VII. Thus it is possible in a great many cases to simplify motor installations by the omission of the usual starting compensator.

There seems to be virtually no limit to the size of squirrel-cage motor that can be connected to a network system of the type under consideration. Table V shows that it is possible to connect motors of 150 to 300 hp. rating. In such cases there will also be other loads equal to or greater than the motor load so that the transformers will be larger, thereby reducing the voltage dip when starting.

From Table VI it is seen that it is possible to connect 50 to 100-hp. motors midway between transformers; but in such cases there will also be other motors and the lighting load in addition, making the total load sufficient to locate a transformer vault on the premises or very close by in the street. It thus seems that it would be a rare occurrence to connect such large motors so far from the transformer. Smaller motors, such as 20 to 35 hp., are easily supplied from midway points even with high-reactance transformers, 1000 ft. spacing, and the 80 per cent starting tap.

Full-voltage starting is possible with all motors up to 10 hp., and up to 20 hp. when the special design is used. Where standard reactance transformers are used, full-voltage starting may be applied to all motors up to 30 hp., and up to 40 hp. with the special design. Where the motors are located at the transformer, full-voltage starting is always possible in sizes up to 20 hp. or up to 30 hp. with the special design. Under the most favorable conditions the standard 75-hp. motor and the special design 100 hp. motor may be started at full voltage.

Some cases may be encountered where flicker is not caused by the initial step of starting current but by the inrush current at the subsequent throwover of the compensator switch. This may be due to the use of a compensator starting tap too small for the type of load on the motor, such as a pump load. Fig. 19, which is a typical speed-current curve at full voltage, will be employed in the following example: Suppose the motor will start on the 50 per cent tap, which gives an inrush current of 167 per cent normal. The motor torque on this tap, however, will bring the motor to only 80 per cent speed. The compensator is then thrown over to line voltage. The curve in Fig. 19 indicates that the inrush current at this speed will be nearly 400 per cent, which may be just enough to cause flicker. Now if the tap is changed to the 65 per cent point to give additional torque to bring it to nearly full-load speed, say 95 per cent, the starting current will be about 285 per cent which may be sufficiently lower than the value causing objectionable flicker. The great majority of motor applications will attain full-load speed before throwover, so that the current will be well below the starting current.

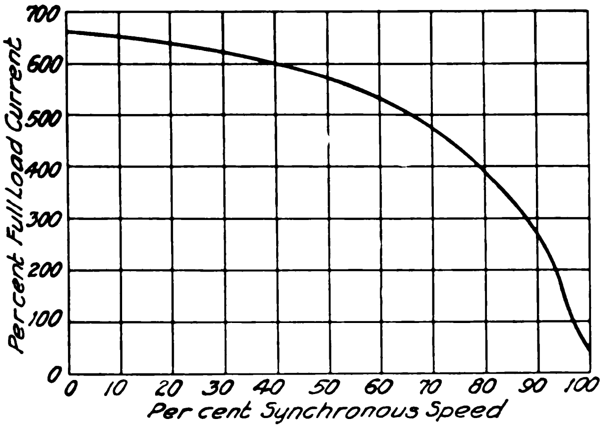

Fig. 19. The Full-voltage Speed-current Characteristic of a 50-hp. Squirrel-cage Motor Running at an Average Speed of 1200 r.p.m.

It therefore seems, from the foregoing considerations, that such network systems can very well take care of virtually all squirrel-cage motor applications utilizing standard starting compensators. There are, however, some network systems that use smaller transformers and smaller cables, making ordinary compensator starting unsuitable in many cases. In these cases resistance starters are utilized, by means of which the current is gradually increased until the motor starts.

(To be continued)

Load Ratio Control

PART II

FIELDS OF APPLICATION

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THE subject of load ratio control can hardly be considered complete without examining somewhat in detail the various fields of application for these equipments in order to give at least a general idea of what is being accomplished by their use. Certain industrial applications, such as to electric furnaces, where, on account of the very great voltage range desired, ratio control is found to be economical in spite of the fact that the kv-a. to be handled in individual units is not relatively large, are not being considered in this article. This omission is deliberate, however, since it seems that interest is being particularly centered at present in application to the control of interconnected central stations. In approaching this subject it is realized that practical experience is required to best determine positively what these load ratio control equipments are good for on given systems, and what are their limitations. At most, it is here possible to approach the subject only from a theoretical angle, and to leave the practical considerations to others who have given the matter considerable thought and whose experience may enable them to point out the cases, if any, wherein actual results are found inconsistent with the present treatment.

The more it is tried to determine just what these equipments are doing to help interconnected systems to operate economically, the more it may be wondered how interconnected systems have been able to get along so well without them. Previously, the induction regulator, which is the only piece of apparatus performing about the same function, was being used in large units in this field only to a very limited extent, and therefore it seems a reasonable question to ask how these systems operated *without* ratio control equipments rather than how they operate *with* them.

The interconnecting of central stations is made for the general purpose of increasing the reliability of service, and also to improve the load-factor of the system. Both of these objects mean that it should be possible for a given load at one point to be supplied from a number of distant stations in the interconnected system. Theoretical analysis of the conditions involved, however, shows that serious limitations are imposed on the operation of these interconnections by the necessity of keeping the voltages at various points of the network constant, or approximately constant, for all conditions of load demand. This limitation, in many cases, may very seriously restrict the amount of kv-a. which can be delivered from a distance to a given point, and, in addition, prevent obtaining as economical a division of energy and current between the various stations.

The simplest qualitative statement of the problem is to say that the presence of impedance of the interconnecting line between stations means that it is impossible to adjust the power-factor of the current in the line for its most economical value without obtaining serious voltage regulation in the line. For example, let us consider two central stations *A* and *B*, Fig. 15, each furnishing a load from its local low-voltage bus, and connected together by means of a high-voltage line and two transformers. The interconnecting line between the two stations can be operated in either of the following ways:

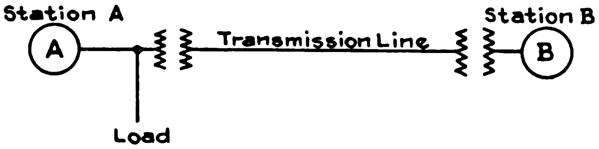

Fig. 15. Single-line Diagram of Two Central Stations Interconnected by a High-voltage Line and Transformers

(1) The phase-angle of the current in the line may be adjusted for the most desirable power-factor for line current at all loads, holding the voltage constant at one point only, and allowing voltages at all other points to vary.

(2) The power-factor of the current in the line may be disregarded, except perhaps for one particular load, and bus voltages maintained constant by means of the generator fields.

(3) The phase-angle of the current in the line may be adjusted for the most economical power-factor at all loads, and the resulting regulation of line and transformer neutralized by means of ratio control.

The purpose of the transmission line is to permit load from station *A* to be delivered to bus *B*, and *vice versa;* and the ideal condition is that the combined kv-a. of the two stations *A* and *B* may be delivered to either bus without affecting the voltage of either. Although the transmission line impedance prevents this, it is possible to assume that voltage is maintained constant at all loads on both buses by generator field current control only and, with this assumption in mind, to find the operating limitations introduced by transmission line impedance.

It is convenient for the purpose of the problem to consider that the impedance drop (IZ) through the transmission line between buses A and B is mathematically resolved into two components, as shown in Fig. 16, one (IX) an impedance drop at right angles to the transmission line current, the other (IR) an impedance drop in phase with the transmission line current. Each of these drops can be analyzed separately and independently of the other. The essential difference between these two components is that the impedance drop perpendicular to line current may vary through a wide range without affecting the premises of the problem, *i.e.*, constant potential in both buses, whereas the in-phase impedance drop cannot vary at all without introducing variable voltage on one of the buses. Any value whatsoever may be assigned to the in-phase drop, but it cannot vary.

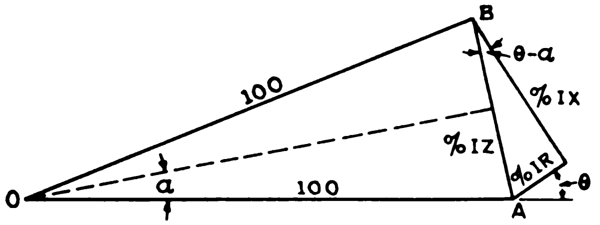

Fig. 16. Vector Diagram Showing the Voltage Relations and the Impedance of the Transmission Line Between Stations A and B, Fig. 15

Taking first the perpendicular impedance drop, it is to be found what kind of current can flow in the system and result only in perpendicular drops in the line. The conditions which govern this current flow are shown in the voltage diagram (Fig. 16), in which OA is the receiver bus voltage, OB the sending bus voltage, and AB the impedance drop in the line. The triangle, OAB is isosceles with impedance drop AB as the base of the triangle. The diagram to scale represents about 38 per cent impedance drop. A smaller value of impedance would make the drop AB relatively smaller, but the general conclusions derived from this triangle would not be affected. The important point to note from this voltage diagram is the resistance component (IR) of the impedance drop which determines the phase angle of the current in the transmission line with respect to the receiver bus voltage OA. This current obviously leads the bus voltage by an appreciable angle θ. This angle (θ) will be somewhat smaller for lighter loads and also smaller as the ratio of reactance to resistance of the transmission line becomes less. It is, however, always leading the receiver voltage.

The corresponding current relations are shown in Fig. 17 in which the horizontal line is the receiver bus voltage and OB is the current in the transmission line, leading the receiver voltage by the angle θ, as determined by the voltage diagram in Fig. 16. The total load current on bus A is OA; BA is the current in the generator near the load, and therefore OC, the arithmetical difference between OA and AB, is the contribution of the far generator and the transmission line to the load, although actually it is loaded an amount equal to OB. For example, assuming that two central stations each have a rating of 50,000 kv-a., and assuming a load of 85 per cent power-factor and about 33 per cent impedance drop in the line, it would be possible to deliver a maximum load of about 60,000 kv-a. on either bus; but the transmission line and far generator are only 20 per cent effective as far as the delivery of kv-a. is concerned. The generator currents in the diagram are made equal to each other, although in the general case, this is not necessary. The currents can be varied in any way whatever, provided that the angle θ is maintained, as determined by the voltage diagram.

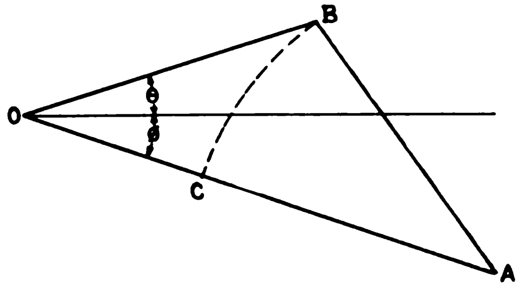

Fig. 17. Diagram of the Current Relations in the Case Illustrated in Fig. 15

It is not to be inferred that this is actually a practical operating condition. However, this situation would be encountered, if it were attempted to operate the system under the following combination of conditions:

(a) Constant voltage on the two buses at all loads;
(b) Voltage control by generator field alone, and
(c) Power to flow equally in either direction.

Conditions can be improved without changing the premises (a) and (b) of the problem by allowing to flow, in addition to the currents shown, a constant current that produces an impedance drop in phase with the line voltage. As long as this current remains invariable, it produces only a line drop and no variation in bus voltages. Since this current is constant, it must be a circulating current between the two generators, and therefore wattless. By suitably choosing the value of this current, the full-load conditions of current division may be considerably improved by bringing currents OB and AB more nearly into phase with each other; and thus the effectiveness of the distant generator is increased. The diagrams in Figs. 18 and 19 illustrate this condition. Here the current triangle OAB is the same as before; but upon it has been imposed the constant wattless current BX which, by circulating between the two generators, gives a resultant current triangle

OXA. The point X may be chosen anywhere on the vertical through B but within the triangle; although once chosen it is a fixed point as far as operation is concerned. It should be remembered that the only currents actually flowing in the system are given by the triangle OXA. What we actually have done is to resolve the real current in triangle OXA which flows at full load into two mathematical components, one component consisting of the current, in triangle OBA, which varies with the load and produces in the transmission line perpendicular impedance drops only; and the other component, a wattless current (BX), which is unvarying and which produces a constant voltage drop in the line. Now as the variable currents forming triangle OBA decrease, the current BX becomes increasingly prominent, and at no load, at which time OBA disappears, BX is the only current left. So BX is the no-load current in the transmission line.

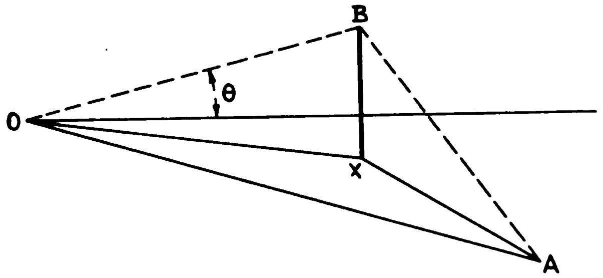

Fig. 18. Current Diagram Illustrating Conditions for Constant Line Drop

Note that, in order to simplify the problem, such residual currents as transformer magnetizing currents, or capacitance currents have been neglected. The current BX multiplied by the line reactance gives the line drop, which is constant for all loads between the two buses A and B.

Although it is evident that the full-load operating conditions are very much improved by the addition of current BX, nevertheless, in adding this current, several things are sacrificed: First, the arrangement improves the situation for power flowing in one direction only, and in fact makes conditions impossible for reverse-power flow. Second, at light loads, this wattless circulating current becomes prominent, and adds greatly to the light-load losses in both generator and transmission line, and therefore, excepting in the cases where the load-factor is very high, these extra light-load losses cannot be neglected. Third, the line drop which is determined by the line BX reduces the kv-a. delivered over the line to the load, and thereby partially neutralizes the good effects produced by bringing the two generator currents closer together in phase.

There seems to be no other way than the one just described in which the system can be operated with constant potential at both buses when the voltage is controlled by generator field currents alone. The best that can be done is to choose the line BX (Figs. 18 and 19) for the most desirable operating conditions, having a given load-factor in mind; and it is evident that BX, selected for an average daily load condition, means inefficient or undesirable operating conditions at both maximum load and at light-load periods. It also means giving up reverse-power flow.

This inflexibility is overcome by allowing the current BX unrestricted variation, so as to obtain the most effective current division at any time or condition of load, and for any normal or abnormal generating condition, and to overcome the resulting variable line drop by varying the transformer ratio by means of ratio control equipments. The line BX now becomes a measure of the voltage control necessary, for by multiplying the current BX by the line reactance there is obtained the variable voltage to be neutralized.

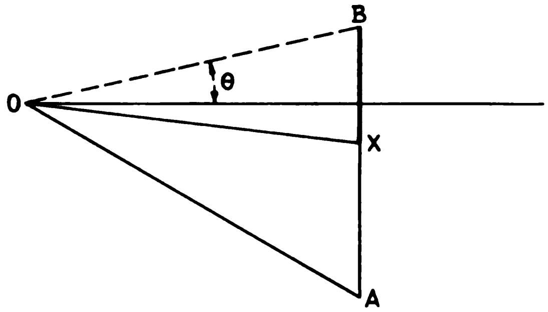

Fig. 19. Current Diagram Illustrating the Conditions Imposed by a Synchronous Condenser

The foregoing treatment may seem a very elaborate method of saying that by means of transformer ratio control, currents can be allowed to vary in any manner whatever, and at the same time, voltages on the buses kept constant; but the rather elaborate explanation gives a more comprehensive idea of their value.

Referring once more to the current diagram shown in Fig. 18, it will be seen that the throttle setting or the energy delivered by the prime movers to the generators determines the location of point B and the vertical line BX, in the horizontal direction. Thus opening the throttle at the generator near the load, moves point B to the left and conversely, closing the throttle shifts B to the right. At the extreme position, *i.e.*, when the generator near the load is not furnishing any energy, point B is at the extreme right, as shown in the diagram in Fig, 19, which represents the case of a synchronous condenser floating on the end of the line near the load.

The function of a synchronous condenser, by adding current AX to the line, is to improve the power-factor of the circuit; and at the same time this improves the line regulation. The rating of the condenser is given by AX, and the no-load lagging current furnished by the condenser, which the line must carry to maintain constant voltage, is given by BX.

The point X can be chosen so as to obtain the best average condition of operation, but, just as in the case of the two generators, X being chosen, it becomes fixed. Therefore, the same inflexibility of operation exists using the synchronous condenser to hold voltage constant as with two generating stations connected together, in that the point X cannot be adjusted for the best condition for every load without introducing regulation; and also the light-load losses are increased. It is clear from this diagram that the function of a synchronous condenser and ratio control are quite distinct. The primary function of the condenser is to improve the power-factor on the line at the heavier loads and by so doing, increase the kv-a. output that can be delivered, or decrease the energy losses in the line. Transformer ratio control, on the other hand, can only neutralize the regulation drop in the line. The combination of the two gives the maximum amount of flexibility, since it permits the condenser to adjust the line power-factor for best conditions for all loads, whereas, when the condenser alone is used, it is possible to adjust the line power-factor for only the average load.

The relative value of the condenser and ratio control equipments depends largely upon the following conditions: first, whether the transmission line copper losses are relatively high, which is the case for relatively long lines; second, whether the power-factor of the load is relatively low; and third, whether the load-factor is relatively high. In such cases the synchronous condenser is of greater value than ratio control; but, conversely, ratio control equipments are found to be of greater value, first, when the line losses are inherently low, such as in shorter lines; secondly, when the power-factor of the load is relatively high, and thirdly, when the load-factor is relatively low. At the one extreme, it becomes preferable to use the synchronous condenser exclusively, and, at the other, ratio control exclusively. For intermediate cases, it would appear that a combination of the two is desirable.

From all of the considerations as herein set forth, the following general conclusions may be drawn:

(1) That the proper function of synchronous apparatus on the interconnected systems is to control the division of current and their phase angles. This cannot be properly done if they are called upon to maintain voltages constant.

(2) That ratio control apparatus is ideally suited for maintaining constant voltage; and by relieving the synchronous apparatus of this duty, the flexibility, efficiency and overall apparatus economy of systems is very greatly increased.

It is desirable to consider briefly the conditions involved when the transmission line forms a loop. In an open-ended line, it is evident that if the current in the various sections of the line are to be independently variable, a resultant voltage, which is the summation of the impedance drops in the lines, will exist between the two ends of an open loop. This voltage varies not only in amount, but also may vary widely in phase. It is short-circuited if the loop is closed, resulting in a circulating current through the loop, the value of which is equal to the voltage short-circuited, divided by the impedance of the entire loop. This circulating current changes the values of the currents in every section of the loop. In order to be able to maintain the same current flexibility as exists in the open-ended line, it is necessary to insert in the loop a voltage which is equal to and opposite in phase to the summation of the impedance drops around the loop. This can only be accomplished by means of two voltage-control equipments, one providing the proper variable in-phase and the other providing the proper variable quadrature voltage. It must also be possible to vary the in-phase and quadrature voltages independently of each other.

In general, therefore, a transmission line loop in which current flexibility is desired, together with N points maintained at constant potential requires N in-phase voltage ratio controls and one quadrature voltage control.

Whether this complete current flexibility is necessary or desirable is another question, depending upon several conditions. The desirability of loop control is determined by the question of whether the circulating current flowing around the loop when loop control is not used is objectionable. For example, this circulating current may result in a severe overload in a portion of the loop, which, especially when underground cables are employed, is prohibitive on account of the resulting overheating. In addition, the circulating current may introduce excessive energy losses. If operation with minimum line losses within the loop is the only requisite, several other facts are of interest. The division of current in a loop, automatically, is such as to result in minimum line copper losses for any distribution of load when the impedance characteristics of the loop are such that in every portion, the ratio of resistance to reactance is the same. Therefore, in loops where the resistance and reactance of each section has been adjusted so as to maintain constant ratio, quadrature voltage control is not needed to obtain minimum copper losses within the loop.

Concluded

New Features in the Construction of Outdoor Switch Houses

By J. H. POWERS
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THE increasing demand for electric service in outlying districts has brought up many problems in connection with reliable and economical methods of distribution. The use of weatherproof housings for feeder equipment has been one solution and a common practice for some time, each year bringing about improvements which have proved a benefit to both the operating company and the consumer. Recent developments have been made particularly in the direction of neater appearance, strength, accessibility, exclusion of dust and moisture, ventilation, and lower costs. In the past year or two the improvements applied to this type of equipment have resulted in a very noticeable increase in demand.

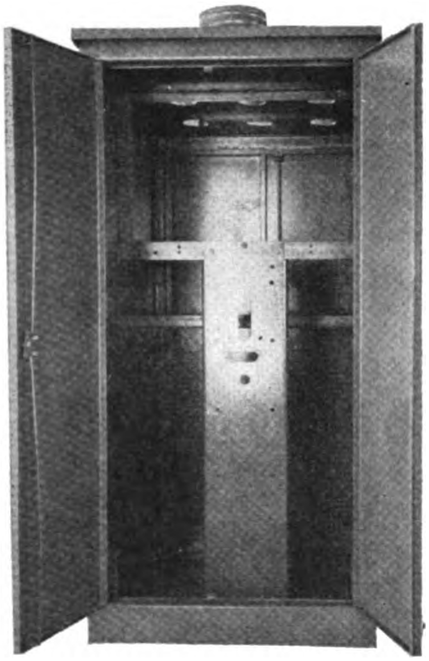

Fig. 1. A New Type of Outdoor Switch House of Great Mechancial Strength and Weatherproof Construction

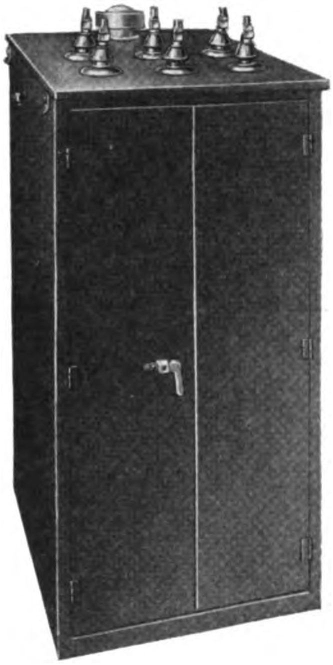

Fig. 2. The Exterior of an Outdoor Switch House of the Improved Type, Showing the Neat Arrangement of Its Component Parts

Mechanical Details

Some of the newer features incorporated in the latest designs of switch houses may be of interest. In place of the older type of construction, utilizing thin sheet iron attached to an angle iron framework, the new houses are made of heavy flanged plates welded together. An example of this new type appears in Fig. 1. The doors are made of the same heavy material with a cup-shaped flange; and are so mounted as to be flush with the house surface when closed, as shown in Fig. 2. Being in two sections, the doors occupy the minimum amount of ground space when open and give full view to the entire equipment. A rolled felt seat for both the front and rear doors insures protection for the equipment against dust and moisture. These doors are fitted with a lever which fastens them in the closed position at one operation and also permits a padlock being attached.

A heavy, flanged, upright member, drilled to mount any of the usual combinations of breakers and operating mechanisms, forms the bedplate of the house, at the same time preventing vibration and adding materially to the rigidity of construction. This universal feature allows the houses to be manufactured on a quantity basis and stocked for various combinations of manual and automatic equipment. Double angles welded to the sides of the house form a rigid support for the current transformers. All cross-member obstructions in the front door opening are now eliminated, thus providing free access to the operating mechanism and potential transformers and exposing the instrument and relay equipment on the panel to full view. Lifting lugs are attached near the top of the housing exterior to facilitate handling.

Complete ventilation is furnished by a screened hole in the floor and a Pullman car type ventilator mounted on the roof of the house. The oil circuit breakers are directly connected to their operating mechanisms to insure dependable operation. This simplifies the mechanical details of the equipment as a whole and facilitates repair and inspection of the apparatus.

All seams joining the flanged members of the house are ground smooth and, as there are no rivet or bolt heads exposed on the exterior, corrosion is reduced to a minimum and painting is made easier.

Types and Sizes

These switch houses are now built in four standard types, their voltage rating and overall dimensions being given in Table I. The first three types are designed for mounting on a concrete base or other support at ground level; and the fourth type, which contains a small oil circuit breaker, is adapted to mounting on a steel structure.

The use of quantity production methods results directly in lower costs and prompt shipments. Nearly any combination of manual or automatic equipment can be mounted in these houses without change of the interior steel work. This feature is also advantageous when it is necessary to make changes in the field occasioned by an increase in the feeder capacity. Where conditions demand multiple circuits, two or more houses may be joined together as in Fig. 3.

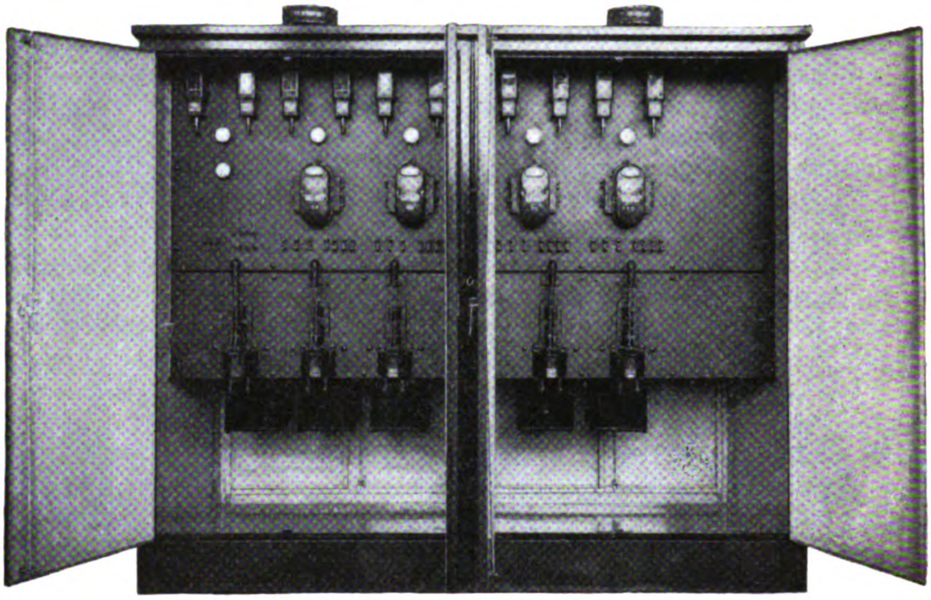

Fig. 3. A Multiple Arrangement of Switch House Units which Readily Provides for Increased Capacity

TABLE I

Voltage	Width	Depth	Height
2300–6600	42 in.	54 in.	90 in.
13200	54 in.	54 in.	90 in.
2300–13200	54 in.	60 in.	102 in.
2300	36 in.	28 in.	36 in.

In addition to the standard houses now manufactured, two types of three-circuit houses have been developed for the Central Illinois Public Service Co. The smaller is for 2300-volt manual equipment, including oil circuit breakers. The larger is for 11460/6600-volt manual or automatic equipment, including suitable oil circuit breakers. In Fig. 4 is shown an illustration of the method of housing the latter.

To further facilitate installation, all switch houses are made self-contained, with the equipment completely assembled, wired, tested, and ready to operate. The advantages of this feature are obvious, particularly in view of the rapidly-expanding business of the public utility companies which are often called upon to make power connections on short notice. The self-contained design also allows the houses to be readily shifted from one location to another when conditions require it.

No small part of the yearly increasing demand for automatic a-c. reclosing feeders is made up of these equipments mounted in outdoor switch houses for installing in isolated districts. With this type of equipment, thus well protected from the elements, only occasional visits of inspection are required in maintaining good service at the remote points of a large system.

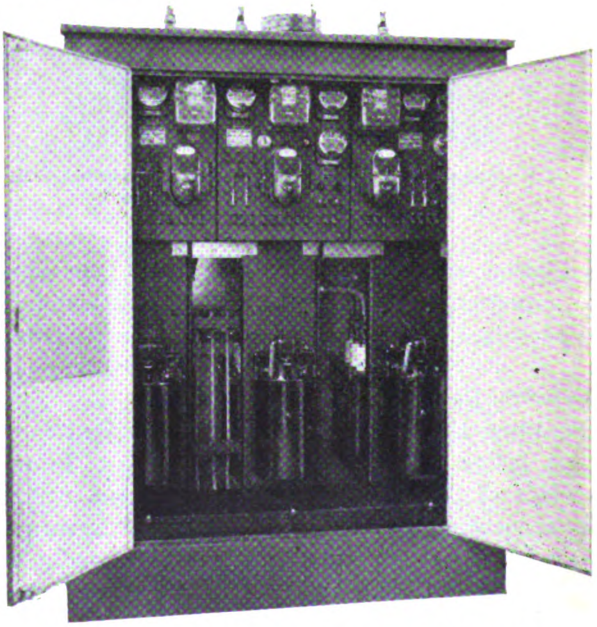

Fig. 4. An Installation of Three Automatic A-c. Reclosing Feeders in an Outdoor Switch House

The Application of Oxygen and Hydrogen to Industrial Operations

PART III

METHODS OF OXYGEN MANUFACTURE AND IMPORTANCE OF THE BY-PRODUCT GASES

By F. P. WILSON, Jr.
Manager's Staff, Schenectady Works, General Electric Company

THE exceptional growth in the application of fusion welding and oxygen cutting of metals has resulted in an enormous annual consumption of the gases used in these processes throughout the country. One of the principal factors which has contributed to the building up of the vast system of commercial production and distribution as it exists today has been the use of oxygen and acetylene in cutting and gas welding.

Acetylene and the electric arc have until recently been supreme in their respective fields of application in welding and cutting. Now, however, the economic supremacy of the former has been seriously challenged in welding as well as in cutting to an extent which may result in an economic adjustment in the present system of gas manufacture and distribution.

The atomic-hydrogen flame has demonstrated characteristics which indicate its great value in many welding applications, and in many instances molecular hydrogen or city gas can replace acetylene as the fuel gas in cutting. Oxygen, the active agent in metal-cutting operations, will undoubtedly be used for years to come in this application and in constantly increasing quantities.

Because of the annual value of the gases used in many industries today, and the impending economic changes just over the horizon, the need is emphasized for a general knowledge of the relation existing between the process of manufacture and the use of these various gases.

Large consumers of gases for welding and cutting may be potential gas producers and, should economics justify the installation of equipment, consumers should be in a position to judge the relative merits of the apparatus available and determine which process would best suit the requirements with respect to the following factors: The kinds of welding and cutting gases now used and possible substitutes. The effect of such substitutes on the selection of a process for manufacturing these gases. The economic need of consumer manufacture. The need of endeavoring to establish an economic balance between the gases used and the process selected for their manufacture. The possibilities of applying these gases in industrial operations in which their use is now limited or non-existent.

Oxygen can be economically produced from the air by means of air liquefaction and subsequent fractionation, a physical process; or by the electrolysis of water, an electrochemical process. In the former case nitrogen is a by-product in the ratio of four volumes to one volume of oxygen. In the latter case hydrogen is a by-product in the ratio of two volumes to one of oxygen. The purity in both cases is 99 per cent or better, but the electrolytic process gives higher purity for both gases when produced simultaneously. The importance of such purity differential is not great for general industrial operations. Another commercial method of producing hydrogen in large quantities is by the liquefaction and fractionation of water gas. This process, although used in Europe, is not in general use in this country and will not be discussed at this time.

The operation of the air liquefaction process is based on the difference in boiling points of the main constituents: Nitrogen — 195 deg. C., Argon — 186 deg. C., Oxygen — 183 deg. C. The air is a mechanical mixture composed of nitrogen, oxygen, argon, neon, xenon, krypton and other rare gases. Nitrogen is present in the greatest volume and amounts to approximately 79 per cent, oxygen about 20 per cent, and argon about 1 per cent. The last-named gas boils at only 3 deg. lower than oxygen, and therefore in the oxygen produced by this process the impurity tends to be argon rather than nitrogen.

Liquefaction Process

There are two fundamental processes for the separation of air into its constituents by liquefaction and fractionation. One was developed by Linde and the other by Claude. In the former, the refrigeration and liquefaction of the air is produced by the internal energy of free expansion when the air is released through a needle valve with an accompanying pressure drop of approximately 3000 lb. The second method utilizes the external energy of air compressed to about 700 lb. per sq. in. against a piston in an expansion engine, thus cooling the air by the dissipation of heat in the form of mechanical energy. While the latter permits slightly greater power efficiency in plant operation, the former is in more general use and is the process which will be discussed here.

The production of liquid air by free expansion through a nozzle is due to the Joule-Thomson effect, and the cooling can be expressed by the following formula:

$$d = 0.276\,(p_1 - p_2)\left(\frac{272}{T}\right)^2$$

where d = cooling effect, T = absolute temperature of expansion, and p_1 and p_2 are the pressures in atmospheres before and after expansion.

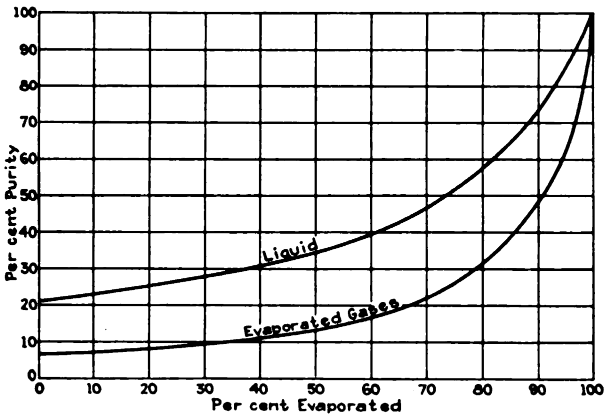

Fig. 17. Evaporation of Liquid Air, According to Dr. Linde

Relative oxygen content in the remaining liquid.
Relative oxygen content in the evaporated gases.

The cooling caused by adiabatic expansion* in a cylinder against a piston is given theoretically by the formula:

$$\frac{T_0}{T_1} = \left(\frac{P_1}{P_2}\right)^{0.29}$$

where T_0 = absolute temperature of compressed air, T_1 = absolute temperature of expanded air, P_1 = pressure in atmospheres of compressed air, and P_2 = pressure in atmospheres of expanded air. $T_0 - T_1$ = cooling in degrees centigrade.

If these two formulas are applied, it will be seen that with the Joule-Thomson effect a drop from 40 atmospheres to one gives a cooling of 10 deg. C., whereas in the adiabatic expansion such a drop gives a cooling of 175 deg. C. However, when a regenerative cooling system is used, the efficiency of the Joule-Thomson effect is greatly increased in the liquefaction columns.

Liquefaction of air, or the conversion from a gaseous to a liquid state, has long been the first step in obtaining oxygen from the air. It is accomplished by a reduction of temperature but the extent of the reduction required is great. It is lessened by increasing the pressure upon the air to be liquefied. This liquefaction step is followed by one of partial revaporization, called fractionation, in which the gases given off from the liquid are rich in nitrogen, because of its greater volatility, and the remaining liquid is richer in oxygen than the original liquid. But by simple fractionation the remaining liquid does not become pure or nearly pure oxygen until shortly before the complete vaporization of the original liquid, with the consequent loss, in the vapors, of practically all the oxygen. Rectification was added to recover some of the oxygen from the waste gases resulting from fractionation. By this process the vaporization of the liquid is brought about in the bottom of a rectification column wherein the warm vapors ascending from the boiling pool and passing up the column are brought into contact with a cold descending stream of oxygen-nitrogen liquid which reliquefies the less volatile of the ascending vapors, oxygen, causing it to join the descending stream and again to enter the vaporizing pool, while the heat of the vapor and the latent heat set free by the liquefaction of the oxygen vaporize an equivalent quantity of the more volatile constituent, nitrogen, of the descending liquid. The products of liquefaction, fractionation, and simple rectification are liquid oxygen and an effluent waste gas containing 93 per cent nitrogen and 7 per cent oxygen.

The comparative difficulty of separating air into its constituents compared to the fractional distillation of oils is at once apparent from observing Linde's curve in Fig. 17. It is obvious from the curve that it would be impossible to economically separate the nitrogen from the oxygen by a single-phase evaporation.

A diagrammatic layout of a liquefaction plant is shown in Fig. 18. Prior to the admission of air to the fractionation column, its carbon dioxide content must be removed. This removal is essential because it solidifies at comparatively high temperatures (−14 deg. C.) and would obstruct the heat interchanger or the column itself. Its removal may be accomplished by a chemical reaction between the carbon dioxide and a potassium or sodium hydroxide solution brought into intimate contact with the air by means of spraying towers.

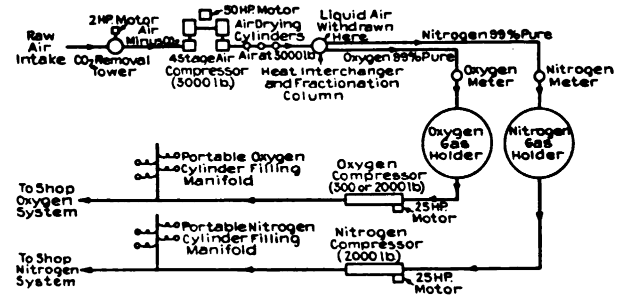

Fig. 18. Diagrammatic Layout of Liquefaction Plant

Capacity 1000 cu. ft. of oxygen and 4000 cu. ft. of nitrogen per hour.

$$2\,KOH + CO_2 \rightarrow K_2CO_3 + H_2O$$

The air which is the only raw material needed for making oxygen or nitrogen by this process, after

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*If the temperature of air be allowed to fall unchecked during expansion, without transference of heat either by radiation or heating devices, the pressure falls faster than the volume increases and the expansion is adiabatic.

If the temperature of the air be kept constant, heat being added as fast as it disappears, the expansion is isothermal.—Editor treatment in the carbon dioxide removal towers, is drawn into a specially designed air compressor capable of operating at a pressure of 3000 lb. per sq. in.

Considerable moisture is removed from the air at the various traps on the compressor and the remainder is taken out of the air as it passes through drying cylinders placed in the line running from the compressor to the heat interchanger. These drying cylinders form an essential part of the equipment. If the moisture were not removed it would freeze in the heat interchanger or column and ultimately compel the plant to be shut down for thawing out. Some producers remove moisture by means of auxiliary refrigeration. Any oil that is carried over from the compressor is removed from the system by means of an oil purger placed directly ahead of the drying cylinders. By this means also the chemical drying material is kept free from oil and thus its effective life prolonged.

From the drying cylinders the air is delivered to the heat interchanger where regenerative cooling is made possible by passing the ingoing air countercurrent to the outgoing expanded gases. The ingoing air and outgoing gases should not have a greater temperature difference than 5 deg. F. As the air passes through the interchanger, it is progressively cooled to the point of expansion where the dissipation of heat in the form of internal molecular energy produces liquefaction.

The liquid air so produced passes into the fractionation column (Fig. 20) where through concomitant liquefaction and evaporation the gaseous nitrogen, because of its greater volatility, is driven to the top of the column and oxygen in liquid form is dropped to the bottom of the column. The oxygen in liquefying gives up heat of condensation, which in turn causes the nitrogen to evaporate, and the concomitant effect of the nitrogen evaporating is to produce further drop in temperature and more liquid oxygen.

The gaseous nitrogen and liquid oxygen pass out of the column into the heat interchanger where they absorb heat from the incoming compressed air. The ratio of specific heats of the expanded gases and the compressed air is such that the interchange of heat is facilitated, and the outgoing gases finally emerge from the interchanger at room temperature and pass to their respective gasholders.

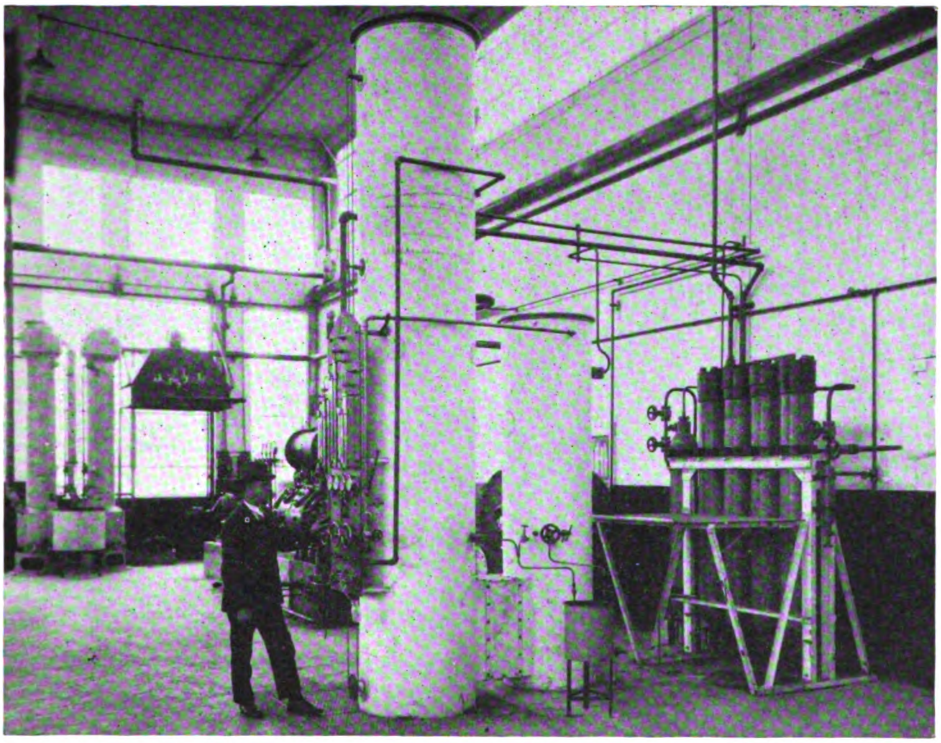

Fig. 19. Liquefaction Plant at the Schenectady Works of the General Electric Company. From left to right, the carbon dioxide towers, high-pressure compressor, liquefaction and fractionation column, heat interchanger column, oil purger and air-drying bottles

It is obvious that the attainment of such low temperatures requires some time, usually a period of from two to eight hours or more from the time of starting a warm plant to actual production of gas. The factors determining the length of this period are: the efficiency of the insulation on the interchanger and column, the pressure drop at the expansion valve, the steadiness of the operating pressure, the delivery of the air compressor, and the room temperature. The temperature of the air when entering the heat interchanger is a vital factor in the operation of the plant insofar as its efficiency in producing liquid is concerned, and for this reason any variation in room temperature is of concern.

The efficiency of this apparatus is measured in terms of cubic feet of oxygen produced per kilowatt-hour. This efficiency depends on the factors mentioned in the preceding paragraph, as well as on the conversion efficiency of the column, the volumetric efficiency of the compressor, the design of the heat interchanger, and the efficiency of removal of moisture and CO_2 from the air before it enters the interchanger and column. The oxygen produced per kilowatt-hour may vary from 25 to 50 cu. ft., depending upon the quantity and purity of the oxygen produced.

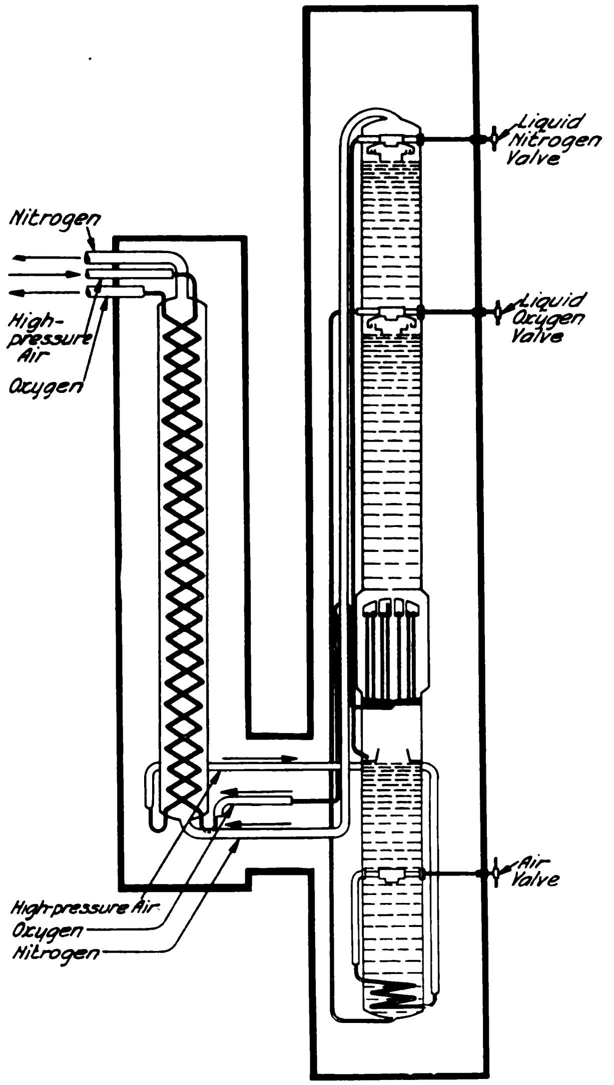

Fig. 20. Section Through a Heat Interchanger Column and a Liquefaction and Fractionation Column

Gauges, gauge piping, and details of joints omitted for clarity.

Liquid air, or more correctly liquid oxygen, can be withdrawn from the liquefaction column in any quantity within the limits of the equipment's capacity for making liquid. Such withdrawal represents a loss of "cold" which must be compensated for by increasing the pressure drop at the expansion valve. Fig. 21 shows the effect of withdrawing liquid oxygen from a plant having a capacity of about 4000 cu. ft. of oxygen per hour.

The comparative simplicity of the liquefaction process is quite apparent and the hazards involved are not great.

Electrolytic Process

The electrolytic method of producing oxygen and hydrogen is based on the phenomenon of electrolysis of water, and is followed out according to the scheme outlined in Fig. 22. A basic chemical is added to distilled water to decrease electrical resistance and the reactions are carried on in special equipment called cells, Fig. 24. The solution or electrolyte ionizes into negative and positive ions. If sodium hydroxide is used as the base, this compound breaks up into positive sodium ions and negative hydroxide ions. The water, which may be written HOH, breaks up into positive hydrogen ions and negative OH ions. The subsequent passage of an electric current through this ionized solution carries the positively charged ions to the cathode or negative terminal and the negatively charged ions to the anode or positive terminal. Subsequent electrochemical reactions result in the formation and collection of gaseous hydrogen at the cathode and gaseous oxygen at the anode.

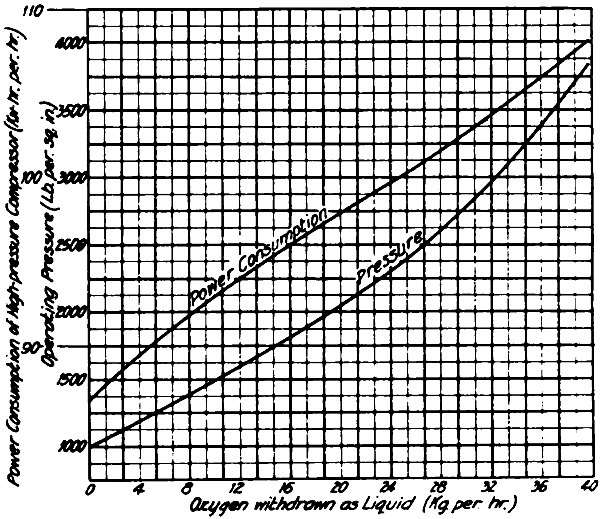

Fig. 21. Curves Showing Power Consumption of High-pressure Compressor and Operating Pressure Necessary to Permit Withdrawal of Various Amounts of Liquid Oxygen from a Liquefaction Column

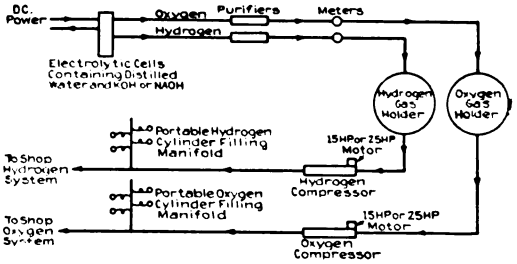

Fig. 22. Diagrammatic Layout of Electrolytic Plant

An asbestos diaphragm divides the cell into two compartments and mixing of oxygen and hydrogen is prevented by action of the surface tension of the liquid on the diaphragm. A dry diaphragm will not prevent the passage of gas through it and for this reason the proper liquid level must be maintained.

The theoretical efficiency of electrolytic equipment can be calculated by means of electrochemical equivalents which in their application to the electrolysis of water give the following: 16.2 divided by the number of volts taken per cell will give the number of cubic feet of hydrogen produced per kilowatt-hour; 8.1 divided by the voltage taken per cell will give the number of cubic feet of oxygen produced per kilowatt-hour. The volumes thus obtained are as of 760 mm. pressure and 20 deg. C. temperature. Inasmuch as the oxygen and hydrogen are produced simultaneously, the sum of their respective outputs per kilowatt-hour must be taken in comparing the electrolytic process to any other on an efficiency basis.

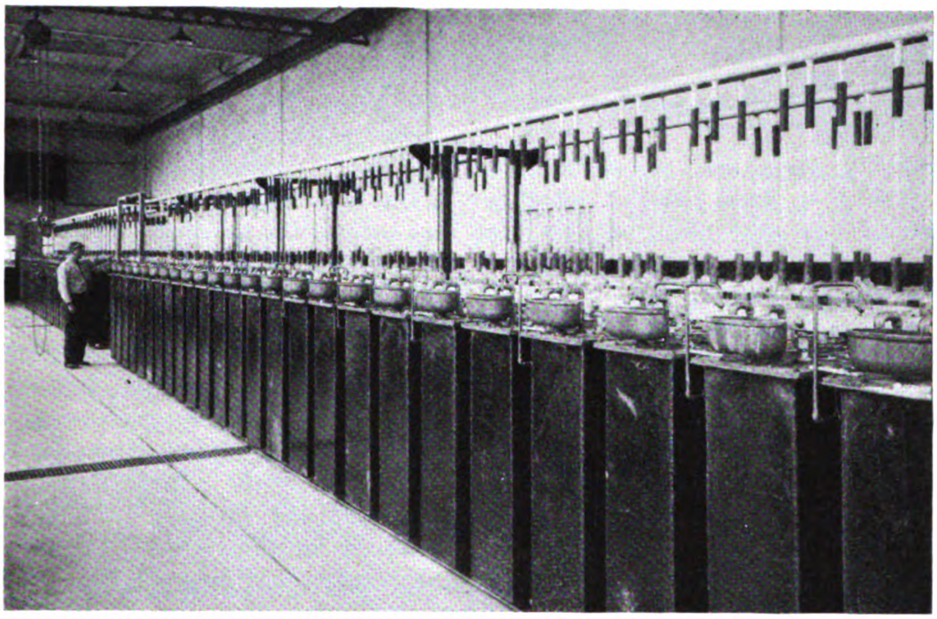

Fig. 23. Electrolytic Cell Room at the Schenectady Works of the General Electric Company. Each cell is fed with electrical energy from the positive and negative direct-current bus bars immediately above it and delivers oxygen and hydrogen separately through the glass tubes to the gas mains

Factors Influencing Choice of Process

The ideal market, whether local or national, will permit of operating liquefaction and electrolytic equipment in parallel, as shown in Fig. 25. The products of such a plant would be oxygen, hydrogen, and nitrogen. At present there are but few local or national markets that have been sufficiently developed to permit of operating such a plant on a balanced cycle.

For oxygen production alone, electrolytic equipment is usually not suitable unless the load is very small or power is exceptionally cheap. However, in the light of the possible use of atomic hydrogen for welding, molecular hydrogen as a fuel gas in cutting, and other rapidly growing uses for hydrogen, the economic situation is greatly altered. The electrolytic cells with their by-product hydrogen might be admirably adapted to such conditions.

The capital charges for a given oxygen capacity are greater with the electrolytic equipment, and so is the power consumption, but such apparatus must not be judged on the basis of its oxygen output alone. The hydrogen output must also be considered. If no market for hydrogen exists and if no market can be created, then the installation of electrolytic equipment is hardly justified.

The liquefaction process produces a by-product for which there is no national market and few local markets today. The fixation of atmospheric nitrogen in the manufacture of ammonia consumes enormous quantities of nitrogen, but the present system of compressed gas manufacture would not permit an economical distribution of the by-product nitrogen to the fixation plants. The transportation costs would be prohibitive. Therefore, except in isolated cases, the oxygen produced from the air must bear the total cost of plant operation. While oxygen produced electrolytically bears only one-third of the cost of plant operation, the application of both the gases produced must be such as to admit of their economical utilization if the electrolytic process is to compete with the liquefaction process of oxygen manufacture. Atomic-hydrogen welding or other industrial uses of hydrogen afford an excellent means of balancing the output of the electrolytic plant and enable this process to compete effectively with the other.

In addition, the electrolytic plant offers excellent opportunities for correcting the power load-factor by utilizing off-peak power, which brings down the cost of production considerably. The flexibility of operation obtained with such equipment is very important for the smaller loads which would not admit of continuous operation of the liquefaction equipment.

For years one of the greatest economic objections to the electrolytic process of oxygen manufacture has been the lack of market for the by-product hydrogen. As the national oxygen load increased and the use of acetylene displaced hydrogen in cutting, the excess of the latter became greater and the economic result of such an unbalanced condition was to force the electrolytic process out of commercial oxygen manufacturing activities.

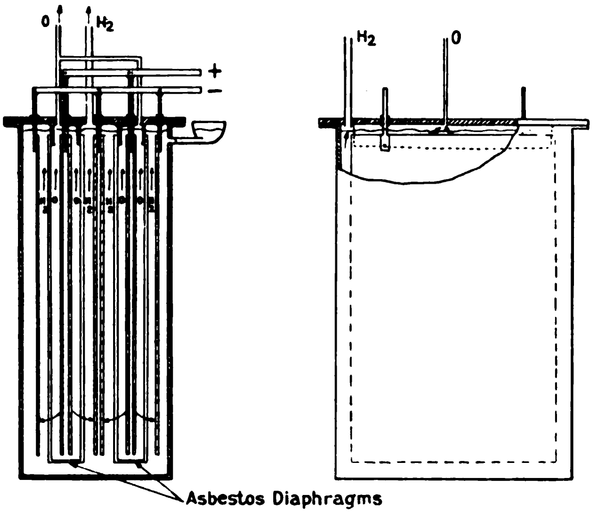

Fig. 24. Section Through an Electrolytic Cell
Minor details omitted for clarity.

Hydrogen can be, and has been, used as a fuel gas in cutting for years but until recently it could not be used in welding. Acetylene being used as a fuel in welding prompted its extended application to cutting thus reducing the number of kinds of gas required. Now the situation is subject to change.

The cutting process uses more oxygen than it does fuel gas and is obviously a poor market for electrolytic products because hydrogen is produced two to one by volume and cutting operations use at least two to one of oxygen. It is quite apparent that if much cutting is to be done the hydrogen would be greatly in excess and wasted. However, if as much welding is being done as there is cutting, the situation is changed. The use of hydrogen in atomic-hydrogen welding, as well as other applications which do not involve the use of oxygen, tends to bring the hydrogen load up to a point where a balanced condition might be possible.

TABLE VII

Efficiency of Electrolytic Oxy-hydrogen Cells Based on Electrochemical Equivalents

Energy Efficiency in Per Cent	CUBIC FEET OF GAS PER KW-HR. AT 20 DEG. C., 760 MM. PRESSURE			Corresponding Voltage per Cell at 100 Per Cent Current Efficiency *
	Both Gases	Hydrogen	Oxygen	
100	15.87	10.58	5.29	1.496
95	15.07	10.05	5.02	1.575
90	14.28	9.52	4.76	1.661
85	13.49	8.99	4.50	1.760
80	12.70	8.46	4.24	1.995
75	11.90	7.94	3.96	2.137
70	11.11	7.41	3.70	2.3
65	10.32	6.88	3.44	2.49

* The current efficiency is 100 per cent when no products other than oxygen or hydrogen are produced and when there is no rediffusion of the gases or ions throughout the electrolyte or other losses.

In addition to balancing the plant load, the use of this process of welding where possible effects further economies over oxyacetylene welding which should be credited to the electrolytic process of manufacture. Therefore when hydrogen can be used in welding, cutting, or as the reducing atmosphere in brazing and annealing furnaces, a balanced condition is produced which warrants serious consideration of the use of electrolytic equipment for the production of oxygen and hydrogen in place of liquefaction equipment for the production of oxygen alone.

TABLE VIII

BASIC VALUE FOR ELECTROLYTIC CELLS

Electrochemical equivalent of oxygen (at 20 deg. C. and 760 mm. pressure) per 1000 amp-hr.	7.9144 cu. ft.
Electrochemical equivalent of hydrogen (at 20 deg. C. and 760 mm. pressure) per 1000 amp-hr.	15.8266 cu. ft.
Electrochemical equivalent of oxygen	0.2984 grams per amp-hr.
= 0.007373 cu. ft. (at 0 deg. C. and 760 mm. pressure) per amp-hr.	
Electrochemical equivalent of hydrogen	0.0376 grams per amp-hr.
= 0.014747 cu. ft. (at 0 deg. C. and 760 mm. pressure) per amp-hr.	
Electrolysis of 1 gal. of water at 20 deg. C. 760 mm. pressure will produce approximately	89.13 cu. ft. of O_2 178.26 cu. ft. of H_2

One Faraday = 96,500 coulombs (ampere-seconds).
Density H_2 = 0.005,621 lb. per cu. ft. at 0 deg. C. 760 mm. pressure.
Density O_2 = 0.089,22 lb. per cu ft. at 0 deg. c. 760 mm. pressure.
Coefficient of expansion of oxygen per deg. C. between 0 and 100 deg. C. = 0.003,668,1 at 760 mm. pressure.
Same for hydrogen = 0.00366.

It is at once apparent that the economics of the application of the gases is an excellent means of determining which manufacturing process to use. No selection of equipment should be made until the possibility of economically exploiting hydrogen has been investigated. The market for by-product hydrogen in one case and nitrogen in the other bears a vital relation to the process used, as well as to the operating characteristics of the equipment itself. The selection should not be based on relative power efficiencies nor necessarily on total costs of plant operation. The final decision should be made with consideration to the possible economic application of the by-product gases and to the effect of that phase of activity on the total costs. The manufacture of these gases should be closely allied with their susbsequent use, and the selections of processes of manufacture should be made with respect to the overall efficiency of cutting and welding or other possible gas applications.

Recently developed liquefaction columns permit the simultaneous production of high purity nitrogen and oxygen. Such an improvement, resulting in low-cost nitrogen, should stimulate new uses for this inert gas in metallurgical operations. The creation of such nitrogen markets would thus reduce the cost of liquefaction oxygen; but it will be some time before such markets can be developed to have an appreciable effect on the ultimate cost of oxygen to the consumer who is buying it. The cost of distribution will remain a predominating factor until radical changes are made in the distribution system as it exists today.

TABLE IX

COMPARATIVE DATA FOR 1000-CU. FT. PER HR. OXYGEN PLANT

	Liquefaction Process	Electrolytic Process
Oxygen produced per kw-hr. (theoretical)	25 cu. ft.	5.29 cu. ft.
Oxygen produced per kw-hr. (actual)	24 cu. ft.	4.00 cu. ft.
Initial cost for producing equipment	$25,000	$50,000
Non-productive time required to get plant into full production	2 to 8 hr.	Almost instantly*
Number of hours per day plant must run to be most economical	24	Depends on type of cell
Rate of depreciation	10%	10%
By-products produced (theoretical)	100 cu. ft. N_2	10.58 cu. ft. H_2
By-products produced (actual)	88 cu. ft. N_2	8.00 cu. ft. H_2
Equipment area required including aisles, etc.	800 sq. ft.	2500 sq. ft.†
Ratio of labor cost per M. cu. ft.	100%	100%
Relative incidental material costs per M. cu. ft.	100%	90%
Relative maintenance costs per M. cu. ft.	100%	90%
Purity of oxygen	99.0%	99.5%
Purity of by-product	99.0% N_2	99.5% H_2
Connected load for production only	55 hp. approx.	330 hp. approx.
Ratio of other costs per M. cu. ft.	100%	100%

* A short time, together with an increased power consumption, is necessary to bring the electrolyte to normal operating temperature.

† Includes area for motor-generator sets. This varies with the type of cell but a sacrifice in operating efficiency results if floor space is favored at the expense of other design features of the cell.

Should atomic-hydrogen welding or hydrogen cooling for electric generators and the use of hydrogen atmospheres in electric brazing and annealing furnaces become quite general, it is possible that electrolytic oxygen equipment may again fill an economic need and re-establish itself in the commercial gas field as a serious competitor of liquefaction oxygen equipment.

Tables VII, VIII, and IX are included to make possible a comparison of the performance characteristics of electrolytic and liquefaction oxygen plants.

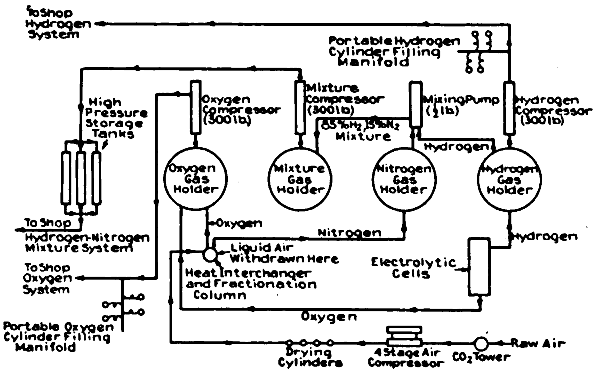
To Shop Hydrogen System
Portable Hydrogen Cylinder Filling Manifold
High Pressure Storage Tanks
Oxygen Compressor (300 lb)
Mixture Compressor (300 lb)
85% H₂, 15% N₂ Mixture
Mixing Pump (½ lb)
Hydrogen Compressor (300 lb)
Hydrogen
Oxygen Gas Holder
Mixture Gas Holder
Nitrogen Gas Holder
Hydrogen Gas Holder
To Shop Hydrogen-Nitrogen Mixture System
Oxygen
Nitrogen
Hydrogen
Liquid Air Withdrawn Here
Heat Interchanger and Fractionation Column
Electrolytic Cells
To Shop Oxygen System
Oxygen
Portable Oxygen Cylinder Filling Manifold
Drying Cylinders
4 Stage Air Compressor
CO₂ Tower
Raw Air

Fig. 25. Diagrammatic Layout of Combined Liquefaction and Electrolytic Plant

Judged on the basis of oxygen requirements only, for annual oxygen loads of over 2,000,000 cu. ft. per year, the liquefaction process seems preferable. For annual oxygen loads of less than 2,000,000 cu. ft., the electrolytic method is probably more suitable, provided of course that uses can be found for the by-product hydrogen. In view of the importance of the possible exploitation of the by-product gases, it is believed that each process has a field of application which will become of increasing importance to industrial consumers of these gases when a true realization is obtained of the economic factors involved.

(To be continued)

Industrial Electric Heating

PART V: THE CORELESS INDUCTION FURNACE

By N. R. STANSEL and E. F. NORTHRUP
General Electric Company — Ajax Electrothermic Corporation

THE use of the principle of the transfer of energy by electromagnetic induction from one electric circuit to another, *i.e.*, the transformer function, is the same in the coreless induction furnace as in the core-type induction furnace. In both types we have three successive energy conversions, *viz.*, from electric to electromagnetic, to electric, to heat.

The transition from the core-type induction furnace to the coreless induction furnace is represented by (A), (B), and (C), Fig. 74. In (A) is illustrated the principle of the core-type induction furnace; the distinguishing features of this type being an iron core for the magnetic circuit and a single-turn closed loop for the secondary electric circuit. With the removal of the iron core, as in (B), we have in principle the coreless induction furnace, which in elementary form is illustrated by (C) in Fig. 74. The fundamental parts consist only of the charge to be heated and a primary coil.

If a given voltage (E_p) is applied to the primary coil, Fig. 74 (A), the magnetic flux set up is proportional to the ampere-turns of the coil and to the magnetic conductivity of the path of the flux, and is independent of the frequency of the source of the current in the coil. The voltage (E_s) induced in the secondary circuit is proportional to the rate of change in the magnetic flux interlinking that circuit, *i.e.*, to the frequency of the circuit supplying the primary coil.

With the removal of the iron core, the conductivity of the path of the magnetic flux is reduced to the magnetic conductivity of air with the corresponding reduction in the flux. Hence for any value of frequency which may have been considered in the preceding paragraph the current in the secondary circuit—which is proportional to the secondary voltage—will be reduced in the same ratio. We can, however, with no other change in conditions raise the frequency at which the current is furnished in the primary coil until the secondary voltage, and hence the secondary current, has the same value as before. Thus in the first case, *viz.*, that of the core-type induction furnace, we deal with a large magnetic flux and a slow change in the flux, *i.e.*, a low frequency; and in the second case—that of the coreless induction furnace—with a small magnetic flux and a rapid change in the flux, *i.e.*, a high frequency.

In Fig. 74 (C) the secondary coil of definite form, indicated in Fig. 74 (B), is replaced by a secondary circuit of indefinite form, *i.e.*, a simple body of metal in a non-metallic non-electrical conducting container or crucible. For the induced currents in the charge we depend upon the penetration of the alternating magnetic flux into the body of metal. The flux density in the body of metal (or charge) falls off rapidly from the side surface towards the vertical axis, as shown by the logarithmic curve C in Fig. 75, so that even at a comparatively short distance from the side surface the flux density is very small. We can substitute for the actual area of the magnetic path within the charge an equivalent area, $l \times p$, Fig. 75, which with a uniform flux density would give the same total magnetic flux as exists in the body of metal. The value of p thus defined is termed the "depth of penetration" of alternating magnetic flux into the charge. Steinmetz (27) has shown that

$$p=\frac{3570}{\sqrt{f\lambda\mu}} \qquad (15)$$

in which

p = depth of penetration in cm.
f = frequency
λ = electrical conductivity
μ = magnetic permeability.

Thus, in accordance with the equation it is seen that the "depth of penetration" of an alternating magnetic flux is inversely proportional to the square roots of the frequency of the magnetic flux and of the two properties of the substance, electrical conductivity and magnetic permeability. Table XXI gives values of p for several metals and alloys.

As the e.m.f. developed with a given frequency at any point within the mass of a charge of metal is proportional to the magnetic flux at that point, the current densities measured along the horizontal axis of the charge will give in graphic form a logarithmic curve similar to curve C of Fig. 75. Hence in lieu of referring to the depth of penetration of alternating magnetic flux into the charge we can refer to the depth of penetration of the currents which circulate in the charge.

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(27) Steinmetz: "Transient Electric Phenomena and Oscillatings," Chap. VI, 1909 Ed.

The following table facilitates the location of material hitherto published in this serial:

Part	*Issue*	*Figs.*	*Equations*	*Examples*	*Tables*	*Footnotes*
Part I	Oct. 1927, p. 488	1 to 23	(1) to (8b)	1 to 3	I to III	(1) and (2)
Part II	Nov. 1927, p. 551	24 to 43	(9) and (10)	4 to 6	IV to VII	(3) to (13)
Part III	Dec. 1927, p. 600	44 to 57	(11) and (12)	8 to 11	VIII to IX	(14) and (15)
Part IV	Mar. 1928, p. 125	58 to 73	(13) and (14)	......	X to XX	(16) to (26)

The total current in the charge is the product of the average current density and the cross section of the path of the current. The value p now represents the average current density in the charge and the area $l \times p$ is the cross sectional area of a conductor equivalent in electrical resistance and reactance to these values for the actual path of the current in the charge. From the values of p in Table XXI and from the chart given in Fig. 76 it will be noted that in the coreless induction furnace the development of heat in a metal charge is largely concentrated in a comparatively thin shell of metal around the charge. This shell in effect forms the secondary circuit of the transformer (Cf. (B) in Fig. 74).

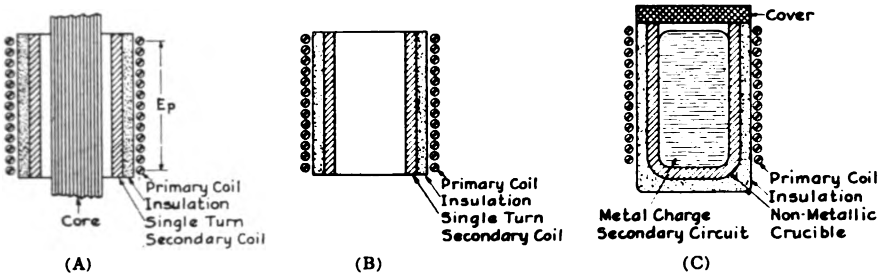

Fig. 74. Diagrams Illustrating the Transition from the Core-type Induction to the Coreless Induction Furnace

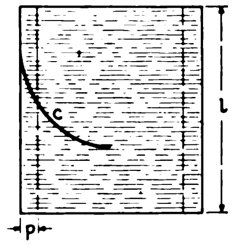

Fig. 75. The Logarithmic Gradient of Alternating Flux Penetration in a Body of Metal

When an alternating current through the primary coil, as that shown in (C) Fig. 74, reaches its maximum value, I_m, the total electromagnetic energy stored in the space contiguous to the coil is

$$W = \frac{L I_m^2}{2} \qquad (16)$$

in which

W = watt-seconds

L = coefficient of self-induction of the coil

I_m = maximum value of the current during the half-cycle.

A certain part of this energy enters the charge of metal. This is

$$W' = \frac{\theta \, L \, I_m^2}{2} \qquad (16a)$$

in which

θ = the fraction of the total electromagnetic energy which is absorbed by the charge.

TABLE XXI

VALUES OF p IN CENTIMETERS

Frequency	25		60		500		1000		10,000	
	Solid	Molten	Solid	Molten	Solid	Molten	Solid	Molten	Solid	Molten
Copper	0.92	3.22	0.60	2.12	0.20	0.70	0.15	0.51	0.05	0.16
Zinc and 78-22 Brass	1.75	4.13	1.13	2.66	0.39	0.93	0.28	0.65	0.09	0.21
60-40 Brass	1.75	4.74	1.13	3.05	0.39	1.06	0.28	0.75	0.09	0.24
Wrought Iron	0.70	....	0.05	....	....	....	0.01	....	0.004	....
Cast Iron	0.50	....	0.33	....	....	....	0.08	....	0.025	....

For a frequency f the corresponding input of power into the charge of metal is:

$$P' = 2f\,W' \qquad (17)$$

or by the substitution of Equation (16a) into (17)

$$P' = \theta \, L \, f \, I_m^2 \qquad (17a)$$

and assuming sine wave forms, $I_m^2 = 2\,I^2$,

$$P' = 2\,\theta \, L \, f \, I^2 \qquad (17b)$$

From Equation (17) it is seen that with all other conditions unchanged the rate at which energy is absorbed by the charge of metal, *i.e.*, the rate at which the metal is heated, is proportional to the frequency of the current in the primary coil. This is a basic principle of the coreless induction furnace.

The value of the factor θ of Equation (17b) depends upon the coupling between the primary and secondary circuits and approaches its maximum value as the coupling approaches the theoretically perfect relation between these two circuits.

As previously stated (Part IV) a portion of the heat developed in the secondary circuit of an induction furnace is lost by flow outwards through the furnace walls, and the maximum temperature to which a charge of metal can be raised with a given power input P' depends upon the thermal resistance between the charge and the surrounding atmosphere. This thermal resistance, *i.e.*, the heat insulation, between the charge and the primary coil of a coreless induction furnace, shown in (C), Fig. 74, functions as a one-way valve (leaking to a certain extent) in the path of the flow of energy into and out of the charge. The tighter this valve—the higher the thermal resistance—the higher the maximum temperature which can be obtained.

The ideal case of heating is that of raising the temperature of a body without a loss of heat from the substance. For such a case the time-temperature curve, assuming a constant rate of heat input and no change of state while heating, is a straight line, curve *C* in Fig. 77, the slope of which for a given constant power input depends upon the thermal properties of the substance being heated. However, as soon as the temperature of a body rises above the temperature of its surroundings heat flows from the body; and the time-temperature curve bends more and more to the right of the tangent curve *C* as the heat flow from the body increases with the rise in temperature of the body, as curve *A*, Fig. 77. This time-temperature curve is logarithmic, and has a horizontal asymptote the ordinate of which is the temperature of the body at which the rate of heat loss from the body is equal to the rate of heat absorption by the body, *i.e.*, the temperature of the body becomes constant at the maximum temperature attainable under the given conditions.

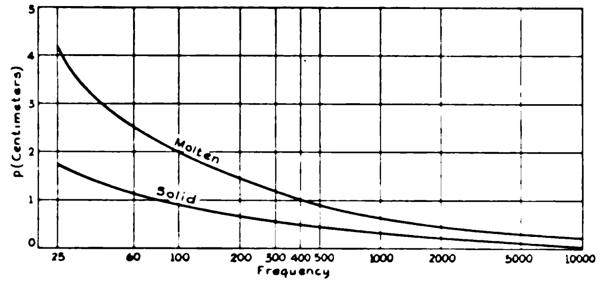

Fig. 76. Relation of the Penetration Factor of Alternating Magnetic Flux and Frequency in Melting 78-22 Brass

Consider again the body of metal which forms the charge in Fig. 74 (C). If the thermal resistance between the charge and the primary coil be higher than that assumed in the case represented by the time-temperature curve *A*, Fig. 77, the body can be raised to a higher maximum temperature by the same power input to the body and we will have another time-temperature curve, *B*. What is more important in practice, however, is that the body in the latter case can be raised to some given temperature (lower than the maximum temperature attainable in that case) within a shorter time. In lieu of increasing the thermal resistance, which with a coreless induction furnace would probably mean increasing the thickness of the heat-insulating material thereby decreasing the coupling and decreasing the value W'' in Equation (17), we can increase the value of P' of Equation (17) by raising the frequency (thus leaving W'' unchanged) and the result is the same, *i.e.*, the body is raised to the given temperature in a shorter time. From another viewpoint, the higher the thermal resistance of the materials surrounding the body—the tighter the one-way valve—the lower may be the frequency required to obtain a given temperature in the given body within a given time.

The minimum thickness of the wall of the container of the charge is fixed by the need for mechanical strength. Hence in order to obtain a reasonable value of the fraction θ, the space between the crucible and the primary coil is limited. Therefore the thickness of the wall of material around the crucible—and therefore the thermal resistance between the charge and the surrounding atmosphere—is a compromise between the selection of the frequency and the acceptable rate of heat loss; and this with particular reference to the high temperatures of metal-melting service. Under the practical conditions of furnace construction the maximum temperature attainable in coreless induction furnaces of small and moderate sizes by the use of a low frequency, say 60 cycles, are low—much lower, in fact, than the melting-points of many metals and alloys. Hence this is one of the reasons for the operation of this type of furnace at frequencies which are higher than the standards for light and power circuits.

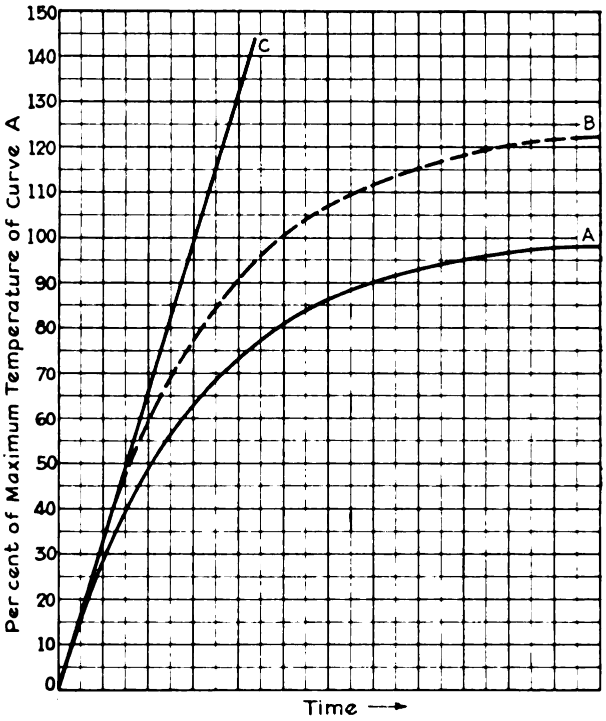

Fig. 77. Time-temperature Curves Representing Various Conditions Under which Heating Takes Place

With the coreless induction furnace the significance of frequency as expressed in Equation (17) is of moment also with reference to the conversion efficiency of the furnace. The total heat loss from the charge by flow through the walls during the heating period is the product of the average rate of this heat flow and of the time required for the heating operation. The average rate of heat flow outward depends only upon the difference between the initial and final temperatures of the charge. Hence the less the time required for heating the less is the total heat loss per charge. As, with no other change, the time required for heating depends only upon the value of P', Equation (17), and as this in turn depends only upon the value of the frequency, the higher the frequency the higher will be the conversion efficiency of the furnace.

TABLE XXII

VALUE OF p IN CENTIMETERS

Frequency	25		60		500		1000		10,000	
Temperature: Deg. C	0	1000	0	1000	0	1000	0	1000	0	1000
Material: Graphite	20	21	13	13.6	4.5	4.7	3.2	3.3	1.0	1.04

Coreless induction heating can be applied to a wide variety of shapes of charges since there is no magnetic core in the coreless induction furnace to require a certain configuration of parts. However, in melting service, the need for a container for a molten metal makes the cylindrical shape of charge practically universal. With this in mind in addition to the foregoing considerations, it will be evident that the factors which determine the selection of frequency for a coreless induction furnace for a given melting service are:

(*a*) The depth of penetration
(*b*) The diameter of the charge
(*c*) The temperature required
(*d*) The degree of heat insulation obtainable
(*e*) The amount of power required
(*f*) The cost of the electrical equipment.

An analysis of the factors (*a*) and (*b*) made by Northrup(28) is expressed by the equation:

$$f_{min.} = \frac{25 \times 10^8 \times \rho}{d^2} \qquad (18)$$

in which

$f_{min.}$ = the minimum value of the frequency
ρ = the resistivity of the charge, ohms per centimeter cube
d = diameter of the charge in centimeters.

For metal-melting service the value of ρ in Equation (18) should be the resistivity of the metal or alloy in the molten state. This equation states only the minimum value of frequency for any given metal and diameter of charge. The value of frequency to use in any given case should be as much higher as may be permissible by the other factors which influence the choice of frequency.

Example 12: With reference to the factors (*a*) and (*b*) assume a charge of 60-40 brass. From Fig. 64 the value of ρ is 44×10^{-6} ohm. The diameter of the charge is assumed to be 25 cm.

$$f_{min.} = \frac{25 \times 10^8 \times 44}{625 \times 10^6} = 176 \text{ cycles}$$

Example 13: Assume that a frequency of 480 cycles is available for melting the alloy of Example 12.

$$480 = \frac{25 \times 10^8 \times 44}{d^2 \times 10^6}$$

This gives 15 cm. as the minimum diameter of the charge for a 480-cycle furnace. The diameter can be as much larger as the economics of furnace construction permit.

Equation (18) states in general that the smaller the diameter of the charge the higher must be the minimum value of the frequency. Thus for small charges, for which the energy required for melting is small, a high frequency is indicated. For large charges and correspondingly high power inputs the frequency may be much lower than that needed for small charges.

If the charge in a coreless induction furnace is magnetic material, *e.g.*, a ferrous alloy, the high magnetic conductivity of the path of the magnetic flux in the charge aids the heating of the charge until the temperature of the metal reaches the critical temperature ranges, beyond which such material is non-magnetic.

It has been proposed to use a lower frequency, *e.g.*, 60 cycles, for heating ferrous charges up to the critical temperature range and then to continue the heating of the charge by the use of a suitable high frequency. This method may or may not be practicable depending upon the conditions in each case.

A substance that is not an electrical conductor, *e.g.*, glass, cannot, of course, be heated directly by the induction method. For such materials the crucible is made of an electrical conducting material of high electrical resistance, *e.g.*, graphite, and the wall of the crucible serves as the secondary circuit in which heat is developed. The charge is heated by the flow of heat by conduction into the mass of the charge. In such cases the surface thermal resistances oppose the heating of the charge up to the melting point of the material. As far as our knowledge of surface thermal resistances goes at the present time, the thermal resistance at the junction of a solid and a liquid is nil.

Table XXII gives the values of the depth of penetration (p) for graphite. It will be noted that for this class of material the value of p is not affected to any great extent by temperature.

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(28) Northrup: "Inductive Heating," *Journal of The Franklin Institute*, February, 1926.

The temperature gradient in heating non-electrical conducting charges is from the wall of the crucible into the charge. Hence for the melting of such charges the wall of the crucible must be carried at a higher temperature than that of the charge. This is the reverse of the condition obtained when heating metallic charges; and the conversion efficiency of the furnace is somewhat lower than when heating a metallic charge.

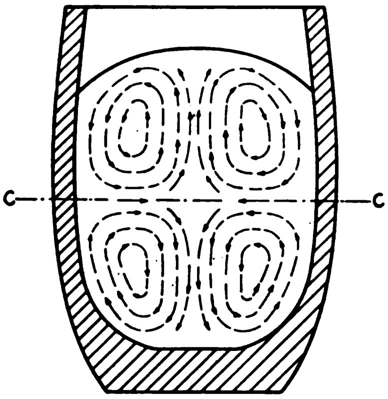

Fig. 78. The Stirring Effect Occasioned by the Circulation Within the Mass of Molten Metal Being Heated by Induced Currents

The center line of charge and of primary coil is indicated by line *c-c*.

A charge of material—either metallic or non-metallic—is seldom a one-piece solid. The more finely divided the charge, the higher will be the thermal resistances in the path of heat flow from the outer shell of the charge (in which the heat is developed) into the body of the charge. In case of metal charges, a subdivision of the charge affects also the electrical resistance of the secondary circuit (while the charge is in the solid state) by the introduction of contact resistances in the paths of the induced currents. However, contact resistances between pieces of the charge tend to make each piece of metal a separate secondary conductor; also the depth of penetration is proportional to the square root of the resistivity of the metal (Equation 15), so that on the whole, the effect of subdividing a metal charge as regards heating may be favorable or unfavorable, depending upon the resistivity of the metal and the size of the pieces. In general, subdividion of the charge does not limit the application of the coreless induction furnace for melting service. For example, it is found that "cabbage" charges of non-ferrous metal scrap are melted with peculiar facility in this type of furnace.

As soon as the molten state is reached in the melting of metals in a coreless induction furnace the electrodynamic forces in the liquid conductor, together with the force of gravity, cause motion or circulation in the molten mass. This movement of the molten metal in a crucible is indicated by the arrows in Fig. 78. Thus the molten charge is stirred automatically. The stirring is proportional in degree to the power input to the furnace and, being thus under control, can be adjusted to any degree desired to suit the metallurgical phase of the melting operation. For the refining of a molten charge the power input to the furnace is reduced to that required for holding the metal temperature, and during refining the metal can be held in a quiescent state or stirred to whatever degree desired by regulating the power input to the furnace.

The power input to the charge (while in the molten state) is given(29) as

$$P' = \frac{0.45\,\theta\,E\,I}{\sqrt{\theta^4+2-2\,\theta^2}} \tag{19}$$

in which

P' = input to the charge in watts.

E = voltage applied to the primary coil of the furnace.

I = current in the primary coil.

θ = as before, the fraction of the total electric magnetic energy absorbed by the charge.

The power input to the primary coil of a coreless induction furnace of the general arrangement shown in Fig. 74 (C) is

$$P = \frac{P'}{\psi_c} \tag{20}$$

in which ψ_c = conversion efficiency of the furnace, expressed in per cent.

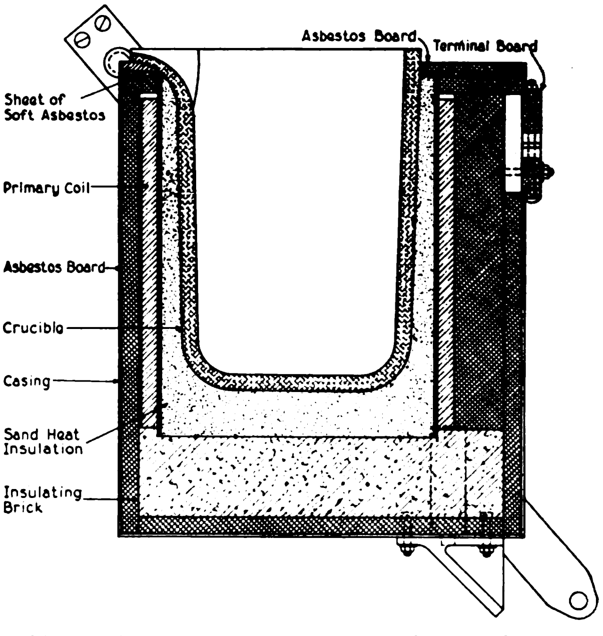

Fig. 79. A Sectional Diagram of a Coreless Induction Furnace

Then Equation (2) becomes

$$P = \frac{0.45\,\theta\,E\,I}{\psi_c\sqrt{\theta^4+2-2\,\theta^2}} \tag{21}$$

Equation (21) states that for the theoretical coupling between the primary and secondary circuits represented by $\theta = 1.0$, the power-factor of the primary circuit of the furnace with a molten metal charge would have the value 0.45; *i.e.*, the power-factor of this circuit in any actual case will be some value less than 0.45.

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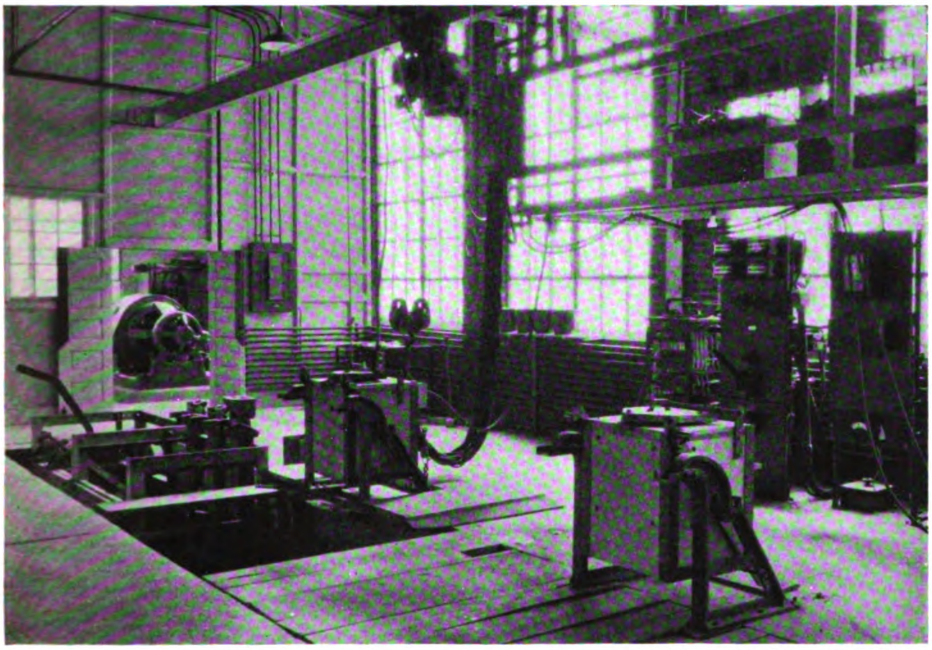
(29) Northrup: "Electric Heating by Ironless Induction." GENERAL ELECTRIC REVIEW, November, 1922, p. 656.

The power-factor of the primary circuit of a coreless induction furnace depends upon the ratio of the squares of the diameters of the charge and of the primary coil, the frequency, and the electrical properties of the charge. For less than full charges of metal the degree of coupling will vary with the height of the metal in the crucible. The smaller the charge the less the value of the fraction θ and the more this relation tends to lower the power-factor. On the other hand, the smaller the charge the higher is the resistance of the secondary circuit. The result of these two opposing factors is that, for a given metal or alloy, the power-factor of the furnace circuit does not vary over a wide range for different weights of charges. If the crucible is used as the secondary circuit, as in melting non-conducting materials, the power-factor of the furnace circuit is not affected by the weight of the charge. The value of the fraction θ is selected to suit conditions in each case. For metal-melting furnaces an average value of θ is 0.61 when the charge is in the molten state and the corresponding value of the power-factor, from Equation (21), is 0.23. If in all cases the power-factor of the furnace circuit is corrected to the degree desired for average operating conditions, there will be but little need for adjustments of the corrective effect for varying weights of charges.



(Courtesy of Ajax Electrothermic Corporation)

Fig. 80. A 150-kw. Coreless Induction Furnace Installation, Showing the Frequency Changer in the Background

While some metals are more amenable to the use of the coreless induction furnace for melting than others (see Table XVI) this type of furnace is not limited in its application to metal-melting service. The electrodynamic forces in the charge after the molten state is reached is not a limiting factor, and a high power input to the charge is permissible throughout the entire range of the time-temperature curve of the operation. The freedom in this respect makes practicable the rapid melting of charges and gives rise to the term "high-speed melting," often used with reference to this type of furnace. The high conversion efficiency—in terms of lbs. per kw.-hr.—resulting from rapid melting is an important factor in the economy of melting service; but more often the high rate of production is a more important factor in the economy of manufacturing operations.

The only temperature limitation of this type of furnace is the temperature limit of the material of the container of the charge. However, as previously noted, the maximum attainable temperature in any given case is that fixed by the maximum input of power into the charge and the degree to which the charge can be insulated against the loss of heat. The container for the charge of a coreless induction furnace is a crucible of the ordinary form or a crucible-shaped cavity formed in a refractory. Hence with this type of furnace the refractory problem is in its simplest form. With the coreless induction furnace the heating operation starts with a cold charge (or with a molten charge if there is any occasion for such procedure) and if the charge does not expand while cooling, it may be allowed to freeze while in the furnace if it is not convenient to pour after melting. The crucible can be sealed during the melting process unless there are materials (water, oil, etc.) in the charge which give off gases when heated. If the crucible is closed the atmosphere in contact with the surface of the metal can be made neutral or reducing as may be needed in any particular case. Obviously various modifications of the crucible to suit particular conditions can be made; one of these is the use of a sealed crucible for melting in a vacuum.

The diagram in Fig. 79 illustrates the construction of the small and moderate sizes of single-phase coreless induction furnaces for melting service. For these sizes the frequencies range from 500 to 2000 cycles and voltages from 400 to 2000 volts, both according to the kind of service and the dimensions of the furnace. A typical installation of a furnace of this type is shown in Fig. 80.

Perhaps the most interesting feature in the design of this furnace is the single-layer primary coil, shown separately in Figs. 81 and 82. The single-layer design is based upon the following considerations:

(*a*) The strength of the magnetic field is proportional to the ampere-turns of the coil. The amperes in a given coil are limited by the rate at which heat can be dissipated from the coil and by the insulation between turns. Neglecting the space required for insulation, the I^2R heating effect for a given number of ampere-turns is independent of this relation between the number of turns and number of layers. For example, if in a given coil space we use two layers instead of one layer we would have twice the number of turns, four times the ohmic resistance, and one half the current; but the I^2R loss would be unchanged, *i.e.*,

$$\left(\frac{I}{2}\right)^2 4\,R=I^2R$$

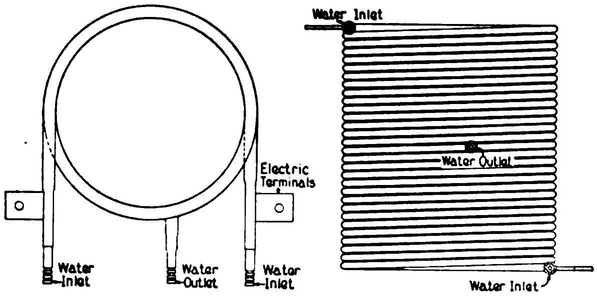


Fig. 81. A Primary Coil of the Water-cooled Type for a Coreless Induction Furnace

(*b*) The voltage at the primary coil terminals is comparatively high and the electrical insulation of the turns of the coil becomes more and more a problem, both as to the insulation property and space occupied, as the number of layers in a given coil space is increased.

(*c*) With high frequencies the distributed capacity will be increased with consequent increase in heat loss by the use of a multiple-layer coil.

(*d*) It is essential to bring the primary coil as close to the charge as thermal conditions will permit. The thermal resistance between the charge and the primary coil is limited by both the space and the available materials for heat insulations so that the primary coil must serve as a dissipator for a large part of the heat loss of the furnace. A single-layer coil makes simple the carrying away of the heat received by the coil from the furnace and the I^2R loss of the coil by the use of a hollow conductor and a flow of water through the coil. For a given frequency, the maximum rate of heating is a matter of the maximum permissible ampere-turns of the primary coil. For a given coil the maximum ampere-turns is determined by the voltage applied to the terminals which in turn is limited by the electrical insulation of the coil. With water cooling the coil is maintained at a comparatively low temperature; and the electrical insulation can be made of high degree so that the rate of the energy transferred at coil will be correspondingly high. In practice, using moderate pressures, the temperature rise of the water passing through the coil does not exceed 25 deg. C. and the average temperature of the coil is below 50 deg. C. However, water cooling of the primary coil is not always necessary. If not, the primary coil as before is a single-layer coil made of flat copper strap, wound edgewise.

The selection of the crucible depends upon the nature of the charge to be melted. As previously noted the crucible may be self-contained or may be formed as a cavity in a refractory material, acid or basic as required. For carbon-free melting of ferrous alloys with pouring temperatures around 1500 deg. C. (2732 deg. F.) a self-contained crucible made of moulded silica sand is often used. The space between the crucible and the primary coil is packed with silica sand as shown in Figs. 79 and 82. As the inner diameter of this type of crucible increases with successive melts new crucible material forms on the outer side of the crucible and this action materially prolongs the life of the crucible.

The furnace shown in Fig. 80 is arranged for pouring by tilting, the mechanism being a more or less conventional arrangement of parts. The coreless induction furnace is flexible in design as regards the provision for removal of the molten metal and standard types include, according to the size:

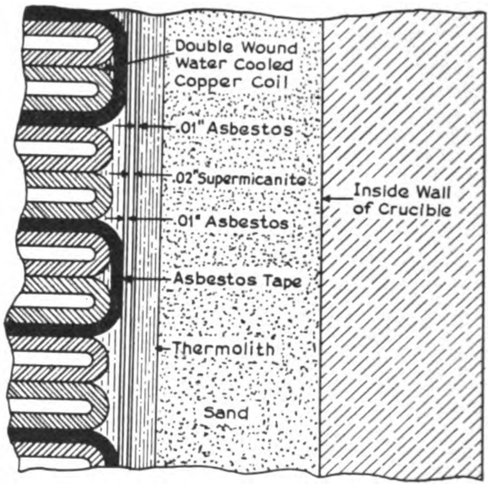


Fig. 82. An Enlarged Section of a Double-wound Single-layer Water-cooled Primary Coil for a Coreless Induction Furnace

(*a*) Portable furnaces
(*b*) Stationary furnaces for dipping
(*c*) Drop coil furnaces. The furnace structure is raised and lowered around a stationary crucible.
(*d*) Lift coil furnace. The reverse of the drop coil crucible.
(*e*) Tilting furnaces with center of rotation through the spout.
(*f*) Large furnaces arranged for tapping.

The circuit diagram of a coreless induction furnace installation is shown in Fig. 83. This diagram applies only to the larger size installations in which rotating apparatus is used to supply the high-frequency current and in which capacitors are connected in parallel with the primary coil of the furnace. For small furnaces the high-frequency current is supplied by a mercury discharge gap oscillator. This apparatus, together with rotating high-frequency generators, high-frequency capacitors, switching apparatus and methods of power control for coreless induction furnace installations will be included in the subject matter of a later part of this serial under the title "Electrical Apparatus for Electric Heating Equipment."

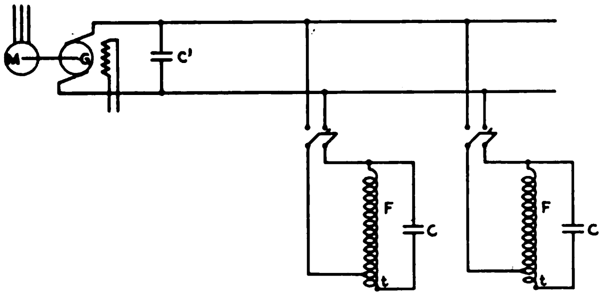


Fig. 83. The Diagram of Connections for a Single-phase Coreless Induction Furnace Installation of Two (or more) Furnaces in Parallel

M—Motor
G—Single-phase separately-excited generator
C—Capacitor for each furnace
C'—Line capacitor, which may or may not be used depending upon specific conditions
t—End section of furnace coil in series with capacitor.

The conversion efficiency, operating efficiency and melting rate of a coreless induction furnace depends—as in the case of other types of furnaces—upon the size of the installation, nature of charge, pouring temperature, and upon the method of operation. The following data are typical of the results obtained with a small unit in commercial service melting a particular alloy:

Charge	322 lb.	36 per cent iron 36 per cent nickel 5 per cent copper 23 per cent scrap
Crucible		11 in. inside diameter 16½ in. inside depth
Size of generator		150 kw.
Frequency		2000 cycles
Voltage		900 volts
Power-factor of circuit to furnace		1.0 (average)
Melting time		30 min.
Melting rate		644 lb. per hr.
Temperature of cooling water at exit		45 deg. C.
Kw-hr. per charge		67.5
Conversion efficiency of furnace alone		420 kw-hr. per ton
Conversion efficiency of unit, *i.e.*, including frequency changer at 72 per cent efficiency		585 kw-hr. per ton

The extent of the present use of the coreless induction furnace in installations of 50 kw. and over in metal-melting service is shown in Table XXIII. In addition to the installations noted in this table there are a large number of small furnaces both in this country and abroad which have been in service for a number of years.

In accordance with the logical mode of development of new types of apparatus, the design and technique of the coreless induction furnace was first developed along the more simple lines of the single-phase non-shielded furnace. As two or more such furnaces can be balanced on a polyphase circuit this imposes no handicap on the full utilization of the rotating type high-frequency generator. Moreover, the external magnetic field of small and moderate sizes of furnaces is not a serious matter. With the growth in the use of this type of melting equipment there has arisen the demand for large units; and with this demand three additional phases of design are readily apparent; *first*, the furnace must be made a polyphase unit; *second*, the external magnetic field is not permissible; and *third*, the problem of mechanical strength in the furnace design. The solution of the problems thus presented form a second chapter in the treatment of the coreless induction furnace and will be the subject of a later article in this series. For the present it will suffice to say that furnaces of this type for melting steel are available in sizes of one ton and upward.

An analysis of the conditions in each case is essential for the economic use of heat in manufacturing. This holds true for an understanding of the possibilities of the coreless induction furnace as a melting unit. Coreless induction heating applied to melting or to other heating service is not a substitute for other methods of electric heating save where its use is economically justified. This justification may come about through a variety of conditions so that each case must be studied with reference to its application. While the use of high frequencies for induction heating is comparatively new with respect to the general use of electric heat, the application has extended over a considerable period of time both in this country and abroad; and the field to which it is suited in an economic sense is fairly well defined. This field, as regards melting of metals, lies mainly in the production of high-grade ferrous alloys of uniform composition from melt to melt, the production of pure metals and alloys of all kinds, for the production of high pouring temperatures, and for rapid melting in all classes of melting service.

TABLE XXIII

Installation	Size	Frequency	Voltage	Metal Melted
1	50	. . .	. . .	Steel
2	50	. . .	. . .	Steel
3	60	960	900	Steel
4	60	960	440	Steel
5	100	480	1800	Silver
*6	100	500	1800	Steel
*7	100	1800	1900	Nickel
*8	100	500	. . .	Nickel
9	150	2000	900	Steel
10	150	2000	900	Steel
*11	150	2200	. . .	Steel
*12	150	800	900	Steel & brass
13	1500	480	1800	Nickel-silver

*Installations in foreign countries. All others in the United States.

(*To be continued*)

Heaviside's Operational Calculus as Applied to Engineering and Physics

Part VII: Procedure When the Expansion Theorem Cannot be Used Because All Roots are not Different or Some Roots are Zero
Part VIII: Simplifications which Can be Made When the Roots are Conjugates
Part IX: Problem Involving Mutual Inductance
Part X: The Effect of Suddenly Changing the e.m.f. in a System or of Short-circuiting a Network
Part XI: The Effect of Short-circuiting Certain Sections of a Network and of Introducing New Impedances by *Closing* a Switch
Part XII: The Effect of Suddenly Changing the Circuit Constants by *Opening* a Switch
Part XIII: Heaviside's Shifting

By DR. ERNST JULIUS BERG
Consulting Engineer, General Electric Company
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CONSIDER first the case when one root is zero. Such a condition would arise if for instance it were desired to find the number of coulombs supplied by a storage battery after the switch was closed on an inductive circuit.

The current then is $i=\frac{E}{r+pL}1$

and the number of coulombs is $\int_0^t idt$, or $\frac{i}{p}$

$$\therefore \text{Coulombs}=\frac{E}{p(r+pL)}1$$

In this case $Z_{(p)}=p(r+pL)$ and one root is zero. Thus the Expansion Theorem could not be applied.

The procedure then is to determine by the Expansion Theorem the value of $\frac{E}{r+pL}1$ that is: $\frac{E}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$.

Then, since $\frac{1}{p}$ means integration between the limits zero and t, we get

$$\text{Coulombs}=\int_0^t \frac{E}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]dt.$$

Therefore, when one root is zero an integration has to be resorted to.

In general if $i=E\frac{Y_{(p)}}{p(p-\alpha)\,(p-\beta)\ldots}1$, one root is zero. In this case determine the value of $i_1=E\frac{Y_{(p)}}{(p-\alpha)\,(p-\beta)\ldots}1$ and later find i by integration, that is: $i=\int_0^t i_1\,dt$

When two or more roots are equal, the simplest way is to assume that they differ slightly in which case the Expansion Theorem can be applied. The correct answer is later obtained by finding the value of these expressions in t as the roots approach equality, or else working out suitable new operators, as will be shown below.

Consider the circuit shown in Fig. 10, consisting of a resistance, inductance, and condenser in series. Then obviously the operational solution for the current is:

$$i=\frac{E}{r+pL+\frac{1}{pC}}1 \tag{21}$$

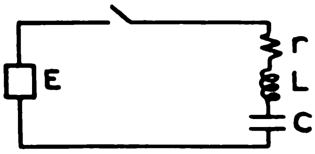


Fig. 10

If the equation is left in its present form

$$Y_{(p)}=1$$

$$Z_{(p)}=r+pL+\frac{1}{pC}$$

Thus $Z_{(p)}=0$ gives

$$\left.\begin{aligned}p_1&=-\alpha+j\omega\\ p_2&=-\alpha-j\omega\end{aligned}\right\}\text{ when }\frac{1}{CL}>\frac{r^2}{4L^2}$$

where

$$\alpha=\frac{r}{2L},\qquad \omega=\sqrt{\frac{1}{CL}-\alpha^2}=\sqrt{\omega_0{}^2-\alpha^2}$$

$$\therefore\ \alpha^2+\omega^2=\omega_0{}^2\text{ if }\omega_0=\sqrt{\frac{1}{LC}}$$

$$d\frac{Z_{(p)}}{dp}=L-\frac{1}{p^2C}$$

$$\therefore\ p\frac{dz}{dp}=pL-\frac{1}{pC}=\frac{p^2LC-1}{pC}$$

$$p\frac{dZ}{dp} \text{ for } p=p_1 \text{ then becomes } +\ 2j\omega L$$

and

$$p\frac{dZ}{dp} \text{ for } p=p_2 \text{ becomes } -\ 2j\omega L$$

$$\frac{Y_{(0)}}{Z_{(0)}}=0$$

$$\therefore i=E\left[\frac{\epsilon^{(-\alpha+j\omega)t}}{2j\omega L}-\frac{\epsilon^{(-\alpha-j\omega)t}}{2j\omega L}\right]=\frac{E}{\omega L}\epsilon^{-\alpha t}\left[\frac{\epsilon^{j\omega t}-\epsilon^{-j\omega t}}{2j}\right]$$

$$=E\frac{\epsilon^{-\alpha t}}{\omega L}\sin \omega t$$

It is readily proved that if $\frac{1}{CL}<\frac{r^2}{4L^2}$, then

$$i=E\frac{\epsilon^{-\alpha t}}{L\beta}\sinh \beta t$$

where

$$\beta=\sqrt{\frac{r^2}{4L^2}-\frac{1}{CL}}$$

The case of equal roots, that is, when $\frac{1}{CL}=\frac{r^2}{4L^2}$. As these approach equality, ω approaches zero, thus $\sin \omega t$ approaches ωt

$$\therefore i=E\frac{\epsilon^{-\alpha t}}{\omega L}\omega t=Et\frac{\epsilon^{-\alpha t}}{L}$$

But we could have obtained the same relation in another way. Equation (21) might also be written

$$i=\frac{pCE}{rpC+p^2CL+1}\mathbf{1}$$

or
$$i=\frac{pCE}{CL\left(p^2+\frac{r}{L}p+\frac{1}{CL}\right)}\mathbf{1}=\frac{pE}{L\left(p^2+\frac{r}{L}p+\frac{1}{CL}\right)}\mathbf{1}$$

$$=\frac{pE}{L(p+\alpha)^2}\mathbf{1}$$

The question then, is, what is the meaning of

$$\frac{E}{L}\frac{p}{(p+\alpha)^2}\mathbf{1}\ ?$$

Note that
$$\frac{d}{d\alpha}\frac{p}{p+\alpha}=-\frac{p}{(p+\alpha)^2}$$

but
$$\frac{p}{p+\alpha}\mathbf{1}=\epsilon^{-\alpha t}$$

$$\therefore \frac{d}{d\alpha}\frac{p}{p+\alpha}\mathbf{1}=-t\,\epsilon^{-\alpha t}$$

$$\therefore \frac{p}{(p+\alpha)^2}\mathbf{1}=t\,\epsilon^{-\alpha t}$$

and
$$i=\frac{E}{L}t\,\epsilon^{-\alpha t}$$

In the example calculated above which gave a pair of conjugate roots the calculation was not very laborious. This was largely due to the fact that $Y_{(p)}$ was unity. This is, of course, not always the case.

PART VIII: SIMPLIFICATION WHICH CAN BE MADE WHEN THE ROOTS ARE CONJUGATES

In general, the operational equation may be written:
$$i=E\frac{Y_{(p)}}{Z_{(p)}}\mathbf{1}=E\frac{Y_{(p)}}{p^2+a_1p+b}\mathbf{1}$$

which under certain conditions give two roots,

$$-\alpha\pm j\omega.$$

It can easily be proved that when the roots of $Z_{(p)}=0$ are conjugates, $p\frac{dZ}{dp}$, for $p=p_1$ and for $p=p_2$, must also be conjugate, and so also the two values of $Y_{(p)}$. And finally the two expressions of $\frac{Y_{(p)}}{p\frac{dZ}{dp}}$ are conjugates.

Let the complex expression of $\frac{Y_{(p)}}{p\frac{dZ}{dp}}$ for $p=-\alpha+j\omega$ be $\underset{\cdot}{A}=c+jc_1$, or $\underset{\cdot}{A}=A\epsilon^{j\delta}$, where $A=\sqrt{c^2+c_1^2}$ and $\tan\delta=\frac{c_1}{c}$. Then the corresponding relation for

$$p=p_2=-\alpha-j\omega \text{ is } \underset{\cdot}{B}=c-jc_1, \text{ or } \underset{\cdot}{B}=A\epsilon^{-j\delta}.$$

Therefore the two terms under the summation sign become

$$A\epsilon^{j\delta}\epsilon^{(-\alpha+j\omega)t}+A\epsilon^{-j\delta}\epsilon^{(-\alpha-j\omega)t}$$
$$=A\epsilon^{-\alpha t}\left[\epsilon^{j(\omega t+\delta)}+\epsilon^{-j(\omega t+\delta)}\right]$$
$$=2A\epsilon^{-\alpha t}\cos(\omega t+\delta)$$

$$\therefore i=E\left[\frac{Y_{(0)}}{Z_{(0)}}+2A\epsilon^{-\alpha t}\cos(\omega t+\delta)\right] \tag{22}$$

This method will be demonstrated below.

Find the voltage consumed by the inductance L in Fig. 10 after the switch is closed.

We have $i=\dfrac{E\mathbf{1}}{r+pL+\frac{1}{pC}}$

$$\therefore e_l=pLi=\frac{pLE}{r+pL+\frac{1}{pC}}\mathbf{1}=\frac{p^2E\mathbf{1}}{p^2+p\frac{r}{L}+\omega_o^2}$$

$$=\frac{p^2E}{p^2+2\alpha p+\omega_0^2}\mathbf{1}$$

$Z=0$ gives $p=-\alpha\pm j\omega$ where $\omega=\sqrt{\omega_0^2-\alpha^2}$ and $\omega_0^2=\frac{1}{CL}$

$$\frac{dZ}{dp}=2\,(p+\alpha)\;\therefore\;p\frac{dZ}{dp}=2p\;(p+\alpha)$$

and

$$\frac{Y_{(p)}}{p\dfrac{dZ}{dp}}=\frac{p}{2\,(p+\alpha)}.$$

For $p=-\alpha+j\omega$ this becomes

$$\frac{-\alpha+j\omega}{2\,(-\alpha+j\omega+\alpha)}=\frac{-\alpha+j\omega}{2j\omega}=\frac{j\alpha+\omega}{2\omega}$$

$$\therefore A=\frac{\sqrt{\alpha^2+\omega^2}}{2\omega}\underline{|\delta}\text{ where }\tan\delta=\frac{\alpha}{\omega}$$

or

$$A=\frac{\omega_0}{2\omega}\epsilon^{j\delta}$$

$$\therefore e_l=E\left[\frac{Y_{(0)}}{Z_{(0)}}+\frac{\omega_0}{\omega}\epsilon^{-\alpha t}\cos\,(\omega t+\delta)\right]$$

but

$$\frac{Y_{(0)}}{Z_{(0)}}=0$$

$$\therefore e_l=E\frac{\omega_0}{\omega}\epsilon^{-\alpha t}\cos\,(\omega t+\delta).$$

For $t=0$,

$$e_l=E\frac{\omega_0}{\omega}\cos\delta=E\frac{\omega_0}{\omega}\frac{\omega}{\omega_0}=E.$$

PART IX: PROBLEM INVOLVING MUTUAL INDUCTANCE

Fig. 11 is a diagram of two circuits having mutual inductance for which the conventional relation is of course,

$$e_1=i_1r_1+L_1\frac{di_1}{dt}+\frac{1}{C_1}\int i_1dt+M\frac{di_2}{dt}$$

$$0=i_2r_2+L_2\frac{di_2}{dt}+\frac{1}{C_1}\int i_2dt+M\frac{di_1}{dt}.$$

Thus under Heaviside conditions

$$E\mathbf{1}=i_1\left(r_1+pL_1+\frac{1}{pC_1}\right)+i_2pM=i_1a_1+i_2b$$

$$0=i_2\left(r_2+pL_2+\frac{1}{pC_2}\right)+i_1pM=i_2a_2+i_1b.$$

From the latter equation $i_2=-\frac{i_1b}{a_2}$

$$\therefore E\mathbf{1}=i_1a_1-i_1\frac{b^2}{a_2}=i_1\frac{(a_1a_2-b^2)}{a_2}$$

$$\therefore i_1=\frac{Ea_2}{a_1a_2-b^2}\mathbf{1}$$

and

$$i_2=-\frac{Eb}{a_1a_2-b^2}\mathbf{1}$$

In these equations

$$a_1=r_1+pL_1+\frac{1}{pC_1}$$

$$a_2=r_2+pL_2+\frac{1}{pC_2}$$

$$b\;=pM$$

Special Case

The condensers are omitted in each circuit.

$E=500$ volts $\quad r_1=100$ ohms $\quad r_2=20$ ohms

$L_1=0.4$ henrys $\quad L_2=0.1$ henrys

$M=0.6\sqrt{L_1L_2}=0.12$ henrys

Verify that the operational solution for the primary current is:

$$i_1=500\frac{20+0.1\,p}{2000+18\,p+0.256\,p^2}\mathbf{1}$$

and that when the Expansion Theorem has been applied

$$i_1=5\,[\,1-0.412\,\epsilon^{-138t}-0.6\,\epsilon^{-565t}]$$

The secondary current is: $i_2=-5.5\,[\epsilon^{-138t}-\epsilon^{-565t}]$

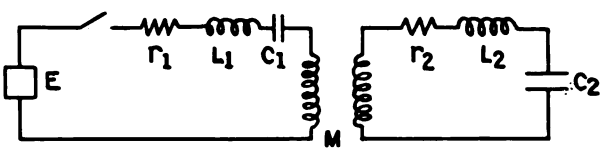


Fig. 11

Problem

If instead of applying a storage battery of 500 volts to the primary circuit a 60-cycle alternator with the same maximum voltage were used, and if it so happened that the switch were closed at the instant when the e.m.f. wave passed through its zero value, the voltage wave would be 500 sin $377t$.

The operational equation for the primary current would then be:

$$i_1=500\frac{377\,p}{p^2+377^2}\times\frac{20+0.1\,p}{2000+18\,p+0.0256\,p^2}\mathbf{1}$$

and the numerical value after the Expansion Theorem is applied would be:

$$i_1=3.06\sin\,(\omega t-41.3°)+0.66\,\epsilon^{-138t}+1.365\,\epsilon^{-565t}$$

PART X: THE EFFECT OF SUDDENLY CHANGING THE E.M.F. IN A SYSTEM OR OF SHORT-CIRCUITING A NETWORK

Originally switch s_1 in Fig. 12 is open and s_2 closed.

At $t=0$ switch s_1 is closed when a current

$$i=\frac{E_1}{r+pL}\mathbf{1}=\frac{E_1}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$$

flows in the circuit.

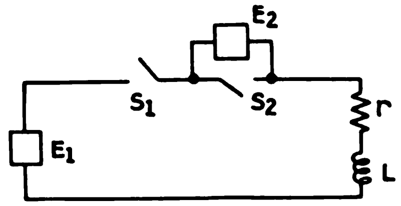


Fig. 12

At $t=t_1$ switch s_2 is opened which introduces an additional voltage E_2. This voltage causes a current

$$i_2=\frac{E_2}{r+pL}\mathbf{1}=\frac{E_2}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$$

to flow which is superimposed on i_1 and begins of course at $t=t_1$.

If E_2 is equal but opposite to E_1 then i_2 is negative as shown by the dotted line in Fig. 13 and the resultant current is shown by the heavy line.

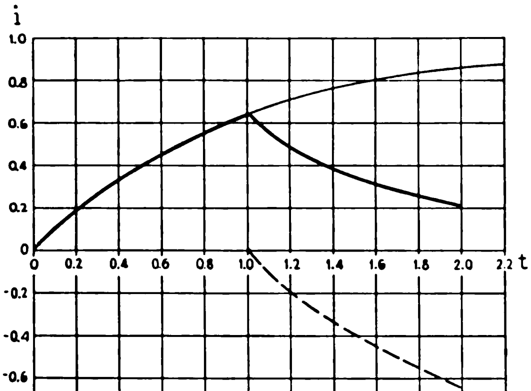


Fig. 13

Up to time t_1 the current is $\frac{E_1}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$; after that t_1 it is $\frac{E}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]-\frac{E}{r}\left[1-\epsilon^{-\frac{r}{L}(t-t_1)}\right]$.

Obviously after $t=t_1$ there is no battery e.m.f. in the circuit, so that the circuit is short-circuited upon itself.

Special Case

If the short-circuit takes place some time after the battery has been connected so that the system has reached steady state, then the current due to the original e.m.f. is $E\frac{Y_{(0)}}{Z_{(0)}}$

and the current due to the fictitious negative voltage is

$$-E\left[\frac{Y_{(0)}}{Z_{(0)}}+\Sigma\frac{Y_{(p)}}{p\frac{dZ_{(p)}}{dp}}\epsilon^{pt}\right]$$

where t is counted from the time of short-circuit. Therefore the current after the short-circuit is the sum of these two or

$$i=-\Sigma\frac{Y_{(p)}\epsilon^{pt}}{p\frac{dZ_{(p)}}{dp}}$$

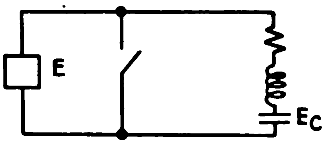


Fig. 14

The rule is therefore: Write out the operational solution covering the initial condition (the starting condition), solve this equation by the Expansion Theorem, omit the constant term, and reverse the sign before the summation term.

In this particular circuit it is easily seen that after the short-circuit $i=\frac{E}{r}\epsilon^{-\frac{r}{L}t}$ but $\frac{E}{r}$ is the current that existed when the short-circuit took place. Thus the original current dies down according to the exponential law.

Another Example

Find the discharge current of a condenser initially charged to a potential difference E_c from a battery of voltage E in the circuit represented in Fig. 14.

$$i=-\frac{E\mathbf{1}}{r+pL+\frac{1}{pC}}=\frac{E_c\mathbf{1}}{r+pL+\frac{1}{pC}}. \tag{23}$$

Find the voltage across the condenser after the discharge has begun.

$$e_c=\frac{i}{pC}=\frac{E_1}{pC\left(r+pL+\frac{1}{pC}\right)}\mathbf{1}$$

where the transient term alone is used after the solution from the Expansion Theorem has been found.

At first glance Equation (23) certainly seems strange. It contains $E_c\mathbf{1}$ yet we know that the voltage of the condenser is going to decrease most likely through oscillations until it becomes zero. It is far from a unit function.

It must be remembered that Equation (23) was not derived under any assumptions of a steady condenser voltage. It happened to come out that way through mathematical reasoning.

It is interesting to note that in the case of a discharge of a condenser, the current is identical to the original charging current except for reversed direction.

Other Examples

The oscillations in mechanical systems are similar to those in electric circuits. When dealing with springs of negligible mass compared with the mass of the moving body, and when the frictional force is proportional to the velocity, the equations become identical.

The dotted line in Fig. 15 is the position of equilibrium. A force has been applied to the mass so that it has been displaced a distance x_0. Find the position of the mass when the force is suddenly removed.

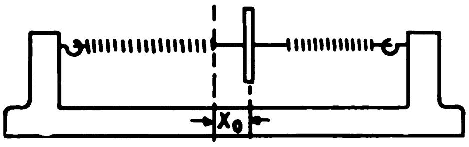


Fig. 15

Let the friction force be $fv = f\dfrac{dx}{dt}$

Let the force necessary to compress $= kx$

The force necessary to accelerate the mass is $= M\dfrac{d^2x}{dt^2}$.

Then if a steady force F is applied to bring the mass to position x_0 the following relation exists:

$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + kx$$

or, with Heaviside's notation,

$$F\,\mathbf{1} = M\,p^2x + fpx + kx = x\,(p^2M + pf + k)$$

$$\therefore x = \frac{F}{p^2M + pf + k}\mathbf{1} \qquad (24)$$

If the numerical relations between M, f, and k are suitable, the mass will reach its final position after a number of oscillations and the final value $x_0 = \dfrac{Y_{(0)}}{Z_{(0)}}$ which corresponds to $p = 0$ will be $\dfrac{F}{k}$.

After the force is removed, the motion will be the resultant of two motions: one the original due to $+F\,\mathbf{1}$ and the other due to $-F\,\mathbf{1}$.

$$\text{Thus } x = \frac{Y_{(0)}}{Z_{(0)}} - \left[\frac{Y_{(0)}}{Z_{(0)}} + \Sigma\frac{Y_{(p)}\epsilon^{pt}}{p\dfrac{dZ}{dp}}\right] = -\Sigma\frac{Y_{(p)}}{p\dfrac{dZ}{dp}}\epsilon^{pt} \qquad (25)$$

Equation (24) may be written

$$x = \frac{\dfrac{F}{M}}{p^2 + p\dfrac{f}{M} + \dfrac{k}{M}}\mathbf{1}$$

$$Z_{(p)} = 0 \text{ gives } p = -\frac{f}{2M} \pm \sqrt{\frac{f^2}{4M^2} - \frac{k}{M}}$$

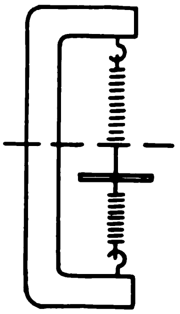


Fig. 16

If the constants are such that $\dfrac{k}{M} > \dfrac{f^2}{4M^2}$

then $\qquad p = -\alpha \pm j\omega, \qquad$ where $\alpha = \dfrac{f}{2M}$,

$$\omega = \sqrt{\omega_0^2 - \alpha^2} \text{ and } \omega_0^2 = \frac{k}{M}$$

$$\frac{dZ}{dp} = 2p + 2\alpha = 2(p+\alpha),$$

thus $\qquad p\dfrac{dZ}{dp} = 2p\,(p+\alpha) \qquad Y_{(p)} = \dfrac{F}{M}$

$$\therefore \frac{Y_{(p)}}{p\dfrac{dZ}{dp}} = \frac{F}{2pM\,(p+\alpha)}$$

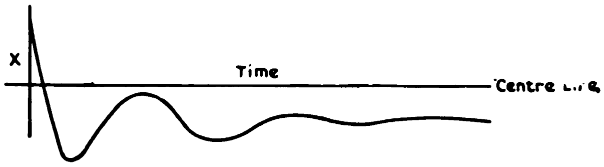


Fig. 17

$$\frac{Y_{(p)}}{p\dfrac{dZ}{dp}} \text{ for } p = -\alpha + j\omega = -\frac{F}{2M\omega\,(\omega + j\alpha)}$$

$$= -\frac{F\,(\omega - j\alpha)}{2M\omega\,(\omega^2 + \alpha^2)} = \frac{F\,(-\omega + j\alpha)}{2M\,\omega\,\omega_0^2}$$

Thus, referring to Part VIII,

$$A = \frac{F}{2M\,\omega\,\omega_0} \text{ and } \tan\delta = \frac{+\alpha}{-\omega}$$

δ therefore lies in the second quadrant and

$$x = -\frac{F}{M\,\omega\,\omega_0}\epsilon^{-\alpha t}\cos(\omega t + \delta)$$

At $t=0$ $\quad x=-\dfrac{F}{M\omega\omega_0}\cos\delta$

$$=-\frac{F}{M\omega\omega_0}\left(-\frac{\omega}{\omega_0}\right)=\frac{F}{M}\frac{M}{k}=\frac{F}{k}=x_0$$

At $t=\infty$, $x=0$. The oscillation begins at $x=x_0$ and ends in the middle position.

If the springs are vertical as shown in Fig. 16, the equilibrium position would be below the center line due to the weight of the disk.

Problem

Suppose that a force F greater than the weight had originally been applied so that the position of the disk were that indicated above the dotted line and that the force F had been suddenly removed. Give the equation of x_1 the distance of the disk from the center line after the force was removed. In this case the following relation holds during the first part of the cycle, that is during the time that the mass is being raised

$$F-W=M\frac{d^2x}{dt^2}+f\frac{dx}{dt}+kx$$

$F-W$ being the force available for accelerating the mass, overcoming the resistance, and compressing the spring.

$$\therefore\ (F-W)\ \mathbf{1}=x\ (p^2M+fp+k)\ \text{or}$$

$$x=\frac{(F-W)\ 1}{p^2M+fp+k}\mathbf{1} \tag{26}$$

The final position $x_0=\dfrac{F-W}{k}$

During the second part of the cycle, that is when the external force has been removed and the system is free to oscillate under the action of the force of gravity alone, we can apply the same reasoning or, briefly, we find a new fictitious force F' and calculate x_1 due to that force, then add to this x_0 to get the answer. F' must be such that when added to the original force $F-W$ the result will be $-W$.

$$\therefore\ F'+F-W=-W \qquad \text{or } F'=-F$$

therefore $\qquad x_1=-\dfrac{F}{p^2M+pf+k}\mathbf{1}$

The solution of this equation using the Expansion Theorem is the sum of a constant term and an exponential term.

It is:

$$x_1=-\frac{F}{k}-\frac{F\epsilon^{-\alpha t}}{M\omega\omega_0}\cos(\omega t+\delta)$$

$$\therefore\ x=x_0+x_1=\frac{F-W}{k}-\frac{F}{k}-\frac{F\epsilon^{-\alpha t}\cos(\omega t+\delta)}{M\omega\omega_0}$$

$$=-\frac{W}{k}-\frac{F\epsilon^{-\alpha t}\cos(\omega t+\delta)}{M\omega\omega_0}$$

For $\quad t=0, \quad x=-\dfrac{W}{k}+\dfrac{F}{M\omega_0^2}=\dfrac{F-W}{k}$

for $\quad t=\infty, \quad x=-\dfrac{W}{k}$

The chart would be similar to that shown in Fig. 17 if $F=2W$.

PART XI: THE EFFECT OF SHORT-CIRCUITING CERTAIN SECTIONS OF A NETWORK AND OF INTRODUCING NEW IMPEDANCES BY CLOSING A SWITCH

Consider as an illustration the circuit in Fig. 18. Switch S is closed after the original current is steady; find the equation of the current after the switch is closed.

Before the switch is closed, a steady current $I=\dfrac{E}{r+r_1}$ flows in the circuit and an e.m.f. $e=Ir_1$ is consumed by resistance r_1. The e.m.f. itself is obviously $-Ir_1$ and this e.m.f. exists across the switch blade. After the switch is closed there is no e.m.f. across the switch blade, so that closing the switch is equivalent to adding suddenly at the switch an e.m.f. $+Ir_1$. The sum of the e.m.f. across the switch blade is therefore zero.

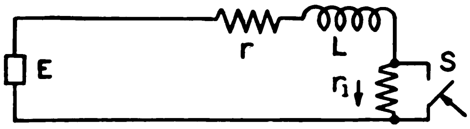


Fig. 18

This e.m.f. $+Ir_1\mathbf{1}$ causes a certain current to flow in the circuit so that the acutal current after the switch is closed is the algebraic sum of the original current and the transient current due to an e.m.f. $Ir_1\mathbf{1}$ at the switch.

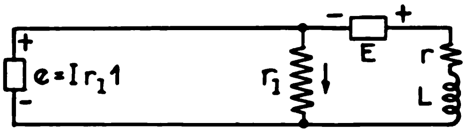


Fig. 19

It is advantageous to redraw the diagram as shown in Fig. 19 (the resistance of the original battery is assumed as zero).

The transient current in circuit $r+pL$ is

$$i=\frac{Ir_1}{r+pL}\mathbf{1}=\frac{Ir_1}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$$

The original current was I and was in the same direction as the transient. Thus the total current is

$$i = I + I\frac{r_1}{r}\left[1 - \epsilon^{-\frac{r}{L}t}\right]$$

At the moment that the switch closes, $t=0$, the current begins at its original value and then rises to a final value $\frac{E}{r}$ at $t=\infty$.

In the other branch, the transient current due to e.m.f. Ir_1 rises at once to $I\frac{r_1}{r_1}=I$ and this current flows as seen in the diagram in opposite direction to that originally in the resistance. Therefore the current in r_1 falls at once to zero after the switch is closed.

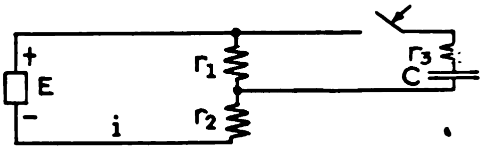


Fig. 20

Fig. 20 illustrates a similar case where in closing a switch the constants of the circuit are changed. The method of solving the problem is practically the same in the previous case. A fictitious e.m.f. is introduced between the switch blades, the value of which is the e.m.f. consumed in the shunted branch just prior to closing the switch. This fictitious e.m.f. causes currents to flow in the network and these currents added to the original currents give the actual current.

Fig. 21 is Fig. 20 redrawn for convenience of analysis.

Problem

Find current i after the switch is closed. Comparing the diagrams, it is readily seen that current i is the current in resistance r_2.

The resistance operator for the entire circuit, Fig. 21, is

$$z = r_3 + \frac{1}{pC} + \frac{r_1 r_2}{r_1 + r_2}$$

The total transient current, $i_0 = \frac{I r_1}{z}\mathbf{1}$

This current divides through the two resistances so that

$$i_2 = \frac{r_1}{r_1+r_2} i_0$$

$$\therefore i_2 = \frac{Ir_1^2}{r_1+r_2} \quad \frac{pC(r_1+r_2)}{pCr_3(r_1+r_2) + r_1 + r_2 + pCr_1r_2}\mathbf{1}$$

$$= \frac{Ir_1^2 pC}{r_1 + r_2 + pC(r_1r_2 + r_1r_3 + r_2r_3)}\mathbf{1}$$

and the actual current i is the sum of the original and this transient.

That is:

$$i = \frac{E}{r_1+r_2}\left[1 + \frac{pCr_1^2}{r_1 + r_2 + pC(r_1r_2 + r_1r_3 + r_2r_3)}\right]\mathbf{1}$$

The first term is simply $\frac{E}{r_1+r_2}$. The second may be

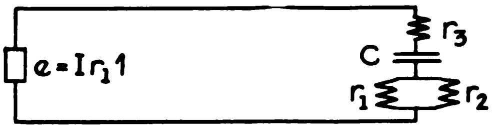


Fig. 21

solved either from the operator directly or by the Expansion Theorem. The second term may be written

$$\frac{Ap}{B+Dp}\mathbf{1} = \frac{A}{D}\,\frac{p}{p+\alpha}\mathbf{1}$$

where

$$\alpha = \frac{B}{D}$$

or

$$\frac{Ap}{B+Dp}$$

$$= \frac{A}{D}\epsilon^{-\alpha t} = \frac{Cr_1^2}{C(r_1r_2 + r_1r_3 + r_2r_3)}\epsilon^{-\alpha t}$$

$$\therefore i = \frac{E}{r_1+r_2}\left[1 + \frac{r_1^2}{r_1r_2 + r_1r_3 + r_2r_3}\epsilon^{-\frac{r_1+r_2}{r_1r_2+r_1r_3+r_2r_3}t}\right]$$

For $t=0$

$$i = \frac{E(r_1+r_3)}{r_1r_2 + r_1r_3 + r_2r_3}$$

For $t=\infty$

$$i = \frac{E}{r_1+r_2}$$

PART XII: THE EFFECT OF SUDDENLY CHANGING THE CIRCUIT CONSTANTS BY OPENING A SWITCH

Referring to Fig. 22, assume that the circuit has reached its permanent condition before switch S is opened. Before the switch is opened a steady current I flows through the switch. After the switch is opened zero current flows through the switch.

If, therefore, we substitute for the mechanical opening of the switch a certain peculiar e.m.f. in the switch, which at all times after $t=0$ gives a steady current $-I$, we have accomplished the same result. Fig. 22 might advantageously be redrawn, as shown in Fig. 23.

The current caused by the peculiar e.m.f. divides in the two circuits in parallel, so that the current

i_1 in r_1 and the total current $-I$ are related as the admittances.

$$\frac{i_1}{-I}=\frac{\frac{1}{r_1}}{\frac{1}{r_1}+\frac{1}{r+pL}}$$

$$\therefore i_1=\frac{r+pL}{r+r_1+pL}(-I)\,\boldsymbol{1}$$

Note that unit function $\boldsymbol{1}$ is inserted. This is evident because the original current in the switch was steady and was $+I$. In order that the current shall be zero, $-I$ must have unit shape.

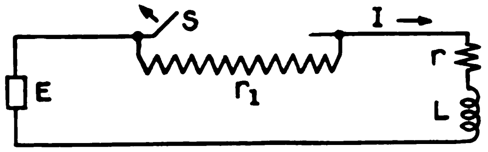


Fig. 22

In this particular case the initial current $I=\frac{E}{r}$, so that the current flowing in the circuit is

$$i_1=-\frac{E}{r}\,\frac{r+pL}{r+r_1+pL}\,\boldsymbol{1}$$

For $t=0$ $\qquad i_1=-\frac{E}{r}$

the minus sign showing that the current in r_1 is in opposite direction to the current I, thus forms the return current after the switch is opened.

The solution of the equation is easily found to be

$$i_1=-\frac{I}{r+r_1}\left[r+r_1\,\epsilon^{-\frac{r+r_1}{L}t}\right]$$

This is the total current flowing, because before the switch was opened there was no current in r_1.

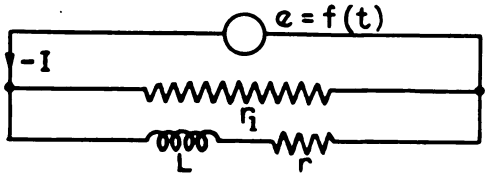


Fig. 23

If the switch had been shunted by a condenser, as in Fig. 24, the following relation is at once apparent:

$$\frac{i_c}{-I}=\frac{pC}{pC+\frac{1}{r+pL}}$$

$$\therefore i_c=-\frac{E}{r}\,\frac{pC\,(r+pL)}{rpC+p^2CL+1}\,\boldsymbol{1} \tag{27}$$

An interesting problem is the so-called buzzer excitation used with wave meters in a fundamental circuit such as shown in Fig. 25. Here the equation of the current after the magnet has opened the circuit is obviously the same as in the above case.

Solution of (27) by the Expansion Theorem. The expression is really the sum of the two following terms:

$$\frac{pA_1}{p^2+2\alpha p+\omega_0^2}\,\boldsymbol{1}\text{ and }\frac{p^2B_1}{p^2+2\alpha p+\omega_0^2}\,\boldsymbol{1}$$

in which $\qquad A_1=-\frac{E\,Cr}{r\,C\,L}=-\frac{E}{L}$

and $\qquad B_1=-\frac{E\,C\,L}{r\,C\,L}=-\frac{E}{r}$

Let $\qquad 2\alpha=\frac{rC}{CL}=\frac{r}{L}$ and $\omega_0^2=\frac{1}{CL}$

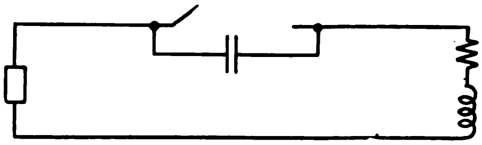


Fig. 24

In both cases $Z_{(p)}=0$ gives $p=-\alpha\pm j\omega$ when oscillation conditions prevail $\left(\text{that is when } \frac{1}{CL}>\frac{r^2}{4L^2}\right)$ and $\omega^2=\omega_0^2-\alpha^2$,

$$p\frac{dZ}{dp}=2p\,(p+\alpha)$$

and $\frac{Y_{(p)}}{p\frac{dZ}{dp}}=\frac{A_1}{2\,(p+\alpha)}$ for the first term

and $\frac{p\,B_1}{2\,(p+\omega)}$ for the second.

Thus $\frac{Yp}{p\frac{dZ}{dp}}$ for $p=p_1=-\alpha+j\omega=\frac{A_1}{2j\omega}=-j\frac{A_1}{2\omega}$ for the first term and $=\frac{B_1}{2\omega}(\omega+j\alpha)$ for the second.

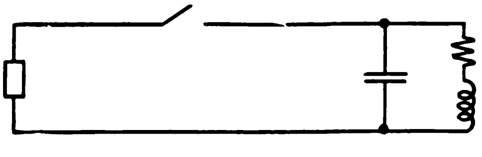


Fig. 25

Referring then to Equation (22), in Part VIII:

A for the first term is $=\sqrt{0^2+\frac{A_1^2}{4\omega^2}}=\frac{A_1}{2\omega}=-\frac{E}{2L\omega}$

and A for the second term is $\frac{B_1}{2\omega}\sqrt{\omega_2+\alpha^2}=-\frac{E\,\omega_0}{2\,r\omega}$

For the first term $\cot\delta=\frac{0}{-\frac{A_1}{2\omega}}$ or $\delta=-90°$

for the second term $\tan\delta=\frac{\alpha}{\omega}$.

$\frac{Y_{(0)}}{Z_{(0)}}$ in both cases is zero, therefore

$$i_c = -\frac{E}{L\omega}\,\epsilon^{-\alpha t}\cos(\omega t - 90°) - \frac{E}{r}\frac{\omega_0}{\omega}\,\epsilon^{-\alpha t}\cos(\omega t + \phi)$$

where $\tan\phi = \frac{\alpha}{\omega}$

$$= -\frac{E}{r}\,\frac{\epsilon^{-\alpha t}\omega_0}{\omega}\sin(\omega t + \phi_1)$$

where $\tan\phi_1 = \frac{\omega}{\alpha}$

For $t=0$

$$i_c = -\frac{E}{r}\frac{\omega_0}{\omega}\frac{\omega}{\omega_0} = -\frac{E}{r}$$

The condenser current is the entire original current at $t=0$. It forms its return.

In solving this problem two following useful operators have been obtained:

$$\frac{A_1 p}{p^2+2\alpha p+\omega_0^2}\,\mathit{1} \text{ and } \frac{B_1 p^2}{p^2+2\alpha p+\omega_0^2}\,\mathit{1}.$$

PART XIII: HEAVISIDE'S "SHIFTING"

Heaviside writes, on page 294, Vol. II,

$$f(p)\,u\,\epsilon^{at} = \epsilon^{at} f(p+a)\,u \qquad (28)$$

where $f(p)$ is a function of p, usually written $\frac{Y_{(p)}}{Z_{(p)}}$, and u is some function of t. It may, for instance, be $\sin(\omega t+\alpha)$ or merely the unit function $\mathit{1}$, depending upon the problem.

Proof

First let $f(p)$ be p. Then the left-hand side of Equation (28) becomes

$$p(u\,\epsilon^{at}) = \frac{d}{dt}(u\,\epsilon^{at}) = \epsilon^{at}\frac{du}{dt} + au\,\epsilon^{at}$$

$$= \epsilon^{at}\left[\frac{du}{dt} + au\right] = \epsilon^{at}\,(p+a)\,u$$

which satisfies the right-hand member of Equation (28).

Similarly

$$p^n\,(u\,\epsilon^{at}) = \epsilon^{at}\,(p+a)^n\,u$$

It is seen that Equation (28) holds for $f(p)=p^n$. It will therefore hold for any function of p which can be expanded in a power series. It can be easily shown that the rule also applies for $\frac{1}{p}$.

To illustrate the use of this relation, assume that when switch S is closed in Fig. 26 a voltage $E\,\epsilon^{-\beta t}$ is applied to the inductive circuit. Find the voltage consumed by the inductance.

If unit e.m.f. had been impressed,

$$i = \frac{E\,\mathit{1}}{r+pL}$$

and $e_i = pLi = \frac{pLE}{r+pL}\,\mathit{1} = E\frac{p}{p+\alpha}\,\mathit{1}$, where $\alpha = \frac{r}{L}$.

Put $f(p) = \frac{p}{p+\alpha}$

and $u = E\,\epsilon^{-\beta t}\,\mathit{1}$

then $\frac{p}{p+\alpha}$ should operate on $E\,\epsilon^{-\beta t}\,\mathit{1}$

or

$$e_i = \frac{p}{p+\alpha}\,(E\,\epsilon^{-\beta t})\,\mathit{1} \qquad (29)$$

But we do not always know how to operate on a function of t and therefore some transformation frequently must be made. The theorem given above says that $\epsilon^{-\beta t}$ can be shifted outside of the operator if, in $f(p)$, that is $\frac{p}{p+\alpha}$, p is changed to $p-\beta$.

$$\therefore\ e_i = E\,\epsilon^{-\beta t}\cdot\frac{p-\beta}{p-\beta+\alpha}\,\mathit{1} = E\,\epsilon^{-\beta t}\frac{Y_{(p)}}{Z_{(p)}}\,\mathit{1}$$

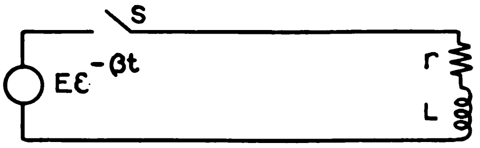


Fig. 26

$\frac{Y_{(p)}}{Z_{(p)}}$ (by the use of the Expansion Theorem)

$$= \frac{1}{\alpha-\beta}\left[\alpha\,\epsilon^{-(\alpha-\beta)t} - \beta\right]$$

$$\therefore\ e_i = \frac{E}{\alpha-\beta}\left[\alpha\,\epsilon^{-\alpha t} - \beta\,\epsilon^{-\beta t}\right] \qquad (30)$$

As a matter of fact in this simple case this transformation was not necessary, because it is known that $(E\,\epsilon^{-\beta t})\,\mathit{1}$ in Equation (29) can be written $E\frac{p}{p+\beta}\,\mathit{1}$

Therefore the solution with every term in p is

$$e_i = \frac{p^2\,E}{(p+\alpha)\,(p+\beta)}\,\mathit{1}$$

which could have been solved by the Expansion Theorem, and would have given the solution shown in Equation (30).

The shifting operation of course works both ways and sometimes it is an advantage to reverse the operation. Thus assume that we have an equation

$$\frac{p}{p-\alpha}1 \qquad (31)$$

This could be written

$$\epsilon^{-\beta t}\epsilon^{+\beta t}\frac{p}{p-\alpha}1 \qquad (32)$$

$\epsilon^{+\beta t}$ can be shifted to the right, that is back of a new operator, if, in the new operator, p is changed to $p-\beta$.

Then Equation (32) becomes

$$\epsilon^{-\beta t}\cdot\frac{p-\beta}{p-\beta-\alpha}(\epsilon^{+\beta t}1) \qquad (33)$$

Since $\epsilon^{\beta t}1=\frac{p}{p-\beta}1$, Equation (31) becomes

$$\epsilon^{-\beta t}\frac{p-\beta}{p-\beta-\alpha}\,\frac{p}{p-\beta}1=\epsilon^{-\beta t}\frac{p}{p-\beta-\alpha}1$$

Thus $$\frac{p}{p-\alpha}1=\epsilon^{-\beta t}\frac{p}{p-\alpha-\beta}1, \text{ and} \qquad (34)$$

for $\beta=\alpha$ $$\frac{p}{p-\alpha}1=\epsilon^{-\alpha t}\frac{p}{p-2\alpha}1 \qquad (35)$$

Note that in shifting to the left we retain the sign of the exponential, while in shifting to the right we reverse it,

Thus

$$\frac{p}{p+\alpha}(\epsilon^{-\alpha t}1)=\epsilon^{-\alpha t}\frac{p-\alpha}{p+\alpha-\alpha}1=\epsilon^{-\alpha t}\frac{p-\alpha}{p}1$$

$$=\epsilon^{-\alpha t}\left(1-\frac{\alpha}{p}\right)1=\epsilon^{-\alpha t}(1-\alpha t)1 \qquad (36)$$

and $$\epsilon^{-\alpha t}\frac{p}{p+\alpha}1=\frac{p+\alpha}{p+2\alpha}(\epsilon^{-\alpha t}1)=\frac{p+\alpha}{p+2\alpha}\cdot\frac{p}{p+\alpha}1$$

$$=\frac{p}{p+2\alpha}1=\epsilon^{-2\alpha t}1 \qquad (37)$$

$$\frac{1}{Z_{(p)}}1=\epsilon^{-\alpha t}\epsilon^{+\alpha t}\frac{1}{Z_{(p)}}1=\epsilon^{-\alpha t}\frac{1}{Z_{(p-\alpha)}}(\epsilon^{+\alpha t}1)$$

$$=\epsilon^{-\alpha t}\frac{1}{Z_{(p-\alpha)}}\cdot\frac{p}{p-\alpha}1 \qquad (38)$$

For $\frac{1}{Z_{(p)}}=\frac{p}{\sqrt{p+\alpha}}$ we get from Equation (38)

$$\frac{p}{\sqrt{p+\alpha}}1=\epsilon^{-\alpha t}\frac{p-\alpha}{\sqrt{p-\alpha+\alpha}}\cdot\frac{p}{p-\alpha}1$$

$$=\epsilon^{-\alpha t}p^{\frac{1}{2}}1=\frac{\epsilon^{-\alpha t}}{\sqrt{\pi t}}1 \qquad (39)$$

This relation is explained in Part XXIII.

Thus

$$\frac{\alpha}{\sqrt{p+\alpha}}1=\int_0^t\frac{\epsilon^{-\alpha t}}{\sqrt{\pi t}}dt$$

$$=2\sqrt{\frac{t}{\pi}}\,\epsilon^{-\alpha t}\left(1+\frac{2\alpha t}{1\cdot 3}+\frac{(2\alpha t)^2}{1\cdot 3\cdot 5}+\cdots\right) \qquad (40)$$

Another Example

A certain problem gives the following operational solution:

$$y=\sqrt{\frac{p}{p+2\alpha}}1 \qquad (41)$$

Find the numerical value of y.

The general method outlined in Part I would be to write

$$y=\sqrt{\frac{1}{1+\frac{2\alpha}{p}}}1=\left(\frac{1}{1+\frac{2\alpha}{p}}\right)^{\frac{1}{2}}1$$

and then to expand

$$\left(1+\frac{2\alpha}{p}\right)^{-\frac{1}{2}}1$$

in an infinite series; thus

$$y=\left(1-\frac{1}{2}\left(\frac{2\alpha}{p}\right)+\frac{1\cdot 3}{2\cdot 4}\left(\frac{2\alpha}{p}\right)^2-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\left(\frac{2\alpha}{p}\right)^3\cdots\cdots\right)1$$

$$=\left(1-\alpha t+\frac{1\cdot 3\cdot 4}{2\cdot 4}\,\frac{\alpha^2t^2}{\underline{|2}}-\cdots\cdots\cdots\right)1$$

$$=\left(1-\alpha t+\frac{1\cdot 3(\alpha t)^2}{(\underline{|2})^2}-\frac{1\cdot 3\cdot 5(\alpha t)^3}{(\underline{|3})^2}\cdots\right)1 \qquad (42)$$

Equation (41) may be written

$$y=\left(\frac{p}{p+2\alpha}\right)^{\frac{1}{2}}1=\epsilon^{-\alpha t}\epsilon^{+\alpha t}\left(\frac{p}{p+2\alpha}\right)^{\frac{1}{2}}1$$

Now shift $\epsilon^{\alpha t}$ to the right and change operator Substituting for p, $p-\alpha$

$$\therefore y=\epsilon^{-\alpha t}\left(\frac{p-\alpha}{p+\alpha}\right)^{\frac{1}{2}}(\epsilon^{+\alpha t}1)$$

$$=\epsilon^{-\alpha t}\left(\frac{p-\alpha}{p+\alpha}\right)^{\frac{1}{2}}\frac{p}{p-\alpha}1=\epsilon^{-\alpha t}\frac{p}{\sqrt{(p+\alpha)(p-\alpha)}}1$$

$$=\epsilon^{-\alpha t}\frac{p}{(p^2-\alpha^2)^{\frac{1}{2}}}1=\epsilon^{-\alpha t}\frac{1}{\left(1-\frac{\alpha^2}{p^2}\right)^{\frac{1}{2}}}1 \qquad (43)$$

This gives

$$y=\epsilon^{-\alpha t}\left[1+\frac{1}{2}\,\frac{\alpha^2}{p^2}+\frac{1\cdot 3}{2\cdot 4}\,\frac{\alpha^4}{p^4}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\,\frac{\alpha^6}{p^6}+\cdots\cdots\right]1$$

$$=\epsilon^{-\alpha t}\left[1+\frac{1}{2}\,\frac{\alpha^2t^2}{2!}+\frac{1\cdot 3\,\alpha^4t^4}{2\cdot 4\quad 4!}+\frac{1\cdot 3\cdot 5\,\alpha^6t^6}{2\cdot 4\cdot 6\quad 6!}+\cdots\right]1$$

$$=\epsilon^{-\alpha t}\left[1+\frac{\left(\frac{\alpha t}{2}\right)^2}{1!^2}+\frac{\left(\frac{\alpha t}{2}\right)^4}{2!^2}+\frac{\left(\frac{\alpha t}{2}\right)^6}{3!^2}+\cdots\right]1$$

$$=\epsilon^{-\alpha t}J_0(i\alpha t)\,1 \qquad (44)$$

$J_0\,(i\alpha t)$ is the Bessel function of the 0th order with the imaginary argument $i\alpha t$. Heaviside uses the notation $I_0\,(\alpha t)$.

In order to illustrate a point that may cause some confusion, another example will be given.

Consider an inductive circuit to which is applied a voltage $E\,\epsilon^{-\beta t}\cos\omega t$ at $t=0$. The conventional relation would then be:

$$E\,\epsilon^{-\beta t}\cos\omega t = ir + L\frac{di}{dt} = i(r+pL)$$

It has been shown in Part VI that $\cos\omega t = \dfrac{p^2}{p^2+\omega^2}\,1$

thus $$i = \frac{E}{r+pL}\left(\frac{p^2}{p^2+\omega^2}\,\epsilon^{-\beta t}\right)1$$

Here we desire to shift $\epsilon^{-\beta t}$ outside and the rule apparently was to do so while at the same time p is changed to $p-\beta$.

It must be remembered however that $\dfrac{p^2}{p^2+\omega^2}$ converts the cosine wave into a unit function. This feature has nothing to do with the resistance operator $\dfrac{1}{r+pL}$.

It is the "resistance operator" on the unit function which is to be changed, obviously not the unit function itself. Thus we write

$$i = E\,\epsilon^{-\beta t}\frac{1}{r+L\,(p-\beta)}\cdot\frac{p^2}{p^2+\omega^2}\,1$$

The procedure is then to solve for

$$i_1 = \frac{1}{r+L(p-\beta)}\cdot\frac{p^2}{p^2+\omega^2}\,1$$

and then multiply the result by $E\,\epsilon^{-\beta t}$ to get i. Referring to Equation (37) $Z_{(p)}=0$ gives three roots $p_1=\beta-\dfrac{r}{L}$, $p_2=+j\omega$, and $p_3=-j\omega$ so that

$$i_1 = A\,\epsilon^{\left(\beta-\frac{r}{L}\right)t} + B\,\epsilon^{j\omega t} + C\,\epsilon^{-j\omega t}$$

The last two terms combine to the permanent term as described in Part VI. The coefficient for the remaining term is $\dfrac{Y_{(p)}}{p\,h_2\,(p)\,h_1'\,(p)}$ for $p=p_1$ where $h_2(p)=p^2+\omega^2$, and $h_1=r-L\beta+pL$, and $Y_{(p)}=p^2$

$$\therefore A = \frac{p^2}{pL\,(p^2+\omega^2)} = \frac{p}{L\,(p^2+\omega^2)}.$$

For $p=p_1$ it is $$\frac{\beta-\dfrac{r}{L}}{L\left[\left(\beta-\dfrac{r}{L}\right)^2+\omega^2\right]} = \frac{\beta\,L-r}{(\beta L-r)^2+\omega^2\,L^2}$$

The permanent term of i_1 corresponds to an impedance $(r-\beta L+j\omega L)$ or

$$Z=\sqrt{(r-\beta L)^2+\omega^2\,L^2}$$

$$\therefore i_1 = \frac{E}{Z}\cos\,(\omega t-\delta) + \frac{E\,(\beta L-r)\,\epsilon^{\left(\beta-\frac{r}{L}\right)t}}{(\beta L-r)^2+\omega^2\,L^2}$$

where $\tan\delta = \dfrac{\omega L}{r-\beta L}$.

Therefore $$i = \frac{E\,\epsilon^{-\beta t}}{Z}\cos\,(\omega t-\delta) - E\,\frac{r-\beta L}{Z^2}\,\epsilon^{-\frac{r}{L}t}$$

$$= \frac{E}{Z}\left[\epsilon^{-\beta t}\cos\,(\omega t-\delta) - \epsilon^{-\frac{r}{L}t}\cos\delta\right]$$

The same result could of course have been obtained without "shifting" by the use of the operator for $\epsilon^{-\beta t}\cos\omega t$ given in Part VI. In that case the operational solution is:

$$i = \frac{p\,(p+\beta)}{(p+\beta)^2+\omega^2}\cdot\frac{1}{r+pL}\,1$$

(To be continued)

Floodlighting in Christ Church Cathedral

By I. E. COX
St. Louis Office, General Electric Company

THE sculptured altar and reredos of Christ Church Cathedral in St. Louis have been successfully illuminated with floodlighting projectors.

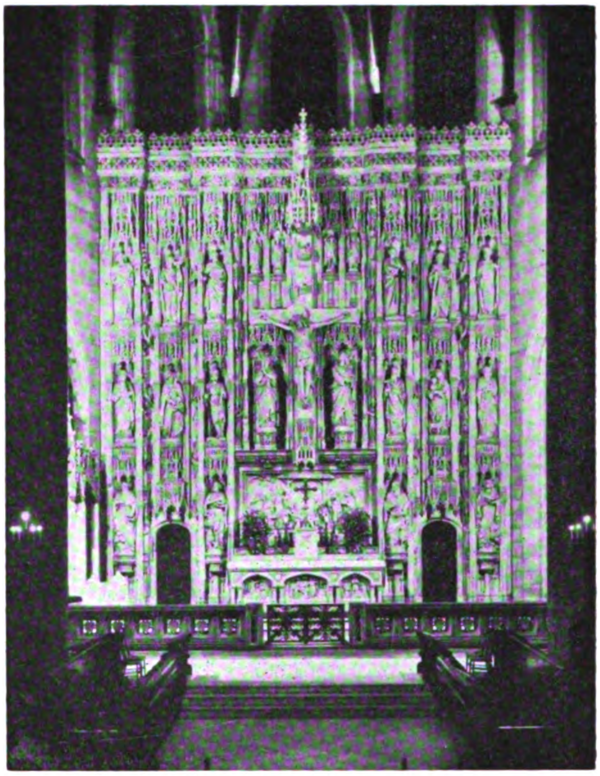
The reredos is thirty-five feet, seven inches high from the floor level to the tip of the central spire, and weighs 160 tons. The altar and reredos are made of Caen stone, an oölitic limestone of such purity that it is light cream in color and practically flawless.

This stone was obtained from the ancient subterranean quarries situated near the city of Caen in Normandy (France), the birthplace of William the Conqueror. It has been quarried continuously since the days of the ancient Romans who learned its characteristics from the natives. When quarried it is soft, similar to soapstone, but on exposure it hardens until it is as hard as limestone, as we know it. It is therefore a very desirable material for sculpturing, and has been used for this purpose widely in Europe and especially in England.

The stone after being selected and quarried was shipped to the sculptor's studio in Exeter (England), and after being cut was shipped to St. Louis (Mo.), in 230 pieces. It was erected, a process requiring four months, and dedicated in 1911.

This old Cathedral is in the heart of the city and buildings have sprung up around it, cutting off what little light was thrown on the reredos and altar. A lighting arrangement using a row of 146 60-watt lamps with reflectors was installed around the inside of the sanctuary arch. This required about 8.76 kw. of energy, but failed to illuminate the altar properly. The equipment was then replaced by four Form L20 General Electric floodlight projectors equipped with 200-watt lamps. Far better illumination has thus been secured with but one-eleventh the former energy consumption. The lenses selected are stippled glass which give a soft diffused light in keeping with the sacred feeling which this old church inspires.

Christ Church Cathedral, St. Louis (Mo.)


Altar and Reredos as Illuminated by Floodlights

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Am. Weld. Soc. Jour., Dec., 1927; v. 6, pp. 1-76 (sup.)
("Information and instructions for the making of arc welds," compiled by the Educational Committee of the A.W.S.)

Design for Welded Roof Trusses. William Dalton.
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(A short account of tests made at Rensselaer Polytechnic Institute on G-E welded specimens.)

Carrier-Current Communication, Broadcast

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(Short article on the methods and problems of supplying "broadcast" programs over telephone lines.)

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Heating of Rubber-Insulated Wires and Cables. L. Blin Desbleds.
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("Results of tests to establish permissible current densities.")

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Approximate Solution for Electrical Networks When These are Highly Oscillatory. E. A. Guillemin.
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Automatic Drive of Centrifuges.
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(Describes equipment used in the operation of a new dry-dock at Esquimalt, British Columbia.)

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Course in Electrical Engineering. Vol. 1: Direct Currents. Ed. 2. Chester L. Dawes. 589 pp., 1927, N. Y., McGraw-Hill Book Co., Inc.

Essentials of Transformer Practise. Ed. 2. Emerson G. Reed. 401 pp., 1927, N. Y., D. Van Nostrand Co.

Gummifreie Isolierstoffe. Arthur Sommerfeld. 103 pp., 1927, Berlin, Julius Springer.
(This little German booklet deals with the subject of hot- and cold-molded electric insulating materials, not including those made of rubber or rubber compositions. A brief historical review is followed by a chapter on the raw materials necessary and on the machinery and methods involved in their fabrication. Other chapters discuss the classification, properties and testing of these insulating materials, hints to the manufacturer and user of them, and standards of the Verein Deutscher Elektrotechniker pertaining to such products. The treatment throughout is confined to German practice and German materials.)

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(As used in this work, the term "thermionic phenomena" is intended to denote the phenomena of electrical conductivity exhibited in the neighborhood of substances at elevated temperatures. The author tells us that, though these phenomena have been recognized for many years, our knowledge of the subject is by no means complete. It is the function of this little book to assemble in one place the more important facts of thermionic emission as we now know them. To this end the author gives us chapters on the theory and experimental results of electron emission in a vacuum; on velocity distribution, energy exchange and saturation curves of electron emission; on the emission of ions by heated salts; and on various applications of thermionic phenomena to the measurement of high vacua and to rectification.)

Theorie der Wechselstromübertragung. Hans Grünholz. 222 pp., 1928, Berlin, Julius Springer.
(This German text dealing with the fundamental principles of high-tension, long-distance, alternating-current transmission is the outcome of the author's experiences in designing the first German 220-kv. line. The treatment is primarily graphical, the author reviewing some of the better-known earlier graphic solutions and presenting several new ones. A chapter of some twenty pages is devoted to actual examples of the applications of the author's methods to transmission line design. Supplementary sections present various constants and summarize the formulas discussed in the text proper. A brief list of bibliographic references is appended.)

Type FK-139

A Partial List of G-E Oil Circuit Breakers

Type	Voltages	Current-Carrying Capacities
INDUSTRIAL TYPES		
FP-115	110 to 550	50
FP-110	110 to 550	50
FK-20	2500	60 to 300
MANHOLE AND POLE TYPES		
FP-104	7500	200
FP-7	4500 to 15000	100 to 200
FKO-37	15000 to 37000	400 to 600
SWITCHBOARD TYPES		
FK-13	3300	200
FK-5	600 to 7500	200 to 800
FK-35 and 35Y	600 to 7500	400 to 800
FK-12	15000	300 to 800
FK-12B	4500	1500
FK-132A and B	7500 to 15000	400 to 1200
FK-32C	7500 to 15000	600 to 1200
STATION TYPES		
FK-34A	4500	2000
FK-25	600 to 2500	3000 to 5000
FK-53B	15000 to 25000	400 to 2000
FK-54	7500	600 to 1600
FH-103, 203	7500 to 35000	500 to 2000
FH-206, 209	7500 to 35000	500 to 4000
FHD-17	15000	600 to 5000
FHD-21	15000	600 to 4000
FK-230A	15000 to 25000	600 to 3000
FHK-230B	15000 to 37000	600 to 3000
FK(O)-136A	25000 to 110000	400 and 800
FK(O)-236A	15000	400 and 800
FK(O)-139A	15000 to 154000	600 to 1200
FHK(O)-139B	7500 to 154000	600 to 1200
FHK(O)-139C	15000 to 220000	600 to 2000

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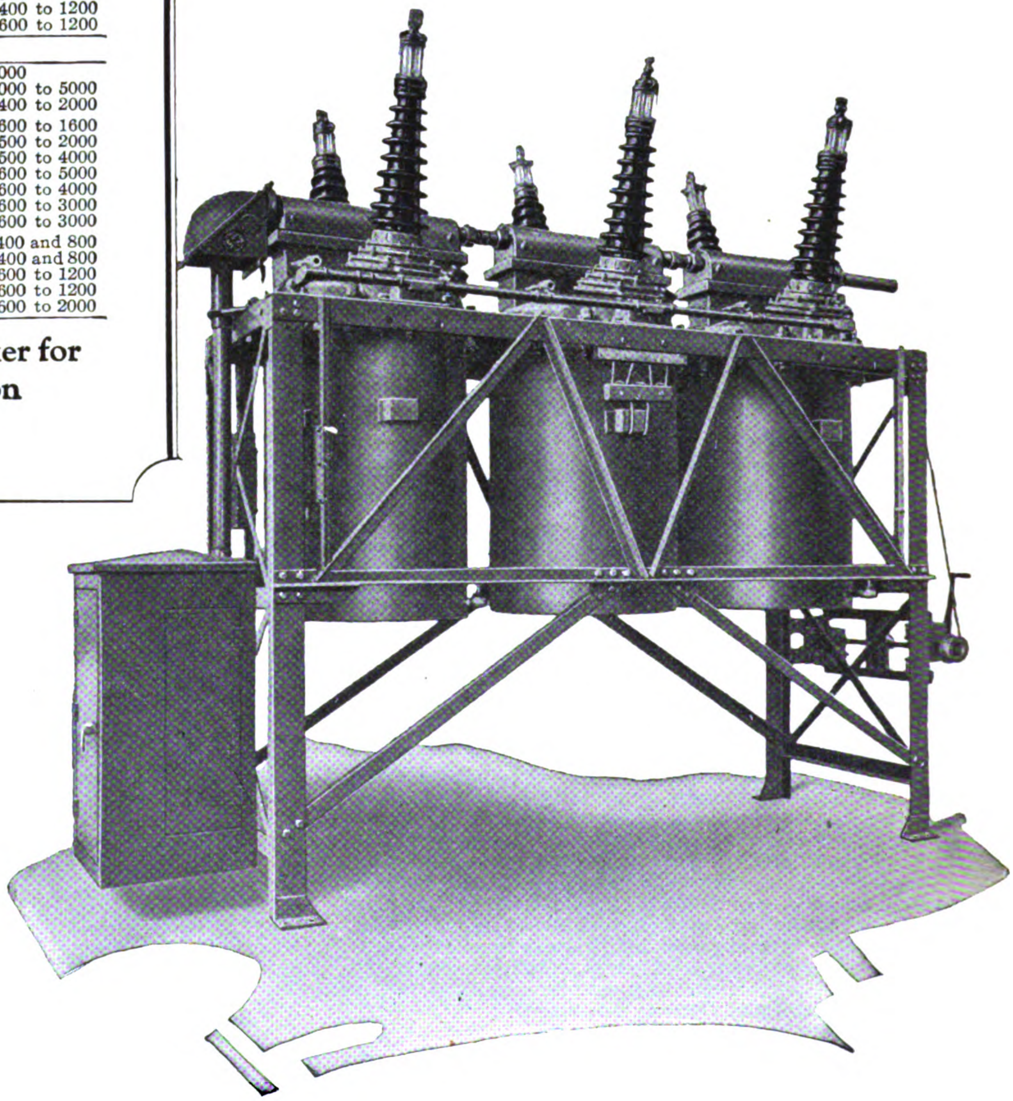

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GENERAL ELECTRIC REVIEW

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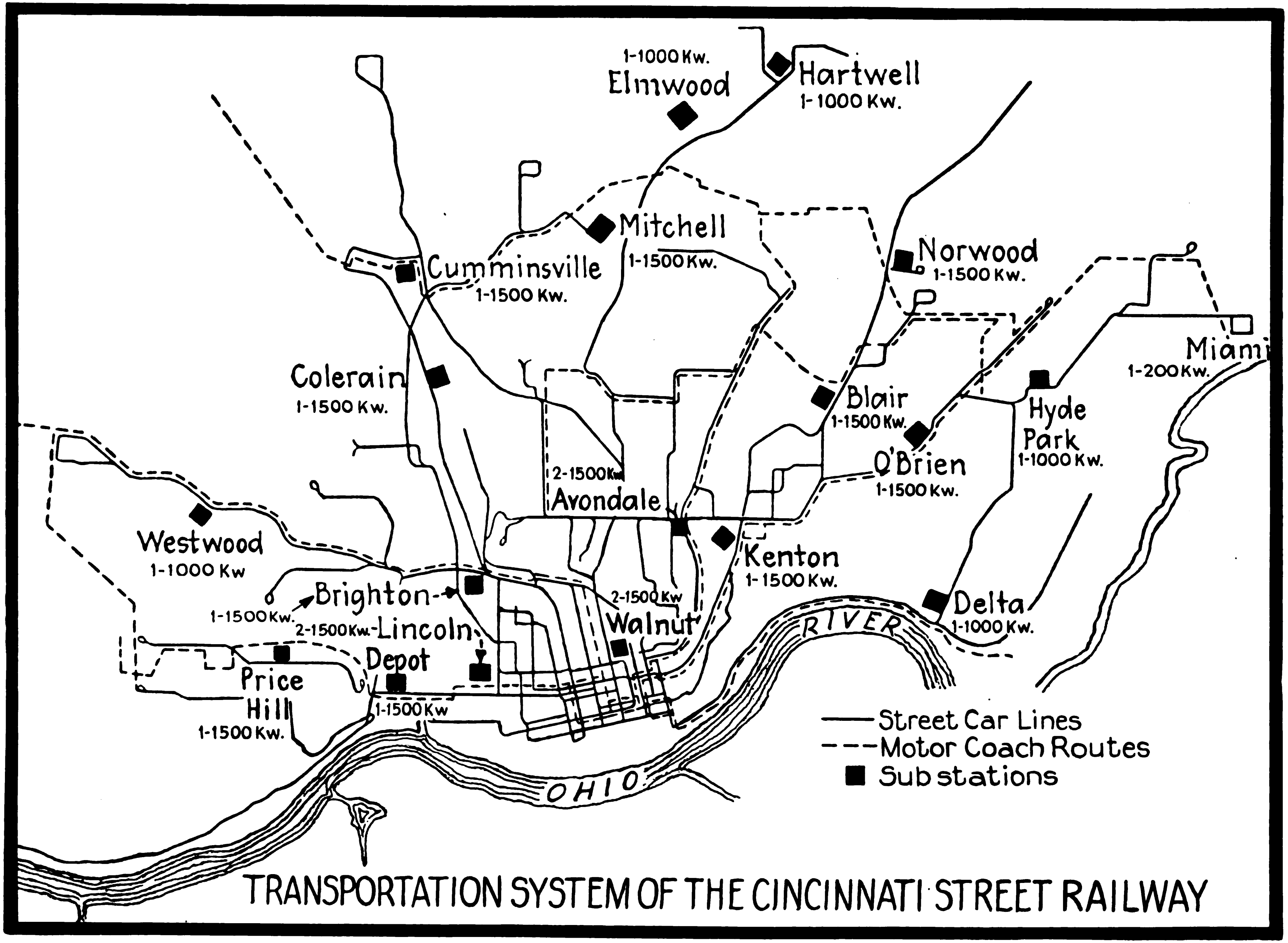

TRANSPORTATION SYSTEM OF THE CINCINNATI STREET RAILWAY

GENERAL ELECTRIC REVIEW

Vol. 31 May, 1928 No. 5

AND NOW 100 PER CENT AUTOMATIC STREET RAILWAY POWER SUPPLY

Only a short retracing of memory is needed to recall the time when a small group of adventurers in engineering discovered how to make a synchronous converter operate automatically. At that time it seemed almost incredible that the duties of a trained operator could possibly be performed by inanimate electrical devices of copper and iron fashioned into odd shapes and sizes. Was it indeed not asking too much of Nature's tools to furnish a substitute for man's ability to watch, to reason from his observation and experience, and then to do the proper thing that is so essential to the successful operation of costly electrical apparatus serving important public requirements?

The first automatic station—which by chance served an electric railway—was eminently successful; and since then there has been no interruption in the steady growth of automatic applications to synchronous converters and condensers, motor-generator sets and waterwheel generators of all sizes and voltages.

Initially, the motive behind automatic operation was mainly that of economy, for the elimination of the labor charges of station operators represented a potential saving of proportionally large amount. If the annual capital charges and maintenance of the additional apparatus necessary for automatic operation amounted to less than the labor charges for attendants under corresponding manual operation, an economical installation investment for automatic equipment was evident. The problem in this respect was relatively simple and rather clear-cut, with the result that utilities quickly took advantage of this new means to meet increasing labor costs.

Often the application of a new idea for a certain purpose afterward reveals additional advantages of an unexpected character. These secondary advantages may even assume an importance equal to or greater than the primary benefit. In this respect automatic operation of synchronous machines was no exception, as revealed by an investigation which shows that the all-around reliability secured by the elimination of the human element, with its uncertainty and inability to furnish unfailing protection to the station equipment under all operating conditions, is considered to be of importance comparable to the saving of wages by automatic operation.

Concurrent with the growth of automatic applications, and prompted by them, there have been developed the supervisory control and telemetering systems for operating in conjunction with non-attendant stations. Formerly, the remote supervision of electrical apparatus was accomplished through multiple-control circuits, and for short distances the investment cost of this scheme was entirely reasonable. However, when stations were installed over extended areas, and it became desirable to enable a centralized dispatcher to exercise direct control over each station and also receive indications from many of their devices, such as rotating machines, incoming-line and distribution-feeder switches and meters, the use of multiple remote-control circuits became prohibitively costly.

Modern supervisory and telemetering equipment by means of simple devices and relatively few control wires to each station permits a system dispatcher to exercise immediate control over each important piece of operating equipment on the system and it furthermore keeps him advised as to what is happening in each station. Indications as to whether the station door is locked is an example of the degree to which he may be kept informed of the conditions under his jurisdiction.

In Mr. Peters' article in this issue will be found a full description of a notable automatic installation now being completed by the Cincinnati Street Railway Company. Every station, nineteen in all, is an independent automatic substation capable of functioning in accordance with its load requirements without attendants or external supervision. By means of supervisory control, however, the system dispatcher can quickly assume control over any or all stations and then can release control back to automatic operation when desirable. In addition, the dispatcher has continuous indication of the status of the equipment in each station as well as curve-drawing and indicating meter readings brought to him from each station by the remote-indicating system.

The Cincinnati Street Railway in its commendable desire to furnish superior transportation is now completing an extensive program of rehabilitation that includes motive equipment, track, shops, and power supply. Deliberate study has been made of each branch of operation for the purpose of securing the maximum reliability, speed, and operating economy. Only that has been accepted which will contribute to giving its patrons the most effective and modern transportation service. The decision of the executives of this important Railway to make their power supply system completely automatic is surely a tribute to the present development of automatic operation.

Protection of Station Equipment on High-voltage Transmission Lines

Analysis of Stress Caused by Impulse Voltage—Relationship of Insulator Flashover to Protection—Operating Characteristics and Location of Arrester

By K. B. McEACHRON
Lightning Arrester Engineering Dept., Pittsfield Works, General Electric Company

To obtain a proper understanding of the principles underlying the protection of transmission line terminal apparatus from the effects of transient overvoltages it is necessary to consider the three major elements of the problem, *viz.*, the nature of the overvoltage, the characteristics of the insulation being protected, and the arrester itself.

Difficulty is frequently experienced in interpreting the effects of transient voltages on transmission lines and apparatus. This article is written in the attempt to remove at least some of the mystery surrounding such occurrences. Much of the information contained has resulted from the use of the cathode-ray oscillograph. An article describing a new cathode-ray oscillograph will appear in a forthcoming issue of this magazine.—EDITOR

THE IMPULSE

Overvoltages Due to Lightning

As a result of studies(1) of transients on transmission circuits, it appears that switching transients are not usually responsible for dangerously high potentials, and therefore this article will be confined entirely to the transients that appear as a result of lightning storms, although the same general principles apply to waves caused by switching.

It is recognized that transient overvoltages on lines may appear as the result of electrostatic induction, in which case the lightning does not strike the line, or the direct stroke in which either the main flash or perhaps a side flash terminates on the conductor.

Of these two the direct stroke is more dangerous only if it occurs at points where apparatus is located, as the potential is higher and the wave front probably steeper and the duration may be greater than for the induced transient. If however the direct stroke occurs out on the line at sufficient distance from apparatus so that the wave is cut off by the flashover of an insulator string, then the direct stroke may be less dangerous to terminal apparatus than the induced transient of lower potential. This statement may seem paradoxical, but it is nevertheless true as will be shown later.

The maximum induced potential has been estimated(2) to be 100 kv. per foot of height of conductor. It has been demonstrated during the lightning seasons of 1926-1927 that voltages high enough to flash over 14 disks of line insulation can appear on transmission circuits during a lightning storm. The records taken at the time indicated potentials of 1800 to 2000 kv., which gives a gradient of at least 40 kv. per foot since the average height of the line was 50 ft.; but whether these values represent direct or induced voltages is not definitely known. The actual induced potential attained in any given case will depend on the height of line, the change of the potential gradient as the result of the lightning flash, and the time required for the change in gradient to be completed. If the discharging cloud is immediately overhead and the time to discharge it sufficiently short, the potential under the middle of the cloud will be equal to the change in gradient times the height. That the time of discharge is important will be understood when it is remembered that as soon as a change in field takes place, no matter how small, the induced potential will begin to move out along the line in both directions, from under the cloud, and unless the cloud discharges faster than the released charge can run along the line no high potentials will be built up on the conductor under the cloud.

The maximum potential which may reach apparatus depends not only on the maximum potential induced but also on the location of apparatus with respect to the point of induction. This condition is shown in Figs. 1 and 2 where perpendicular waves are used to illustrate the principles involved. (In practice, absolutely perpendicular wave fronts do not occur but the sloping wave fronts can be resolved into a series of perpendicular fronts for the purposes of calculation.)

Assume that an electrostatic charge has just been released having a maximum potential of E and a length of 5000 ft., Fig. 1a. At zero time, all of the energy is in the electrostatic field and is equal to $\frac{E^2C}{2}$. If the distributed electrostatic capacity to earth is represented as a series of capacitors connected between line and earth, then for 5000 ft. the capacitors are charged to E volts and those at points out from under the charge, as at x and y, have zero potential. Just as soon as the wave is released, the capacitor at w and z discharge into x and y, and since the capacities are assumed to be equal the potential drops to half and a current flow is established. The next capacitor back of w then discharges into the capacitor beyond x, which process is continued on both ends of the wave until when one microsecond has elapsed the conditions are as shown in Fig. 1b. Here a traveling wave of $\frac{E}{2}$ volts has moved forward 1000 ft. and a similar wave has traveled backward 1000 ft.

---

(1) "Klydonograph Surge Investigations," by J. H. Cox, P. H. McAuley, and L. Gale Huggins, *A.I.E.E. Journal*, May, 1927.

(2) "Lightning," by F. W. Peek, Jr., *Journal Franklin Institute*, Jan. 1924.

After 2.5 ms. (microseconds) the potential at the midpoint A reaches a value of $\frac{E}{2}$ volts and immediately afterward drops to zero. The waves now separate and after 5 ms. of elapsed time are as shown in Fig. 1c. The waves are separated by a distance of 5000 ft. Each wave has a length of 5000 ft. and potential of $\frac{E}{2}$. Each wave, since the electromagnetic and electrostatic energies of traveling waves must be equal, has one quarter of the original energy in its electrostatic field and the same amount in the magnetic field. The current flowing in the conductor is equal to $\frac{E}{2}$ divided by the surge impedance $\sqrt{\frac{L}{C}}$. The wave front of current and potential are the same. It should be noted that a current flows through the earth in the direction opposite to that in the line, and that this current represents the current to charge or discharge the capacity of the line as the front or the tail of the wave passes any given point.

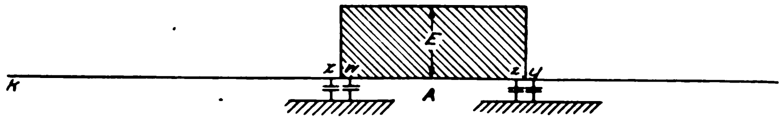

Fig. 1a. Zero Time.

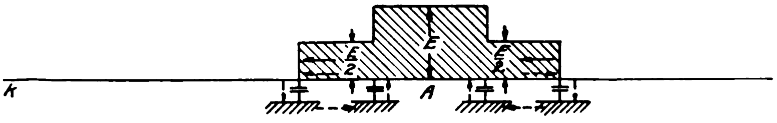

Fig. 1b. After 1 ms.

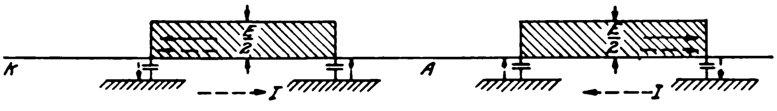

Fig. 1c. After 5 ms.

Fig. 1. Diagram of Traveling Waves, in Both Directions, Resulting from the Release of a Bound Charge on a Transmission Line

As a wave travels along a line, its potential will increase or decrease depending on the value of the surge impedance. If the capacity becomes larger or the inductance smaller, the voltage of the surge decreases; or, conversely, if the capacity decreases or the inductance increases, the voltage will increase and the current decrease. At the open end of a line the surge impedance becomes infinity and the voltage becomes double and the current zero. If at the end of the line the conductor is connected to the earth through zero impedance the current becomes double and the voltage zero.

In Fig. 2 is shown the effect on the traveling wave of having the induced potential originate at the end of the line. The same reasoning applies here as in Fig. 1; and it is to be noted especially that the wave becomes twice as long, but has the same potential and current as the traveling waves considered in Fig. 1 although both the electrostatic and electromagnetic energies are doubled.

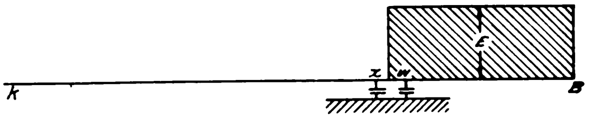

Fig. 2a. Zero time

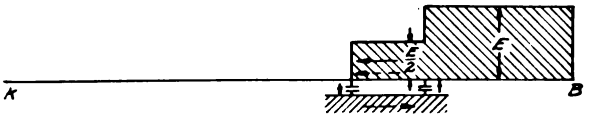

Fig. 2b. After 1 ms.

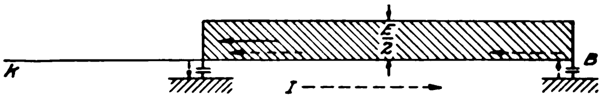

Fig. 2c. After 5 ms.

Fig. 2. Traveling Wave, in One Direction, Resulting from the Release of a Bound Charge at the End of a Line

Assume that apparatus is connected at the point A in Fig. 1, while in Fig. 2 it is connected at point B. In the first case the time of application of E volts was 2.5 ms. while in the second case it is twice as long, and apparatus which might stand the shorter time of application might fail under the longer time of application when located at the end of the line.

It is interesting to note that if either of the waves considered in Figs. 1 and 2 traveled to some point k

where the line ended and a complete reflection could take place, double potential at the point of reflection would exist for a length of time equal to the length of the wave, which in Fig. 1 is 5 ms. and in Fig. 2 is 10 ms.

Such traveling waves once started in a line travel back and forth, losing a certain amount of energy due to the resistance of the conductors together with losses in various leakage paths, such as corona, leakage over insulators, etc., until the energy is completely dissipated. It is well to keep in mind that, while a flashover of an insulator string near a piece of apparatus may offer a path of very low resistance to ground, energy must be dissipated through resistance in the form of heat to absorb the transient. Such low impedance to earth might prevent the failure of the insulation of the nearby apparatus; but if the resistance of the arc be taken as zero, the original wave is merely sent back out over the transmission circuit with reversed polarity. Actually, the resistance of the arc is not zero and some energy is expended in the arc. This means that the wave is reflected with decreased amplitude.

In practice, perpendicular wave fronts probably do not exist, except perhaps in the case of the flashover of a string of insulators when the potential across the string may drop from perhaps a million or more volts to a low value in the time it takes the arc path to change from an infinite resistance to that of the arc. Such insulator flashovers give rise to a very steep wave of potential traveling back over the transmission line with reversed polarity.

Based on the space distribution of potential under a cloud, the writer has assumed(3) that at least 5 ms. would be required for a traveling wave to reach its crest value if the apparatus being considered were not located under the discharging cloud. In case the cloud were directly over the apparatus, the maximum potential could be reached in approximately one microsecond.(3)

No estimate is offered for the condition of direct stroke as no data are yet available which would determine this value. It seems likely however, according to present theories, that its wave front is likely to be shorter than one microsecond.

It is probable that in practice many kinds of waves are actually obtained, with different crest potentials and wave fronts and tails. For the induced wave the minimum length of front may be taken as one microsecond and the length of the tail as 10 ms., or approximately two miles.(4) Longer wave fronts and longer tails may be found in service as a result of a longer time of discharge than assumed or as a result of the influence of other factors.

Insulator Flashover

It is well known but not always remembered that the flashover of a string of insulators is a function of both time and potential. The shorter the time of application, the higher the potential required to cause flashover. Such a curve of insulator flashover takes a hyperbolic shape and is represented by the curve in Fig. 3a.

Generally the potentials and the time of application of lightning voltages to apparatus insulation are limited by the flashover value of the line insulation, if of sufficient magnitude to be dangerous, although the flashover of an insulator does not necessarily mean that other apparatus has not been overstressed. Of all of the different forms of impulses, at least three general types deserve mention. Assume first, as illustrated in Fig. 3b, a wave of potential which rises to its crest calue in one microsecond but whose crest voltage would be considerably above the flashover value of the insulator for a one microsecond wave. As a result, flashover occurs on the front and the impulse is cut off after less than one thousand feet of the impulse has passed, in fact this distance might be 500 ft. or less. This wave representing comparatively

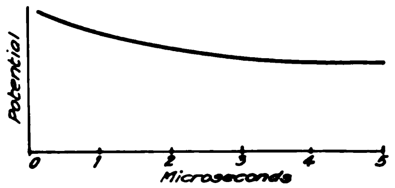

Fig. 3a. Insulator flashover volt-time curve

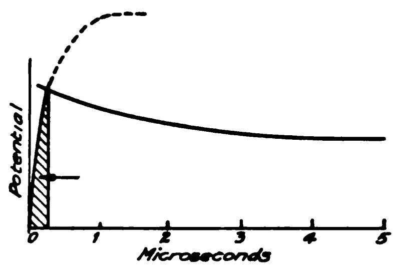

Fig. 3b. The shaded area represents the part of a traveling wave remaining after insulator flashes over on the front of the wave

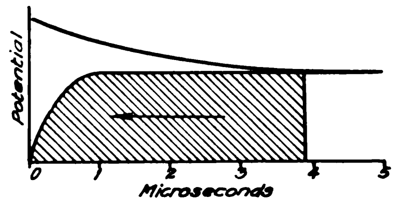

Fig. 3c. Traveling wave that does not reach insulator flashover value for several microseconds

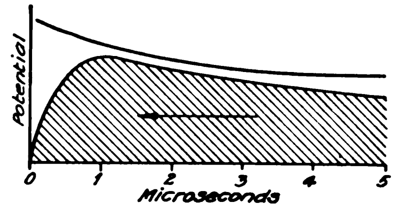

Fig. 3d. Traveling wave which does not reach insulator flashover value

Fig. 3. Insulator Flashover as a Means of Protection

little energy passes to the apparatus, where it may or may not do any damage. If the various capacities to ground of the bus structure, bushings, and transformer are sufficient (and they very often are), no trouble will be experienced with the apparatus from this type of overvoltage wave as the energy of the portion of the transient is not sufficient to charge the capacity of the apparatus to a dangerous value. This type of wave might result from a direct stroke far enough from the station so that induced effects in the station, caused by the overhead cloud, do not alter the situation. Such flashovers of line insulators close to the station are known to have taken place without damage to station apparatus.

---

(3) "Time, Voltage, and Current Characteristics of Lightning Arresters," by K. B. McEachron, GENERAL ELECTRIC REVIEW, October, 1926.

(4) These values are based on a cloud 1000 ft. high and two miles in extent.

In Fig. 3c is shown another form of wave which may be responsible for many of the troubles with station apparatus. This wave has so low a crest potential that the insulator does not flash over for several microseconds. In this case the energy of the wave passing to the apparatus is considerable, and may charge the various capacities to ground to such values of potential that bushings may flash over or other apparatus fail. A transformer may first be considered to act as a capacity and later as an inductance. Either way it is regarded, the potential will finally reach approximately double value if the incoming wave is maintained for a long enough period of time, which may be only a microsecond or so. Other structures, such as bushings and bus structures, will extend this time somewhat.

With this form of wave the insulator flashes over and the station apparatus may fail, which again is a condition which has been observed in practice.

The third form of wave, given in Fig. 3d, is one which has such a rate of decrease of potential that the insulator flashover is never reached, either on the front or on the tail. This wave also may contain sufficient energy so that, upon being reflected, the station apparatus is charged to dangerous potentials causing the flashover of bushings or damage to terminal equipment. Such troubles have been observed, but less frequently than the other types of failure.

The distance between the apparatus and the insulator string is the fundamental reason why there can be a difference of potential between them. If they are but a short distance apart, the potential across the apparatus could not be much different from that on the insulator string, since the insulator would cut off the reflection as well as the original impulse which, with considerable distance in between, it fails to do before reaching a high value. This statement applies equally to any means for decreasing the potential, such as a lightning arrester, although with the type of arrester to be described the distance can be greater than with an insulator string with reduced insulation, because the arrester's potential during discharge can be kept at a considerably lower value than would be tolerated in an insulator string, or other form of gap.

THE INSULATION

Lightning arresters are installed for the purpose of protecting the station apparatus from transients due to lightning. Usually the transformer is the most important piece of apparatus, and arrester design is determined largely by the strength of the transformer against the action of impulse voltages.

Considered from the standpoint of transient overvoltages, the transformer is a rather complicated combination of inductances and capacities. For steep waves, the voltage distribution through the winding is determined by the relative capacities; while for long tails or sloping fronts, the distribution will be determined by the inductance, and these distributions may be quite different. Thus it may be seen that the stress on the insulation at any given point in the structure will be dependent on the form of the applied impulse. Certain oscillations of a complicated nature may occur within the structure, and their nature will depend to a considerable degree on whether the transformer is grounded.

The insulation in the transformer consists of oil and solid insulation, both of which exhibit hyperbolic volt-time failure characteristics. Thus as the time of application of potential becomes shorter, the voltage necessary to cause failure becomes higher. This characteristic is also possessed by the transformer bushing and the other insulating structures, although differing somewhat in degree.

As a whole, the transformer may be considered as a capacity for the first few microseconds. The transformer bushing has a certain electrostatic capacity—usually much smaller than that of the transformer. Before a voltage reflection can take place at a transformer, it is necessary to charge the electrostatic capacity of the transformer and bushing and nearby insulation to the voltage of the reflection. This takes time, and energy is stored in the dielectric at the expense of the voltage of the transient, which means that the incoming transient, if extremely short (having been cut off by an insulator flashover), might not be able to charge the capacity of the transformer and other apparatus to a dangerous value. Other types of waves, as shown in Figs. 3c and 3d, could raise the potential of the transformer high enough either to cause the bushing to flash over or to damage the transformer winding.

Solid insulation may be damaged by impulses which do not cause an actual failure. Above a certain potential, which according to Peek⁽⁵⁾ is about 80 per cent of its single-shot impulse strength, the effects of repeated applications of potential are cumulative and in general the principle of both the time of application and the value of the impulse being kept as small as possible is well worth consideration.

---

(5) "Protection from Lightning," by F. W. Peek, Jr., International High Tension Conference, Paris, July, 1927.

High-voltage transformers are not immune to damage by lightning, just because they operate on a high-potential system. From the standpoint of reliability of service, such a system requires high line insulation and while it is also true that the transformer requires high insulation, yet the ratio between line insulation and transformer insulation is not materially different than at 44 or 66 kv. The chief difference lies in the fact that the number of disturbances of sufficient magnitude to cause trouble on a high-voltage system is less than on the lower voltage circuits, and therefore the chances taken are not so great on the high-potential system. When however the increased value of the apparatus and service at the higher potentials is taken into account, the cost of a lightning arrester is often justified if a single transformer failure is averted.

THE ARRESTER

The simplest form of arrester is a gap connected between each line conductor and the earth. This may take the form of reduced insulation by the use of closer spacing of arcing rings, but such a method of protection results in a power arc following the lightning discharge, which usually means an interruption to service. With modern lightning arresters adequate protection may be obtained without such interruption to service.

The so-called valve-type arresters of which the oxide-film arrester is a good example have two basic principles in their operation, one determining the protection offered by the arrester and the other the ability of the arrester to prevent the flow of follow-current from the system after the impulse has passed. The protection characteristic is largely determined by the ability of the arrester to decrease its resistance as the current increases, so that with a large flow of current the potential across the arrester is not much different than with a small flow of current. As the impulse current decreases, following the discharge of a lightning surge, the valve-type arrester has the characteristic of so increasing its resistance that little or no system current follows.

In designing an arrester of the above type there are two operating limits to be recognized: one, the impulse failure of the insulation being protected; and, the other, the failure of the arrester itself from the flow of system current. It is probable that, to be satisfactory, the arrester should limit the potential applied to a transformer to not more than 80 per cent of its ultimate strength and an additional factor of safety which would limit the potential to 50 or 60 per cent is highly desirable. On the other hand, a sufficient number of operating elements, which take the form of cells in the oxide-film arrester, are required so that at normal operating system potentials the current taken by the arrester is of the order of milliamperes. Unfortunately however it is necessary to design somewhat more liberally than for normal system potential. Because of certain conditions, such as sudden loss of load on hydroelectric systems, very considerable increases in system voltage may occur making necessary a larger and more expensive arrester than would otherwise be necessary. This increase in the size of the arrester also reduces the protection for, with a given impulse current flowing, the voltage across the arrester varies directly with the number of cells or units.

Characteristics of the Oxide-film Arrester

The operation of lightning arresters under impulse conditions is best determined by the use of the cathode-ray oscillograph, with which it is possible to study the arrester's operation in times as short as a millionth of a second or less. Although it has not yet been possible to obtain oscillograms of the operation of the arresters in service on *actual* lightning, yet it is reasonably certain that the laboratory tests indicate the order of protection afforded by the arresters.

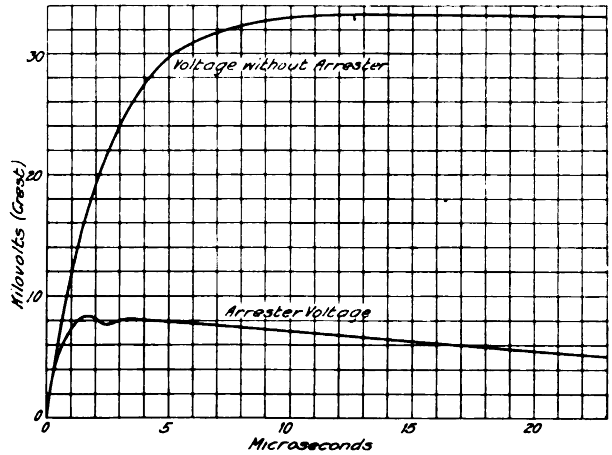

Fig. 4. Volt-time Characteristic of Eight Oxide-film Cells in Series, After Eight Years of Service, Compared with Volt-time Curve Without Arrester

In the laboratory a bank of capacitor units is charged by the use of kenotrons and transformers. By properly employing capacity, resistance, and inductance in the discharge circuit, the wave front, length of the wave, and the character of the current wave are controlled. When studying the operation of arresters the circuit is so arranged that the voltage rises to 63 per cent of its maximum value in 3 ms. In Fig. 4, volt-time curves with and without the arrester are given. The lower curve shows that the maximum voltage across the arrester is obtained in about 1.5 ms.

Using an impulse as described, volt-time, ampere-time, and volt-ampere curves (Fig. 5) were taken on the same eight cells, which curves taken together give a complete picture of the protective characteristics of the arrester. Of these, the volt-ampere and volt-time characteristics are the most important. When compared with data on the strength of transformers, the volt-time curve shows for instance that a 110-kv. arrester will permit a voltage equal to only about 60 per cent of the transformer's ultimate strength. The volt-time curve further shows that the series gap plays no appreciable part in determining the maximum potential allowed by the arrester under impulse conditions. That this must be true is clear from a consideration of the comparatively high electrostatic capacity of the cells compared to the series gap. Even with the highest voltage arresters, the electrostatic capacity of the cells is several times that of the series gap, so that, since the potential divides inversely as the capacity, most of the applied potential of the transient appears on the series gap, which breaks down without measurable lag. At the time this occurs, the voltage across the entire arrester is but slightly in excess of the voltage required to cause gap breakdown. Since the impulse voltage required to cause breakdown is substantially the same as the 60-cycle breakdown (crest value), which in turn is only a small percentage above the line-to-line potential, the normal gap breakdown will be found to have no influence on the potential allowed by the arrester under impulse conditions. The gap with this arrester serves its intended purpose of keeping the arrester disconnected until needed without having any appreciable effect on the arrester's protective characteristics.

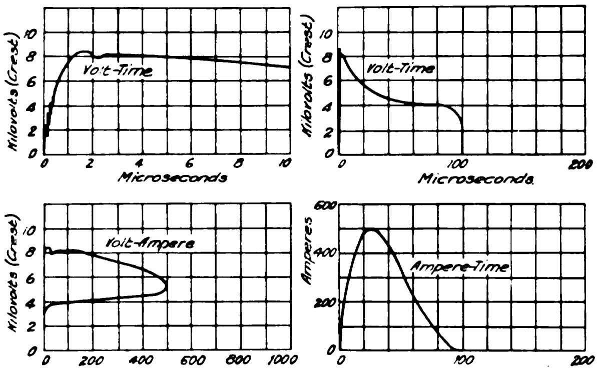

Fig. 5. Impulse Characteristic Curves of an Arrester Consisting of Eight Oxide-film Cells That Had Been in Service Eight Years

The potential allowed by the arrester depends on the rate of voltage application rather than on the amount of potential applied, assuming of course that sufficient potential is applied so that breakdown occurs on the front of the wave. If waves of sloping front and long tails are applied, the insulation fails at a lower crest potential but the arrester also operates at a correspondingly decreased potential, so that the factor of safety of the insulation against failure is not materially reduced.

The volt-ampere curves of oxide-film arresters show that the potential across the arrester decreases with increasing current and thus the maximum voltage across the arrester is unaffected by the amount of current or the length of time that it flows. Thus the maximum voltage of the arrester is independent of the length of the traveling wave, and depends only on conditions up to the time when current flows through the arrester in appreciable quantity. The volt-ampere curve also shows the potential across the arrester when the impulse current ceases, which potential is a measure of the arrester's ability to prevent the flow of troublesome follow-current. These characteristics are shown in Fig. 5.

Tests made with an impulse generator supplying a crest potential of over 1000 kv. on different numbers of cells in series show a straight-line relationship between potential and number of cells, which justifies the extrapolation of the cathode-ray oscillograph results to high-voltage arresters. Similar tests at lower voltage have confirmed this straight-line ratio. The potential across the arrester was measured with the sphere gap. Tests to determine the strength of transformer insulation have also been made with the same impulse generator.

It is possible by continued application of very severe impulses in rapid succession to increase the maximum voltage allowed by an oxide-film arrester to a considerable degree above that found on the first impulse applied to a new cell. Such an accelerated life test does not correspond very closely to conditions in operation, and results obtained in this manner must be used with caution. The maximum potential of a new cell for the first application of the standard wave used when taking the cathode-ray oscillograms as described in a preceding paragraph will be practically identical with the arrester's 60-cycle rating.

Because of the lack of knowledge of all the conditions affecting the application of transient voltages caused by lightning, the only real test of the arrester's ability to maintain its characteristics in service is to test arresters which have experienced several years of service. This has been done; tests being made on cells which have had eight or nine years' service in different parts of the country, some from Colorado, some from Cuba, and some from New England. The tests showed the arresters were in excellent condition to furnish protection. Typical cathode-ray oscillograms are given in Fig. 5, showing the volt-ampere characteristics of eight oxide film cells taken from a high-voltage arrester that had been in service for eight years. These oscillograms indicate a maximum potential of slightly less than 1.1 kv. crest per cell on the standard laboratory wave which would probably be equivalent to 1.6 to 1.8 kv. per cell for the steepest waves which are likely to occur in service. These cathode-ray oscillograms indicate that the arrester composed of these cells is still giving good protection.

A good lightning arrester of the type described in the foregoing must furnish not only protection but must always be able to stop the flow of follow-current under any usual operating condition. To determine the factor of safety against failure of the arrester itself, tests must be made which apply an impulse while the arrester is connected to a power supply. The impulse is applied to the arrester at a point on the 60-cycle potential wave which has been found by test to be most dangerous for the particular type of arrester being tested. In the case of the oxide-film arrester, the impulse is applied at about 80 deg. on the 60-cycle wave. The power voltage applied to the arrester is varied until a value is obtained which will not allow the arrester to stop the flow of follow-current. The arrester designer uses this voltage as the failure voltage of the arrester, and so adjusts the number of cells or units until it is certain that under no operating condition will the system potential applied to the arrester equal the failure voltage of the arrester.

Many factors need to be considered when making the failure tests on different arresters. The point on the 60-cycle wave at which it is most dangerous for the impulse to hit varies with different arresters. The constants of the power supply circuit have an influence on the arrester failure, some arresters failing easier with high inductance in the supply circuit while others fail more readily with low inductance. Each type of arrester must be studied under a variety of conditions as the arrester in service may be subjected to many different conditions.

Location of Arrester

In some installations the arresters are not so located as to give the maximum protection to the transformers. It is necessary when studying this phase of the problem to note that the wave front of the impulse may be short and that distances of only 100 or 200 ft. may be of importance. The transformer will receive the maximum protection from the arrester when the arrester is located close to the transformer, assuming that the arrester ground and the transformer ground are both tied to the same ground bus.

In case it seems necessary to locate the arrester on the bus at some distance from the transformer, it may be desirable to so shield the bus that it cannot pick up charges between the arrester and the transformer, as these might damage the transformer. This shielding, which should take the form of some overhead grounded conductor, is probably necessary only for long buses. When the arrester is located on the incoming lines, the same argument for shielded bus structure applies.

When the arrester is some considerable distance from the transformer the same reasoning that was used in a preceding paragraph with reference to reflections with incoming waves also applies, and illustrates the desirability of keeping the arrester and transformer as close together as practicable if the very best service of the arrester is to be obtained. A condition of this kind is illustrated in Fig. 6. For the sake of simplicity, a perpendicular wave front is assumed and the transformer is considered as a capacity to illustrate the principles involved. This wave has a crest potential of E kv. entering the station as a traveling wave. Its length is measured by the time it is permitted to pass any insulator before the insulator flashes over and cuts off the tail of the wave. This time is determined from the volt-time characteristic of the insulator similar to that shown in Fig. 3a.

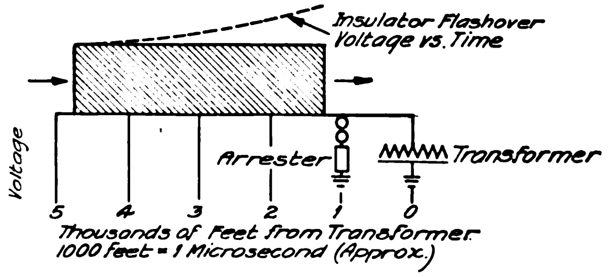
Insulator Flashover Voltage vs. Time
Voltage
Arrester
Transformer
5 4 3 2 1 0
Thousands of Feet from Transformer
1000 feet = 1 Microsecond (Approx.)

Fig. 6a. 0.25 ms. before wave reaches arrester

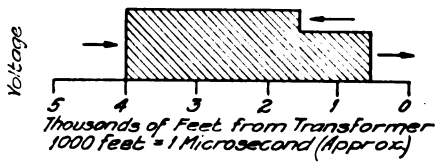
Voltage
5 4 3 2 1 0
Thousands of Feet from Transformer
1000 feet = 1 Microsecond (Approx.)

Fig. 6b. 0.5 ms. after wave reaches arrester

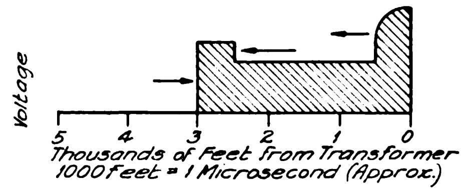
Voltage
5 4 3 2 1 0
Thousands of Feet from Transformer
1000 feet = 1 Microsecond (Approx.)

Fig. 6c. 1.5 ms. after wave reaches arrester

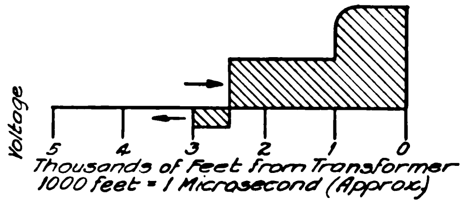
Voltage
5 4 3 2 1 0
Thousands of Feet from Transformer
1000 feet = 1 Microsecond (Approx.)

Fig. 6d. 2.0 ms. after wave reaches arrester

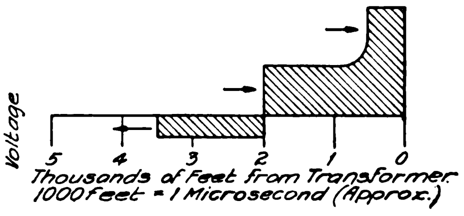
Voltage
5 4 3 2 1 0
Thousands of Feet from Transformer
1000 feet = 1 Microsecond (Approx.)

Fig. 6e. 2.5 ms. after wave reaches arrester

Fig. 6. Effect of Arrester Location on the Duration of Transformer Stress

Under very steep wave-front conditions, the insulator concerned in Fig. 6 will flash over in a time that is short compared to 1 ms. at 1.5 E kv. but will withstand E kv. for 3½ ms. before flashing over. Its time-voltage characteristic is plotted in Fig. 6a as the dotted line above the wave (reversed with respect to left and right from that in Fig. 3a), and this shows that the wave is 3½ ms. long, or in other words, even though the insulator did flash over it has allowed 3500 ft. of impulse to pass by before the flashover took place.

Five-tenths of a microsecond after the front of the wave reaches the arrester, Fig. 6b, a traveling wave of 0.3 E kv. originating at the arrester has moved to the left a distance of 500 ft. and the original incoming wave has moved to the right a distance of 500 ft. having been reduced in value to 0.7 E kv. by the arrester.

This wave reaches the transformer and because of the electrostatic capacity of the transformer the wave is modified to some extent as a result of charging the capacity represented by the transformer. After 1.5 ms., the voltage across the transformer has built up to 1.4 E kv. and a reflected wave of this potential travels from the transformer, as shown in Fig. 6c, until it reaches the arrester, as shown in Fig. 6d, at which time a potential-reducing wave of 0.7 E kv. starts from the arrester and travels toward the transformer, as shown in Fig. 6e.(6) A potential of 1.4 E kv. exists at the transformer until this wave of reduction reaches the transformer. Thus during a total time of 2 ms. a voltage much in excess of that allowed by the arrester has been applied to the terminals of the transformer.

A voltage of double the arrester voltage may therefore be applied to the transformer for a time equal to twice the time it takes electricity to travel from the arrester to the transformer, which may be sufficient to cause failure of the transformer under extreme conditions. If the arrester had been placed 100 or even 200 ft. away from the transformer, the stress on the transformer insulation (both from the standpoints of voltage and time) would have been very much less indeed.

As previously mentioned, in practice waves are not perpendicular, and the transformer is not the simple capacity assumed; but if the wave is sufficiently long, so that the full reflection voltage of the transformer has time to reach the arrester before the end of the wave arrives at the arrester, the time that reflection voltage is applied to the insulation will be as given in Fig. 6. It is probable that many of the incoming waves do not have sufficient lengths of tail to cause the various capacities to be charged to dangerous values. However, it is well worth while to install the arrester as near to the transformer as possible. If this is done and other conditions are normal, the record of protection for the arresters will be even better than it has been in the past.

The connection between the arrester, the line, and the ground should be as short as possible, as shown by the following analysis. Assume an impulse traveling to a station which has a lightning arrester connected between line and ground at the point where the line enters the station. Assume, also, that the distance between the point where the lightning arrester taps off the line and the earth, measured along the arrester connection and the ground wire is 100 ft. Even if the arrester were shorted by a copper strap, 200 ft. of original impulse would pass by the arrester tap before the potential would begin to be reduced by the effect of the earth connection. This comes about because of the time required for the wave to travel to the earth and back up to the line conductor. The steeper the incoming waves the more important will be the length of connection between the line and ground through the arrester. It is worth while to keep these connections as short as possible and to keep the amount of exposure of conductors between the arresters and the equipment as little as possible.

In designing a high tension system careful consideration should be given to all of the factors affecting the protection of the system against lightning. It is necessary to consider the effect of high towers, long spans, high line insulation, transformer and bushing strength and to so balance these that no part of the system will be unduly stressed. For any given system the effect of the ground wire and lightning arrester in reducing potentials on the transmission line and on the apparatus may be approximately evaluated at least. As our knowledge of actual conditions during lightning storms increases due to the use of the new measuring instruments now available it will be possible to determine more accurately the exact value of these devices in service.

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(6) The wave of suppressed potential at the left of the tail of the original wave in Figs. 6d and 6e is the continuation of the 0.3 E kv. wave which started at the arrester and traveled to the left as shown in Figs. 6b and 6c.

Second 1200-lb. Turbine Unit for Milwaukee

The Milwaukee Electric Railway and Light Company will soon put into operation a second turbine-generator unit designed for 1200-lb. steam pressure. This decision to add to its existing high-pressure equipment was the result of 15 months' operation of the first 1200-lb. unit, during which period this unit shared the station load during a considerable portion of the time.

The steam conditions will be 1200-lb. gauge, 750 deg. F., and the turbine will exhaust into the regular station steam mains at 300 lb. pressure. The generator will be rated 7700 kw., three phase, 60 cycles, 13,800 volts, 90 per cent power-factor.

The first unit installed by the Milwaukee Electric Railway and Light Company was rated at 7000 kw. and was placed in service in October 1926.

Arc-welded Steel-frame Factory Building at the West Philadelphia Works of the General Electric Company

Application of Welding to Building Construction—General Problems—Description of the Factory Building—Details of Welding in Fabricating Shop and Field

By FRANK P. McKIBBEN
Consulting Engineer, Black Gap, Pennsylvania

> *When presenting this article as a paper before the A.S.C.E. at Philadelphia on the fourteenth of last March, Prof. McKibben recommended the general use of welding in bridges and buildings, but stated that such use should be confined to responsible designers, fabricators, and engineers who will apply it only where it has well-defined advantages over any other known method of building and exlude it where not suitable.*
>
> —EDITOR

MAN progresses slowly; but he progresses. His efforts as exhibited in the structural field during the last half century may be summarized approximately by decades as follows:

First. During the decade between 1880 and 1890 there was heard much discussion among American engineers concerning relative merits of pin-connected and riveted trusses, but by the latter date it was conceded that the riveted truss for most, but not all, uses was superior; and the pin-connected truss which had been the standard in America surrendered its prestige to the European riveted type.

Second. By 1900 the riveted steel frame for tall buildings was no longer a curiosity. It was a standard construction of recognized merit.

Third. The decade ending in 1905 witnessed the remarkable increase in the use of concrete for structural building purposes. Many concrete buildings and some concrete bridges collapsed during those early days; but man nevertheless had progressed. Concrete buildings and bridges had come to fill a desideratum and to stay.

Fourth. Considére's classic experiments on spirally-reënforced columns resulted in a great step forward in the use of stronger columns, so that by 1910 or thereabouts we were erecting large reënforced concrete buildings characterized by flat slabs, then an innovation, and by spirally-reënforced columns.

Fifth. And now we are witnessing another transitional stage wherein welded steel structures are coming into the structural field, at first by such gradual steps as to be almost imperceptible; but now the movement is going forward by leaps and bounds. Welded steel frames possess some good distinctive characteristics; and without doubt will find an appropriate place in a structural fabric consisting of the useful elements of pins, rivets, reënforced concrete, and welded joints.

In 1877 the first public demonstration of welding steel by the electric arc was shown in an experiment before the Franklin Institute in Philadelphia. It is somewhat fitting that there should be described, on the fiftieth anniversary of that event, the first electrically welded steel building erected in Philadelphia.

The Use of Welding in Structures

Since about 1915 somewhat over thirty-five steel building frames have been fabricated by welding; twenty-five miscellaneous structures including large gas or oil tanks, towers and such important constructions as plate girder and truss bridges, have been entirely built new as welded structures or have been extensively strengthened or modified by this process; and approximately fifteen ships and barges have been welded throughout.

The Latin expression *festina lente*, meaning "make haste slowly" surely is suitable as a motto for the conservative and careful engineer; but the array of welded structures included in the foregoing list of about seventy-five examples is sufficient evidence that welding has well passed the experimental stage and established its place in the family of useful tools.

Some Problems to be Solved

One is no longer concerned about the safety or desirability of carefully designed and properly built welded structures. In many cases the utilization of this type of structure distinctly lowers the cost of construction; the noiseless welding process commends itself alike for structures being built in close proximity to hotels and hospitals, and to office buildings—in fact, in any thickly settled community; the elimination of unnecessary gusset plates is noteworthy; economies resulting from absence of holes in tension members are worth while; often considerable time may be saved in making drawings; the very great reduction in number of templets at the fabricating shop is marked.

Naturally, several unsolved problems appear. Among these are:

(1). The revision of building codes in those cities where no provision is made for welding. Such codes do not specifically exclude welding, but they do not specifically include it, because it is new. Building codes should be revised to include the issue of welding permits by responsible municipal bureau heads to responsible fabricators.

(2). Another interesting need is a simple practical series of accomplishment tests for use in employing welders on important works. In some cases the welding inspector is commissioned to test applicants seeking work by watching .their proficiency in maintaining a steady arc of proper length, in depositing a fillet on vertical as well as horizontal surfaces, in making a fillet of specified length, and in securing proper penetration on each of two plates welded together. This last can be easily tested by arranging the plates in such manner as to be wedged apart, thus breaking the fillet. Already a study of this problem has been delegated to a duly appointed committee.

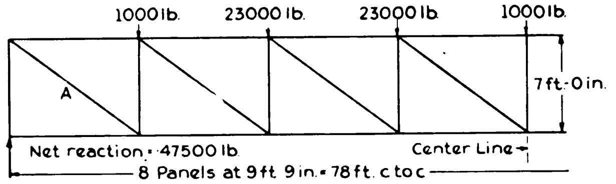

Fig. 1. Skeleton Diagram of a Welded Roof Truss Showing Gross Dimensions and Loading

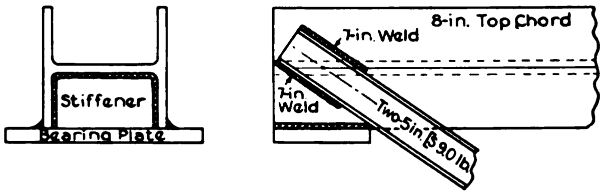

Fig. 2. A Detail of Welded Construction at the Connection of an End Diagonal to the Top Chord

(3). More extensive use of jigs in frabricating shops is necessary to reduce cost of assembling. The efficacy of jigs was shown to a marked degree in assembling the small triangular skylight truss frames of the building which is to be described later in this article. The saving in unit cost is great where a large number of similar objects are to be made. In other words, the use of jigs eliminates templets and results in further saving of assembling costs.

(4). Automatic welding machines depositing continuous or intermittent fillets offer great opportunities for securing cost reduction and better welds. For example, machines which weld simultaneously the flange on both sides of a plate girder web give more uniform quality of fillets, more speed and lower costs than manual welding; and because automatic machines use continuous wires for electrodes the time and wire lost in the frequent electrode changing of hand welding are eliminated.

(5). Standardization of details, such as connections of one beam to another or of beams to columns, is needed. If this be worked out now, while the welding program is just starting, much time will be saved. Such connections are very different from similar ones in riveted construction. In this respect, definite work is being done on the problem of adapting welding structural practice to general use.

Some Details of Design

Example A. Given a roof truss of the type and with the loading shown in Fig. 1, having a top chord made of one 8-in. H section with web horizontal, proportion the end diagonal, *A*, and welding connecting it to top chord, using 16,000 lb. per sq. in. for tension in diagonal and 3000 lb. per linear inch shearing value for ⅜ in. by ⅜-in. triangular fillets. The truss is symmetrical about its center line.

Thus are computed:

Tension in end diagonal $= \dfrac{47{,}500 \times 12}{7} = 81{,}400$ lb.

Net area required in end diagonal $= \dfrac{81{,}400}{16{,}000} = 5.1$ sq.in.

Use two 5-in. channels, 9.0 lb. per ft. (5.26 sq. in.)

Length of ⅜-in. fillet required $= \dfrac{81{,}400}{3000} = 27.1$ in.

Use four fillets, each 7 in. long, as shown in Fig 2.

The 7-in. fillet of Fig. 2 is ordinarily indicated on the detail drawing as "7 W" if it be shop, and "7 F W" if field welded.

Example B. Given a welded plate girder, span 40 ft. c. to c. under a uniform load of 4000 lb. per linear foot over entire span. Assuming web 40 in. by $\frac{5}{16}$ in., proportion the tension flange using 16,000 lb. per sq. in. as tension in the member.

Whence:

Maximum moment at center

$$= \frac{4000}{8} \times (40)^2 \times 12 = 9{,}600{,}000 \text{ in. lb.}$$

Required net area in tension

$$= \frac{9{,}600{,}000}{40\ (16{,}000)} - \tfrac{1}{8} \times \tfrac{5}{16} \times 40$$

$$= 15.0 - 2.1 = 12.9 \text{ sq. in.}$$

Use one plate 16 in. by $\frac{13}{16}$ in., or two plates giving the same area.

Notice that this simple calculation for the tension flange reveals a double saving, first, in that no allowance need be made for rivet holes; and, second, that as no vertical row of holes is punched in the web, the web equivalent for the welded girder is larger and the flange correspondingly smaller than in the riveted girder. In many girders, however, the greatest saving can be secured in using as stiffeners single flat bars with one edge welded directly to web, thus eliminating not only fillers but also those legs of angle stiffeners which lie parallel with the web. (See

Fig. 3.) No plate girders were used in the building described herein, but this example is given here to illustrate the reasons for economies which result from welded girders.

Example C. Given a single angle (Fig. 4) in direct stress such as tension; to find the proper lengths of fillets to connect the angle to connecting piece, using tension at S lb. per sq. in. and longitudinal shear on fillet at f lb. per linear inch.

Using 0.6 as a factor to allow for eccentric loading, the required area of one angle $=\frac{P}{0.6S}$, where $P=$ pull in angle, and $S=$ allowable unit tension. Then, by taking moments about outer edge of angle leg, the length of fillet required along back of angle is $Y=\frac{Pb}{(a+b)f}$; and the length X for the fillet along outer edge of leg is $\frac{Pa}{(a+b)f}$. Naturally, these lengths added give $\frac{P}{f}$, as they should.

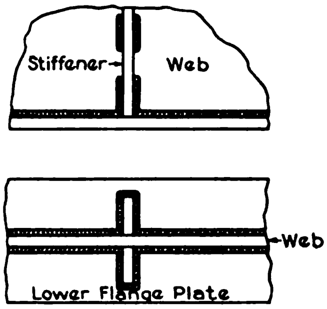

Fig. 3. A Drawing in Plan and Elevation Illustrating the Use of Flat Plates Instead of Angles as Girder Stiffeners

GENERAL DESCRIPTION OF BUILDING

The arc-welded steel-frame building that is now being completed at the General Electric Company's West Philadelphia Works consists of a headhouse or transept parallel with Elmwood Avenue, and two main aisles at right angles to it. The headhouse is 78 ft. wide and 171 ft. long with a vertical clearance of 43 ft. One aisle is 59 ft. wide and 474 ft. long with a vertical clearance of 34 ft. from the ground floor to the lower chords of the roof trusses, while the other aisle is 79 ft. wide and 474 ft. long with a reënforced concrete gallery floor, in all but the two westerly bays, nearly midway between the ground-floor level and the lower chords of the roof trusses. The gallery floor is 20 ft. above the ground floor, and is connected by an enclosed passageway to a similar gallery in another building. These aisles and the headhouse comprise a building approximately 138 ft. wide by 552 ft. long with a total ground area of 80,150 sq. ft.

When completed, the building will be used for the manufacture of arc-welded steel tanks of various kinds and sizes.

Concrete spread footings in clayey soil serve as foundations for brick walls, steel columns, and for the reënforced concrete columns supporting the gallery. The ground floor throughout consists of a wooden-block surface on a concrete base on cinder fill, and the gallery floor also is wooden block on flat-slab construction. The outside walls are of brick; and the roof of gypsum slab cast in place. The building will be well provided with natural light from unusually large windows in all external walls, and from four lines of sawtooth skylights, two lines in each main aisle.

Steel Framing

The steel frame consists of steel columns, made of Carnegie beams of a new type, between which are welded transverse trusses of the Pratt type with parallel chords in the two main aisles, but with inclined top chords in the headhouse trusses. The latter trusses have spans of about 77 ft. center to center of columns and vary in depth from 8 ft. 4 in. at one end to 6 ft. 4 in. at the other to carry the sloping roof of the headhouse. One main aisle has roof trusses 7 ft. deep with six panels each 9 ft. 9 in. making the span 58 ft. 6 in. The other main aisle has 78-ft. trusses 7 ft. deep with eight panels of 9 ft. 9 in. As all truss chords in the main aisle are horizontal and as the main purlins rest on the top chords of these trusses, roof slopes for drainage are provided for by sloping the secondary purlins.

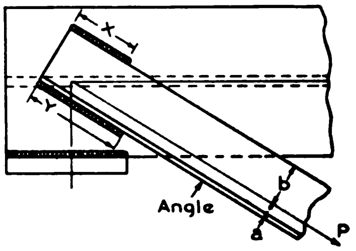

Fig. 4. Diagram Showing Dimensions Used in Calculating the Welded Connection for an Angle

In general, each column has a 1½-in. cap plate welded directly to its web and flanges, but the 1½-in. base plate was in most cases not welded to the column. Each anchor bolt was connected to each column by passing between the column flange and a short angle standing vertically with outer edges of both legs welded to the face of the column flange. The anchor bolt nut bears against a plate washer resting on the upper end of the angle.

The bays in the main aisle are 24 ft. to 25 ft. while those in the headhouse are from 25 ft. 7 in. to 29 ft. 3 in. wide.

The design of the roof trusses is characterized by the use of one 8-in. Carnegie beam for each top and bottom chord with flanges vertical, to the outer surfaces of which are welded channel diagonals and to the inner surfaces 7-in. I-beam verticals. This welding of the diagonals and verticals directly to the chord flanges avoided the use of over 1200 gusset plates in the trusses. Another feature is the absence of lattice bars, not only in all trusses but generally throughout the building. In only two members in the entire building were lattice bars used.

The fillet welds, of triangular cross-section with base and altitude generally ⅜ in. each, are subjected to longitudinal shear, and the unit shearing stress used in the design was 3000 lb. per linear inch for ⅜-in. fillets. As the critical shearing section for a ⅜ by ⅜-in. triangular fillet is 0.265 in., this stress corresponds to a shear 11,300 lb. per sq. in. In comparison with many tests, this represents a factor of safety of at least four.

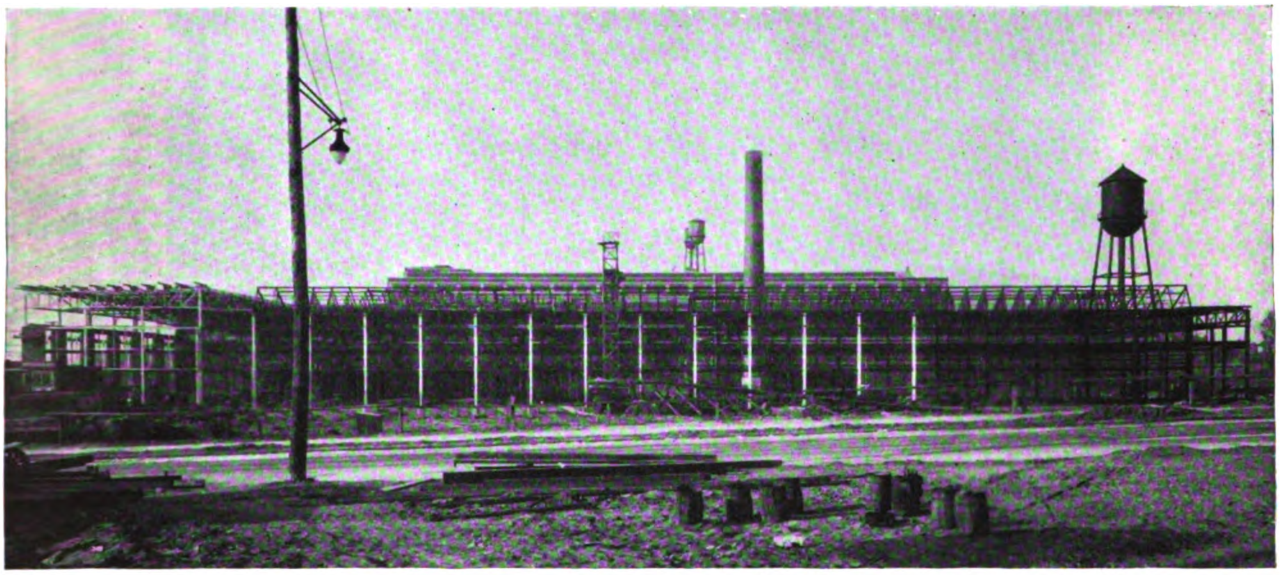

Fig. 5. A General View of the New Welded Building at Philadelphia

A roof load of 60 lb. per sq. ft. was used in the design, divided equally between live and dead load, applied not only to the level portions of the roof but also to the sloping surfaces of skylight monitors. The unit stresses in the steel members were those of the Philadelphia building code.

Horizontal bracing, consisting of diagonal rods with Carnegie beam struts, is provided in the plane of the truss lower chords in all bays of the headhouse and in the 58 ft. 6 in. main aisle where cranes operate; but in the 78-ft. main aisle with concrete gallery, lower-chord bracing is used only in four bays. Horizontal top-chord bracing is provided in four bays of both main aisles and in two headhouse bays. Vertical longitudinal bracings were also used in several bays to resist crane stresses.

Provisions are made for the following cranes: In the headhouse, 10-ton bridge cranes with span of 73 ft. 5½ in.; in the 59-ft. main aisle, 10-ton bridge cranes with span of 55 ft. and 2-ton wall cranes on either side; in one end of the 79-ft. aisle one 5-ton bridge crane with span of 46 ft. 2 in. All bridge crane runway girders are Carnegie beams generally supported directly on top of the auxiliary crane columns. The only exception is at intersection of the headhouse and the 59-ft. main aisle where the 10-ton headhouse crane girders are supported on a deep welded truss carrying not only the crane but the roof and clerestory loads as well.

The wall cranes are supported on brackets carried by the auxiliary crane columns and main columns. As these cranes exert horizontal forces at both upper and lower levels, as well as vertical forces at the lower levels, their runway supports are not simple.

In the headhouse are three interesting structural features; first, supporting one transverse headhouse 77-ft. truss on two longitudinal wall trusses at the connection of this new building to an existing adjacent one; second, supporting one end of one transverse headhouse 77-ft. truss at the center of a 58 ft. 6 in. truss at the intersection of the headhouse and one main aisle; third, supporting the headhouse crane runway girders by welding them to cantilever ends of two needle beams supported by a 58 ft. 6 in. truss at the intersection of headhouse and the main aisle.

Welding

The use of welding resulted in a considerable saving of steel, in a reduction in the cost of the steel frame, and in the elimination of the noise of riveting. Having a total of 989 tons of steel, of which 745 tons passed through the welding shop of the American Bridge Company at Trenton (N. J.), this building is one of the heaviest among welded structures. It is also unique in that it exceeds all other welded buildings in the use of trusses.

At the Trenton fabricating shop five welders were generally used, each employing a single-operator motor-generator set consisting of a generator, control panel, motor, starter, and reactor assembled on a base. At the building site in West Philadelphia two welders were used, each supplied with a machine brought from Trenton and made portable by mounting on simple hand trucks.

When depositing a $\frac{3}{8}$ by $\frac{3}{8}$-in. triangular fillet, the arc voltage was about 23 to 25 and the current 175 amp. for $\frac{3}{16}$-in. G-E welding electrodes, Type F. Direct current was used and was under the control of the operator, the current being changed by shifting the brushes, employing low voltage for light work and higher voltages for heavy work.

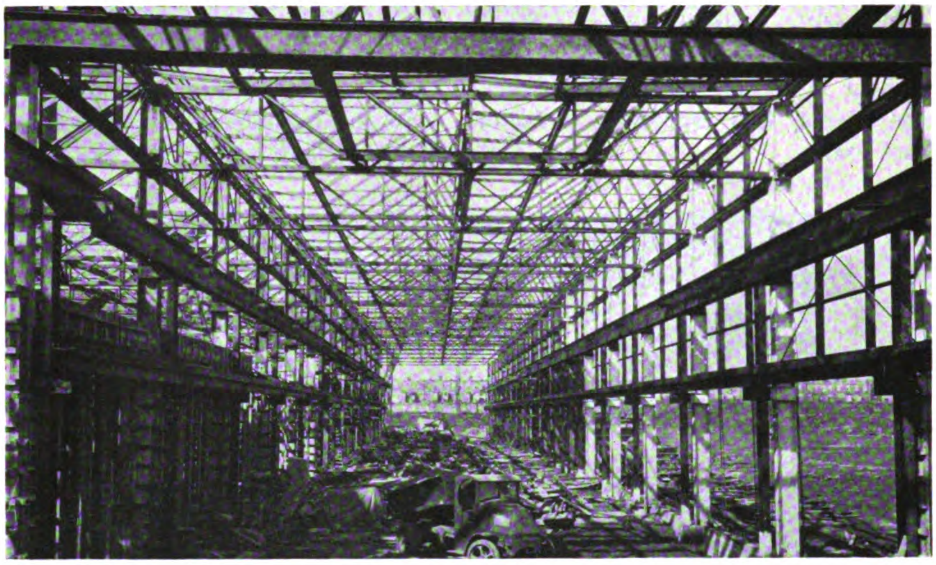

Fig. 6. A View of the Steel Work of the Welded Building, Looking Lengthwise

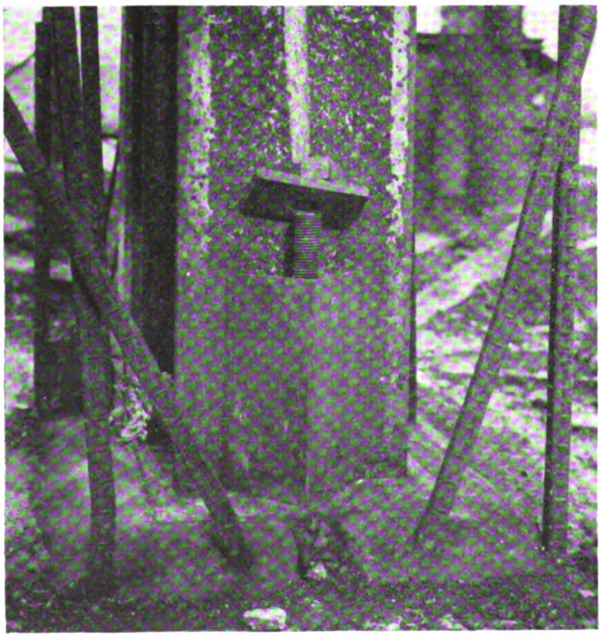

Fig. 7. A Detail of the Anchor-bolt Fastening of a Column When Welded Construction is Utilized

In the fabricating shop a series of 18-in. I-beams from 21 ft. to 24 ft. long were supported about 9 ft. apart on the shop floor and connected together with a steel bar welded to each beam. The positive terminal was bolted to the web of one of these beams, and the negative connected to the $\frac{3}{16}$ by 24-in. electrode in the electrode holder held in operator's hand.

The 206 skylight trusses were nearly alike and were assembled by clamping the members in jigs which expedited the process greatly. Jigs for small triangular trusses can be made by welding flat plates to skids, then welding small angle clips to the plates so that when the truss members are laid against the clips

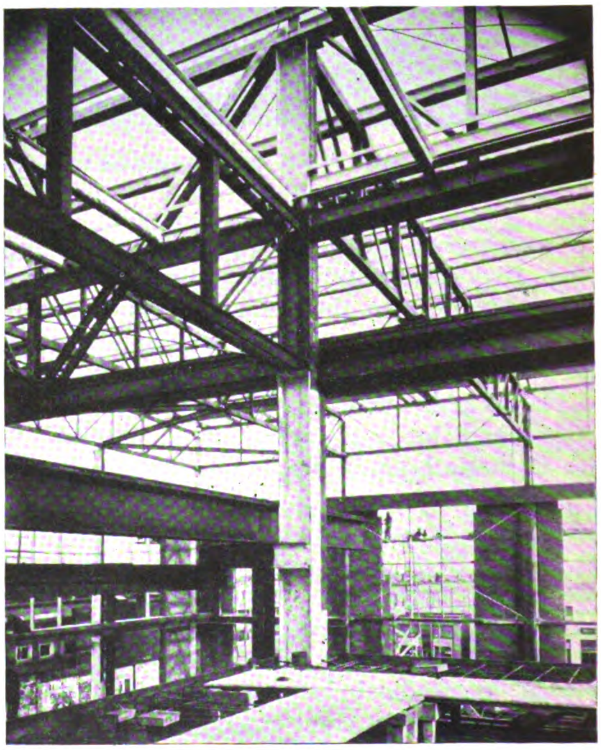

Fig. 8. The Intersection of Main Building and Transept, Showing Simplicity of Construction at the Ends of Heavy Members

the truss is quickly assembled and clamped, then either welded in place or removed for welding.

The large trusses were put together by first laying the bottom chord against a long 8 by 8-in. angle welded to the I-beam skids, then the top chord spaced and clamped to skids. Finally verticals and diagonals were clamped to the chords and the joints welded. All trusses were assembled and welded lying flat on the skids.

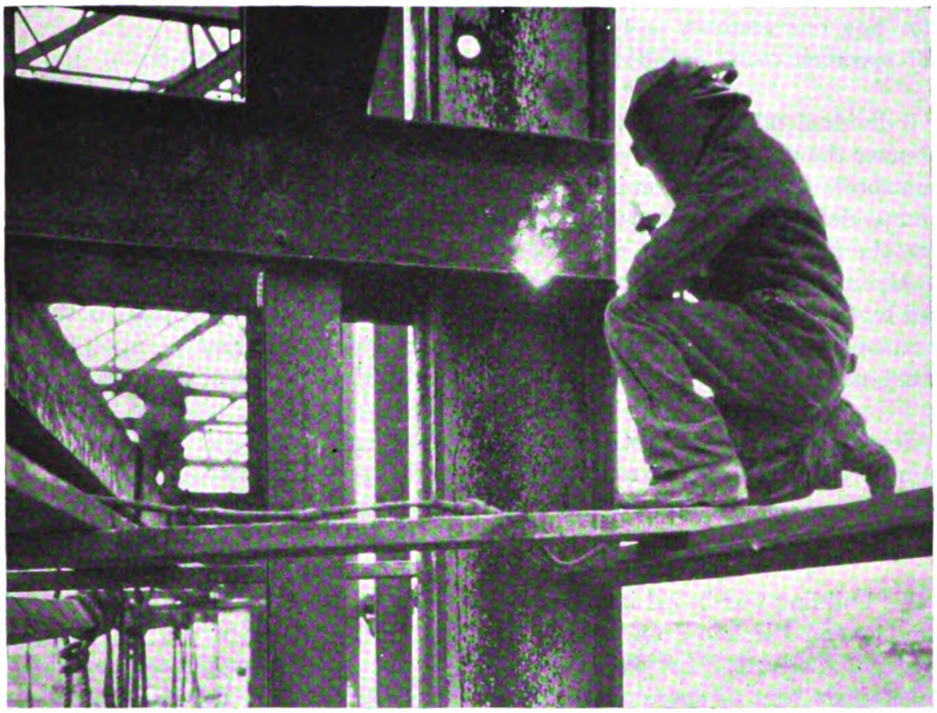

Fig. 9. A Detail of Welded Building Construction Showing a Welder Fastening the End of a Girder to a Column

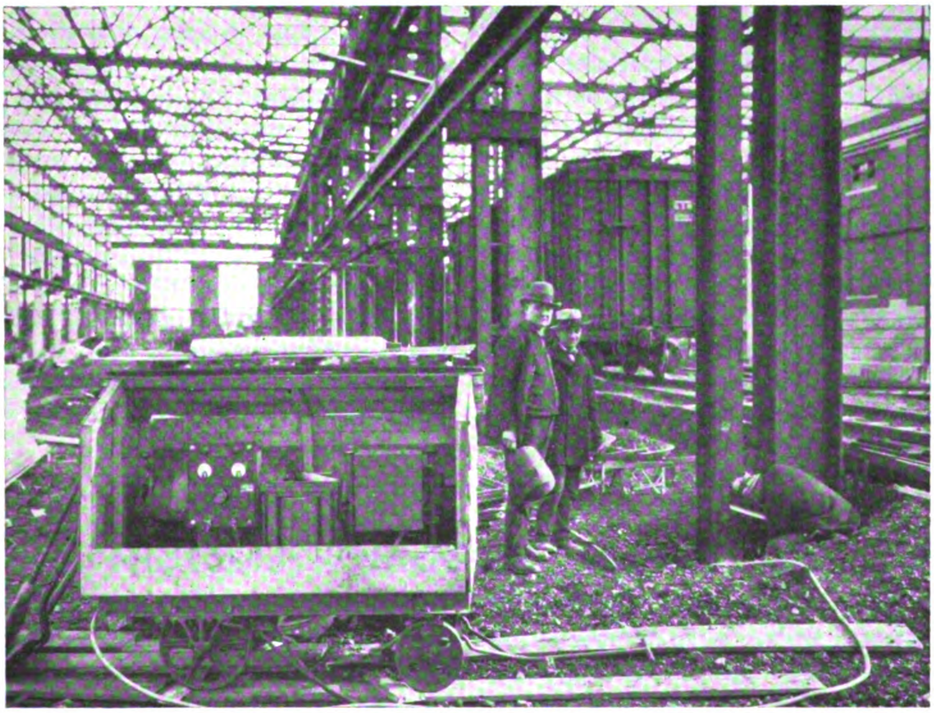

Fig. 10. Fastening a Crane Column to Its Base. A Portable Welding Set is Shown in the Foreground

All lengths of welds specified on the drawings were marked on the assembled steel; and it is interesting to compare the lengths of fillet welds on the web members, specified on the drawings of one of the trusses in the 78-ft. aisle, with actual measured lengths of welds on the truss:

(1). Total actual length of fillets is 633¼ in. compared with 598 specified.

(2). Maximum excesses of actual over specified for any one member is 5½ in. or 46 per cent of specified weld; and 6¼ in. or 52 per cent for another fillet.

(3). Maximum deficits of actual as compared with specified for any one member is 1½ in. or 6 per cent of specified weld; and 3½ in. or 8.8 per cent.

(4). Some few individual fillets are as much as 10 per cent shorter than specified.

This comparison shows that inspection is necessary but that maximum deficits are not sufficient to appreciably reduce the factor of safety.

It is probable that, in truss work, costs can be materially reduced by the use of jigs for rapid assembling of the truss members, and by specifying on the drawings the actual rather than imaginary working points.

The shop fabrication and field erection of the building required nearly two months for each of these operations.

Tests

The design of this building was based in part upon tests made for the General Electric Company by the Rensselaer Polytechnic Institute. The plates were of such sizes and thicknesses that at the ultimate loads the stresses in the plates were much below the elastic limit. For specimens in tension, the ⅜ by ⅜-in. triangular fillets of varying lengths gave an average longitudinal shearing strength of 13,300 lb. per linear inch of fillet; whereas, compression specimens with varying lengths of ⅜ by ⅜-in. fillets gave from 17,800 to 15,800 lb. ultimate shearing strength per linear inch of fillet.

It is apparent that the 3000 lb. per linear inch used in designing this building gives ample security.

At Trenton three 58 ft. 6 in. trusses were temporarily erected for testing, being supported and braced and erected in the same relative positions that they were to occupy finally in the completed structure. Test loads consisting of I beams were then applied to the top chord joints through purlins until the center truss carried a total load equivalent to 1½ times the load for which it was designed. As the total design dead load was 30 lb. per horizontal sq. ft. and the live load the same amount, this test load was equivalent to the full dead load plus twice the live load.

The vertical deflection at the center top chord joint checked very closely with the calculated deflection. Deflections were observed until the truss was carrying the design load, when the central deflection was 0.52 in., then continued until the truss supported the design dead load plus double the design live load at which time the deflection reached 0.78 in. As the loads were removed, the deflection decrements were almost exactly equal to the increments under increasing loads. Finally, upon removal of all applied leads, the truss occupied exactly its initial position showing that no slipping in the joints had taken place under the test.

Before any welder was allowed to do work on this building, either at the fabricating shop or at the building site, he was given certain tests by a capable welding inspector and his work was subsequently watched by the inspector. A comparison in accomplishment was secured at Trenton by having each of the five welders make two test pieces consisting of two 6 by ⅞-in. main plates spliced by two 5 by ½-in. plates with 10 in. of ⅜ by ⅜-in. fillets on each side of the splice. These ten specimens were then tested and showed that, while the two specimens made by any individual welder showed remarkably close results, the variation between the lowest average and the highest, *i.e.*, between the poorest and the best welder, was about 22 per cent of the lowest. The lowest values, however, were practically the same as those used in design; and it is the lowest value that controls, not the average. Of course, two specimens per welder are not many, but taken in connection with other tests and accomplishments of these five shop welders, it is believed that the difference in the specimens represents shop variations. These are no greater than exist in concrete tests or in riveted joint tests.

The New Ford Has Many Welded Parts

The increasing use of welding in automobile manufacture is demonstrated by the large number of arc-welded parts in the new Ford car. Among these welded constructions are the rear axle housing, the steering gear casing, the radius rod, the spare tire carrier, the drag link for the front axle, the axle rods; and it is said that the welding of several other parts is being contemplated. To carry on this work the Ford Company has installed about 30 General Electric automatic arc-welding equipments in the past few months.

Low-voltage A-c. Networks

PART IV: LOAD DIVISION

By D. K. BLAKE
Central Station Engineering Department, General Electric Company

As a consequence of non-uniformity of spacing of loads connected to a network system, the spacing of transformers cannot be uniform; and to secure a uniform cable layout is difficult because of the manner in which the city streets are usually laid out. It is therefore necessary to give particular attention to the question of load division in order to avoid overloading cables, transformers, and feeders. The problem is simply one of balancing the impedances of the cable, transformer, and feeder circuits. Since it is not very practicable to vary the impedance of the cables, the transformer location and sometimes the reactance have been adjusted to suit local conditions.

Transformer Reactance

In Fig. 20 is represented a section of a network wherein each transformer is fed from a different feeder. Now, if feeder *No. 3* is removed from service, the transformers connected to feeders *No. 1* and *No. 2* will divide the load formerly carried by *No. 3* according to the ratio of the impedance of the transformers to the impedance of the cables between transformers *No. 1* and *No. 2*. For convenience, the cable reactance is taken as unity in Table VIII and the transformer reactance is varied showing how the load division is affected. There is little to be gained by increasing the transformer reactance above the cable reactance.

TABLE VIII

RELATIVE REACTANCE		TRANSFORMER LOAD IN PER CENT	
Cable	Transformer	Feeder Circuit No. 1	Feeder Circuit No. 2
1	0.25	16.5	83.5
1	0.50	25.0	75.0
1	0.75	30.0	70.0
1	1.00	33.0	67.0
1	1.25	35.8	64.2
1	2.00	40.0	60.0
1	3.00	42.8	57.2

Suppose that the transformers of Fig. 20 are 300-kv-a. banks, of 5 per cent reactance, spaced 500 ft. apart, and that there are three 500,000-cir. mil cables in one duct. Then the additional load thrown on transformer *No. 2* is 77 per cent of that formerly carried by *No. 3*; and *No. 1* carries 23 per cent extra. If the reactance of the transformers is increased to 10 per cent then the division of the additional load is 68 per cent and 32 per cent respectively. The foregoing neglects the primary feeder impedance which will give a slightly better division of load.

It will be observed that the real effect of the additional reactance is to permit the transformer to carry 8 per cent additional load. Some engineers consider it more economical to underload the transformers 8 per cent than to pay the extra cost of the special 10 per cent reactance transformer. Furthermore, it is doubtful if transformers are actually loaded so close to their carrying capacity that a reduction of 8 per cent would be of any value. It seems likely that the ordinary addition of transformer capacity to keep ahead of load growth provides sufficient margin. Therefore, considering the extra cost, the delay incurred in securing the equipment, and the higher regulation of the 10 per cent reactance transformer, it seems that as a general practice standard transformers ought to be suitable.

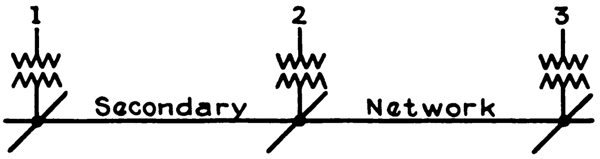

Fig. 20. A Schematic Network Showing Transformers Fed by Separate Feeders

After the network is installed a survey of load division can be made and additional reactance can be inserted where particular conditions demand it. This additional reactance may be obtained very economically and conveniently by means of cable-type reactors made of iron punchings as shown in Fig. 21. These reactors may be obtained in 1 per cent units for the 300-kv-a. bank of transformers. The smaller the bank of transformers the more units are necessary to obtain the required reactance because of the smaller cable current. In connecting such reactors into the circuit it is necessary to break the lead sheath to avoid short-circuiting the reactor. This may be avoided by putting the reactor in the neutral conductor and applying tape from the copper to the lead. The use of bakelite varnish between layers will make a watertight joint. One company short circuits the reactors with a fuse designed to blow when the transformer is overloaded. Thus better regulation is obtained when the reactor is not in actual operation. The reactors have an air-gap which must be lined up, when more than one is used, to prevent overheating caused by adjacent units shunting some of the flux around the air gap.

When large buildings are supplied from two or more feeders as shown in Fig. 22, it is desirable to place reactors between the transformers to reduce unequal load division due to unequal voltage regulation of the primary feeders. The cable-type reactors are utilized for this purpose. Another important condition requiring the use of reactance is that in which the network is fed at two different voltages such as 2300/4000 volts from a step-down substation and at 13,200 volts from the generating station as illustrated in Fig. 4.(3) High-reactance transformers or cable-type reactors are used on the transformers supplied from 13,200 volts in order to balance the impedance of the 13,200/4000-volt step-down transformer at the substation. It is obvious that if 10 per cent reactance, 2300/4000-volt network transformers are used, it would be necessary to have at least 15 per cent in the 13,200-volt transformers. Such a reactance is too high for the desired regulation. Therefore, it is important that the 2300-volt transformers be of standard reactance, such as 3.5 per cent, so that more than 10 per cent will not be necessary for the 13,200-volt transformers. If the 2300/4000-volt transformers are to be a very small part of the total network it may be better to accept the condition of slight underloading than to improve the load division for a time and be handicapped later with high-reactance 13,200-volt transformers.

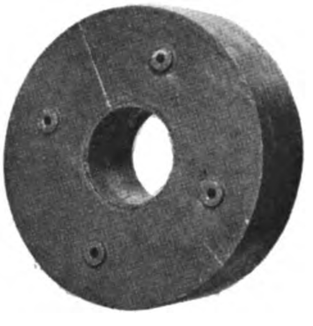

Fig. 21. A Cable-type Reactor Built Up of Iron Punchings

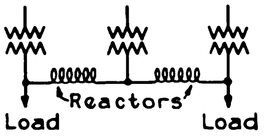

Fig. 22. Diagram of Feeders and Transformers Between which Reactors Are Connected for the Improvement of Load Division

Parallel Operation of Induction Regulators

It is necessary to consider also the influence of induction regulators on load division when they are connected in feeder circuits supplying network systems. It is the usual practice to control the induction regulators of ordinary radial-feeder circuits by a contact-making voltmeter in combination with a line-drop compensator. The function of the contact-making voltmeter is to raise or lower the regulator voltage when the feeder voltage falls below or rises above its setting. The function of the line drop compensator is to increase (in effect) the setting of the contact-making voltmeter as the load increases in order to hold the voltage approximately constant at the load.

If the contact-making voltmeter only were used, the load on the feeders supplying the network would divide approximately according to the impedance of the feeders as long as the contact-making voltmeters have approximately the same setting. When, however, the line-drop compensator is added, there is danger of one regulator going to "full buck" and the other to "full boost," with the resultant circulating current unbalancing the loads on the feeders and also the loss of the regulating features. This may be caused by one regulator getting ahead of the other (because of slight differences in contact-making voltmeter settings and regulator operating characteristics) producing a circulating current component flowing out to the network in its feeder and in from the network in the other feeder. This resulting compoment of circulating current, acting through the line drop compensator, will cause the leading regulator to further advance and the lagging regulator to go in the opposite direction. This will continue until the leading regulator reaches the full-boost position and the lagging regulator reaches full-buck position.

It is necessary, therefore, to have some method of controlling the regulators so that they cannot go to their limit positions and produce a circulating component. Mechanical interconnection is a remedy, but this is objectionable because of the complications involved in running a shaft system between the circuits. Furthermore, mechanical interconnection does not permit individual compensation for each feeder and would not be applicable in the few cases where it is desirable to have a separate line-drop compensator setting for each feeder. There are several schemes available for interconnecting the line-drop compensator and contact-making voltmeter circuits in such a manner as to permit individual settings of compensators and for preventing the regulators' getting far enough apart to cause objectionable circulating currents. There are just two of the schemes in general use for network systems and these will be described.

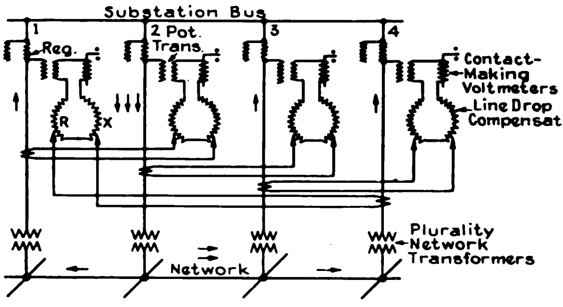

Fig. 23. The Diagram of Connections for a Scheme of Operating Regulators which Provides a Sequence of Adjustments of Circuit Conditions

The first scheme in general use on network systems has the current transformer of feeder *No. 1* connected to the compensator of feeder *No. 2*, the current transformer of feeder *No. 2* connected to the compensator of feeder *No. 3*, and so on for all the feeders; and the last feeder would have its current transformer connected to the compensator of feeder *No. 1*. This is illustrated schematically in Fig. 23. Suppose there is no load on the network and that *No. 2* regulator is advanced by hand to cause a circulating current which divides equally among the other three feeders on its return as shown by the arrows. The current in *No. 2* will cause regulator *No. 3* to go in the boost direction while the other regulators will go in the opposite direction. When the current in *No. 3* passes through zero, *No. 4* regulator will pass through neutral and then go in the boost direction after the current in *No. 3* flows out to the network. Likewise *No. 1* regulator will be in the buck position until the current in *No. 4* passes through zero. The regulator in *No. 2* will continue to go from the boost to the buck position until the current in *No. 1* goes to zero. The regulator in *No. 3* will not return to the neutral position until the current in *No. 2* passes through zero.

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(3) Fig. 4 occurs in Part I, which appeared in the February, 1928, number of the REVIEW.

It would be quite a difficult and lengthy matter to thus follow this sequence through until the regulators finally return to the neutral position, but enough of the process has been given to show that the adjustment takes place in a sequence and that the regulators each move back and forth a few times until they all finally have come to rest on the neutral position. It is evident, however, that under usual operating conditions the regulators cannot get very far apart and that they will take such positions as to cause approximately the same current in each feeder. If feeder *No. 2* has more impedance than the others, it will have a normal position ahead of the others in order to compensate for the greater impedance. Therefore, this scheme successfully satisfies the requirement of approximately equal load division with feeders having different lengths and impedances.

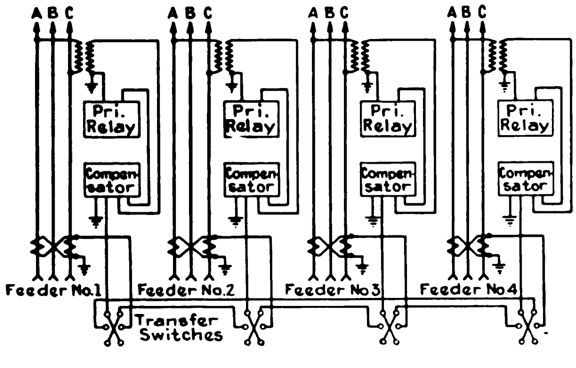


Fig. 24. The Connection of Transfer Switches in a Parallel-feeder Regulator Circuit

In applying the scheme to a network system it is necessary to provide transfer switches, operated by the feeder oil circuit breaker, in order to transfer the connection of the current transformer in, say, feeder *No. 2* from the compensator of *No. 3* to that of *No. 4* when feeder *No. 2* is removed from service. The connections of these transfer switches are shown in Fig. 24.

The other scheme now being used is shown in Fig. 25. The usual connections of current transformer, potential transformer, contact-making voltmeter and line-drop compensator are shown with the addition of an impedance (*Z*) and current transformer (*T*). The current transformer (*T*) and impedance (*Z*) are usually in the form of an additional compensator with a 1:1 current transformer built in. The elements *T* and *Z* are so connected in the usual circuit that the drop across *Z* is opposite in direction to that in the usual compensator. The combinations of *Z* and *T* of all the circuits are connected in series by means of a pilot wire. When all the regulators are together the load current sets up a circulating current in the pilot wire and current transformer *T* but no current flows through *Z*. Therefore, as long as the regulators are together the control circuits perform just as though there were no interconnection and the impedance (*Z*) were not present.

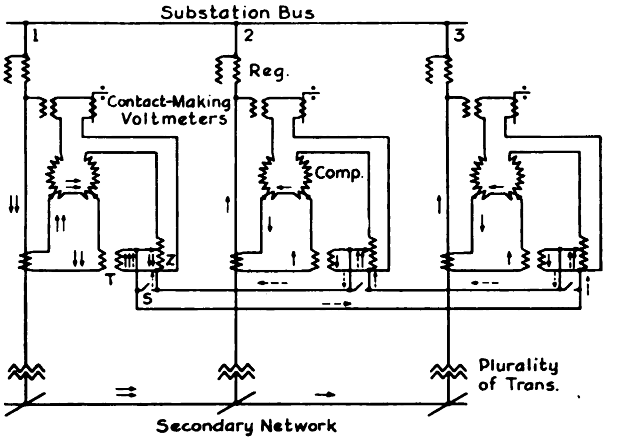


Fig. 25. The Diagram of Connections for a Scheme of Operating Regulators in which Adjustments of Circuit Conditions Are Made Simultaneously

If, however, *No. 1* regulator gets ahead of the others a circulating current is set up in the feeders which flows out to the network over *No. 1* and in from the network through *No. 2* and *No. 3*. Since the current in feeder *No. 1* is opposite to and greater than that in *No. 2* and *No. 3*, all the current does not circulate as before. Instead, part goes through *Z1* and part through the pilot wire and *Z2* and *Z3* of the other feeders as shown by the dotted arrows. The division of the current depends on the number of the feeders; and in this case about two-thirds goes through *Z1* and one-third through the pilot wire, *Z2*, and *Z3*. This is because *Z2* and *Z3* in series are in parallel with *Z1* and, therefore, the secondary current of *T1* divides inversely as the impedance. The current in *Z1* opposes the compensator *No. 1* so that regulator *No. 1* cannot stay in the boost position but is returned toward neutral by the contact-making voltmeter because the regulator is boosted above the voltmeter setting and the compensator effect is two-thirds neutralized by *Z1*. At the same time regulators *No. 2* and *No. 3* go in the boost direction because of the pilot wire current shown by the dotted arrow. The circulating current flowing in the feeder circuits have no effect on regulators *No. 2* and *No. 3* because the full arrows show that this current flows through *Z2* and *Z3*, completely neutralizing that flowing through their compensators. Therefore, *No. 1* regulator undergoes twice as much correction as *No. 2* and *No. 3* which is the proper proportion to quickly and effectively bring all the regulators together and eliminate the circulating current. After the circulating current is reduced to zero the contact-making voltmeter returns the regulators to neutral or to the position determined by the load current through the compensator. When a feeder is removed from service, *T* and *Z* are short circuited by the switch *S*.

The first scheme makes adjustments in a sequence while the second adjusts all regulators simultaneously in the proper proportion. The former requires double-throw transfer switches while the latter requires short-circuiting switches. Both schemes permit each feeder to have individual line-drop compensator adjustments.

When networks are supplied from feeders from different substations or from both substations and generating station, as illustrated in Fig. 4,(3) it is of course not feasible to use either scheme of interconnection between regulators at different stations. It is probable that the reactance (resistance is usually negligible) of the transmission circuits and power transformers will be high enough to keep the regulators stable without interconnection schemes. Such schemes are not necessary where the total reactance compensated for is less than the total reactance of the loop formed by the feeders, transformers, and network. If two feeders supplying a network have 3 per cent reactance, and the transformers and the network are respectively of 5 per cent and 10 per cent reactance, then the loop reactance will be 23 per cent. If each regulator compensator is set at 11.5 per cent or above, the regulators will be unstable and an interconnection scheme becomes necessary. If each regulator is set lower, say at 10 per cent, then they will be stable and an interconnection scheme unneccessary. This may be explained by the aid of the vector diagram in Fig. 26. Vector *oa* represents the setting of the contact-making voltmeter. Now suppose one regulator is run to the full-boost position by hand; then the circulating current will run the regulator in the other feeder to full buck. The first regulator has voltage *od* applied to the contact-making voltmeter and compensator circuit. The circulating current set up in the feeders will cause a voltage drop in the compensator of the first regulator which subtracts from the impressed voltage to give the resultant voltage applied to the contact-making voltmeter. A setting greater than half the loop impedance will subtract the voltage *db* giving the resultant *ob* on the contact-making voltmeter.

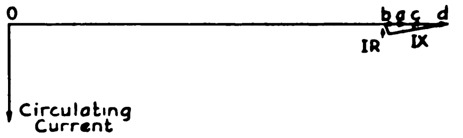


Fig. 26. Vector Diagram of Conditions Determining the Stability of Regulator Operation

This value (*ob*) is below the voltmeter setting and consequently the boosting contacts will be closed, and the regulator will stay in the boost position. Therefore, the regulators will be unstable. A compensator setting lower than half the loop impedance will run the second regulator only part way to full buck. The resultant circulating current will subtract the voltage *dc* giving the resultant *oc* on the contact-making voltmeter of the first regulator. This value (*oc*) is above the voltmeter setting and consequently the lowering contacts will be closed and the regulator returned to the neutral position. It is obvious that the regulators will be stable, for the contact-making voltmeter always has complete control. It is believed, therefore, that where two or more substations supply network systems, it will be possible to operate bus or feeder regulators without pilot wire interconnection between substations because of the uncompensated reactance in the power transformers and transmission circuits.

(*To be continued*)

Railroad Electrification Reduces Damage by Smoke

Since the electrification of the Chicago Terminal of the Illinois Central Railroad (described in detail in the April 1927 issue of this magazine) the Chicago Art Institute has found that the amount of soot and cinders falling on the roof of its building has been reduced by two-thirds. During the year prior to the electrification some 13 tons were removed from the roof; whereas during the following year (1927) there were only about four tons, these resulting from the remaining local combustion activities. In consequence of there now being less smoke to seep into the building, the Institute has been able to reduce its maintenance staff and its work of preserving objects of art has become less difficult. This experience in connection with a single building furnishes an indicative measure of the greater cleanliness enjoyed also by the surrounding neighborhood.

Cincinnati Street Railway Company Leads in Power System Rehabilitation

Extent of Project—Power Purchased—Nineteen Substations, All Fully Automatic—Supervisory Control System Additional—Operating Features

By F. W. PETERS
Railway Engineering Department, General Electric Company

Member companies of the A.E.R.A. recently gave the following weights to the conspicuous advantages of automatic stations: improved labor conditions, 12; reduced operating expenses, 12; improved service continuity, 11; greater reliability, 10; greater protection to equipment, 8. A new and notable automatic installation is here described in an A.I.E.E. paper the author read last February at Cincinnati.—EDITOR

THE modernization of the power supply of the Cincinnati Street Railway is one of the outstanding developments of the street-railway industry. In no other city of metropolitan proportions has this development progressed to 100 per cent automatic operation, all under the direct control of a centrally located dispatcher through a supervisory and remote metering system.

This new power and distribution arrangement covers an area having a maximum radius of 8½ miles from the city's business center, throughout which are distributed 19 automatic substations. These station locations have been carefully selected to properly serve the present railways as well as the anticipated growth of the city and railway system.

Each station is automatic in all respects. No attendants are required and the doors are locked from the outside. Whenever a demand for power arises at any point, as represented by cars operating on some part of the system, the station serving that district will commence operation and continue to deliver power so long as the load demand exists, after which it will shut down and wait until it is again called upon to function. Should one of the station d-c. feeder circuits become involved in trouble, during a period of operation, the feeder will open automatically and then will reclose automatically only after the cause of trouble has been removed. Likewise, should irregularities occur—such as single-phase or prohibitively low a-c. supply voltage, d-c. overload, or overheated apparatus—the station will properly protect itself. In other words, each station is a responsible and dependable element in the system, will deliver power whenever required to do so, and is continuously checking for any unusual conditions against which it may become necessary to protect itself.

Even though each substation is an independent and self-contained unit, it has been considered desirable to superimpose over all 19 stations a supervision by a central dispatcher. Through the medium of relatively few control wires and suitable automatic selector equipment, he will at all times be informed whether a station is operating (if a two-unit station, whether one or two units are operating); the open, closed, or auxiliary bus position of each of the 87 feeders; together with the metered ampere load and bus voltage on each substation of the entire system. Also, in conjunction with this supervisory system the dispatcher may use his private telephone to communicate with the Union Gas and Electric Company dispatcher and maintenance men or inspectors who periodically will visit each station.

Briefly then, an area is being considered throughout which are located 19 self-dependent substations over which a dispatcher has complete supervision and a full knowledge as to the load carried and what is occurring in each station.

Over a period of many years the railway company generated its own 25-cycle power at the Pendleton plant. Some few years ago, three automatic and three manually operated 60-cycle substations were placed in operation with power supplied by the Union Gas and Electric Company. First-hand experience with automatic-type stations prompted the decision to apply these over the entire system. The basis for such action lay in the economy of automatic over manual operation, due to the elimination of unnecessary light-load losses and station attendants' wages, together with the ability of the automatic equipment to continue operating through more trying conditions than a manual equipment and at the same time protect itself far more accurately than standard manual stations.

A résumé of the conditions against which an automatic station will protect itself may be itemized as follows:

On the High-tension Side
Undervoltage
Overload or ground
Single or reverse phase
Phase-current unbalance
Transformer overload, ground or failure.

On the Converter Equipment

Excessive machine temperature
Incomplete starting sequence
Loss of field
D-c. overload
Reverse polarity
D-c. reverse current
D-c. grounding or flashover
Load-limiting resistor overheating
Overheated bearings
Over speed.

On the Feeders

Short circuits; opening and automatically reclosing as soon as the cause of the short-circuit is removed.

Location of Substations

Obviously stations should be located where they may most efficiently serve their load demands in respect to both present and future requirements.

The plan now in progress calls for the abandonment of one substation, the remodeling of eight, and the construction of eleven new substations. The locations and capacities have been determined on the basis of of spot maps set up to show the car distribution throughout the city at times of minimum headway or maximum service, together with a knowledge of car-energy consumption when loaded and on grades.

Three of the downtown stations will each contain two 1500-kw. converters. Ten stations will contain one 1500-kw. converter, 5 stations will contain one 1000-kw. converter, and one station a 200-kw. converter. Several station buildings have been made large enough and provisions incorporated for the installation of an additional unit when required by the system's growth.

Substation Ventilation

In the days of manually operated stations, the problem of station ventilation rarely caused concern. If a substation became uncomfortably warm, the attendant opened windows and doors. Since electrical apparatus may well operate within the temperatures measured by man's comfort, no one worried about or provided particular means of ventilation. With the advent of automatic substations, where windows are somewhat of a detriment and consequently reduced in size or eliminated altogether and where doors are closed and locked, an adequate means of ventilation became necessary.

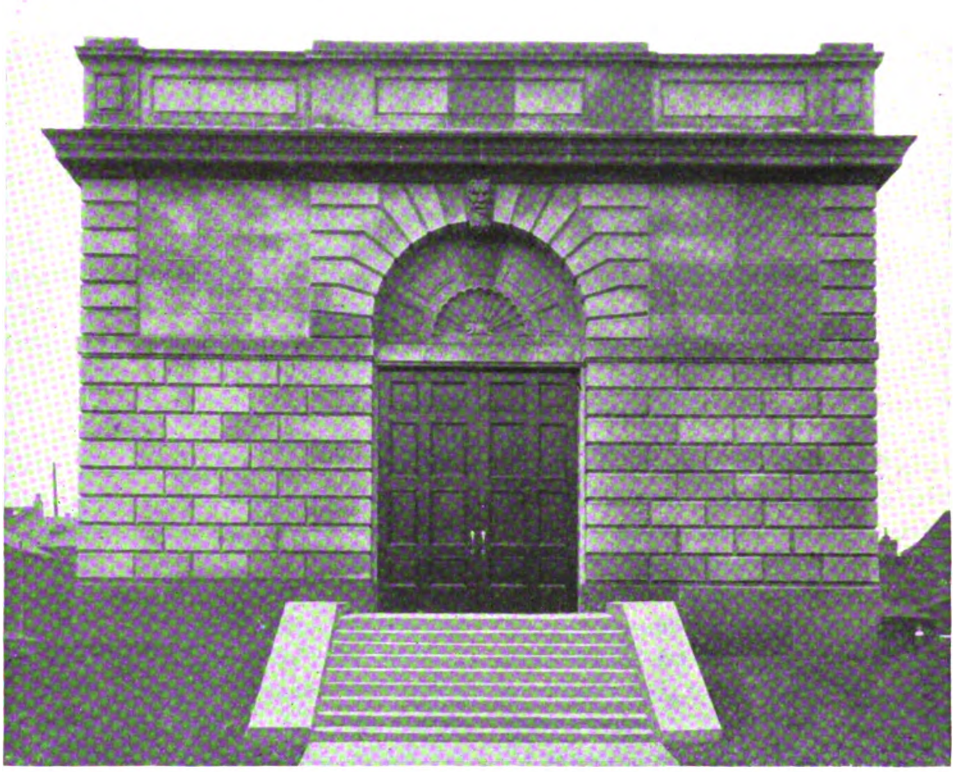


Fig. 1. Lincoln Park Substation, Containing Two 1500-kw. Automatic Converter Equipments

No comprehensive study of air circulation or vitiation in substations had been made until the American Electric Railway Association in 1924 sponsored an exhaustive report backed up with extensive tests to cover the subject. Briefly, the results evolved from this study are:

(*a*) Air is discharged from converter armatures at velocity heads up to 2000 f.p.m. in a rather narrow band on each side of the magnet frame, in a plane parallel to the frame.

(*b*) Air enters the armature at the commutator and collector ends, in a direction parallel to the shaft at velocities up to 500 f.p.m.

With these facts established, it developed that cool air should be brought into the station through ducts to floor gratings located just outside the converter base close to the bearing pedestals. This system delivers cool air to the machine at the point where it naturally draws it into the armature.

Since the warm air is discharged from the machine at high velocity, it is proper that roof ventilators should be located in the line of this velocity so that velocity head, temperature head, and any suction set up by wind passing through the ventilator will all be working in series to eject heated air from the building.

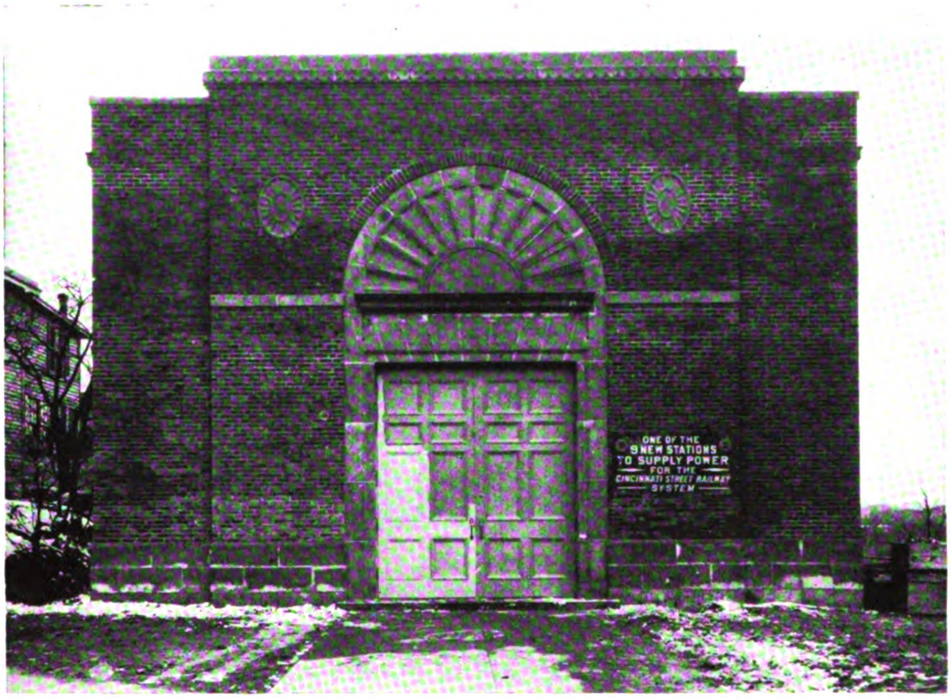


Fig. 2. Westwood Substation, Containing One 1000-kw. Automatic Converter Equipment

The new buildings now nearing completion have all been designed with these principles in mind; and to insure continuity of operation in the hottest weather, provision has been made to conduct to the outside the heated air discharged through the base of the machine. The transformers are placed on a grill work through which cool air is delivered to the transformer base and flows upward through the tubular radiator pipes and then on up to a ventilator placed overhead where the temperature head again assists the ventilator suction in discharging heated air from the room.

Building Construction

The buildings deserve initial reference to their conspicuously improved appearance, both inside and out, with respect to the conventional substation design and construction. There are no duplicate exterior designs; each has been treated in a substantial and artistic manner with the result that one is reminded of attractive community banks or public library buildings.

The interiors are faced with glazed brick, apparatus is arranged and installed in a manner to promote reliable operation, facilitate maintenance, and present a generally pleasing appearance, all of which prompts the safe prediction that this property will be one of the outstanding show places in this country for modern street railway power operations.

All equipment is located on the main floor in one room with no galleries or second floors. A half basement serves only for cable runs and for bringing cool air to the floor gratings. The sub-construction is of reënforced concrete; the walls are of hollow brick with glazed brick inside facing; the building fronts are made decorative with various materials to produce the pleasing effect desired.

Substation Apparatus

Each double-unit station has two incoming 13,200-volt 60-cycle a-c. lines whereas each single-unit station has one incoming line with provision for a second line upon the addition of a second unit. Relay protection on these lines has been designed in conjunction with the Columbia Engineering & Management Corporation's staff so as to insure coördinated operation between the two systems.

The a-c. supply enters the building through cables leading to a fireproof, fabricated, oil switch compartment which houses the current and potential transformers, disconnecting switches, fuses, and main oil circuit breakers. Lead-covered cable between the switch structure and the main power transformer enters the latter through side-type oil-filled potheads with no exposed high-voltage conductor.

The transformers are of the outdoor, three-phase high-reactance core type with interleaved disk winding and tubular radiators. The primary taps may be changed by means of an operating lever extending through the transformer cap plate. The low-voltage leads are brought out the side through weatherproof bushings at an angle to accommodate connections to the starting and running contactor panel setting close to the transformer.

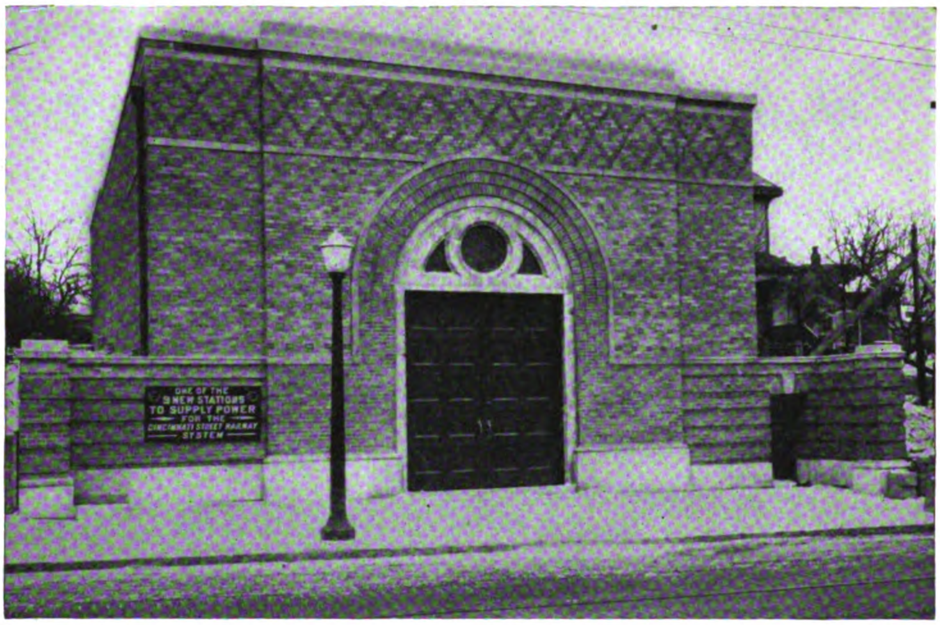


Fig. 3. Colerain Substation, Containing One 1000-kw. Automatic Converter Equipment with Provision for a Second Unit

The converters incorporate features of the most modern design. They are built with high-reluctance commutating poles, are rated at 600 volts at the d-c. end, and both the 1000-kw. and 1500-kw. units (which operate at 900 and 720 r.p.m. respectively) are capable of carrying 150 per cent load for two hours and 200 per cent load momentarily. They each have a compound field winding, but the shunt field and armature have been so designed that they may be operated without undue heating as a full-compound or a full-shunt machine or anywhere between these limits by adjustment of the series-field shunt. The principle underlying this latter feature of operation merits considerable comment, but briefly it is this:

In the downtown section or area where traffic lights, pedestrians, and vehicular traffic naturally slow up car movement, motormen must operate on controller resistance points a major portion of the time. They rarely have opportunity to operate free running with all motor resistance out of circuit.

Under such conditions it becomes unnecessary to maintain a rigid 600-volt trolley potential because this voltage is being reduced by the car resistors before it reaches the traction motors, and consequently the excess voltage represents wasted energy. Tractive effort which determines rates of acceleration exerted by series traction motors is a function of current only, but speed after the accelerating period is a function of voltage. Excessive voltage during acceleration is useless, and high voltage becomes effective only upon reaching the free-running portion of the motor curve in any duty cycle.

If shunt characteristic converters supply power to congested areas, they automatically adjust themselves to the voltage requirements of the car equipment. At night or non-rush periods where cars can do some free running and the load demand in current is relatively low, the shunt converter delivers its highest voltage, permitting high car speeds. During congestion, as represented by the rush hours, load demands are maximum, streets are crowded, cars are slowed down because of this congestion and the

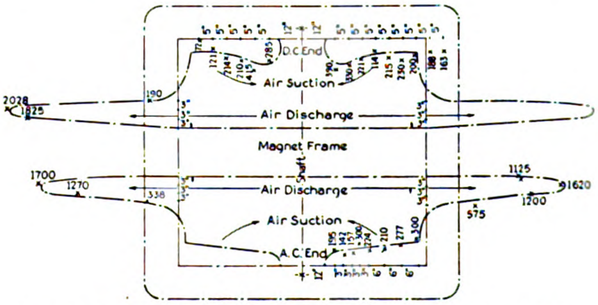


Fig. 4. Diagram Showing Velocity at which Air Enters and is Discharged from a Railway Synchronous Converter

converter inherently delivers a lower voltage and thereby decreases the amount of energy the motorman throws away in his motor resistors.

In outlying sections, where congestion is a minimum, cars can travel as fast as trolley voltage permits and here it is desirable to maintain a rigid trolley voltage, which is done by using the full series field to effect a flat voltage characteristic. Between these two extremes of maximum and minimum congestion, there is a gradation of trolley voltage requirements which may be accommodated by partially shunting the converter series field.

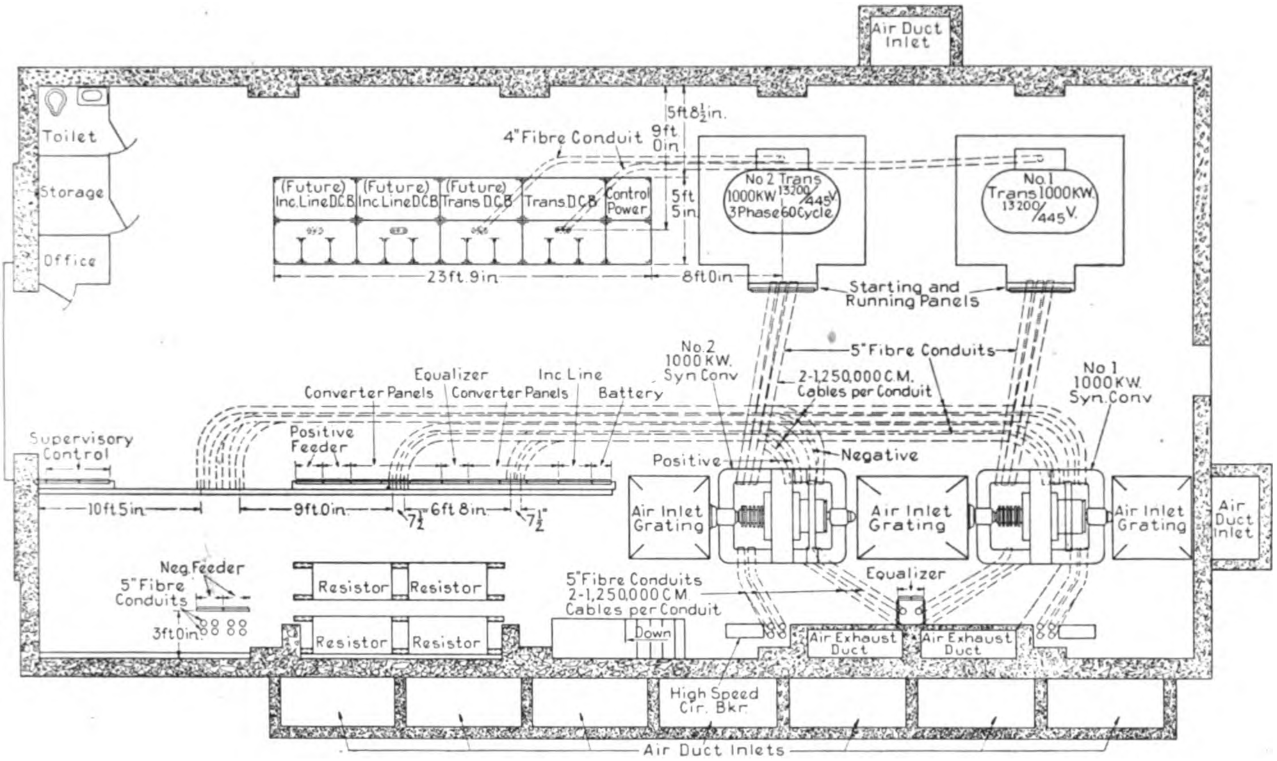


Fig. 5. Floor Plan Layout of the Double-unit Automatic Stations.

There is another distinct operating advantage in this method of operation. If in the downtown section an unusual occurrence results in many cars demanding more power from a given station than it can well accommodate, its shunt characteristic continues to drop the voltage in an effort to limit its power output, thereby permitting surrounding stations which are less heavily loaded to reach in and assist the station that is struggling with its load.

Returning to the converters proper, the machines have a rolled-steel magnet frame to insure uniform permeability, which is sometimes questionable in cast-magnet frames. Radial-type d-c. brush rigging fully enclosed with insulating and flameproof material, together with flash barriers between brush-holders, reduces to a minimum the possibility of flashover between brush-holders or arcing damage to the rigging. The collector-end brush-holders have a micrometer-type construction, thereby permitting minute adjustment throughout brush life with consequent control of this item of maintenance.

The equipment necessary to automatic operation mounted on the converter frame includes the motor-driven brush-operating mechanism which lifts all but the pilot brushes during the starting operation; bearing temperature relays which automatically shut down the machine in the event of overheated bearings; and a grounding relay which in the event of insulation failure or flash to ground immediately shuts down the unit.

The stations have been designed to reduce noise so frequently objectionable to surrounding residents or establishments. To this end the converter bases are set on cork mats to minimize the transmission of vibrations to the building structure and the setting up of noise.

Automatic Switching Equipment

Briefly, the station functions in an automatic sequence as follows: While the converter is idle, a voltage relay is continuously measuring voltage on the d-c. bus which is always energized through feeder ties from some adjoining station. If the d-c. bus voltage as measured by this relay drops to a predetermined point, it signifies the coming of a load which will demand power from that particular station. After a brief delay, to eliminate false starts because of temporary short-time loads, the voltage relay transmits an indication to start operation. Immediately the oil circuit breaker is closed, thereby energizing the main power transformer. A motor-driven drum controller rotates and contact fingers energize control circuits to close the starting contactors which connect one-half normal a-c. voltage to the converter collector rings for starting. Next, when synchronous speed is reached, the converter is polarized by momentarily connecting its fields to the station control storage battery. Then it is self-excited and the starting contactors are opened and the running contactors closed, connecting full a-c. voltage to the converter, the brushes are lowered and the machine is ready to be connected to the d-c. bus to deliver power. Up to this time many other performances have taken place. Various relays have been at work to check the a-c. voltage to make certain it is of proper value and that no accident has occurred causing single or reverse phase or unbalanced current.

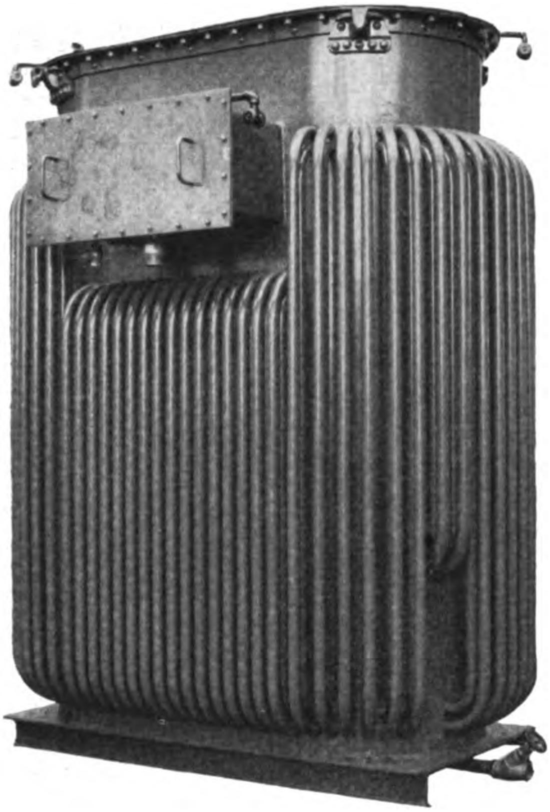


Fig. 6. Exterior View of Converter Transformer Showing the Side-type Oil-filled High-tension Potheads

The successive connections are delayed until it is proper to make the next. Speed, voltage, and polarity are re-checked and finally the converter is connected to the bus, first through a load-limiting resistor and then direct in a total time of 25 to 40 sec., depending on the conditions.

Should overload or any other unusual conditions arise, the equipment can shift the load or do whatever is necessary to permit operation being continued but should conditions be too severe the station will shut down. After the trouble has passed the converter will re-start, but if the trouble is such as to warrant an inspection before re-starting the station will remain shut down.

High-speed breakers are connected in the negative side of each converter. These open upon short-circuit in 0.007 sec., which precludes current building up to a high value before the converter is cut off the bus.

Automatic Feeders

Power is delivered to the cars through feeder circuits and trolley wires. All of the feeder equipments are controlled individually by automatic reclosing breaker equipment located at the point where they leave the station bus. If a short-circuit or severe overload occurs on any feeder, its circuit breaker opens to isolate the trouble. Shortly after the breaker opens, certain relays begin to test the circuit and so

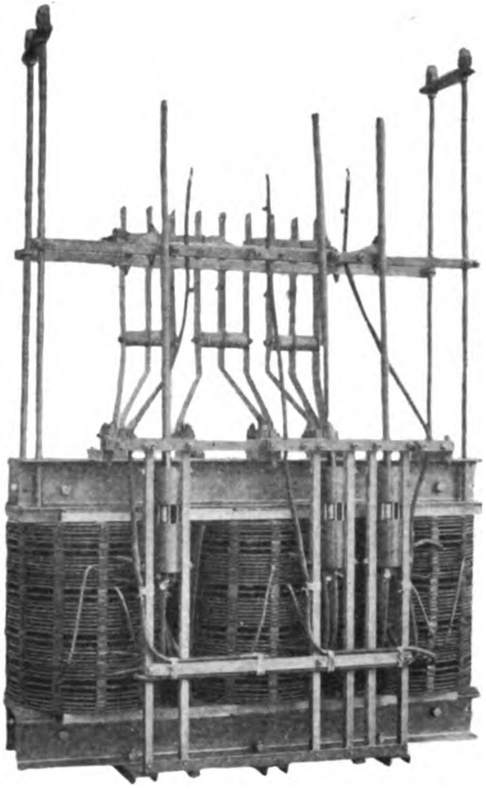


Fig. 7. Interior View of Converter Transformer

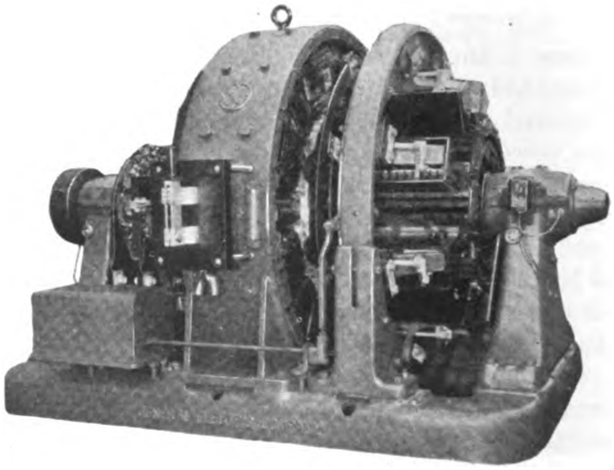


Fig. 8. Synchronous Converter with Automatic Equipment Installed

long as the troublesome conditions remain the breaker stays open. When the trouble is cleared the relays will cause the breaker to reclose and normal service is then resumed.

In several important stations, the arrangements are such that an auxiliary feeder bus may be installed. This feature and its application to the system is somewhat unusual and interesting. The auxiliary bus is normally isolated and de-energized, but the dispatcher is free to disconnect one or more feeders from the regular station bus and connect it to the auxiliary bus. Bearing in mind that practically all feeders originate at one station bus and terminate at an adjoining station bus, the dispatcher by proper manipulation through the auxiliary bus may transfer a feeder load from its normal station supply to some adjoining station. In other words, if a given station for some reason is overloaded it is possible to transfer one or more feeders to some other station which may not be overloaded. Such flexibility is a most desirable operating advantage since it permits of transferring an overload to some definite place where it can be properly handled. Fundamentally, railway traffic moves in waves and on each side of the traveling peak-points there usually are stations that are in position to handle more load.

The scheme is equivalent to adding additional converter capacity to a system but at a greatly reduced cost for station equipment.

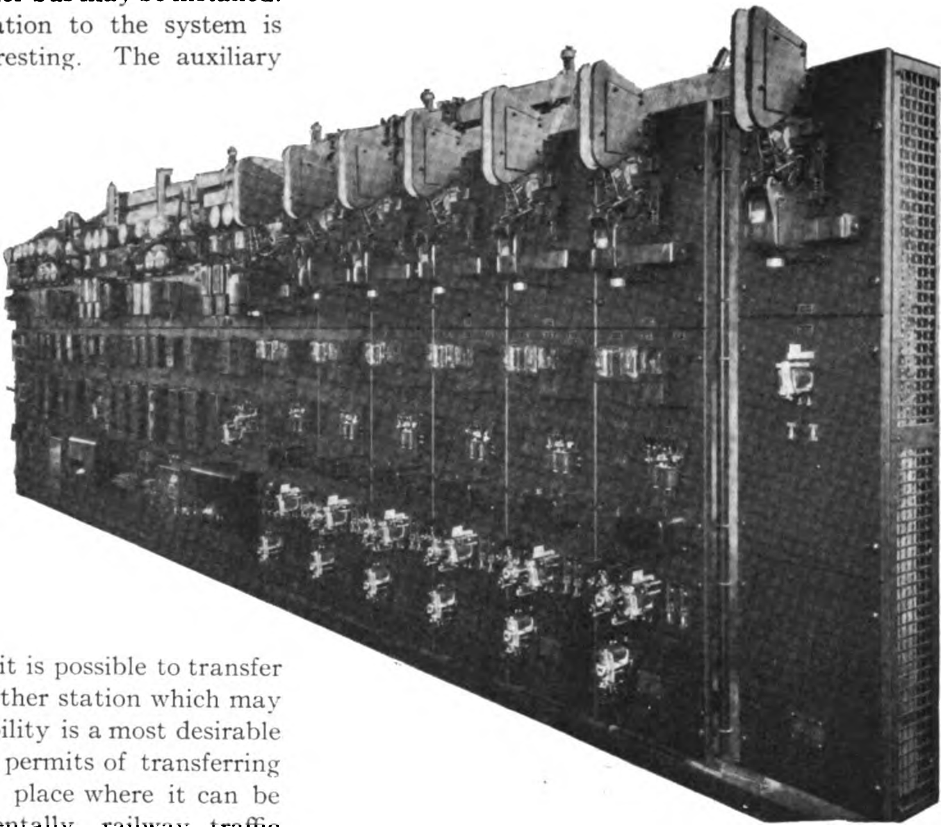


Fig. 10. Automatic Switching Panels for Two Converters and Six Automatic Reclosing Feeder Equipments

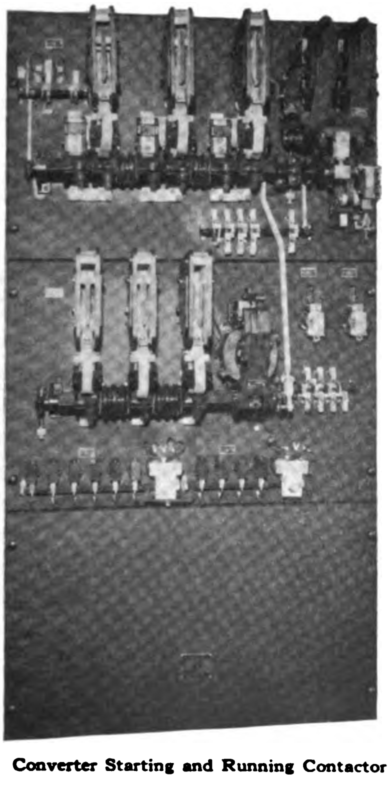


Fig. 9. Converter Starting and Running Contactor Panel

Supervisory Control

The supervisory control for this system is the most extensive thus far undertaken. Requirements are such that existing standard devices are entirely suitable but their application has been modified to the speed of operation necessary where 19 stations are involved, each of which has many functions to control

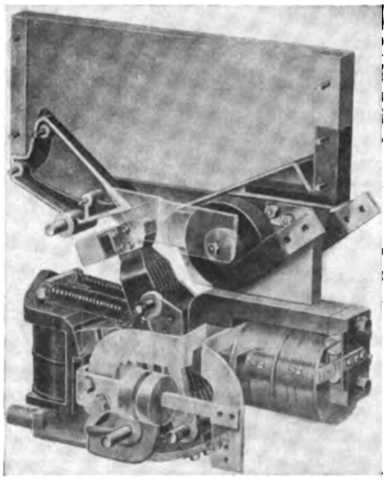


Fig. 11. Phantom View of High-speed D-c. Circuit Breaker

and indicate. In general, each station has its own individual control. This promotes reliability because an accident to the control is thereby confined to one station only. In the dispatcher's office there is a semicircular board each panel of which carries the control and remote metering for some one station. From this panel four control wires radiate to each station for supervisory control and indication. Over these wires, by means of control buttons, the dispatchers may perform the following operations:

(1). Start or stop each synchronous converter.

(2). Release each converter to automatic operation.

(3). Open or close each incoming line oil circuit breaker.

(4). Open or close each d-c. feeder.

(5). Release each d-c. feeder to automatic operation.

(6). Transfer feeders to the auxiliary bus (when installed).

By means of red, white, and green lights on the dispatcher's board, he has a continuous indication of:

(1). Running or stopped position of each converter.

(2). Whether load-limiting resistor contactors are opened or closed.

(3). Sequence in which converters will start in double-unit stations.

(4). Indication whether each high-tension line is available for duty.

(5). Open or closed position of each incoming line-circuit breaker.

(6). Open or closed position of each d-c. feeder.

(7). Indication whether each d-c. feeder is connected to the main bus or to the auxiliary bus (when installed).

(8). Indication whether the station control battery is being charged.

(9). Indication that the station lockout circuit has shut down the station.

(10). Indication whether the station doors are locked or unlocked.

As an example of performance, suppose the dispatcher wishes to start a station independent of its automatic functioning, he first goes to the particular panel of the board controlling that particular station. By turning a key and pulling a button he will, within a maximum of five seconds, establish a direct circuit between his board and the relay to be operated in the distant station. A yellow light will appear which gives him a check that the circuit has been correctly established to the distant relay. He is then free to push a button which closes the relay and a red light appears to show the operation has been correctly performed.

If during operation some automatic device operates in a distant station, the dispatcher is immediately advised by change in color, red to green or vice versa, of the light on his board associated with that device and a bell and white lamp calls attention to the change.

The outstanding feature of this supervisory control is the complete knowledge it gives the dispatcher at all times of every important function in each station, even to whether the station door is locked. Also a station is absolutely under his control to regulate system operation to best advantage. He may transfer load from one station to another for reasons of load limitation or economy of operation, or, should the fire chief order all lines cut out in a fire area, the dispatcher can do so more quickly than it would take to give verbal instructions.

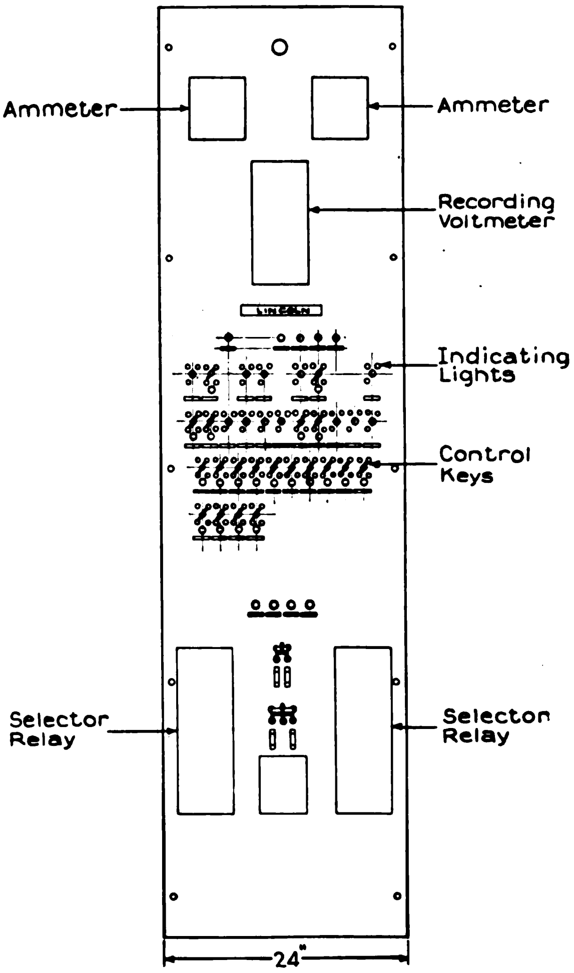


Fig. 12. Drawing of a Dispatcher's Supervisory Panel for a Two-unit Converter Station

The relays establishing a control circuit operate within a minimum of $\frac{1}{5}$ sec. and a maximum of 5 sec., which gives approximately $2\frac{1}{2}$ sec. per operation as the average. After the control circuit has been established, the operation of the device and indication back is almost instantaneous and is dependent only on how fast the dispatcher can push the operating button.

On each panel in the dispatcher's office is a recording and indicating ammeter and voltmeter which gives a record of the ampere load on each converter in the system and the bus voltage for each substation. This provides a permanent record of all load conditions and permits an accurate study to effect economy of operation. Remote metering, together with the private telephone system, requires four wires between the dispatcher's office and each substation.

Various Methods of Connecting Indicating Wattmeters

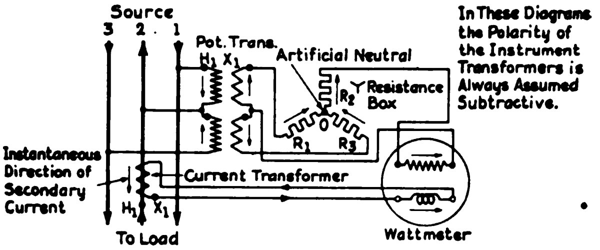
PART I

Three-phase Three-wire Circuits—Three-phase Four-wire Circuits—Single-phase and Polyphase Instruments—Various Combinations of Current and Potential Transformers—Power-factor and Voltage Balance Considerations

By JOHN AUCHINCLOSS
Switchgear Engineering Department, Philadelphia Works, General Electric Company

*About a year ago we published an article on methods of synchronizing and another on transformer polarity and connections, both by Mr. Auchincloss. Their usefulness to operating engineers was later evidenced by many appreciative comments we received. It is with pleasure therefore that we have succeeded in securing this article of a similar character from the same author.—*EDITOR

WHILE much has been written on energy measurement for billing purposes by means of the integrating watt-hour meter, comparatively little information has been published regarding the measurement of power for other purposes by means of the so-called indicating wattmeter. Since no billing is done from the latter method of measurement, operating companies may sometimes be tempted to sacrifice accuracy for the sake of avoiding added investment in large instrument transformers. While in some unusual cases this attitude may be justified, it should not be taken without a thorough understanding of the lowered accuracies inherent in such combinations, and afterward the operator should not permit everyday usage to cause him to forget these limitations. These methods will be described in Part I of this article, together with their shortcomings in accuracy; and other methods of greater accuracy will be detailed in Part II. In both parts, only three-phase power circuits and connections for instruments of the dynamometer type will be considered.



In These Diagrams the Polarity of the Instrument Transformers is Always Assumed Subtractive.

Fig. 1. Single-phase Wattmeter with One Current Transformer and Two Potential Transformers Connected to Measure Power in a Three-phase Three-wire Circuit

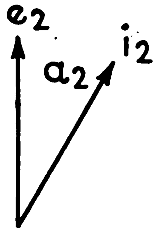


Fig. 2. Vector Relationship Determining the Instrument Torque for the Connections Illustrated in Fig. 1

THREE-PHASE, THREE-WIRE CIRCUITS

Neutral, Isolated or Grounded for Protection Only

(*No current carried over neutral*)

Two of the simplest methods of measuring power in this class of circuits are:

(a) Single-phase instrument with one current transformer and two potential transformers.

(b) Single-phase instrument with two current transformers and one potential transformer.

The connections for Method (a) are shown in Fig. 1. In the Y resistance box, the resistors R_1 and R_3 must each be equal in value to the combined resistance of R_2 plus the instrument potential coil, otherwise the point O would not be a true neutral. The torque is proportional to $e_2 i_2 \cos \alpha_2$, as shown in Fig. 2, where e_2 is the leg voltage, i_2 the current in line 2, and α_2 the angle of displacement, if any, between them. This torque is proportional to the power flowing in one line only. If however the powers in all three lines are equal, *i.e.*, if $e_1 i_1 \cos \alpha_1 = e_2 i_2 \cos \alpha_2 = e_3 i_3 \cos \alpha_3$ it is clear that if the scale of the instrument be multiplied by 3 it will read the total three-phase power directly. Since it measures the power in line 2 only, however, its readings will be in error if the powers in the other two lines differ from that in line 2. Due to the fact that only one current transformer is used, this method does not lend itself readily to relaying.

The connections for Method (b) are shown in Fig. 3. Here two current transformers are used and therefore are available for relays or other devices which may be connected *inside* the cross-connection.

As will be seen from Fig. 4 (a), the current through the meter is the vector sum of the currents in phases 1 and 3, one of these currents being reversed. Since this reversal changes the displacement angle between them from 120 to 60 deg., the resultant current I_γ is $\sqrt{3}$ times the current in lines 1 or 3. If 5 amp. represents the normal full-load current in these lines, then the current coil of the instrument should be designed to carry continuously $5\times\sqrt{3}$, or 8.66 amp. It will also be noted that the resultant current I_γ is, at unity power-factor with balanced currents and voltages, exactly in phase with the line voltage E_{3-1}, and the torque is $\sqrt{3}Ei$, which is the true power on a balanced three-phase system at unity power-factor. (1)

For unbalanced currents at power-factors other than unity, the torque is:

$$E_{3-1}\,[i_1 \cos (30^\circ-\alpha_1)+i_3 \cos (30^\circ+\alpha_3)] \qquad (1)$$

which reduces to:

$$E_{3-1}\,(0.866\, i_1 \cos \alpha_1+0.5\, i_1 \sin \alpha_1 \\ +0.866\, i_3 \cos \alpha_3-0.5\, i_3 \sin \alpha_3) \qquad (2)$$

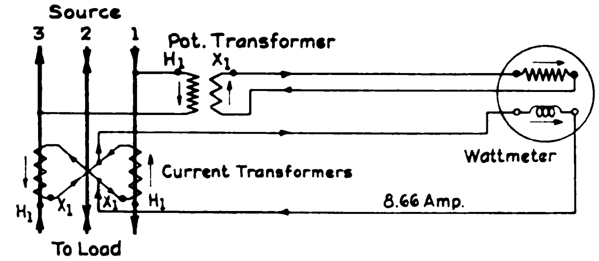


Fig. 3. Variation of the Connections in Fig. 1 to Employ but One Potential Transformer and Two Current Transformers

Now the torque developed by the standard two-element meter (which is known to be correct for any condition of unbalance or power-factor) is, from Fig. 5:

$$E_{1-2}\,[i_1 \cos (30^\circ+\alpha_1)]+E_{2-3}\,[i_3 \cos (30^\circ-\alpha_3)] \qquad (3)$$

which reduces to:

$$E_{1-2}\,(0.866\, i_1 \cos \alpha_1-0.5\, i_1 \sin \alpha_1) \\ +E_{2-3}\,(0.866\, i_3 \cos \alpha_3+0.5\, i_3 \sin \alpha_3) \qquad (4)$$

Comparing Equations (2) and (4) it will be observed that the sine terms have opposite signs, *i.e.*, where a sine term is positive in (2) it is negative in (4) and *vice versa*. Consequently, on unbalanced currents the torque of the single-phase instrument shown in Fig. 3 is not the same as that developed by the polyphase instrument shown vectorially in Fig. 5; and since the latter is known to be correct, the former must be erroneous.

If, however, we have the condition $E_{1-2}=E_{2-3}=E_{3-1}=E$, then Equation (2) becomes

$$E\,(0.866\, i_1 \cos \alpha_1+0.5\, i_1 \sin \alpha_1 \\ +0.866\, i_3 \cos \alpha_3-0.5\, i_3 \sin \alpha_3) \qquad (5)$$

and (4) becomes

$$E\,(0.866\, i_1 \cos \alpha_1-0.5\, i_1 \sin \alpha_1 \\ +0.866\, i_3 \cos \alpha_3+0.5\, i_3 \sin \alpha_3) \qquad (6)$$

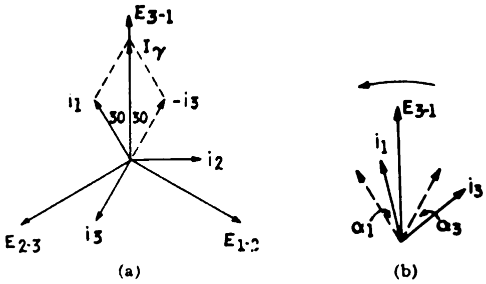
Furthermore, if the watt load is also balanced, *i.e.*, $i_1=i_3=i$ and $\alpha_1=\alpha_3=\alpha$, then (5) becomes

$$E\,(0.866\, i \cos \alpha+0.866\, i \cos \alpha) \qquad (7)$$

and (6)

$$E\,(0.866\, i \cos \alpha+0.866\, i \cos \alpha) \qquad (8)$$

Since Equations (7) and (8) are now exactly alike we may conclude that, in general, the torque of the single-phase instrument agrees with that of the polyphase instrument only when the currents and voltages are balanced. For example, assume a single-phase load of *i* amperes connected across phases 1 and 3 of a three-phase circuit. By reference to Fig. 3, it will be evident that the current in the meter coil derived from this load is 2*i* amperes; and consequently the reading on the meter is twice as high as it should be because it is proportional to 2 *Ei* cos α instead of *Ei* cos α. This conclusion does not, of course, deny that there may be certain sporadic conditions of unbalance in which the single-phase instrument reads correctly but *in general* balanced conditions are necessary for correct readings. This does not necessarily imply unity power-factor, however.



(a) (b)

Fig. 4. Vector Relationships Determining the Torque for the Connections Shown in Fig. 3

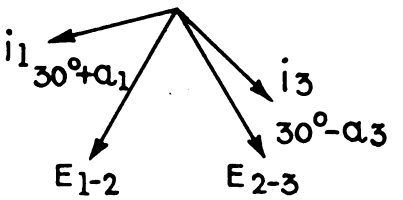


Fig. 5. Vector Relationship for the Torque in a Standard Two-element Meter

Summarizing these two methods we find the results very similar. Neither can be used on a system where the currents and voltages are unbalanced among the phases; and the choice of either for balanced conditions will depend solely upon whether current or potential transformers are the most expensive for a particular installation. In comparing costs, however, the adaptability of Method (b) to the use of protective relays should not be overlooked.

Another connection sometimes suggested for measuring power by means of a single-phase wattmeter is shown in Fig. 6 (a), while the current and potential relations are as shown in Fig. 6 (b). From the latter diagram it is apparent that the scale reading may be multiplied by 2 to give a correct indication, since the torque at unity power-factor is:

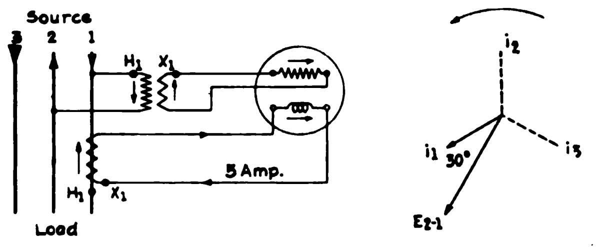
---

(1) Throughout this article *e* will be used to denote a leg voltage or voltage from *line to neutral* while *E* will represent a voltage between lines.

$$Ei \cos 30° \text{ or } 0.866\, Ei$$

Nevertheless, at power-factors other than unity the indications are decidedly incorrect. For example, if the rotation be as indicated by the arrow, then at 30 deg. lead the torque will be porportional to Ei which, with the multiplier of 2, will give a total reading of $2Ei$, whereas in reality it should be only:

$$\sqrt{3}\, Ei \cos 30° \text{ or } 1.5\, Ei$$



(a) (b)

Fig. 6. Single-phase Instrument with One Current Transformer and One Potential Transformer Connected to Measure Power in a Three-phase Three-wire Circuit

At 30 deg. lead, therefore, the meter indication is 33.3 per cent high. On the other hand, at 30 deg. lag the torque is only:

$$Ei \cos 60° \text{ or } 0.5\, Ei$$

This quantity, multiplied in the scale by 2, gives a total reading of Ei which, since the true power is 1.5 Ei, is 33.3 per cent low. This connection is, therefore, correct only at unity power-factor and (since but one current and one voltage are measured) only on balanced conditions of both voltage and current. These limitations of power-factor and voltage and current balance reduce the application of this connection to a very restricted field.

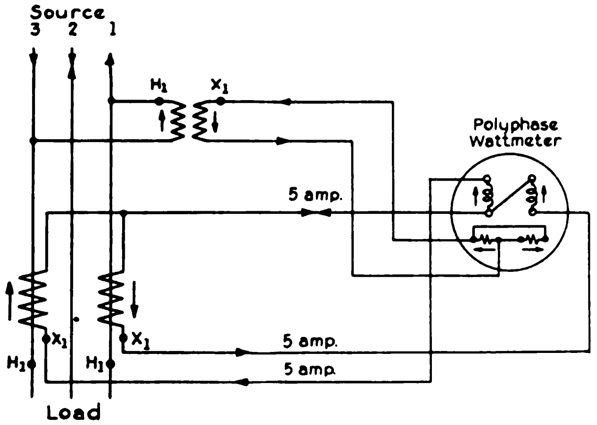


Fig. 7. Standard Polyphase Wattmeter with One Potential Transformer and Two Current Transformers Connected to Measure Power in a Three-phase Three-wire Circuit

Another method which has been employed on high-voltage lines, utilizes a standard polyphase wattmeter, but, on account of the cost of high-voltage potential transformers, only one of these has been used. The connections are as shown in Fig. 7.

In this method the potential coils are connected in multiple and one current coil is reversed. This reversal is necessary to bring the current in phase 3, Fig. 8, to a 30-deg. displacement from the voltage E_{3-1} instead of its normal displacement of 150 deg.

The torque developed by this instrument is:

$$T = E_{3-1}\left\{[i_1 \cos(30° - \alpha_1)] \\ + [i_3 \cos(30° + \alpha_3)]\right\} \qquad (9)$$

$$= E_{3-1}\,(i_1 \cos 30° \cos \alpha_1 + i_1 \sin 30° \sin \alpha_1 \\ + i_3 \cos 30° \cos \alpha_3 - i_3 \sin 30° \sin \alpha_3) \qquad (10)$$

$$= E_{3-1}\,(0.866\, i_1 \cos \alpha_1 + 0.5\, i_1 \sin \alpha_1 \\ + 0.866\, i_3 \cos \alpha_3 - 0.5\, i_3 \sin \alpha_3) \qquad (11)$$

which we find is the same as Equation (2) thereby showing that the polyphase instrument connected as in Fig. 7 is at the same disadvantage as the single-phase instrument shown in Fig. 3, in that it is dependent upon voltage and current balance.

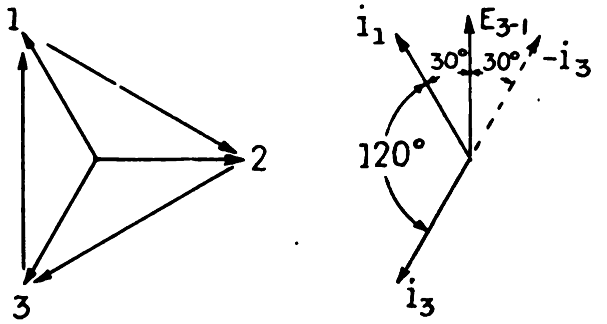


Fig. 8. Vector Diagram Showing the Occasion for Having Reversed One Current Coil in Fig. 7

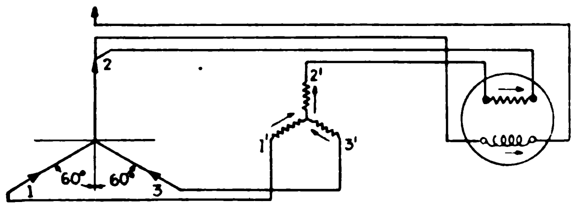


Fig. 9. Diagram Illustrating the Polarity Relationship in a Single-phase Wattmeter Connection

It will be noted that both methods require two current transformers and one potential transformer. The auxiliary equipment in both cases is therefore the same, but the single-phase instrument with its one element gives practically the same result as would a polyphase instrument with its two elements.

If relays are used, however, the wiring will cost slightly more for the single-phase meter inasmuch as the relays must be connected inside the cross-connections, thereby requiring a four-wire run to the switchboard in place of a three-wire run for the polyphase meter.

While the theory of operation of these instruments may be perfectly clear from the vector diagrams accompanying the various connection diagrams, the reasons for the actual electrical connections may not be so readily apparent. In connecting up any permanent-magnet type of instrument on a direct-current circuit or a multi-coil type instrument on an alternating-current circuit, it is obvious that to obtain a positive deflection on the scale a certain definite polarity relationship must be established between the permanent magnet and operating coil in the first case and between the several operating coils in the latter. Alternating-current ammeters and voltmeters in general, having but one operating circuit, are immune from any polarity requirement.

On the other hand the wattmeter, which must measure both current and voltage, requires two operating coils and to obtain a positive deflection on the scale it is obvious that the required instantaneous direction of current through both coils must be known. From Fig. 1 it is evident that, in this particular instrument, if at the instant chosen the left-hand potential stud is positive, the left-hand stud of the current coil must also be positive. The relationship will result in a positive deflection over the scale when both coils are energized. If the relative polarity of either coil were reversed, the deflection would be in the opposite direction or below the zero mark.

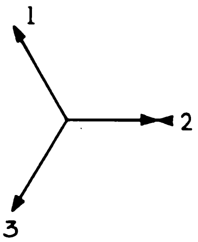


Fig. 10. Phase Position 90 deg. Earlier Than That Shown in Fig. 9

To determine this relationship, it is desirable to assume the system to be "frozen" at a particular instant in the voltage cycle. For example, in Fig. 9, at unity power-factor we assume a stationary condition at the instant when phases 1 and 3 are positive and phase 2 negative. We will have i_1 cos 60 deg. or 0.5 i_1 and i_3 cos 60 deg. or 0.5 i_3 flowing toward the load and i_2, or 0.5 $i_1+0.5\ i_3$, returning to the source of power.

While this condition will be continuously changing, it is obvious that a relationship established at any particular instant will hold true for any other instant throughout the cycle. For example, at 90 deg. earlier in the cycle, as shown in Fig. 10, the current in phase 2 and consequently the current in the current coil of the meter will be zero, (2) but so also will be the current through the potential coil for at this particular instant the potential of phase 3 is equal and opposite to that in phase 1 and the voltage cross leg 2 is zero.

The torque developed by the polyphase two-element meter on a three-phase three-wire circuit has already been considered, as in Fig. 5 and Equation (3). The connections for this instrument are illustrated in Fig. 11. To obtain a positive deflection of the pointer it is essential, due to the polarity of the meter, that the relative instantaneous directions of the currents in the current and potential coils be as shown in this diagram. The particular instant chosen is that at which phases 1 and 3 are of equal value and positive to phase 2. At this moment, currents will flow through the primaries of the potential transformers from *a* to *b* and from *d* to *c*. Since the polarity of these transformers is subtractive,* the secondary currents will flow from *b* to *a* and from *c* to *d*; consequently the direction through the potential coils of the meter will be from *a* to *b* and from *d* to *c*.

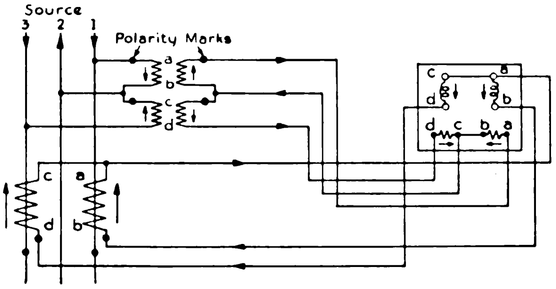


Fig. 11. Two-element Polyphase Wattmeter with Two Potential Transformers and Two Current Transformers Connected to Measure Power in a Three-phase Three-wire Circuit

At this same instant the direction of the currents in the primaries of the current transformers will be from *a* to *b* and from *c* to *d* which, since their polarity is also substractive, will result in corresponding secondary currents from *b* to *a* and from *d* to *c*. Consequently, to obtain a positive summation of the torques of both elements of the meter, the ends *b* and *d* of the current transformer secondaries must connect to studs *b* and *d*, respectively, of the meter, with studs *a* and *c* connected to the common point *a* and *c* of the transformers.

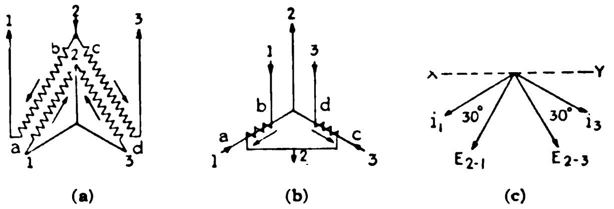


Fig. 12. Unity Power-factor Balanced-load Phase Positions for the Connections Shown in Fig. 11

The phase positions for these connections at unity power-factor and balanced load are shown in Fig. 12 (a), (b) and (c). Fig. 12 (a) and (b) are combination vector and connection diagrams intended as connecting links between Figs. 11 and 12 (c). Here it will be noted that the arrows in the vector diagram, Fig. 12 (c), are all below the horizontal line, *x*—*y*, and consequently the forces in the meter are all positive. If, however, the actual connections are made as shown in Fig. 13, *i.e.*, with the ends *b* and *d* of the current transformer secondaries connected to studs *a* and *c*, respectively, of the meter, the resulting deflection would be negative. The vector diagram of the meter forces would then be as shown in Fig. 14, in which the total torque, instead of being $Ei \cos 30° + Ei \cos 30°$ or $1.732\ Ei$ becomes

---

(2) Double arrow in vector and connection diagrams indicates instant of no current flow.

*A complete description of the method of determining additive and subtractive transformer polarity, and of the resulting influence upon transformer connections, appeared in the article, "Notes on Transformer Polarity and Connections": Parts I and II, by J. Auchincloss, in the November and December, 1926, issues of this magazine, pages 783 and 862 respectively—EDITOR.

$$\begin{aligned} & Ei \cos 150° + Ei \cos 150° \\ &= -Ei \cos 30° + (-Ei \cos 30°) \\ &= -1.732\ Ei. \end{aligned}$$

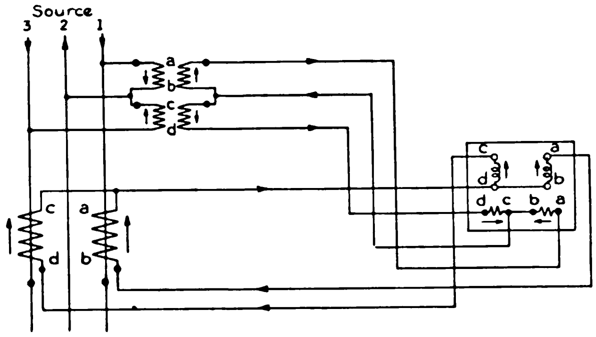


Fig. 13. Incorrect Polarity Variation of the Connections Shown in Fig. 11

The total torque is therefore negative. Obviously, if the connections to the potential coils were reversed, instead of those to the current coils, the same negative torque would result.

Still another erroneous connection of this meter is shown in Fig. 15. The torque developed with this connection at unity power-factor and balanced load is as shown in Fig. 16.

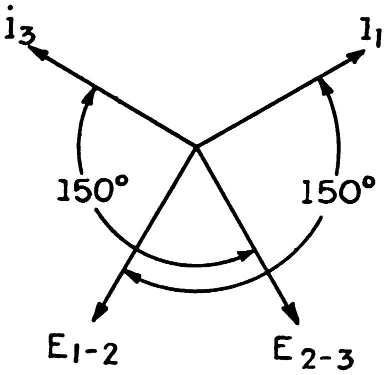


Fig. 14. Vector Relationship for the Torque of the Instrument in Fig. 13

Under the conditions stipulated the forces set up in the respective elements of the circuit diagram (as shown in this article) give rise to a balance of torque, or the total torque is zero, which is determined as follows:

$$\begin{array}{lcl} \text{Right-hand Element} & & \text{Left-hand Element} \\ Ei \cos 30° & + & Ei \cos 150° \\ = 0.866\ Ei & - & 0.866\ Ei \\ = 0 & & \end{array}$$

Under other conditions it will be either positive or negative, depending upon the sign of the algebraic sum of the two individual torques. For example, if the meter is operating on a balanced load at a power-factor of 86 per cent lag, corresponding to a displacement angle of 30 deg., the vector relations will be as shown in Fig. 17, and the total torque will be:

$$\begin{array}{lcl} \text{Right-hand Element} & & \text{Left-hand Element} \\ Ei \cos 60° & + & Ei \cos 180° \\ = 0.5\ Ei & - & Ei \\ = -0.5\ Ei & & \end{array}$$

which would give a negative deflection of the pointer.

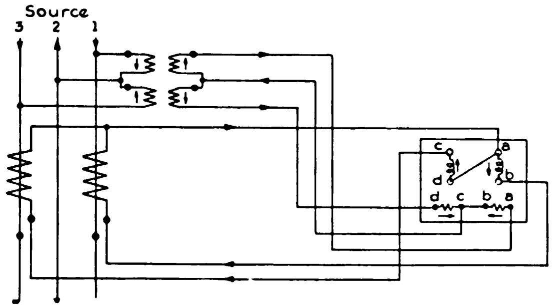


Fig. 15. Another Incorrect Variation of the Connections Illustrated in Fig. 11

The general case, assuming balanced voltages, may be derived from Fig. 18 as follows:

$$\begin{array}{lcl} & \text{Right-hand Element} & & \text{Left-hand Element} \\ T = E & \{i_1 \cos (30° + \alpha_1) & + & i_3 \cos (150° + \alpha_3)\} \\ = E & (i_1 \cos 30° \cos \alpha_1 - i_1 \sin 30° \sin \alpha_1 & + & i_3 \cos 150° \cos \alpha_3 - i_3 \sin 150° \sin \alpha_3) \\ = E & (0.866\ i_1 \cos \alpha_1 - 0.5\ i_1 \sin \alpha_1 & - & 0.866\ i_3 \cos \alpha_3 - 0.5\ i_3 \sin \alpha_3) \end{array}$$

which, when $i_1 = i_3 = i$ and $\alpha_1 = \alpha_3 = \alpha$, becomes

$$T = -Ei \sin \alpha.$$

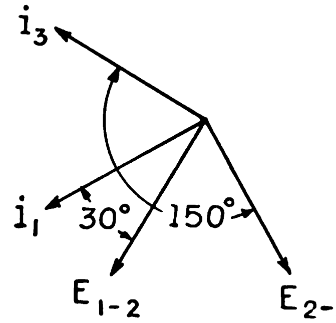


Fig. 16. Vector Relationship for the Torque of the Instrument in Fig. 15 Under Conditions of Unity Power-factor and Balanced Currents

This gives a negative deflection proportional to the sine of the displacement angle.

If an approximately true estimate of the total power is obtainable (by voltmeter, ammeter and power-factor meter, or by other means) an incorrect wattmeter connection may frequently be detected by a consideration of the foregoing equations or from others similarly derived.

As long as no power is carried over the neutral wire, the total three-phase power can be accurately measured by the two-element wattmeter when connected as shown in Fig. 11. It is axiomatic that the total power consumed on an n-phase circuit must be the sum of the powers consumed in the individual phases. Consequently, on a three-phase circuit the total power must be the sum of the powers flowing over each phase, or $e_1 i_1 + e_2 i_2 + e_3 i_3$. To measure these three quantities, however, three wattmeters would be required the potential coils of which must be energized by the three star-voltages, e_1, e_2, and e_3. Since however on a three-phase three-wire system, the entire load is carried between phases, and the neutral, being not required, is seldom available, there is a decided advantage in measuring the power in terms of the delta-voltage, E, instead of the star-voltage, e. Now the equation for the total power in terms of the star-voltages, $e_1 i_1 + e_2 i_2 + e_3 i_3$, may be transformed into terms of the delta-voltages as follows:

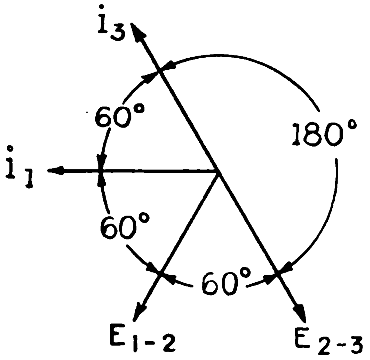


Fig. 17. Vector Relationship for Same Conditions as in Fig. 16 Except That the Power-factor is 86 Per Cent Lagging

$$
\begin{aligned}
e_1 i_1 &+ e_2 i_2 + e_3 i_3 \\
&= e_1 i_1 - e_2 i_1 + e_2 i_1 + e_2 i_2 + e_3 i_3 - e_2 i_3 + e_2 i_3 \\
&= i_1 (e_1 - e_2) + e_2 (i_1 + i_2 + i_3) + i_3 (e_3 - e_2)
\end{aligned}
$$

which, since $i_1 + i_2 + i_3 = 0$, becomes $i_1 (e_1 - e_2) + i_3 (e_3 - e_2)$

$$= E_{1-2}\, i_1 + E_{3-2}\, i_3$$

or in scalar form, at unity power-factor,

$$E_{1-2}\, i_1 \cos 30° + E_{3-2}\, i_3 \cos 30°$$

The total power therefore may be measured by means of two wattmeters, either separately or connected to a common shaft.

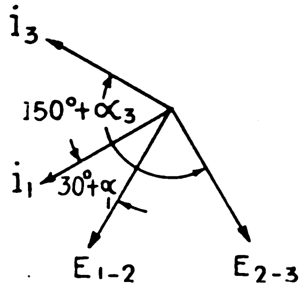


Fig. 18. General Relationship for Balanced Voltage Conditions in Fig. 15

THREE-PHASE FOUR-WIRE CIRCUITS

Power Carried Over the Neutral

When however power is carried over the neutral, or fourth wire, the equation $i_1 + i_2 + i_3 = 0$ no longer holds, and the instrument must measure in accordance with the fundamental equation $e_1 i_1 + e_2 i_2 + e_3 i_3$. This obviously requires the use of three meters, either individual or combined on a common shaft. If however the three voltages, e_1, e_2 and e_3, are reasonably well-balanced, a fairly accurate reading may be obtained by using a two-element meter equipped with four current elements, one of which is really a single element split in two sections, one section operating with the voltage, e_1, and the other with the voltage e_3. It is therefore simply a three-current-coil device having one coil in two sections. The connections for this instrument are shown in Fig. 19, and the vector diagram in Fig. 20.

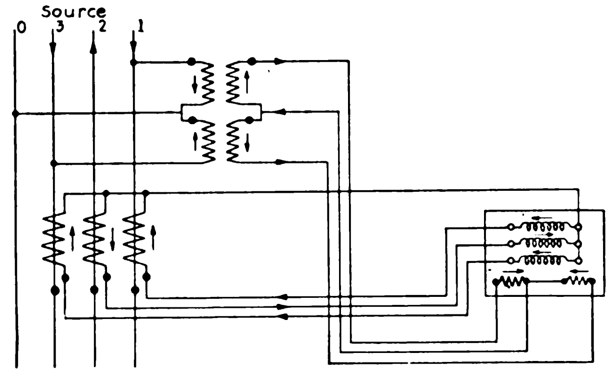


Fig. 19. Three-current-coil Two-potential-coil Instrument with the Same Number of Instrument Transformers Connected for Measuring Power in a Three-phase Four-wire Circuit

From this vector diagram, the torque, at unity power-factor, is:

$$
\begin{aligned}
e_1 i_1 &+ e_1 i_2 \cos 60° + e_3 i_2 \cos 60° + e_3 i_3 \\
&= e_1 i_1 + 0.5\, e_1 i_2 + 0.5\, e_3 i_2 + e_3 i_3
\end{aligned}
$$

but if the voltages are balanced:

$$0.5\, e_1 + 0.5\, e_3 = e_2$$

so that the torque may be written:

$$e\,(i_1 + i_2 + i_3) \tag{12}$$

If however the voltages are not balanced the meter reading is obviously not correct, but only by the amount by which the torque of the middle coil (phase

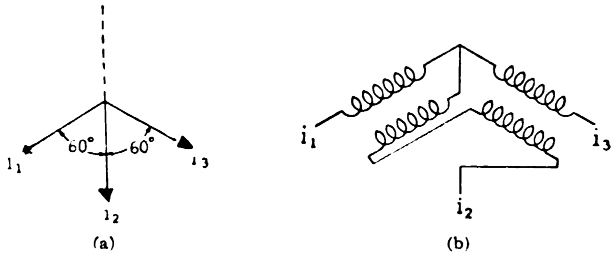


Fig. 20. Vector Relationship Diagram of Connections Shown in Fig. 19

2) is erroneous, for the currents in phases 1 and 3 are operating with their true voltages e_1 and e_3 respectively. Since the current in phase 2 is responsible for approximately only one-third of the total torque, any error arising from it is greatly diminished in the total reading.

Another method for measuring three-phase four-wire power, but employing an ordinary polyphase wattmeter equipped with two current and two potential coils, is shown in Fig. 21. Here the current transformers are connected in delta and since there are three of them they are affected by the currents in all three phases and the currents external to the delta are the vector summations of the currents flowing in the lines. Any current flowing over the neutral wire is measured by the current transformer through which it passes and thence carried to the wattmeter over the delta-connection. For example, if the current flowing over line 2 is greater than that passing over lines 1 and 3 (single-phase load) this difference will flow over the neutral line, O. This excess current, however, will also appear in the delta-connection to the meter and be duly measured. In other words, instead of the three individual currents being measured by three meter coils, as in the previous method, their vector summation is measured by two meter coils; but as in Fig. 3, these coils must be designed to carry $\sqrt{3} \times 5$ amp., instead of the regular 5 amp. carried by the three-coil meter just described. The torque developed is shown in Fig. 22. It will be noted from Fig. 21 that the connections to the coil energized by current X is reversed. This is done to obtain a positive torque with the potential e_1 as is shown by the vector diagram in Fig. 22, otherwise the torques of the two elements would be in opposition and the pointer would measure their difference. By this reversal the current X is displaced (at unity power-factor) from e_1 by 30 deg. while the current Y is displaced 30 deg. from potential e_3. X is made up of line currents i_1 and i_2, while Y is made up of i_2 and i_3.

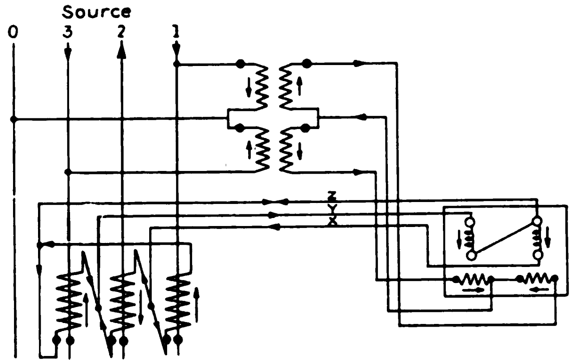


Fig. 21. Connections of the Two-coil Polyphase Wattmeter Used in Place of the Three-coil Instrument in Fig. 19

The torque on a balanced load at unity power-factor is therefore:

$$e_1 X \cos 30° + e_3 Y \cos 30°$$

but $X = Y = \sqrt{3}\ i$ and $e_1 = e_3$ so that this expression becomes $2\sqrt{3}\ ei \cos 30° = 3\ e\ i$, or on an unbalanced load it is: $e_1\ i_1 \cos \alpha_1 + e_1\ i_2 \cos (60° - \alpha_2) + e_3\ i_2 \cos (60° + \alpha_2) + e_3\ i_3 \cos \alpha_3$ which, when $e_1 = e_2 = e_3$, becomes

$$e\ (i_1 \cos \alpha_1 + i_2 \cos \alpha_2 + i_3 \cos \alpha_3).$$

This, it will be noted, is the same as Equation (12) developed for the three-current-coil meter previously described. The meter under present consideration therefore labors under the same disability, in that for extreme accuracy it is dependent upon equal voltages on all three phases. One disadvantage of this method lies in the fact that if ammeters are to be connected to the same current transformers, the delta-connection must be made beyond them. In other words, the ammeters or other 5-amp. devices must be connected inside the delta. Obviously, the reading of an ammeter connected outside the delta would be meaningless so far as any measure of the line current is concerned, since it would be a composite of two line currents which would vary both with the power-factor and current balance-factor. This would mean that if the instruments were mounted on a switchboard, the interconnection between the current transformers would have to be made at the switchboard instead of at the transformers, thereby necessitating a run of six leads from the transformers to the switchboard instead of the four required by the three-coil meter.

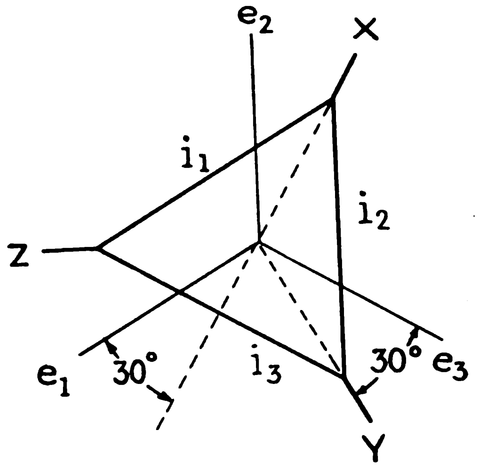


Fig. 22. Diagram Showing the Occasion for Having Reversed the Connections to the Coil Energized by Current X in Fig. 21

Since both these methods measure only two of the leg-voltages they are, as previously stated, dependent for their accuracy upon balanced voltage conditions. For accuracy on unbalanced voltages, all three leg-voltages would have to be measured, necessitating the use of three potential transformers instead of two, together with a meter equipped with three potential windings. This latter method of measurement would obviously cost much more; and while the expense may be justified in the case of the integrating watt-hour meter, the readings of which are used for billing purposes, it will seldom find justification for indicating instruments only.

(To be continued)

Balloons—Balloons

By L. C. PORTER and A. C. ROY
Engineering Department, Edison Lamp Works, General Electric Company

LIGHT for decoration is being used with ever-increasing effectiveness both indoors and out, at Christmastide, and at midsummer lawn parties. For such purposes, soft lighting, rich in colors, is most suitable because the lighting is but one of the attractions and should reveal the others to advantage, not compete with them. Light of this character can be produced most readily through the use of large-size sources, a new and unique type of such a source being the ordinary ten-inch rubber balloon into which a Christmas-tree lamp has been inserted.

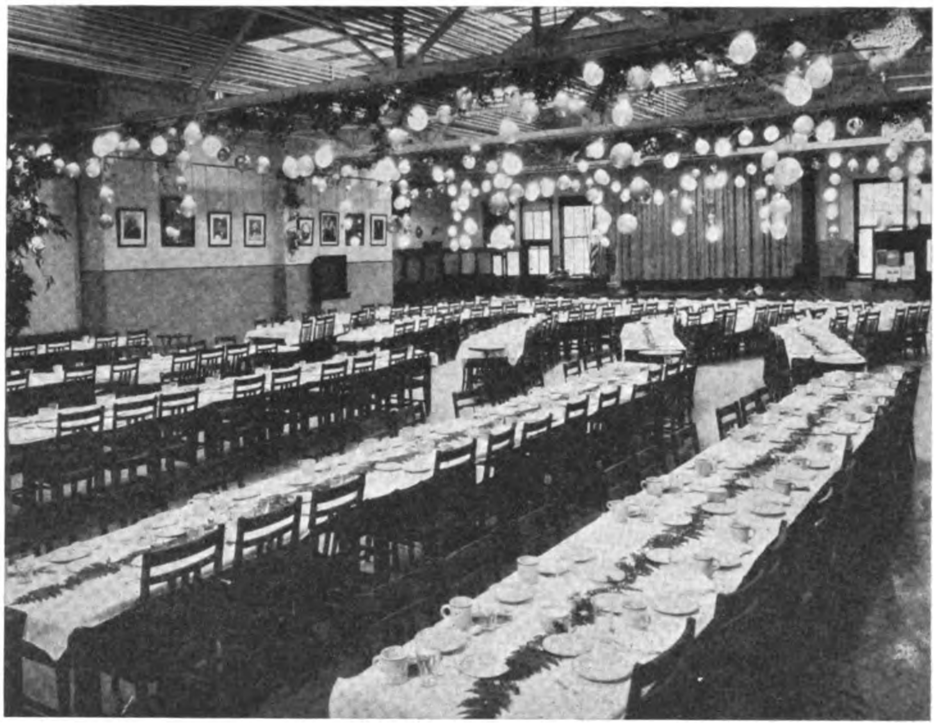


Fig. 1. A Decorative Scheme of Lighting for a Banquet

Last summer the spacious grounds of the Broadmoor Hotel at Colorado Springs were lighted in this manner. In one court the balloons were hung in vertical strings from the limbs of the trees, the lower ends of the strings being free to swing in the wind, as shown in the front cover illustration of this issue. Each individual string was operated through a flasher button, and the combination of swinging and flashing was very attractive. Fifty to sixty flashes a minute has been found to be the most effective rate for decorative lighting of this character; but flashing had best be confined to outdoor installations, as indoors it may prove somewhat annoying.

The ordinary eight-lamp, series, Christmas-tree strings using cone-shaped lamps are the most suitable for use with balloons. For best effects, red lamps should be used in red balloons, yellow lamps in yellow balloons; green balloons will require white lamps, and in blue balloons clear lamps should be used, otherwise the absorption of light would be too great.

In large installations or other cases where it would be unduly bothersome to match the color combinations of the balloons and lamps, clear bulb lamps can be used throughout. In fact, the bright spot of light caused by the filament of the clear lamp is considered by some observers to be more attractive than the more uniform illumination of the balloon by a colored lamp. The use of clear-bulb lamps has the further advantage that it lessens the tendency of the rubber to stick to the bulb when inserting the lamp in the balloon.

When making an installation of lighted balloons, much time and effort can be saved by first "lamping" and testing all the strings. Occasionally a defective string or an open-circuited lamp will have been included, and if so it will be much easier to locate and replace before the balloons have been put on the lamps.

The next operation is to inflate each balloon to somewhat less than full diameter, about nine inches, thereby allowing for later expansion due to the heat of the lamp. With the neck of the balloon pinched tight between the thumb and forefinger of the left hand, the point of the lamp bulb can be inserted into the open end of the neck with the right hand. Both hands are then used to work the neck of the balloon over the lamp bulb until the entire bulb is inside the balloon and the neck of the balloon is bunched around the lamp between the shoulder of the bulb and the end of the socket. (This operation can be accomplished more readily if the lamp has been dipped into water just prior to inserting it in the rubber neck). It is not necessary to tie the neck of the balloon to prevent air leakage, for the stretching of the neck around the lamp makes an air-tight joint.

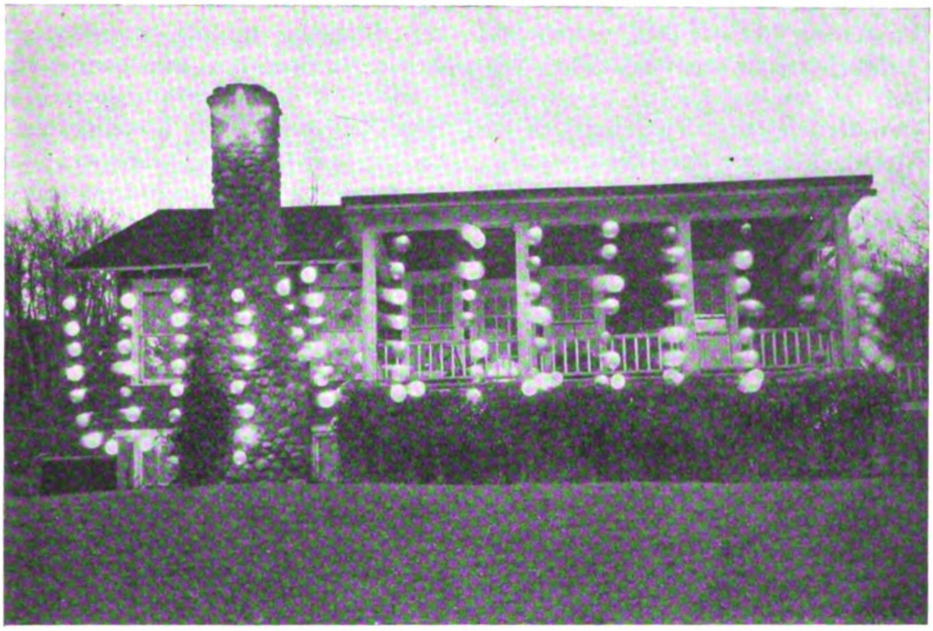


Fig. 2. Strings of Lighted Balloons Employed in Outdoor Ornamental Illumination

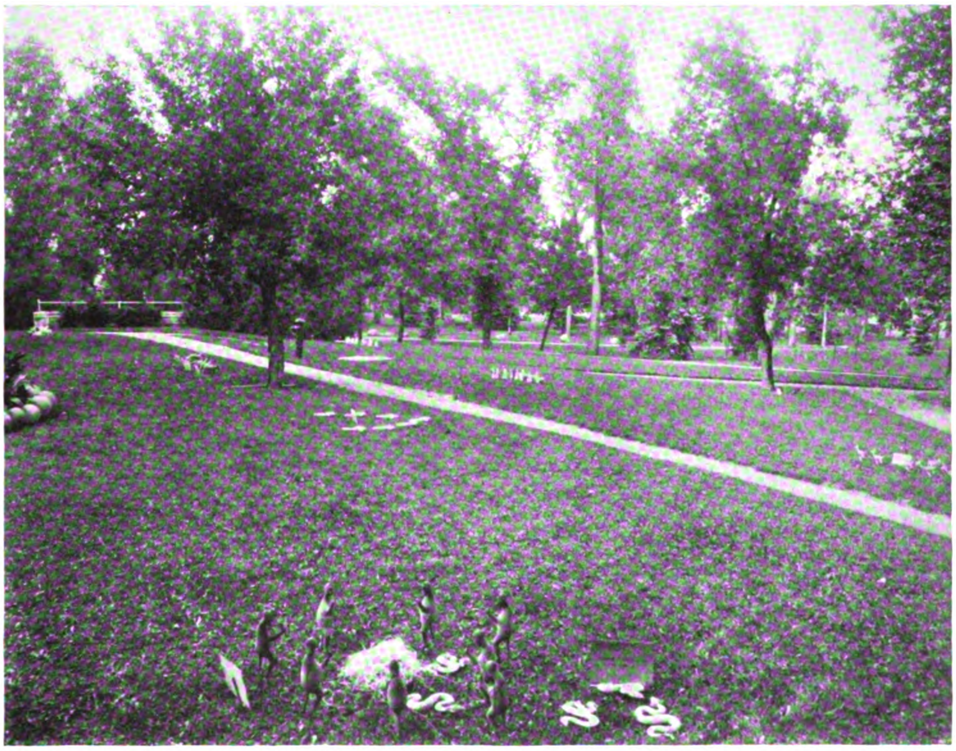


Fig. 3. A Lawn Decorated by Groups of Lighted Balloons of Unusual Shape and Also Spherical Balloons Bordering Flower Plots as Shown at the Left

The work of putting the balloons on the lamps can be done in a surprisingly short time. For example, one of the writers has inflated and put on 147 balloons in less than an hour in a case where strings were all in place and most of the attaching had to be done from a ladder.

The complete dining room installation shown in Fig. 1 was accomplished by four men in about three hours.

A new installation of lighted balloons should be burned for a couple of hours before needed, as this will furnish opportunity to weed out the few balloons which have small pin-holes that will deflate them and also any lamps that burn out prematurely. The remaining balloons and lamps are likely to continue in service for a considerable time. The length of life of the balloons depends of course upon their location, and to some extent upon whether they are of a light or dark color. The light-colored balloons usually last longer, probably due to less heat absorption.

When installed indoors, balloons have remained inflated for more than two weeks; but outdoor installations, exposed to the hot summer sun, may require a new set of balloons every day. An installation of some 100 balloons for outdoor Christmas lighting (Fig. 2) required relatively few renewals between Christmas and New Year's Day, though some of the balloons shrunk considerably in size. Apparently, mid-winter weather is not as severe on the thin rubber as is the hot summer sunshine. In this particular installation it was later found much more effective to outline the house than to have all the strings in a row or hanging straight down as shown in Fig. 2.

When ordinary eight-lamp Christmas-tree strings are to be used outdoors on direct-current circuits, and the lamps have been screwed into their sockets, the combination of lamps and sockets should be dipped in a bath of paraffine wax that is only a little above the melting point so that when the lamp and socket is removed the coating of wax will solidify quickly. This will prevent rain running down into the socket around the leading-in wires and eliminate the possibility of arcing. When the strings are used on alternating-current circuits, this precaution need not be taken.

Multiple, outdoor, Christmas-tree lighting outfits, which use larger lamps, are not satisfactory for balloon lighting. The lamps are too big to go through the necks of the balloons and the heat they generate rapidly deteriorates the rubber.

An interesting departure from spherical balloon lighting can be made with rubber balloons representing donkeys, rabbits, monkeys, alligators, frogs, devils, reindeer, etc., as shown in Figs. 3 and 4.

Inserting the lamps in balloons of such special forms is not as conveniently accomplished as in the case of the ten-inch spherical balloons, and usually it necessitates cutting a foot, hand, or tail off the animal to provide adequate entrance for the bulb. A rubber band stretched tightly several times around the amputated limb and lamp socket will seal the balloon, though it has been found advisable also to take precaution to double over upon itself the small neck and rubber plug that is used when inflating the balloon and to wind an additional rubber band around it.

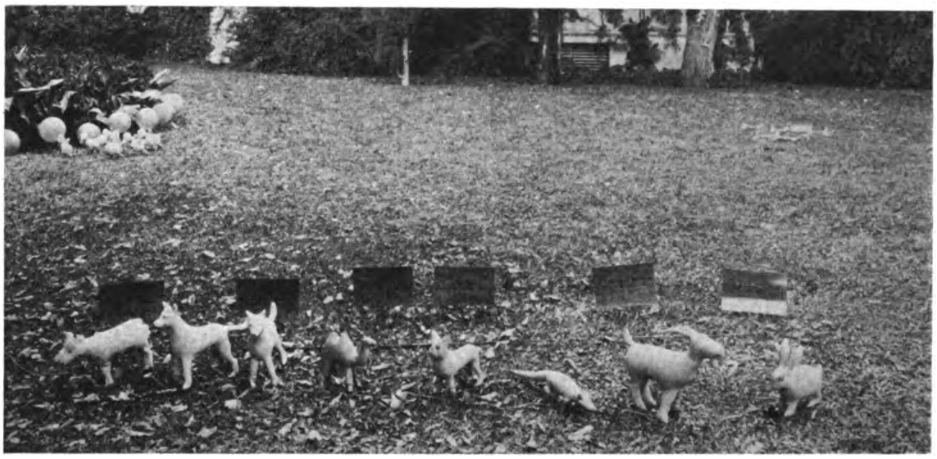


Fig. 4. A Menagerie Comprised of Rubber Balloons of Special Shapes Lighted by Lamps Within

If the animals are to be placed so that they will be viewed from one side only, nearly as good a lighting effect can be obtained and much trouble saved by simply placing a lamp behind the balloon.

In any installation of a number of strings, it is advisable to have a few reserve strings complete with lamps and balloons for then any string that goes out of service can be quickly replaced and the defective string repaired at leisure. Replacing the strings where they are hung from trees is most easily accomplished by the use of cords running through iron rings which are tied to the tree limbs, thus permitting the entire balloon string to be lowered conveniently. The slack cord can be wound on two nails preferably driven into a stick tied to the tree trunk to avoid injuring the tree. This feature may be noted in the installation that is illustrated on the front cover of the magazine.

The beauty of the big soft-colored balloons and the ease with which the strings can be draped about and simply attached to the nearest outlet, should make this form of decorative lighting very popular.

Heaviside's Operational Calculus as Applied to Engineering and Physics

Part XIV: **Duhamel's Integral**
Part XV: **General Equations Pertaining to Transmission Lines. Open and Short-circuit Conditions of "Ideal" Cable**
Part XVI: **An Alternator Suddenly Connected to an "Ideal" Cable**
Part XVII: **Transmission Line Having All Four Constants R, L, G and C**
Part XVIII: **Transmission Line with Impedance at the Sending and Receiving Ends**
Part XIX: **Two or More Transmission Lines of Different Constants Connected Together by an Impedance or Shunted at Their Junction by an Impedance**
Part XX: **A Battery Connected at Some Point of a Cable**

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THE use of Duhamel's integral is often very advantageous when dealing with an impressed e.m.f. which is not of "unit function" shape. It is especially convenient when dealing with problems involving "distributed" constants, as is the case in transmission line or heat calculations.

The procedure then is: Find first the solution in the case of unit function; let this be $\varphi(t)$. Let $\varphi(0)$ be the value $\varphi(t)$ when $t=0$. Let the impressed e.m.f. be $e(t)$, and let $e(u)$ be $e(t)$ when u is substituted for t. Then one form of the solution is

$$e(t)\,\varphi(0)+\int_{u=0}^{u=t} e(u)\,\frac{d}{dt}\varphi\,(t-u)\,du \qquad (45)$$

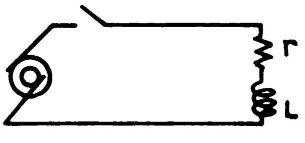

Fig. 9 *(Repeated)*

Another form is given in Equation (46), and in Part XXVIII will be given a number of derivations from this formula.

Before giving the proof of Equation (45), it may be well to illustrate its use in a particular case. Find the current if an e.m.f. $e(t)=E\sin(\omega t+\varphi)$ is impressed on an inductive circuit as shown in Fig. 9.

The solution with unit e.m.f. is

$$i=\frac{1}{r+pL}\mathit{1}=\frac{1}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$$

thus
$$\varphi(t)=\frac{1}{r}\left[1-\epsilon^{-\frac{r}{L}t}\right]$$

$$\varphi(0)=0$$

$$\varphi(t-u)=\frac{1}{r}\left[1-\epsilon^{-\frac{r}{L}(t-u)}\right]$$

$$\frac{d}{dt}\varphi(t-u)=\frac{1}{L}\epsilon^{-\frac{r}{L}(t-u)}$$

$$e(u)=E\sin(\omega u+\varphi)$$

$$\therefore i=\int_{u=0}^{u=t}E\sin(\omega u+\varphi)\,\frac{1}{L}\,\epsilon^{-\frac{r}{L}(t-u)}\,du$$

$$=\frac{E}{L}\epsilon^{-\frac{r}{L}t}\int_{u=0}^{u=t}\epsilon^{\frac{r}{L}u}\sin(\omega u+\varphi)\,du$$

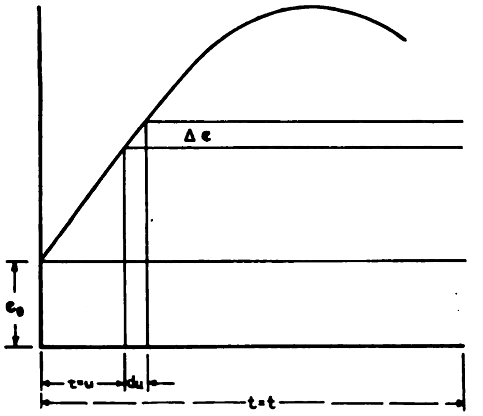

Fig. 27

$$=\frac{E}{L}\left[\frac{\sin(\omega t+\varphi-\delta)-\epsilon^{-\frac{r}{L}t}\sin(\varphi-\delta)}{\sqrt{\frac{r^2}{L^2}+\omega^2}}\right]$$

$$=\frac{E}{Z}\left[\sin(\omega t+\varphi-\delta)-\epsilon^{-\frac{r}{L}t}\sin(\varphi-\delta)\right]$$

where $\quad Z=\sqrt{r^2+\omega^2L^2}$, and $\tan\delta=\dfrac{\omega L}{r}$

The explanation of this integral which was given almost one hundred years ago by Duhamel is perhaps best made by referring to Fig. 27. Let the e.m.f. be represented by the curved line. The curved line can be approached by assuming that at $t=0$ an e.m.f. e_0 suddenly is impressed. A short time afterward another rectangular e.m.f. e_1 is added, sometime after an e.m.f. e_2, etc., all these being of "unit" shape. The current at time t is then the sum of the currents due to e_0 beginning at $t=0$, e_1 beginning at $t=t_1$, etc. The current due to e_0 is of course $e_0\,\varphi(t)$

where $\varphi(t)$ is the solution with unit e.m.f. impressed at $t=0$. The current due to the small e.m.f. $\triangle e$, which begins at $t=u$, is obviously $\triangle e\,\varphi(t-u)$. Therefore the total current at $t=t$, is

$$i=e_0\,\varphi(t)+\sum_{u=0}^{u=t}\triangle e\,\varphi(t-u)$$

But $$\triangle e=e(u+du)-e(u)$$

and thus $$\frac{\triangle e}{du}=\frac{e(u+du)-e(u)}{du}=\frac{d}{du}e(u)$$

Therefore $$\triangle e=\frac{d}{du}e(u)\,du$$

$$\therefore i=e_0\,\varphi(t)+\int_{u=0}^{u=t}\varphi(t-u)\frac{d}{du}e(u)\,du \tag{46}$$

This then is one solution which is sometimes convenient to use. The integral may be written

$$\int_0^t U dV=\Big[UV\Big]_0^t-\int_0^t V\,dU$$

Let $U=\varphi(t-u)$ and $dV=d\,e(u)$ or $V=e(u)$

Then $$\int_0^t\varphi(t-u)\frac{d}{du}e(u)\,du$$

$$=\Big[e(u)\,\varphi(t-u)\Big]_{u=0}^{u=t}-\int_{u=0}^{u=t}e(u)\frac{d}{du}\varphi(t-u)\,du$$

$$=e(t)\,\varphi(0)-e(0)\,\varphi(t)+\int_{u=0}^{u=t}e(u)\frac{d}{dt}\varphi(t-u)\,du$$

and the new expression of Equation (46) becomes

$$i=e_0\varphi(t)+e(t)\,\varphi(0)-e_0\varphi(t)+\int_{u=0}^{u=t}e(u)\frac{d}{dt}\phi(t-u)\,du$$

$$=e(t)\,\varphi(0)+\int_{u=0}^{u=t}e(u)\frac{d}{dt}\varphi(t-u)\,du$$

which was to be proved.

There are thus in general three ways in which problems can be solved when an arbitrary e.m.f. is impressed upon a circuit.

1st: It may be possible to convert the e.m.f. into an operator on $\mathbf{1}$.

2nd: If the arbitrary e.m.f. contains an exponential, Heavisides' "shifting" may be used.

3rd: Duhamel's integral may be used.

To illustrate the three methods, consider an e.m.f. $E\,\epsilon^{-\beta t}\,\mathbf{1}$ impressed on a circuit of resistance and capacity in series: find the current.

The resistance operator is $r+\dfrac{1}{pC}=\dfrac{pCr+1}{pC}$

Method No. 1

Since $E\,\epsilon^{-\beta t}\,\mathbf{1}=E\dfrac{p}{p+\beta}\,\mathbf{1}$ the operational solution is then

$$i=\frac{E\,pC}{pCr+1}\times\frac{p}{p+\beta}\,\mathbf{1}=EC\frac{p^2}{(pCr+1)\,(p+\beta)}\,\mathbf{1}$$

which, solved by the Expansion Theorem, gives

$$i=\frac{EC}{1-\beta Cr}\left(\frac{\epsilon^{-\frac{t}{Cr}}}{Cr}-\beta\,\epsilon^{-\beta t}\right)$$

Method No. 2

$$i=\frac{pC}{p\,Cr+1}\,(\epsilon^{-\beta t}\,E)\,\mathbf{1}$$

Shift $\epsilon^{-\beta t}$ outside of the operator and get

$$i=EC\,\epsilon^{-\beta t}\,\frac{p-\beta}{(p-\beta)\,Cr+1}\,\mathbf{1}=EC\,\epsilon^{-\beta t}\,\frac{Y_{(p)}}{Z_{(p)}}\,\mathbf{1}$$

$\dfrac{Y_{(p)}}{Z_{(p)}}$ by the Expansion Theorem is

$$\frac{1}{1-Cr\beta}\left[\frac{\epsilon^{\left(\beta-\frac{1}{Cr}\right)t}}{Cr}-\beta\right]$$

$$\therefore i=\frac{EC}{1-Cr\beta}\left[\frac{\epsilon^{-\frac{1}{Cr}t}}{Cr}-\beta\,\epsilon^{-\beta t}\right]$$

Method No. 3

The solution with unit operator is $i=\dfrac{pCE}{pCr+1}\,\mathbf{1}$

$$\therefore i=\frac{E}{r}\,\epsilon^{-\frac{1}{Cr}t}$$

Put $\varphi(t)=\dfrac{1}{r}\,\epsilon^{-\frac{1}{Cr}t}$. Then $\varphi(t-u)=\dfrac{1}{r}\,\epsilon^{-\frac{1}{Cr}(t-u)}$

$$\frac{d}{dt}\varphi(t-u)=-\frac{1}{Cr^2}\,\epsilon^{-\frac{1}{Cr}(t-u)}$$

$$e(u)=E\,\epsilon^{-\beta u},\quad e(t)=E\,\epsilon^{-\beta t}\quad\text{and }\varphi(0)=\frac{1}{r}$$

$$\therefore i=E\frac{\epsilon^{-\beta t}}{r}+\int_{u=0}^{u=t}-E\frac{\epsilon^{\beta u}}{Cr^2}\,\epsilon^{-\frac{1}{Cr}(t-u)}\,du$$

$$=E\frac{\epsilon^{-\beta t}}{r}-\frac{E}{Cr^2}\,\epsilon^{-\frac{1}{Cr}t}\int_{u=0}^{u=t}\epsilon^{\left(\frac{1}{Cr}-\beta\right)u}\,du$$

$$=\frac{EC}{1-Cr\,\beta}\left[\frac{\epsilon^{-\frac{1}{Cr}t}}{Cr}-\beta\,\epsilon^{-\beta t}\right]$$

In connection with the use of Duhamel's integral, certain expressions seem to appear frequently. It is therefore advantageous to record them for reference.

$$\epsilon^{-\alpha t}\int_{u=0}^{u=t}\epsilon^{\alpha u}\sin\,(\omega u+\delta)\,du$$

$$=\frac{\sin\,(\alpha t+\delta-\varphi)-\epsilon^{-\alpha t}\sin\,(\delta-\varphi)}{\sqrt{\alpha^2+\omega^2}}$$

$$\epsilon^{-\alpha t}\int_{u=0}^{u=t}\epsilon^{\alpha u}\cos(\omega u+\delta)\,du$$

$$=\frac{\cos(\alpha t+\delta-\varphi)-\epsilon^{-\alpha t}\cos(\delta-\varphi)}{\sqrt{\alpha^2+\omega^2}}$$

$$\text{where}\quad \tan\varphi=\frac{\omega}{\alpha}$$

or $$\epsilon^{-\alpha t}\int_{u=0}^{u=t}\epsilon^{\alpha u}\sin(\omega u+\delta)\,du$$

$$=\frac{-\cos(\omega t+\delta+\psi)+\epsilon^{-\alpha t}\cos(\delta+\psi)}{\sqrt{\alpha^2+\omega^2}}$$

and $$\epsilon^{-\alpha t}\int_{u=0}^{u=t}\epsilon^{\alpha u}\cos(\omega u+\delta)\,du$$

$$=\frac{\sin(\omega t+\delta+\psi)-\epsilon^{-\alpha t}\sin(\delta+\psi)}{\sqrt{\alpha^2+\omega^2}}$$

$$\text{where }\tan\psi=\frac{\alpha}{\omega}$$

PART XV: GENERAL EQUATIONS PERTAINING TO TRANSMISSION LINES. OPEN AND SHORT-CIRCUIT CONDITIONS OF "IDEAL" CABLE

Heaviside devotes only a few pages out of about one thousand to circuits of concentrated resistance, inductance, capacity, and mutual inductance. The remaining pages deal largely with cable problems and more particularly with "ideal cables" where the numerical work becomes fairly simple.

Consider a transmission line as shown in Fig. 28.

Let the constants per unit length of transmission distance be R, L, C, G.

The well-known relations between current and e.m.f. at any point, distant x from the end, is

$$\frac{\partial i}{\partial x}\,dx=eGdx+Cdx\frac{\partial e}{\partial t}\tag{47}$$

and

$$\frac{\partial e}{\partial x}\,dx=iRdx+Ldx\frac{\partial i}{\partial t}.\tag{48}$$

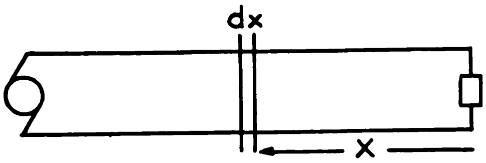

Fig. 28

From these we derive

$$\frac{\partial i}{\partial x}=Ge+C\frac{\partial e}{\partial t}=Ge+pCe=e\,(G+pC)\tag{49}$$

$$\frac{\partial e}{\partial x}=Ri+pLi=i\,(R+pL)\tag{50}$$

and from these

$$\frac{\partial^2 e}{\partial x^2}=(R+pL)\,(G+pC)\,e=n^2e\tag{51}$$

where n is a function of p, that is, of t, but not of x. The solution of this equation is well known. It is

$$e=A_1\,\epsilon^{nx}+A_2\,\epsilon^{-nx}\tag{52}$$

$$=A_1(\cosh nx+\sinh nx)+A_2(\cosh nx-\sinh nx)$$

$$=(A_1+A_2)\cosh nx+(A_1-A_2)\sinh nx$$

$$e=K_1\cosh nx+K_2\sinh nx\tag{53}$$

where K_1 and K_2 are determined from the terminal conditions.

Since from Equation (49)

$$\frac{di}{dx}=(G+pC)\,e=Ye$$

it follows that $$i=Y\int edx$$

$$\therefore i=\frac{Y}{n}\,[K_1\sinh nx+K_2\cosh nx]\tag{54}$$

For these equations

$$Y=G+pC$$

and in subsequent equations we shall write $Z=R+pL$

so that $n^2=YZ$ therefore $\frac{Y}{n}=\frac{n}{Z}$ (55)

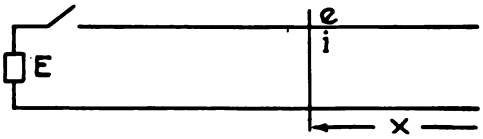

Fig. 29

Special Cases

(*1*) *Open Line* (*Fig. 29*).

Then $i=0$ for $x=0$; $e=E$ for $x=l$.

$$\therefore E=K_1\cosh nl+K_2\sinh nl$$

$$0=\frac{Y}{n}\,[K_1\sinh 0+K_2\cosh 0].$$

Since cosh 0 is 1, it is evident that K_2 must be zero.

$$\therefore e=K_1\cosh nx$$

$$\therefore E=K_1\cosh nl,\text{ or }K_1=\frac{E}{\cosh nl}$$

$$\left.\begin{aligned}&\text{and}\quad e=E\,\frac{\cosh nx}{\cosh nl}\,\mathbf{1}\\&\text{and}\quad i=\frac{Y}{n}\times E\,\frac{\sinh nx}{\cosh nl}\,\mathbf{1}\end{aligned}\right\}\begin{array}{c}\text{under}\\\text{Heaviside}\\\text{conditions.}\end{array}\tag{56}$$

The method of solving equations of this type by the Expansion Theorem will be shown later.

(2) *Short-circuited line* (*Fig. 30*).

For $x=0$, the potential difference $=0$. Therefore $e=0$ for $x=0$. For $x=l$, $e=E$.

$$\therefore\ 0=K_1 \cosh 0+K_2 \sinh 0$$

$$\therefore\ K_1=0$$

and $e=K_2 \sinh nx$. For $x=l$, $e=E$

$$\therefore\ K_2=\frac{E}{\sinh nl}$$

$$\left.\begin{aligned} \therefore\ e &= E\frac{\sinh nx}{\sinh nl}\mathbf{1} \\ i &= \frac{Y}{n}E\frac{\cosh nx}{\sinh nl}\mathbf{1} \end{aligned}\right\} \begin{array}{l}\text{under}\\ \text{Heaviside}\\ \text{conditions.}\end{array} \qquad (57)$$

It is not difficult to solve Equations (56) and (57) in the most general case, that is, with transmission lines having resistance, inductance, capacity, and leakage, as will be shown in Part XVII.

At first in order to show more clearly the method employed, the application will be made to what Heaviside calls an "ideal cable." A cable which has insignificant leakage and inductance and therefore only two constants, resistance and capacity per unit length.

No such cable can exist, but many cables closely approach this condition. The most serious objection to the study of such cable is that the equations do not show the velocity of propagation of the current and the voltage waves. The advantages are that the results obtained are satisfactory for all practical purposes and that the mathematical transformations are few.

Open-circuited Case: Referring to the open-circuited case and letting Fig. 29 represent such an *ideal cable*—or transmission line—, we have from Equation (56)

$$e=\frac{E \cosh nx}{\cosh nl}\mathbf{1}$$

and $$i=E\frac{Y}{n}\frac{\sinh nx}{\cosh nl}\mathbf{1}$$

The problem is to solve these operational relations by the Expansion Theorem. While it is not necessary, it is usually best to introduce trigonometric relations rather than hyperbolic. Engineers, at any rate, are far more familiar with trigonometric functions than with hyperbolic functions.

In Part XXVIII it will be shown that $\cosh ja = \cos a$ and $\sinh ja = j \sin a$

If, therefore, we write $n^2=-m^2$ then $n=\pm jm$

and $\cosh nx=\cosh(\pm jmx)=\cos mx$

and $\sinh nx=\sinh(\pm jmx)=\pm j \sin mx$

$$\therefore\ e=E\frac{\cos mx}{\cos ml}\mathbf{1}$$

$Z_{(p)}=0$ gives $\cos ml=0$ thus $ml=\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ or $\frac{2s-1}{2}\pi$

and $$m=\frac{2s-1}{2}\frac{\pi}{l} \qquad (58)$$

where the first value of s is 1, the next 2, etc. We could of course also have written $ml=\frac{2s+1}{2}\pi$, in which case the first term corresponds to $s=0$, the next $s=1$, etc.

The question may be raised: is it permissible to use the Expansion Theorem? Perhaps one of the roots of p is zero. If so, $\frac{Y_{(p)}}{Z_{(p)}}$ would be infinity when we substitute $p=0$.

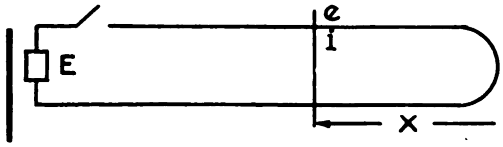

Fig. 30

To test it we have $n^2=(R+pL)\ (G+pC)$

for $p=0$ $n^2=RG$ or $n=\pm\sqrt{RG}$

$$\therefore\ \frac{Y_{(p)}}{Z_{(p)}}=\frac{\cosh x\sqrt{RG}}{\cosh l\sqrt{RG}} \text{ for } p=0.$$

This is not infinity.

In our case $G=0$, and thus $\frac{Y_{(p)}}{Z_{(p)}}=\frac{\cosh 0}{\cosh 0}=1$, for $p=0$.

Thus we conclude that the Expansion Theorem can be used. Let the problem be to find the voltage difference between the two lines and the current flowing at any point x (measured from the end of the line) at any time t after a storage battery of voltage E has been connected to this "ideal" open-circuited cable.

$$\frac{dZ}{dp}=\frac{dZ}{dm}\frac{dm}{dp}$$

but from Equation (51) $-m^2=pCR$ and thus $-2m\ dm = CR\,dp$, or $\frac{dm}{dp}=-\frac{CR}{2m}$. Also $p=-\frac{m^2}{CR}$ and $\frac{dZ}{dm}=-l\sin ml$.

Therefore

$$p\frac{dZ}{dp}=\left(-\frac{m^2}{CR}\right)(-l\sin ml)\left(-\frac{CR}{2m}\right)=-\frac{ml}{2}\sin ml \qquad (59)$$

$$=+\frac{ml}{2}(-1)^s \qquad (60)$$

since $\sin ml$ for $s=1$ is $+1$.

$\sin ml$ for $s=2$ is -1 etc.

Thus $e=E\left[1+\sum_{s=1}^{s=\infty}\frac{2\,(-1)^s}{ml}\cos mx\,\epsilon^{-\frac{m^2}{CR}t}\right]$

Since $\frac{Y_{(0)}}{Z_{(0)}}$ has been shown to be equal to 1, we get when substituting the value for m

$$e=E\left[1+\frac{4}{\pi}\sum_{s=1}^{s=\infty}(-1)^s\frac{\cos\frac{2s-1}{2}\cdot\frac{\pi x}{l}}{2s-1}\,\epsilon^{-\frac{(2s-1)^2\pi^2}{4\,CRl^2}t}\right] \quad (61)$$

To find the current, substitute the values of Y and n in Equation (56).

$$\frac{Y}{n}=\frac{n}{Z} \quad \text{since } n^2=YZ \quad \text{thus } \frac{Y}{n}=\frac{n}{Z}=\frac{\pm jm}{R}$$

$$\therefore i=-E\frac{m}{R}\,\frac{\sin mx}{\cos ml}\,\mathbf{1}$$

$p\frac{dZ}{dp}$ in this case obviously becomes $\frac{Rml}{2}(-1)^s$, $\frac{Y_{(0)}}{Z_{(0)}}=0$

Thus $$i=\sum_{s=1}^{s=\infty}-\frac{2E}{Rl}(-1)^s\sin mx\,\epsilon^{-\frac{m^2}{CR}t} \quad (62)$$

where m is definied by Equation (58).

At the battery $x=l$ $\therefore e=E$ for all values of t, and

$$i=-\frac{2E}{Rl}\left[-\epsilon^{-\frac{\pi^2}{4\,CRl^2}t}+\epsilon^{-\frac{9\pi}{4\,CRl^2}t}+\epsilon^{-\frac{25\pi^2}{4\,CRl^2}t}\cdots\right] \quad (63)$$

At the very moment $t=0$ this series becomes

$$i=-\frac{2E}{Rl}[-1+1-1+1\cdots\cdots]$$

which is indefinite. However Equation (63) converges very rapidly with the smallest values of t, so that we can write with a very fair degree of accuracy

$$i=\frac{2E}{Rl}\,\epsilon^{-\frac{\pi^2}{4\,CRl^2}t}$$

If l is large, the current which is initially large persists for a considerable time; if l is short, the current dies down rapidly.

Thus in the case of a short ideal line the battery current is

$$i=\frac{2E}{R_0}\,\epsilon^{-\frac{\pi^2}{4\,R_0C_0}t}$$

where R_0 and C_0 are the *total* line resistance and capacity.

In a circuit consisting of concentrated resistance and capacity the current is

$$i=\frac{E}{R_0}\,\epsilon^{-\frac{1}{C_0R_0}t}$$

It is thus evident that an "ideal" line or cable may with fair degree of accuracy be represented by placing the line capacity in the middle of the line. The initial current is, for all practical purposes,

$$i=\frac{2E}{Rl}$$

Short-circuited Case: Consider next a short-circuited "ideal" cable as is represented in Fig. 30.

Substituting m for n as in the previous case, Equations (57) become

$$\left.\begin{aligned} e&=E\frac{\sin mx}{\sin ml}\mathbf{1} \\ i&=E\frac{m}{R}\,\frac{\cos mx}{\sin ml}\mathbf{1}\end{aligned}\right\} \quad (64)$$

$$Z_{(p)}=0 \text{ gives } ml=0,\ \pi,\ 2\pi\cdots\cdots \text{or } \frac{s\pi}{l} \text{ or } m=\frac{s\pi}{l} \quad (65)$$

where $s=0$ $s=1$ $s=2$ etc.

Again we have to question whether $p=0$ is one root. $n^2=(R+pL)(G+pC)$. As p approaches zero, n also approaches zero,

$$\therefore \frac{\sinh nx}{\sinh nl}=\frac{nx}{nl}=\frac{x}{l}$$

which is not infinity. Therefore $p=0$ is not a root and the Expansion Theorem can be used. From Equation (65) $m=0$ when $s=0$ and with this value we get the constant term $\frac{Y_{(0)}}{Z_{(0)}}=\frac{x}{l}$. Therefore when using $\frac{Y_{(0)}}{Z_{(0)}}$ we should begin the series with $s=1$

In solving for e, $\frac{dZ}{dp}=\frac{dZ}{dm}\,\frac{dm}{dp}=(l\cos ml)\left(-\frac{CR}{2m}\right)$

and $p\frac{dZ}{dp}=\left(-\frac{m^2}{CR}\right)(l\cos ml)\left(-\frac{CR}{2m}\right)=\frac{ml\cos ml}{2}$

$$\therefore e=E\left[\frac{x}{l}+\sum_{s=1}^{\infty}\frac{2\sin mx\,\epsilon^{-\frac{m^2}{CR}t}}{ml\cos ml}\right]$$

or since $\cos ml=-1,\ +1,\ -1$, etc., and $m=\frac{s\pi}{l}$

$$e=E\left[\frac{x}{l}+\sum_{s=1}^{\infty}(-1)^s\frac{\epsilon^{-\frac{s^2\pi^2}{CRl^2}t}\sin\frac{s\pi x}{l}}{s\pi}\right] \quad (66)$$

By a similar calculation the current is found to be

$$i=\frac{E}{Rl}\left[1+2\sum_{s=1}^{\infty}(-1)^s\,\epsilon^{-\frac{s^2\pi^2}{CRl^2}t}\cos\frac{s\pi x}{l}\right] \quad (67)$$

At the end of the line $x=0$, and there

$$i_r=\frac{E}{Rl}\left[1+2\sum_{s=1}^{\infty}(-1)^s\,\epsilon^{-\frac{s^2\pi^2}{CRl^2}t}\right]$$

In Fig. 31, Curve *A*, is shown the values of the current at the end of such a short-circuited cable. It also represents the current quite accurately if the receiving device is of small resistance compared with that of the cable.

The unit of time taken is $T=\frac{2l^2CR}{\pi^2}\log\frac{4}{3}$. This particular value was suggested by Kelvin and gives a rapidly converging series.

The exponential term becomes $\epsilon^{-\frac{s^2\pi^2}{CRl^2}\frac{2l^2CR}{\pi^2}\log\frac{4}{3}}$

$$=\epsilon^{-2s^2\log\frac{4}{3}}=\left(\frac{3}{4}\right)^{2s^2}$$

$$\therefore i_r=\frac{E}{Rl}\left[1+2\sum_{s=1}(-1)^s\left(\frac{3}{4}\right)^{2s^2}\right]$$

The particular values used pertain to a telephone cable No. 19 B.&S. 100 miles long where

$$R \text{ (per loop mile)} = 88 \text{ ohms}$$

$$C \text{ (per loop mile)} = 0.054 \text{ m.f.}$$

The unit of time *T* becomes then $\frac{2.77}{1000}$ sec.

Curve C shows the shape of the current if the battery had been connected to the cable during a time interval *T* after which the cable had been grounded at the battery. Curves *D*, *E*, *F*, etc. show the received current if the battery had been connected during intervals of $2T$, $3T$, etc. before grounding.

The method of arriving at these curves should be apparent from the discussion in Part X.

Curve *C* is the sum of the original current beginning at $t=0$ and a reversed similar current beginning at $t=T$ as shown in Curve *B*.

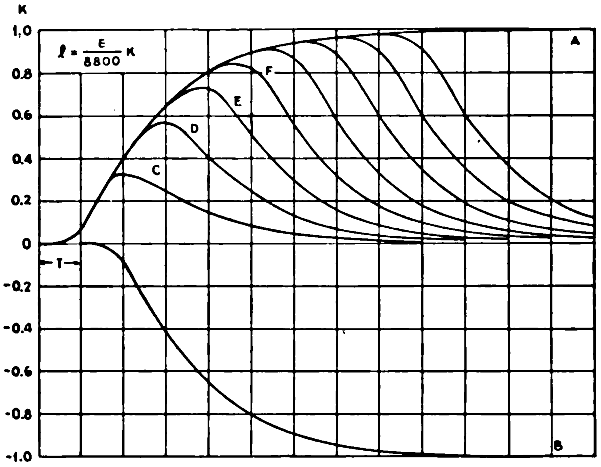

Fig. 31. Current at Receiving End of Closed No. 19 B.&S. Telephone Cable

Curve *A*: Current due to applying *E* at $t=0$
Curve *B*: Current due to $-E$ after an interval *T*
Curve *C*: Actual current which is the sum of the two
Curves *D*, *E*, *F*, etc: Resultant current when the battery is grounded after intervals $2T$, $3T$, and $4T$, etc.

Constants of Cable:
$C=0.054$ m.f. per mile of distance
$R=88$ ohms per mile distance
$T=$ unit of time taken as the "Retardation"
$t=\frac{2lCR}{\pi^2}\log\frac{4}{3}$
$T=\frac{2.77}{1000}$ for $l=100$ miles

PART XVI: AN ALTERNATOR SUDDENLY CONNECTED TO AN "IDEAL" CABLE

Let the problem be to find the value of the voltage at any time and place on a short-circuited cable, Fig. 32, if the impressed e.m.f. is $E\cos\omega t$ and if the switch is closed at $t=0$.

We may proceed by either of two routes: by the use of operator $\frac{p^2}{p^2+\omega^2}1=\cos\omega t$ (from Part VI) or by the use of Duhamel's integral (from Part XIII).

In the first case

$$\underset{\cdot}{e}=E\frac{\sinh nx}{\sinh nl}\,\frac{p^2}{p^2+\omega^2}1 \text{ or } E\frac{\sin mx}{\sin ml}\,\frac{p^2}{p^2+\omega^2}1 \qquad (68)$$

$Z_{(p)}=0$ gives two roots $\pm j\omega$ and a series of roots $m=\frac{s\pi}{l}$, as discussed in Part XV. The permanent term corresponds to the two roots $\pm j\omega$, and it is usually simplest to determine that term in vector form by introducing $p=j\omega$ in the operational solution for the unit e.m.f. as has been shown in Footnote (2), Part IV.

$$\underset{\cdot}{e}=E\frac{\sinh \underset{\cdot}{nx}}{\sinh \underset{\cdot}{nl}}, \text{ but } n^2=pCR=j\omega CR$$

thus

$$\underset{\cdot}{n}=\sqrt{j}\,\sqrt{\omega CR}=\sqrt{\omega CR}\,\underline{|45^\circ}=\sqrt{\frac{\omega CR}{2}}\,(1+j)=a(1+j)$$

Therefore $$\underset{\cdot}{e}=E\frac{\sinh\,[ax\ (1+j)]}{\sinh\ [al\ (1+j)]}$$

but the magnitude of $\sin\,(a+jb)=\sqrt{\sinh^2 a+\sin^2 b}$ and the slope is obtained from the relation $\tan\varphi=\cosh a\tan b$

Thus $$\underset{\cdot}{e}=E\sqrt{\frac{\sinh^2 ax+\sin^2 ax}{\sinh^2 al+\sin^2 al}}\ \underline{|\varphi_1-\varphi_2}$$

where $\tan\varphi_1=\cosh ax\tan ax$ and $\tan\varphi_2=\cosh al\tan al$

or $$e_p=E\sqrt{\frac{\sinh^2 ax+\sin^2 ax}{\sinh^2 al+\sin^2 al}}\cos\,(\omega t+\varphi_1-\varphi_2)$$

To calculate the transient term it is best to resort to the method shown in Part VI.

Let $$h_{1(p)}=\sin ml \text{ and } h_{2(p)}=p^2+\omega^2$$

It is then necessary to find

$$\frac{Y_{(p)}}{p\,h_{2(p)}\,h_1{}'_{(p)}}$$

$$p\,h_1{}'_{(p)} = pl\cos ml\,\frac{dm}{dp}, \text{ or since } -m^2 = pCR$$

$$p\,h_1{}'_{(p)} = \frac{ml}{2}\cos ml$$

and $$p\,h_{2(p)}\,h_1{}'_{(p)} = \left(\frac{m^4}{C^2R^2}+\omega^2\right)\frac{ml}{2}\cos ml$$

$$= (m^4+\omega^2C^2R^2)\,\frac{ml}{2}\cos ml$$

$$Y_{(p)} = Ep^2\sin mx = E\,\frac{m^4}{C^2R^2}\sin mx$$

$$\therefore \frac{Y_{(p)}}{p\,h_{2(p)}\,h_1{}'_{(p)}} = \frac{2\,E\,m^3\sin mx}{l\cos ml\,(\omega^2C^2R^2+m^4)}$$

Fig. 32

thus $$e_t = \sum_{s=1} \frac{2\,E\,(-1)^s\,s^3\pi^3\sin\dfrac{s\pi x}{l}}{\omega^2\,C^2\,R^2\,l^4+s^4\pi^4}\,\epsilon^{-\frac{s^2\pi^2}{CRl^2}t}$$

and $$e = e_p + e_t$$

In using Duhamel's integral it is again best to drop the terms pertaining to the permanent condition and to use the integral only for the transient term. It should be noted however that it is *not necessary* to do so. It is merely less laborious and the result obtained is in a familiar form.

The solution with unit function was shown to be

$$e = E\left[\frac{x}{l}+2\sum_{s=1}(-1)^s\,\frac{\epsilon^{-\frac{s^2\pi^2}{CRl^2}t}\,\sin\dfrac{s\pi x}{l}}{s\pi}\right]$$

Note that the only term involving t is the exponential; everything else is constant as far as t is concerned.

$$\therefore \varphi(t) = k\,\epsilon^{-\alpha t}+\frac{x}{l}$$

$$\varphi(t-u) = k\,\epsilon^{-\alpha(t-u)}+\frac{x}{l}$$

$$\frac{d}{dt}\varphi(t-u) = -k\alpha\,\epsilon^{-\alpha(t-u)}$$

$$e(u) = E\cos\omega u$$

$$e(t) = E\cos\omega t$$

$$\varphi_{(0)} = \frac{x}{l}+\sum(-1)^s\,\frac{\sin\dfrac{s\pi x}{l}}{s\pi}$$

One part of the permanent term is therefore

$$E\,\varphi_{(0)}\cos\omega t$$

Referring to Equation (45), Part XIV, the second term becomes

$$e = \int_{u=0}^{u=t} -k\alpha\,\epsilon^{-\alpha(t-u)}E\cos\omega u\,du$$

$$= -k\alpha\,\frac{\cos(\omega t-\varphi)-\epsilon^{-\alpha t}\cos\varphi}{\sqrt{\alpha^2+\omega^2}}, \text{ where } \tan\varphi = \frac{\omega}{\alpha}$$

Part of this expression belongs to the permanent term. The transient part is

$$e_t = \frac{E\,k\alpha\,\epsilon^{-\alpha t}\cos\varphi}{\sqrt{\alpha^2+\omega^2}}$$

$$= \frac{E\,k\alpha^2}{(\alpha^2+\omega^2)}\,\epsilon^{-\alpha t} = \sum_{s=1}\frac{E(-1)^s}{s\pi}\sin\frac{s\pi x}{l}\,\frac{s^4\pi^4}{C^2R^2l^4}\,\frac{C^2R^2l^4}{s^4\pi^4+\omega^2C^2R^2l^4}$$

which by substituting becomes

$$e_t = \sum_{s=1}\frac{2\,E\,(-1)^s\,s^3\pi^3\sin\dfrac{s\pi x}{l}\,\epsilon^{-\frac{s^2\pi^2}{CRl^2}t}}{s^4\pi^4+\omega^2C^2R^2\,l^4}$$

PART XVII: TRANSMISSION LINE HAVING ALL FOUR CONSTANTS R, L, G, AND C

Let the problem be to find the current in a short-circuited line at any time and place after a storage battery has been connected to the circuit.

The general operational solution as shown in Part XV is

$$i = E\,\frac{Y}{n}\,\frac{\cosh nx}{\sin nl}\,\mathbf{1} = E\,\frac{n}{Z}\,\frac{\cosh nx}{\sinh nl}\,\mathbf{1} \qquad (69)$$

$$= E\,\frac{m}{Z}\,\frac{\cos mx}{\sin ml}\,\mathbf{1} = E\,\frac{m\cos mx}{(R+pL)\sin ml}\,\mathbf{1} \qquad (70)$$

Since $Z_{(p)}$ is a product, $(R+pL)\sin ml$, one root will be $p_1 = -\frac{R}{L}$. The other will be a series of values corresponding to $\sin ml = \text{zero}$, or $ml = s\pi$, or $m = \frac{s\pi}{l}$, beginning with $s=1$, since the value corresponding to $p=0$ is taken care of in the $\frac{Y_{(0)}}{Z_{(0)}}$ term.

The solution then will contain three sets of terms, the first corresponding to $\frac{Y_{(0)}}{Z_{(0)}}$, the second corresponding to the root $p_1 = -\frac{R}{L}$, and the third a series, or

$$i = E\,\frac{Y_{(0)}}{Z_{(0)}}+\frac{Yp}{ph_1{}'(p)\,h_2(p)}\,\epsilon^{p_1t}+\sum\frac{Yp}{ph_2{}'(p)\,h_1(p)}\,\epsilon^{pt}$$

(1). Find $\frac{Y_{(0)}}{Z_{(0)}}$.

Since $n^2 = (R+pL)(G+pC)$, $n = \sqrt{RG}$ for $p=0$ (71)

and $Z_{(0)} = R$

$$\therefore \frac{Y_{(0)}}{Z_{(0)}} = E\sqrt{\frac{G}{R}}\,\frac{\cosh x\sqrt{RG}}{\sinh l\sqrt{RG}} \qquad (72)$$

(2). Find the second term.

Let $h_1(p) = R + pL$ and $h_2(p) = \sin ml$

Then, referring to Part VI,

$$ph_1'(p)\,h_2(p) = p\,L\,\sin ml$$

Thus

$$ph_1'(p)\,h_2(p) \text{ for } p = p_1 = -\frac{R}{L} \text{ becomes } -R\sin ml$$

$$Y(p) = m\cos mx$$

but
$$\begin{aligned} -m^2 &= (R+pL)\,(G+pC) \\ &= p^2CL + p(RC+LG) + RG \end{aligned} \tag{73}$$

If we substitute $p = p_1 = -\frac{R}{L}$ we get

$$-m_1^2 = \frac{R^2}{L^2}CL - \frac{R}{L}(RC+LG) + RG = 0 \quad \text{or } m_1 = 0.$$

Thus $\dfrac{Y(p)}{ph_1'(p)\,h_2(p)} = \dfrac{0}{0}$ We must therefore find the value of this expression as m approaches zero.

It is $\dfrac{Y(p)}{ph_1'(p)\,h_2(p)} = \lim\limits_{m \to 0} \dfrac{m\cos mx}{-R\sin ml} = -\dfrac{1}{Rl}$

Thus the second term becomes $-\dfrac{E}{Rl}\epsilon^{-\frac{R}{L}t}$ (74)

(3). $\sin ml = 0$ gives $m = \dfrac{s\pi}{l}$

but $-m^2 = p^2CL + p(RC+LG) + RG$

thus

$$p = -\frac{RC+LG}{2\,CL} \pm \sqrt{\frac{(RC+LG)^2}{4\,C^2L^2} - \frac{RG+m^2}{CL}} = -\alpha \pm j\beta \tag{75}$$

where
$$\alpha = +\frac{1}{2}\left(\frac{R}{L} + \frac{G}{C}\right) \tag{76}$$

and
$$\beta = \sqrt{\frac{m^2}{CL} - \frac{1}{4}\left(\frac{R}{L} - \frac{G}{C}\right)^2} \tag{77}$$

α depends, as seen, upon the line constants only; β depends upon them and the particular terminal condition also. In this case, the case of a short-circuited line, $\sin ml = 0$ gave $m = \frac{s\pi}{l}$. Often, however, especially when the terminal condition is complicated, it is necessary to resort to graphs in order to find the root of m.

This will be illustrated in Part XVIII.

We next proceed to find $ph_2'(p)\,h_1(p)$

$$h_2'(p) = \frac{d}{dp}\sin ml = \frac{d}{dm}\sin ml\frac{dm}{dp} = l\cos ml\frac{dm}{dp} \tag{78}$$

but $-m^2 = p^2CL + p(RC+LG) + RG$

$$\therefore \frac{dm}{dp} = -\frac{2\,p\,CL + RC + LG}{2m}$$

$$\therefore h_2'(p) = -l\cos ml\frac{CL}{2m}\left[\left(\frac{R}{L} + \frac{G}{C}\right) + 2p\right]$$

$$\therefore ph_2'(p)\,h_1(p) = -pl\cos ml\,(R+pL)\frac{CL}{2m}\left[\left(\frac{R}{L} + \frac{G}{C}\right) + 2p\right]$$

thus

$$\frac{Y(p)}{ph_2'(p)\,h_1(p)} = \frac{2E\,m^2\cos mx}{-pl\cos ml\,(R+pL)\,CL\left[\frac{R}{L} + \frac{G}{C} + 2p\right]}$$

By substituting $m^2 = \frac{s^2\pi^2}{l^2}$ and $p = -\alpha + j\beta$, one term results, which is multiplied by $\epsilon^{(-\alpha+j\beta)t}$

By substituting $p = -\alpha - j\beta$, another similar term results which is multiplied by $\epsilon^{(-\alpha-j\beta)t}$.

The two terms combine and the solution of the third term is

$$\left.E\sum_{s=1}\frac{2(-1)^s\cos\frac{s\pi x}{l}\epsilon^{-\alpha t}}{s^2\pi^2 + RG\,l^2}\left[\beta\left\{\frac{s^2\pi^2}{L} + \frac{1}{2}G\left(\frac{R}{L} - \frac{G}{C}\right)\right\}\sin\beta t - G\cos\beta t\right]\right\} \tag{79}$$

The solution is the sum of the three terms given in Equations (72), (74), and (79).

PART XVIII: TRANSMISSION LINE WITH IMPEDANCE AT THE SENDING AND RECEIVING ENDS

It has been shown that when the transmission line is either grounded or open, no difficulty exists in finding the roots $z_{(p)} = 0$. It often becomes quite difficult however, if the terminal conditions are not as simple.

Heaviside discusses such cases in various parts of his book. Since, however, his treatment is somewhat different from that which will now be given, it will be well to illustrate one or two cases. Let us take, for instance, the case shown by Heaviside on page 143 (Vol. ii), illustrated here in Fig. 33, in which the long horizontal lines may be the insulated conductor in a cable whose outside sheath is grounded. z_1 and z_2 are concentrated impedances.

From previous consideration it is evident that, as far as the transmission line itself is concerned, the following relations exist:

$$\left.\begin{aligned} e &= K_1\cosh nx + K_2\sinh nx \\ i &= \frac{Y}{n}[K_1\sinh nx + K_2\cosh nx] \end{aligned}\right\} \tag{80}$$

For $x = 0$,

$$e = e_2 = i_2z_2 = K_1(1+0) = K_1$$

for $x = l$,

$$e = e_1 = K_1\cosh nl + K_2\sinh nl$$

But $E = e_1 + i_1 z_1 \quad \therefore E = i_2 z_2 \cosh nl + K_2 \sinh nl + i_1 z_1$ for $x = 0$,

$$i_2 = \frac{Y}{n}[0 + K_2] \quad \therefore K_2 = \frac{ni_2}{Y}$$

For $x = l$,

$$i_1 = \frac{Y}{n}\left[i_2 z_2 \sinh nl + \frac{n}{Y} i_2 \cosh nl\right]$$

From these relations we obtain

$$E = i_2 z_2 \cosh nl + \frac{n}{Y} i_2 \sinh nl$$

$$+\frac{Yz_1}{n} i_2 \left[z_2 \sinh nl + \frac{n}{Y} \cosh nl\right]$$

$$= i_2 \left[(z_1 + z_2) \cosh nl + \frac{1}{n}[z + Yz_1 z_2] \sinh nl\right]$$

$$= i_2 A \quad \therefore i_2 = \frac{E}{A}. \quad \text{Here } z = R + pL$$

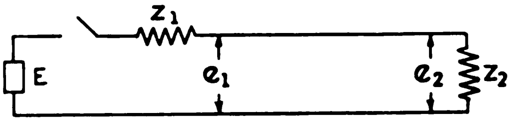

Fig. 33

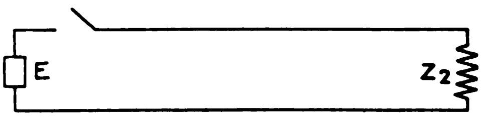

Fig. 34

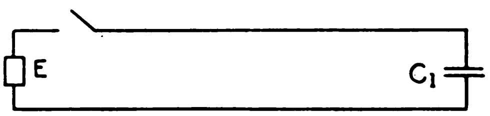

Fig. 35

and from Equation (80) $e = i_2 z_2 \cosh nx + \frac{ni_2}{Y} \sinh nx$

Thus

$$e = \frac{z_2 \cosh nx + \frac{n}{Y} \sinh nx}{(z_1 + z_2) \cosh nl + \frac{1}{n}(z + Yz_1 z_2) \sinh nl} E1 \qquad (81)$$

From Equation (80)

$$i = \frac{Y}{n} i_2 \left[Z_2 \sinh nx + \frac{n}{Y} \cosh nx\right]$$

$$= \frac{\frac{z_2}{z_0} \sinh nx + \cosh nx}{(z_1 + z_2) \cosh nl + \frac{1}{n}[z + Yz_1 z_2] \sinh nl} E1 \qquad (82)$$

where $$z_0 = \frac{n}{Y} = \frac{z}{n}$$

Special case $z_1 = 0$, illustrated in Fig. 34.

$$\therefore e = \frac{z_2 \cosh nx + \frac{n}{Y} \sinh nx}{z_2 \cosh nl + \frac{z}{n} \sinh nl} E1$$

$$= \frac{z_2 \cosh nx + z_0 \sinh nx}{z_2 \cosh nl + z_0 \sinh nl} E1 \qquad (83)$$

From Equation (82), for $z_1 = 0$,

$$i = \frac{\frac{z_2}{z_0} \sinh nx + \cosh nx}{z_2 \cosh nl + z_0 \sinh nl} E1 \qquad (84)$$

At the receiving end $x = 0$.

$$\therefore e_2 = \frac{1}{\cosh nl + \frac{z_0}{z_2} \sinh nl} E1 \qquad (85)$$

and

$$i_2 = \frac{1}{z_2 \cosh nl + z_0 \sinh nl} E1 \qquad (86)$$

This current will be calculated in order to show the procedure when the roots $z_{(p)} = 0$ cannot readily be obtained except by graphical means.

Assume again, for the sake of simplicity, that we deal with the "ideal" cable.

$$\therefore n^2 = pCR$$

If $m^2 = -n^2$ then $m^2 = -pCR$ and $p = -\frac{m^2}{CR}$.

Let the terminal impedance be a condenser of capacity c_1, as in Fig. 35.

$$\therefore z_2 = \frac{1}{pc_1} = -\frac{CR}{c_1 m^2} \text{ and } z_0 = \frac{z}{n} = \frac{R}{\sqrt{pCR}} = \sqrt{\frac{R}{C}} \frac{\sqrt{CR}}{jm} = \frac{R}{jm}$$

Thus, since $\cosh jml = \cos ml$ and $\sinh jml = j \sin ml$

$$i_2 = \frac{1}{-\frac{cR}{c_1 m^2} \cos ml + \frac{R}{m} \sin ml} E1$$

$$= \frac{\frac{c_1}{CR} m^2}{-\cos ml + \frac{c_1}{C} m \sin ml} E1$$

$z_{(p)} = 0$ gives $\tan ml = \frac{C}{c_1 m}$.

What is wanted then are all possible values of m, m_1, m_2, m_3, etc., which satisfy the relation $\tan ml = \frac{C}{c_1 m}$.

Fig. 36 illustrates the method of obtaining these roots in a particular numerical case. An "ideal" cable (100 miles long) of resistance 88 ohms per mile, of capacity 0.054 mf. per mile, is shunted at the distant end by a condenser c_1 of the same capacity as that of the entire cable. Find the equation for the current at the distant end when the cable is suddenly connected to a source of constant potential E.

The procedure used in finding the roots is to assign values to ml then plot the corresponding tangent curves and the graph $\frac{C}{c_1 m}$.

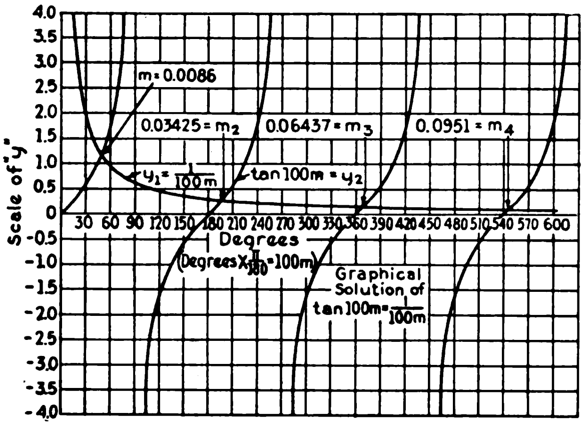

Fig. 36

Where these curves intersect, the roots are obviously located. These roots appear to be $m_1 = 0.0086$, $m_2 = 0.0343$, $m_3 = 0.0643$, etc. To find i_2, the current at the receiving circuit, we proceed in the usual way.

In this case (the "ideal" cable)

$$n^2 = (R + pL)\,(G + pC) = pCR \quad \therefore -m^2 = pCR \quad (87)$$

$$p\frac{dz}{dp} = p\frac{dz}{dm}\frac{dm}{dp}$$

$$\frac{dz}{dm} = l \sin ml + \frac{c_1}{C}\,[ml \cos ml + \sin ml]$$

from Equation (87)

$$\frac{dm}{dp} = -\frac{CR}{2m}$$

$$\therefore p\frac{dz}{dp} = \frac{m}{2}\left(l + \frac{c_1}{C}\right) \sin ml + \frac{c_1}{2C} m^2 l \cos ml$$

$\frac{Y_{(0)}}{Z_{(0)}}$ corresponds to the steady case. Since we have a condenser at the end of the line, the final direct current is of course zero, thus $\frac{Y_{(0)}}{Z_{(0)}} = 0$. Or, it can be seen directly by making $p = 0$, which means $m = 0$.

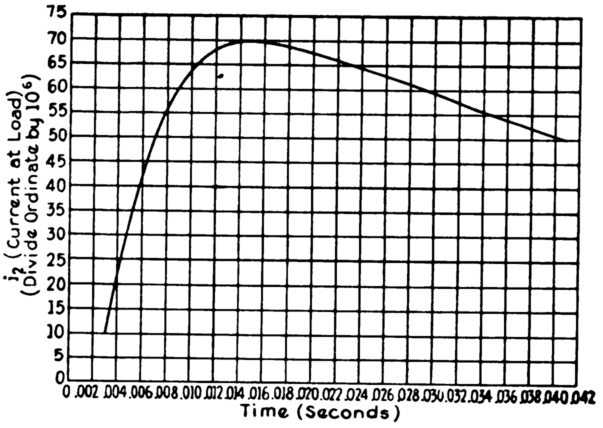

Fig. 37

The solution for the current at the receiving end of the line is therefore:

$$i = \sum_{m_1 m_2 m_3} \frac{\frac{c_1}{CR} m^2 \epsilon^{-\frac{m^2}{CR}t}}{\frac{m}{2}\left(l + \frac{c_1}{C}\right) \sin ml + \frac{c_1}{2C} m^2 l \cos ml} E$$

$$= \frac{2E}{RL} \sum_{\alpha} \frac{\alpha \cos \alpha}{\alpha + \sin \alpha \cos \alpha} \epsilon^{-\frac{\alpha^2}{RCl^2}t}$$

where $\alpha = ml$ and α is obtained from equation $\tan \alpha = \frac{C}{c_1 m} = \frac{Cl}{c_1 \alpha}$. Fig. 37 gives the current at the end of the line for $E = 1$.

PART XIX: TWO OR MORE TRANSMISSION LINES OF DIFFERENT CONSTANTS CONNECTED TOGETHER BY AN IMPEDANCE OR SHUNTED AT THEIR JUNCTION BY AN IMPEDANCE

Consider first the case of a shunted impedance as shown in Fig. 38, in which case, for the sake of simplicity, it has been assumed that the line is short circuited at the receiving end.

The impedance or leak Z' is introduced at a point distant l_1 from the generator. Let the constants pertaining to the left-hand and right-hand sides have indices 1 and 2, respectively, and let the problem be to find the current at any point in either line.

Consider first the right-hand side. The conditions existing there are those of a short-circuited transmission line discussed in Part XV. That is

$$e = \frac{E \sinh n_2 x}{\sinh n_2 (l - l_1)} \mathbf{1}$$

and

$$i = \frac{Y}{n_2} E \frac{\cosh n_2 x}{\sinh n_2 (l - l_1)} \mathbf{1}$$

where $l - l_1$ is the length of the right-hand line.

Thus $$\frac{e}{i}=\frac{n_2}{Y_2}\frac{\sinh n_2 x}{\cosh n_2 x}$$

At the leak, $x=l-l_1$

$$z=\frac{e}{i}=\frac{n_2}{Y_2}\frac{\sinh n_2(l-l_1)}{\cosh n_2(l-l_1)} \tag{88}$$

We can therefore replace the right-hand short-circuited line by an impedance z placed at the junction. This impedance is in parallel with impedance Z'; thus the joint impedance, which will be denoted by Z_2, is

$$Z_2=\frac{z\,Z'}{z+Z'}$$

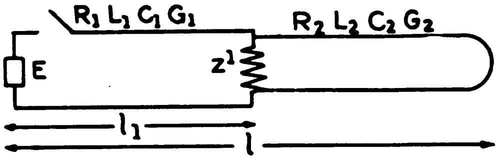

Fig. 38

The current and voltage at any point on the left-hand transmission line of length l_1 can therefore be found from equations already worked out (see Part XVIII, Equations (83) and (84)) where we deal with a line of constants having index 1, and where x is counted to the left from the junction point.

At the end of this line, which is the junction point in this problem, we find from Equation (85) in Part XVIII that the voltage e_j is

$$e_j=\frac{1}{\cosh n_1 l_1+\dfrac{Z_1}{n_1 Z_2}\sinh n_1 l_1}E\mathbf{1}$$

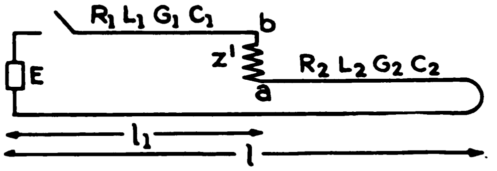

Fig. 39

Thus for the right-hand line the currents and voltages are

$$e=\frac{1}{\cosh n_1 l_1+\dfrac{Z_1}{n_1 Z_2}\sinh n_1 l_1}\,\frac{\sinh n_2 x}{\sinh n_2(l-l_1)}E\mathbf{1} \tag{89}$$

and

$$i=\frac{1}{\cosh n_1 l_1+\dfrac{Z_1}{n_1 Z_2}\sinh n_1 l_1}\times\frac{Y_2}{n_2}\,\frac{\cosh n_2 x}{\sinh n_2(l-l_1)}E\mathbf{1} \tag{90}$$

Consider next two transmission lines of different constants (indices 1 and 2) connected together through an impedance Z' as shown in Fig. 39, where again, for the sake of simplicity, the method is illustrated by the particular case of a short-circuited line.

It is evident that, up to point a, the impedance is z as shown in Equation (88). The impedance up to point b is therefore $Z_2=Z'+z$.

Again the values of the voltage and current for the left-hand side are obtained from Equations (83) and (84) in Part XVIII, where we deal with a line of length l_1 and of constants having index 1.

At the end of this fictitious line represented in Fig. 40, the voltage and current can be obtained from Equations (85) and (86) in Part XVIII.

Let $Z'+z=Z_2$

Thus $$e_2=\frac{1}{\cosh n_1 l_1+\dfrac{Z_1}{n_1 Z_2}\sinh n_1 l_1}E\mathbf{1}$$

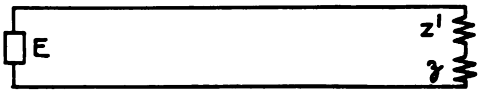

Fig. 40

$$i_2=\frac{1}{Z_2\cosh n_1 l_1+\dfrac{Z_1}{n_1}\sinh n_1 l_1}E\mathbf{1}$$

The voltage available across the second transmission line, the right-hand side of Fig. 39 is, $e_3=i_2 z$.

Thus for the right-hand side, remembering that we now deal with a short-circuited line of length $l-l_1$

$$e=\frac{z}{\cosh n_1 l_1+\dfrac{Z_1}{n_1 Z_2}\sinh n_1 l_1}\times\frac{\sinh n_2 x}{\sinh n_2(l-l_1)}E\mathbf{1}$$

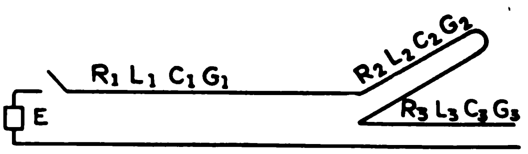

Fig. 41

and

$$i=\frac{z}{\cosh n_1 l_1+\dfrac{Z_1}{n_1 Z_2}\sinh n_1 l_1}\times\frac{Y_2}{n_2}\,\frac{\cosh n_2 x}{\sinh n_2(l-l_1)}E\mathbf{1}$$

Suppose three transmission lines of different constants are connected together. Fig. 41 illustrates such a case, a simple case, chosen purposely to illustrate the method to be used.

Branch No. 2 is short circuited, branch No. 3 open circuited. Find the current and voltage at any point of the network.

Each of the two branch lines is converted to impedances at the junction point. These are converted to one single impedance placed at the end of the main line, after which currents and voltages in the main line can be calculated from Equations given in Part XVIII. Of special interest is the voltage at the junction point. Having obtained this voltage the currents in the branch lines are readily calculated.

Thus the equivalent impedance at the junction point of the open-circuited line is

$$z_3=\frac{e}{i}=\frac{n_3}{Y_3}\,\frac{\cosh n_3 l_3}{\sinh n_3 l_3}\quad \text{(See Part XV)}$$

and the equivalent impedance of the short-circuited line is

$$z_2=\frac{n_2}{Y_2}\,\frac{\sinh n_2 l_2}{\cosh n_2 l_2}$$

The combined impedance is $z=\frac{z_2\, z_2}{z_2+z_3}$.

This is then the impedance at the end of the main line and takes the place of Z_2 in Equations (83), (84), (85), (86) of Part XVIII and the conditions in that line are at once obtained; x is of course counted from the junction point toward the left.

Of special interest is e at the end of this fictitious line. It is

$$e=\frac{1}{\cosh n_1 l_1+\frac{Z_1}{n_1}\sinh n_1 l_1}\,E\,1$$

Therefore the current and voltage in the open-circuited line is

$$e=\frac{1}{\cosh n_1 l_1+\frac{Z_1}{n_1}\sinh n_1 l_1}\times\frac{\cosh n_3 x}{\cosh n_3 l_3}\,E\,1$$

$$i=\frac{1}{\cosh n_1 l_1+\frac{Z_1}{n_1}\sinh n_1 l_1}\times\frac{Y_3}{n_3}\,\frac{\sinh n_3 x}{\cosh n_3 l_3}\,E\,1$$

For the short-circuited branch

$$e=\frac{1}{\cosh n_1 l_1+\frac{Z_1}{n_1}\sinh n_1 l_1}\times\frac{\sinh n_2 x}{\sinh n_2 l_2}\,E\,1$$

$$i=\frac{1}{\cosh n_1 l_1+\frac{Z_1}{n_1}\sinh n_1 l_1}\times\frac{Y_2}{n_2}\,\frac{\cosh n_2 x}{\sinh n_2 l_2}\,E\,1$$

PART XX: A BATTERY CONNECTED AT SOME POINT OF A CABLE

In Fig. 42 is represented a cable having a grounded sheath. The left-hand end of the cable is grounded; the right-hand end is open circuited. It is desired to find the potential difference between the cable and the sheath after battery E has been inserted.

Referring to the general equations given in Part XV, and remembering that in them x is reckoned from the end of the line, we get for the right-hand side

$$V_1=K_1\cosh n\,(l-y)+K_2\sinh n\,(l-y)$$

$$i_1=\frac{Y}{n}\left[K_1\sinh n\,(l-y)+K_2\cosh n\,(l-y)\right].$$

For $y=l$, $i_1=0$, therefore $K_2=0$.

$$\therefore\ V_1=K_1\cosh n\,(l-y).$$

For the left-hand side

$$V_2=K_3\cosh ny+K_4\sinh ny$$

When $y=0$, $V_2=0$, and therefore $K_3=0$.

$$\therefore\ V_2=K_4\sinh ny.$$

At $y=D$ the difference of potential between V_2 and V_1 must be the voltage of the battery. Therefore,

$$K_1\cosh n(l-D)-K_4\sinh nD=E.$$

Since

$$\cosh\alpha\cosh\beta+\sinh\alpha\sinh\beta=\cosh(\alpha+\beta),$$

it is seen by inspection that this condition exists if

$$K_1=\frac{\cosh nD}{\cosh nl}E \text{ and } K_4=-\frac{\sinh n(l-D)}{\cosh nl}E^{(4)}$$

Thus

$$\left.\begin{aligned}V_1&=\frac{\cosh nD}{\cosh nl}\cosh n\,(l-y)\,E\,1\\ V_2&=-\frac{\sinh n\,(l-D)}{\cosh nl}\sinh ny\,\mathrm{E}\,1\end{aligned}\right\}\qquad(91)$$

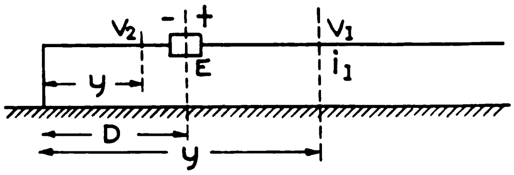

Fig. 42

Heaviside gives this solution in a slightly different form on page 140 (Vol. ii). He uses m instead of n and trigonometric terms instead of hyperbolic.

$$\left.\begin{aligned}&\text{Since} && m^2=-n^2,\ n=jm\\ &\text{and since} && \sinh jm\alpha=j\sin m\alpha\\ &\text{and} && \cosh jm\alpha=\cos m\alpha\end{aligned}\right\}\qquad(92)$$

$$\left.\begin{aligned}V_1&=\frac{\cos mD\cos m\,(l-y)}{\cos ml}\,E\,1\\ V_2&=\frac{\sin m\,(l-D)}{\cos ml}\sin my\,E\,1\end{aligned}\right\}\qquad(93)$$

(*To be continued*)

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(4) These constants can also be obtained as follows:

$V_1=K_1\cosh n(l-y)$, $\quad i_1=\frac{Y}{n}K_1\sinh n\,(l-y)$

$V_2=K_4\sinh ny$, $\quad i_2=\frac{Y}{n}K_4\cosh ny$

For $y=D$, $V_1-V_2=E\,1$ and $i_1+i_2=0$

From these relations K_1 and K_4 can be determined.

The Application of Oxygen and Hydrogen to Industrial Operations

PART IV

SELECTION OF A FUEL GAS FOR METAL CUTTING

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THE interest that is being shown at this time in the great variety of fuel gases adaptable to use in a cutting torch suggests the following discussion of the economic factors involved in the selection of a gas for this purpose.

It should be emphasized that although the functions of the preheating flame and the oxygen cutting jet must be synchronized, they are quite separate and distinct. The fuel flame furnishes the ignition heat to permit the initial starting of the cut; and the subsequent reaction of the steel and oxygen would make the process self-sustaining from a thermal standpoint were it not for a small heat loss through radiation. Therefore, after once starting a cut, the importance of the preheating flame is greatly reduced. For this reason and under certain economic conditions, there are many fuels which lend themselves to this field of application.

The various factors on which a selection must be based are all interdependent. Each is important and the ultimate choice of gas must be based on a thorough study and correct interpretation of all the factors.

Gas characteristics, unit gas costs (costs per cubic foot), relative gas consumptions, cutting speeds, and similar data can be tabulated but their relation to the economic problem as a whole must be given proper consideration before any definite conclusions can be reached.

Nature of Cutting Work

For the purpose of facilitating this discussion, we might classify the cutting phase of the problem as follows:

(1). Light-metal sections
(2). Heavy-metal sections
(3). Constructional work
(4). Destructional work
(5). Work involving frequent starts and short cuts
(6). Work involving infrequent starts and long cuts
(7). Production work where continuity prevails
(8). Emergency work where speed is the main factor
(9). Hand cutting
(10). Machine cutting.

It is generally known that the fuel gas having the higher flame temperature permits greater speed on light-metal sections, in contrast to the heavier sections where the rate of cutting is governed largely by the rate of penetration of the cutting oxygen jet itself. Therefore, acetylene with its high flame temperature is usually more effective in cutting metal of light section where the other factors do not mitigate against its use. Such advantage, however, is almost inversely proportional to the thickness being cut. As a rule, acetylene is more effective in destructional work, for cutting where frequent starts must be made, and for emergency work. For production work, machine cutting, riser cutting, and constructional work, other fuels are in most cases superior to acetylene from an economic viewpoint. For extremely heavy cutting, hydrogen is more effective than any other fuel gas available. Machine cutting, where smooth cuts are required, is an excellent application for fuels other than acetylene.

Segregation of Cutting Activities

The number of torches being operated at one permanent geographical location is a very important factor, for the reason that if a large part of the cutting is performed at scattered points the use of pipe-line distribution of gas cannot be resorted to economically. If tanked gas must be used to give the desired degree of portability, acetylene is the preferable gas because more heat units can be delivered to the job with the minimum amount of tank handling and subsequent expense. On the other hand, if cutting is being done at any fixed point and in sufficient volume to warrant a pipe-line installation, then hydrogen, coal gas, coke-oven gas, water gas, or city gas become attractive substitutes for acetylene. Riser cutting in a steel foundry, where several torch operators are grouped, offers an excellent application for fuel gases other than acetylene.

Preliminary Survey

Before attempting to substitute some other fuel gas for acetylene, each industrial should make the following analyses:

Acetylene

(1). Annual cost of acetylene and volume used for cutting.

(2). Source of supply and method of distribution.

(3). If dissolved acetylene is being used, do economic conditions warrant the installation of acetylene generators?

(4). In what other applications is acetylene used? In what quantities and at what annual cost?

(5). What is the relative quantity and costs of acetylene used for cutting and for welding or any other applications?

Oxygen

(1). Annual cost of oxygen and volume used for cutting.

(2). Annual cost of oxygen and volume used for welding.

(3). Source of supply and method of distribution.

(4). Do economic conditions warrant the installation of an oxygen plant?

(5). In the event a plant were installed, can oxygen be piped to the points of consumption or must tanks be used?

Substitute Fuel Gases Available

The substitute gas might be hydrogen, city gas (water gas, coke-oven gas, or carburetted water gas), natural gas, or one of the numerous other compressed fuels.

Hydrogen is an excellent cutting fuel but as a rule its use is not advantageous unless the consumer has a producing plant, the supply from which is essential to some other manufacturing process. In those localities where very cheap power is available to permit the cheap generation of hydrogen, this gas might be the preferable cutting fuel. Such instances, where economic conditions favor an electrolytic-plant installation, are not sufficiently numerous at the present time to make hydrogen a general substitute for acetylene. Its cost when purchased in cylinders is comparatively high so that, although it is an excellent fuel, its availability feature is deficient, thus excluding it from the general discussion of this subject.

Natural gas is not generally available, so although this gas also can be used as a fuel in cutting its application must necessarily be restricted to certain geographical localities.

City gas and acetylene are available at almost any city in the country, with the former possessing somewhat of an advantage in this respect.

The following are a few representative questions to which the industrial must find favorable answer before any change in cutting fuel gas is made:

(1). What will be the replacement cost for the available substitute fuel gas in the event that it is adapted to the work?

(2). What are the main requirements for the cutting fuel in the particular cutting operation under consideration?

(3). How do the substitute fuels measure up to such requirements?

(4). What will be the effect of the substitution on the consumption of oxygen and its subsequent cost? (If the consumer operates his own oxygen plant this factor is of less importance.)

(5). In the event city gas is being considered as the substitute, what percentage of carbon dioxide and nitrogen does it contain? These inert gases if present in excessive quantities interfere with the attainment of ideal cutting conditions.

(6). How and under what pressure is the substitute gas delivered? Is it sufficient for the process? If not, how can the requisite pressure be secured? The pressure required depends upon the type of cutting torch used. A pressure torch requires from 15 to 50 lb. per sq. in., an injector torch from ½ to 5 lb. per sq. in.

Present Method of Gas Distribution

It may be possible that the gas in use is properly adapted to the work, but tanks might be eliminated in favor of pipe lines. If the volumes used are sufficiently great, consumer manufacture might be resorted to in the case of oxygen, hydrogen, or acetylene. Where gas is purchased in tanks, the loss is usually greater than when pipe-line distribution is used. Manifold systems might prove desirable, or the installation of acetylene generating equipment or hydrogen generating equipment might be advantageous. The path to economy is not dependent alone upon a change in the fuel gas used, but also upon the method of procurement.

Time Study of Cutting Activities

To determine the relative importance of cutting speed, a time study of the operations should be made. In most cases, cutting speed is not nearly as important a factor as is commonly supposed for only a nominal percentage of the operator's time is spent in actual cutting and the more expensive fuel does not always come up to its potentialities in practice. The time study in Table X shows the relative unimportance of cutting speed.

TABLE X

TIME STUDY OF RISER CUTTING IN STEEL FOUNDRY

(Using Hydrogen as Fuel)

Three men work 8 hr. per day	24 labor-hours
Time spent in actual cutting	9 labor-hours
Time spent in preheating prior to actual cutting	2 labor-hours
Time spent in handling tanks	3 labor-hours
Time spent in handling castings, risers, etc.	10 labor-hours

From these figures it will be observed that an increase of 100 per cent in cutting speed would effect a decrease of only 18.5 per cent in labor-hours. In discussions on this subject, the importance of cutting speed should not be stressed without due regard to the cost of its attainment and its relative importance to the problem as a whole.

Basis for Efficiency

Various plants have various criterions for determining the efficiency of their cutting operations. Of course, the ideal objective is maximum results with minimum cost, but the two cannot always be reconciled and the acceptable standard of performance might be one of the following:

(1). *Speed of cutting*, the purpose here being to save labor expense or accomplish results in a minimum time as in an emergency. The speed although obtained at an increased gas cost may be justified through the realization of other economies

(2). *Low gas cost.* In this case the relation of labor and material is such that a reduction in the latter is the desirable procedure. Low gas cost does not necessarily mean increased labor expense per inch of cut. As a rule, where the cheaper fuels are serviceable, the labor cost per inch cut remains constant

(3). *Results obtained.* In machine cutting, smooth faces and edges are ordinarily desirable. City gas or hydrogen permits the attainment of such results with greater facility than does acetylene

(4). *Safety.* This is a real consideration and even though the other factors might favor a more dangerous gas, sometimes a loss in efficiency might be charged against accident insurance

(5). *Relation to other operations*, such as welding. By using the same gas for both (in this case atomic hydrogen or acetylene would be the competitors), economies can at times be realized.

Cost of Oxygen and Source of Supply

Inasmuch as the fuel gas requires a pure oxygen supply and inasmuch as various fuel gases require various quantities of oxygen for their combustion, it is apparent that the oxygen factor is quite important. As a rule, the least expensive fuel gases involve the use of a slightly greater quantity of oxygen, so that a cheap source of oxygen supply has a very important bearing on any proposed substitution. The net result is determined by comparing the economy obtained through fuel gas substitution with the increased cost of oxygen if there is such an increase.

Analysis of Operating Costs for Acetylene and for City Gas

The effect which a variation in unit gas costs (cost per cu. ft.) has upon the total relative gas costs is clearly shown by the curves in Fig. 26.

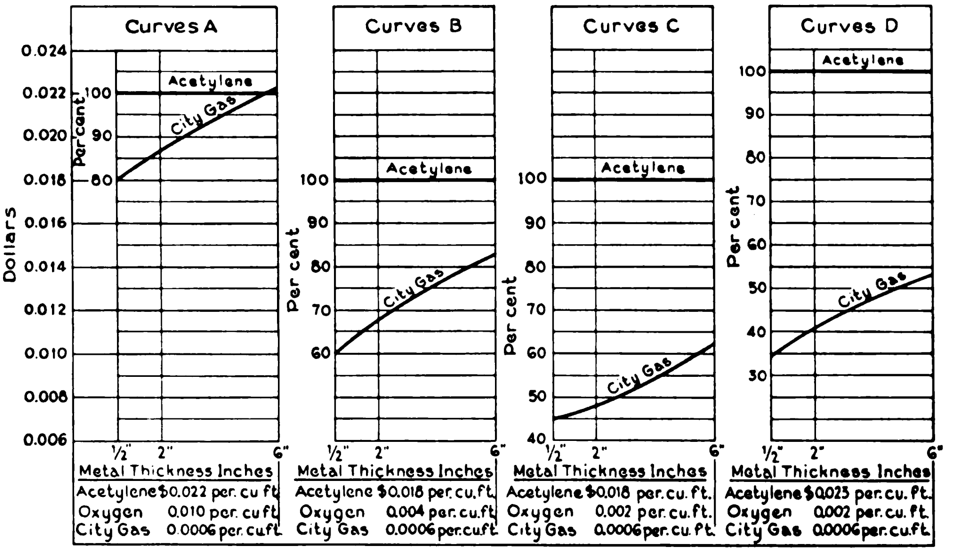

Fig. 26. Relative Total Oxygen and Fuel Gas Costs When Cutting with Oxy-city Gas Compared with Oxy-acetylene Taken as 100 Per Cent in Each Case

Curves *A*, *B*, *C*, and *D* are all based on the same gas consumption figures but, as indicated in the space below the curves, a different unit gas cost is used. By the use of these various unit costs, it is possible to translate gas consumption data into terms of relative costs which are more nearly representative of general conditions. It is of course obvious that due to the increased oxygen consumption which is experienced when cutting with city gas, the cost of oxygen per cubic foot is a vital factor in the ultimate attainment of economy. The economy obtainable through the use of city gas increases as the unit cost of oxygen decreases, as is clearly indicated by the curves. In Curves *D*, with oxygen at $0.002 per cu. ft., an average saving in total gas costs of approximately 60 per cent is indicated when city gas is employed. This is an extreme case however for while the Curves *D* are based on a reasonable price for acetylene, the exceptionally low price for oxygen is attainable only by those consumers who operate their own oxygen producing plants under ideal economic conditions.

Curves A

These curves are based on unit gas costs which generally prevail among those consumers who are buying dissolved acetylene and oxygen in tanks. They undoubtedly apply to conditions where the volumes of acetylene and oxygen used are insufficient to warrant the installation of generating equipment for either gas. The comparison is probably representative of general conditions and is therefore of greater general interest than the others in the group.

Curves B

This pair of curves is based on average unit gas costs which prevail when the consumer produces his own acetylene and oxygen in quantities large enough to justify consumer-plant operation. This comparison indicates the economies when cutting with city gas and manufactured oxygen compared with the use of manufactured acetylene and manufactured oxygen.

Curves C

In these curves the unit cost for acetylene is based on its generation by the consumer. The unit oxygen cost is based on its manufacture by the consumer in large quantities which permit the attainment of a low cost per unit especially when the by-products can be utilized.

Curves D

These curves are based on conditions which are more or less special. In them it is assumed that the quantity of acetylene used for cutting is insufficient to warrant its manufacture by the consumer, and that the price for dissolved acetylene would be approximately $0.023 per cu. ft. Other compressed gas conditions might be such that oxygen would be available at $0.002 per cu. ft. These figures are representative of conditions which actually exist at several industrial establishments and the values are therefore by no means hypothetical.

The relation which the cost of oxygen bears to the problem of fuel-gas substitution emphasizes the importance of a thorough knowledge of oxygen manufacturing economics. Increased oxygen consumption results in an increased expense which is directly proportional to such increased consumption when oxygen is being purchased. With the consumer-operated plant, the unit cost of oxygen decreases almost proportionately as the load increases. Therefore the possibility exists that where consumers cannot afford to use city gas because of the prevailing market price of oxygen, they might install an oxygen plant and realize a two-fold saving—first on the cost of oxygen direct and second through the use of a cheaper fuel gas.

Comparative Tests on a Laboratory Basis

A laboratory test to determine the relative value of various cutting fuel gases is difficult to conduct and is more difficult to interpret in terms of shop practice.

An economic survey can be made to advantage, however, and the results should conclusively indicate whether some change of fuel gas should be made and the overall costs that will be obtained in actual practice. In this way, and at comparatively small expense, the information secured will automatically take into consideration all the special circumstances existing at the plant.

Conclusions

Wherever conditions are such that city gas fulfils the economic requirements, its use results in marked savings. Acetylene has a distinct field of application; so has city gas; it remains for the user to determine which gas best meets his particular needs, all things considered. City gas is not a cure-all, neither will it completely displace acetylene as a cutting fuel.

The attainment of economy in using a fuel gas depends upon how closely results in practice correspond to expected gas consumption figures. Experience indicates that excessive pressures, oversize tips, and excessive preheating flames are the general rule. The desire for speed has more or less obscured the cost per inch cut. When a costly fuel gas is used, the loss through variation from expected gas consumption figures is proportionately greater than in the case where a cheap gas is employed. It cannot be denied that wastage and loss of gas do occur in this application, and that the financial burden which they entail is proportional to the price paid per unit quantity.

Comparative data of the cutting speeds of two gases cannot be directly translated into comparative labor-hours required for production. As shown in the time study in Table X, the elapsed labor-hours and the actual cutting labor-hours are quite different. Any advantage through increased speed of cutting depends more or less on this relation between elapsed time and actual cutting time. For this reason, relative labor cost figures have no general value outside of the particular test to which they apply.

As previously mentioned, little general value can be attached to data based on very special conditions such as apparatus used, cutting speeds, temperature of metal being cut, personal equation, unit gas costs, characteristics of city gas used, etc.

It is therefore apparent that there are so many factors involved in the proper selection of a cutting fuel gas which factors are closely allied with the problem of oxygen procurement that an empirical solution is not possible. Each problem must be solved as a separate proposition and the solution approached with an open mind.

(*To be continued*)

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Condensed references to some of the more important recent articles in the technical press, and to new books of interest to the industry, as selected by the General Electric Main Library.

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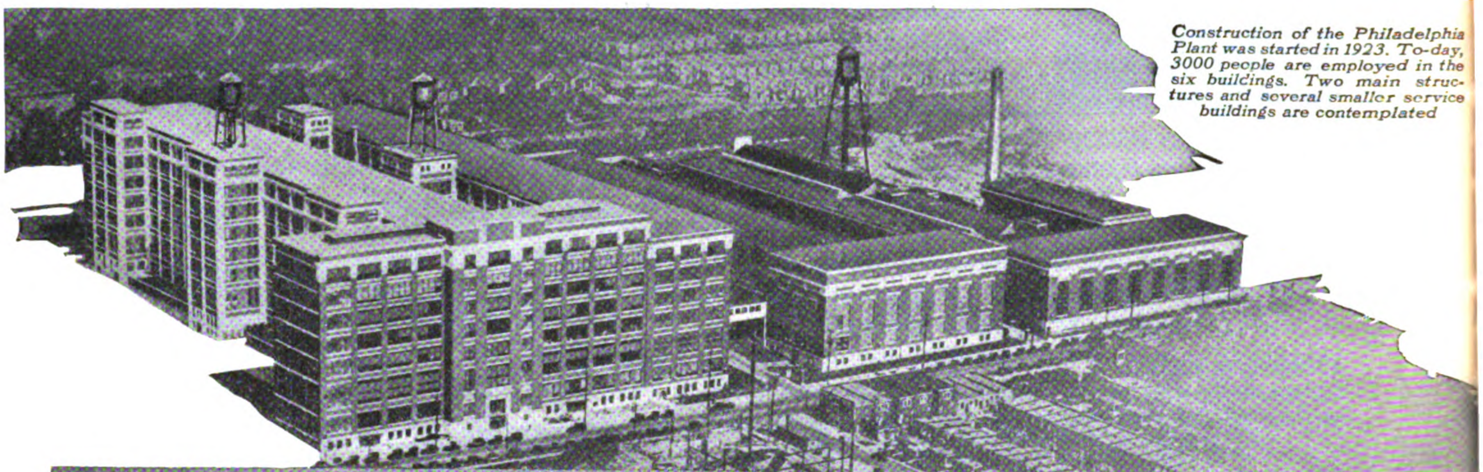
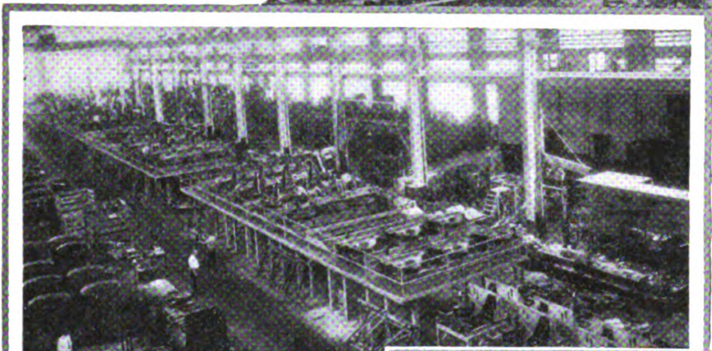
Every modern method for

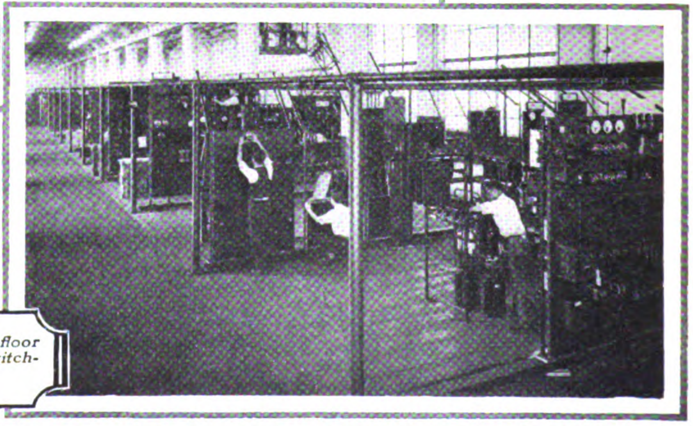


Construction of the Philadelphia Plant was started in 1923. To-day, 3000 people are employed in the six buildings. Two main structures and several smaller service buildings are contemplated



Partial view of the large oil circuit breaker assembly

One of the main drafting rooms



Part of the assembly floor devoted to automatic switching equipment

GENERAL

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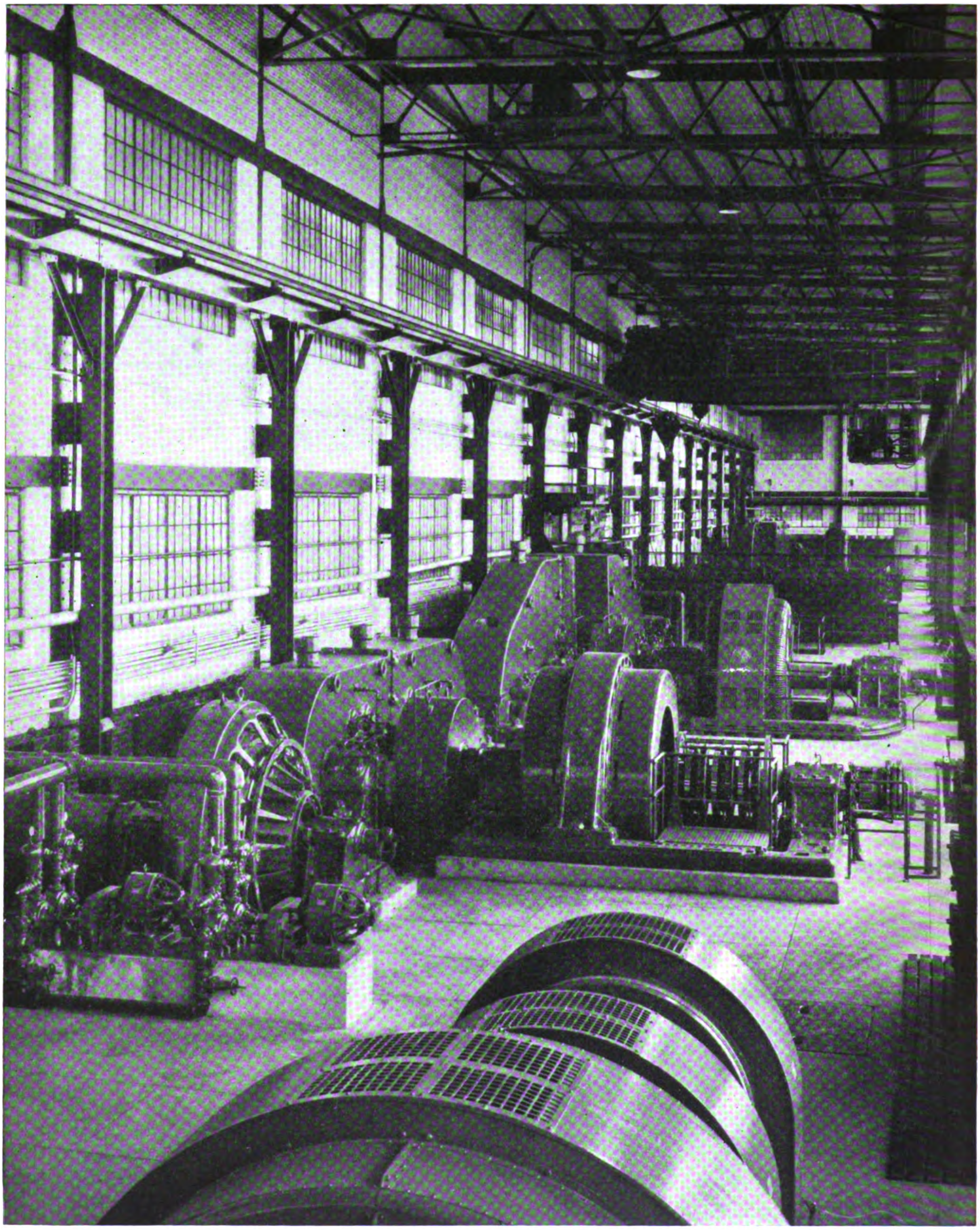

THREE TYPES OF MOTORS FOR ROLLING STEEL

Motor room of a continuous skelp and sheet bar mill. From left to right are shown a 2600-hp. 275/320-r.p.m. 600-volt D-c. motor, a 6700/5000/3320-hp. 500/375/250-r.p.m. 6600-volt 25-cycle Scherbius-controlled Induction motor, a 6500-hp. 187.5-r.p.m. 6600-volt 25-cycle Synchronous motor, and (in the background) a 4000-hp. 83-r.p.m. 6600-volt 25-cycle Synchronous motor. The Scherbius speed-regulating motor-generator set stands in the foreground

GENERAL ELECTRIC REVIEW

Vol. 31 June, 1928 No. 6

ELECTRIC DRIVE IN THE IRON AND STEEL INDUSTRY

The annual convention of the Association of Iron and Steel Electrical Engineers at Chicago, on June 25–29, will furnish the best opportunity of the year for collectively considering the many problems that are encountered in turning the wheels of the vast industry represented. As a contribution to the Convention, and for the information also of those interested subscribers who will be unable to attend, the editors of the REVIEW have set aside a portion of this issue for appropriate articles. In introducing them, it was believed that a review of the trend in the prominent divisions of the general subject would be desirable.

The present hand-to-mouth practice of buying steel requires that the mills be equipped to deliver a wide range of products within a single day, or sometimes within an hour; and to secure this necessary flexibility, they must be capable of operating over a considerable range in speed. For such drives, direct-current motors are available and also induction motors equipped with suitable speed-regulating equipment such as the Kraemer or Scherbius type. Which of these types of drives to use on a particular mill is a matter for individual study, but as a general rule it can be stated that where only one drive is involved the adjustable-speed induction motor should be given favorable consideration. On the other hand if a single mill requires several drives, as is the case with the modern continuous-strip mill, direct-current motors will usually be found preferable with respect both to first cost and to flexibility of operation.

The trend in direct-current drive of continuous mills has been toward the provision of one or more motor-generator sets to supply power to each individual mill, thus making available the advantage of the generator-voltage-control system of starting the mill motors. In an installation of this character, the motor and generator fields should each be separately excited and a reversing potentiometer type of generator-field rheostat may be employed, thereby permitting the excitation to be decreased to zero and then if desired built up again in either direction all without opening the field circuit or operating a reversing switch. Thus the mill operator, by controlling this one rheostat, can start the mill forward or backward, adjust its speed to the needs of the moment, or stop it.

Within the past two or three years, synchronous motors have been applied to main-roll drives with unqualified success. The starting characteristics of this type of motor have been so improved that they are now staisfactory for most types of mills. The primary reason for this application lies in the ability of the synchronous motor to improve the power-factor of the supply system. Compared with the induction motor, its efficiency is higher in the usual case and its first cost is lower particularly in large low-speed units. Thus for any constant-speed mill drive, the possibility of using a synchronous motor should not be overlooked.

The ventilation of main-drive motors and motor rooms has not been given sufficient attention. There seems to be a tendency to crowd the electrical equipment into just as small a space as possible; and in modern mills this concentration of power is making more and more evident the need for adequate ventilation.

All the losses that occur in the motors and other apparatus are converted into heat; and, while modern electrical apparatus is highly efficient, these losses are by no means negligible. For example, the motors and motor-generators supplying power to a 48-in. hot-strip mill have full-load losses that generate heat at a rate equivalent to that obtained from the perfect combustion of five tons of coal per 24 hours, and all this heat is liberated in the motor room. Of course the motors are not fully loaded all the time but the losses do not decrease in direct proportion to the decreased load.

Most motors will cool themselves satisfactorily provided the air around them is kept reasonably cool, the maximum desirable ambient temperature being usually considered as 40 deg. C. (104 deg. F.) If the motor room is large and high, with adequate openings for ingress and egress of air, and the apparatus in the room is not congested, natural ventilation may be sufficient. Even under these conditions, however, the comfort of the operators can be greatly increased and possibly the life of the apparatus lengthened if artificial ventilation is provided.

It is difficult if not impossible to give definite rules as to how to secure sufficient ventilation because of the varied installation and operating conditions encountered. In some cases, cool clean air is simply forced into the basement and allowed to rise through the opening within the base of each power unit. In other instances, enclosing shields are added to the motors so that the air forced up through the base is caused to traverse the machine in a definite path. This second method of cooling a machine is more efficient than the first, but both are open to the objection that the heated air from the machine is discharged into the motor room to the discomfort of the operators and the detriment of any apparatus which is not connected to the ventilating system. An ideal system of ventilation would force cleaned cool air into the apparatus and then discharge the heated air outside the motor room. In cool weather, the arrangement should make possible the discharge of a portion of the hot air into the room in order to maintain a suitable room temperature.

It is a comparatively simple matter to build synchronous or induction machines so that they will draw air in around the shaft and discharge it through an opening in the frame to a duct that may be either above or below the machine. Direct-current machines, however, do not lend themselves quite so readily to such treatment, but a volute housing of sheet metal and supporting angles has been developed which quite satisfactorily collects the heated air and discharges it at the desired point. This housing encloses the magnet frame and extends radially inward close to the periphery of the armature, at the extreme end of the winding.

So-called anti-friction bearings have been used in a few main-roll motors but it is difficult to see any justification for them in such service. The maintenance of sleeve bearings is practically negligible because the bearing pressures are always low since the size of the bearing is determined by the diameter of the shaft necessary to transmit the torque rather than by the weight to be carried. The sleeve bearings are usually of the split ball-seated type and can readily be removed for repair or replacement if necessary. A like operation on an anti-friction bearing at the coupling end of a large motor first necessitates lifting out the armature and pressing off the coupling, and afterward replacing them. So far as can be determined there is little if any difference in the running friction losses of the two types of bearings in this service. The starting resistance of anti-friction bearings is of course lower but this is of no moment in the case of a main-roll motor.

For auxiliary drives, it has been customary to employ series motors except where there was danger that the motor would reach an excessive speed during the lightly loaded portions of its operating cycle. One reason for the preference for the series motor is the greater torque it makes available for a given percentage overload in armature current. However, the series motor does not possess as great an advantage in this respect as is commonly supposed. For example, compare a series motor with a compound-wound motor which has sufficient shunt field to limit its no-load speed to 150 per cent of its full-load speed. While both machines of course exert the same torque at 100 per cent armature current, at 150 per cent current the series motor develops but 10 per cent more torque than the compound-wound motor and at 200 per cent current the torque of the series motor exceeds that of the compound motor by only 20 per cent.

Another reason for favoring the series motor is that it does not require as many leads from the control panel, for it has no shunt field. This absence of a shunt field also prevents all possibility of shunt field failure. On the other hand, there is small likelihood of such failure in a compound machine if the shunt coils are wound with asbestos-insulated wire.

The compound motor has several advantages which are now bringing it into increasing use for almost all kinds of auxiliary drives. The most important advantage is that this type of motor, unlike the series motor, will not run away at light loads, straining both itself and the mechanism to which it is connected. Reversing tables have usually been driven by series motors and the friction load of the journal bearings of the table has been sufficient to hold the speed to a safe value. But now, some tables are equipped with roller bearings. With such a table, a motor of fairly large capacity is still required for rapid acceleration, on account of the inertia of the table, but if a series motor is used it may reach a very high speed and when plugged the whole mechanism may be severely strained. The obvious choice is to employ a compound-wound motor and thus reduce the maintenance expense of both the mechanical and electrical equipment.

Another tendency of today is to make the operation of auxiliaries as nearly automatic as possible. In those cases where a drive must be accurately stopped at a predetermined point in its cycle, it is advantageous to employ dynamic braking and the compound motor can be dynamically braked much more easily than can the series motor. The shunt field of the compound machine is always excited, whereas the series field of a series motor must be separately excited during the braking operation and this requirement complicates the control and delays the action.

Regardless of whether the auxiliary motors be of the series or the compound type, they are usually built totally enclosed with the idea that they will be installed where they will be exposed to much dirt and dust. Several mill operators have discovered however that it pays to force clean cool air into those motors which are subject to the hardest service. Not only does this ventilation keep the motor cooler and thus permit it to carry greater loads, but it prevents the infiltration of dirt and dust by reason of the air pressure within the motor being somewhat above atmosphere.

The effect of forced ventilation in decreasing the maintenance cost of auxiliaries is so marked that it seems probable it will be used in many installations, particularly as standard motors are already available with threaded openings from which the pipe plugs may be removed and air pipes attached.

The performance records of anti-friction bearings in steel-mill auxiliary motors seems to justify their application in this service, and standard motors for this purpose are now being built with anti-friction bearings of several types.

Magnetic control equipments using definite time-limit acceleration are now in widespread use for auxiliary drives. With this system of control, the motor is always accelerated within a definite time, unless of course the load is so great as to trip the overload relay. When the drive is stiff, as on a cold Monday morning, the current peaks will be high, but as soon as the drive becomes free the accelerating peaks will drop to their normal value thus preventing the electrical and mechanical equipment's being unnecessarily punished throughout the day.

The benefits of automatic control are not confined to auxiliary drives, however. A great many motor-generator sets are being installed with pull-button automatic switching equipment. The starting of even a large synchronous motor-generator set is now accomplished by the operator's merely pulling a single control switch, after which the various devices function entirely automatically under the control of suitable relays until the set is on the line.

Extensive applications are also being made of automatic reclosing feeder equipments on the 250-volt direct-current circuits that supply steel-mill auxiliaries. By automatically restoring service just as soon as the cause of the interruption has been removed, these equipments save the moments that are valuable in many phases of mill operation.

The Iron and Steel Industry is by far the largest industrial consumer of electric power, not altogether because of its many single drives of thousands of horsepower but because each one of these drives is in a line of production that requires the services of hundreds of auxiliary motors of widely varied sizes. Taken in its entirety a steel mill is in reality a gigantic machine, in the design and operation of which the electrical engineer deserves a large share of credit.

H. A. Winne

Electric Drive for the Bethlehem Wide-flange Structural Mill at Lackawanna

Blooming, Roughing, and Finishing Mills—Five Driving Motors—3500 to 17,150 Hp. (Max.)—Three Flywheel Motor-generator Sets—Power Supply—Control of Operation

By F. D. EGAN
Electrical Superintendent, Lackawanna Plant, Bethlehem Steel Company

IN April, 1927, the Bethlehem Steel Company put in operation at its Lackawanna plant one of the largest structural mills in this country. The mill is designed to roll wide-flange columns, girders, and beams. It is comprised of a 54-in. reversing blooming mill; a roughing mill with main stand of horizontal and vertical rolls, and supplementary stand with horizontal rolls only; and a finishing mill similar to the roughing mill and with the same layout of rolls.

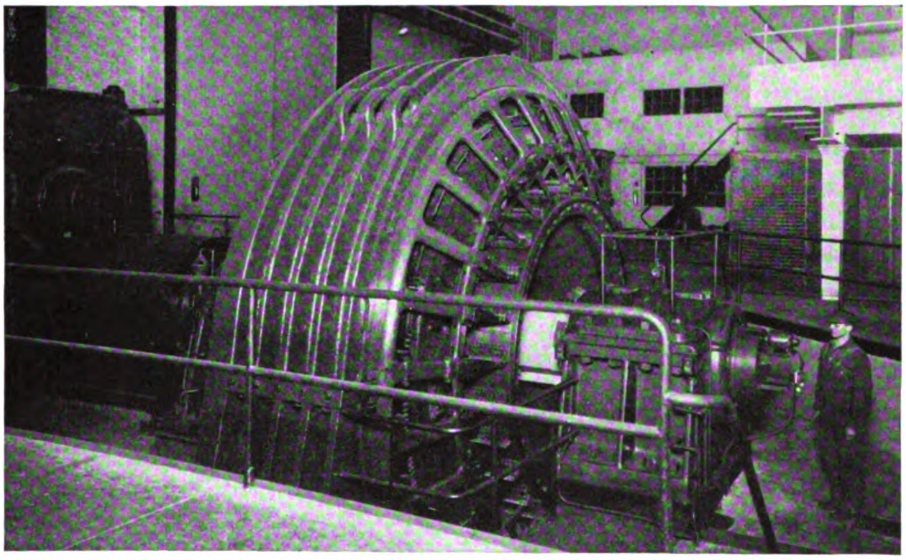

Fig. 1. The 7000-hp. (17,150-hp. Max.) Single-armature Reversing Blooming-mill Motor

Blooming Mill

The 54-in reversing blooming mill is designed to roll large ingots (up to 18 tons or more) into blanks to feed the roughing mill next in line. It is driven by a 7000-hp. (continuous) 50 deg. C. rise force-ventilated 750-volt, 40/80-r.p.m. direct-current motor. At the circuit breaker setting this motor develops 17,150 hp. at 37½ r.p.m., which corresponds to a torque of 2,400,000 lb. at one foot radius. This motor is shown in Fig. 1. It is 21 ft. wide, 22 ft. long, weighs 244 tons, and is the largest direct-current motor ever built by the General Electric Company. The mill-end bearing is 30 by 60 in., and the pedestal is fitted with a thrust collar for taking the end thrust of the mill. The bearing pedestal has eight foundation bolts in the sides parallel with the shaft, and four foundation bolts across the front next to the coupling. These latter bolts are provided to enable the pedestal to withstand any upsetting force caused by a heavy thrust from the mill.

Ventilating air is forced into the enclosed coupling end of the motor and thence through the motor and out across the commutator.

Direct current is furnished for the single-armature blooming-mill motor by the flywheel motor-generator set shown in Fig. 2. The wound-rotor induction motor is rated 5000 hp., 6600 volts, 375 r.p.m., and is arranged for speed variation by means of a slip regulator, Fig. 3, in order to make effective use of the energy stored in the flywheel. The two shunt-wound direct-current generators connected in parallel are each rated 3000 kw., 750 volts. The cast-steel flywheel is 15 ft. 3 in. in diameter, weighs 50 tons, and at synchronous speed the set has a total stored energy of 187,000 hp.-sec. The set is designed as a self-ventilated unit, but the spare mill-motor blower is so connected that forced ventilation may be used on the set if desired, and also used for ventilating the mill motor in case of a breakdown of the regular ventilating equipment.

Excitation is furnished by a four-unit motor-generator set, Fig. 4, made up of a 200-hp., 750-r.p.m. 220-volt induction motor direct connected to three (75, 50 and 15-kw.) 250-volt direct-current generators. The 75-kw. unit furnishes excitation for the blooming-mill motor, the 50-kw. unit excites the fields of the two 3000-kw. generators, and the 15-kw. unit (which is compound wound and self-exciting) furnishes power for the control circuits and excitation for the fields of the other two exciters.

The speed of the blooming-mill motor between standstill and 40 r.p.m. is controlled by varying the generator voltage, and between 40 and 80 r.p.m. by varying the motor-field excitation. The generator voltage is varied by changing the resistance in the generator-field circuits through contactors actuated by a foot-operated master switch. The motor-field excitation is varied by changing the resistance in the field of the motor-field exciter by means of contactors actuated by the same master switch. The resistances are so proportioned that smooth acceleration takes place throughout the range. The master switch reverses the motor by reversing the excitation of the generators, the voltage having been dropped so that there is very little field current at the instant of reversal. The sequence of contactor operations is fixed; and the minimum time that must elapse between steps is so fixed that the operator cannot cause trouble by any improper manipulation of the master switch.

The slip regulator previously mentioned in connection with the induction motor on the main set forms a Y connection for the rotor windings. Resistance in the form of a bicarbonate of soda solution is interposed in the three legs and the distance between the submerged electrodes in each leg is varied by a torque induction motor. This motor receives its power from current transformers in the leads to the motor whose slip it regulates. The regulator is subject to hand control through an auxiliary counterweight that is handled by a small auxiliary motor controlled by a push-pull button on the switchboard. Whenever the set is shut down, the maximum resistance is automatically inserted in the secondary of the induction motor in readiness for the next start.

When the blooming-mill motor encounters a heavy load, the load is transferred back to the induction motor and the consequent current inrush brings about an increase in the resistance of its secondary circuit through the action of the slip regulator. A drop of 10 per cent in speed allows the set to give up 35,000 hp-sec. of stored energy, or enough to drive the blooming-mill motor at full rated load (7000 hp.) for five seconds. When the load is relieved, the secondary resistance is reduced and the induction motor brings the set back to speed thus replenishing the stored energy. When the blooming-mill motor is quickly brought to rest, its stored energy is converted to direct current which reacts on the generators and causes them to "motor" thus assisting the induction motor in restoring the speed.

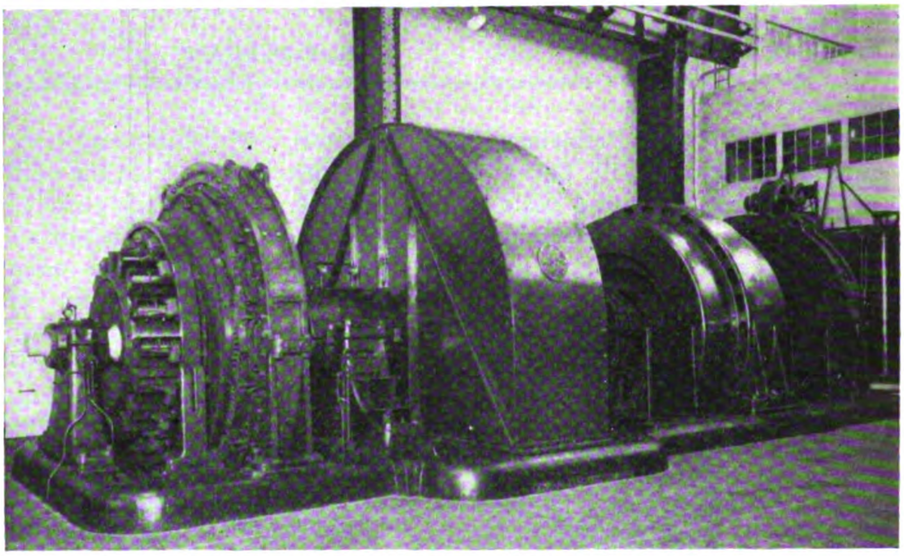

Fig. 2. The Flywheel Motor-generator Set which Furnishes Direct-current Power to the Motor Shown in Fig. 1

Unless otherwise provided for, the speed fluctuations of the flywheel set would bring about corresponding voltage fluctuations in the output of the generators. A small generator with separately excited fields is mounted in a bracket and geared to the shaft of the set; and its output acting through a voltage

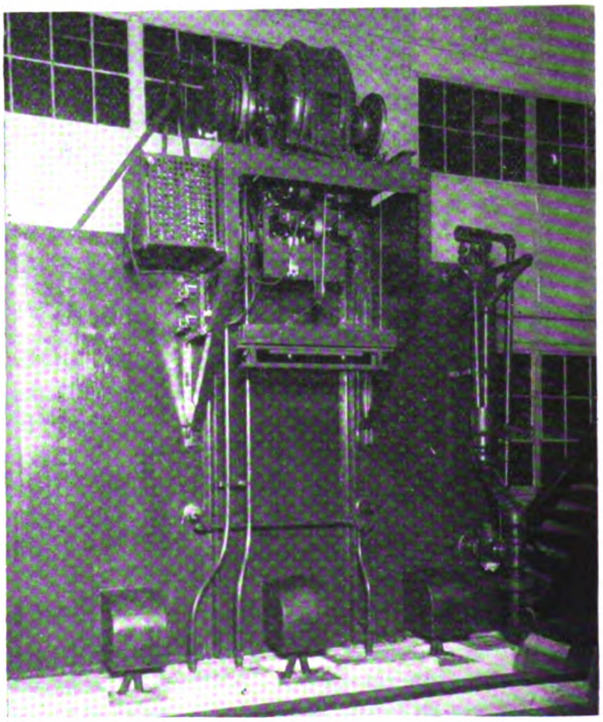

Fig. 3. The Slip Regulator for Controlling the Speed of the Wound-rotor Induction Motor of the Set Shown in Fig. 2

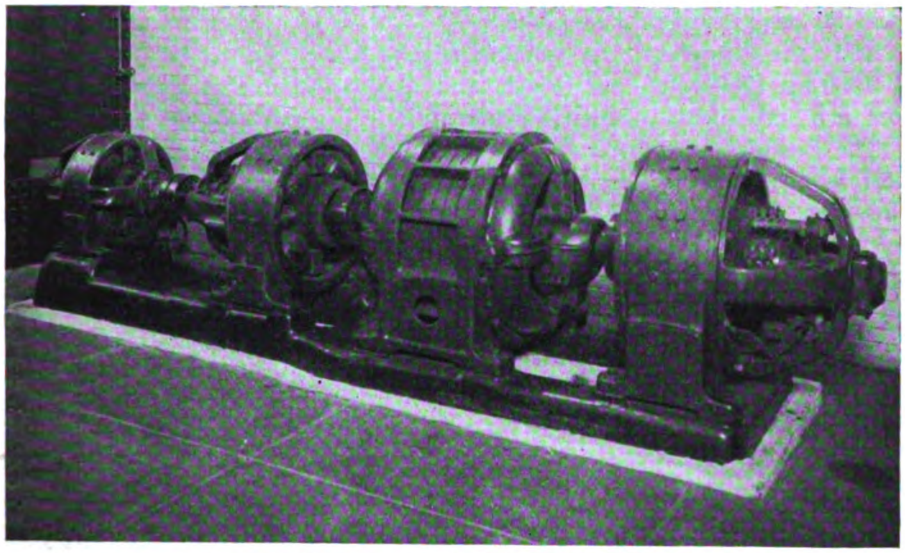

Fig. 4. The Four-unit Motor-generator Set for Furnishing Excitation and Control Current for the Blooming-mill Drive Equipment

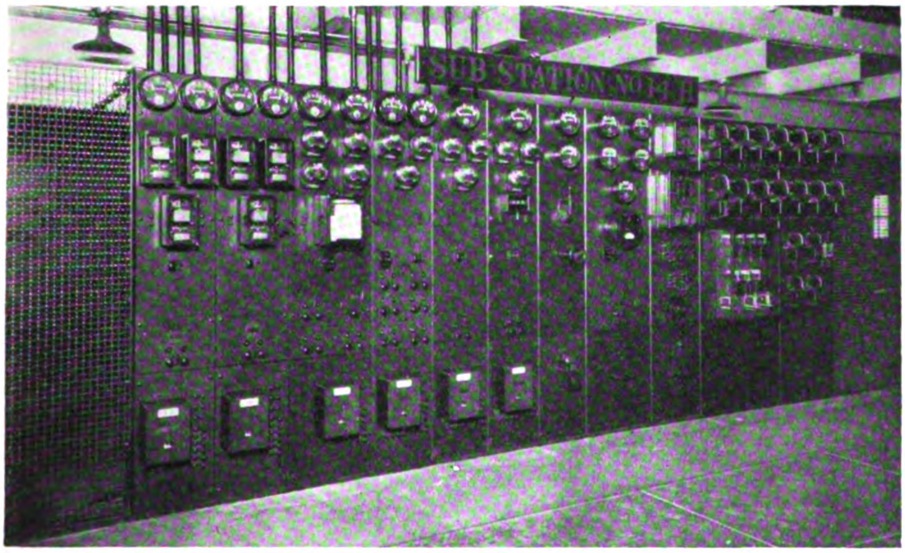

Fig. 5. The Panels for Controlling the A-c. Breakers in the Circuits Feeding the Blooming-mill Drive

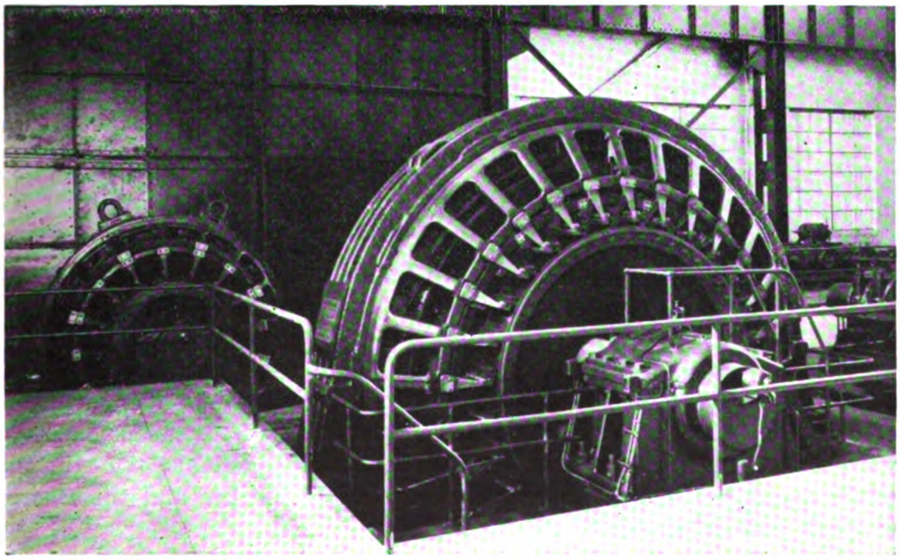

Fig. 6. The Roughing-mill Main and Supplementary Motors of 7000 and 1500 Hp. (14,900 and 3500 Hp. Max.) Respectively. The finishing mill is driven by duplicate equipment

regulator automatically controls the field of the exciter for the generators. By this means a drop in speed of the motor-generator set is met by an increase in the field excitation of its generators, thereby holding constant voltage on the d-c. power circuit except as it may be purposely varied by means of the master switch.

The switchgear in the blooming mill is connected through oil breakers to two incoming lines from the main plant loop system, and consists of forward and reverse oil circuit breakers for the motor driving the flywheel set, an oil circuit breaker for the auxiliary transformer bank, and two outgoing radial-feeder oil circuit breakers to the motors driving the bloom shear pumps. The panels controlling the a-c. breakers, Fig. 5, are lined up with the panels for controlling the main motor and are nine in number. In the operator's pulpit is a pedestal on which is mounted the indicating instruments and control switches for the main d-c. circuit breaker.

The oil circuit breakers are of 1200-amp. rating and have a one-second carrying capacity of 100,000 amp., a five-second capacity of 60,000 amp., and a ¼-second interrupting capacity of 52,300 amp. at 6600 volts. Lightning protection is provided on both the incoming and outgoing lines by three-phase 15,000-volt oxide-film lightning arresters. The choke coils of 1¼-in. diameter solid-copper conductor are rated 1200 amp. at 15,000 volts.

Roughing and Finishing Mill

The roughing mill and finishing-mill drives are duplicates and only one will be described. The main rolls are driven by a 7000-hp. (continuous) 50 deg. C. rise 750-volt 65/100-r.p.m. motor; and the supplementary rolls are driven by a 1500-hp. 750-volt 65/225-r.p.m. motor. Both motors are force-ventilated and are shown in Fig. 6. The greater speed range of the supplementary motor is necessary to compensate for various ratios between the main-roll and supplementary-roll diameters. The circuit breaker setting will allow the main motor to develop 14,900 hp. at 62.5 r.p.m. and under like conditions the supplementary motor can develop 3500 hp. at 61 r.p.m.

The motor-generator sets for the roughing and finishing mills, Fig. 7, are similar to that for the blooming mill except that they have three generators instead of two, the third being required for the operation of the supplementary mill motor. The sets are over 50 ft. long, 17 ft. wide and weigh 250 tons each. The stored energy of each set at synchronous speed is 190,000 hp-sec.

Each mill requires a five-unit induction motor-generator set for excitation. The high-reactance squirrel-cage motor, rated 150 hp. 750 r.p.m. 220 volts, is thrown directly on the line for starting. One 50-kw. 250-volt direct-current generator excites the fields of the 7000-hp. motor and a 15-kw., 250-volt unit excites the fields of the 1500-hp. motor. The fifth unit is a 1½-kw. generator used for field compounding. A two-unit induction motor-generator set common to the roughing and finishing mills furnishes a constant-potential circuit for control operation and for exciting the fields of the exciters.

Both the main and supplemenatry motors are controlled from a single foot-operated master switch; and once they are adjusted they operate at the same relative speeds throughout the entire speed range. A pedestal having the necessary speed indicators, ratio meter, and control switches is mounted in the pulpit adjacent to the operator.

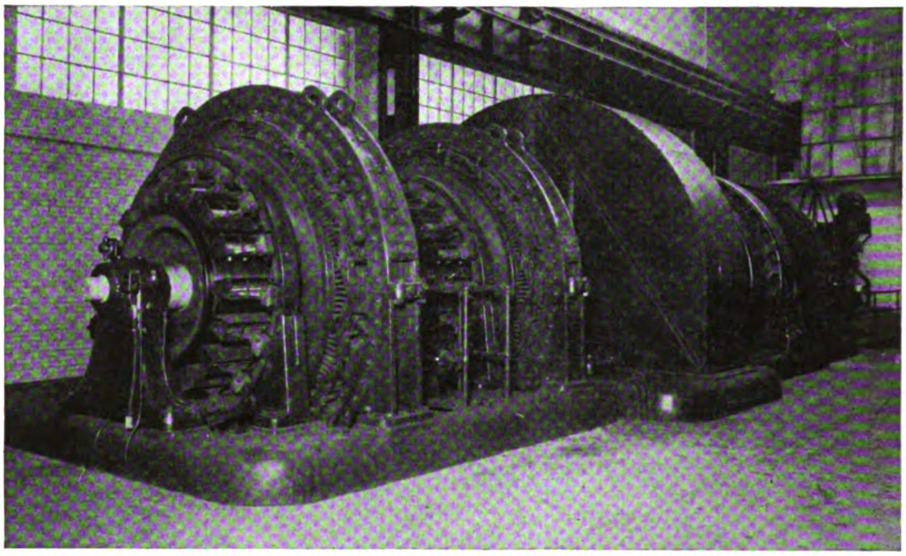

Fig. 7. The Four-unit Motor-generator Set That Supplies Direct-current Power to the Roughing-mill Motors. A similar set furnishes power to the finishing-mill drive

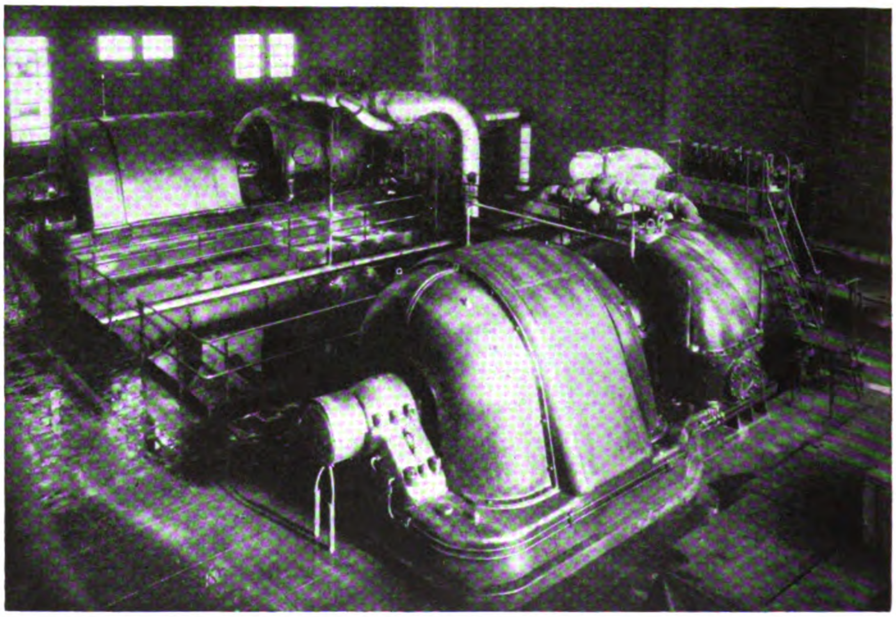

Fig. 8. The Turbine-generator Units, 7500 and 12,500-kw. Capacity, which in Parallel with Purchased Power Supply the Mill

Power Supply

A portion of the power supply at this plant is purchased, and the remainder is generated by a steam-turbine station, Fig. 8, that operates in parallel with the power company's system. Purchased power is received at 60,000 volts, 3 phase, 25 cycles, and is stepped down to 6600 and 2200 volts through two banks of 15,000-kv-a. transformers for 6600-volt service and a single bank of 6000-kv-a. for 2200-volt service. The steam station contains two turbine-generator units, one of 12,500 kw. and the other of 7500 kw.

The transformer banks in the public-service company's lines furnish the block load of power at about

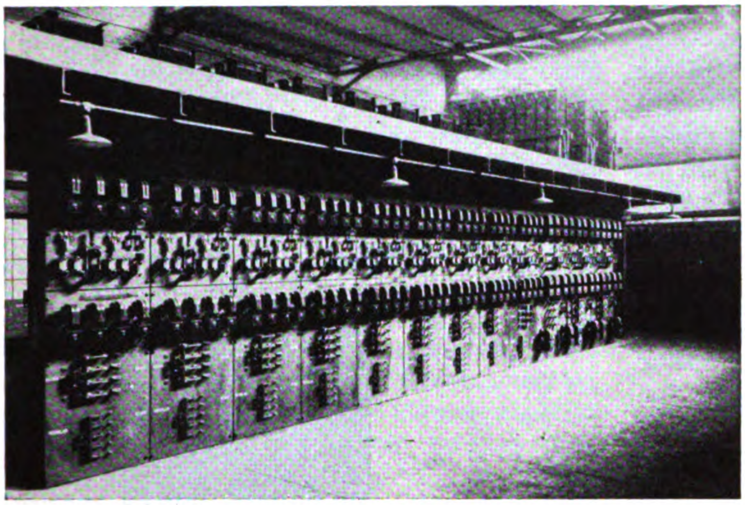

Fig. 9. Eight 300-amp. Auxiliary Control Panels and a Spare, and Two 600-amp Panels and a Spare, with the Resistors Mounted Above

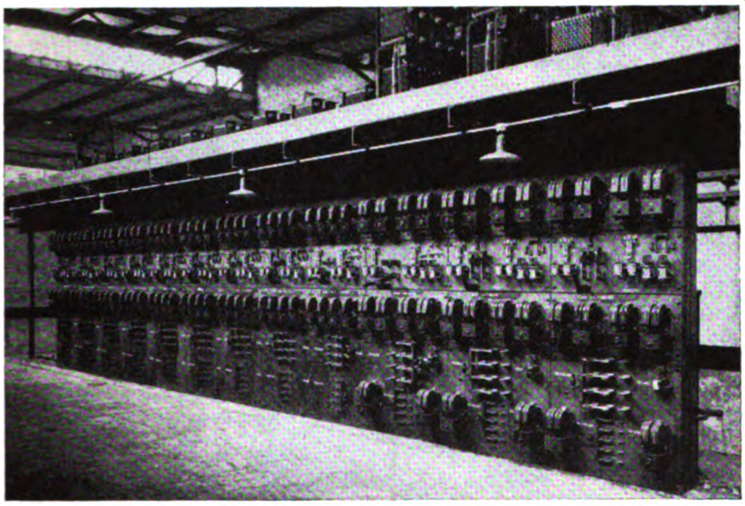

Fig. 10. The Same Control Board as Shown in Fig. 9, but Viewed from the Other End

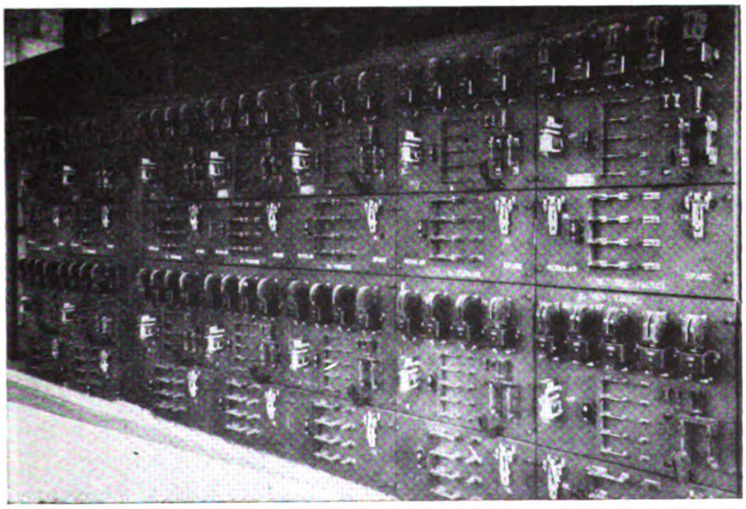

Fig. 11. Motor Control Panels for the Furnace Doors, Valves, etc., with the Equipment Mounted in Two Rows

85 per cent load-factor; while the steam station, controlled by a peak-taker, furnishes the necessary peak-power supply. At times the load drops so low that the generators of the turbine units act as synchronous condensers.

The transmission system consists of four circuits from the transformer station feeding into one corner of a loop system, with the steam station feeding into another corner at a point diagonally opposite. The different substations and mill-motor rooms are fed from this loop system, the power feeding into the bus and out again to the next station. This arrangement gives power supply to any substation from two opposite directions and assures a positive service.

Two circuits supply the blooming mill motor room and four circuits feed the roughing and finishing mill motor rooms. Power for the mill auxiliaries is supplied from a substation in the finishing mill motor room. Three 400-kv-a. 6600/220-volt transformers supply the a-c. auxiliary motors; and two 1000-kv-a. synchronous motor-generators supply the d-c. auxiliary motors. The d-c. bus is also tied to the d-c. loop-transmission system.

There are 283 motors and generators aggregating 84,310 equivalent full-load horsepower involved in the driving of this structural mill; and the motor room that houses the roughing and finishing mill motors, together with the equipment for driving an adjacent 35–28-in. mill contains 85,000 hp. in motors and generators.

Auxiliary Control

The control panels are mounted in groups of eight or more in well-designed brick control houses and have a clearance of at least 48 in. back and front. Where possible, the panel houses are located near or in the motor rooms for the main mill drives, which arrangement makes possible the use of washed air for ventilation. Figs. 9 to 12 show front views of a group of 12 controls, nine of which are rated at 300 amp. and three at 600 amp. The steel framework to which the controllers are attached also provides a support for the resistance banks, with provision for walking in front of and between each set of resistor units. This arrangement gives easy access and good ventilation to the grids, and facilitates neat wiring between the controllers and resistors.

With each group of control panels there is mounted a spare controller in line with the regular controller boards, and in case of trouble this spare board can be cut in to serve in place of any of the regular boards by throwing the four-pole main and five-pole master switches on the base of the regular controller from the left or "regular" position to the right or "spare" position as shown in Fig. 13.

One of the master switches is shown in Fig. 14. These switches are mounted at a 50-deg. angle on a receptacle box, which has two rows or sets of female receptacles. The switches can be removed by releasing two thumb-nuts and lifting the switches from the receptacle. Two sets of receptacles are provided in each box, the set at the left is connected to the middle of the five-pole switch on the regular control board, and the set at the right is connected in parallel with the right-hand receptacles in the other boxes, and the entire group to the spare side of the five-pole control switch on the spare panel.

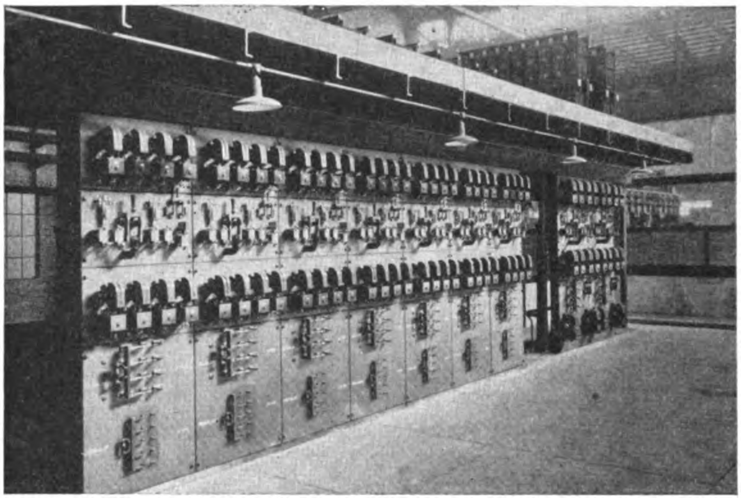

Fig. 12. Another Group of Panels for Controlling Auxiliary Drives. The transfer switches on these panels are thrown to the "Regular" position

The master switches and receptacles are well designed, rugged and heavy, and the switches can be mounted on 7¾-in. centers, which gives the operator easy reach over a bank of eight. In front of each master switch is mounted a safety switch which can be used to open the control circuits to their respective boards in case it is necessary for any of the repairmen to oil or repair the drives between the rolling of the bars.

The control panels are all 90 in. high, the 150 and 300-amp. panels are 32 in. wide and the 600 and 900-amp. panels are 40 in. wide. The apparatus on all the boards is lined up both vertically and horizontally and presents a symmetrical appearance.

The panels like that in Fig. 13 are equipped with four directional contactors that operate in pairs and are in series with a line contactor. These place three series breaks in the motor circuit and open both sides of the line. Three additional accelerating contactors are used to cut out the three steps of resistance and four magnetic-time relays give fixed-time acceleration and plugging. In each side of the line is a calibrated, instantaneous, overload relay so connected as to open the undervoltage relay which in turn opens the control circuit of the panel. These overload relays can be reset only by bringing the master switch to the off position. In addition to them, the equipment includes a double-pole knife switch, provided with a lockout device for the main line; a double-pole fused knife switch for the control circuit; a four-pole double-throw knife switch for transfer of the motor lines; a five-pole double-throw knife switch for transfer of the control wires, and an ammeter shunt connected in the armature circuit of the motors.

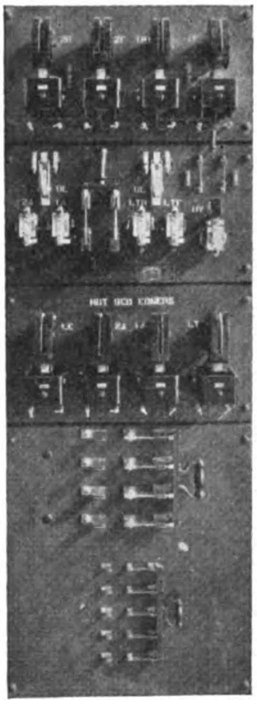

Fig. 13. A Close-up of One of the Auxiliary-control Panels with Its Transfer Switches Thrown to the "Spare" Position

Operation of Control

The operating coil of the line contactor is in series with the operating coils of the directional contactors. This contactor opens both sides of the line when the master switch is in the off position. On overload, voltage failure, or opening of the safety switch at the master switch, all these contactors will open; thus making three breaks in series in the motor circuit.

The plugging relays are connected for time delay and are normally open, their operating coils being in series across the line. If the master switch is thrown to the forward position, the forward contactors close, which causes the motor to start with the total resistance in the motor circuit. One of the forward contactors short circuits the operating coil of the first accelerating relay, which at the end of a set time interval makes a circuit, closing the first accelerating contactor. This in turn short circuits the coil of the next accelerating relay and after a time delay the armature of this relay will drop, closing its contacts and energizing the second accelerating contactor, etc.

The relays are simple, strong, and rugged in construction and have no pins, dash pots, etc., to corrode and prevent the devices from functioning in accordance with their settings. The friction is a very small percentage of the operating torque. The contactors are quick in action and have sufficient blowout capacity to interrupt high currents without burning their contacts or arc chutes.

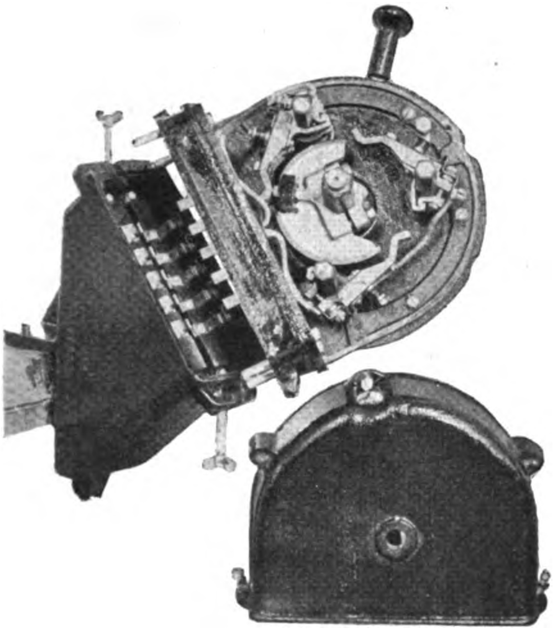

Fig. 14. Master Switch with Cover Removed and with Switch Partially Separated from Its Base to Show How a Replacement Can be Readily Made

The resistance banks were built at the steel plant, and consist of cast grids assembled in rolled-steel frames. Only one size of grid is used in each frame; and with terminals equally spaced each size grid bank is then interchangeable with one of the same number in any part of the plant. The grid banks are connected in series or parallel in accordance with the resistance required. Only four sizes of grids are used, and with this arrangement of frames they take care of practically all sizes of motors up to 500 hp. direct-current.

The set of one-line schematic diagrams in Figs. 15, 16 and 17 shows the wiring of two regular control panels and a spare panel, connected to their respective motors and master switches, and the change necessary to transfer from the regular control panel to the spare panel and from the regular master switch to the spare master switch. This scheme can be used with any number of panels but three only are shown for simplicity. With the switches thrown as indicated in Fig. 15, the regular master switch No. 1 operates through the left-hand row of receptacles in box No. 1, over the regular master cable, through the control transfer switch to panel No. 1, which controls motor No. 1 through the motor transfer switch. The equipment to control the second motor operates similarly and the spare panel is idle.

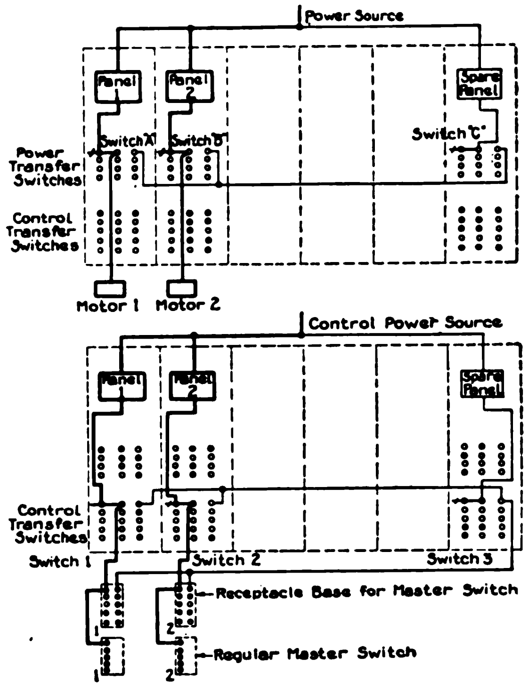

Fig. 15. Control Scheme Layout—Normal Operation. Regular panels, master switches and lines in use. Spares idle

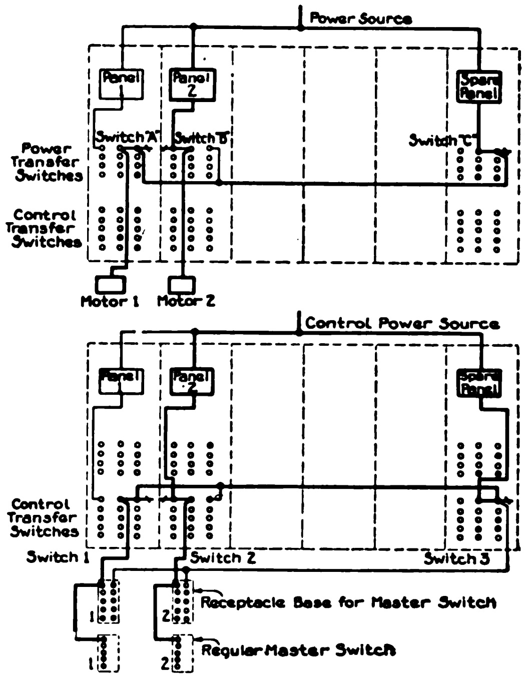

Fig. 16. Emergency Operation. Spare panel and spare lines connected to motor No. 1

If panel No. 1 becomes inoperative, the electrician throws the motor transfer switch and the control transfer switch to the right, Fig. 16. The regular master switch now operates motor No. 1 on the spare panel. When the trouble has been eliminated, the switches on panel No. 1 are thrown to the left and the motor is then controlled from regular panel No. 1, leaving the spare panel free for use in the next case of trouble.

Assume that trouble is experienced with the regular control cable No. 1. Removing the regular master switch No. 1 from the receptacle and replacing it with the spare master switch, then throwing the control transfer switch on the spare panel to the right and the motor transfer switch on panel No. 1 to the right, connects the spare master switch over the spare cable with the spare panel to operator Motor No. 1, Fig. 17.

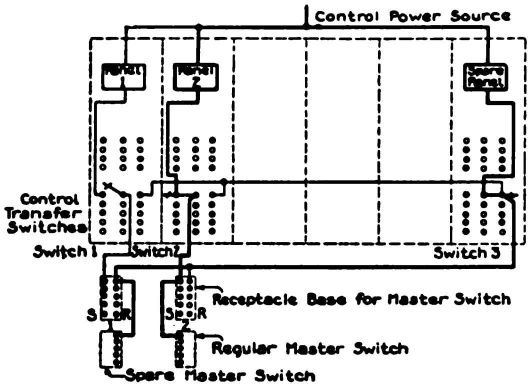

Fig. 17. Emergency Operation. Spare master switch in place. Panel No. 1 and normal lines for motor No. 1 completely disconnected

The master switch is arranged for one-point control, and for connecting the master switch and the safety switch to the control panel a six-conductor multicolored cable is provided.

The control cable has red, white, blue, green, black, and yellow covered wires. The red wire is used only between the control switch and panel and safety switch, the white wire from the safety switch to the master switch, and the blue wire from the master switch to the undervoltage relay. These are the three "hot" wires. The green and the black wires are directional, the green closing the forward and the black the reverse contactors. The yellow is a spare wire.

The motor leads from the center of the double-throw four-pole switch on the panel are numbered from 1 to 4 from the top down. They come out of the condulet cover in the same order and connect as follows: No. 1 to the top armature wire, No. 2 to the bottom armature wire, No. 3 to top field wire and No. 4 to bottom field wire at the motor. The No. 4 wire is a direct line through the line contactor and is connected to the grounded side of the plant system. This prevents voltage strain from grounding on the motor fields and brake coils. All wiring and connections on the different panels are symmetrical.

Fig. 18 is a schematic layout of the 54-in. reversing-mill tables, each driven by two 100-hp., 480-r.p.m., 230-volt series-wound motors. The motors are connected to the same shaft and are electrically connected in series to one control panel. There are two such tables, one in the front and one at the back of the mill. The forward motion of the front table and the reverse motion of the back table are controlled by one master switch *A*. The reverse motion of the front table and the forward motion of the back table are controlled by a second master switch *B*.

With a bar on the front table the operator throws the master switch *A* forward, which operates the front table forward and carries the bar into the rolls. When it has passed through the rolls the bar drops out on a dead back table, nosed in close ready to come back. The operator then throws master switch *A* to reverse, which allows the front table to drift to a stop and starts the back table in the reverse direction, carrying the bar back into the rolls and when through the rolls it drops out on a dead front table. This keeps the bar close to the mill, runs the tables about 50 per cent of time and avoids plugging. When the bar is rolled, the master switches *A* and *B* are thrown in the same direction, which operates both tables in the same direction and carries the bar away from the mill.

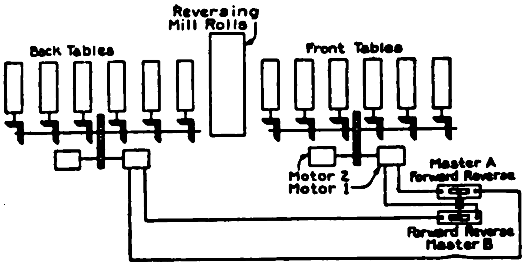

Fig. 18. Schematic Layout of the Mill and Elementary Connections of the Master Switches

The permanent installation of spare equipment obviously increases the first cost slightly, but a review of the advantages shows that the expense is justified. This system practically eliminates all delays due to failure in any part of the controller. In case of trouble, the maintenance man, by the simple operation of throwing switches, quickly provides a new control panel, resistor, and set of control cable leads. He can then locate the trouble and remove it without delaying the mill. Also, when inspecting panels the maintenance men may cut out any single panel and change the tips or operating coils or adjust the relays without interfering with the mill schedule in any way.

The New 84-in. Tandem Plate Mill of the Lukens Steel Company

General Description of Plant and Products—Electrical Equipment—Roughing and Finishing Mills—Heat-treating and Charging Operations—Finishing Processes in Plate Production

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IN 1927 the Lukens Steel Company put into operation an 84-in. plate mill, replacing a mill of the same width built in 1871. The new mill is of the Tandem type, with a 2-high reversing rougher, and a 4-high reversing finishing stand. It is designed to produce plates, either blue-annealed or black, in thicknesses ranging from 0.109 in. to ½ in., and in widths up to 72 in., and lengths up to 35 ft. Plates are rolled from either bottom-poured ingots or slabs, depending upon weights of finished pieces. Slabs and plates are handled mechanically through all the successive steps of heating, rolling, annealing, cooling, inspecting, shearing, weighing, and shipping. One exception to this is the "sketch" shearing, which is done by hand.

The Lukens Steel Company operates, in addition to a 48-in. universal mill, four mills for producing sheared plate, *viz.*, a 206-in. (the largest plate mill in the world), a 140-in., a 112-in., and the 84-in. mill here described. The average gauge produced on this mill is naturally comparatively light; therefore the problem of design was resolved largely into a matter of providing for the handling of a large number of light pieces. With this in view, the mill was laid out very compactly, with all equipment geared to as high a speed as practicable. The general design and installation was handled by the United Engineering and Foundry Company. A diagrammatic plan of the equipment is shown in Fig. 1.

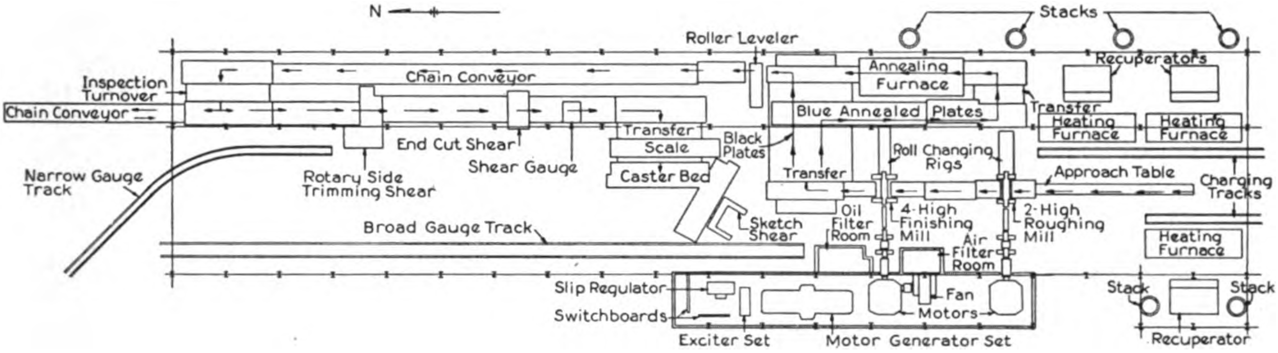

Fig. 1. Plan of the 84-in. Mill of the Lukens Steel Company, Showing Location of Units and the Line of Production in the Mill

Electrical Equipment for Main Roll Drive

Electrical equipment for the main drive of the mill, consisting of a flywheel motor-generator set driven by a 3000-hp. 2200-volt 3-phase 60-cycle 600-r.p.m. induction motor, a 49,000-lb. plate steel flywheel, and three 1050-kw. 330-volt d-c. generators as well as the two main reversing mill motors was furnished by the General Electric Company. The total stored energy of the rotating parts at 600 r.p.m. is 100,000 hp.-sec. These three generators are all duplicate machines. One of them is used singly to drive the roughing-mill motor, and the other two, with armatures in series and fields in parallel, are used to drive the finishing-mill motor.

The starting and speed regulation of this flywheel set is accomplished by means of a slip regulator. The normal control of the regulator is automatic; but hand control also is available at the main switchboard by means of a push-button switch. To provide the remote control of this regulator, a small d-c. motor, driving a winch, raises or lowers a heavy auxiliary counterweight which may be caused to bear on the electrode mechanism. The motor is so interlocked with the oil circuit breakers that when both breakers are open the electrodes are automatically raised to their highest position, but when either breaker is closed the mechanism tends to lower the electrodes to their lowest position. When the current taken by the motor increases to a point corresponding to the operating value of the torque motor, the lowering is retarded.

Provision is made such that if the electrodes are not in the upper position it is impossible to close the oil circuit breakers. When stopping the motor-generator set, the oil circuit breaker is opened and an auxiliary switch on the breaker starts the pilot motor, allowing the main counterweight to raise the electrodes to their highest position. Two 800-amp., 15,000-volt oil switches are provided for control of this motor-generator set. If it becomes necessary to stop this set quickly the motor can be plugged without danger to the equipment.

The exciter set consists of a 150-hp. 220-volt induction motor driving a 50-kw. constant-potential exciter and two 30-kw. variable-potential exciters. In case of trouble with these exciters it is possible to supply excitation for the generators from the 250-volt d-c. shop circuit and use the 50-kw. exciter in place of either of the 30-kw. exciters.

A 300-kv-a. bank of transformers furnishes power for the 150-hp. motor driving this exciter set, and for a 50-hp. 230-volt induction motor which drives a Clarage fan, furnishing 45,000 cu. ft. of air per min. at a pressure of three inches on water gauge, for cooling the main drive motor. Air for cooling the mill motors is taken from the mill through an air filter, provision having been made for the recirculation of a portion of the air from the motor room so that in cold weather the operator has control of the temperature in this room.

The roughing-mill motor is rated 1200 hp. 330 volts d-c. at 25/50 r.p.m. Owing to the fact that the ingots or slabs were very short, it was found that in actual service it was not necessary to take advantage of speeds above 25 r.p.m.

The finishing-mill motor is an exact duplicate of that used on the roughing mill, and has the same continuous and overload torque characteristics. By applying 660 volts to the armature it is possible to run it at double speed. Its rating is 2500 hp. at 660 volts and 53/80 r.p.m. Both of these motors will develop repeatedly an operating torque of 600,000 lb-ft., or in emergency cases, 700,000 lb-ft.

Two field-control panels are furnished for controlling the polarity and voltage of generators and the strength of the motor fields, automatic control of the motor fields being provided by two load-control regulators. An elementary diagram of connections for the 2500-hp. reversing mill drive is shown in Fig. 2.

The two 6-point master switches which control the mill motors are located in the operator's pulpit between the two mills, where the screw-down and table controllers are also located. In this pulpit also are located two meter pedestals, each mounting an ammeter and tachometer. All a-c. starting is controlled from the main switchboard in the motor room on which are mounted meters, relays, etc.

In addition to the regular ring oiling of the motors and generators, a circulating oil system supplies oil to all bearings on the main drive motors and bearings on the 3000-hp. motor-generator set. To this system is connected an oil centrifuge through which the oil is passed periodically for purification.

Roughing Mill

The 2-high reversing roughing mill is of the usual design, except that it is equipped with a United differential screw-down, which permits keeping a plate square without "cornering," even though rolled from a tapered ingot. The rolls are 34 in. by 84 in.

A Geer universal spindle is used for driving the top roll, and a Fast flexible coupling is provided between the pinion stand and driving motor.

Finishing Mill

The finishing stand is of the widely discussed 4-high roller-bearing type and is the widest mill now being used equipped with roller bearings on the backing-up rolls. For ten years, the Lukens Steel Company has operated a 4-high 206-in. reversing mill with complete success, and was the first to order a roller bearing type mill for rolling steel. The working rolls, 23 in. by 84 in., are driven by a reversing motor, while the backing-up rolls, 40 in. by 84 in., are driven through the friction from the working rolls. The top-roll spindle, main flexible coupling, and differential screw-down are duplicates of corresponding parts on the roughing mill. Oil-tight pinion housings are bolted directly to the foundations, the pinion and necks being lubricated by a force-feed oiling system. One unusual feature is that guides and roll chucks for each working roll are cast in one piece, thus making it possible to set up a pair of rolls outside the mill, and with a hydraulic roll-changing device to make a complete roll change very quickly.

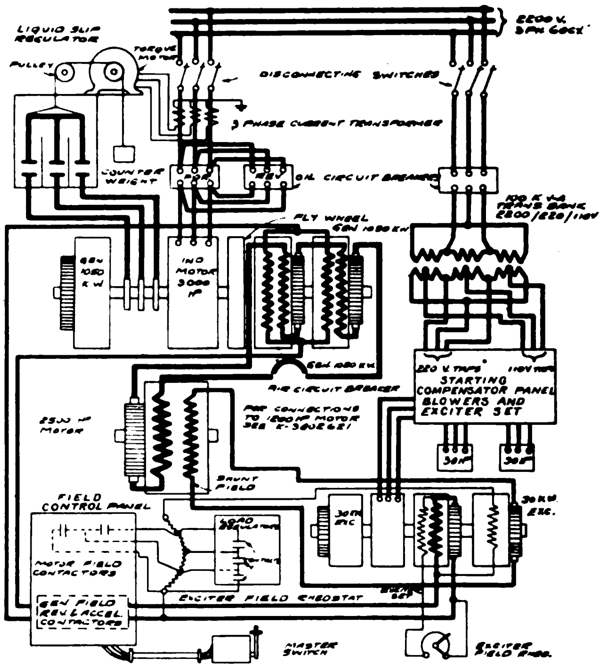

Fig. 2. Elementary Diagram of Connections for the 2500-hp. Reversing Mill Drive in the 84-in. Mill

The roller bearings exert two distinct influences: first, in the reduction of power; and second, in the production of flatter plates, through the avoidance

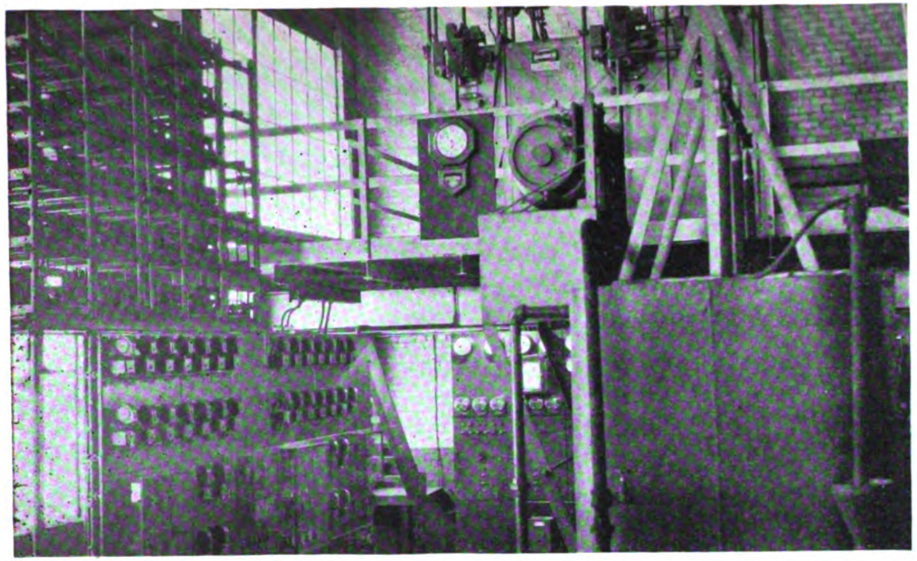

Fig. 3. A Front View of Switching and Control Equipment, Including 2200-volt Oil Switches, Slip Regulator, D-c. Control Boards, and A-c. Switchboard

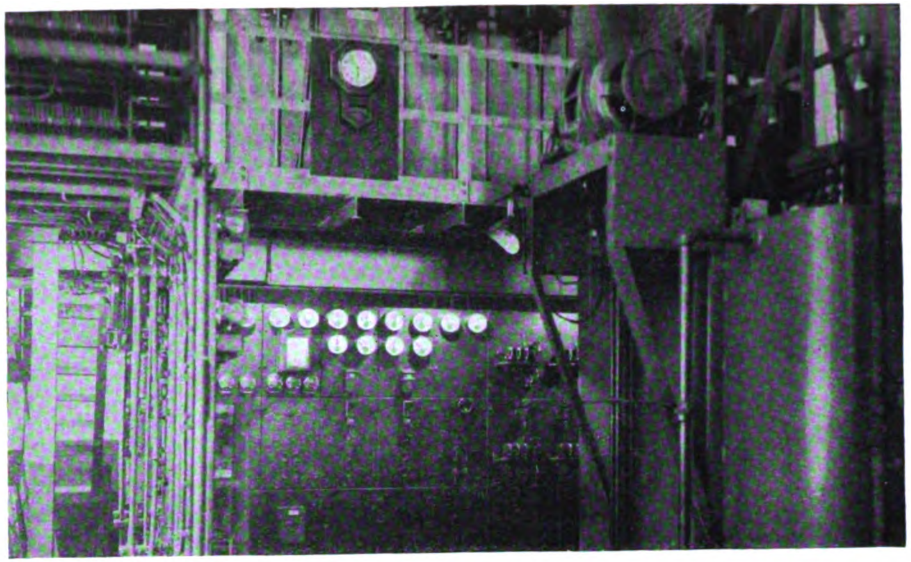

Fig. 4. Another View of Control Equipment, Showing the Back of the D-c. Control Board, the Slip Regulator, and the A-c. Switchboard

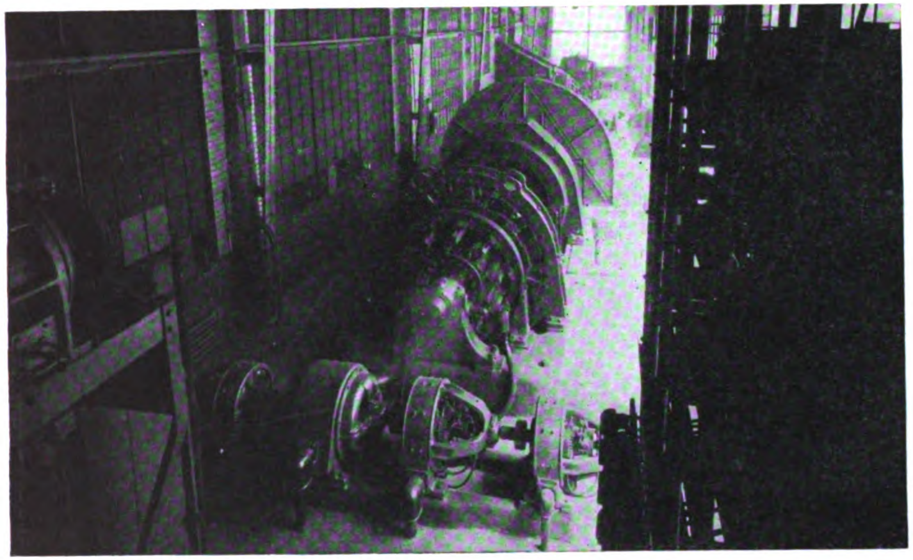

Fig. 5. The Motor Room as Seen from the Oil Switch Platform. Here are shown the exciter set, the motor-generator set, and the mill motors

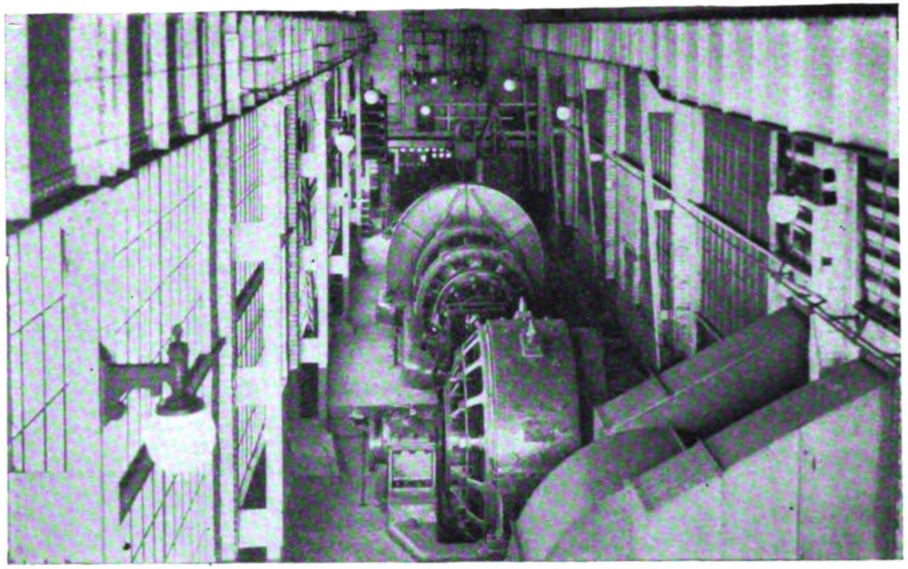

Fig. 6. A View from the End of the Motor Room, Showing in Order the Finishing-mill Motor, the Motor-generator Set, and Control-switching Equipment

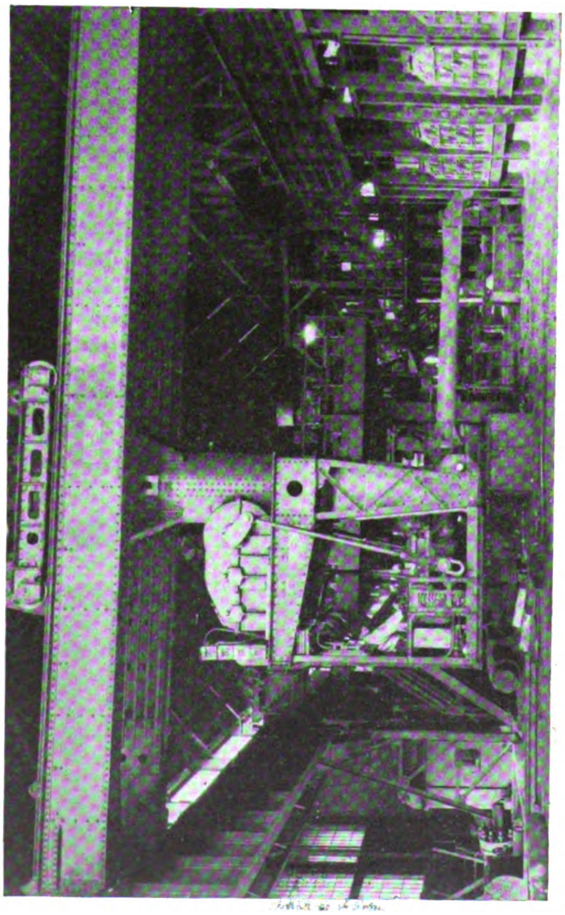

Fig. 7. The Charging Crane by Means of which Charges Are Transferred from Cars to the Oil-fired Heating Furnaces Shown at Right

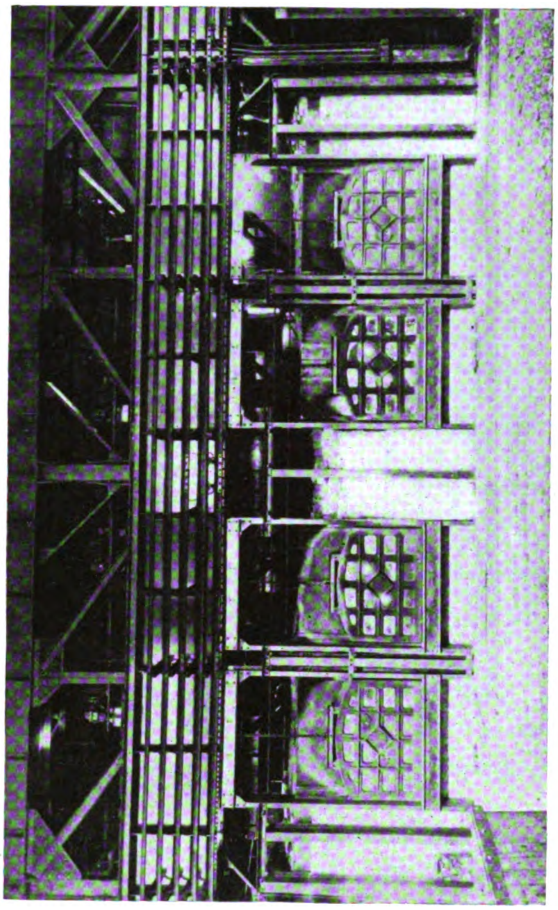

Fig. 8. A Front View of the Heating Furnaces, Showing Electrically-operated Door Hoists and Sectionalized Collector Rails

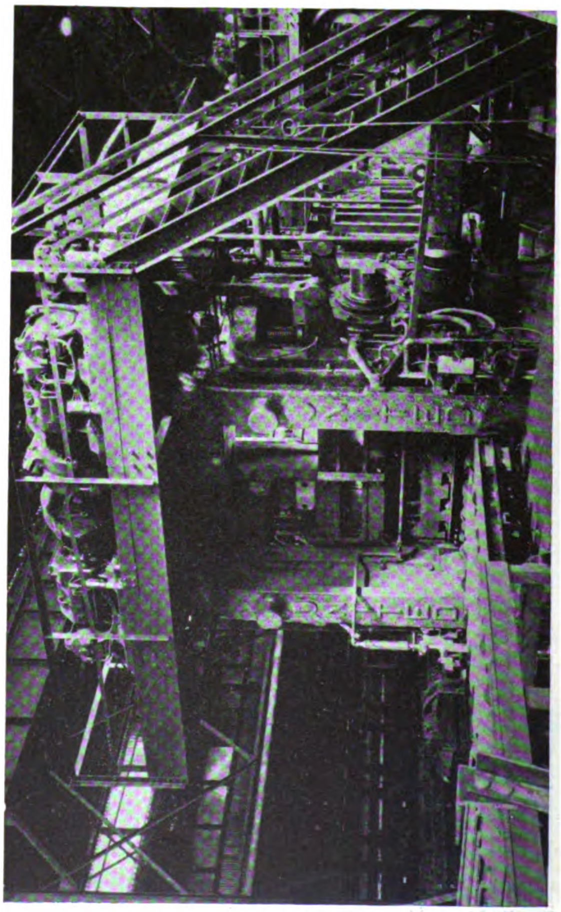

Fig. 9. The 4-high Finishing Mill which is Equipped with Roller Bearings and Embodies Other Features of Unusual Interest

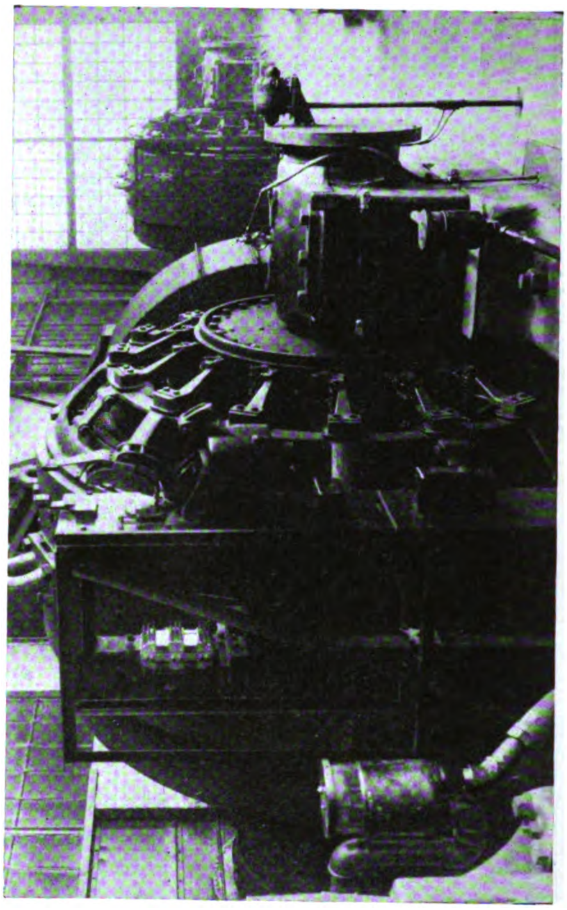

Fig. 10. The Main Mill Motors. The motor in the foreground drives the 4-high finishing mill; the other the 2-high roughing mill

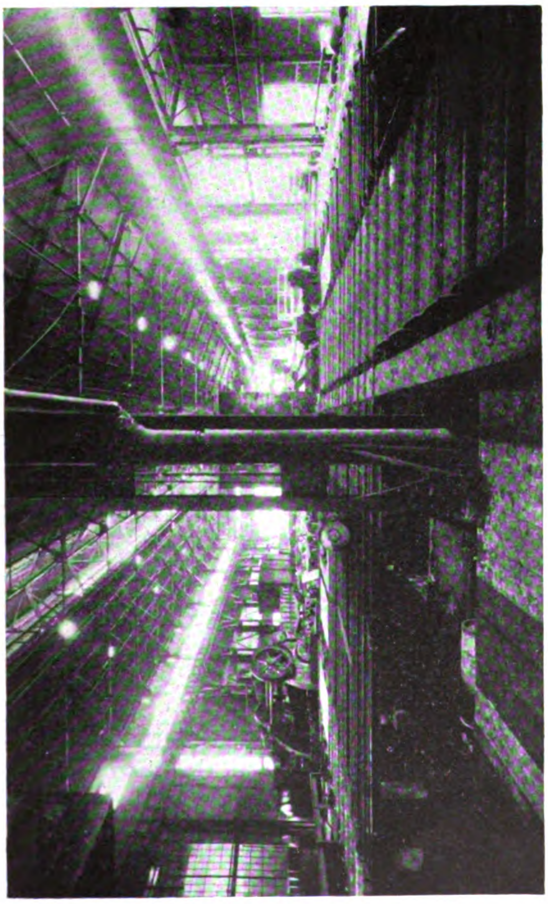

Fig. 11. A View Along the Line of Production, Showing Roll Leveller, Conveyors, Rope Transfer and Sketch Shear

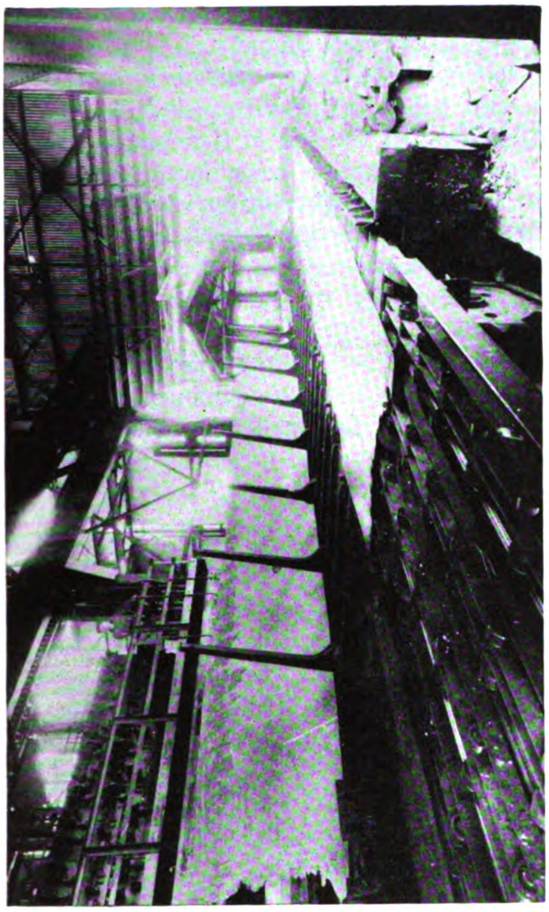

Fig. 12. The End of the Chain Conveyor Where Plates Are Turned Over for Inspection. At the left is the control gallery

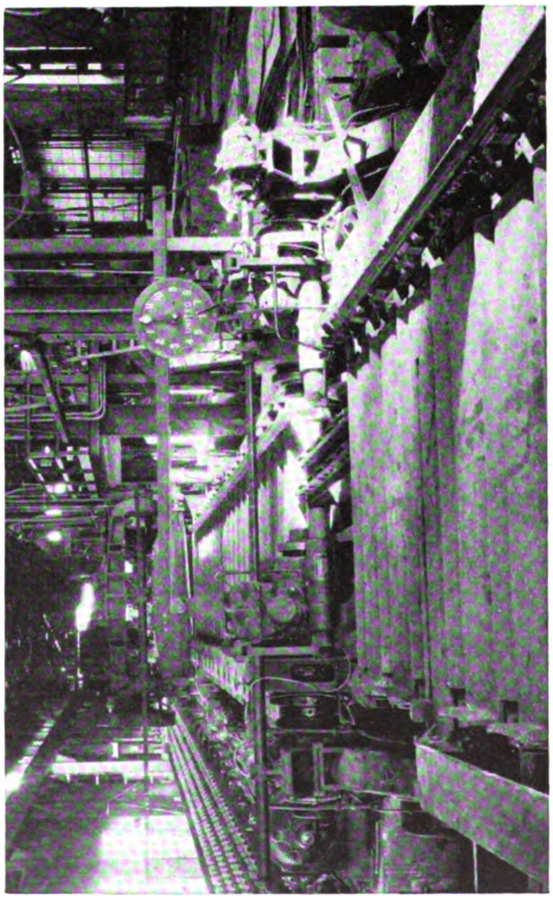

Fig. 13. The Approach Table with Plate Clamp Guide for Holding Plates Square with the Shear. Here also is shown width gauge and end cut shear

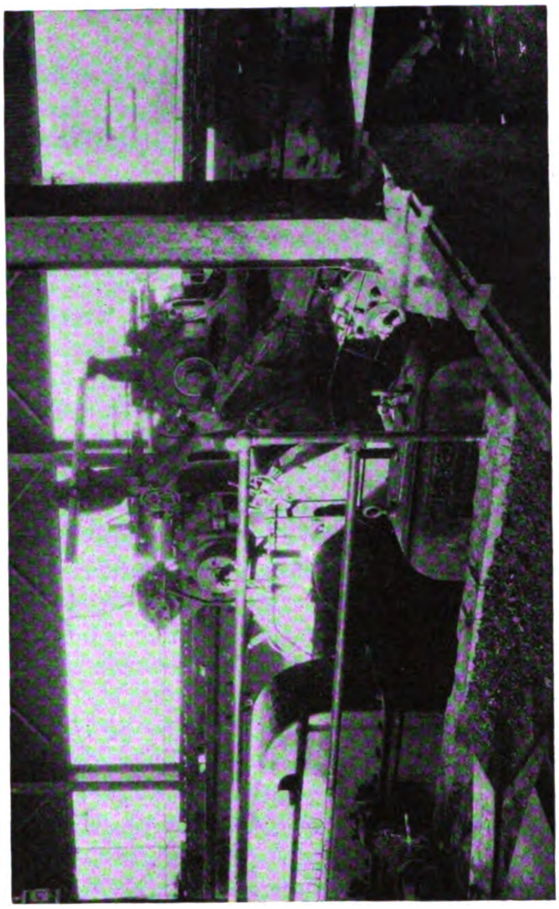

Fig. 14. The Rotary Shear on which the Side Scrap is Trimmed from the Plates. Guides are provided for side scrap before it reaches the chop shear

of changing roll contour by heat generated in neck friction.

The finishing mill was designed and built complete in the shops of the United Engineering and Foundry Company. Mill tables were built by the Treadwell Engineering Company.

Heating and Charging

As shown on the plan (Fig. 1), semi-finished material enters the furnace room on two tracks located immediately in front of the heating furnaces. Tracks are low enough to permit charging and drawing operations to be carried on over the tops of loaded cars.

The heating furnaces are of the oil-fired recuperative double-chamber type, built by the George J. Hagen Company. They are equipped with Hagen high-pressure type burners, with Amco Tile Recuperators, which are set entirely above ground. Factors governing the arrangement and design of the furnaces were: the small space available; the frequent flood stages of the famous Brandywine Creek, which is but a few feet away; and the small quantities in which plates of any one size are produced.

Furnaces are charged and drawn by an overhead charging crane, built by the Wellman-Seaver-Morgan Company. In the heating section, the doors on the furnaces are electrically operated. By means of sectionalized collector rails, these doors are controlled by the charging crane operator. Lewis Automatic Door Hoists are installed on platforms located over the furnaces.

Annealing

If black plates are being produced, they are transferred from the finishing mill directly to the roller leveller; while if blue-annealed plates are being produced, they are transferred to tables as shown in Fig. 1 and through the annealing furnace before reaching roller leveller.

The tables on the entry and delivery sides of the annealing furnaces are driven by motors that have a 4/1 speed ratio. The annealing furnace is of the roller-bottom type, and was built by the Costello Engineering Company. It is fired with low-pressure type oil burners, and is thermostatically controlled. A unique feature of the furnace is that rollers are carried on foundations independent of the furnace proper, and are equipped with roller bearings, and driven through worm gearing. A unit worm-gear reduction is provided for each roller, the worm shafts being connected end to end through flexible couplings, to form in effect a continuous side shaft; and with other flexible couplings the gear shaft is connected to each roller.

Leveller

The roller leveller is equipped with 13 levelling rolls, and a pair of pinch rolls at entry and delivery sides. Each roller is driven through a universal spindle from an enclosed gear box. The machine was built by R. S. Newbold & Sons Company.

The cooling, or layout, conveyor is of the usual roller chain type, and was furnished by the Link Belt Company. At the end of the cooling conveyor, plates are turned upside down for inspection, then turned a second time right side up, which two motions transfer from the cooling conveyor to the shear tables. The inspection turnover and the shear approach tables were built by the Birdsboro Steel Foundry and Machine Co.

Standard d-c. mill-type motors are used on most of the auxiliary drives. Flexible couplings are used on all auxiliary motor drives except mill tables and screw-downs.

Shearing

Side scrap is removed on a pair of rotary shears, designed and built by United Engineering and Foundry Company. An unusual feature in design of this shear is a bottom knife so large in diameter as to approximate a flat knife, whereby it was sought to avoid a "set" in the edge of the sheared plate caused by ordinary rotary shears. The designer's anticipations in this respect have been fully realized. As a part of the machine itself the shear is provided with scrap choppers for cutting up the side scrap as it is removed from the plate.

The end shearing is done on an ordinary "guillotine type" shear, although the arrangement is unusual in that the length of plate is cut to a mechanical gauge, rather than to chalk marks on the plate; and the tables are so designed as to hold the plates perfectly square with the shear. Leaving the end cut shear, the plates are transferred into the shipping room, and over the scale. The shear tables, the end-cut shear, and final transfer were built by the R. S. Newbold & Sons Company.

In addition to the arrangement for continuous shearing, the mill is equipped with a side shear for cutting sketches, and with a circle shear for cutting circular outlines of various diameters.

Conclusion

In any plant which includes such innovations as one of the first 4-high roller bearing mills; an altogether new type of rotary shear; provision for shearing plates to a mechanical gauge; and drive of both roughing and finishing stands with d-c. reversing motors supplied with power from one motor-generator set—some features must be worked out in operation. The "children's diseases" in this case were contracted as usual; but, as usual also, did not prove fatal. Today the equipment, mechanical and electrical, is acceptably reliable; and the products of the mill in all respects are meeting the company's expectations.

Service Records of Auxiliary Motors in the Steel Mills

Detailed and Accurate Maintenance Data Essential to Economical Production—Pre-mill Type, Early Mill Type, and Modern Mill Type Motors—Average Service of Bearings, Armature Coils, Commutators, and Shafts

By A. C. CUMMINS
Chairman, Standardization Committee, Association of Iron and Steel Electrical Engineers

IT is customary to consider the blast furnace as the unit of production in steel plants that use raw materials in the form of ore, coke, limestone, and scrap, and produce steel in a finished marketable form. An investigation of the auxiliary motor equipment of a number of mills reveals the fact that over 500 motors are required to drive the machinery for such a production unit.

It is generally assumed that the demand for steel in the United States requires the operation of approximately 225 blast furnaces. There must, therefore, be about 110,000 auxiliary motors in regular operation in the steel industry. These motors range in size from 1 to 150 hp. and have an average rating of about 25 hp. They are of many different types, furnishing varying, constant, and adjustable-speed service. They operate an average of 6500 hr. per year, as compared to 3000 hr. per year in other industries. Assuming that the cost of inspecting and maintaining these motors is $30 per year, the total annual upkeep bill must be on the order of three and a third million dollars. It therefore is desirable to consider the maintenance characteristics of these motors in order to show the importance of service records in analyzing motor performances in the industry.

While many recent mill installations are completely equipped with motors of modern design there are still many auxiliary motors in service which were designed before the severity of steel-mill duty was fully appreciated and which therefore are not well suited to the tasks they are called upon to perform. Such motors will be referred to in this discussion as the Pre-mill Type Motor. The design of this motor was based on the early non-commutating pole railway motor, and while it possessed mechanical and electrical characteristics which fulfilled the requirements of the street railway service of that day, it proved inadequate when installed in steel-mill reversing service with the crude controllers in use at that time.

The pre-mill type motor was followed in 1905 by the development of the so-called Mill Type. This motor was a decided improvement and remedied many of the weaknesses that existed in the earlier design. It possessed a shaft of ample dimensions, the commutation was improved, the frames were very much heavier in construction, and the feet were so reënforced that they rarely broke in service. Oil-ring bearings were substituted for waste-packed and grease-fed bearings. While the change in type of bearing was a great improvement, it introduced a new difficulty, *viz.*, these bearings permitted oil to leak into the motor and damage the insulation. This motor will be referred to hereafter as the Early Mill Type.

In 1909 and 1910, additional refinements were made, and what is termed the Modern Mill Type Motor appeared on the market.The modern mill type motor differs chiefly from the early mill type by the addition of commutating poles to improve commutation, and changes in oil-ring bearing design which greatly reduced the amount of oil reaching those parts of the motor where it could damage insulation. These motors could be depended upon to give reliable service under severe operating conditions. The curves and discussion which follow are limited to the three types of motors, mentioned, *viz.*, Pre-mill Type, Early Mill Type, and Modern Mill Type.

In 1915, engineers began the study of applying so-called anti-friction bearings to motors in steel-mill service as a means by which difficulties with oil could be eliminated. As there was a possibility of eventually selling at least 200,000 bearings at an average price of $40—an $8,000,000 potential market—roller-bearing manufacturers were quick to seize this field for sales, and enthusiasm for anti-friction bearings became quite general. It was customary to estimate that anti-friction bearings would eliminate 50 per cent of all motor failures. On the other hand, motor manufacturers were not so enthusiastic. They believed the ultimate possibilities of the sleeve bearing had not been developed and that additional maintenance costs would be added on account of the greater cost of the anti-friction bearing. It was therefore not long until better sleeve-type bearings were designed. Today, the user may select either type with the assurance that troubles arising from bearings will be a source of fewer interruptions and much less expense than at any previous time in the history of the mill-type motor.

The most recent development in steel-mill auxiliary motors is the so-called standardized mill-type motor, designed with the idea of procuring uniform ratings, external dimensions, and such other features as are necessary to make motors of a given rating interchangeable as units, regardless of the name of the maker. At the present time, one manufacturer has on the market a new line of motors built to conform to these standardized dimensions. Others will probably follow when business conditions justify the redesign of their existing lines. The ratings of these standardized motors and their dimensions were developed by a joint committee of the Association of Iron and Steel Electrical Engineers and the various mill-type motor manufacturers. Except for the bearings, standardized mill-type motors do not possess any basic features that differ from the mill-type designs of 15 years ago. It was the endeavor of those responsible for selecting the standardized features to adopt ratings and dimensions that would permit development of logically designed motors meeting the most severe requirements of the industry, and at the same time to provide a more versatile motor which could be used in some applications usually considered as requiring the installation of the the general-purpose type of motor.

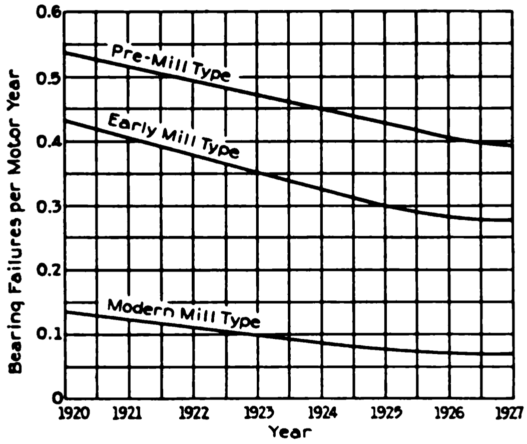

Fig. 1. Bearing Failures Per Motor-year

At the time the Committee of the A. I. & S. E. E. were selecting the standardized mill-motor dimensions, it was necessary to make studies of motor service records in order to determine the weaknesses in existing designs and to endeavor as far as possible to attempt their correction. A questionnaire was sent to all of the steel manufacturers using mill-type motors, asking for information on questions relating to service. Detailed reports were also procured from several of the larger mills to be used for checking replies to the questionnaire.

The results of this investigation have never been published and it is the thought of the writer that curves illustrating the service performance of mill-type motors in reversing duty might be of interest. The data collected originally covered a five-year period, but recently additional data have been secured which makes it possible to plot curves covering a service period of eight years, with a fair degree of accuracy.

Before going into a detailed consideration of the various curves and figures that follow, it is necessary to emphasize the fact that operating conditions in individual installations of auxiliary mill-type motor equipment differ so widely that it is impossible to verify average figures by comparison with single motor installations. However, it will be found that records covering groups of 100 or more installations will closely check with the curves, provided the same relation of motors in reversing crane service and reversing mill-table service is maintained, *viz.*, 40 per cent mill tables, 60 per cent cranes.

There are five principal causes of failure in mill-type motors in steel-mill service, as follows:

(*a*). Bearing failures.
(*b*). Armature-coil failures.
(*c*). Commutator failures.
(*d*). Shaft failures.
(*e*). Field failures.

In Fig. 1 it will be noted that bearing failures in the modern mill-type motor are extremely few, and that an average life of something like 10 to 12 years per motor per set of bearings can be expected. This does not mean, however, that in 10 or 12 years the bearings may not have been the real cause of other failures. The longer bearing life shown by the modern motor is partially due to the greater proportion of anti-friction bearing motors included in this class, as all early mill-type motors which have been equipped with anti-friction bearings have been included in the modern mill-type classification. It is also interesting to note that the bearing life of a modern mill-type motor is about six times greater than that of a pre-mill type motor. There has been an increase in bearing life in all three types of motors during the past eight years; amounting to some 25 per cent for the pre-mill type motor, 35 per cent for the early mill type, and about 47 per cent for the modern mill type.

Fig. 2 presents the history of the armature-coil failures in the three different types of motors. An increase of 35 per cent in the life of armature coils has been made in the pre-mill type motor, 30 per cent in the early mill type, and 43 per cent in the modern mill type. It will be noted that the expected life of armature coils in the modern mill-type motor is nearly three times that in the pre-mill type. This, in the opinion of the writer, is due to the fact that the pre-mill type motors are wound with coils having cotton insulation, and accordingly are more easily damaged by heat and oil. Furthermore, the modern mill-type motor has better bearings and hence less oil finds its way into the windings. It is also noteworthy that in the three types of motors approximately the same relation in armature-coil life existed in 1920 as exists in 1927. As approximately the same increase in life is evidenced in all three types, there is n indicated improvement in steel-mill maintenance shop practice.

Another hazard to long life of the mill-type motor is the failure of commutators. There has been greater increase in commutator life than in any other one feature during the eight-year period. This increase is shown in Fig. 3. It is not easy to explain the improvement, as it seems to be equally large in both the most obsolete and the most modern motors. An oil-resisting cement was developed by several manufacturers to prevent damage to commutator insulation, and it is understood that this is quite generally used. This cement may be responsible for part of the increase in commutator life. The practice of buying commutators factory assembled on bushings, instead of assembling them in the steel-mill shops, may also be responsible for part of the improvement. However, data are not available to show what proportion of commutator replacements are made in this manner. Another possible reason is the more general use of carbon brushes specially selected for the motor on which they are used.

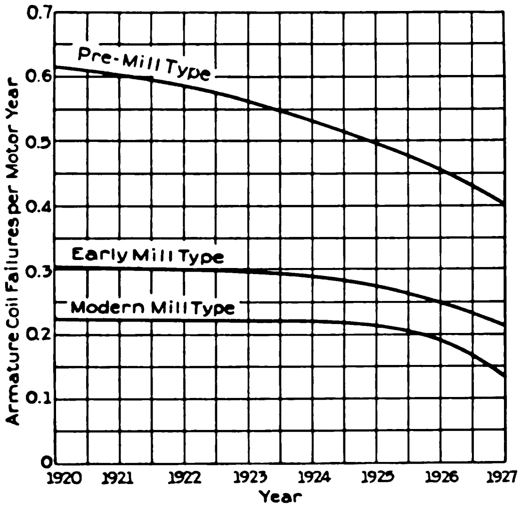

Fig. 2. Armature-coil Failures Per Motor-year

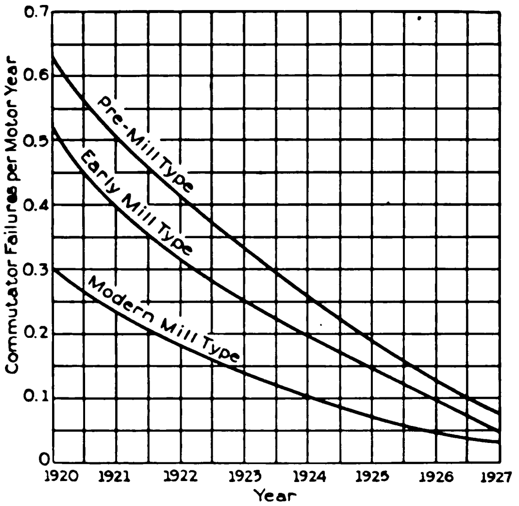

Fig. 3. Commutator Failures Per Motor-year

Shaft failures represent such a small item that further reduction will not greatly affect the service a motor will render. However, the curves in Fig. 4 show the pre-mill type motor to be lacking when compared with the modern mill-type motor. The records indicate that pre-mill type motors will break about 10 times as many shafts as the modern mill-type motors and about four times as many shafts as an early mill-type motor.

The data collected on field failures proved so erratic that it was impossible to plot any curves of field life with assurance of accuracy. This condition can best be explained by the fact that an armature failure may result in an interruption to production primarily caused by a field failure. In such cases, the repairmen may replace the damaged field but report armature failure as the primary cause of the trouble. This difficulty in presenting field-failure data emphasizes the necessity of accurate records, honestly kept.

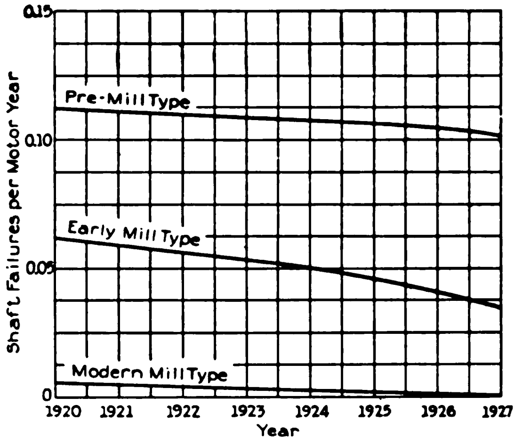

Fig. 4. Shaft Failures Per Motor-year

Figs. 5, 6, and 7 show graphically the relation of causes of failure in each of the three different types of motors. In the pre-mill type (Fig. 5) armature-coil failures are the predominating source of trouble. Bearing failures are in second place. It is apparent that the method of attacking the maintenance problem in these motors is to devote special attention to eliminating obsolete sleeve-type bearings and if possible the use of fire-proof armature-coil insulation as a substitute for cotton.

Fig. 6 is a similar set of curves for the early mill-type motor. In this case bearing failures predominate again and armature coils are not far behind. The importance of better bearing design is again emphasized in this set of curves.

In Fig. 7, service curves of modern mill-type motors are shown. Armature coils are lasting 7 to 8 years, bearings over 10 years, and commutators over 20 years. Armature-coil life has improved in the modern mill-type motor in the last three years, largely due to increase in the number of modern mill-type motors equipped with improved bearings. The recommendation for further improvement in the modern motor is to concentrate on eliminating oil from the windings, either by improved oil-tight sleeve bearings or by anti-friction bearings.

Maintenance engineers have been discussing the matter of increased life of motors after bearings have been changed from the sleeve to the anti-friction type.

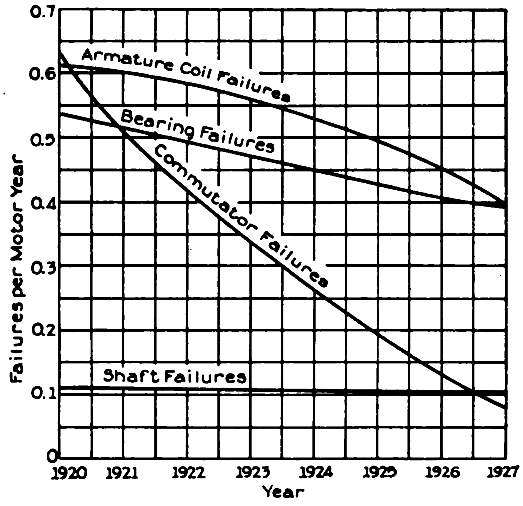

Fig. 5. Pre-mill Type Motor; Relation of Causes of Failure

The curve in Fig. 8 is the result of an effort to answer this question. All the motors whose records are plotted are either pre-mill or early-mill types, and are in extremely hard reversing-mill table service. They are not all equipped with magnetic control. These severe-service applications were selected for securing the data because sleeve-bearing motors previously in this service had a short life and it was hoped that definite data could be accumulated in a relatively short time. The resulting curve (Fig. 8) is practically a straight line indicating that regardless of the life with sleeve bearings, 3⅙ times longer life per failure may be expected with anti-friction bearings. The improvement would be much less if the motors changed to anti-friction bearings had originally been fitted with sleeve bearings of the most up-to-date design. In the writer's opinion the replacement of sleeve bearings as originally supplied in modern mill type motors by anti-friction bearings is difficult to justify from an investment standpoint.

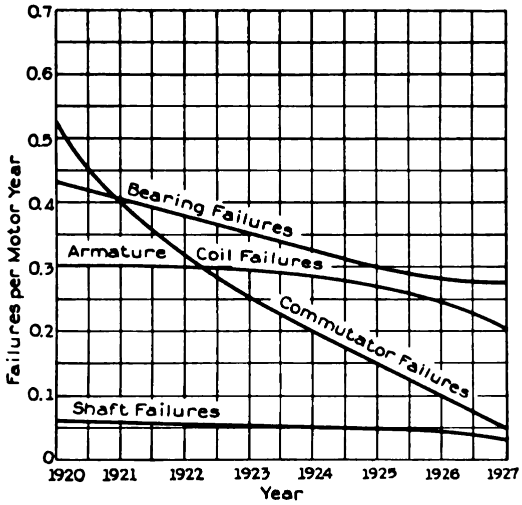

Fig. 6. Early Mill-type Motor; Relation of Causes of Failure

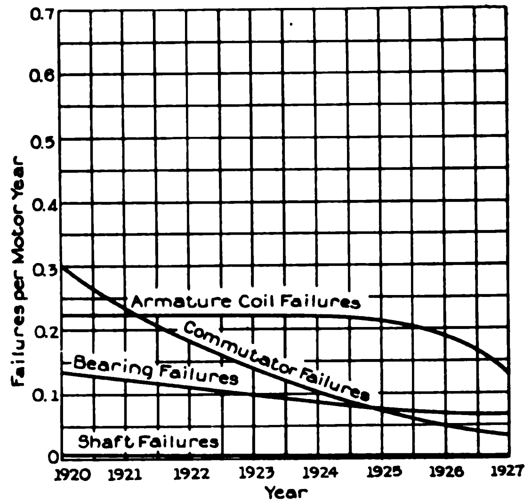

Fig. 7. Modern Mill-type Motor; Relation of Causes of Failure

No definite rules can be given to determine policies in replacing bearings. However, experience indicates that the following statements can be used as a guide:

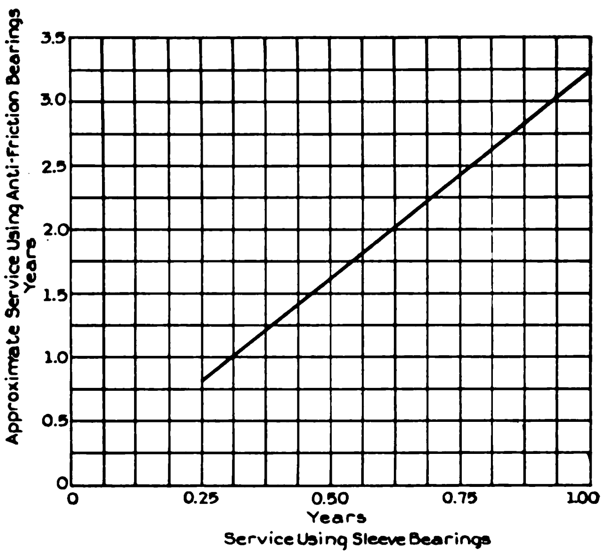

Fig. 8. Relation of Motor Service Using Sleeve Bearings and Anti-friction Bearings as Applied to Pre-mill and Early Mill-type Motors

(1). On pre-mill and early mill type motors, space limitations necessitate replacing the old type of sleeve bearings with anti-friction bearings.

(2). In the modern mill type motor, experiments indicate that improved sleeve-type bearings can be substituted for the bearings originally furnished, and that improvement in service life will result. The cost of installing improved sleeve-type bearings will be much less than the cost of installing anti-friction bearings.

(3). In purchasing new mill type motor equipment, anti-friction bearings can be procured at but slightly greater cost than sleeve bearings, and they are worth the additional cost. However, care should be taken in selecting the type of anti-friction bearings used, particularly if end thrust is developed in the armature shaft.

Next, we come to the expected life per failure including failures due to all the causes listed and also due to other minor troubles which occasion service interruptions, but which do not result in shop repairs. These data are shown in the form of a curve in Fig. 9. In 1920, the modern mill-type motor was giving 19.6 months' life per service failure; in 1927, slightly over 25 months', an increase of approximately 27 per cent. In 1920 the early mill-type motor delivered 9 months' service per failure, while in 1927 this had been increased to 13½ months', or an increase of 50 per cent. The pre-mill type motor in 1920 gave only four months' service per failure. In 1927 this had been increased to 9.6 months', or an increase of 140 per cent. On a percentage basis, the improvement in service per failure is much greater in the pre-mill type motor than in the modern mill type. This is logical, as equipment which causes most trouble always secures concentrated attention, and hence, as the older types of motors seriously interrupt production, the correction of their weaknesses receives special attention.

Improvements in mill-type motor maintenance have not resulted in tangible savings in maintenance costs in direct proportion to the increase in length of motor service per failure. In many instances such improvements have been effected by the use of higher grade materials, and the adoption of maintenance practices which have required the spending of more money. However, material reductions in maintenance costs have been effected, and the improvements in the reliability of service that these motors have rendered has resulted in production savings which, if evaluated would show that most excellent progress in the reduction of steel-mill costs has been accomplished.

It has been suggested that much of the improvement in operating performance has been obtained by improvements in shop practice. To those who are familiar with the steel industry in its early days, it is not hard to recall that failures were so frequent that the difficulty of maintaining continuity of production pushed the shops to their uttermost ability. At one time it was the prevalent practice to make patchwork repairs of the crudest nature. It has required years to train shop organizations to realize that any repair but a most thorough one is a service liability; that dipping, baking, and impregnating are processes not to be classed as manufacturing frills but an absolute necessity to reliable and economical mill-motor maintenance. This lesson has been learned by the industry and there has been a gradual improvement in the class of work turned out by the shops until, at the present time, many shops in steel plants are turning out repair work which compares quite favorably with new equipment leaving the manufacturers' test floors. Undoubtedly the shopmen deserve great credit for the improvement that has been achieved.

The question of replacing obsolete motors with modern equipment is one that frequently arises. An inspection of the curves in Fig. 9 would indicate that the much longer life per failure rendered by the modern motor would well justify the replacement of obsolete equipment. Unfortunately, it frequently happens that the investment required to make such changes is not merely that required to purchase new motors, because new bedplates, foundations, and mechanical drives must be provided as well, and the purchase price of the motor is but a small part of the total expenditure required. Under such conditions, it is often difficult to justify the capital expenditures necessary for replacements of this nature on the basis of savings in maintenance.

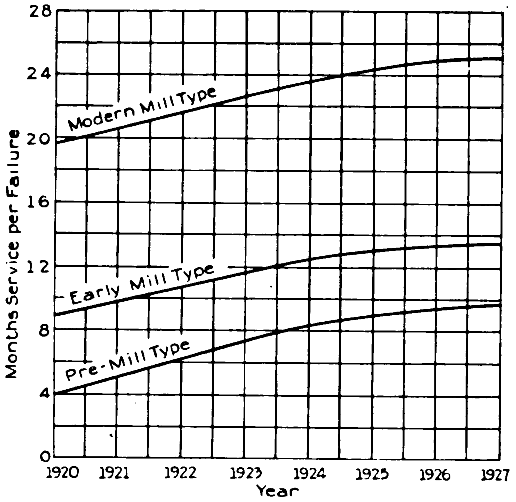

Fig. 9. Months' Service Per Failure (from all causes) in Mill Duty

However, motors that frequently cause production delays always have excessive maintenance costs—usually 10 or even 20 times the average. If proper records are available it is usually a simple matter to justify the investment required to effect a replacement and the whole manufacturing process profits as a result. It is therefore the duty of every maintenance engineer to keep records of troubles with such accuracy and detail that he can learn where unreliable high-cost installations exist in the production trains. If this is done, there is no question that replacements can be justified, provided the records are dependable and the costs correct.

The principal field for future increases in service life is not so much in improved equipment designs as in a more general realization by the entire maintenance organization that preventive measures in maintenance are superior to corrective ones. Proper vigilance on the part of those responsible for the inspection of mill apparatus to anticipate trouble before damage is done should be the aim of all maintenance engineers. The curves indicate that some progress has been attained. Just recently one well-known steel-mill maintenance engineer made the statement that it could be proved that one efficient man in the field will save the work of three men in the shop. This statement does not seem to be an exaggeration, and it is probable that future improvements in motor service will be along the line of better maintenance practices, rather than radical changes in design.

Pot-type Electric Furnaces for Lead Hardening

A NEW pot-type electric furnace for lead hardening has been developed in four sizes to operate at temperatures up to 1650 deg. F.

Although capable of doing the same amount of work as previous designs of pot-type electric furnaces, it consumes less current. It utilizes the standard method of submerging the work in a bath of molten lead, and is particularly suitable for hardening and tempering chisels, files, drills, taps, dies, and other small tools or parts entering into the manufacture of a high-grade product.

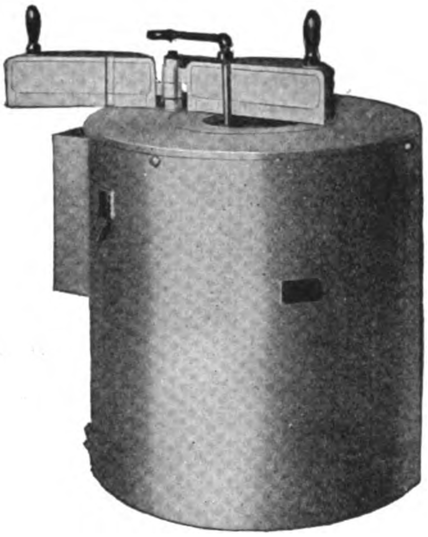

Fig. 1. General View of One of the Furnaces

The four sizes cover a wide range of production conditions. The smallest has a maximum capacity of 75 lb. of steel per hour at 1500 deg. F., an inside pot diameter of eight inches and depth of 12 in., with an overall diameter of 32 in. and a working height of 32 in. The connected load of this size furnace is 11 kw., furnished at 75 volts by single-phase transformers that step down the 220-, 440-, or 550-volt service to the working voltage.

The 16-kw. size has a maximum capacity of 125 lb. of steel per hour at 1500 deg. F.; the inside dimensions are 12 in. diameter and 18 in. depth; the working height is 37 in. and outside diameter 38 in.; and the power supply voltage is 110, single-phase. A third size has a maximum capacity of 250 lb. of steel per hour at 1500 deg. F.; the inside dimensions are 16 in. diameter and 18 in. depth; the working height is 45⅜ in. and the outside diameter 46¾ in.; the connected load is 29 kw.; and the power supply is 220 volts, single phase or direct current. The largest size has a maximum capacity of 250 lb. of steel per hour at 1500 deg. F.; the inside dimensions are 16 in. diameter and 24 in. depth; the working height is 45⅜ in. and the outside diameter 46¾ in.; the connected load is 29 kw. and the power supply 220 volts, single-phase or direct current.

As a result of the inner melting pot not extending over the top of the furnace, very little heat is lost by being conducted out by the pot. The following figures of savings show the energy consumption of this type of furnace as compared with a construction where the pot extends to the outside and is supported by a top cover casting:

SIZE OF POT		Approximate Savings in Kw-hr. Per Hr. at 1500 Deg. F.
Diameter, In.	Depth, In.	
8	12	1.5
12	18	2.5
16	18	3.5
16	24	3.5

The principal features of the furnace are: (1) minimum current consumption; (2) uniform quality of product; (3) lack of contact of pot with shell or cover of furnace; (4) use of direct-heat unmuffled heating unit of the metallic-resistor type; (5) ease of removal of pot; (6) improved working conditions; and (7) reduced fire hazard.

Application of Synchronous Motor-generator Sets to Steel Mill Reversing Drives

Types of Reversing Drives—Power Requirements—Comparison of Synchronous M-g. Set and Flywheel Set—Description of a Typical Synchronous Installation—Results of the Operation of a New Set

By L. A. UMANSKY
Industrial Engineering Department, General Electric Company

THE new electric drive, recently installed by the Standard Seamless Tube Company for driving their 32-in. reversing bar mill at Ambridge (Pa.) is extremely interesting to all who follow the progress of electric power in the steel industry. The interest of this drive can not be attributed to its exceptional size—its capacity is only moderate, as reversing mill drives go—but is due to certain radical departures from conventional lines heretofore considered essential for equipments of this nature.

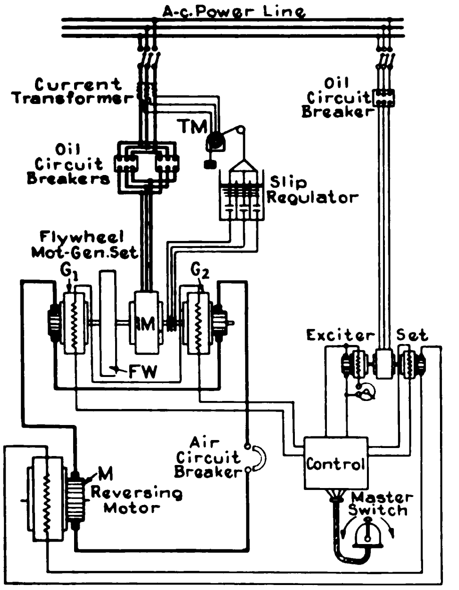

Fig. 1. Elementary Diagram of a Typical Conventional Electric Drive for a Reversing Rolling Mill

M—Reversing motor G_1, G_2—D-c. Generators
IM—Slip-ring induction motor *FW*—Flywheel
TM—Torque motor

This installation marks another success of the synchronous motor in the industrial field. Not only is it a definite accomplishment, but it also is a forerunner of future developments.

Conventional Steel Mill Reversing Drives

The well known layout, typical of electric drives used with reversing steel mills, is schematically represented in Fig. 1. The reversing motor (*M*) is shown supplied with power from two generators (G_1 and G_2). This motor is started in either direction and is accelerated by changing the polarity and strength of the generator field and, further, by weakening its own field. Single- or double-unit motors, and one or two or more generators, connected in series or in multiple, as the case may be, were—and are still—used in different combinations.

Common to all cases was the method of driving the motor-generator sets of these generator-voltage control equipments. Slip-ring induction motors were universally used, and heavy flywheels were always provided, as part of the sets. The slip regulators (*SR*), usually of the liquid type, automatically inserted under heavy loads an additional resistance in the secondary of the induction motor (*IM*); the latter would then tend to slow down and the flywheel (*FW*) would give out part of its stored energy and thus relieve the motor (*IM*) and the power line from peak loads. The reversing motor itself and the d-c. generators are, obviously, not assisted by the flywheel and should be designed to withstand the peaks. This layout is too well known to be elaborated here any further.

A flywheel motor-generator set became a "fixture" of any steel mill reversing drive. It seems as if everyone has taken for granted that any reversing mill load is something from which the power line and the motor (*IM*) should be, by all means, protected by a flywheel, lest some trouble be invited.

This was quite true some 20–25 years ago, when steel mill reversing drives were first brought out. It is unquestionably still true in a majority of cases; but it is as wrong to generalize this rule as any other.

The capacity of power systems has grown by leaps and bounds in recent years. A momentary peak of, say, 5000 kw. imposed by a single mill drive, was previously considered as something quite serious, as far as the power lines are concerned. Now in many steel mills, regardless of whether they generate their own power or purchase it from a public utility, such a load peak would be regarded as a mere "drop in the bucket," which may just as well be passed on to the line, together with other peaks on the same system.

Many non-reversing mills, previously driven by induction motors and equipped with flywheels "to smooth out the peaks," would now be free from any flywheels. A growing number of them are now driven by synchronous motors.

Thus arose the question: Why not forego the flywheel on the reversing mill motor-generator set?

Then, why not use a synchronous motor for driving this set? Such questions are logical and legitimate. It remained to find a suitable case to analyze it and to give the proper answer by facts.

The new reversing mill, laid out by the Standard Seamless Tube Company in the Fall of 1926, happened to be this case.

Power Requirements of the 32-in. Mill

The products of this Steel Company are seamless tubes, pierced and rolled from round billets. To produce these rounds, the company purchases heavy square billets, up to 13½ by 13½-in. size, and reduces them by rolling on a 32-in. reversing bar mill to squares of suitably smaller cross-section. These squares are then rolled in the same heat into rounds on the following 22-in. three-high mill. After this the rounds are reheated and enter the piercing mills.

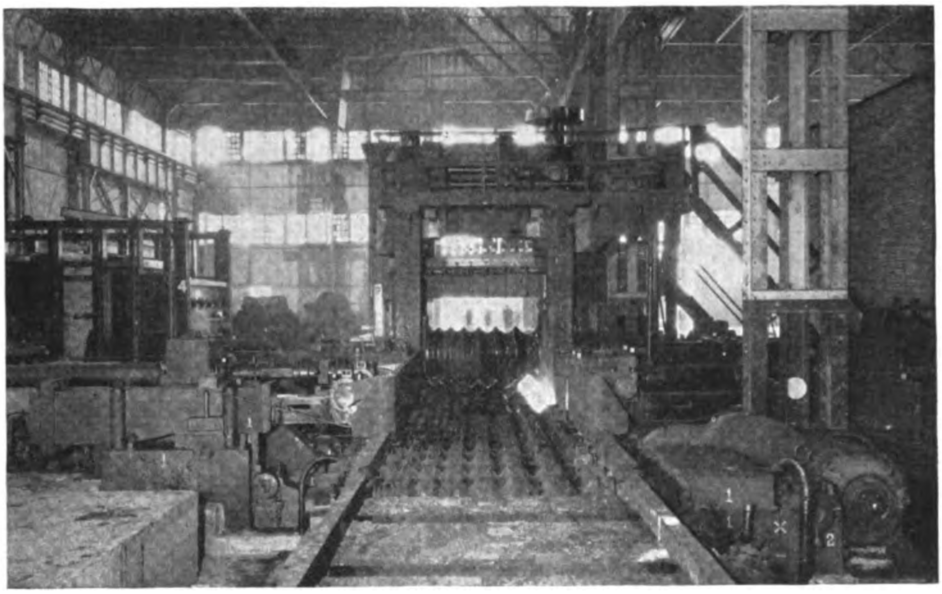

Fig. 2. View of the 32-in. Reversing Bar Mill, Driven by a 2150-hp. Motor. The latter is mounted behind the brick partition at the right. Mill tables, side-guards, screw-downs and other auxiliaries are electrically driven

The final cross-section to which the square billets are reduced on the reversing mill depends, of course, on the size of the seamless tubes into which these billets will be ultimately converted. Seven- and six-in. squares represent the average, with the four by four-in. section being close to the minimum. An hourly output of 90 tons of finished weight is the average mill capacity. The mill consists of a single stand of rolls. Not more than one billet is in the mill at any given time.

The general view of the 32-in. reversing mill itself is given in Fig. 2.

A study of the power requirements has shown that the driving motor should have a maximum operating torque of over 500,000 lb-ft., with an emergency torque of about 600,000 lb-ft. The motor selected is capable of developing this torque up to its basic speed of 43 r.p.m.; higher speeds, up to 86 r.p.m., are obtainable by motor field weakening, with a correspondingly reduced torque. The continuous capacity of the reversing motor is 2150 hp. at 43–86 r.p.m. and 550 volts.

To furnish power to this motor there were selected two 1000-kw. 720-r.p.m. 275-volt generators connected in series. The required maximum capacity of the generators, corresponding to the motor torque of 600,000 lb-ft. at 43 r.p.m., is about 4000 kw.

Synchronous M-g. Set vs. Flywheel Set

The next question was to decide whether the two generators were to be driven in a conventional way by an induction motor with a flywheel, or whether a synchronous motor could be used to an advantage.

An induction motor, protected by a flywheel from high peak loads, could have in this case a smaller mean effective (R.M.S.) capacity than a synchronous motor. For example, a 1700–1800 hp. induction motor could be selected as against a synchronous motor of about 2800 hp.; the latter will readily stand peak loads up to 4000 kw. or more at the d-c. end of the set. This increase of the capacity of the driving motor does not appreciably increase the cost of the equipment, however, as synchronous motors of a given rating are usually less expensive than induction motors of the same capacity and speed. Moreover, the elimination of the flywheel with its large bearings is a direct saving. Neither is any slip regulator required; this reduces still further the first cost and maintenance.

The d-c. generators, driven at a constant speed by a synchronous motor, require lighter fields than identical generators comprising part of a flywheel set; in the latter case a certain drop of speed is essential to get the desired benefit out of the flywheel, and therefore a special regulating equipment must often be provided to boost the generator excitation as the speed of the set drops off.

Absence of flywheel also means that no special means need be at hand to bring the set rapidly to rest in an emergency; plugging or dynamic braking usually stops a flywheel set in two or three minutes; a synchronous motor-generator set comes to rest in even a shorter time after the oil circuit breaker is tripped. In addition to these, of course, the power-factor corrective feature stands foremost in favor of the synchronous motor driven set.

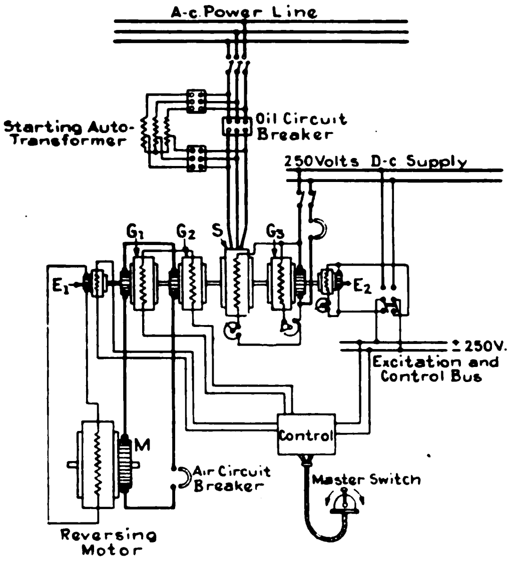

Fig. 3. Elementary Diagram of Connections of the Electrical Equipment Used to Drive the 32-in. Reversing Bar Mill of the Standard Seamless Tube Company and to Furnish 250-volt d-c. Power for Auxiliaries

M—Reversing motor G_1, G_2, G_3—D-c. generators
S—Synchronous motor E_1, E_2—Exciters

Actual Layout of the Electrical Drive

A number of additional advantages of the synchronous type of drive will be made plainer when the layout of the electrical equipment is briefly explained.

At the same time this reversing drive was considered, this steel company needed an increase of their 250-volt d-c. capacity. It seemed very likely that a 1000-kw. motor-generator set would have to be purchased.

Now, it was found that this additional capacity is needed at the same periods of the day that the reversing bar mill (and, therefore, the seamless mills) are running. There was no reason, therefore, for not mounting a third, duplicate 1000-kw. generator on the set and to operate it at 250 volts. It is far cheaper to suitably increase the capacity of the driving synchronous motor from 2800 kv-a. to 4200 kv-a. than to provide a separate 1400-kv-a. motor for the additional generator. Elimination of additional high-voltage oil circuit breaker equipment was another net gain.

This arrangement would hardly be feasible, were a flywheel set used for the reversing drive because, with the speed of the set continuously varying, the voltage of the third generator could not be held within reasonable limits; nor could it be made to parallel successfully with the 250-volt system unless a separate regulating means were provided. Thus the advantage of this simple, three-generator layout is fully realized because the synchronous motor was selected to drive the set.

The elementary diagram of connections of these machines is shown in Fig. 3. The three duplicate 1000-kw. 250/275-volt generators, G_1, G_2, and G_3, are driven by a 4200-kv-a. 0.8-p-f. 720-r.p.m. 2200-volt synchronous motor (S). Two of these generators, G_1 and G_2, connected in series, furnish 550 volts to the reversing motor (M). The third generator (G_3) energizes the general-purpose 250-volt bus; it also furnishes excitation to the synchronous motor (S).

Two 250-volt exciters (E_1 and E_2) are mounted, one at each end of the set. Exciter E_1 provides the excitation of the reversing motor, and this is controlled through the exciter field. Exciter E_2 operates at constant voltage, energizing the control bus and the fields of G_1 and G_2; the latter are controlled directly by contactors and resistors.

Such mounting of the exciters was again greatly facilitated by the fact that the main set is driven by a synchronous motor. Were it a flywheel set, the exciters thus mounted would need a separate voltage regulator; and more likely than not a separate motor-driven exciter set would be used, as shown in Fig. 1. This would require an additional oil switch, again increasing the initial cost.

It is interesting that the third general-purpose generator (G_3) serves as a "spare" to the other two generators—a spare which does other work in its spare time. If, in the future, the mill requirements should increase to such an extent that a higher operating speed will be essential, this can be readily provided by connecting the third generator (G_3) in series with G_1 and G_2; with approximately 820 volts applied to the motor (M), it will be rated about 3200 hp. at 65/130 r.p.m.

Effect on Cost of Power

The steel plant of the Standard Seamless Tube Company does not generate its own power, but purchases it from a large public utility. It was therefore advisable, before the application of a synchronous motor-generator set to this reversing drive was finally decided on, to study its effect—if any—on the cost of purchased power. Likewise, the opinion of the Power Company's engineers on the whole subject was sought.

The power rates included in this case the usual items of demand charge, energy charge, and the power-factor clause.

With a synchronous motor-generator set the maximum peak load of the reversing drive within its capacity is transmitted to the a-c. line as an approximate 4500-kw. peak. With a flywheel set, of the type shown in Fig. 1, the instantaneous peak value could be effectively limited by the slip regulator to approximately 2000 kw. Now, with a possibility that the peak load will be more than doubled, should not one expect the demand charge and, therefore, the power bill to go up? This was not, however, the case.

The "maximum demand" as defined in practically every power contract is not the maximum *instantaneous peak*, but the integrated demand over a definite period of time, *e.g.*, 5, 15, or 30 minutes. In other words, this demand represents the maximum amount of kilowatt-hours consumed in any, say, 15-min. interval. As the rolling mill load peaks are of a very short duration (a few seconds at a time) they do not affect in the least the integrated demand. The latter merely depends on the amount of useful work done at the mill during this 15-min. period, and on the overall efficiency of the electric drive. With a flywheel dispensed with, with its windage and friction losses and the slip regulator losses eliminated, the synchronous drive is naturally more efficient than a flywheel set drive. Hence the integrated demand, if anything, is reduced.

The power-factor clause was next considered. It was estimated that a synchronous motor on this drive would improve the power-factor of the whole plant to the extent of changing from an average of about 75 per cent to about 95 per cent lagging. Any improvement of the power-factor above the 75 per cent value entitled the steel company to a suitable rebate. A simple analysis has shown that the expected power-factor correction will reduce the total monthly gross bill by about 5 per cent. This saving alone is sufficient to write off in about five years the cost of the complete electrical equipment for the new reversing mill. This is not an altogether bad investment, considering that the same electrical machines are doing the rolling work, in addition to the power-factor correction.

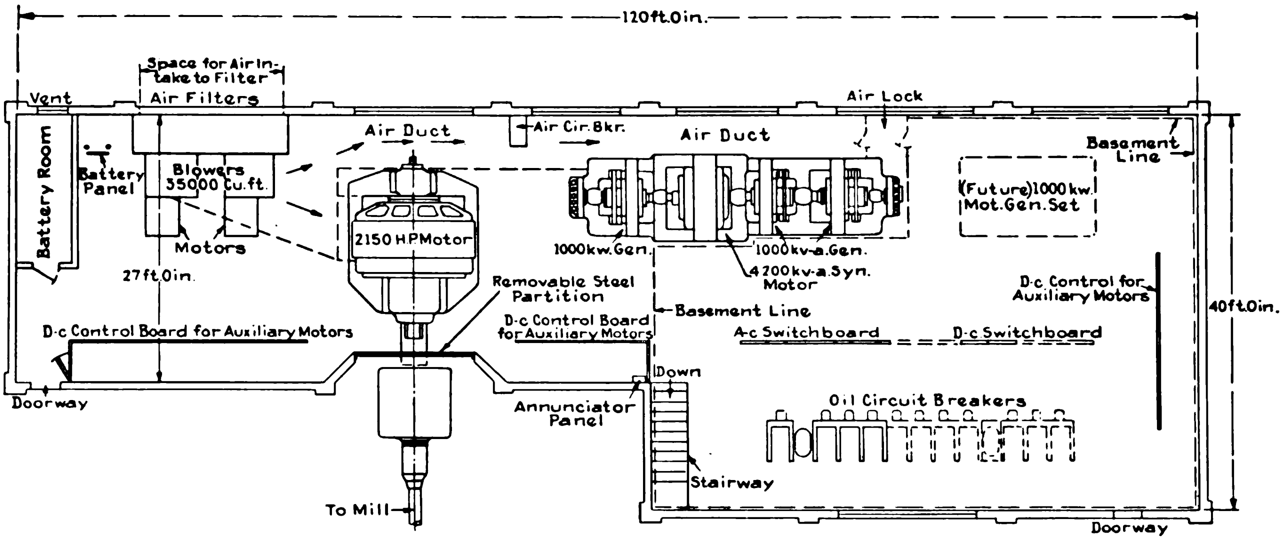

Fig. 4. Layout of the Motor Room at the 32-in. Reversing Mill

But the problem should have been considered from still another and broader angle. A power contract is, of course, binding on both parties to it. There was no doubt that the steel company could increase the instantaneous peaks, improve the power-factor, and obtain the corresponding reduction of power bills, remaining within the letter and the spirit of the contract. But, leaving the legal aspects aside, the question was whether the increased peak loads would or would not inconvenience in any way the power company.

The engineers of the latter were, therefore, approached and asked as to what they would rather have: An instantaneous peak load of 4500 kw. with an average integrated monthly power-factor of 95 per cent; or a 2000-kw. peak load with a 75 per cent power-factor? Needless to say, the power company's choice was in favor of the former conditions.

This helped to finally settle the question; and the order for the first electric steel mill reversing drive with a synchronous motor generator set was placed early in December of 1926.

Installation

The motor room plans were prepared by the electrical manufacturers and the construction and erection work was carried on by the steel company.

Fig. 4 gives the general layout of the electrical equipment and Figs. 5 and 6 represent several views of the motor room. It is separated from the mill by a brick wall. The motor and the set are supplied with filtered air provided by two motor-driven blowers. Contactor panels for auxiliary drives are also housed in the same room.

The layout is compact, yet ample clearances are left around the machines and control apparatus to make all parts accessible. Likewise, provision was made for possible future extension.

This installation is a good example of the evergrowing tendency on the part of the steel mills to well house and protect electrical equipment and to reduce, in this manner, the maintenance costs to a negligible amount.

The equipment was put in service in the latter part of 1927 and has been in continuous operation ever since. It is interesting to see how closely were the expectations realized.

Fig. 8 is reproduced from a high-speed wattmeter chart (1) showing the typical input to the 4200-kv-a. synchronous motor, while the mill is rolling the heaviest billets (13½ in. by 13½ in.). The general-purpose generator (G_3) was then carrying a fairly constant load of approximately 500 kw.; the combined maximum peak load was 3800 kw. on the 2200-volt line. As noted previously, with a flywheel set for the reversing mill, the combined peak would be about 1800–2000 kw.; thus the use of a synchronous motor increased the instantaneous demand by some 2000 kw.

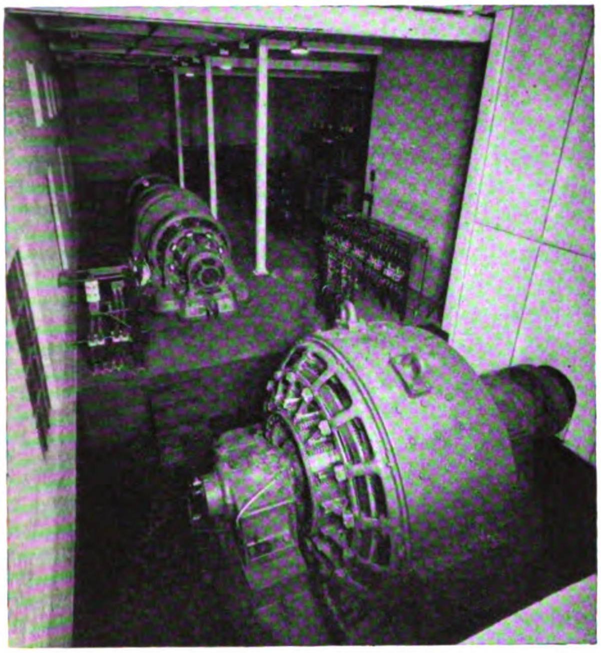

Fig. 5. General View of the 32-in. Mill Motor Room. The 2150-hp. 43/86-r.p.m. 550-volt reversing motor is in the front; the synchronous motor generator set is seen in the middle background

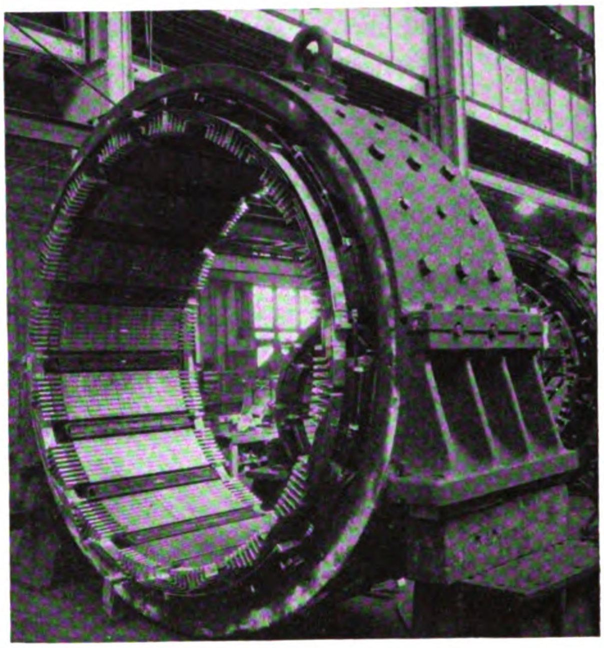

Fig. 7. Shop View of the 2150-hp. 43-r.p.m. Reversing Motor During Assembly

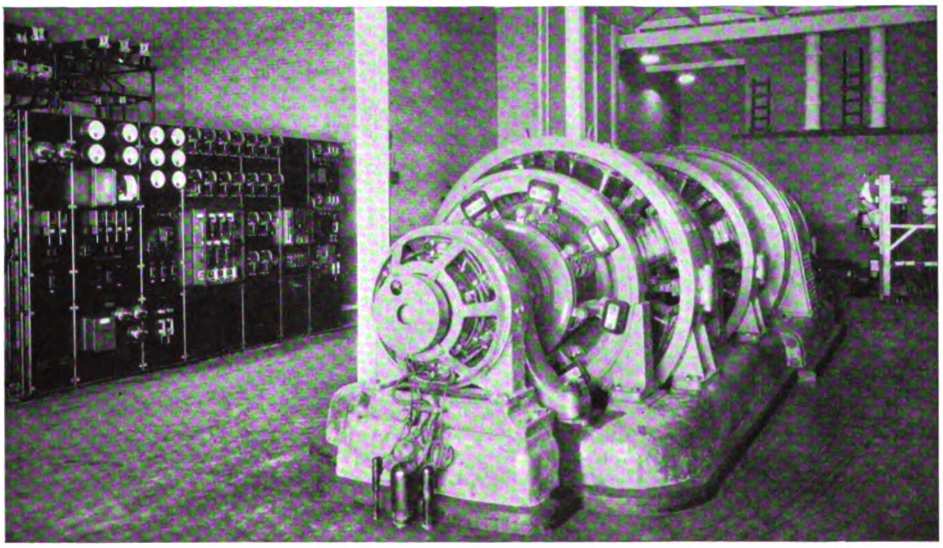

Fig. 6. Close-up View of the Synchronous Motor Generator Set. Switchboard and the reversing mill control are seen at the left

If this were the only large electric drive at the steel plant and the principal user of power, then such increase of momentary peak loads might under certain conditions cause some concern. For instance, the capacity of the transformer bank, or of a transmission line could be the limiting factor, possibly imposing such conditions as to require a flywheel type of drive.

Such a case, although conceivable, does not frequently occur. In the greatest majority of steel mills with an electrically-driven reverser there are usually a number of other motor drives; for instance, at the plant in question there are installed a 5000-hp. synchronous motor (see Fig. 9), a 3500-hp. induction motor, several 1600-hp. and 800-hp. motors, all driving rolling mills—not counting many smaller units. The larger motors, 3500 hp. and 5000 hp. respectively, drive piercing and expanding mills; the character of the load on these mills is such (a single pass of a tube through the rolls may last 20 to 30 sec.) that the application of a flywheel is out of question. Hence, the load is imposed directly on the a-c. system; this is something which is usually accepted as a matter of fact. Now, if the peak loads imposed by 3500-hp. or 5000-hp. non-reversing drives are acceptable, there is no reason why the maximum estimated 4400-hp. peak load of the reversing motor should not be equally acceptable. The actual operating experience has fully proven this contention.

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(1) Thanks are extended to Mr. O. Meineke, Electrical Superintendent of the Standard Seamless Tube Company, for furnishing this graphic record.

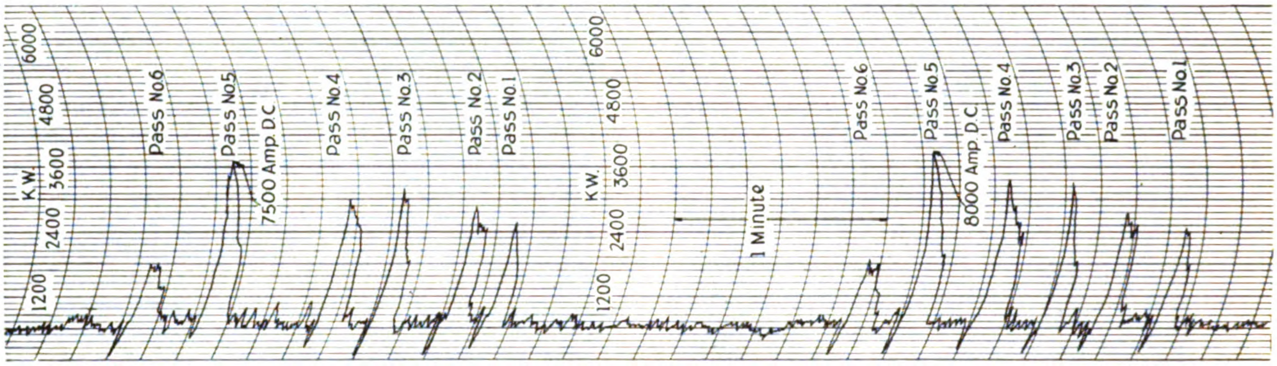


Fig. 8. High-speed Wattmeter Chart, Showing the Power Input to the 4200-kv-a. Synchronous Motor Driving the Set During Heavy Rolling. The peak load is, in this case, close to 4000 kw., of which about 700 kw. corresponds to the load on the general-purpose generator G_3, shown in Fig. 3

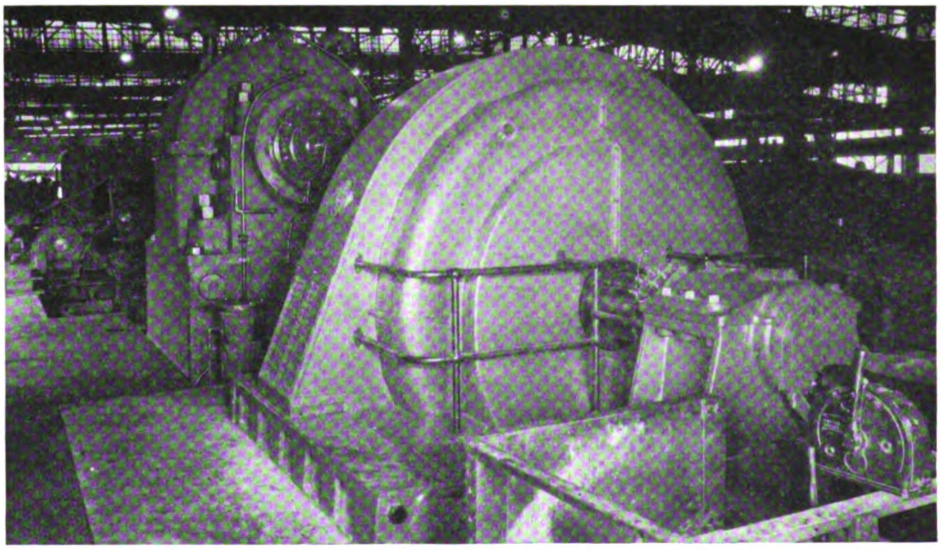


Fig. 9. A 5000-hp. 240-r.p.m. Synchronous Motor Driving the 51-in. Tube Expanding Mill at the Standard Seamless Tube Company, and Operating on the Same 2200-volt Power Line as the Motor-generator Set of the Reversing Mill. This is the largest seamless tube mill drive in the world

Conclusions

What was found to be true at the Standard Seamless Tube Company may be applied with equal reason to many other steel mills. In fact, this example of a reversing drive with a synchronous motor-generator set was quickly followed during 1927 in at least two cases, both by the General Electric Company and by other electrical manufacturers. This first installation is, therefore, particularly interesting and noteworthy as it is undoubtedly a forerunner of still larger reversing drives similarly equipped.

The power systems are still growing; their ability to absorb high-peak loads increases. It is but natural to expect that the day is not far distant when to provide even the largest reversing blooming mills with synchronous motor generator sets will be justified. Flywheel sets will still be built and used but their application will be a matter of judgment and not merely a question of habit.

Some Recent Developments in Motor Control

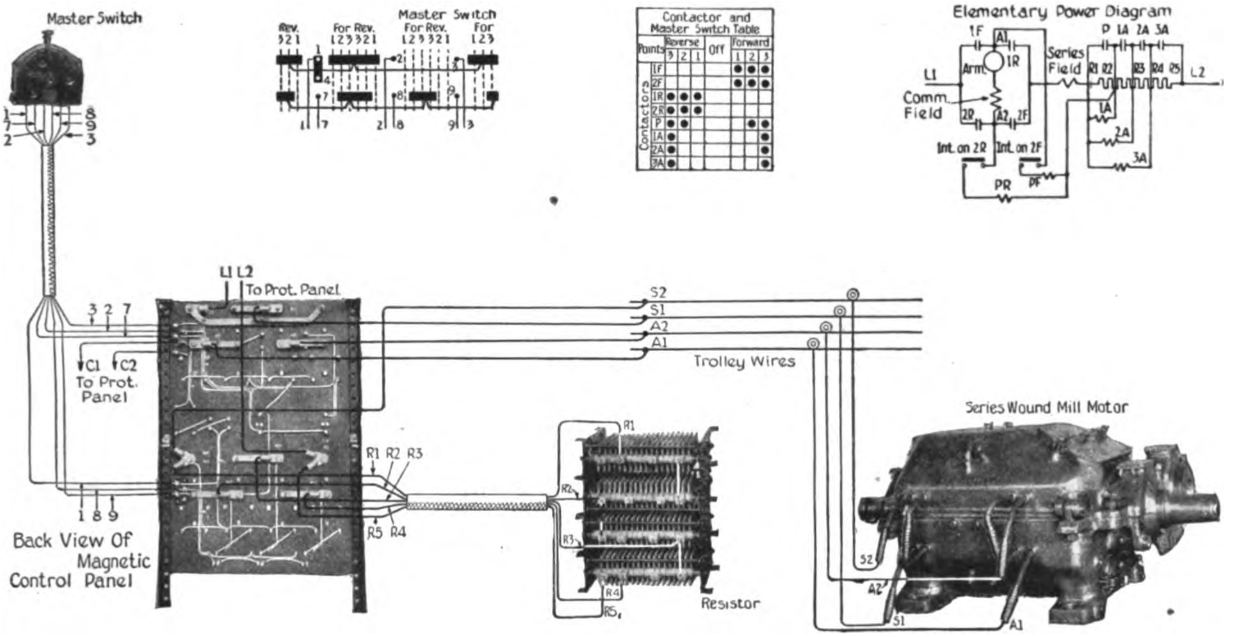
Primary Purpose of Control to Make Motor Work—Trend from Current-limit to Time Control—Non-reversing, Reversing, and Plugging Operations—A-c. and D-c. Equipment

By B. W. JONES

Industrial Control Engineering Department, General Electric Company

INDUSTRIAL control as a branch of electrical engineering is comparatively young. Twenty years ago, the control for industrial motors was fairly simple and consisted principally of manually operated devices. More recently, the development of this branch of engineering has centered around magnetic control, following which its advance has been more rapid. In the early days, one purpose of control seemed to be to protect the motor by preventing it from taking too much current during the accelerating period. Accordingly, the current-limit system was developed for both a-c. and d-c. motors. This system was ideal for protecting the motors because it prevented them from drawing more than a predetermined current during the accelerating period. If the load happened to be greater than that for which the control was set, the motor would not fully accelerate; but this fault was charged against the load and not the control.

When industrial loads became more varied and more exacting, the definition of control as a means for protecting motors did not seem to meet all conditions, and therefore a more adequate one was sought. Accordingly, control came to be considered as a means for making a motor do the required work up to the limit of its ability. With this viewpoint of the function of control, it was necessary to change from the current-limit system to one that more nearly accomplished the desired results. The system selected was one which accelerated the motor as a function of time, *i.e.*, it inserted a specified time delay between the accelerating steps. As will be pointed out later, this system has proved more adaptable for the miscellaneous types of loads than did the current-limit system.



Contactor and Master Switch Table

	Points	Reverse 3	Reverse 2	Reverse 1	Off	Forward 1	Forward 2	Forward 3
Contactors	1F					●	●	●
	2F					●	●	●
	1R	●	●	●				
	2R	●	●	●				
	P	●	●				●	●
	1A	●						●
	2A	●						●
	3A	●						●

Fig. 1. Diagram of Connections and Sequence Chart for Magnetic Time-delay Control of Steel-mill Auxiliary Motors

Time control has become very generally applied and will evidently remain. Control equipment of this type has been developed for both d-c. and a-c. motors, the latter including wound-rotor, squirrel-cage, and synchronous motors. Some details of these equipments will be described.

In general, control can be classified under two heads, *viz.*, reversing and non-reversing control. For controlling the acceleration of a motor, a time-delay device is interposed between each accelerating contactor and its successor. This allows a definite interval of time for the motor to respond in speed between the closing of each accelerating contactor. If the motor is lightly loaded it will more readily respond in speed than if heavily loaded. Therefore, the current peak taken by a time-controlled motor will vary directly as the load, and this is the desired result. To protect against excessive torques or currents, overload relays are used. For d-c.

motors where excessive torques may result if the load becomes jammed, an instantaneous relay is used for protection; but for squirrel-cage motors a temperature overload relay is used because the motor will take a fairly high current to start but the torque is inherently limited and the only precaution necessary is against overheating. Therefore, these combinations will cause the motor to accelerate any load which is within its capability and also will protect the motor against excessive torques or overheating.

If the control provides reversing connections, such as are used for steel-mill auxiliaries where the motor is plugged for stopping, there enters another factor into the control besides the simple time function. Sometimes the motor is electrically connected in the reverse direction (called "plugging") while it is rotating rapidly in the forward direction, and at other times the motor will be at rest when the reverse connection is made. When the motor is plugged, the total resistance needed in series with the armature of the motor may be 150 per cent ohms,(1) but if the motor is at rest only one-third of this resistance is needed. It is therefore obvious that the two-thirds resistance could not be short circuited as a function of time because it would cause an undue loss of time for each cycle of operation.

Reference to Figs. 1 and 2 will show how this problem has been solved for d-c. control. Its action can be defined as accelerating as a function of time but stopping as a function of current. Relays *PF* or *PR* are connected across the armature and a section of the resistor. If the motor is at rest, the voltage across the coil of the relay will be the *IR* drop of both the armature (which is small) and of the resistor which will be sufficient to close the relay and allow the plugging contactor to close. But if the motor is running in one direction and it is then reversed or "plugged," the counter e.m.f. of the armature will oppose the *IR* drop of the resistor and the voltage across the relay coil will be the difference between the armature voltage and the *IR* drop. This resultant voltage will be something between zero and the voltage that results from the standstill condition of the motor as just described.

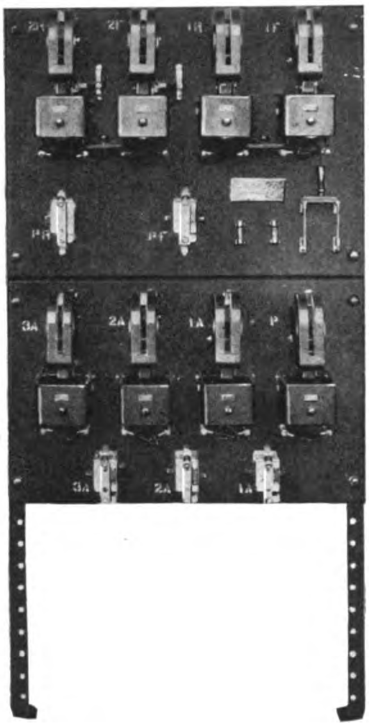


Fig. 2. Front View of the Control Panel Shown in Fig. 1

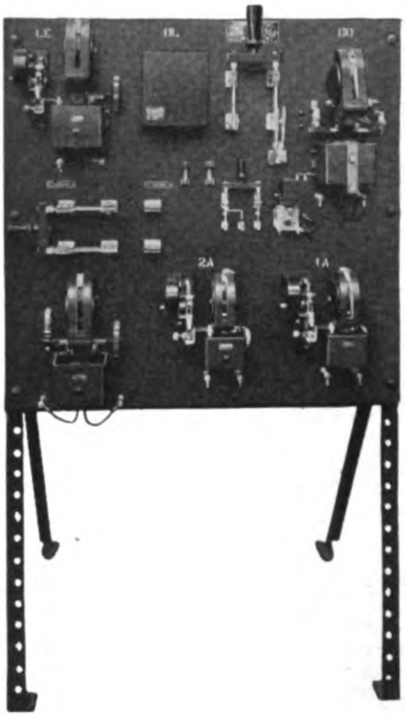


Fig. 3. Control Panel Showing the Use of Escapement-type Timing Relays Mechanically Connected to Resistor Contactors

When relays *PF* or *PR* have closed as a function of the counter e.m.f. of the motor, and have allowed the plugging section of the resistor to be short circuited by contactor *P*, the accelerating contactors are allowed to be closed consecutively under the control of the magnetic time-delay relays *1A*, *2A*, etc. The coil of relay *1A* being connected across the plugging section of the resistor, and the coil of relay *2A* being connected across the plugging and the first accelerating sections of the resistor, are both energized when the motor is first connected to the line and pick up and open their normally closed contacts. When the plugging contactor closes under the control of relay *PF*, or *PR*, the coil of the first accelerating magnetic time-relay *1A* is short circuited. The short circuiting of this relay coil causes the coil to be deënergized but the flux decreases slowly, thus holding the armature closed for an interval of time after the short circuiting of the coil. This interval of time is adjustable from one-quarter of a second to at least one second for each relay, which is sufficient for accelerating most loads.

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(1) The 100 per cent basis of this valve is the armature-circuit resistance that will permit only 100 per cent current to flow when line voltage is applied to the motor at standstill.

For the few cases where one second per relay is not a sufficient delay for the accelerating contactors, another timing relay is available. Fig. 3 shows this relay which is mechanically operated by the resistor contactors and can be adjusted over a range of two to eight seconds. Its principle of operation is that of an escapement. The device is used quite extensively for controlling the acceleration of a-c. motors. The *PF* and *PR* relays for controlling the plugging contactor would remain as has been described. Fig. 4 shows an oscillogram of the line current, the counter e.m.f. of the armature, and the resultant voltage impressed on the *PF* or the *PR* relay coil when used with an 80-hp. 230-volt series motor and when plugged from one direction of rotation to rest and then accelerated to full speed in the other direction. This illustration furnishes a graphic picture of the important factors in controlling a d-c. motor.

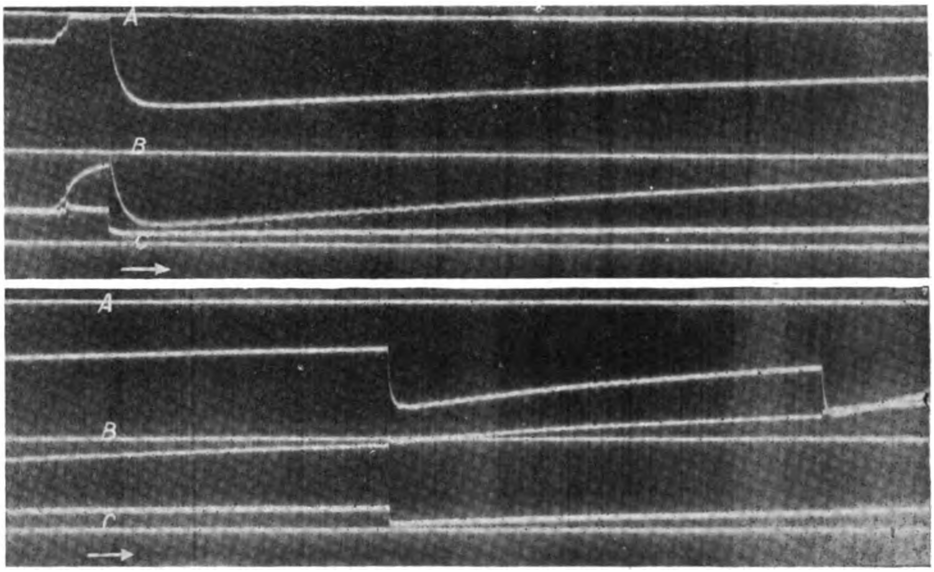


Fig. 4. Oscillogram Taken While Plugging a Series Motor to Rest Followed by Acceleration to Full-speed in the Reverse Direction

Curve *A:* Line current. Curve *B:* Counter e.m.f. of the motor.
Curve *C:* Resultant of the counter e.m.f. and the drop across part of the resistor which is impressed on the coil of the reversing relay.

The outstanding and interesting features of this system of d-c. motor control are as follows:

(1). By the use of two independent plugging relays, *PF* and *PR*, maximum protection is provided during the instant of plugging. When the motor is running in one direction the *PF* relay is active but the *PR* relay is inactive, though in the proper position to give protection when the equipment is suddenly reversed. No racing for the proper position takes place, because the relay is already in the correct open position.

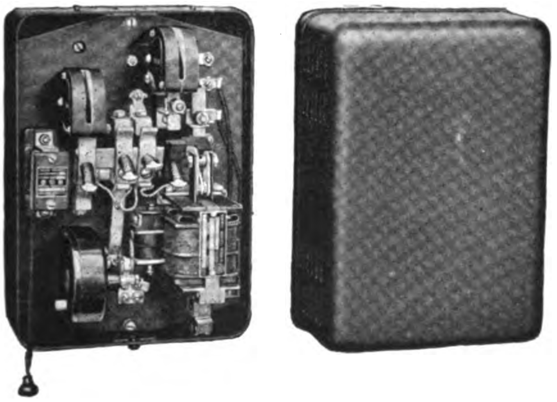


Fig. 5. A Three-point Non-reversing Magnetic Controller for Accelerating Small Motors as a Function of Time

(2). By insulating from the reversing section the section of the master controller which governs the resistor contactors, these contactors cannot be reënergized by a quick throwover of the controller. If insulation were not present the resistor contactors might be held closed by a quick throwover of the controller and this would plug the motor without resistance in the circuit and thus cause a flashover on the commutator. By using two plug relays and one insulated section on the controller, maximum protection is secured against this fault in operation.

(3). Since the relays *PF* and *PR* can be adjusted for pickup over a wide range of voltage, the plugging contactor can be closed before the motor comes to rest. This is an important time-saving feature because the motor would otherwise tend to come to rest quite slowly. If the counter e.m.f. of the motor had to reverse before the plugging contactor could reverse, much time would be lost on a rapid duty cycle.

(4). All the relays are the same in essential details and each has only one coil and therefore no polarity requirements. Also, since shunt coils are used, the relays are the same for all sizes of motors.

(5). Since the relays have no mechanical relationship to the contactors which they govern, the reversing and the resistor contactors can all be duplicates.

(6). The principles used make the entire scheme of operation exceedingly simple and effective.

For small motors (20 hp. and less) a small, compact starter has been developed. This equipment is shown in Fig. 5 and the same combination for reversing in Fig. 6. This type of equipment provides two

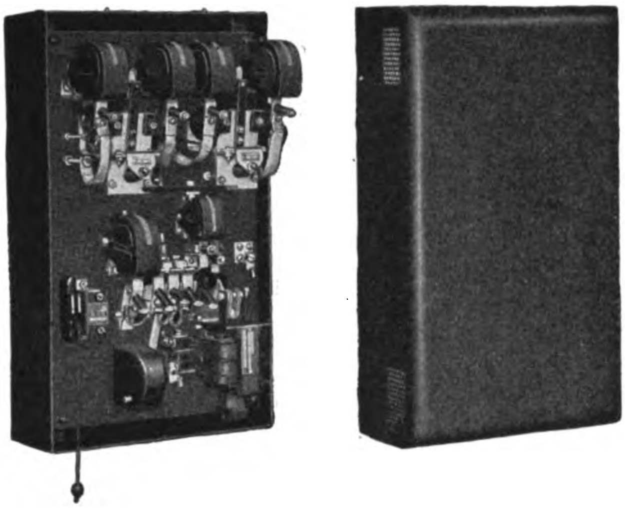


Fig. 6. A Four-point Reversing Controller for Accelerating a Small Motor as a Function of Time, Dynamically Braking it to Rest, and Operating the Motor in Either Direction of Rotation.

or three accelerating resistor divisions together with dynamic braking, provided the motor has a shunt field. These accelerating steps are cut out of circuit by a time action which is obtained by an ingenious escapement device. One magnet places the fingers under a bias to close but they are retarded by this escapement device. For a reversing equipment, reversing contactors are placed outside the starter circuit. Instead of plugging the motor as has been described, it is dynamically braked to rest. Of course, this applies only to motors having a shunt field. This operation also brings the motor to rest as a function of the counter e.m.f. and accelerates it as a function of time.

Since wound-rotor a-c. motors do not have a counter e.m.f. which can be used for operating a plugging relay in a manner similar to that of the d-c. motor, a different system has been devised employing timing relays which give a time interval when needed but which cause no delay when the motor starts from rest.

For accelerating, each contactor except the last one carries an escapement timing relay similar to those just described, except that the contacts are normally open and close at a short interval of time after the contactor closes. This timing is adjustable from two to eight seconds and governs the succeeding contactor.

To accelerate a squirrel-cage induction motor it is either connected directly to the line or is first connected to partial voltage until it has reached nearly full speed and then connected to line voltage. If the partial-voltage scheme is used, it is best to keep on partial voltage for a specified time and then to throw on full voltage. The practice of providing a time function instead of a current function to control this transfer from partial to full voltage is preferable because at partial voltage the motor frequently does not have much excess torque for acceleration; and if some variable such as increased load or low voltage exists, the motor might otherwise remain on partial voltage and cause overheating of both the motor and the starter. A time-governed control allows a sufficient time, under normal conditions, to accelerate the motor and if the motor has not reached the expected speed the motor should be connected to full voltage so that full speed can be reached. Fig. 7 shows a time-controlled automatic compensator.

A synchronous motor is in principle a squirrel-cage induction motor while it is accelerating from rest to 95 per cent speed, and therefore should be equipped with the same time-controlled starter. The excitation should be applied after the motor has reached a specified speed, as determined either by a relay that operates on the induced field current and frequency or by a definite-time relay that will have operated by the time the motor has attained this speed. Fig. 8 shows a synchronous-motor starter.

Test and experience have shown that a time function is a very suitable and practical means for controlling the acceleration of all kinds of motors and loads. The reason why it has not been used more generally in the past has been due to the lack of practical and dependable timing devices. Much progress has recently been made along this line and therefore time-functioned control is coming into extensive use.

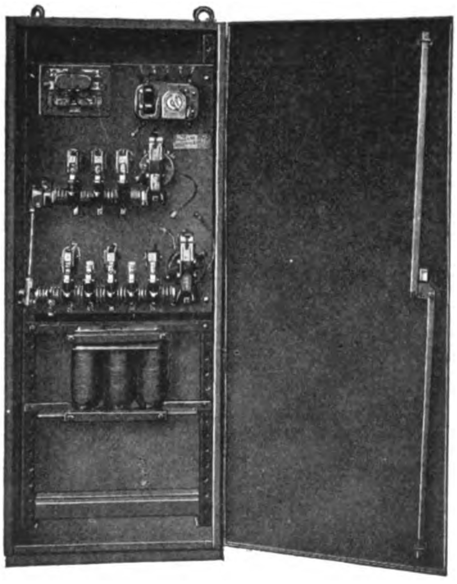


Fig. 7. Automatic Time-controlled Compensator for Squirrel-cage Induction Motors

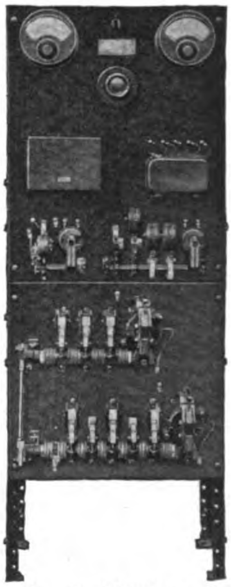


Fig. 8. Partial-voltage Synchronous-motor Starter in which the Starting Contactor is Held Closed as a Function of Time and the Field Contactor is Closed as a Function of the Induced Field Current

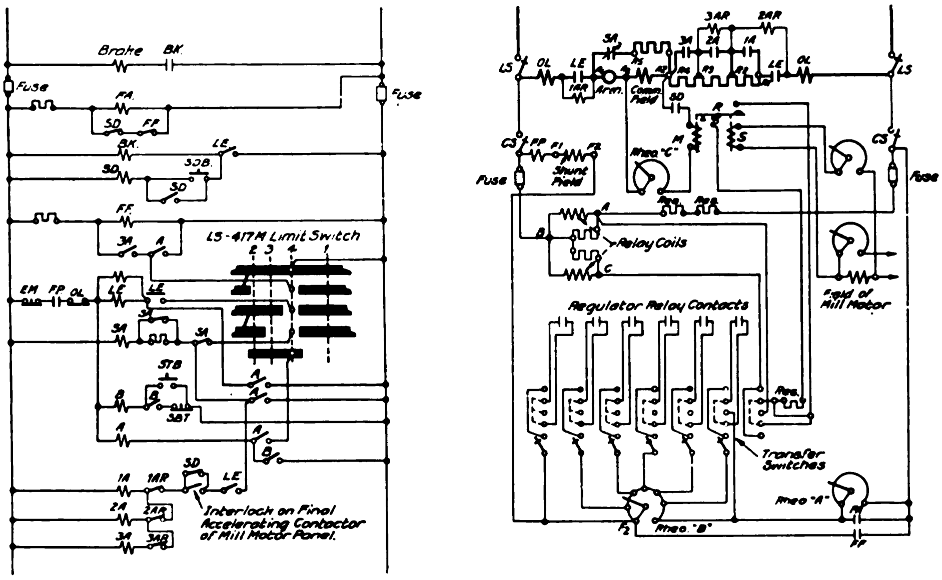
The Cold-rolled Strip-steel Reel for the Central Alloy Steel Corporation

Duty Cycle—Taking Up the Slack—Maintaining the Strip Tension—Stopping at Fixed Angular Position—Connection Diagram of Control Equipment

By A. SUKSDORF

Industrial Control Engineering Department, General Electric Company

A COLD-STRIP reel is a device for coiling cold-rolled strip steel as it passes from the cold-strip mill stand, or the last stand if there are several operating in tandem. An equipment for controlling the drive of such a reel has been recently built for The Central Alloy Steel Corporation. The motor employed is of 25 hp. and the speed is adjustable over a 4:1 range.



Sequence of Operation of Devices																							Operations Performed
L.S.	C.S.	F.P.	1AR	2AR	3AR	F.F.	FA	STB	B.	A.	SA.	L.E.	BK.	SB.	1A	2A	3A	SDB	SD	R.	SBT		
																					●		Stand Still
●			●	●	●																●		L.S. Closed
●	●	●	●	●	●	●	●														●		C.S. Closed
●	●	●	●	●	●	●	●	●	●												●		STB. Pushed
●	●	●	●	●	●	●	●		●	●	●	●	●	●							●		ACC. To Basic Speed
●	●	●		●	●	●	●		●	●	●	●	●	●	●						●		
●	●	●			●	●	●		●	●	●	●	●	●	●	●					●		
●	●	●				●	●		●	●	●	●	●	●	●	●	●				●		
●	●	●					●		●	●	●	●	●	●	●	●	●				●		ACC. To Max. Speed
●	●	●							●	●	●	●	●	●	●	●	●				●		
●	●	●							●	●	●	●	●	●	●	●	●	●	●	●	●		SDB. Pressed
●	●	●					●		●	●	●	●	●	●	●	●	●		●	●	●		Operating Speed
●	●	●					●			●	●	●	●	●	●	●	●		●	●			SBT Pressed
●	●	●					●				●	●	●	●	●	●	●		●	●	●	1 Position of LIT.	Slow Down
●	●	●				●	●				●	●	●	●	●	●	●		●	●	●	2	
●	●	●				●	●					●	●	●					●	●	●	3	
●	●	●	●	●	●	●	●														●	4	Stopped

LS = Line Switch
CS. = Control Switch
LE = Line Contactor
SA = Armature Shunting Contactor
1A-3A = Accelerating Contactors
BK = Solenoid Brake Contactor
OL = Overload Relay.
1AR-3AR = Accelerating Relay
FP = Field Protective Relay.
FA = Field Accelerating Relay.
FF = Full Field Relay.
AB. & SD = Control & Relaying Contactors.
R. = Strip Tension Regulator
SB. = Solenoid Brake
STB = Start Button
SBT. = Stop Button
SDB = Slow Down Button
LIT. = Limit Switch
EM = Emergency Stop Button

Fig. 1. Circuit Connections and Operating Sequence of the Control for the Reel Drive

The functions of the control and the conditions of operation are as follows: To start and accelerate the motor to maximum speed; then, after the slack is taken out of the strip, to slow down the motor to the strip speed. The motor must hold constant tension on the strip while coiling and must also hold tension on the strip while the mill is shut down. In shutting down, the mill must stop at the same definite angular position each time.

To start the reel the operator steps on the "Start" button (all push-buttons except the emergency stop button are foot-operated) and the motor is accelerated to its basic speed by contactors, sequenced and actuated by magnetic time relays. These contactors cut out sections of the starting resistor. From basic speed, the motor is further accelerated by two field relays that successively cut resistance into the motor-field circuit, thereby bringing the motor to maximum speed which is somewhat higher than the strip speed in order to take the slack out of the strip. The maximum speed to which the motor accelerates is adjustable and is determined by a hand-operated rheostat. After nearly all the slack has been taken up, the operator steps on the "Slow" button. This short circuits a field rheostat and decreases the speed of the motor. The strip-tension regulator then comes into operation to keep a constant tension on the strip.

This strip-tension regulator is essentially a current-regulating device that controls the field current and thereby the speed of the motor in such a manner as to maintain the correct tension on the strip. The regulator is operated by two coils in opposition to each other. The main coil is connected across the commutating field of the reel motor; therefore the influence of this coil is directly proportional to the armature current of the motor. The other coil, which is in series with an adjustable rheostat that is mechanically connected to the field rheostat of the mill motor, is connected across the field of the last mill motor. Its influence is proportional to the setting of the field rheostat of the mill motor.

The regulator does not handle the motor-field current directly but operates a relay that controls the field current through six relay contacts, each of which operates a section of the motor rheostat. Any change in the speed of the reel motor will affect the tension in the strip and therefore cause more or less current to flow through the armature circuit. This current acts on the main coil of the regulator to increase or decrease the field current. The cycle of operation of the regulator is comparatively rapid and therefore causes the motor to hold constant tension.

With the equipment in normal operation and the mill motor shut down with a strip in the mill, it is essential that the reel motor hold the strip in tension at reduced torque. This is accomplished by a normally open interlock on the last accelerating contactor of the mill motor panel. The opening of the interlock deënergizes all the accelerating contactors on the reel-motor panel and places resistance in the armature circuit of the reel motor which is then stalled and held on the line with 50 to 100 per cent of full-load torque.

When the mill motor is again started, and the interlock on the final accelerating contactor closes, the circuit is made for the accelerating contactors of the reel motor to close when actuated and sequenced by the time-limit accelerating relays. Under this condition of acceleration the regulator still continues effective, while the field relays remain ineffective.

When the drive is to be stopped, the operator steps on the "Stop" button; and the limit switch then starts to function. Part of the circumference of the limit switch is used to slow down the motor before it is taken off the line and the brake set. If the "Stop" button is pressed while this section of the limit switch is effective the reel will be carried around another revolution, so that the motor will always start to slow down at the same angular position of the limit switch. In the process of slow-down, increased field current is first put on the motor, next the series or starting resistance is cut into the armature circuit and a resistor connected in parallel with the armature. The limit switch is adjustable and cam-operated, so that the starting point of slow down may be adjusted to suit the conditions of operation. By the time the motor has reached the stopping position, it has slowed down to practically creeping speed. At the "Off" position of the limit switch, the motor is taken off the line and the solenoid brake set. The slowing down of the motor to a creeping speed and the setting of the brake insures the reel's always being stopped in the same angular position with the jaws ready for the next strip.

In case of serious operating trouble the "Emergency Stop" button is pressed, which immediately takes the motor off the line and sets the brake thus bringing the reel quickly to rest without regard to the angular position of the stop.

In Fig. 1 is shown a simplified connection diagram of the control circuit, the main motor and field circuits, and also a chart that indicates the sequence operation of the different contactors.

New Type Transformer for Post-top Mounting

A new form of street-lighting transformer especially constructed for mounting on the top of a trolley pole has simplified the installation problem by eliminating the necessity of drilling the pole for the secondary leads. It also eliminates the unsightly appearance caused by mounting side-bracket transformers.

A support just below the top of the transformer carries insulators to which the street-lighting cable is dead ended. The primary leads are connected to these, while the secondary leads go through the pole and out into the units through the brackets, thus leaving no exposed wires.

A Study in Welded Plate Girders

Problem of Designing a Plate Girder—Step-by-Step Solution—Comparison of Weights of Welded and of Riveted Construction

By FRANK P. McKIBBEN
Consulting Engineer, Black Gap, Pennsylvania

A PLATE girder is a structural steel member consisting of a steel plate, called a web plate, to which other steel shapes such as angles and plates are attached along its two parallel edges to form flanges. Its function in a building or a bridge is to support loads or pressures acting transversely to its length. Briefly, a plate girder is a steel beam composed of a web and two flanges. Until recently, riveting was the sole means of connecting the flanges to the webs, but now welding is beginning to supersede riveting.

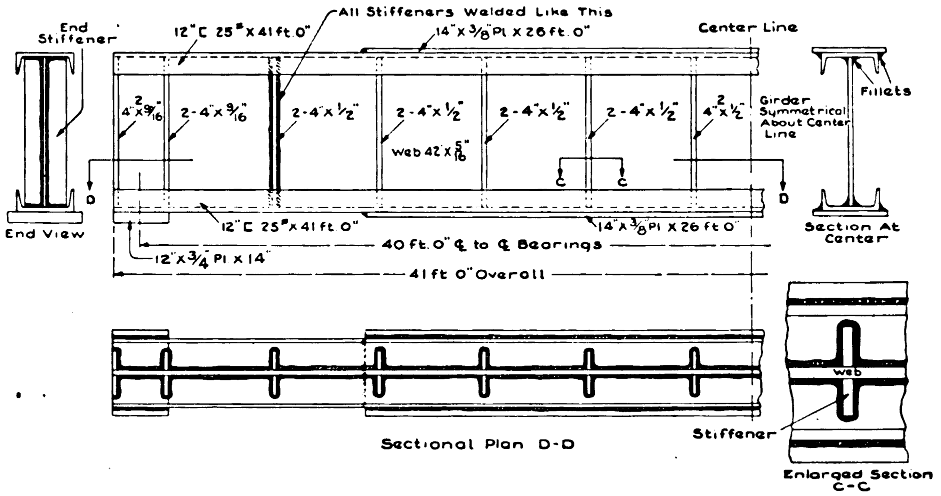


Fig. 1. One-half of Welded Girder Designed to Carry 4000 Pounds Per Linear Foot with a Span of 40 ft. 0 in. Center to Center of Bearings; Design No. 4

Note that ⅜-in. by ⅜-in. continuous fillets are used in place of intermittent fillets determined by the calculations.

As the design of welded plate girders differs somewhat from that of usual riveted forms, the complete calculation for a typical welded girder is here presented together with a comparison between the weights of several welded and riveted types.

The problem succinctly stated is: Design an electrically welded girder, of 40-ft. span, to be supported at its ends, and to carry a uniformly distributed load of 4000 pounds per linear foot including the girder itself; and compare the result with riveted girders performing a similar function.

The following assumptions are permissible:

Shear on gross area of web, 10,000 lb. per sq. in.
Tension in flanges, 16,000 lb. per sq. in.
Compression flange assumed to be braced laterally.
Shear on welded fillet, 9,000 lb. per sq. in. of limiting section. The permissible shearing values of fillets shown in Table I are based on this safe shearing strength of 9000 lb. per sq. in., which is a very conservative value. For structures where capable welders and good inspection are employed, this value can safely be taken at 11,200 lb. per sq. in., giving for a ⅜-in. by ⅜-in. triangular fillet a working unit stress of 3000 lb. per linear inch of fillet.
Rivets, ¾-in. diameter, with shear and bearing at 12,000 and 24,000 lb. per sq. in. respectively.
Girder design is controlled by strength or deflection or both. In what follows it is assumed that only strength need be considered.

WELDED PLATE GIRDER

Design No. 4 (Figs. 1 and 2)

Item A

End shear $= 4000 \times 20 = 80{,}000$ lb.

Required gross area of web $= \frac{80{,}000}{10{,}000} = 8$ sq. in.

A web 42 in. by ¼ in. meets this requirement but on account of web buckling use 42 in. by $\frac{5}{16}$ in.

Item B

The horizontal distance between web stiffeners is given by the formula

$$d = \frac{t}{40}\left(12{,}000 - \frac{V}{A}\right) = \tfrac{5}{16} \times \tfrac{1}{40} \times \left(12{,}000 - \frac{80{,}000}{13.1}\right) = 46 \text{ in.}$$

where d = distance in inches between stiffeners.
t = thickness of web.
V = vertical shearing force at section.
A = gross cross-sectional area of web.

As the value of d from this equation exceeds the web depth of 42 in., the latter will be used as the approximate horizontal distance between stiffeners.

Item C

The cross-sectional area of the tension flange at the center of the span is:

$$\text{Net area} = \frac{M}{h'S} - \tfrac{1}{8}th$$

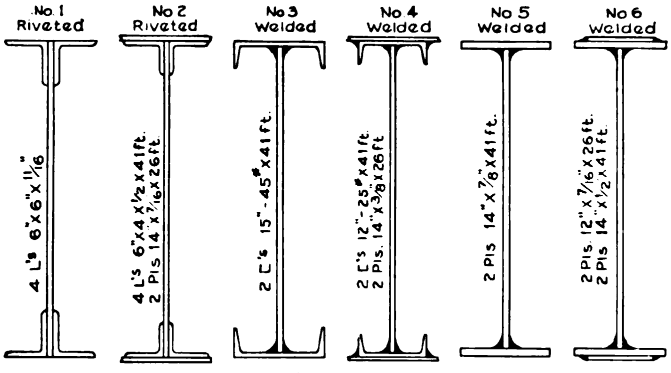


Fig. 2. Central Sections of Girders, Nos. 1 to 6 Inclusive

where M = bending moment at center of span.

h' = effective depth.

S = allowable unit tension.

t = web thickness.

h = web depth.

(Note that in this case the effective depth h' and depth of web, h, are practically the same.)

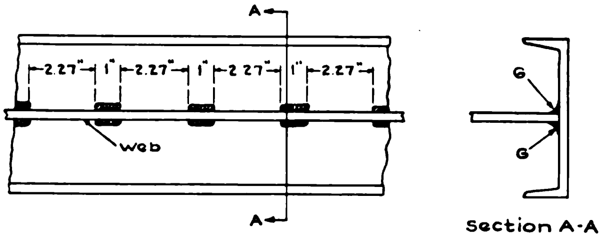


Fig. 3. Plan of Lower Flange Showing Required Spacing for ⅜-in. Fillets, Connecting Web and Flange at Ends of Girder; Design No. 4

$$\text{Net area} = \frac{9{,}600{,}000}{42 \times 16{,}000} - \tfrac{1}{8} \times \tfrac{5}{16} \times 42 = 12.08 \text{ sq. in.}$$

One 12-in. channel (25 lb.) = 7.35 sq. in.

One 14-in. by ⅜-in. plate = 5.25 sq. in.

Total = 12.60 sq. in.

Item D

As the girder is loaded uniformly, the curve of moments is parabolic; hence, one-half length of the 14-in. by ⅜-in. plate equals:

$$\sqrt{\frac{5.25 \times (20)^2}{14.78}} = 12.0 \text{ ft.}$$

Make the plate 26.0 ft. long.

Item E

Since the compression flange is supported laterally it will be made like the tension flange. However, if the ⅜-in. cover plate on the compression flange be welded only along the channel flanges, the distance between these welds would be 12 in. or 32 times the thickness of the cover plate. As the maximum edge distance usually allowable is only eight times the plate thickness, the design as shown is not ideal. This difficulty could be overcome by punching two rows of holes in the compression plate and plugging these with weld material, thus connecting the plate to the channel. However, since the distance in question is an internal and not an edge distance the design shown is considered adequate.

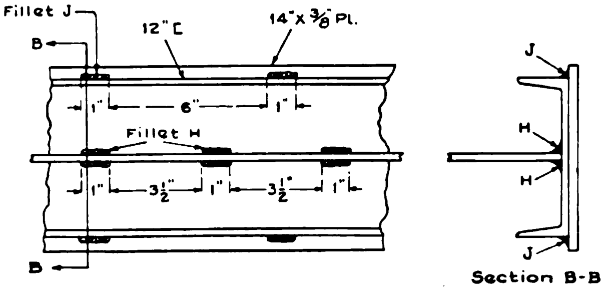


Fig. 4. Plan of Lower Flange Showing Required Spacing for ⅜-in. Fillets, Connecting Web and Channel, also Channel with 14-in. Plate; Design No. 4

Item F

To determine the fillet at G, Fig. 3, proceed as follows: These fillets transmit the longitudinal shear from web to flange. This longitudinal (horizontal) shear $= \frac{VQ}{I}$

where

V = vertical shearing force on web at the section under consideration.

Q = statical moment, about neutral axis of girder, of that part of flange section outside of plane on which longitudinal shear is desired.

I = moment of inertia, about neutral axis of girder, of the entire cross section of web and both flanges at section under consideration.

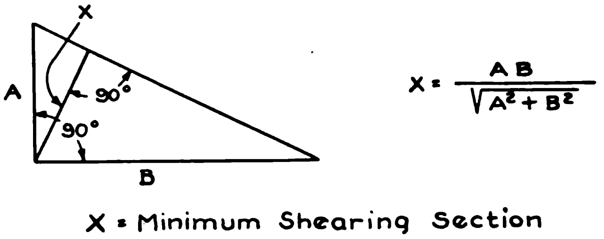

X
A
90°
90°
B
$X = \frac{AB}{\sqrt{A^2 + B^2}}$
X = Minimum Shearing Section

Fig. 5. Cross-section of Triangular Fillet

Thus at end of girder:

$V = 4000 \times 20 = 80{,}000$ lb.

Q = statical moment of one channel
$= 7.35\ (21.53 - 0.68) = 153$

I = moment of inertia of web and two channels

$$\begin{aligned} \tfrac{1}{12} \times \tfrac{5}{16} \times 42^3 &= 1929 \\ 2\,[7.35\ (21.53-0.68)^2 + 4.5] &= \underline{6409} \\ I &= 8338 \end{aligned}$$

$\frac{VQ}{I} = \frac{80{,}000 \times 153}{8338} = 1470$ lb. per linear inch on two fillets marked G in Fig. 3, or 735 lb. for each.

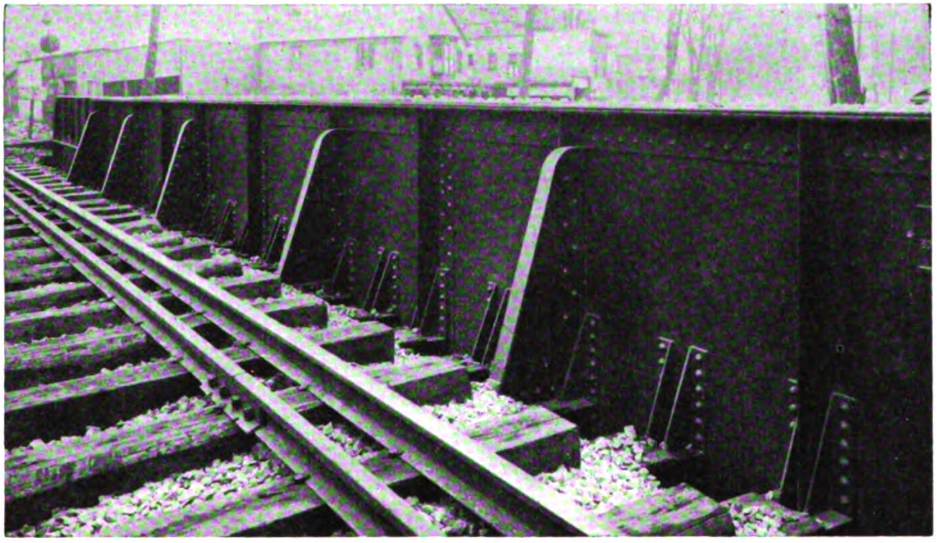


Fig. 6. Half-through Riveted Plate Girder of Railroad Bridge

From Table I the shearing strength of one $\frac{3}{8}$-in. by $\frac{3}{8}$-in. fillet per linear inch is 2400 lb., therefore the spacing required for these fillets at the end of the girder is:

$\frac{2400}{735} = 3.27$ in., arranged as in Fig. 3.

While Fig. 3 shows fillets only one inch long as illustrating the results of these calculations, the writer's experience with welding in shops shows that with one-inch fillets, *i.e.*, one inch in length, the crater remaining at the end of each fillet forms too great a proportion of the short weld. If fillets two inches long were used, the computations would give $\frac{2400 \times 2}{735} = 6.5$ in. This requires fillets 2 in. long alternating with open spaces $4\frac{1}{2}$ in. long. However, continuous fillets are recommended for this girder as shown in Fig. 1.

Item G

Fillets H and J, Fig. 4, at a point 13 ft. from the center of the span where the 14-in. by $\frac{3}{8}$-in. cover plate ends are obtained in a similar manner. Thus:

For fillet H, Fig. 4

$V = 4000 \times 13 = 52{,}000$ lb.

$Q = 7.35\ (21.53 - 0.68) + 5.25 \times 21.72 = 153 + 114 = 267$

$I = 8338 + 2\,[5.25 \times 21.72^2] = 13{,}308$

$\frac{VQ}{I} = \frac{52{,}000 \times 267}{13{,}308} = 1040$ for two fillets H or 520 lb. for one fillet.

Pitch of $\frac{3}{8}$-in. by $\frac{3}{8}$-in. fillets one inch long $= \frac{2400}{520} = 4\frac{1}{2}$-in. (see Fig. 4).

For fillet J, Fig. 4

$\frac{VQ}{I} = \frac{52{,}000 \times 114}{13{,}308} = 450$ lb. per linear inch for two beads or 225 lb. for one. But as the spacing for intermittent welds required for beads J to connect the 14-in. by $\frac{3}{8}$-in. cover plate to the channel is $\frac{2400}{225} = 10.6$ in., a distance so great as to afford insufficient protection against buckling of the $\frac{3}{8}$-in. cover plate on the compression flange, these fillets J will be

TABLE I

Safe Permissible Shearing Values of Fillets for Use in Design

A (Inches) (Fig. 5)	B (Inches) (Fig. 5)	X (Inches) (Fig. 5)	Safe Strengths Per Linear Inch of Fillet at 9000 Lb. Per Sq. In. on Section X (Pounds)
½	½	0.355	3200
⅜	½	0.300	2700
⅜	⅜	0.266	2400
¼	⅜	0.208	1870
¼	¼	0.177	1600

made not to exceed 16 times the thickness or 6 in. as shown in Fig. 4.

Item H

End stiffeners over the bearings may be of four **L**'s 4 in. by 3 in. by $\frac{9}{16}$ in. for riveted, or four flats 4 in. by $\frac{9}{16}$ in. for welded girders, designed as follows. On account of unequal bearing resulting from the girder's deflection let us assume that three-quarters of the end reaction is carried by each pair of bearing stiffeners., *i.e.*, $\frac{1}{2} \times \frac{3}{4} \times 80{,}000 = 30{,}000$ lb. per stiffener.

In thirty years' experience with steel construction, the writer has observed only two cases where stiffeners were weak and in both instances the angle failed by buckling in the outstanding leg near the bottom.

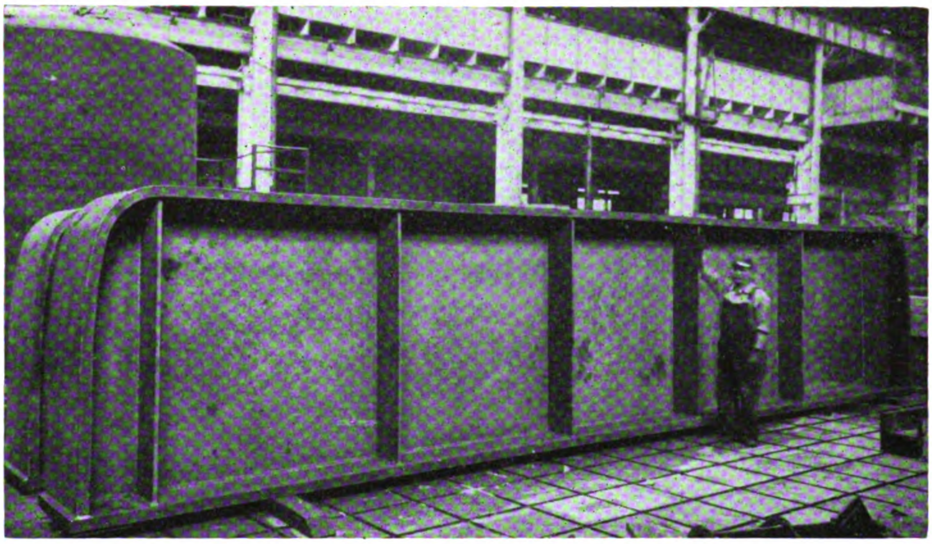


Fig. 7. Fabricated Welded Girders for Upper Bearing Bracket of 19,000-kv-a. Waterwheel Generator

Each stiffener will therefore be designed as a column 8 in. long and only the outstanding leg will be assumed to carry the load of 30,000 lb.

For a 4-in. by $\frac{9}{16}$-in. section the radius of gyration squared about central axis parallel to longer side is:

$$\frac{\frac{1}{12} \times 4 \times (\frac{9}{16})^3}{4 \times \frac{9}{16}} = \frac{27}{1024}$$

and the average permissible unit compression is:

$$\frac{16{,}000}{1 + \frac{8^2}{20{,}000\,(\frac{27}{1024})}} = 14{,}300 \text{ lb. per sq. in.}$$

The average actual unit compression

$$= \frac{30{,}000}{4 \times (\frac{9}{16})} = 13{,}300 \text{ lb. per sq. in.}$$

The 4-in. by $\frac{9}{16}$-in. flat bars or 4-in. by 3-in. by $\frac{9}{16}$-in. **L**'s are satisfactory; the former for use on welded and the latter on the riveted girders.

Item J

The length of $\frac{3}{8}$-in. by $\frac{3}{8}$-in. fillets required to transmit end shear from each end stiffener into the web is $\frac{30{,}000}{2400} = 12\frac{1}{2}$ in.

In Fig. 1, which shows the welded girder, these fillets are extended from bottom to top flange. Continuous, not intermittent welding, has been used

TABLE II

Comparison of Weights of Riveted and Welded Girders

Design No.	Weight (Pounds)	Pounds Saved Over No. 1	Per Cent of No. 1 Saved	Remarks
1	7498	....	....	Riveted
2	6455	1043	13.9	Riveted
3	6282	1216	16.3	Welded
4	5572	1926	25.7	Welded
5	6012	1486	19.8	Welded
6	5467	2031	27.1	Welded

TABLE III

WEIGHT OF WELDED GIRDER, No. 4

Section	Expression	Weight (Lb.)	Remarks
2 Channels 12 in. (25 lb.) by 41 ft. 0 in.	$2 \times 25 \times 41$	2050	Flange
2 Plates 14 in. by $\frac{3}{8}$ in. by 26 ft. 0 in.	$2 \times 17.85 \times 26$	930	Flange
1 Plate 42 in. by $\frac{5}{16}$ in. by 41 ft. 0 in.	44.63×41	1830	Web
8 Flats 4 in. by $\frac{9}{16}$ in. by 3 ft. 6 in.	$8 \times 7.68 \times 3.5$	214	End Stiffeners
20 Flats 4 in. by $\frac{1}{2}$ in. by 3 ft. 6 in.	$20 \times 6.8 \times 3.5$	476	Intermediate Stiffeners
2 Plates 12 in. by $\frac{3}{4}$ in. by 1 ft. 2 in.	$2 \times 30.6 \times 1.17$	72	Bearing Plates
		5572	

throughout even though calculations show the latter to be of sufficient strength. The continuous fillet is more desirable, particularly in the top flange; and it costs little if any more than the intermittent, especially if automatic welding machines be used for the continuous fillets. But if intermittent fillets be used, a minimum length of two inches exclusive of the crater is a desirable practical limitation.

COMPARISON OF WEIGHTS

To compare the weights of welded and riveted girders Table II is presented, showing that a saving of 27.1 per cent in weight of steel results by using welded girder No. 6 instead of riveted girder No. 1, *i.e.*, 27.1 per cent of the riveted girder. But this is not quite a fair comparison because the riveted girder No. 1 has flange angles only, which extend the entire length, whereas in welded girder No. 6 weight has been minimized by cutting off the flange plates to conform with the parabolic moment curve. On the other hand, the riveted girder No. 1 is a type in very common use and the comparison presented is justifiable. Another, and more equitable comparison is furnished by riveted girder No. 2 and welded girder No. 6, as each has the outer flange plates cut to meet the moment requirements. A saving of 6455 − 5467 = 988 lb. or 15.2 per cent is shown for the welded girder No. 6, *i.e.*, 15.2 per cent of the riveted girder.

A more precise comparison of weights would take cognizance of rivet heads and punchings from plates and angles on the one hand, with weights of welding electrodes consumed on the other.

For the sake of completeness, the weight calculations for the welded girder No. 4 are given in Table III. Other weights are similarly computed.

CONCLUSIONS

A study of the results obtained reveals that the saving in weight of welded girders is due to the following factors:

(1). Elimination of all loose fillers.

(2). Use of flat bars instead of angles for stiffeners, thus eliminating the metal in angle legs in contact with the web.

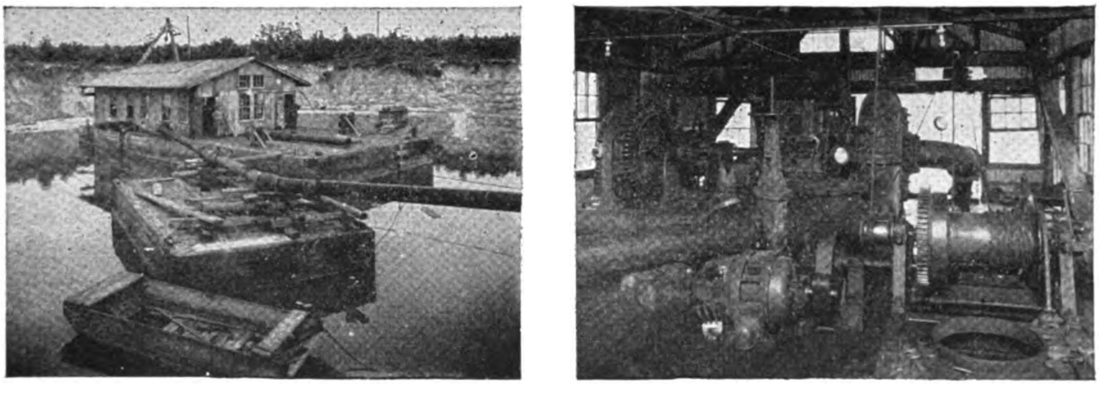
(3). Elimination of rivet holes in tension flanges, because gross areas are used in the welded designs and net areas in the riveted.

(4). Inasmuch as the "web equivalent" in welded girders is one-sixth the web area, while in riveted girders it is only one-eighth, smaller flanges are permissible in welded types.

Thus, based on savings in weight alone, welded girders have great possibilities.

Electrification Reduces Dredging Costs 24 Per Cent

Unusual interest is attached to the results obtained from the conversion of two steam-operated dredges to electric drive. These dredges are the property of the Menantico Sand & Gravel Co.; and they had been equipped with 225-hp. Corliss engines belted to 12-in. pumps employed in excavating sand and gravel for road and building purposes, and also for foundry and sand blast uses. Four thousand-volt synchronous motors of 300 hp. at 450 r.p.m., with full automatic control, were installed to replace the steam engines.



Exterior and Interior Views of One of the Electrified Dredges

These electrically-operated dredges have now been in operation a sufficient length of time to make comparisons available between the superseded steam and the present electric drive. In addition to the realization of those benefits common to electric drive, such as cleanliness, simplicity, safety, and service reliability, the following condensed data from the operating account furnished by the owner show in a rather astonishing manner the further advantages after the conversion from steam power:

Annual running time of pumps.......	Reduced	15	per cent
Labor cost per ton..................	"	56	" "
Unit cost per ton...................	"	24	" "
Annual production..................	Increased	60	" "

Short-time Current Carrying Capacity of Copper Wire

By E. R. STAUFFACHER
Superintendent of Protection, Southern California Edison Company

As POWER systems grow in size the possible concentration of energy at some point where a short-circuit or flashover to ground occurs becomes correspondingly greater; and one of the predominating present-day requirements of electrical apparatus is its ability to withstand short-circuits. This applies also to the manner in which the equipment is installed and utilized, in other words, to the design of the power house, substation, transmission or distribution line. The utilization of copper wire in apparatus or as some connecting link in the scheme for controlling short-circuit currents should therefore be considered not only from the standpoint of continuous carrying capacity but also from the standpoint of its short-time current carrying capacity.

The ability of copper wire to withstand heavy currents for short periods of time, varying in length from a fraction of a second to a few seconds, has been studied; and the results are given in this article. One of the applications where a knowledge of the short-time current carrying capacity of copper wire is of value is where copper wire is used for bonding and grounding the insulator pins of wooden pole transmission lines or distribution lines operating at 10 kv. or above. This bonding is occasionally employed for the purpose of preventing pole-top fires which might occur during a fog or rain as a result of minute current leakage over the insulators, across the crossarms, and down the pole.

In case of a flashover of an insulator to ground, or the simultaneous flashover of two insulators, it will be necessary to have the copper wire used for bonding of such size that it will not be melted by the short-circuit current within the time required to clear the faulty line from the system. This time depends upon the protective relay settings and the speed of operation of the oil circuit breakers connected to the terminals of the line in trouble.

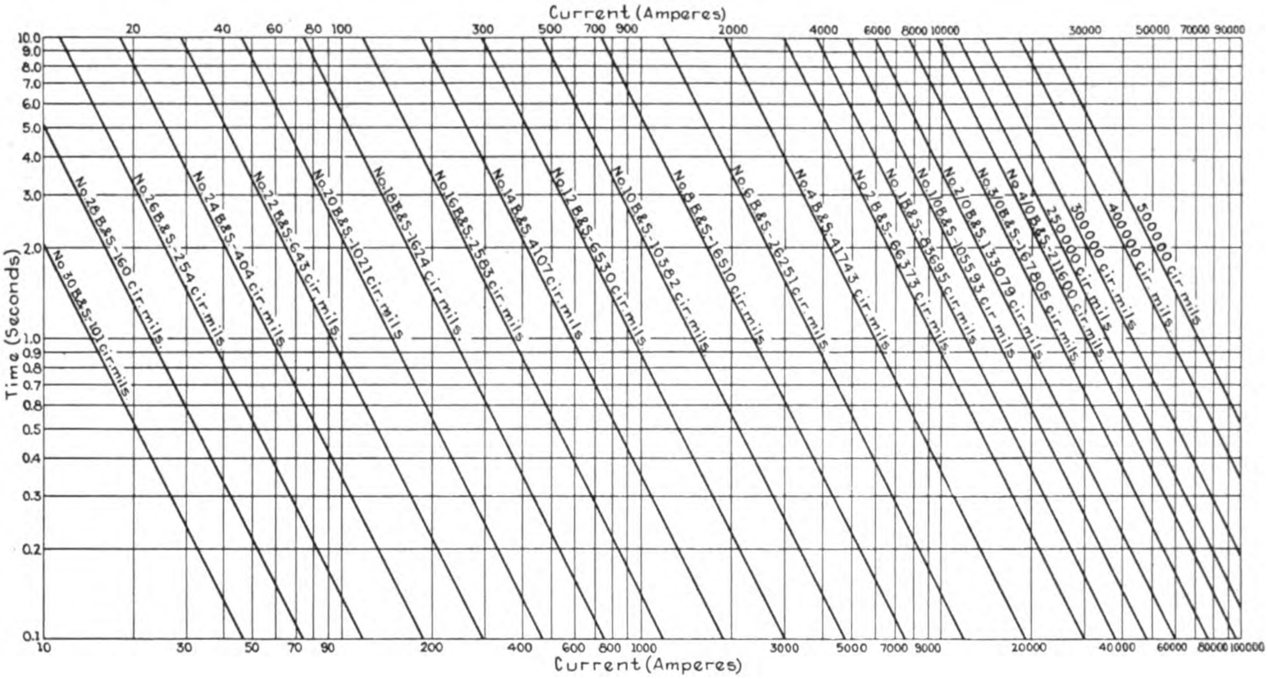


Fig. 1. Chart Showing Ultimate Rating of Copper Conductors with Respect to Current and Time

The consideration of short-time current carrying capacity of a copper conductor is also desirable when laying out the ground busses in generating plants and substations. However, the carrying capacity of the joints, a function of the temperature at which the soldering or brazing will melt, is more of a determining factor in station design than in the bonding of wooden pole transmission line insulator pins.

The values of short-time current required to melt copper conductors, as given by the chart shown in Fig. 1, were calculated by the formula: (1)

$$33\left(\frac{I}{A}\right)^2 S=\log_{10}\left(\frac{t}{274}+1\right)$$

---

(1) This formula was developed by I. M. Onderdonk.

where

I = the current in amperes

A = the cross-sectional area of the conductor in circular mils

S = the time in seconds that the current I is applied

t = the difference in temperature or the temperature rise of the copper.

The calculations are based on a value of 1083 deg. C. for the melting point of copper and on the following assumptions:

(*1*) Ambient temperature of 40 deg. C.
(*2*) Radiation neglected due to short time.
(*3*) Resistance of one cm.^{3} of Cu at 0 deg. C. taken as 1.589 microhms.
(*4*) Temperature coefficient of Cu at 0 deg. C. taken as 1.234.

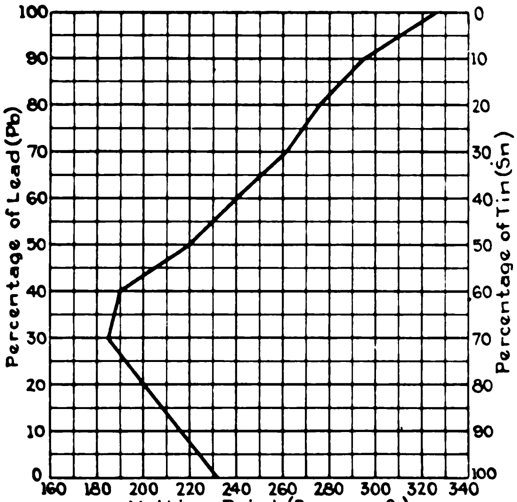


Fig. 2. Melting Points of Tin and Lead Alloyed in Various Proportions

It must be understood that the application of the foregoing formula is limited in that it applies only to short periods of time and that heat storage only in the copper itself is considered. When a long period of time is involved, *i.e.*, in the order of several minutes, radiation becomes a factor and the copper wire would require a longer time to reach melting temperature than indicated by the formula.

The values for the charts were calculated by solving for I as follows:

$$33\left(\frac{I}{A}\right)^2 S = \log_{10}\left(\frac{t}{274}+1\right)$$

$$\left(\frac{I}{A}\right)^2 = \frac{\log_{10}\left(\frac{t}{274}+1\right)}{33\,S}$$

$$\frac{I}{A} = \sqrt{\frac{\log_{10}\left(\frac{t}{274}+1\right)}{33\,S}}$$

$$I = A\sqrt{\frac{\log_{10}\left(\frac{t}{274}+1\right)}{33\,S}}$$

The temperature of the melting point of copper is taken at 1083 deg. C. and the temperature of the air at 40 deg. C.

The temperature difference is then 1083 deg. − 40 deg. = 1043 deg. C.

The time is first taken as 1 second

Then

$$I = A\sqrt{\frac{\log_{10}\left(\frac{1043}{274}+1\right)}{33\times 1}}$$

$$= A\sqrt{\frac{\log_{10} 4.807}{33}}$$

$$= A\sqrt{\frac{0.68187}{33}}$$

$$= A\sqrt{0.02066}$$

$$= A \times 0.14374 \qquad (S = 1 \text{ sec.})$$

In considering a longer time, 10 seconds is taken as the maximum.

Then

$$I = A\sqrt{\frac{0.68187}{33\times 10}}$$

$$= A\sqrt{0.002066}$$

$$= A \times 0.04547 \qquad (S = 10 \text{ sec.})$$

The same formula can be applied where the melting temperature of the soldering materials used in making up the joints in the copper conductor is the determining factor. In the case of ordinary solder, the value of t would be about 183 deg. C. minus the temperature of the air. The curve in Fig. 2, taken from the Smithsonian Physical Tables, shows the melting points of an alloy of tin and lead in a full range of proportions. Where brazing is used, a reasonable value for t is 450 deg. C. minus the temperature of the air.

The author wishes to express his appreciation of the suggestions given by W. W. Lewis and J. E. Clem in making a study of this subject.

The Hall High-speed Recorder

By **CHESTER I. HALL**
Developmental Laboratory, Fort Wayne Works, General Electric Company

THE development of the Hall recorder resulted from an insistent demand from operating companies for an automatic device which would give a complete record of the characteristics developed in an electrical circuit, such as a transmission or distribution line, during and immediately following the occurrence of an accidental fault. Since its development many other applications have been suggested; but its greatest usefulness has been found in connection with transmission systems where it furnishes a record of the condition of the circuit upon the occurrence of unusual disturbances, such as short circuits.

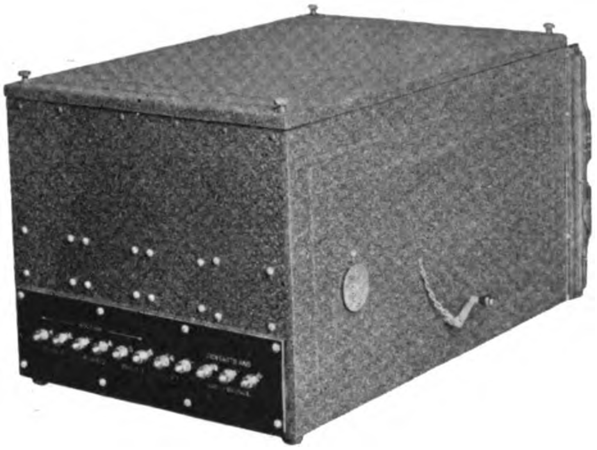


Fig. 1. The Four-element Type High-speed Recorder

The need of a device of this character has been brought forcibly to the attention of engineers many times by the complete failure of transmission systems due to the development of an accidental fault which, though incidental in itself, is the primary cause of breakdown. The point or time at which these disturbances are liable to take place cannot be predetermined, so that in the past it has been impossible to obtain satisfactory data showing the sequence of the various steps causing these failures. With the installation of high-speed recorders at various points on a system, these data can be obtained, and an accurate analysis made of the particular system under these abnormal conditions.

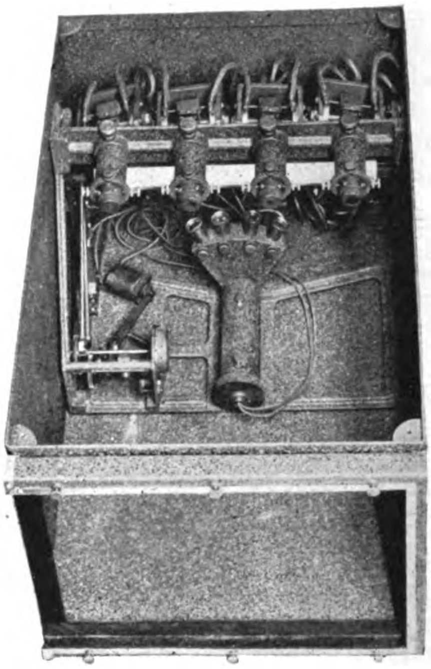


Fig. 2. The Interior Construction of a Four-element Recorder, Showing Light Source, Galvanometers, and Means of Projecting the Record-tracing Beam Onto the Film

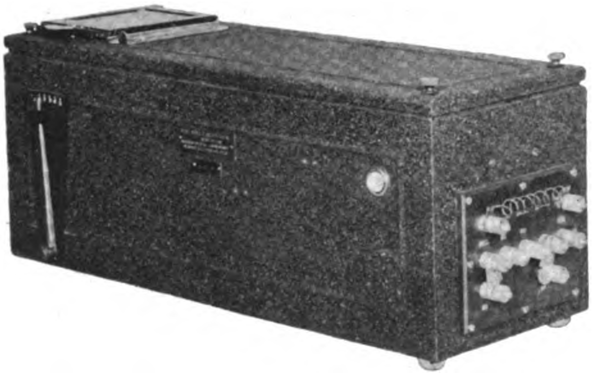


Fig. 3. The Single-element Type High-speed Recorder

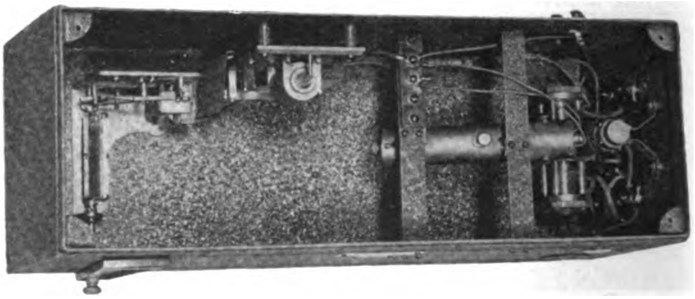


Fig. 4. Interior View of the Single-element Type Recorder

Two principal forms of the High-speed Recorder have been developed. These are the four-element recorder shown in Figs. 1 and 2, and the single-element recorder shown in Figs. 3 and 4. Both of these forms begin to record automatically through the operation of a high-speed relay on the occurrence of a disturbance, the records being made on standard photographic film.

The four-element recorder, as its name implies, has four recording elements and obtains simultaneous records of current, voltage, or other quantities which

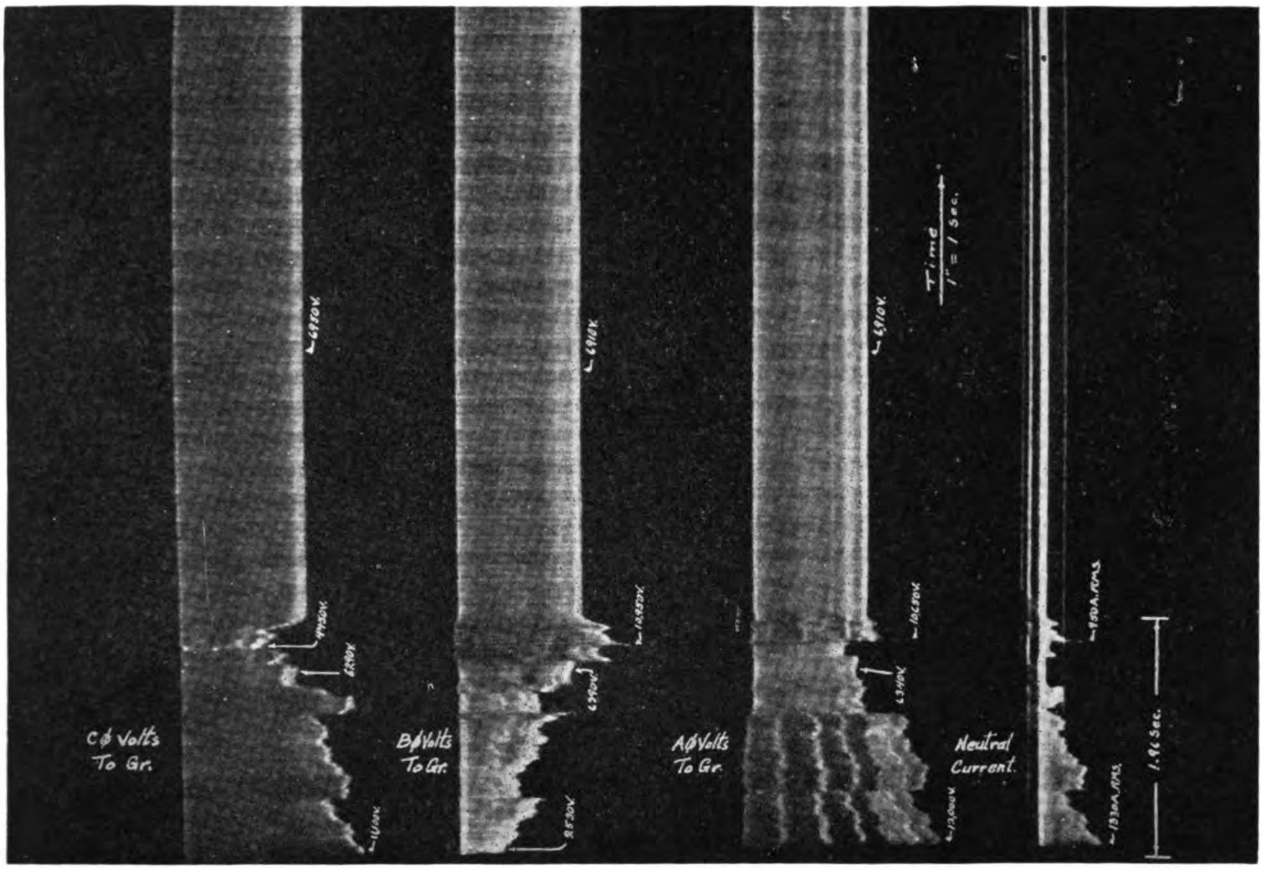


Fig. 5. A Typical Record Obtained by a Four-element Recorder, Showing the Progress of an Arc Started by Lightning in an Overhead Line

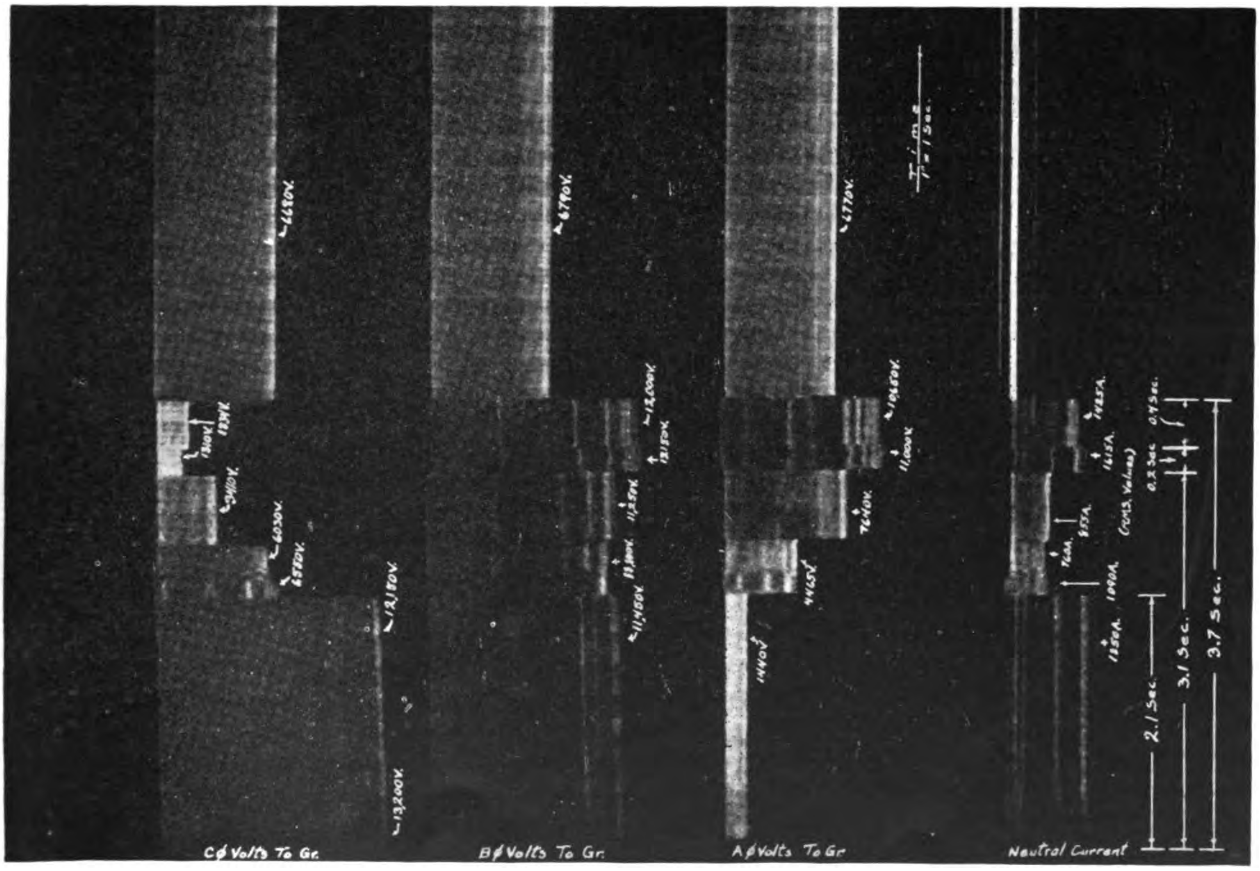


Fig. 6. Another Record Obtained by Recorder Showing the Effects of the Simultaneous Failure of Two Cables Upon Circuit Voltages and Current

it is desired to record, the various records being produced with respect to a time base. In the standard form the device is equipped with three voltage elements and one current element, the measurements giving phase-to-ground potential and neutral ground current. The high-speed relay is usually connected in the neutral ground circuit and starts the recorder when the current reaches a predetermined value. In Figs. 5 and 6 are shown reproductions of actual records from the Crawford Station of the Commonwealth Edison Company.

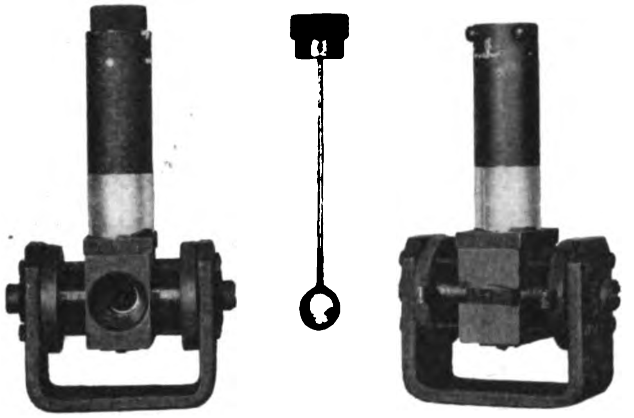


Fig. 7. Detailed Views of One of the Recording Elements

The single-element recorder has been developed both with a stationary recording film, for giving maximum values only, and with a moving film for giving records having a time base. The single-element form may give sufficient data for some system analyses while the four-element form is required to obtain complete data on the more complicated interconnected networks.

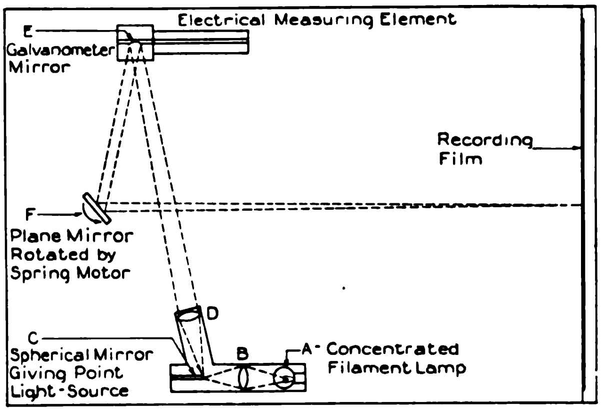

Electrical Measuring Element
E
Galvanometer Mirror
Recording Film
F
Plane Mirror Rotated by Spring Motor
D
C
Spherical Mirror Giving Point Light-Source
B
A
Concentrated Filament Lamp

Fig. 8. A Diagram of the Optical System of the High-speed Recorder

The essentials of the four-element recorder are shown in Fig. 2. Light from a high-intensity automobile lamp is focused by means of a condensing lens upon a highly polished steel reflector having a spherical surface of 20 mils radius. The image of the filament in this spherical mirror is used as a point source of light. Four beams of light from this point are directed through focusing lenses to the four galvanometer mirrors, mounted on the moving elements, from which they are reflected to a plane plate-glass mirror and then focused on the film at the end of the box. The plane mirror is mounted on a shaft which is rotated at constant speed by means of a spring motor, thus moving the beams along the film, giving the time base record. The points of light move across the film in accordance with the motion of the galvanometer mirrors in response to variations of current in the element windings.

Front and rear views of a recording element are shown in Fig. 7. This element consists of a soft iron yoke supporting two adjustable poles which carry the winding. The moving element is a soft iron vane rigidly mounted on the end of a stiff duraluminum rod and set, inclined at an angle of 45 deg., across the flux path between the poles. A stiff support riveted to the vane carries the galvanometer mirror and a pivot for the upper guide bearing. The length of light beam from galvanometer mirror to the film is 24 in., so that only a very small angular motion is necessary to obtain a considerable deflection of the spot on the film. For use on a-c. circuits, the recording elements are connected in the lines to be studied through transformers to obtain the proper ratings. Where d-c. circuits are being studied the transformers are replaced by special external shunts.

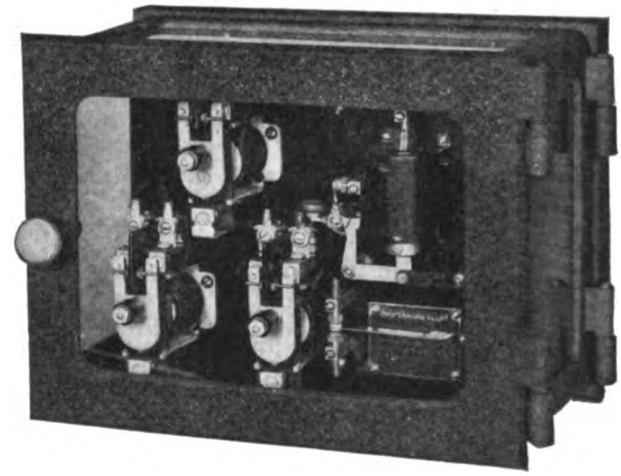


Fig. 9. Operating Relay Used in Connection with Four-element Recorder

In the four-element recorder only one light source is necessary since light beams are reflected at all angles from the hemispherical reflector.

The moving systems of these recorders are of very low inertia and high period of oscillation so that the changes occurring in the circuits may be recorded for each half cycle. In contradistinction to the ordinary indicating or recording element of standard instruments, having a fine hair-spring for furnishing the restraining torque, the elements of the high-speed recorder use the torsional force in the stiff duraluminum rod to return the element to its zero position.

The use of a relatively stiff moving system results in a very reliable element which is portable and not liable to damage in shipment or handling. Light beams and photographic film are utilized in the recording system to eliminate the inertia inherent in other types of recorders.

The optical system, shown diagrammatically in Fig. 8, is entirely different in construction and much more efficient than anything used heretofore for such purposes. The light source is the high-intensity filament of a standard automobile-type bulb, the image of which, after being reduced in size by a condensing lens and a special hemispherical mirror is reflected from the mirror mounted on the moving element and finally accurately focused on the photographic film.

In principle, the optical system of the single-element recorder is the same as that in the four-element device. Where time-base records are required the film (in the single-element type) is moved past the recording position at constant rate by a spring motor.

Both single- and four-element recorders are started automatically by a high-speed relay which releases the motor and energizes the light source from an external battery. The assembly of the high-speed relay and three auxiliary contactors for the four-element device is shown in Fig. 9.(1) Accurate tests on a complete unit show that times of 0.028 to 0.033 sec. are required for the device to begin to record, the difference in times being due to the amount of overload on the relay.

The records from both single- and four-element devices are made on standard size film using standard film holders. In the case of the single-element stationary film the size is 3¼ in. by 4¼ in., giving the possibility of recording four successive readings of maximum values. Where the moving film is used a standard roll film 3¼ in. wide is used, which is moved past the recording position at a rate of one inch per second for a period of 10 sec. after each start, four successive records being possible. The four-element device uses standard 11 in. by 14 in. film in a standard holder which is fastened to the recorder in a manner similar to that in a standard camera. Daylight loading of the recorder is possible in all cases.

The High-speed Recorder is now being used by several of the larger central station companies for studying transmission line problems.(2) Data obtained from these recorders are used in determining the proper settings of the protective relays for the system.

In addition to the applications mentioned above the device is useful in studying many other types of circuits, recording in some cases, for instance, the rate of building up or decay of current in generator fields, the manner of opening fused circuits under various overloads, changes in current value in starting induction motors, tungsten-lamp circuits and many special devices. Visual indications of circuit changes are also made possible by substituting a piece of ground glass for the recording film. The device is not intended to displace the oscillograph as used in wave shape analyses, but is to be considered as a high-speed graphic recorder having low inertia in its recording system.

The adaptability of the high-speed recorder to any type of circuit makes it extremely valuable in connection with investigative work where complex circuits are to be studied. Because of its simplicity, the use of the device does not require a specially-trained operator.

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(1) The high-speed relay has recently been modified somewhat from the construction shown in Fig. 9.

(2) An account of the application of the High-speed Recorder to the line disturbance problems of the Commonwealth Edison Company was given by E. M. Tingley in the paper, "The Hall High-speed Recorder," published in the *Journal of the A.I.E.E.*, January 1928, p. 7.

Oil-electric Locomotive Economical in Service

Evidence of the economy of the oil-electric locomotive is found in a recent analysis made by the Reading Railroad of the operation of a 60-ton 300-hp. locomotive of this type over a period of 256 days. During this time the locomotive traveled 28,038 mi., representing 4,673 hours of service. The total maintenance cost during this period was $2,036.78. In comparison, a steam engine, employed in the same service, cost $3,364.56 to maintain for an equal number of working hours during an equal period of time. It consumed $4,808.52 worth of fuel and lubricating oil. The corresponding costs were $1,703.23 for the oil-electric locomotive.

The oil-electric locomotive was built jointly by the Ingersoll-Rand Company, the American Locomotive Company, and the General Electric Company. It is equipped with a 300-horsepower oil engine driving a standard generator and supplying power to four 95-horsepower traction motors. Standard multiple-unit control is used.

Definition of An Ideal Synchronous Machine and Formula for the Armature Flux Linkages

By R. H. PARK
Engineering General Department, General Electric Company

IN general the flux in every circuit of a synchronous machine depends in a complicated way on the currents in every circuit of the machine and on the relative position of the armature phases and the rotor. However, in the majority of the practical applications of synchronous-machine theory there would be only a small difference in final results between the operation of an actual machine and the operation of a similar machine possessing the same self and mutual armature leakage reactances but ideal to the extent that, as far as concern effects depending on the position of the rotor, each armature winding is sinusoidally distributed.

It follows that a calculation of the performance of this imaginary or ideal synchronous machine will agree in the main with the results of tests on actual machines. Nevertheless, it is desirable to keep in mind the important differences between an actual machine and its imaginary or ideal counterpart. On this account any machines which are imagined to possess the characteristic referred to will be especially characterized here as "ideal synchronous machines."

If the saturation and hysteresis in every magnetic circuit and the eddy currents in the armature iron are neglected, the no-load per-unit(1) space fundamental flux density at the armature surface will be

$$\beta = I_d \cos \Gamma + I_q \sin \Gamma$$

where

Γ = angle along the armature surface measured in electrical degrees from the axis of the rotor.

I_d = per-unit rotor excitation or rotor current in direct axis.

I_q = per-unit rotor current in quadrature axis magnetizing ahead of the poles.

The per-unit m.m.f. due to armature phase currents i_a, i_b, and i_c will be

$$A = -\frac{2}{3}\left[i_a \cos \gamma + i_b \cos (\gamma - 120) + i_c \cos (\gamma + 120)\right] \tag{1}$$

where γ = angle along the armature surface measured in electrical degrees from the axis of phase a.

Normal m.m.f. = the m.m.f. with normal balanced positive phase-sequence currents applied.

Let

θ = position of the axis of the rotor in electrical degrees measured from the axis of phase a.

Then

$$\gamma = \Gamma + \theta$$

Therefore

$$\begin{aligned} A = &-\frac{2}{3}\left[i_a \cos (\Gamma+\theta) + i_b \cos (\Gamma+\theta-120) + i_c \cos (\Gamma+\theta+120)\right] \\ = &-\frac{2}{3} \cos \Gamma \left[i_a \cos \theta + i_b \cos (\theta-120) + i_c \cos (\theta+120)\right] \\ &+\frac{2}{3} \sin \Gamma \left[i_a \sin \theta + i_b \sin (\theta-120) + i_c \sin (\theta+120)\right] \end{aligned} \tag{2}$$

The flux wave due to the impressed m.m.f., A, will contain fundamental and harmonic components. However, on the basis of the assumptions, we are concerned only with the fundamental components of flux since, in the ideal machine assumed, the armature linkages for all space harmonics are zero. This fundamental will in turn have components cos Γ and sin Γ, respectively, over the direct axis of the poles and over the quadrature axis, and there will be a factor of proportionality between each component and the corresponding m.m.f. giving rise to it. Let these factors be p_d and p_q, respectively, in the direct and quadrature axes. Then the wave of per-unit flux is

$$\begin{aligned} \beta = &I_d \cos \Gamma + I_q \sin \Gamma \\ &-\frac{2}{3} p_d \cos \Gamma \left[i_a \cos \theta + i_b \cos (\theta-120) + i_c \cos (\theta+120)\right] \\ &+\frac{2}{3} p_q \sin \Gamma \left[i_a \sin \theta + i_b \sin (\theta-120) + i_c \sin (\theta+120)\right] \end{aligned} \tag{3}$$

But

$$p_d = \frac{p_d+p_q}{2} + \frac{p_d-p_q}{2}, \text{ and } p_q = \frac{p_d+p_q}{2} - \frac{p_d-p_q}{2}$$

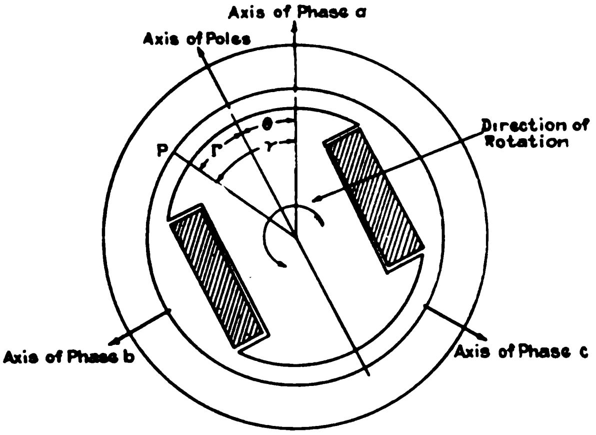
---

(1) The term "per-unit" is employed as a convenient characterization of the method of designation in which quantities are expressed as a decimal fraction of a normal or unit value. The method is analogous to the method of percentage representation of quantities except that the factor one hundred is omitted. The advantage of the method is that the factor 100 does not have to be multiplied in or divided out whenever the operations of division or multiplication are performed. The unit or normal quantities involved in the present article are as follows:

Armature current	$\sqrt{2}$ × rated value
Armature voltage	$\sqrt{2}$ × rated value
Armature flux	value which will generate normal voltage
Field current	value which will produce unit armature flux at no load
Reactance	ratio of normal phase voltage to normal current

whence

$$\begin{aligned}\beta &= I_d \cos \Gamma + I_q \sin \Gamma \\ &\quad - \frac{2}{3}\,\frac{p_d+p_q}{2}[i_a \cos(\Gamma+\theta) + i_b \cos(\Gamma+\theta-120) \\ &\quad + i_c \cos(\Gamma+\theta+120)] \\ &\quad - \frac{2}{3}\,\frac{p_d-p_q}{2}[i_a \cos(\Gamma-\theta) + i_b \cos(\Gamma-\theta+120) \\ &\quad + i_c \cos(\Gamma-\theta-120)] \\ &= I_d \cos(\gamma-\theta) + I_q \sin(\gamma-\theta) \\ &\quad - \frac{2}{3}\,\frac{p_d+p_q}{2}[i_a \cos(\gamma) + i_b \cos(\gamma-120) \\ &\quad + i_c \cos(\gamma+120)] \\ &\quad - \frac{2}{3}\,\frac{p_d-p_q}{2}[i_a \cos(\gamma-2\theta) + i_b \cos(\gamma-2\theta+120) \\ &\quad + i_c \cos(\gamma-2\theta-120)] \qquad (4)\end{aligned}$$



P is any point on armature surface

Fig. 1. Diagram Showing the Angular Relationship of the Axis of Poles, and of Any Point (*p*) on the Armature Surface with Respect to the Axes of Phases *a*, *b*, and *c*

The per-unit linkages in phase *a* due to the flux are equal to the value of β at $\gamma=0$, or

$$\begin{aligned}\psi_a &= I_d \cos\theta - I_q \sin\theta \\ &\quad - \frac{p_d-p_q}{3}\left[i_a - \frac{i_b+i_c}{2}\right] \\ &\quad - \frac{p_d-p_q}{3}[i_a \cos 2\theta + i_b \cos(2\theta-120) \\ &\quad + i_c \cos(2\theta+120)]\end{aligned}$$

The total per-unit linkages in phase *a* are then

$$\begin{aligned}\psi_a &= I_d \cos\theta - I_q \sin\theta \\ &\quad - x_{l_a} i_a + x_m (i_b+i_c) \\ &\quad - \frac{p_d+p_q}{3}\left[i_a - \frac{(i_b+i_c)}{2}\right] \\ &\quad - \frac{p_d-p_q}{3}[i_a \cos 2\theta + i_b \cos(2\theta-120) \\ &\quad + i_c \cos(2\theta+120)] \qquad (5)\end{aligned}$$

where

x_{l_a} = per-unit self leakage reactance of any phase.

x_m = per-unit mutual leakage reactance of any two phases.

Under balanced conditions of operation with normal armature current flowing, there is

$$\begin{aligned}i_a &= \cos t \\ i_b &= \cos(t-120) \\ i_c &= \cos(t+120) \\ i_b + i_c &= -i_a \\ \theta &= t + \theta_o\end{aligned}$$

where θ_o = the value of θ at $t=0$

Then, for $I_d = I_q = 0$ there is

$$\begin{aligned}\psi_a &= -(x_{l_a}+x_m) \cos t \\ &\quad - \frac{p_d+p_q}{2} \cos t \\ &\quad - \frac{p_d-p_q}{2} \cos(2\theta - t) \\ &= -(x_{l_a}+x_m+\frac{p_d+p_q}{2} \cos t \\ &\quad - \frac{p_d-p_q}{2} \cos(t+2\theta_o) \qquad (6)\end{aligned}$$

When $\theta_o = 0$ there is

$$\psi_a = -(x_{l_a}+x_m+p_d) \cos t = -x_d \cos t \qquad (7)$$

where x_d = direct synchronous reactance.

When $\theta_o = 90$ deg., there is

$$\psi_a = -(x_{l_a}+x_m+p_q) \cos t = -x_q \cos t \qquad (8)$$

where x_q = quadrature synchronous reactance.

With zero phase-sequence currents impressed there is

$$i_a = i_b = i_c \qquad (9)$$

$$\psi_a = -(x_{l_a} - 2\,x_m)\, i_a = -x_o\, i_c$$

Thus, on the basis of the assumptions, the following relations exist

$$\begin{aligned}x_{l_a} + x_m + p_d &= x_d \\ x_{l_a} + x_m + p_q &= x_q \\ x_{l_a} - 2x_m &= x_o\end{aligned}$$

and

$$\begin{aligned}p_d - p_q &= x_d - x_q \\ 2\,(x_{l_a} + x_m) + p_d + p_q &= x_d + x_q\end{aligned}$$

After making an algebraic transformation, Equation (5) may be rewritten in the form

$$\begin{aligned}\psi_a &= I_d \cos\theta - I_q \sin\theta \\ &\quad - \left[2\,\frac{x_{l_a}+x_m}{3} + \frac{p_d+p_q}{3}\right]\left[i_a - \frac{i_b+i_c}{2}\right] \\ &\quad - \frac{p_d-p_q}{3}[i_a \cos 2\theta + i_b \cos(2\theta-120) \\ &\quad + i_c \cos(2\theta+120)] \\ &\quad - [x_{l_a} i_a - x_m (i_b+i_c)] \\ &\quad + 2\,\frac{x_{l_a}+x_m}{3}\left[i_a - \frac{i_b+i_c}{2}\right] \qquad (10)\end{aligned}$$

The last two terms may be reduced to the form

$$\frac{x_{l\sigma}-2x_m}{3}[i_a+i_b+i_c]$$

Therefore

$$\begin{aligned}\psi_a=&I_d\cos\theta-I_q\sin\theta\\&-\frac{x_o}{3}[i_a+i_b+i_c]-\frac{x_d+x_q}{3}\left[i_a-\frac{i_b+i_c}{2}\right]\\&-\frac{x_d-x_q}{3}[i_a\cos 2\theta+i_b\cos(2\theta-120)\\&+i_c\cos(2\theta+120)]\end{aligned}\qquad(11)$$

Similarly

$$\begin{aligned}\psi_b=&I_d\cos(\theta-120)-I_q\sin(\theta-120)\\&-x_o\frac{i_a+i_b+i_c}{3}-\frac{x_d+x_q}{3}\left[i_b-\frac{i_c+i_a}{2}\right]\\&-\frac{x_d-x_q}{3}[i_a\cos(2\theta-120)\\&+i_b\cos(2\theta+120)+i_c\cos 2\theta]\end{aligned}\qquad(12)$$

$$\begin{aligned}\psi_c=&I_d\cos(\theta+120)\\&-I_q\sin(\theta+120)-x_o\frac{i_a+i_b+i_c}{3}\\&-\frac{x_d+x_q}{3}\left[i_c-\frac{i_a+i_b}{2}\right]\\&-\frac{x_d-x_q}{3}[i_a\cos(2\theta+120)+i_b\cos 2\theta\\&+i_c\cos(2\theta-120)]\end{aligned}\qquad\mathbf{(13)}$$

where

ψ_a, ψ_b, ψ_c = per-unit phase linkages
i_a, i_b, i_c = per-unit phase currents
I_d = per-unit excitation in direct axis
I_q = per-unit excitation in quadrature axis
x_d = direct synchronous reactance
x_q = quadrature synchronous reactance
x_o = zero phase sequence reactance.

This expression for the per-unit linkages of an ideal synchronous machine was given without proof in a paper, "The Reactance of Synchronous Machines," which was presented before the A.I.E.E. at the Mid-winter Convention, New York City, Feb., 1928, by the writer and Mr. B. L. Robertson.(2) The expression has been found useful in the analysis of synchronous machines.

---

(2) Attention is called to a related paper, "Synchronous Machines: IV," delivered at the same convention by R. E. Doherty and C. A. Nickle.

First Central Station to Operate Entirely at 1200-lb. Pressure

For the first time in history a central station will be built with high-pressure boilers to supply the entire steam demand. This station will be built at Deepwater (N. J.), by the American Gas and Electric Company and the American Electric Power Corporation, Stevens and Wood, Inc., engineers. It will have an initial generating capacity of 118,500 kw.

There will be two main cross-compound turbine sets, each rated at 53,000 kw. Each set will consist of a 12,500-kw. high-pressure unit operating at 3600 r.p.m., and a 41,700-kw. low-pressure unit operating at 1800 r.p.m. The initial steam conditions will be 1200 lb. gauge pressure, and 725 deg. F. total temperature. The high-pressure units will exhaust against a back pressure which varies in proportion to the steam flow, the steam being returned to the boilers for reheating to 725 deg. The low-pressure units will exhaust against a back pressure of one inch absolute.

A third 12,500-kw. high-pressure single-cylinder turbine will be installed, which is a duplicate of the high-pressure elements of the cross-compound turbines. The exhaust of this unit at about 340 lb. pressure flows into a high-pressure evaporator where it will condense and generate process steam for use in a nearby Du Pont factory. The temperature of this process steam is controlled by passing it through a steam reheater supplied with steam extracted from the high-pressure turbine. By the use of these devices there is occasioned no loss or contamination of high-pressure condensate and the difficulty of having excessive makeup is avoided.

The generators of the main high-pressure units will be rated 12,500 kw., 80 per cent power-factor, 15,625 kv-a., 11,000 volts, while the machines driven by the low-pressure turbines will be rated 41,700 kw., 85 per cent power-factor, 49,058 kv-a., 11,000 volts. All generators will have direct-connected exciters.

The station has been designed for high efficiency at all loads. The variation in turbine efficiency between rated load and half load will be less than 1½ per cent, with the point of maximum efficiency at about 70 per cent of rated load.

Up to the present time, high-pressure boilers have been run in conjunction with boilers designed for 300 or 400 lb. pressure, the high-pressure installation furnishing only a part of the total steam required. However, after a thorough study of the dollar economy of various steam pressures and cycles of operation, 1200 lb. initial pressure was decided upon for the Deepwater station, as combining in the highest degree the necessary reliability with high fuel economy and flexibility of the units.

The Charles A. Coffin Foundation Fellowships, 1928-29

THE Charles A. Coffin fellowships have been awarded this year to eight young men preparing to engage, or already engaged, in research dealing with various problems in connection with electricity,. physics, and physical chemistry. These fellowships are provided through the Charles A. Coffin Foundation which was established in honor of the first president and chairman of the Board of Directors of the General Electric Company as a means of encouraging and fostering fundamental research among students of engineering and kindred sciences. It is the purpose of the committee awarding the fellowships to so direct the expenditure of the fund provided by the Foundation that deserving students may be enabled to carry on research work in instances where, without such aid, they would be severely handicapped.

The committee making the awards for the year 1928-29 was composed of Gano Dunn, ex-president of the National Academy of Sciences; Bancroft Gherardi, President of the American Institute of Electrical Engineers; and Robert L. Sackett, Dean of Pennsylvania State College and President of the Society for the Promotion of Engineering Education. In coöperation with the representatives of these institutions, Dr. W. R. Whitney, Director of the Research Laboratory at Schenectady, represented the General Electric Company.

The awards were made as follows to:

Robert F. Bacher, a graduate of the University of Michigan in 1926. Later studies were pursued at Harvard during 1927; and at present Mr. Bacher is a candidate for the degree of Doctor of Philosophy at Michigan. His work in connection with the fellowship will be taken up there, the subject of his research presenting the respective aspects (1) the explanation of the hyperfine structure in bismuth and the reason for the spacing and intensity variation observed in the Zeeman Effect; (2) the distribution of atoms in their low-energy states at normal temperature.

Arthur F. Daniel, a graduate of the University of North Carolina in 1927. Mr. Daniel is now a graduate student there and will be an applicant for the degree of Master of Science in 1928. He is engaged as a teaching fellow in physics and is doing independent research this year. The researches following the award of his fellowship will be carried on at the University of Chicago. For the subject of his investigations he has chosen the study of x-ray or electronic phenomena as applied to the structure of matter.

Neils F. Edlefsen, a graduate of Utah Agricultural College in 1916. Prof. Edlefsen later received the degree of M. A. in Physics from the University of California, in 1923, and is at present Assistant Professor of Physics at Utah Agricultural College. He has been previously engaged in research work and held a fellowship while at the university. His further research in the field of colloid and capillary chemistry will be a study of the variation of surface energy with curvature.

Truman S. Gray, a graduate of the University of Texas in 1927. Mr. Gray is a graduate student at the Massachusetts Institute of Technology, where he will be a candidate for the degree of M. S. in Electrical Engineering this year. The research work following the award of the fellowship will be an experimental and mathematical investigation of the nature of transformer breakdown by means of the cathode-ray oscillograph and the recording integraph. This work will be carried on at M.I.T.

Jamison R. Harrison, a graduate of Tufts College in 1925. In 1927 Mr. Harrison received the degree of Master of Arts from Wesleyan University where he is now engaged in graduate work under a Charles A. Coffin Foundation Fellowship awarded last year. With the renewal of the fellowship he will undertake as a research problem the study of a mode of propagation and determination of the velocity of waves traveling in quartz plates when used in a piezoelectric oscillator. Mr. Harrison has shown exceptional ability in research work.

Neil B. Reynolds, a graduate of Union College in 1924. Mr. Reynolds received in 1926 the degree of Master of Science from Union and is at present engaged in graduate work leading to a Ph.D. degree at Princeton. He has received high recommendation from men with whom he has worked both at Union and Princeton; and his work will be continued under the fellowship at the latter university. The research problem with which he will be concerned is the relation between anode voltage and saturation thermonic emission from pure and composite metallic surfaces. This is a problem which several of the world's most prominent investigators have failed to solve.

Donald M. Tilden, a candidate for the degree of Bachelor of Science at Lombard College in 1928. Mr. Tilden has made chemistry his major study, and has been a student assistant in the organic and quantitative laboratories. His work in connection with the fellowship will be pursued at Iowa State University, his research problem being the free energy of dilution, or adsorption. Mr. Tilden has won honors in general student activities and is a member of Phi Kappa Phi and of Theta Chi Delta.

Lal C. Verman, a 1927 graduate of the University of Michigan. Mr. Verman has been working in research as a graduate student at Cornell in connection with the Charles A. Coffin Foundation Fellowship awarded him last year. The renewal of the fellowship permits the further carrying on of his investigations regarding the polarization of short radio waves as affected by weather conditions.

LIBRARY SECTION

Condensed references to some of the more important recent articles in the technical press, and to new books of interest to the industry, as selected by the General Electric Main Library.

Arc Welding

Arc Welded Pipe Lines. W. L. Warner.
Am. Weld. Soc. Jour., Feb., 1928; v. 7, pp. 40-48.

Arc Welding as a Manufacturing Tool. J. F. Lincoln.
Am. Weld. Soc. Jour., Feb., 1928; v. 7, pp. 21-26.

Welding Trusses for Industrial Buildings. Andrew Vogel.
Weld. Engr., Feb., 1928; v. 13, pp. 33-36.
(From a paper read before the Affiliated Technical Societies, Boston.)

Brushes, Electric

Carbon Brushes; The Problem of Replacements and the Necessity for Standardization. J. B. Lunsford.
Am. Soc. Nav. Engrs. Jour., Feb., 1928; v. 40, pp. 65-89.
(Pertains particularly to brushes as used on electrical machinery in the naval service.)

Car Lighting

Tests Made of Car Lighting. W. R. Cost.
Aera, Mar., 1928; v. 19, pp. 137-141.
(The results of tests made by a committee of the A. R. E. A. on an experimental street car installation.)

Carrier-Current Communication

220-Kv. Carrier-Telephony. Roy B. Ashbrook and Ralph E. Henry.
Elec. Wld., Mar. 10, 1928; v. 91, pp. 495-497.
(Illustrated description of a Southern California Edison Co., installation.)

Daylight

Daylighting in Multi-Story Industrial Buildings. Andrew Vogel and others.
Illum. Engng. Soc. Trans., Feb., 1928; v. 23, pp. 129-173.

Efficiency

Efficiency. John H. Barker.
I. M. E. Proc., No. 4, 1927; pp. 1063-1072.
(A study of efficiencies in general, not only machine efficiency, but that of various natural agencies such as the sun, the tides, the winds, etc.)

Electric Distribution

Cable Laying Improvements. E. S. Byng.
Wld. Power, Mar., 1928; v. 9, pp. 153-157.
(The first of a series of four articles.)

Simplified Distribution Planning. W. R. Bullard.
Elec. Wld., Mar. 10, 1928; v. 91, pp. 499-503.
(Methods of calculating the economic characteristics, particularly transformer spacing and conductor sizes.)

Electric Distribution, Underground

Thirteen Kv. Underground Construction. E. R. Pelster.
Elec. Lt. & Pr., Feb., 1928; v. 6, pp. 28-30, 32-34, 90-91.

Electric Drive—Excavating

Electric Equipment for Power Shovels. David Stoetzel.
Engng. & Min. Jour., Mar. 10, 1928; v. 125, pp. 410-415.
(Presents an illustrated account of motor-generator equipment, motors, control apparatus, etc.)

Electric Lighting—Aviation

Airport Lighting. H. E. Mahan.
Soc. Auto. Engrs. Jour., Mar., 1928; v. 22, pp. 345-350.
("The essential items and general practice in the lighting of airports for night operation.")

Electric Motors, A. C.

Replacing the Misfit A. C. Motors. R. H. Rogers.
Ind. Engng., Mar., 1928; v. 86, pp. 117-120.
(Discusses the choice of a-c. motors for industrial drives.)

Electric Motors—Starting Devices

Automatic Starting Gear for Squirrel Cage Motors.
ASEA-Jour., Feb., 1928; v. 5, pp. 34-39.

Starting Alternating Current Motors. C. W. Falls.
Blast Fur. & St. Pl., Mar., 1928; v. 16, pp. 385-389.
(Considers various methods of starting squirrel cage and synchronous motors.)

Electric Welding

Static and Impact Tests of Electrically Welded Joints. Raymond J. Roark.
Am. Weld. Soc. Jour., Feb., 1928; v. 7, pp. 57-64.
(An account of tests to determine the strength and uniformity of fillet-welded joints.)

Welding and Manufacturing of Large Electrical Apparatus. A. P. Wood.
A. I. E. E. Jour., Mar., 1928; v. 47, pp. 192-195.
(Describes the use of electric welding in the fabrication of large electrical apparatus.)

Electrical Machinery, Synchronous

Synchronous Machines—IV. R. E. Doherty and C. A. Nickle.
A. I. E. E. Jour., Mar., 1928; v. 47, pp. 200-204.
(Abridgment.)

Headlights, Electric

Automobile Headlight Problem. L. B. W. Jolley.
Illum. Engr., Mar., 1928; v. 21, pp. 79-84.
(A paper presented before the International Commission on Illumination, Bellagio, 1927.)

Progress Report on Headlight Investigation. H. H. Allen.
Soc. Auto. Engrs. Jour., Mar., 1928; v. 22, pp. 305-313, 384-385.
(Report of an investigation of automobile headlighting being pursued at the U. S. Bureau of Standards under the sponsorship of the Research Committee of the S. A. E. and the National Automobile Chamber of Commerce.)

Insulators—Testing

Effect of Humidity on the Dry Flashover Potential of Pin-Type Insulators. J. T. Littleton, Jr. and W. W. Shaver.
A. I. E. E. Jour., Mar., 1928; v. 47, pp. 189-192.
(Presents results of tests on porcelain and on Pyrex insulators. Eight bibliographic references, p. 192.)

Load Factor

Electric Heating and Load Factor Improvement. M. O. F. England.
Wld. Power, Mar., 1928; v. 9, pp. 148-152.
(Points out the possibilities of improving the load factor by the encouragement of domestic and other electric water heating load.)

Locomotives, Oil-Electric

New York Central Adopts Diesel-Electric Battery Locomotive.
Oil Engine Power, Mar., 1928; v. 6, pp. 165-167.
(Illustrated description of a combination oil-electric and battery type switching locomotive having G-E electric drive. Also described in *Rwy. Age*, Mar. 3, 1928, pp. 525-527.)

Lubrication and Lubricants

Viscosity of Lubricants Under Pressure. Mayo D. Hersey and Henry Shore.
Mech. Engng., Mar., 1928; v. 50, pp. 221-232.
("An experimental determination of the combined effects of high pressures and temperatures.")

Machinery—Foundations

Large-Unit Foundations. P. D. Freudenberger.
Elec. Wld., Mar. 3, 1928; v. 91, pp. 449-450.
(An illustrated description of the foundation structure for a 60-kw. turbine generator at the Charles R. Huntley station of the Buffalo General Electric Co.)

Measuring Instruments

Harmonic Analyzer for Power Circuits. W. V. Lovell.
Elec. News, Mar. 1, 1928; v. 37, pp. 41-43.

Permeameters

Saturation Permeameter. S. L. Gokhale.
A. I. E. E. Jour., Mar., 1928; v. 47, pp. 196-200.
(Describes the construction, theory, and performance of a permeameter for testing the saturation values of magnetic materials.)

Power Plants, Electric

Three Years of Municipal Diesel Plant Operation. Frank A. Poehlmann.
Oil Engine Power, Mar., 1928; v. 6, pp. 187-191.
("Detailed operating and cost report of Grove City oil-electric central station.")

Pumps, Centrifugal

Characteristic Laws for a Centrifugal Pump with Fluids Other than Water. Hubert Mawson.
I. M. E. Proc., No. 4, 1927; pp. 1037-1045.

Radio Engineering—Amplification

High-Note Loss in Resistance Amplifiers. A. L. M. Sowerby.
Wireless Wld., Feb. 29, 1928; v. 22, pp. 213-216.
(Serial.)

Radio Engineering—Circuits, Coupled

Direct Coupled Detector and Amplifiers with Automatic Grid Bias. Edward H. Loftin and S. Young White.
Inst. Radio Engrs. Proc., Mar., 1928; v. 16, pp. 281-286.
("A composite detecting and amplifying system.")

Radio Engineering—Crystals

Investigation into the Rectifying Properties of Crystals. A. W. J. Cousins and E. L. Damant.
S. Af. I. E. E. Trans., Dec., 1927; v. 18, pp. 196-203.

Radio Engineering—Insulators

Notes on the Design of Radio Insulators. T. Walmsley.
Inst. Radio Engrs. Proc., Mar., 1928; v. 16, pp. 361-372.

Radio Engineering—Reception—Interference

Is Fading Caused by the Motion of the Earth's "Radio Roof?" Raymond A. Heising.
Pop. Radio, Feb., 1928; v. 13, pp. 111-113, 172.
(Author is connected with the Research Dept., Bell Telephone Laboratories.)

Suppressing Radio Interference. A. T. Lawton.
Radio Broadcast, Mar., 1928; v. 12, pp. 379-382.
(On methods of locating local interference.)

Radio Engineering—Transmission

Correlation of Long Wave Transatlantic Radio Transmission with Other Factors Affected by Solar Activity. Clifford N. Anderson.
Inst. Radio Engrs. Proc., Mar., 1928; v. 16, pp. 297-347.
(Includes a bibliography of 15 entries, pp. 346-347.)

Railroads—Electrification

Electricity in Rail Transportation. H. L. Andrews.
Engrs. & Engng., Feb., 1928; v. 45, pp. 27-36.
(From a paper read before the A. S. C. E. A general summary of the situation of railroad electrification in the U. S., particularly the economic phases. Compares steam and electric operation.)

Refrigeration

Sulphur Dioxide. Charles W. Johnston.
Refrig. Engng., Mar., 1928; v. 15, pp. 61-66, 71.
(Describes the manufacture, analysis, testing, physiological effects, etc., of liquid sulphur dioxide, particularly in connection with its use as a refrigerant.)

Spark Discharges

Surge Impulse Breakdown of Air. J. J. Torok.
A. I. E. E. Jour., Mar., 1928; v. 47, pp. 177-181.
(Abridgment. Includes bibliography of 13 entries, p. 181.)

Theory of the Electrical Breakdown of Gases at Atmospheric Pressures. Leonard B. Loeb.
Franklin Inst. Jour., Mar., 1928; v. 205, pp. 305-321.
(On the theory of spark discharges in gases. Reviews the work of previous investigators and includes a list of 39 bibliographic references.)

Spot Welding

Application of Projection and Multiple Welds. W. T. Ober.
Am. Weld. Soc. Jour., Feb., 1928; v. 7, pp. 38-40.
(Pertains to spot welding methods.)

Spot Welding of Aluminum and Its Alloys. W. M. Dunlap.
Am. Weld. Soc. Jour., Feb., 1928; v. 7, pp. 27-38.

Steam Turbines

Developments in European Steam Turbine Design. C. H. S. Tupholme.
Power, Mar. 6, 1928; v. 67, pp. 419-421.

Synchronizing

Condenser-Type Bushing Used with Synchronizing Equipment. E. E. Spracklen and others.
A. I. E. E. Jour., Mar., 1928 v. 47, pp. 205-208.
(Abridgment.)

Telephony

Apparatus Standards of Telephonic Transmission, and the Technique of Testing Microphones and Receivers. B. S. Cohen.
I. E. E. Jour., Feb., 1928; v. 66, pp. 165-203.
(Includes a bibliography of 29 entries, pp. 188-189.)

Vacuum Tubes

Use of Very High-Voltage in Vacuum Tubes. W. D. Coolidge.
A. I. E. E. Jour., Mar., 1928; v. 47, pp. 212-213.

X-Rays

High Speed, High Voltage X-Ray Diffraction Analysis of Metals. Ancel St. John.
Am. Soc. St. Treat. Trans., Mar., 1928; v. 13, pp. 485-492.

X-Ray Examination of Structural Materials. Wheeler P. Davey.
Mech. Engng., Mar., 1928; v. 50, pp. 213-216.

NEW BOOKS

(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW)

Construction Job Management. Charles F. Dingman. 220 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Diesel Engines for Land and Marine Work. Ed. 6. A. P. Chalkley. 320 pp., 1927, N. Y., D. Van Nostrand Co.

Elements of Machine Design; Pt. 1. New ed. W. Cawthorne Unwin & A. L. Mellanby. 531 pp., 1927, N. Y., Longmans, Green & Co.

Introduction to the Metallurgy of Iron and Steel. H. M. Boylston. 571 pp., 1928, N. Y., John Wiley & Sons.

Protective Metallic Coatings. Henry S. Rawdon. 277 pp., 1928, N. Y., Chemical Catalog Co.

Pyroxylin Enamels and Lacquers. Ed. 2. Samuel P. Wilson. 253 pp., 1927, N. Y., D. Van Nostrand Co.

Radio Engineering Principles. Ed. 2. Henri Lauer & Harry L. Brown. 301 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Spherical Harmonics. T. M. MacRobert. 302 pp., 1927, N. Y., E. P. Dutton & Co.

Spur Gears. Earle Buckingham. 451 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Wireless Principles and Practice. L. S. Palmer. 504 pp., 1928, N. Y., Longmans, Green & Co.

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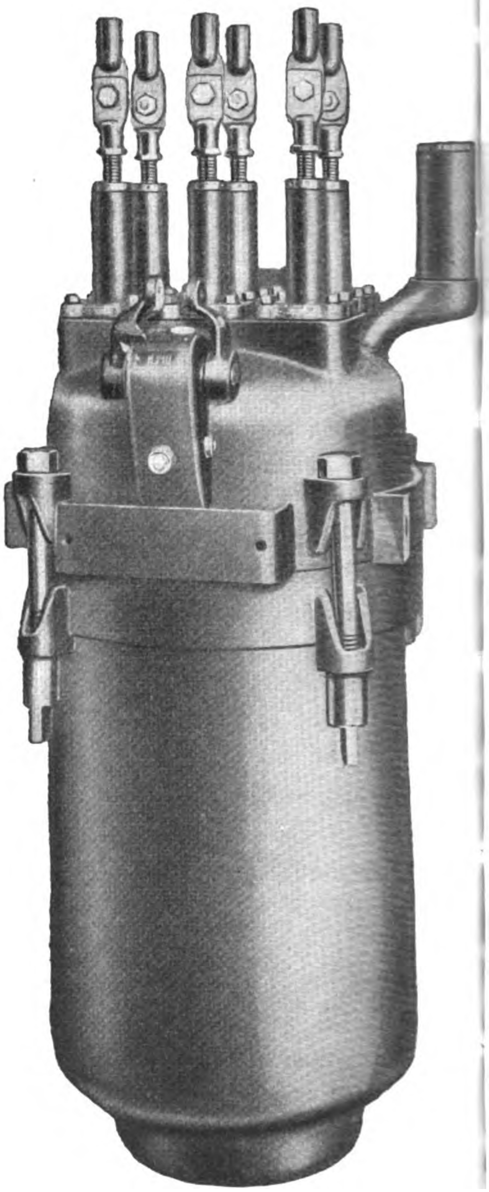

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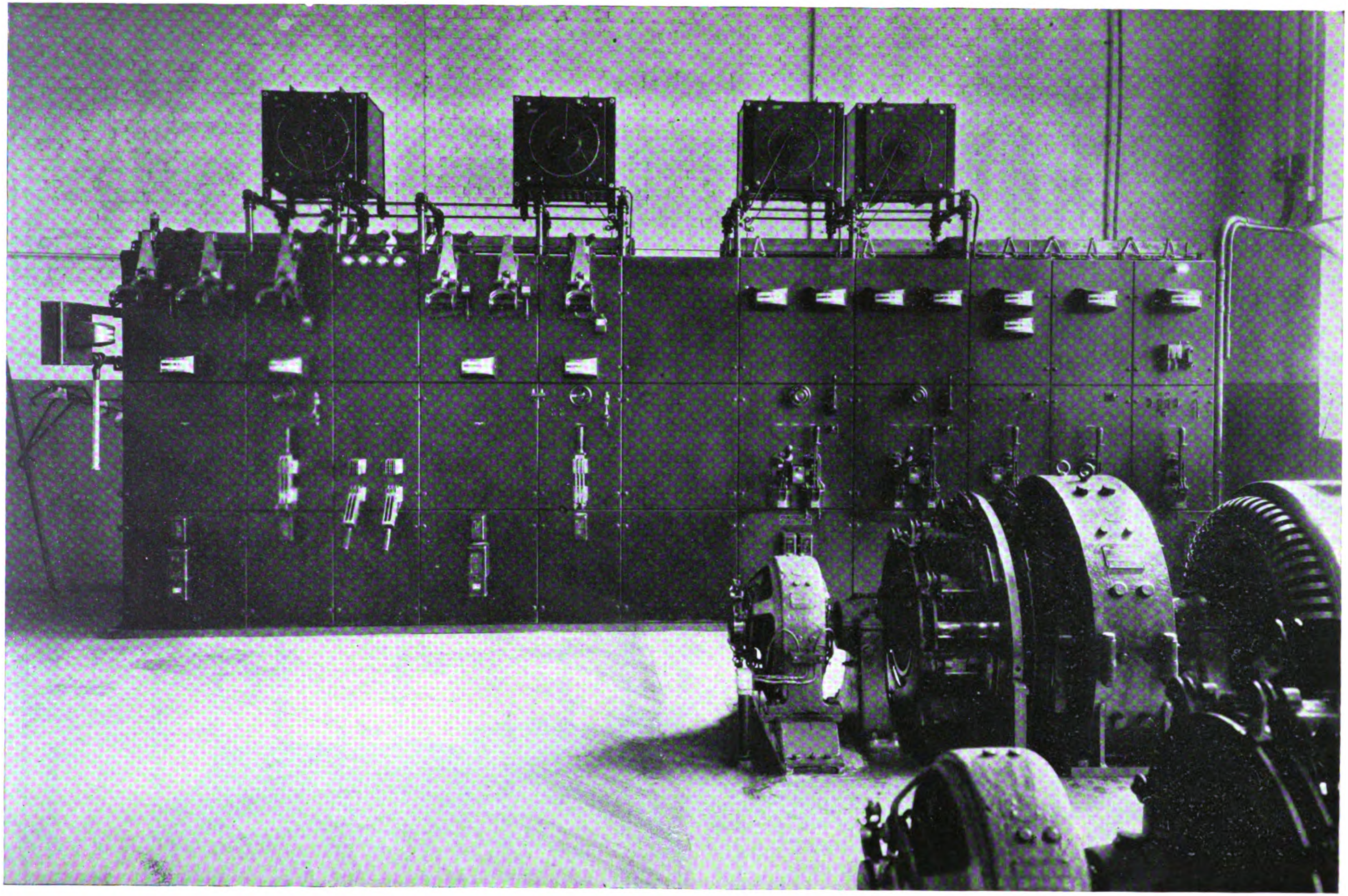

SUPPLYING THE FORCE TO SEPARATE WATER AND AIR INTO THEIR COMPONENT GASES

Motor-generator and control room of a plant for producing oxygen and hydrogen by the electrolysis of water, and oxygen and nitrogen by the liquification and fractionation of air. In manufacturing these gases, abundant use is made of electricity for chemical, power and heating purposes. (*See p. 385*)

GENERAL ELECTRIC REVIEW

Vol. 31 July, 1928 No. 7

ELECTRICITY AND THE COMPRESSED GASES

Power, heat, and light have long been recognized as essential utilities of industry. Their generation, distribution, and application are based on well-defined procedures for which apparatus has been developed to a high state of efficiency. In this branch of engineering a tremendous advance has been made.

At the present time there is an indication that a derived agent may soon enter the ranks of the three that have been mentioned. This newer utility might be listed as "The Compressed Gases." Through modern methods of manufacture and distribution, followed by efficient utilization, these gases offer possibilities of noteworthy consideration in industrial operations. It remains for industry to realize these potentialities and to place this branch of engineering on a basis more nearly comparable to that of the power, heat, and light group.

The compressed gases, so called to distinguish them from illuminating or city gas, include oxygen, hydrogen, and nitrogen as the major units. The extent of the use already made of the compressed gases is indicated by the fact that their production during 1925 had an aggregate value of $56,404,723; this being an increase of about 50 per cent during the preceding five-year period.

The increase in consumption that is taking place is by no means due entirely to expansions in the chemical industries; a considerable share is resulting from other industrials learning of the utility of these gases in their manufacturing operations. For example, mention may be made of the extending industrial applications of but two of these gases—oxygen and hydrogen. Foundries, structural iron works, and machine manufacturers are adopting gas cutting as a substitute for machine tool cutting, followed by electric welding for fabricating the unit assemblies. The demonstrated merits of a hydrogen (or a hydrogen-nitrogen) atmosphere in the operations of brazing and annealing are resulting in additional installations of electric furnaces of this type. Investigations are under way to determine the economic possibilities of securing the advantages which oxygen would afford in iron smelting, the Bessemer process, and the production of ferro alloys.

On a necessarily smaller scale of manufacture, it has been found that in nickel plating a preliminary treatment of the articles in a hydrogen atmosphere improves the quality of the plating. This same gas also facilitates the operation of lead burning.

Gas technique is quite generally understood by the chemical manufacturers. In the various mechanical, structural, and metallurgical industries, however, the knowledge of this subject is usually so incomplete as to make available only a small fraction of the possible benefits that could be derived from a full economic utilization of the gases being considered. The serial in which Mr. Wilson is outlining the various features of "The Application of Oxygen and Hydrogen to Industrial Operations" should therefore be of great interest to industrial executives and engineers who are on the lookout for new means to improve production.

However, this serial possesses an even greater breadth of overall interest for in addition to the consumers of gas it is of direct concern to those who have a part in its manufacture. For instance, the compressed-gas producers will share in the returns that result from additional applications of their products in the mechanical industries.

And, rather unexpectedly to many people, the subject being treated in the serial has a substantial influence on the interests of both the electrical manufacturing and central-station industries, because nearly half of the investment in many compressed gas producing plants lies in electrical equipment. Regardless of whether the consumer purchases or manufactures the gases that he uses, electricity plays a leading part in their production. For instance, an electrolytic process is depended upon largely for the supply of hydrogen, which is accompanied by an output of half as much oxygen. In addition to the cells, this process requires a background of electrical conversion and control equipment of considerable size. The manufacture of nitrogen from the air, with oxygen again as the companion product, is a strictly physical process but one in which electrical energy serves best to supply the power and heating requirements. Then, there are in either process the numerous auxiliary drives for which the electric motor, with its adaptable control, is without a competitor.

As a last or first consideration, depending upon whether one's interest lies mainly in the production or utilization of the compressed gases, the plant for their manufacture offers to the central station a load that is attractive in amount and practically ideal in character. The gas storage capacity of the plant serves as a flywheel that enables varying demands for gas output to be supplied by a steady power input which, combined with continuous day-and-night operation throughout the week, results in high load-factor. The remaining desirable feature—high power-factor—is available through the use of synchronous motors on the main drives, for which they are especially suitable.

Rates of Voltage Build-up Obtainable with Standard Exciters

By H. W. WASHBURN
Engineering General Department, General Electric Company

THE subject of rapid excitation has of late received a great deal of attention from those interested in electric power transmission. Investigations have indicated that the rates of voltage build-up of the exciters are important factors in determining the stability of a transmission system at the time of a short circuit. It was formerly the practice to limit the short-circuit current to as low a value as possible by using line reactors and allowing the terminal voltage of the alternators to drop to a very low value. In this way, the system was protected against excessive currents, but the drop in voltage was liable to cause the synchronous machinery in the system to fall out of step. This instability factor assumed more importance as the systems became larger and as the demand for continuity of service increased. Consequently practice has, in some cases, resorted to obtaining more synchronizing power during short circuits at the sacrifice of protection against excessive currents. That is, transient stability has been obtained by employing exciters with a relatively high ceiling voltage, and a rapid rate of voltage build-up. The possibilities and limitations of this method of maintaining stability have already been pointed out in a recent article(1) on this subject. In the present treatment, data are furnished on the rates of voltage build-up that are obtainable on exciters of standard design. It is hoped that the facts thereby established will be of assistance to those who are interested in excitation speeds.

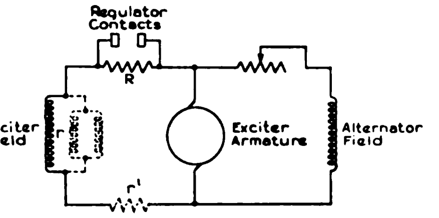

Fig. 1. Diagram of Connections for Self-excitation of Machines in Determination of Voltage Build-up

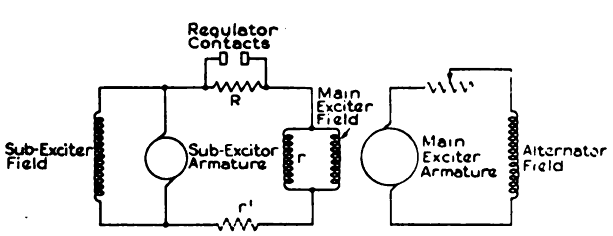

Fig. 2. Connections for Separate Excitation

Calculations

The rate of voltage build-up was determined for twenty-seven standard exciters including horizontal, vertical, and turbine types. The ratings (see Table I) were chosen so as to cover as well as possible the range of capacities and speeds in general use. The rate of voltage build-up of these machines was first calculated for the machines self-excited with all shunt-field coils connected in a single series circuit (as they would normally be operated). A diagram of this connection is given in Fig. 1. A second set of calculations was made for the same machines, self-excited but with the shunt-field coils connected in two parallel circuits as shown by the dotted circuit in Fig. 1. A third set of calculations was made for these machines connected as in Fig. 2, with their shunt-field coils in two parallel circuits, and separately excited from a sub-exciter having a constant voltage equal to the rated voltage of the main exciter.

TABLE I

TYPES AND RATINGS OF MACHINES CHOSEN FOR CALCULATIONS

Type	No. of Poles	Kw.	R.p.m.	Volts
Horizontal	4	5½	750	250
	4	7½	750	240
	4	12½	900	250
	6	30	450	125
	6	60	900	125
	6	60	720	250
	6	75	900	115
	6	80	600	125
	6	96	600	250
	6	145	500	250
Vertical	6	27	90	125
	6	38	100	125
	6	41	120	125
	6	48	400	250
	6	81	514	220
	6	140	300	250
	6	180	375	250
	8	186	300	250
	12	228	75	125
	10	250	120	250
	10	300	157	250
Turbine	6	50	1500	250
	6	100	1500	250
	6	125	1800	250
	6	150	1500	250
	6	175	1800	250

In the self-excited machine the terminal-voltage (*E*) of the machine is applied to the circuit having the shunt field and the resistance (*R*) connected in series (Fig. 1). The resistance (*R*) is alternately shorted out and inserted in the circuit by the regulator contacts.

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(1) "Super-excitation," by D. M. Jones, GENERAL ELECTRIC REVIEW, December 1927, p. 580.

The ratio of the time open to the time closed of these contacts determines the average terminal voltage of the exciter. When the resistance (R) is short-circuited, the voltage available for overcoming the electromotive force induced in the field by the change of the field flux linkages is equal to the terminal voltage (E) minus the ir drop in the field coils and leads (see Fig. 2). This is expressed by the equations:

$$10^{-8}\frac{d\Omega}{dt}=E-ir \qquad (1)$$

where

Ω = flux linkages.
r = resistance of field.
i = field current.

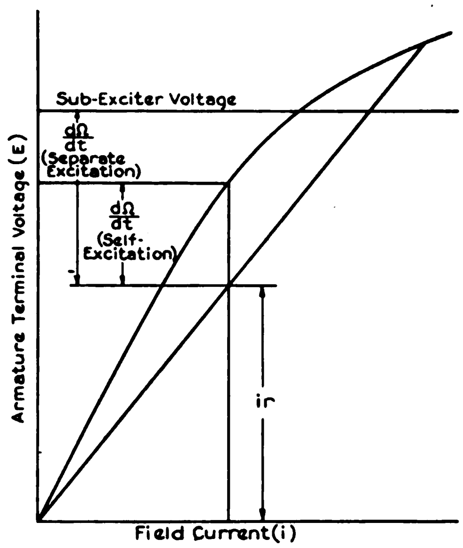

Fig. 3. Saturation Curve of Exciter Indicating the Relative Voltage Drops Caused by the Resistance and Inductance of the Field

The flux linkages (Ω) are equal to the sum of the air-gap and the leakage-flux linkages, *i.e.*,

$$\Omega=\frac{n}{n'}Nk\phi$$

or

$$\frac{d\Omega}{dt}=\frac{n}{n'}Nk\frac{d\phi}{dt} \qquad (2)$$

where

k = ratio of the total flux linkages to the air-gap flux linkages.
ϕ = air gap flux per pole in maxwells.
n = number of poles.
n' = number of parallel paths in field circuit.
N = turns per pole.

At a given speed of the exciter, the armature-induced voltage (E') is proportional to the air-gap flux (ϕ).

$$E'=K\phi$$

or

$$\frac{d\phi}{dt}=\frac{1}{K}\frac{dE'}{dt} \qquad (3)$$

where

K = a proportionality factor.

Substituting Equation (3) in Equation (2),

$$\frac{d\Omega}{dt}=\frac{nNk}{n'K}\frac{dE'}{dt} \qquad (4)$$

Substituting Equation (4) in Equation (1).

$$10^{-8}\frac{nNk}{n'K}\frac{dE'}{dt}=E-ir$$

or

$$\frac{dE'}{dt}=\frac{n'K(E-ir)}{nNk}\times 10^{8} \qquad (5)$$

where

E' = armature-induced voltage.
E = armature-terminal voltage.
i = total field current.
r = total field resistance.
n = number of poles.
n' = number of parallel paths in field circuit.
N = turns per pole.
K = ratio of armature-induced voltage E' to the air gap flux ϕ.
k = ratio of total flux linkages to air gap flux linkages.

Equation (5) gives the rate of voltage rise of an exciter at the instant the terminal voltage passes through the value (E). If this rate be calculated for successive values of terminal voltage, the complete history of the voltage rise from any point to the ceiling voltage is determined.

Calculations were made for values of (E) for 50, 75, 100, and 125 per cent of rated voltage. In order to obtain the slowest probable voltage rise that would occur under operating conditions, shunt machines were assumed to be operating at full load and compound machines at no load. The values of field current (i) were obtained from the no-load saturation curve; the values of the field resistance (r) (at 75 deg. C.), the number of poles (n), the turns per pole (N), and the proportionality factors (K) and (k) were taken from the design data. In making calculations for the conditions of separate excitation, it was merely necessary to replace the value of the armature terminal voltage (E) by rated voltage, all other values remaining the same (see Fig. 3).

The results of tests made on several exciters gave measured rates of voltage build-up of from 5 to 10 per cent less than the calculated rates. In all probability this discrepancy is mainly due to neglecting the demagnetizing effect of the eddy currents in the field yoke. These facts should be considered when studying the curves in Figs. 4 to 13.

Effect of Paralleling Fields

The effect of paralleling the fields may be shown by expressing Equation (5) in another form. The inductance (L) of the field circuit in terms of the constants already defined is:

$$L=\frac{\Omega\times10^{-8}}{i}=\frac{nNkE'\times10^{-8}}{Kn'^2 i_c} \qquad (6)$$

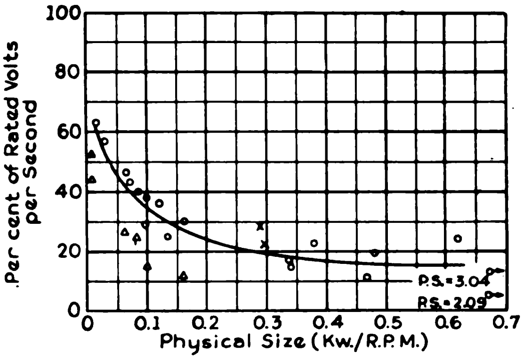

Fig. 4. Excitation 50 per cent rated exciter voltage

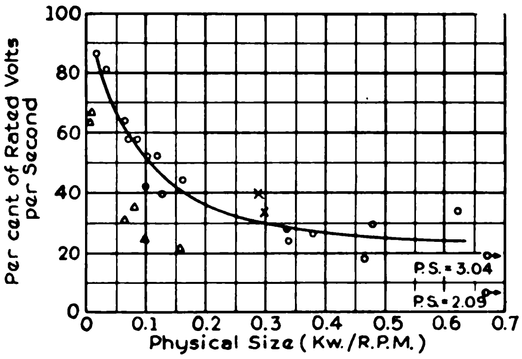

Fig. 5. Excitation 75 per cent rated exciter voltage

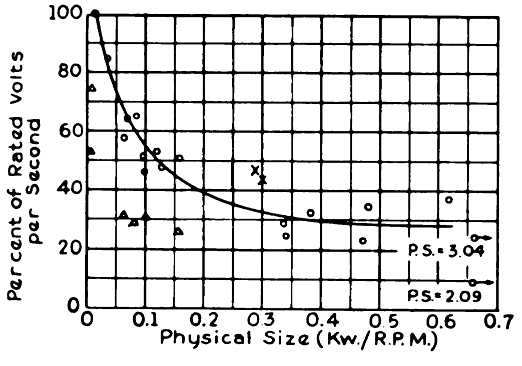

Fig. 6. Excitation full rated exciter voltage

Figs. 4-6. Curves Showing Rate of Voltage Build-up for Self-excited Machines with Field Coils Connected in Series

where

L = inductance of the field circuit in henrys.

i = total field current.

$i_c=\frac{i}{n'}$ = field-coil current.

The field resistance may be expressed in terms of the field-coil resistance

$$r=\frac{nr_c}{n'^2} \qquad (7)$$

where

r = resistance of field at terminals.

r_c = resistance of one field coil.

The time constant of the field circuit obtained from Equations (6) and (7) is:

$$\frac{L}{r}=\frac{Nk}{Kr_c}\times\frac{E'}{i_c}\times10^{-8} \qquad (8)$$

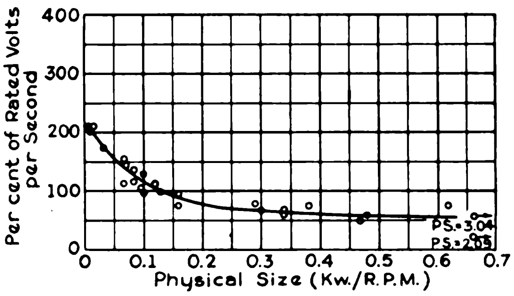

Fig. 7. Excitation 50 per cent rated exciter voltage

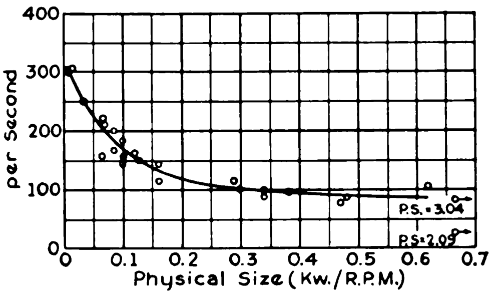

Fig. 8. Excitation 75 per cent rated exciter voltage

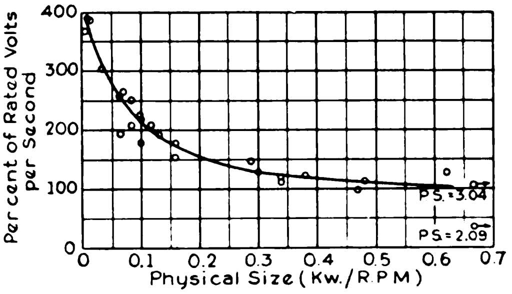

Fig. 9. Excitation full rated exciter voltage

Figs. 7-9. Curves Showing Rate of Voltage Build-up for Self-excited Machines with Field Coils Connected in Two Parallel Circuits

The voltage (V) available per coil for overcoming the electromotive force induced in the field by the change of field flux linkages is:

$$V=\frac{(E-ir)\,n'}{n} \qquad (9)$$

From Equations (5), (8), and (9) the following expression is obtained for the rate of voltage build-up:

$$\frac{dE'}{dt}=\frac{E'}{i_c}\times\frac{V}{r_c\left(\frac{L}{r}\right)} \qquad (10)$$

It is interesting to note from these latter equations the underlying cause for the increase in the rate of voltage build-up as a result of paralleling the fields. In the right-hand member of Equation (8) all terms are constant for a given machine except the term $\frac{E'}{i_c}$ and this term is independent of the field-coil connections. The time constant $\left(\frac{L}{r}\right)$ of the field circuit is therefore independent of the number of parallel paths in the field circuit. Again, the only term in the right-hand member of Equation (10) which is altered by paralleling the fields is the voltage (V). It is concluded that the increase in the rate of voltage build-up resulting from paralleling the fields is due only to the increase in the voltage (V), which is the voltage available for overcoming the electromotive force induced by the change of the field-flux linkages.

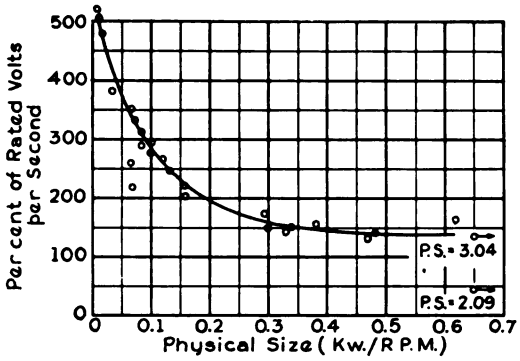

Fig. 10. Excitation 50 per cent rated exciter voltage

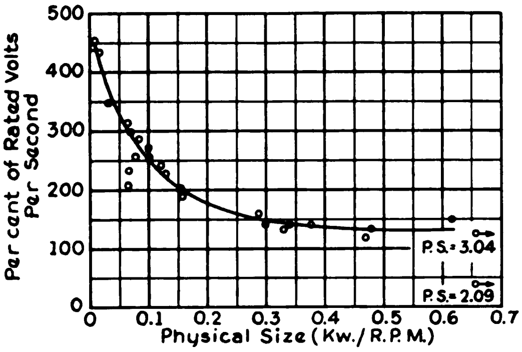

Fig. 11. Excitation 75 per cent rated exciter voltage

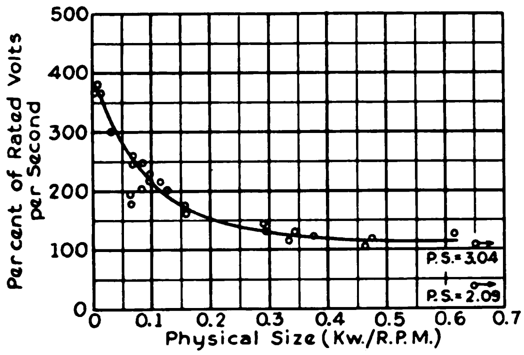

Fig. 12. Excitation full rated exciter voltage

Figs. 10-12. Curves Showing Rate of Voltage Build-up for Separately-excited Machines with Field Coils Connected in Two Parallel Circuits

Characteristics of Exciter Build-up

The calculated rates of voltage build-up were plotted against the factor *kw.*/*r.p.m.*, which is very nearly proportional to the volume or physical size of the machine armature. The curves thus obtained are shown in Figs. 4 to 12. In these curves each plotted point represents a machine. It will be noted that, in the case of normal self-excitation, the points did not group themselves as well as in the case of separate excitation. This leads to the conclusion that the difference would lie in the variation of the field resistance. The machines were therefore classified according to their field *ir* drop for full-load and rated voltage. Those machines having *ir* drops between 55 and 75 per cent of rated voltage were represented by the small circles; those lying above this range by triangles and those lying below by crosses. Reference to Figs. 4, 5, and 6 will show that the circles group themselves fairly well, verifying the conclusion just stated. The curves were drawn so as to represent the mean of the values corresponding to the circles, and show that the following approximate division can be made:

(1). For machines having a ratio of kw. to r.p.m. above 0.3 (large machines), the rate of voltage build-up is approximately the same for the different physical sizes.

(2). For machines having a ratio of kw. to r.p.m. below 0.3 (small machines), the smaller the physical size of the machine, the faster the rate of voltage build-up.

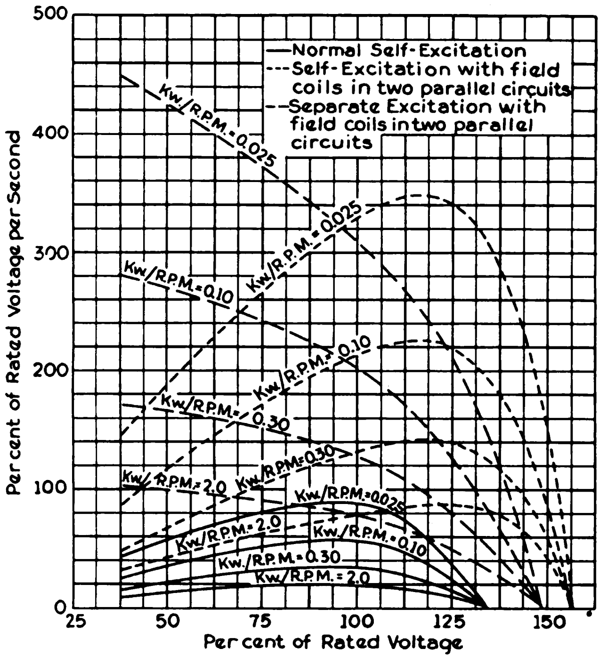

Fig. 13. Composite Curves Showing Rates of Voltage Build-up on Standard Exciters

For convenience in making comparisons, the curves in Figs. 4 to 12 and a few other calculations are summed up by the curves in Fig. 13, which give the complete history of the build-up rate between any voltage limits determined by the operation of the regulator contacts. These curves show that the following comparisons hold true for all types and sizes of machines:

(1). For the larger part of the working range, the instantaneous rate of voltage build-up with self-excitation and paralleled fields is roughly four times that obtained with normal self-excitation (all field coils connected in series).

(2). The instantaneous rate of voltage build-up is greater with separate excitation and paralleled fields than with normal self-excitation by a factor which depends on the exciter armature voltage. This factor has a value of about 8 at 50 per cent and 125 per cent voltage; its minimum value is about 4 and occurs near rated voltage.

In order to avoid the excessive field currents which occur near the ceiling voltage when parallel circuits are employed, it is usually necessary to insert an extra resistance in the field circuit, as in Fig. 1. If a quick-acting relay is used to insert this resistance as soon as a certain field current is exceeded, the speed of build-up indicated by the curves in Fig. 13 would be obtained up to the point of operation of the relay. Above this point the build-up speed would, of course, be reduced. If, however, a resistance is permanently inserted in the field circuit the build-up speed would be reduced over the entire voltage range. In such cases the curves shown in Fig. 13, and the comparison made from them, would be modified to a degree depending on the amount of resistance inserted. For example, consider the extreme case of inserting a resistance which is large enough to bring the maximum field-coil current down to the maximum value obtained with normal self-excitation. The instantaneous build-up speed obtained with self-excitation and paralleled fields will be reduced to just twice that obtained with normal excitation. The instantaneous build-up speed obtained with separate-excitation and paralleled fields will be little reduced at low voltages; but the reduction becomes more pronounced with the increase of voltage until, for voltages above normal, the build-up speed is only about twice that obtained with normal self-excitation.

This article thus far has dealt with the rate of voltage build-up. Another important exciter characteristic, from transient system stability considerations, is its ceiling voltage. The ceiling voltage of standard exciters with normal self-excitation will be roughly 135 per cent of rated voltage. Because of the saturation of the iron and the necessity of limiting the field current, the ceiling voltage with paralleled fields or with separate excitation will be only about 15 per cent of rated voltage above the normal ceiling voltage. The advantage of paralleling the fields or of separately exciting a standard exciter is therefore limited almost altogether to the obtaining of a faster rate of voltage build-up.

Conclusions

(1). For standard exciters with normal self-excitation, the maximum rate of voltage build-up occurs at about rated voltage. This maximum rate is between 10 and 100 per cent of rated voltage per second, depending on the physical size of the machine.

(2). For standard machines with self-excitation, but with the field coils connected in two parallel circuits, the rate of voltage build-up is roughly four times that obtained with normal self-excitation.(2)

(3). For standard exciters with field coils in two parallel circuits and separately excited, the maximum rate of voltage build-up occurs at zero exciter voltage. At rated exciter voltage, this rate is about two-thirds its maximum value and is about four times the maximum value obtained with normal self-excitation.(2)

(4.) The ceiling voltage on standard machines with paralleled field or with separate-excitation is about 15 per cent of rated voltage higher than with self-excitation.

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(2) These comparisons apply only to cases where no extra field resistance is permanently inserted.

How Much Water Flows Over Niagara Falls?

How much water flows over the great Niagara cataract? Engineers measured the flow in the Niagara River in 1917 and concluded that 200,000 cubic feet per second made the drop—the biggest drop of water in America. Since 1917 the level of the water in Lake Erie has lowered slightly, thus reducing the flow over the Falls. In order to determine how much, engineers shot a line acoss the river at Goat Island just above the Falls for the purpose of installing a cable from which soundings of the river depth are made. Thus will be determined the rate of flow, almost to the gallon, and hence calculated how much of the water is put to work making electricity and how much is wasted. About 96 per cent of the river goes over the Horseshoe Falls on the Canadian side. The center of the crest is moving backward eight feet each year.

The Direct-current Transformer Utilizing Thyratron Tubes

By D. C. PRINCE
Research Laboratory, General Electric Company

The transformation of direct current without rotating machinery has for many years afforded a most fascinating problem. Starting with the already available tube rectifier as the latter half of a dc-ac./ac-dc. transformation, pliotrons were a few years ago made to function as inverters and thus complete the solution. In power applications, however, thyratrons are more suitable than pliotrons.
—EDITOR

IN a previous article describing the Inverter(1) it was shown that pliotrons, or high-vacuum three-electrode valves, in suitable circuit arrangements can be made to convert direct current into alternating current in a manner the inverse of that in which a rectifier converts alternating current to direct current. In developing the inverter, the transfer or commutating problem was solved in such a way that the current reverses naturally, leaving to the pliotron grid only the function of holding the current at zero until it should flow again.

The inverter is particularly well adapted for converting direct-current power into alternating-current power at the usual commercial frequencies except for the losses that result from the present rather high space charge drop in the pliotrons. In recent years considerable research by several workers has been undertaken in order to produce and utilize tubes without incurring excessive losses. One of the most interesting of these tubes is the "Thyratron," one form of which consists of a mercury-arc rectifier with a grid added to each anode arm, as shown in Figs. 1 and 2. In operation, the low voltage drop and high efficiency of the mercury-arc rectifier are obtained with valuable control characteristics which are, however, somewhat different from those of the pliotron. While no current is flowing to an anode, the space surrounding that anode remains free of ionization. In this state, the well-known laws of grid action apply as in a high-vacuum tube. As long as the negative electric field due to the grid predominates over the positive electric field due to the anode, no current will flow. If, on the other hand, current is flowing to the anode, the surrounding space is filled with ionization and the grid is no longer effective. The thyratron thus has the characteristic that current can be prevented from starting by a small negative grid voltage but cannot be stopped under the usual conditions of operation by applying a large negative grid voltage. These tubes are, therefore, not suitable for driving oscillating circuits unless a natural stopping of the current is obtained by such methods as was described in the article on inverters.(1)

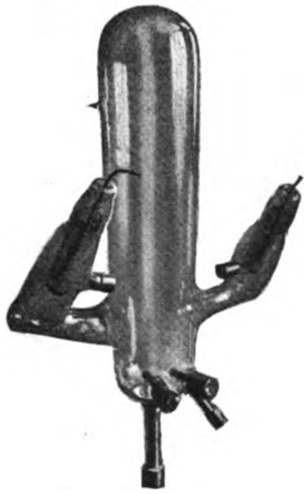
Fig. 1. Thyratron Tube

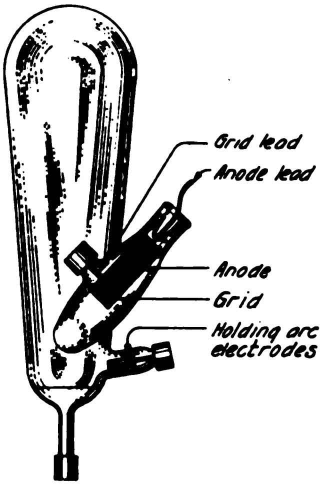

Fig. 2. Sketch of Thyratron Tube with Principal Parts Indicated

More specifically, the current flow is stopped without using grid control. Such a circuit is shown in Fig. 3. It differs from a single-phase high-vacuum tube inverter circuit in that no attempt need be made to keep the grid current proportional to the anode current. As in the high-vacuum inverter, an angular advance must be given to the grid phase. The operation of such a circuit is indicated in Fig. 4. In this illustration, the center points of the transformer windings are used as the points of voltage reference. The potentials e_{a1} and e_{a2} are impressed upon the two anodes a_1 and a_2. Between times t_1 and t_1', the potential of grid g_1 is positive so that current flows to it and to the anode a_1 that it surrounds. During this time, grid g_2 has a negative potential impressed upon it so that no current can start to flow to a_2, the anode which it surrounds.

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(1) "The Inverter," by D. C. Prince, GENERAL ELECTRIC REVIEW, October, 1925, p. 676.

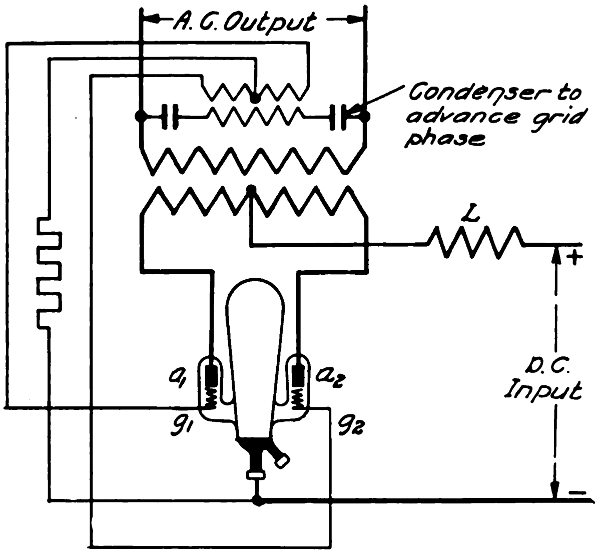


Fig. 3. Thyratron Inverter Circuit

At t_1' the relative polarity of the grids reverses. At this time, anode a_2 is more positive than a_1, and therefore tends to gain current at the expense of a_1 since the total current is held substantially constant by inductance L. Between t_1' and t_2', all the current leaves anode a_1 for a_2 so that at t_2' there is nothing to maintain ionization in the neighborhood of grid g_1 and anode a_1. From t_2' to t_3', the ionization decays and the grid g_1 is able to prevent the current from restarting when anode a_1 becomes more positive than a_2 at t_3'.

At t_1'' the reverse transfer begins to take place and is completed by t_2'', the time between t_2'' and t_3'' serving for de-ionization of the space around a_2.

The wave form of voltage is here assumed to be supplied by the load which may be synchronous machinery or may include static condensers. The operation with static condensers is described in greater detail later.

For purposes of reference, the diagram of a single-phase rectifier is shown in Fig. 5, and the corresponding wave shapes in Fig. 6. A comparison will show a great similarity between the connection diagrams and wave shapes of the inverter and the rectifier. Except during transfer, the inverter and the rectifier have sine waves of counter-voltage and of voltage respectively. Each has a nearly square current wave. The instantaneous powers of the two are therefore of the same form. The variations from the average are absorbed by the inductances and are also of the same form.

It is therefore possible to combine the two circuits as in Fig. 7. In the combination, the alternating-voltage wave form is supplied by condenser C which supplies the transformer excitation that is purposely made large for stability. Since the inverter input power has the same form as the rectifier output power, there is no need for any energy to be stored inside the device except that represented by transformer excitation. That is, the input and output currents neutralize and do not tend to prevent the free swing of the oscillating excitation circuit which acts as a small flywheel. If the power input and output did not balance, a large flywheel would be required, but with the nearly perfect balance very little storage action is necessary.

Since the departures from average power are similar in both inverter and rectifier, the inductances L_1 and L_2 may be wound on the same core provided the turns are in the same ratio as the turns of the main transformer. Were separate inductances used, each would carry a direct-current component and so would have to be made with open cores. When combined on a single core, however, the inverter and rectifier direct currents flow in opposite directions in this two-circuit inductance, so that the saturating effects cancel and a closed core can be used.

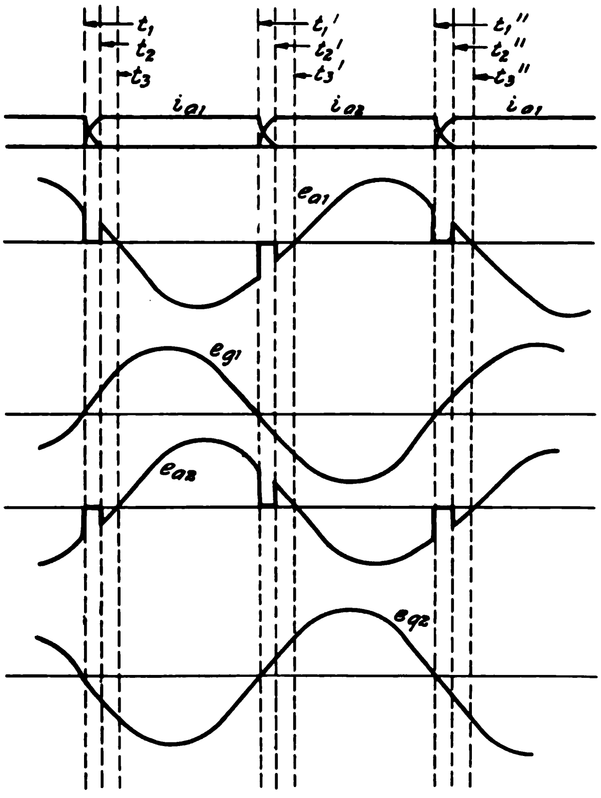


Fig. 4. Wave Shapes of Thyratron Inverter, Showing the Transfer of Current Advanced in Phase with Respect to t_1, t_1', t_1'', etc.

i_{a1} and i_{a2} are the anode currents; e_{a1} and e_{a2} are the anode voltages with respect to the middle of the transformer primary winding; and e_{g1} and e_{g2} are the grid voltages with respect to the center of the winding supplying grid excitation.

Such a closed core does not store energy as does a heavy inductance. Instead, a sudden rise of rectified load accompanied by a corresponding rise in inverter input encounters only the leakage reactance of the combination reactance. When the oscillating circuit voltage is passing through zero, the inverter has no counter-electromotive force, and the entire impressed voltage appears across the inductance. Simultaneously the rectifier has no output voltage so that the entire load must be supplied by the inductance. This device, therefore, transfers power directly from input to output circuits. Conversely, when the oscillating voltage is at its peak, it is higher than the impressed continuous potential. Simultaneously, the rectifier voltage is higher than the counter-electromotive force of its load circuit, so that energy flows back from the rectifier to the inverter circuit.

In effect, the main transformer converts the fundamental energy in sine waves. The combination reactance converts the harmonic energy difference between the continuous input and output and the pulsating energy of the main transformer. The title of harmonic transformer would seem to apply naturally to this piece of apparatus.

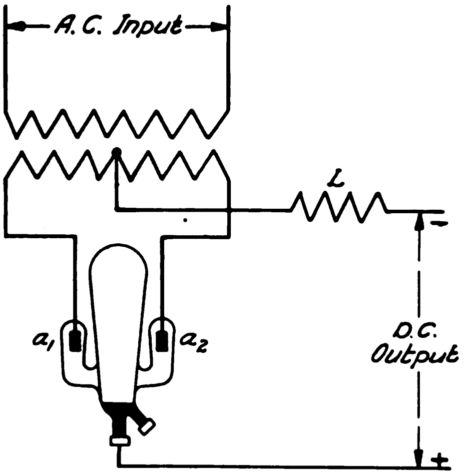


Fig. 5. Mercury Arc Rectifier Circuit

Turning now to the transfer period as shown in Fig. 8, it appears that from t_2 to t_2' the potential difference between inverter anodes a_1 and a_2 is in the correct direction to produce transfer to a_2. At t_1', grid g_2 becomes positive and current can flow to anode a_2. Condenser C promptly discharges, throwing the current all to anode a_2. From t_1' to t_2', the current to anode a_1 is held at zero by the continuing discharge of condenser C which is, however, hastened by the fact that the current is not commutated immediately by the transformer and must, therefore, pass through C until the secondary current is commutated. De-ionization of the space surrounding anode a_1 takes place during this period.

The potential difference between anodes a_1 and a_2 reverses at t_2', but de-ionization is complete and grid g_1 now has a negative potential so that the current cannot return to anode a_1. The potential difference is now correct to transfer current from rectifier anode a_3 to a_4. This transfer takes place between t_2' and t_3', and the free oscillation cycle then recommences. Between t_1 and t_2 and between t_1' and t_2', etc., full-load current flows in condenser C; and even after t_2 and t_2', part of the load current flows through C for a short period.

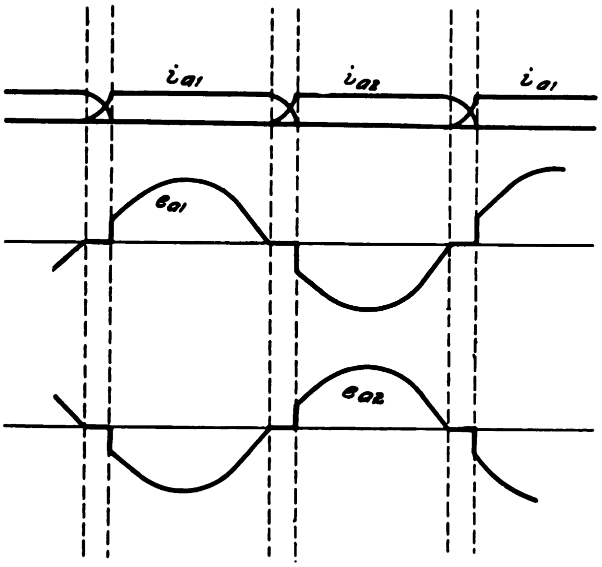


Fig. 6. Mercury Arc Rectifier Wave Forms, Showing Natural Lag of Current Transfer

i_{a1} and i_{a2} are the anode currents; and e_{a1} and e_{a2} are the anode voltages with respect to the middle of the transformer secondary winding.

Fig. 9 shows oscillograms of some of the currents and voltages in different parts of the circuit. Unfortunately, sufficient vibrators were not available so that all traces could be made on one film. Fig. 9 therefore consists of two films. Due to slight irregularities, it was impossible to match these films exactly, but the upright lines are drawn through corresponding points on the various traces. Trace a is the potential between one thyratron anode and the cathode. Trace b shows the corresponding anode current. This trace was taken through a current transformer so that trace b_0, the normal zero line, is in reality the average direct current; and the

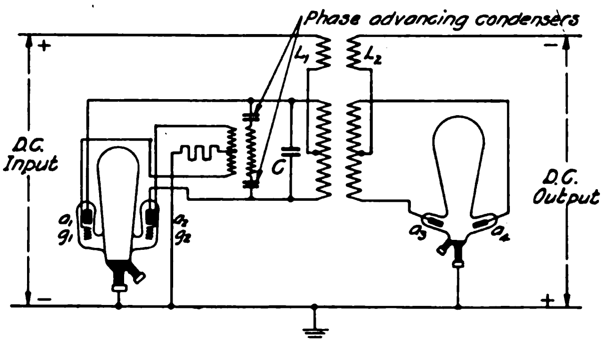


Fig. 7. Direct-current Transformer Circuit, Showing the Combination of Rectifier Circuit (Fig. 5, on right) and Inverter Circuit (Fig. 3, on left)

A small oscillating circuit including condenser C and the exciting reactance of the transformer shown to the right of it supplies the exciting current for this transformer and stores the small amount of energy required for this service.

true zero line *b′* must be drawn on the film. The current is zero while there is a drop across the tube and, when it starts, it jumps to full value while the voltage falls to almost nothing. This accounts for the very high efficiency of these devices, since losses require simultaneous current and voltage drop. Trace *c* shows the current drawn by one grid. The starting of the grid and the anode currents coincide, but, unlike the high-vacuum tube, no further correspondence is necessary. This contributes to the simplicity of circuit design, since no provisions for controlling the grid-current wave shape are required. Trace *d* is the main transformer secondary voltage. Each half cycle shows the usual form of a shock-excited wave with a fairly high decrement. This decrement is probably due mostly to the alternating-current losses, since the output is almost exactly balanced by the input throughout the cycle. Trace *e* is the voltage across the grid transformer. It leads the main transformer voltage in time.

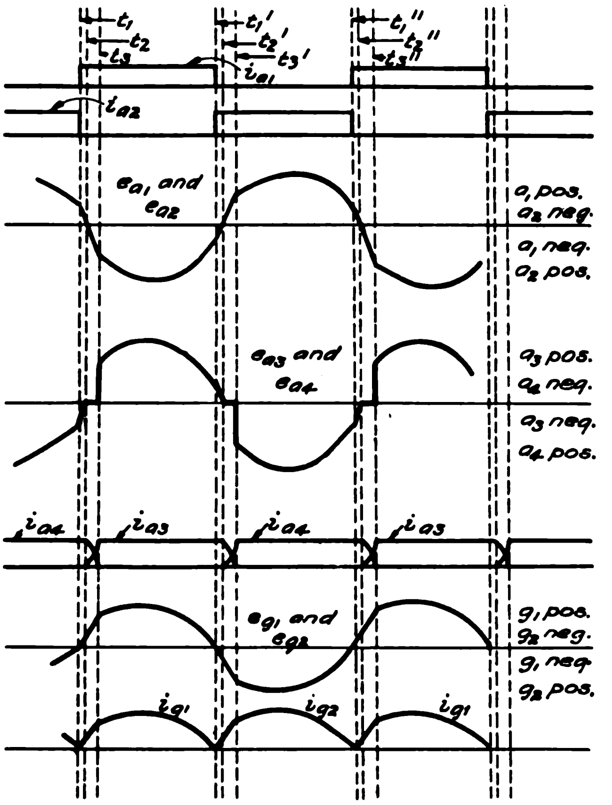


Fig. 8. Wave Shapes of Direct-current Transformer, Indicating the Combination of Those Corresponding to the Inverter and the Rectifier

i_{a1} and i_{a2} are the thyratron anode currents, e_{a1} and e_{a2} the thyratron anode voltages with respect to the middle of the transformer winding. i_{a3} and i_{a4}, and e_{a3} and e_{a4} are the anode currents and voltages of the rectifier. e_{g1} and e_{g2} are the thyratron grid voltages with respect to the mid-point of the exciting winding and i_{g1} and i_{g2} the grid currents.

It will be observed that the time t_1 at which the grid voltage reverses is the time at which a sudden change takes place in each of the other waves. These rapid changes are due to the release of the condenser charge for an instant during which it commutates the inverter current. This is started by the grid which becomes positive, and it stops when commutation is completed. Since there is no reactance in the circuit involved, the action is extremely rapid. At time t_2, the reverse action takes place. A point of particular interest is the negative kick in trace *a*. This is due to commutating the inverter current before the voltage passes through zero, and is depended upon to stop the arc and allow the grid to acquire control.

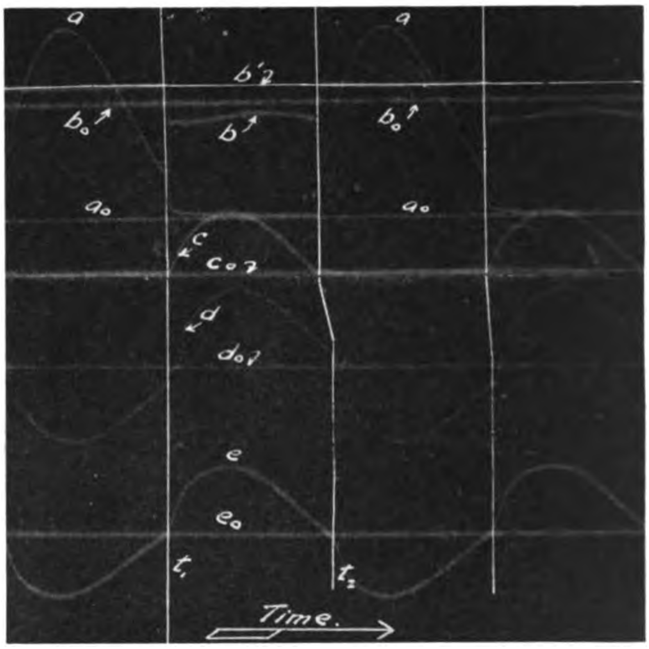


Fig. 9. Oscillograms of the Wave Shapes Obtained with a Direct-current Transformer

a_0—a is the voltage between one thyratron anode and the cathode, b'—b is the corresponding anode current, c_0—c shows the current drawn by one grid, d_0—d is the main transformer secondary voltage and e_0—e is the voltage across the grid transformer.

Not without some difficulty is the imagination restrained from picturing distribution systems of the future using electric valves of different types. It is practically certain that circuits and apparatus of the types described here will be of increasingly greater interest to power system operators.

Maintenance of Gas-electric Motor Coaches

Schedules of Inspection and Maintenance—Man-hours Required for Mechanical and for Electrical Work—Details of Electrical Work—Analysis of Types of Road Failures

By G. W. WILSON
Railway Engineering Department, General Electric Company

THE business problem of any public transportation system is to furnish a fast, reliable, comfortable, and safe service to its patrons, and at the same time make a financial profit on the operation. The solution of this problem can be reached only through the proper choice of equipment and the proper maintenance of this equipment after it has been placed in service. Having secured the type of vehicle best adapted to the purpose, the determination of the proper inspection and maintenance schedules becomes of prime importance.

In the past four years, the modern gasoline-electric motor coach has been developed and approximately 1800 vehicles equipped with this type of transmission have been placed in operation on 30 different properties throughout the United States. The purpose of this article is first to describe briefly the electrical equipment used in a modern motor coach transmission, and then to outline in greater detail the most modern inspection and maintenance schedules for this type of equipment, the road failures that occur most frequently and how these failures are being corrected, and the general results being obtained from the operation of the coaches.

The construction of the electric-drive motor coach is almost identical with that of the mechanical-drive vehicle except for the difference in the power transmission units.

The equipment employed in the electric type of transmission consists essentially of a direct-current self-excited shunt-wound generator (having a small amount of battery teaser field to assure a quick and positive pickup) direct connected to the gasoline engine through some form of flexible drive, and one or more direct-current series-wound motors connected to the driving wheels through a conventional propeller shaft or shafts and proper gearing in the rear axle housing. The control equipment consists of a main controller for connecting the generator to the motors and for determining the directional movement of the vehicle, a potential relay and accelerator switch for controlling the flow of current through the battery teaser field of the generator, and an electric braking resistor. The inherent characteristics of this electrical combination provide an automatic type of transmission that allows the torque at the electric traction motor shaft or shafts to vary from four times to one-half the engine torque with the engine running at full throttle and at its most economical speed. This characteristic permits full engine power to be applied, without interruption, to the driving wheels at all vehicle speeds. An electric-drive vehicle having a given power-weight ratio can thus attain higher acceleration rates than can the conventional gear-drive motor coach. The higher acceleration rate results in a higher schedule speed under most conditions of motor-coach service.

In working out an adequate and economical maintenance system for gas-electric motor coaches, the character of service rendered, the type of vehicle used, and the climatic and road conditions must be taken into consideration, for differences in these factors will materially affect the maintenance and inspection methods and costs. The majority of electric-drive motor coaches are being operated over city streets in a northern climate where stops are frequent, heavy peak loads are carried, and a fast schedule speed is maintained. These vehicles are usually equipped with six-cylinder engines developing 75 to 90 hp. at 1800 r.p.m. The motor coach complete with a single-deck body, seating 29 to 33 passengers, weighs 14,400 to 16,000 lb. without passengers. The same chassis, with a double-deck body that seats 67 to 71 passengers, weighs 18,000 to 21,000 lb. without passenger load.

With this general type of vehicle and character of service to be maintained, the following inspection and maintenance schedule has been worked out by selecting the good points from the schedules employed by a number of transportation systems which operate electric-drive motor coaches. If this schedule is carefully followed, effective, adequate, and economical inspection and maintenance should result. Since the mechanical portions of the vehicle are practically the same as those used in a mechanical-drive vehicle, no attempt will be made to give detailed instructions for the inspection and maintenance of these parts.

The inspection schedule should be divided into three classes and the time for each inspection measured on either a mileage or some other definite basis. It has been found, however, that the mileage basis is entirely satisfactory where the character of service of the whole property is comparable.

1000-mile Inspection

After each 1000 miles of service each part of the vehicle should be inspected and all bearings and wearing parts lubricated. The electric generator and motor commutators should be inspected for loose bars or high mica, the brushes checked for wear and to insure that they are free in the brush-holders. The controller relay, and accelerator switch should be inspected for burned contact fingers and any necessary adjustment made. The braking resistor should be checked for broken grids. This entire inspection should take from 2 to 4 man-hours, about one-half hour of which should be devoted to the electrical equipment.

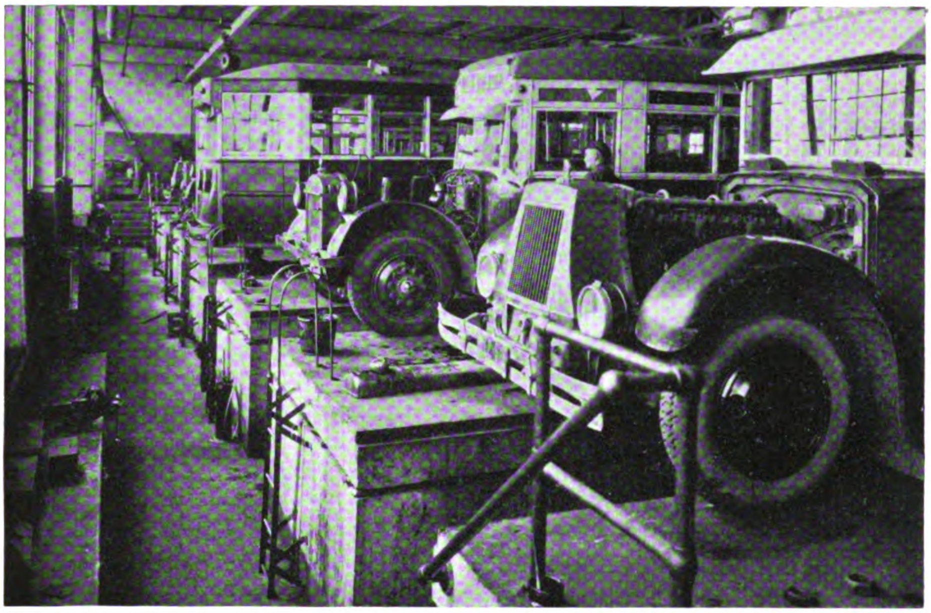


Fig. 1. Motor-coach Pit Arrangement Employed by the Capitol District Transportation Company, Albany, N Y.

2000-mile Inspection

The 2000-mile inspection should comprise a more rigid survey of the mechanical portions of the vehicle for the purpose of detecting and replacing any abnormally worn or defective parts. In addition to the same oiling and greasing as was done at 1000 miles, the engine oil should be changed. The electrical equipment should also be inspected as at 1000 miles; and in addition the generators and motors should be inspected for proper brush-spring tension, any road dirt or dust that has collected inside the machines should be blown out with dry compressed air and the bearings checked for noise. The controller, relay, and accelerator-switch finger pressure should be checked and adjusted if necessary. This inspection should take from 5 to 7 man-hours and approximately 1 hour's work should be spent on the electrical equipment.

30,000-mile Inspection

After 30,000 miles of operation it has been found economical to include certain maintenance work on the vehicle in addition to the regular inspections previously outlined. This work is usually done in the regular inspection garage without removing the units from the chassis. At this time the gasoline engine is completely overhauled and other mechanical portions of the chassis are checked for wear and replaced if necessary. In addition to repeating the work done on the electrical equipment at the 1000 and 2000-mile inspections it may be found necessary, in the case of the earlier types of equipment, to remove the generators from the chassis to turn and slot the commutators. This inspection should take from 10 to 12 man-hours, about two hours of which should be spent on the electrical equipment.

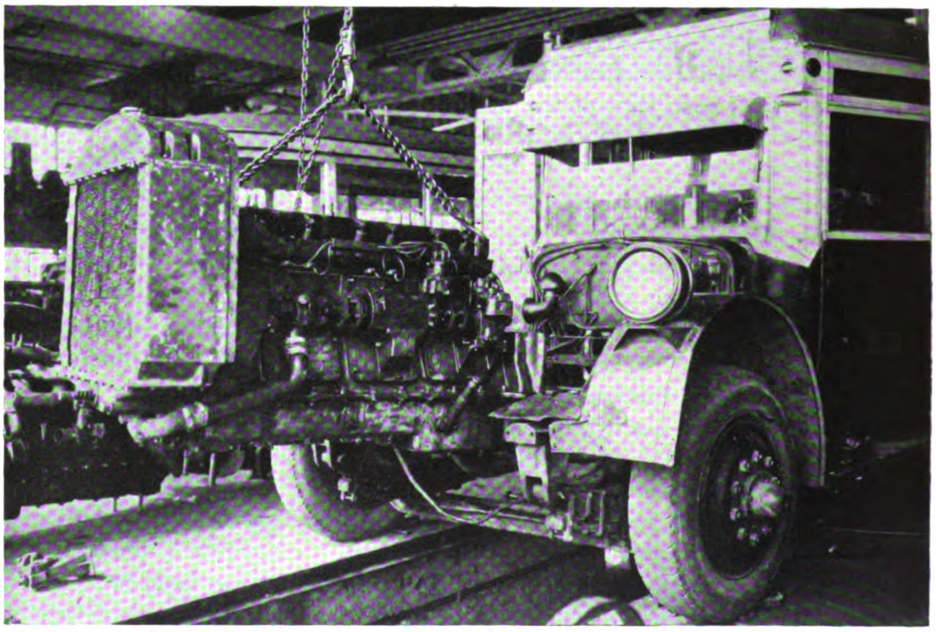


Fig. 2. Method of Removing Engine from Gas-electric Coach

General Overhaul

In addition to the regular inspection periods outlined, it is almost universal practice among gas-electric motor-coach operators to establish a regular overhaul period at which time each individual part is disassembled, checked for wear, and repaired or renewed if required. The length of service between these overhaul periods varies with the operation but a good average figure is 100,000 miles. At this time all electrical units should be removed from the coach, and the motors and generators completely disassembled and each part thoroughly cleaned. The armatures and field coils of the motors and generators should be dipped in a good insulating varnish and thoroughly baked. The commutator should be tightened, turned, and under cut. The ball bearings on all machines should be checked for worn or cracked balls and replaced if necessary. The brush-holder boxes should also be checked for wear. Before assembling these machines each insulated part should be given a high-potential test of twice the normal voltage, and after assembling the complete machine should be given the same high-potential test. The main controller should be thoroughly cleaned and any burned spots on the drum should be dressed up with a fine file. Any fingers that show signs of heating or burning should be replaced. When work is completed on the controller it should be given the same high-potential test that is recommended for the motors and generators.

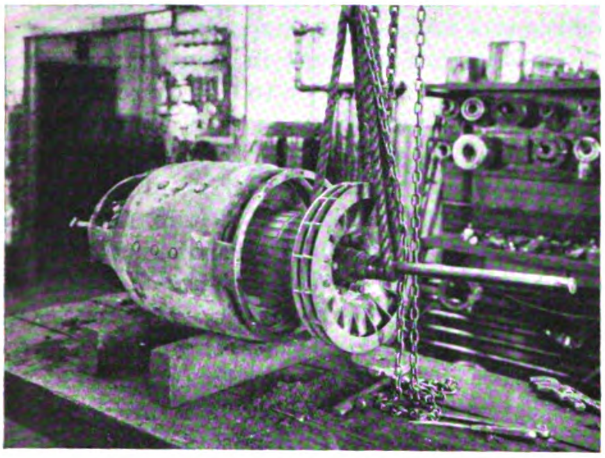


Fig. 3. Removing Armature from a Motor-coach Generator

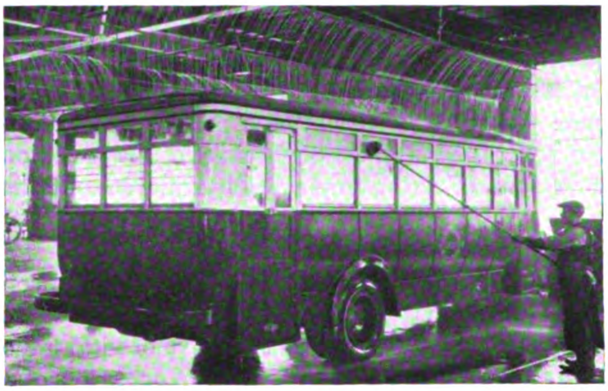


Fig. 5. Garage Facilities for Washing Coach Bodies Quickly and at Low Cost

The contacts, fingers, and springs of the relay and accelerator switch should be replaced. The braking resistors should be checked for broken grids and given the same high-potential test that is recommended for the motors, generators, and controllers. This entire procedure so far as the electrical equipment is concerned should consume from 20 to 30 man-hours. The time taken for doing all of the overhaul work is approximately 8 working days. After the vehicle has been thus overhauled, it is in first-class condition for another 100,000 miles of service with the intervening inspection periods as outlined.

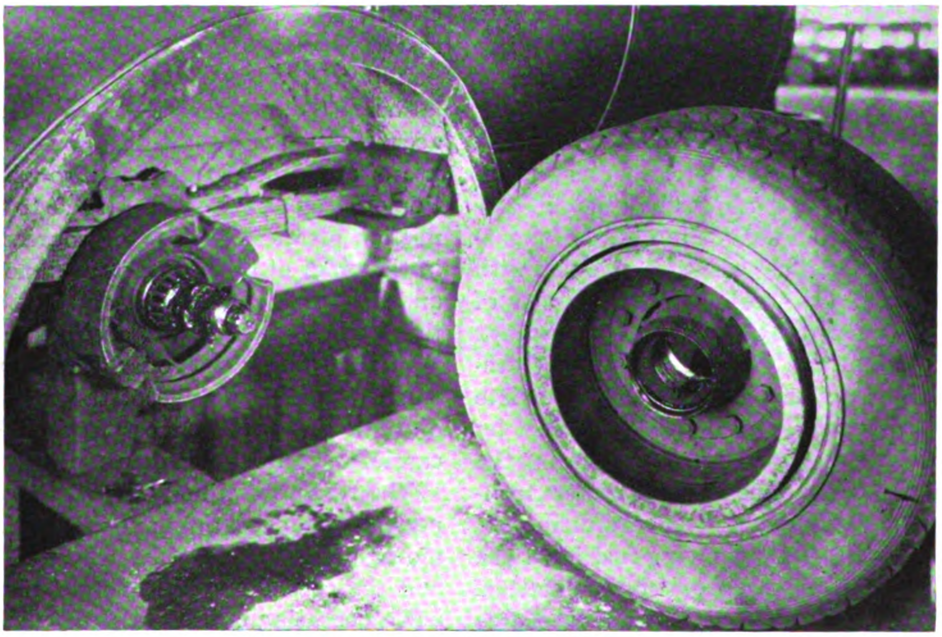


Fig. 4. Wheel Removed for Relining the Brake

The matter of painting has purposely been excluded from this article since it is subject to the desires of the particular operator involved.

Labor Necessary

The number of men necessary to carry out the inspection and maintenance schedule outlined will of course depend on the garage facilities available and the number of vehicles operated. A survey of the operation of 1400 gas-electric drive motor coaches on a number of properties of varied territorial extent has shown that 3.5 vehicles per garage employee is a fairly good average figure for carrying out this work. This includes garage foremen and assistants, mechanics and helpers, electricians and helpers, carpenters, gassers, oilers, greasers, and stockroom employees. An attempt was made during this survey to determine the ratio of electricians per vehicle operated, but it was found impossible because of the variation in the system of rating employees by different operators.

Road Failures

Regardless of the thoroughness of the inspection and maintenance system used, the problem of road delays and their prevention is always of vital importance to the operator. The majority of operators of gas-electric motor coaches keep an accurate record of all road delays and from this record endeavor to correct the troubles that frequently occur and cause delay. Some operators have prepared charts from their road-delay sheets and from these charts have attempted to establish a useful life period for all parts of the vehicle that are a continual source of annoyance. After this useful life period has been established, the plan is to replace the parts before actual failure occurs. While this system has much to recommend it, the question of the useful life of a given part must be checked frequently and new schedules arranged in order to make sure that maximum life is being obtained before the part is discarded; otherwise, this method proves wasteful.

The most frequent of all the road failures of a gas-electric motor coach are those caused by trouble with the gasoline engine. These troubles, in the order of their frequency of occurrence, originate in the spark plugs, ignition system, and gasoline and cooling systems. There seems to be little hope for the immediate correction of the first two kinds of trouble except by the method of determining useful life and replacing the parts before failures occur. The gasoline system troubles are continually being reduced by careful inspection and by keeping all parts carrying fuel from the gasoline tank to the engine free from foreign matter. The cooling system trouble that was a continual source of annoyance on old gas-electric coaches, due to overheating, has almost completely disappeared on later vehicles of this type. The improvement was brought about by the use of larger capacity radiators. However, an occasional cleaning of the radiator is still necessary in order to prevent the adjoining engine from becoming overheated.

Brakes and brake mechanism troubles rate second in causing road delays. These delays are being reduced by the installation of brake-testing equipments in the garages, and by the use of a proper combination of brake lining and drum. The average life of a fabric lining on a single-deck gas-electric motor coach in the service that has been described is between 8,000 and 12,000 miles. The present electric brake is not suitable for service stops but becomes of material value in descending long grades where it is desired to hold the vehicle at a fairly constant speed.

Tire troubles are third in causing road delays. The number of delays due to this cause is being reduced and in many instances such delays have been almost completely eliminated by the use of a tire of adequate capacity and the making of an occasional inspection of the condition of the tire. A careful survey of tire mileage and cost has showed that the average life of a 38-in. by 7-in. high-pressure pneumatic tire on a single-deck gas-electric motor coach is between 15,000 and 20,000 miles, and the average cost is between 1 and 1.6¢ per vehicle-mile. This cost includes a total of six tires per vehicle.

The road delays due to miscellaneous causes of body, chassis lighting, and heating system failures are next in rank but may be materially reduced by careful inspection and proper driving instructions.

The delays due to electric-transmission troubles are fifth in the list and the causes vary with the type of equipment used and the operating schedules. Many of the causes of failure and delay in early designs of electric transmission for motor coaches have been corrected in later designs by the addition of commutating poles in both motors and generators, improved ventilation of the electrical machines, better banding and accurate dynamic balancing of the armatures, and the use of heatproof insulation throughout. A classification of the road delays chargeable to the electric transmission shows the following sources of the troubles, in the order of their importance: motor commutators and banding wire, brushes and brush-holders, insulation failure, relays and accelerator switch, chassis wiring and main controller. These failures are an extremely low percentage of the total under any condition and may be almost completely eliminated by careful inspection and adequate maintenance methods.

The results obtained from the past four years' experience in operating and maintaining gas-electric motor coaches can best be measured by the reliability and the cost of maintenance of this type of vehicle when operating under the service conditions outlined previously. A survey covering 1600 gas-electric motor coaches operated 64 million vehicle-miles in the year 1927 shows that an average of 3800 miles of operation per road delay was realized. This figure includes delays from all causes. The survey further shows an average of 360,000 miles of service per road delay due to the electric transmission.

Although the majority of operators using, or contemplating the use of gas-electric motor coaches, are vitally interested in the maintenance cost of this type of equipment, any actual figures on maintenance costs are of very little value unless the property involved and the operating conditions are known and carefully analyzed. To illustrate this point, it has been found that the operating costs on different properties will vary more than 50 per cent, depending upon the shop methods used, the wage scale, and the thoroughness of the inspection and maintenance system employed. For this reason it is practically impossible to predict the average maintenance cost for a gas-electric vehicle. A careful survey of a number of properties operating this type of equipment has shown that the cost of maintaining this equipment on a vehicle-mile basis is between 3.5 and 5.5¢ per mile for the single-deck type and between 4.2 and 7.0¢ per mile for the double-deck type. These costs include all garage facilities, labor, and materials used for the maintenance of the chassis and bodies and the cost of tires.

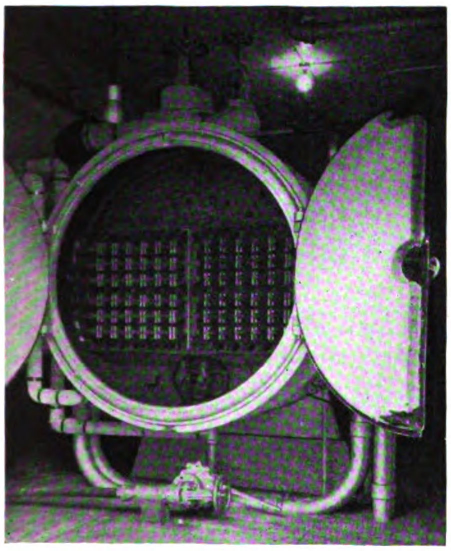
Electric-steam Generator Returns 6 Per Cent on Investment

A savings of $6,000 in a heating season of eight months, or a return of 6 per cent on an investment of $100,000, is estimated by the Central Maine Power Company to be the result of installing an electric steam generator for heating its own and some other private buildings. This company maintains a stand-by steam-electric generating plant at Lewiston (Maine), and hydroelectric power is also available at that point from its plant at Gulf Island (Maine). At Lewiston it is necessary to keep warm the fuel-oil supply and plant equipment, as well as to heat the office building and storehouse. In addition, the Company furnishes heat by contract to a local theatre and to a hotel.

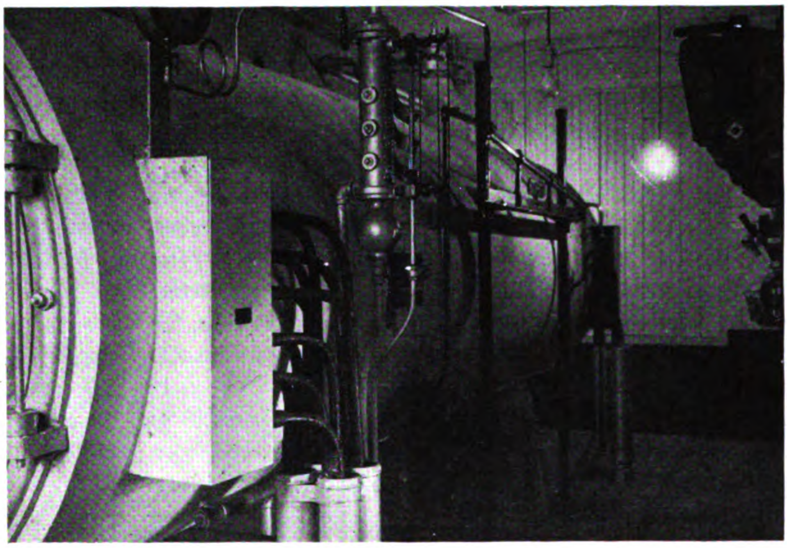
Previous to the installation of the electric-steam generator, heat was supplied by a section of the oil-burning boilers. The electric-steam generator now in use utilizes surplus hydroelectric power generated at the Gulf Island plant. The steam boiler, built by the International Engineering Works, is 66 in. in diameter and 19 ft. in overall length, and contains 72 steel tubes three inches in diameter by 16 ft. long. The boiler was built for a working pressure of 125 lb. and is rated 1200 kw.

The complete electrical equipment was furnished by the General Electric Company and consists of 144 heating units, four control panels, and four pressure governors. The boiler is also equipped with a Copes water regulator, making the operation entirely automatic.

The boiler is designed for 440-volt 3-phase operation, and has a connected capacity of 1200 kw. divided into four circuits of 300 kw. each. An automatic control panel and pressure governor are provided for each 300-kw. circuit, the pressure governors being set a few pounds apart so that the sections will be thrown on or off successively with falling or rising of steam pressure.



End View of the Boiler with Doors Open



Side View of the Boiler, Showing Cables to Terminals, Steam Gauge, Water Column, and Feed-water Regulator

The efficiency of the boiler, as estimated by the owner, is almost 100 per cent; as nearly all the energy is expended inside the boiler, because, with the boiler well insulated, the loss from radiation is practically negligible. The average daily power consumption in winter is approximately 15,000 kw-hr. The Company figures the power consumed by the boiler as surplus, as it would otherwise be wasted over the dam. The only charge made against the boiler, therefore, is the fixed charges on the investment and, on this basis, the saving effected over the oil boiler is roughly $25 per day.

Stability of Synchronous Motors Under Variable-torque Loads as Determined by the Recording Product Integraph

Fundamental Considerations of Torque—Construction and Operation of the Integraph—Solution of Synchronous Torque Problem by Integraph—Approximations and Errors Involved—Mechanical Analogy—Conclusions

By LEO TEPLOW
Graduate, Massachusetts Institute of Technology

> *The human difficulties encountered in the increasingly complex operations of modern life have resulted in the development of remarkably ingenious mechanisms, aptly called "Robots." These were made the subject of an article by Dr. S. W. Stratton and Frank Stockbridge in the "Saturday Evening Post" of January 21, 1928. The application of one of these devices to the solution of a particular problem appears below.*
> —EDITOR

THE development of methods of calculation which may be applied directly to the widespread and increasing use of synchronous motors for driving certain types of loads tends to awaken a new interest in the special problems which accompany such installations. Among these problems the matter of dealing with loads of variable torque assumes great importance as affecting vitally the stability of operation.

One such problem arises in connection with synchronous motor-driven compressors. This and kindred cases present very definite load conditions, but which are often difficult to evaluate accurately both because of the complexity of the functions expressing the load cycle and the difficulty involved in securing accurate practical data, in the form of indicator cards, for example, or the torque characteristic of the compressor.

A large compressor installation is shown in Fig. 1; and Fig. 2 contains a typical torque characteristic to which has been fitted an equivalent curve derived from a four-harmonic Fourier Series. Similar curves furnish the basis for the solutions of the torque problem in synchronous motor operation under variable load conditions, the salient points of which are here outlined.

Analysis of the Problem*

Assume a synchronous motor operating at constant load. Its instantaneous speed, as well as its average speed, will then be synchronous. The rotor will lag a certain constant angle behind the impressed voltage, this angle being dependent upon, but not necessarily directly proportional to, the torque of the load. The torque thus generated by the motor, in this case equal to the load torque, will be called the synchronizing torque, as it makes the motor operate at synchronous speed. If we let T_s be the synchronizing torque per unit angle of lag of the rotor behind the impressed voltage, the equation for the motion of the rotor for the condition given is

$$T_s y = K,$$

where y is the angular lag of the rotor, and K is any constant load torque.

Assume that the load torque is suddenly changed to some smaller value. The motor torque is now greater than the load torque, causing the rotor to speed up. Since such speeding up is liable to throw the motor out of synchronism, it is now universal practice to use amortisseur windings, or squirrel-cage windings, upon the pole faces. These windings are inactive when the rotor runs at synchronous speed; when it runs at any other speed, these windings cut lines of flux which action induces currents in the winding. The effect of these eddy currents is a torque acting in such a direction as to oppose a change in speed of the rotor; *i.e.*, when the rotor is accelerating, the damping torque acts in opposition to the synchronizing torque, and when the rotor is decelerating, the damping torque aids the synchronizing torque. Thus the damping winding tends to keep the rotor in synchronism with the power supply and is an important factor in the study of stability.

Therefore, if the load torque is suddenly decreased, as in the preceding case, the rotor will speed up because of the action of the synchronizing torque, but its acceleration will be opposed by the damping torque, whose magnitude is roughly proportional to the acceleration (or deceleration) of the rotor. Since y is the angular lag of the rotor, the angular acceleration (or deceleration) will be the variation in y per given change in t, or $\frac{dy}{dt}$. If T_d is the magnitude of the damping torque per unit angular slip per second, the magnitude of the induction or damping torque at any instant will be $T_d \frac{dy}{dt}$.

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*TABLE OF SYMBOLS EMPLOYED IN SOLUTION OF PROBLEM

e = Voltage at any instant t.
f = Frequency of oscillation of any system.
f_1, f_2, f_3, f_4 = Any function of t that may be plotted.
$f(t)$ = Shaft torque required by compressor at any instant, in lb-ft.
I = Moment of inertia of motor rotor plus flywheel plus any other attached moving parts.
i = Current at any instant.
K = Any constant load torque.
N = Synchronous speed in r.p.m.
P_s = Synchronizing power; change in input to machine per mechanical radian displacement of rotor.
P_o = Input to motor in kw. at one mechanical radian displacement of rotor.
p = Number of poles.
T_d = Damping torque, in pound-feet per mechanical radian slip per second.
T_s = Synchronizing torque; change in torque in lb-ft. per mechanical radian change in rotor displacement.
WR^2 = Flywheel effect.
y = Angular displacement of rotor with respect to terminal voltage at any instant.

Since acceleration takes place, there is one more factor which must be considered before the complete equation for the motion of the rotor can be written, *viz.*, the "inertia torque" required to accelerate and decelerate the mass of the rotor and connected moving parts. Since it is angular motion that is to be considered, the mass to be accelerated is represented by the moment of inertia (I), and the torque required to give it any acceleration $\frac{d^2y}{dt^2}$ will be $I \frac{d^2y}{dt^2}$. This is the unbalanced torque due to the synchronizing and damping torque on one hand, and the load torque on the other. Consequently, the equation of the rotor torque may be written in terms of the angle of lag as follows:

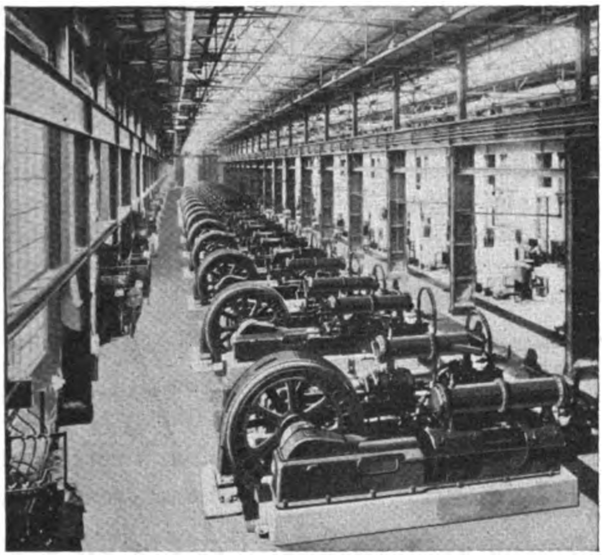
(Courtesy of Ingersoll-Rand)

Fig. 1. A Battery of Direct-connected Synchronous Motor-driven Compressors at Muscle Shoals (Ala.)

$$I \frac{d^2y}{dt^2} + T_d \frac{dy}{dt} + T_s y = f(t)$$

where $f(t)$ is the variable load torque.

This is the basic equation for the rotor of a synchronous motor driving a variable torque load. It is a second order differential equation, whose solution is very arduous if attempted along conventional lines, due in part to the fact that $f(t)$ can not be expressed in any simple mathematical manner. The usual method of analysis has been to expand $f(t)$ into a Fourier Series, and then to solve the equation separately for each harmonic, four being the number ordinarily considered. The method of using four harmonics is laborious, but in most cases sufficiently accurate for practical purposes. The effect of the higher harmonics is usually not large; and this article gives a comparison of the results based upon approximate methods and those obtained from the use of actual compressor torque curves.

Another serious obstacle to the solution of this equation appears in the value of T_s. The synchronizing torque characteristic, as shown in Fig. 3, is not a straight line, but rather approaches very closely a sine curve drawn through the zero and full load points. However, without undue error it may be assumed straight in the range from 15 to 30 deg., within which most motors operate.

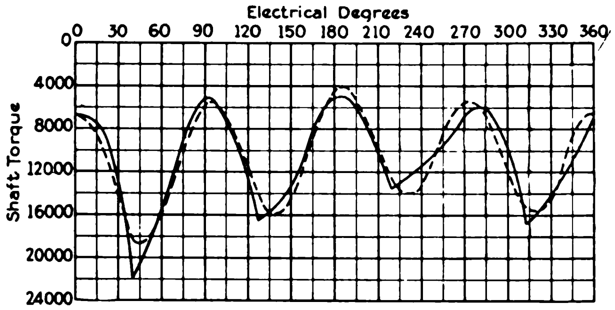

Fig. 2. The Torque Characteristic of a Two-cylinder Double-acting Ammonia Compressor. The dotted curve is the four-harmonic equivalent determined by a Fourier Series analysis

The solution of the problem by the more nearly correct method furnished by the use of the recording product integraph eliminates certain of the larger sources of error in the method usually employed. Moreover, the time and labor involved in obtaining this more exact solution are greatly reduced. Even this solution, however, is not exact, and such assumptions and errors as the integraph method involves will be pointed out later.

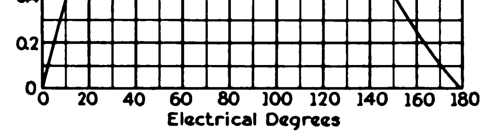

Fig. 3. A Typical Steady-state Synchronous Torque Characteristic of a Low-speed Salient-pole Synchronous Motor

The Integraph

The Recording Product Integraph and Multiplier has been in the process of development at Massachusetts Institute of Technology, under the supervision of Dr. V. Bush, for several years. Originally conceived to help solve the problem of transients in three-phase transmission lines, it has become an invaluable aid in numerous mathematical problems. Since no complete description of the machine, as at present constituted, has yet been published, the following explanation is given.

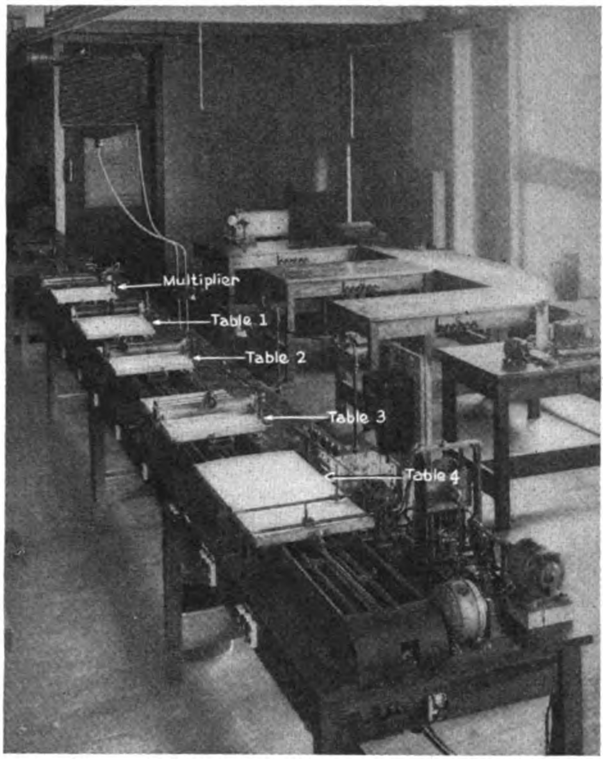

Fig. 4. The Recording Product Integraph, Showing General Arrangement of Parts

The heart of the integraph is an ordinary watt-hour meter as shown in Figs. 4 and 5. It is universally known that a watt-hour meter continually evaluates the instantaneous integral $\int_0^t e\,i\,dt$, the speed of the moving element being proportional to the product of e and i, and the net motion of the moving element or armature being proportional to the integral. In order to obtain the integral of the product of two functions, then, it is only necessary to make the current in the field at all times proportional to f_1, and the current in the armature proportional to f_2. This was done by connecting the field coil and armature to separate slide wires in such fashion that the e.m.f. impressed on them was obtained between a mid-tap and a movable contact on each slide-wire, one of which is shown in Fig. 6. If f_1 and f_2 are now plotted on moving tables (Fig. 4) under their corresponding slide-wires, with the zero axis of the functions under the mid-tap of the slide-wires, and if the movable contacts are made to follow the plotted functions, the e.m.f. impressed upon the watt-hour meter field and armature will be proportional respectively to f_1 and f_2; and the net motion of the armature, assuming that the tables are moving at constant speed, will be proportional to $\int_0^t f_1\,f_2\,dt$. In order to minimize the error due to taking a heavy current from the slide-wires, the field coils were rewound to take the same current as the armature coils.

Since, in order to obtain the instantaneous integral, $\int_0^t f_1\,f_2\,dt$, it would be necessary to read the registration

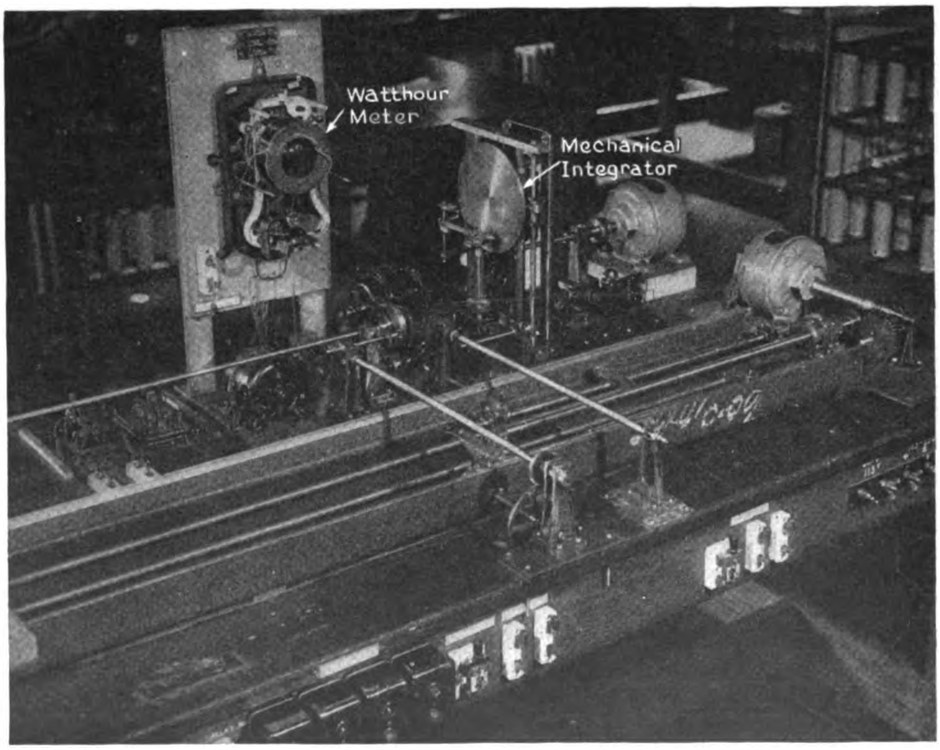

Fig. 5. The Driving and Integrating Elements of the Integraph

of the watt-hour meter at very short, equal intervals of time, an ingenious servo-motor and contactor device were installed, causing the servo-motor to follow the motion of the watt-hour meter armature, and drive a pencil whose displacement was then proportional to the integral.

The next step in the development of the integraph was to split the watt-hour meter field winding into two separate coils, each drawing its e.m.f. from a separate slide-wire. Since the fluxes of the two coils add (or subtract) directly, and since the two slide-wires are independent, it is possible to obtain the integral of the product of two factors, one of which is the sum of two distinct functions, or

$$\int_0^t f_1\,(f_2+f_3)\,dt$$

The next important step in the development of the integraph was the addition of a mechanical integrating device which is shown in Fig. 7. This consists of a large flat disk, mounted in a vertical plane, driven at constant speed by a synchronous motor. This driving disk is in frictional contact on its flat side with a smaller disk mounted horizontally; and the small driven disk may be moved in a vertical plane in such a manner as to vary the distance between the point of contact of the two disks and the center of the driving disk. If this distance varies as $f(x)$, then the net motion of the driven disk will be proportional to $\int_0^t f(x)\,dt$. In actual operation, this distance is automatically made proportional to the first integral, as obtained by the watt-hour meter and its connected, or first stage, servo-mechanism. Therefore the net motion of the driven disk will be proportional to $\int_0^t\int_0^t f_1\,(f_2+f_3)\,dt\,dt$.

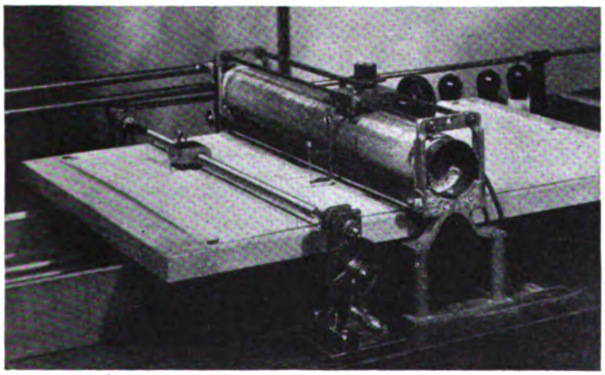
Fig. 6. The Slide-wire Curve Tracing Device by Means of which the Integration Process is Carried Out

The driven disk, in turn, operates a second stage servo-motor, which drives a pencil whose displacement is then proportional to $\int_0^t\int_0^t f_1\,(f_2+f_3)\,dt\,dt$.

Another gear arrangement makes it possible to change the distance between the point of contact of the two disks and the center of the driving disk by hand, this motion being added to the first integral through a differential gear. If this superimposed motion be made proportional to some fourth function f_4, the result will be a solution, in a single operation, of the integral

$$\int_0^t\int_0^t f_1\,(f_2+f_3)dt\,dt+\int_0^t f_4\,dt$$

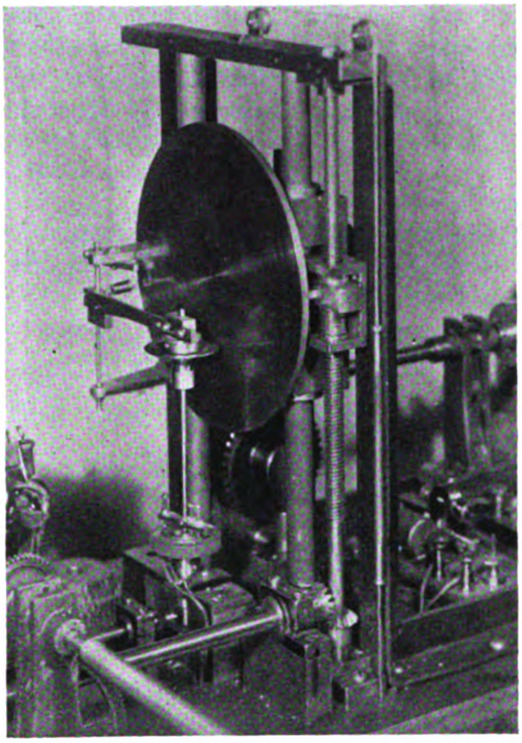
Fig. 7. The Mechanical Integrating Element

The final development of the integraph, which made it possible to solve the present problem with such ease was the installation of three different drives (Figs. 4 and 5) so that each table upon which the functions were plotted could be driven by any one of the three drives. The first drive imparts a constant-speed motion to the attached tables, being driven by a synchronous motor. The second drive, being driven by the first stage servo-motor, gives to its attached tables a displacement proportional to the first integral; and the third drive, driven by the second stage servo-motor, gives to the tables attached to it a displacement proportional to the second integral, or the result.

Thus it is possible to obtain the integral of a function of the result, as, for example,

$$y=\int_0^t\int_0^t (f_1\,y)\,dt\,dt$$

If in the equation

$$y=\int_0^t\int_0^t f_1\,(f_2+f_3)\,dt\,dt+\int_0^t f_4\,dt$$

f_3 and f_4 are attached to the third drive, then we have the solution to the equation

$$y=\int_0^t\int_0^t f_1\,(f_2+f_3\,y)\,dt\,dt+\int_0^t f_4\,y\,dt$$

which, as will be shown later, is the solution to the problem at hand.

A mechanical multiplier is also included as a part of the integraph, by the aid of which it is possible to obtain the solution to the equation and multiply it by some other function, all at a single operation.

The foregoing description merely touches on the salient points of the integraph and multiplier. The skill and nicety of construction, and the numerous ingenious ideas and suggestions which have led to the integraph as it is at present constituted, will be readily appreciated upon consideration of the solutions made possible through its use.

Integraph Solution

It has been shown that the basic equation for the torque of the rotor of a synchronous motor driving a compressor is given by

$$I\frac{d^2y}{dt^2}+T_d\frac{dy}{dt}+T_sy=f(t)$$

This may be written

$$\frac{d^2y}{dt^2}=\frac{1}{I}\left[f(t)-T_sy\right]-\frac{1}{I}T_d\frac{dy}{dt}$$

Integrating this with respect to t, we have

$$\frac{dy}{dt}=\int_0^t\frac{1}{I}\left[f(t)-T_sy\right]dt-\frac{1}{I}T_dy$$

and integrating again, we find that the angle of lag may be expressed as

$$y=\int_0^t\int_0^t\frac{1}{I}\left[f(t)-T_sy\right]dt\,dt-\int_0^t\frac{1}{I}T_d\,y\,dt$$

Before proceeding to the method of solution furnished by the integraph, which is really a continuation of the work(1) of A. R. Stevenson, Jr., on a similar problem, a short résumé of the method(2) employed in the earlier case is given:

Starting with the same basic equation of the form

$$a\frac{d^2y}{dt}+b\frac{dy}{dt}+cy=f(t)$$

the solution was made according to the following brief outline:

"Assume that $f(t)$ begins to act when $t=u$.

Then $$y=\frac{1}{a\omega}\epsilon^{-\beta t}\int_0^t\epsilon^{\beta u}\sin\omega(t-u)f(u)\,du \qquad \text{(a)}$$

or $$z=a\omega y=\epsilon^{-\beta t}\sin\omega t\int_0^t\epsilon^{\beta u}\cos\omega u\,f(u)\,du$$
$$-\epsilon^{-\beta t}\cos\omega t\int_0^t\epsilon^{\beta u}\sin\omega u\,f(u)\,du \qquad \text{(b)}$$

or $$z=\epsilon^{-\beta t}\sin\omega t\left[A+\int_0^t\epsilon^{\beta u}\cos\omega u\,f(u)\,du\right]$$
$$-\epsilon^{-\beta T}\cos\omega t\left[B+\int_0^t\epsilon^{\beta u}\sin\omega u\,f(u)\,du\right] \qquad \text{(c)}$$

where $$A=\frac{I_1(\cos\omega T-\epsilon^{-\beta t})+I_2\sin\omega T}{2(\cosh\beta T-\cos\omega T)} \qquad \text{(d)}$$

and $$B=\frac{I_2(\cos\omega T-\epsilon^{-\beta T})-I_1\sin\omega T}{2(\cosh\beta T-\cos\omega T)} \qquad \text{(e)}$$

where $$I_1=\int_0^T\epsilon^{\beta u}\cos\omega u\,f(u)\,du \qquad \text{(f)}$$

$$I_2=\int_0^T\epsilon^{\beta u}\sin\omega u\,f(u)\,du \qquad \text{(g)}$$

$$\beta=\frac{b}{2a},\ \omega=\sqrt{\frac{c}{a}-\frac{b^2}{4a^2}},\ \text{and } T=\text{time for one}$$

complete period of $f(t)$.

"The procedure is:

(1) Evaluate the two integrals in (b) as functions of time for one period of $f(t)$. As the final result, I_1 and I_2 are obtained.

(2) Calculate the values A and B from the Equations (d) and (e), and add the results to the integrals, as indicated in Equation (c).

(3) Multiply by the factors indicated in (c) and subtract the results. The final result is then z, which, multiplied by a constant factor, gives y."

This process, as appears on the face of it, is extremely laborious, but has the advantage over other methods that $f(t)$ may be used as calculated, and does not need to be analyzed into a Fourier Series, thus eliminating an appreciable, though relatively unimportant error. This method was employed by Mr. Stevenson because of its theoretical interest rather than for the added accuracy; in the present case a solution of the same problem is made on the integraph for the purpose of checking the result he obtained.

For convenience, let the tables of the integraph (see Fig. 4) be numbered in order from left to right, *1*, *2*, *3*, and *4* (not including the multiplier).

One of the factors of the double integral is $1/I$, the other factor being $(f(t)-T_s\,y)$. Accordingly, let slide wire *2* be given a displacement proportional to $1/I$. This setting will be constant, so that table *2* need not be driven at all, and may be disconnected. Plot $f(t)$ on table *1* to a suitable scale. Since $f(t)$ is a function of time, table *1* must be attached to the constant-speed drive. *It should be noted that $f(t)$ need not be expressed mathematically*, since it is necessary only to be able to plot it.

$T_s\,y$ is a function of y. Being plotted on table *3*, this table must be attached to the third drive, thus giving it a displacement proportional to the result.

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(1) "Error Due to Neglecting Electrical Forces in Calculating Flywheels for Reciprocating Machinery Driven by Synchronous Motors," by A. R. Stevenson, Jr., GENERAL ELECTRIC REVIEW, November, 1922, p. 690.

(2) "On the Oscillations of Certain Electrical or Mechanical Systems Due to a Periodic Impressed Force," by Ivar Herlitz, GENERAL ELECTRIC REVIEW, November, 1922, p. 686.

There yet remains the last term of the equation to be taken care of: $\frac{1}{I} T_d\, y\, dt$. Since $T_d\, y$ is a function of y, it, too, must be plotted on a table attached to drive *3*. This is a pure integral, and may therefore be evaluated on the mechanical integrator, as previously described, the factor thus introduced being directly added (algebraically) to the result by means of a hand gear controlling the vertical motion of the driven disk. $T_d\, y$ is plotted on table *3*, and the gear mechanism so operated as to make the pointer follow this plotted line, the desired integral being thus introduced into the final result. The result is drawn on table *4*, which moves at constant speed, by a pencil attached to the gear mechanism driven by the second-stage servo-motor.

In order to actually solve any problem on the integraph as outlined, it is first necessary to calibrate the mechanism and to determine the relative magnitudes of the different displacements. To do this, it is first necessary to analyze the basic equation

$$I\frac{d^2y}{dt^2}+T_d\frac{dy}{dt}+T_s\, y=f(t)$$

If the load be removed, $f(t)=0$, and we have an equation such as would apply to the motion of a damped pendulum, or the oscillation of a damped spring. From previous experience with similar equations, it is possible to write upon inspection that the frequency of such a system is

$$\omega=2\pi f=\sqrt{\frac{T_s}{I}-\frac{T_d^2}{4I^2}}$$

With such quantities as are met with in practice, the second term under the radical sign becomes entirely negligible and, as far as the frequency of oscillation is concerned, the equation may be written

$$w=2\pi f=\sqrt{\frac{T_s}{I}}$$

and

$$f=\frac{1}{2\pi}\sqrt{\frac{T_s}{I}}$$

A sine curve of the desired frequency is obtained by adjusting the slope of T_s and the magnitude of $1/I$, $f(t)$ being set at zero.

The decrement factor remains to be taken into account. By inspection of the basic equation it is evident that the time factor for such a motion is given by

$$t_1=I/T_d.$$

The desired decrement factor is obtained by adjusting the slope of T_d.

Before going further, it might be well to give actual values to the quantities involved. The data used in this problem follow:

A two-cylinder, double-acting ammonia compressor, with cranks at right angles, has a torque characteristic as shown in Fig. 2.

The average torque is 10,710 lb-ft.

It is driven by a 360-kw. 450-hp. 40-pole 220-volt, 3-phase synchronous motor, at 180 r.p.m.

Moment of inertia of rotor plus moving parts, 644.

P_0 of motor is 870 kw.

(P_0 is a fictitious value, representing the input to the motor if it were operated at an angle of one mechanical radian, assuming the T_s characteristic to be a straight line).

$T_d=3400$ lb-ft. per mechanical radian slip per sec. (by test).

$T_s=$ synchronizing torque at a lag of one mechanical radian

$$=\frac{870}{360}\times\frac{450}{2\pi\times180}\times\frac{40}{2}\times33{,}000=634{,}000 \text{ lb-ft.}$$

The lag at the given load of 10,710 lb-ft. is

$$\frac{10{,}710}{634{,}000}\times57.3\times20=19.35 \text{ electrical degrees.}$$

Then the equation may be written

$$644\,\frac{d^2y}{dt^2}+634{,}000\frac{dy}{dt}+3400\,y=f(t)$$

and

$$f=\frac{1}{2\pi}\sqrt{\frac{T_s}{I}-\frac{T_d^2}{4I^2}}=4.99 \text{ cycles per second.}$$

and t, the time for one cycle, is $1/4.99=0.2008$ sec.

Time constant of decrement factor, $I/T_d=644/3400=0.189$ sec.

That is all the preliminary calculation and calibration required. Only one difficulty remains, *viz.*, to decide at what point on $T_s y$ and $T_d\, y$ to start. The initial value of y is not known; therefore $T_s y$ and $T_d y$ must be given the most likely or average values, and several runs must be taken in succession, each beginning where the preceding one leaves off, until the transient is eliminated, and a steady state is achieved; *i.e.*, until the successive result curves become identical. When this is accomplished (it may take six or seven runs) the problem is completely solved.

Each run takes approximately eight minutes, so that the entire series of runs for any given problem will take approximately one hour, in addition to the time required for calibrating the integraph and evaluating the results. Thus the entire problem, including the plotting of the curves, calculation of data, and evaluation of results need not take more than about five or six hours. While the curves are being run off on the integraph, three operators are usually required for the space of about one hour. The calculation of curves, calibration of the integraph, and evaluation of the results require but one person. Thus it is seen that, aside from its greater accuracy, the integraph method saves much time in the solution of such problems.

Results

The result curves of the Integraph solution of the rotor displacement of a synchronous motor driving a compressor are given in Fig. 8. An analysis of these curves is given in Table I, where a comparison is made with the results obtained by Mr. Stevenson.

The initial object of the investigation was to check the integraph method with the best previous solution. In order to do this, the same problem which Mr. Stevenson had solved was attacked; and Curve 1 in Fig. 8, shows the results obtained when the same approximations were used; *viz.*, that T_s is considered constant (the T_s characteristic is assumed to be a straight line) and a Fourier Series four-harmonic equivalent curve is used instead of the true compressor torque curve. Mr. Stevenson's results were believed accurate, as he had performed careful work on the problem. It was therefore gratifying to find that his results were approached very closely. A value of 11.5 electrical degrees, for example, was obtained for the maximum angle of oscillation of the rotor during the cycle, as compared with a value of 11.4 electrical degrees previously obtained for the same angle.

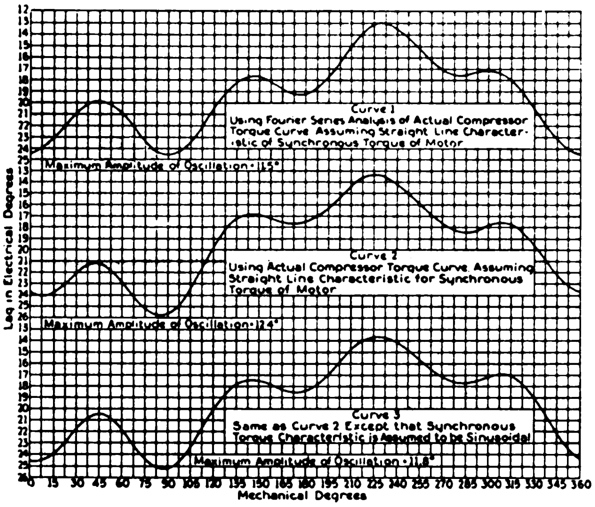

Fig. 8. Rotor Displacement Curves for a Synchronous Motor Driving a Compressor

Satisfied with the accuracy of the work performed by the integraph, the next step was to determine the accuracy of the present method of solving the problem, and to determine the erroneous effect of the assumptions made. The use of a Fourier Series equivalent for the compressor torque curve appeared to the writer to be the cause of the largest error, and a solution was accordingly made, using the same data as for Curve 1, except that the actual compressor torque curve was used. The difference between this and the harmonic curve is quite appreciable as shown by Fig. 2, especially in the region of the sharp peak at the 40-deg. point. The result (Curve 2, Fig. 8) fully confirmed this opinion. It is seen that the maximum amplitude of oscillation of the rotor has become 12.4 deg., an increase of 7.8 per cent. This is an error which is entirely eliminated in the integraph solution, for it is quite as easy to follow the actual torque curve as any other. It is interesting to note how the entire cycle is changed in Curve 2. The effect of the peak at 40 deg. is particularly noticeable, for while in Curve 1 the angle at 90 deg. is but 0.2 deg. greater than the rotor displacement at zero, on Curve 2 the angular lag is 2.1 deg. greater.

TABLE I
COMPARISON OF RESULTS

Angle	Stevenson's Results	INTEGRAPH RESULTS		
		Curve 1	Curve 2	Curve 3
0	24.2	24.3	23.7	24.5
15	23.9	23.3	23.8	24.1
30	22.1	20.9	23.1	20.9
45	20.4	19.9	21.0	20.5
60	21.5	20.9	22.9	22.0
75	23.2	23.9	25.1	24.3
90	24.2	24.6	25.7	25.2
105	22.6	23.2	23.4	23.4
120	19.3	20.5	19.8	20.1
135	16.7	18.2	17.3	17.8
150	16.5	17.3	16.9	17.6
165	17.7	18.8	17.4	18.3
180	18.6	19.2	17.4	18.4
195	17.4	17.7	16.3	17.1
210	14.9	15.0	14.3	14.8
225	13.1	13.2	13.4	13.6
240	13.5	13.5	14.2	14.1
255	15.5	15.4	15.9	15.7
270	17.4	17.1	17.7	17.2
285	17.7	17.5	18.4	17.8
300	16.8	17.2	17.9	17.3
315	17.0	17.9	17.7	17.1
330	19.1	20.4	19.5	19.1
345	22.2	23.2	22.2	22.0
360	24.2	24.3	23.7	24.5
Maximum Amplitude of Oscillation	11.4	11.5	12.4	11.8

It has been noted that the usual synchronizing torque characteristic of a synchronous motor practically coincides with a sine curve drawn through the zero and full load points. (See Fig. 3). Usually this is assumed to be a straight line. In order to determine the magnitude of the error involved, Curve 3 was obtained under the same conditions as Curve 2, except that a sinusoidal synchronizing torque characteristic was used. Here again the difference is appreciable, as the maximum angle of oscillation is found to be 11.8, or a decrease of 5.1 per cent.

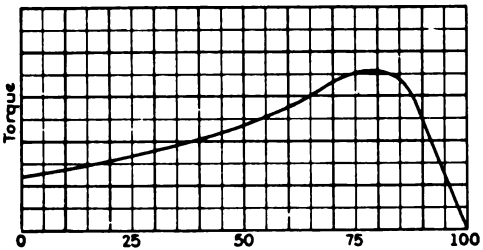

Fig. 9. Typical Damping Torque Characteristic of a Synchronous Motor

It should be noted that the two assumptions usually made cause subtractive errors in this particular case, so that the total error is small; but it is quite possible that the two errors may be in the same direction, thus causing appreciable net errors in the result.

Errors and Assumptions

While the Integraph method of solution of the problem of stability of synchronous machines under variable torque loads is thus seen to be obtained to a very high degree of accuracy, there are, nevertheless, certain approximations made of the actual conditions existing in the motor.

The basic equation used in the solution of the problem, as previously stated, is

$$I\frac{d^2y}{dt^2} + T_d\frac{dy}{dt} + T_s y = f(t)$$

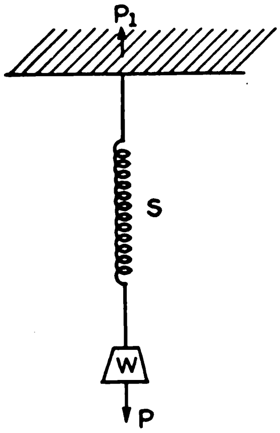

Fig. 10. Mechanical Analogy of Compressor Torque

The moment of inertia may be found by test to as close a degree of accuracy as may be required, and thus the first term of the equation is accurate.

The second term, however, is a deliberate approximation. It is here assumed (1) that the damping torque is proportional to the change in the angle between rotor and field per revolution or, as it is more commonly stated, proportional to the slip; and (2) that this proportionality factor is constant at different speeds. The latter assumption is incorrect, for it is well known that the damping torque characteristic is not a straight line, as may be seen from the typical damping torque characteristic shown in Fig. 9. However, since the variation in speed is very slight (usually not more than one per cent at normal loads) the damping torque characteristic for this variation is essentially a straight line. The other assumption *viz.*, that damping torque is proportional to the acceleration of the rotor with respect to the field, is of more consequence, and is in error, because the damping torque, being due to change in flux relative to the poles, varies not as the total angle of displacement, but as the angle of the flux with respect to the poles. A mechanical analogy will serve to make this clear.

Let us say that the compressor torque is represented by P in Fig. 10, and acts on a spring (s) to which is attached a weight (W) corresponding to the moment of inertia of the rotor, flywheel, etc. Then P_1, the pull on the support, will represent the power taken from a constant-voltage source, where there exists no damping torque.

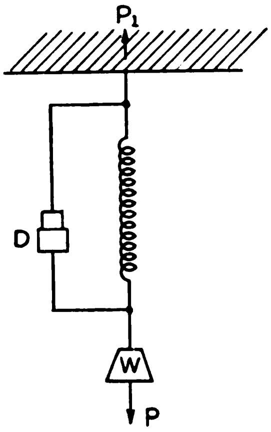

Fig. 11. The Mechanical Analogy of the Conditions Assumed in an Investigation of Compressor Torque

Now, if we put a dashpot (D) as shown in Fig. 11, in parallel with the spring, we will have a mechanical analogy to the conditions as we have assumed them.

The actual state of affairs is more exactly demonstrated in Fig. 12, where the dashpot is in parallel with only a part of the spring, *i.e.*, it is affected, not by the change in the total length of the spring, but by the change in length *ab* only. This is the part which corresponds to the angle of flux with respect to the centers of the poles, while the free part of the spring corresponds to the displacement between internal and external voltages; and the entire spring corresponds to the displacement of the rotor relative to the terminal voltage. Thus, while the angle between internal and external voltages may be changed due to impedance drops in the armature, this may not change the

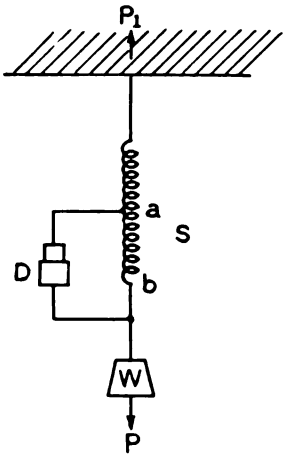

Fig. 12. The Mechanical Analogy of the True Conditions of Compressor Torque in Any Given Installation

angle of the flux with respect to the poles, and therefore will not be affected by the damping torque; the assumption made is that all changes in displacement between internal and external voltage will be affected by the damping torque.

In addition, the field current pulsation which accompanies any pulsation in load causes some change in the synchronizing torque. While this change is not large, it may partly be taken into consideration in the value for the damping torque. To do this, it is necessary to find the damping torque characteristic by test, the motor running as an induction motor, with the field short-circuited. If the damping torque characteristic can be found with pulsating load, it will then include the transient pulsations in the field. This is, however, a refinement that is hardly worth while.

Another effect, which in the analogy is due to the fact that only a part of the spring is in parallel with the dashpot, is that if the frequency of oscillation is increased, the part *ab* would evidently become more and more inactive, leaving most of the pulsation to be taken up by the free part of the spring. This would cause a greater strain per unit length of the spring. In a similar way, if the frequency of oscillation of a motor be increased, the slope of the synchronizing torque may be increased from its steady-state value.

Synchronizing power, P_s, is the rate at which power input to synchronous machine varies with changes in angular displacement. The angular displacement referred to is not that which actually exists between the rotor and the rotating electrical field, but rather that which exists between the rotor and some imaginary field corresponding in phase to the terminal voltage.

Ordinarily P_s is referred to as a constant; as a matter of fact, P_s is constant only for a given set of operating conditions; *i.e.*, for given load, excitation, and frequency of pulsation of the load.

The heavy straight line *ab* in Fig. 13 shows the synchronizing torque as it is commonly assumed to be. The full curved line *a c e* shows the actual synchronizing torque for slowly changing loads. At full load, then, the change of synchronizing power with change in load is given by the tangent at that point, *df*. This is true only for slowly changing loads, however. For sudden changes in load, such as are met with in compressors, the change in power for any change in angular displacement will be given by the dotted line *km*. This change in slope may be explained as follows:

Consider a synchronous motor, operating normally at full load, upon which there is suddenly imposed a heavy load. The armature current will at once increase, occasioning as suddenly an increase in armature reaction which tends to distort the field flux. The field will not permit the flux to be thus changed suddenly, however, so the field current in turn will suddenly increase, but reassumes its previous value along a logarithmic curve. If the armature current continually pulsates, therefore, the field current will continue to oscillate, instead of remaining constant as is usually assumed. It is this pulsating action of the field which tends to change the slope of the synchronizing torque to that shown by the dotted line *km*. This description is only qualitative; a mathematical treatment of the effect of induced field pulsation upon the synchronizing torque has already been capably treated elsewhere. (3)

That the synchronizing torque varies also with the frequency of the pulsation of the load is shown in the treatment of the mechanical analogy appearing earlier in this article.

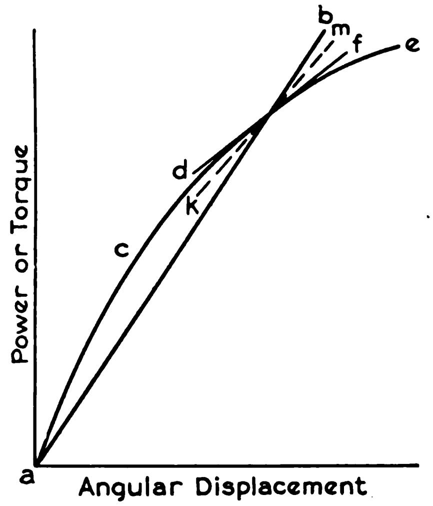

Fig. 13. The Relation of Synchronous Torques to Angular Displacement Under Conditions Ordinarily Assumed for Compressor Calculations

While synchronizing power and synchronizing torque have been used interchangeably in the above presentation they are of course different quantities, their relation being given by

$$T_s = \frac{3520\, P_s p}{N} \text{ lb-ft.}$$

This treatment of the synchronizing torque is given as an aid in understanding its action and the effect of sudden changes in shaft torque upon it. The true synchronizing torque, once found, may be plotted and used in the integraph solution; but for ordinary cases, where the average load on the motor is somewhere near its rated load, it is sufficiently correct to find the synchronizing power at full load. This is always done by the manufacturer, and the constant P_0 is thus obtained. From a study of the synchronous torque characteristics of synchronous motors, it appears that the characterics almost exactly coincide with the sine function, between the region of no load and full load as shown in Fig. 3. Therefore, for most ordinary cases it is sufficiently exact, without making additional tests, to draw a sine curve through the two known points—zero and full load—and to use this as the synchronizing torque characteristic. While it is true that this does not take into account the surges in the field due to sudden changes in load, this can very easily be taken into account in the evaluation of damping torque, as previously outlined.

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(3) "Synchronizing Power in Synchronous Machines Under Steady and Transient Conditions," by H. V. Putnam, *Journal of A.I.E.E.*, December, 1926, p. 1229.

As to the final term in the equation, $f(t)$, it is seen that there is an appreciable error involved when the harmonic analysis equivalent is substituted for the actual torque curve. The curves in Fig. 2 show the difference between the actual torque curve and that obtained by the first four harmonics of a Fourier Series analysis of the actual curve. The integraph solution can always be based upon the actual torque curve, which is obtained much more easily and directly, and, of course, gives the more accurate results.

It should be noted here that practically all solutions attempted so far, with the exception of the laborious method pursued by A. R. Stevenson, Jr., have necessitated a Fourier Series analysis of the true compressor torque characteristic. This means a lot of extra work, and also requires a separate solution for each harmonic evaluated. Moreover, the resulting curves given in Fig. 8 definitely show that, even with the use of four harmonics of an ordinary compressor torque curve, the results obtained are quite different in the case of the equivalent curve from those obtained with the use of the actual compressor torque curve.

Some of the other assumptions made in this method, in calculating the power puslations are:

(1) Armature losses are neglected.
(2) The displacement is assumed proportional to the torque under steady conditions over the range considered.
(3) The direct magnetizing and demagnetizing effects of the current induced in the field winding are not taken into account.

An additional slight discrepancy enters in view of the fact that, with regard to the compressor crank effort, the speed of the shaft is assumed synchronous at all times, when as a matter of fact the instantaneous speed, as has been previously explained, may differ from synchronous speed by one or two per cent, although the average speed is of course equal to the synchronous speed.

Conclusions

The comments accompanying the results given in this article should indicate that the work presented here is merely suggestive of the possible intensive study of this type of problem which is so much simplified by the use of the integraph.

Unfortunately, limited time did not permit carrying this work further, but a brief outline of the next steps which might be pursued will indicate the trend of the work which will probably be done in the near future under the supervision of Dr. Bush at Massachusetts Institute of Technology.

It is a very simple matter to change the calculations to apply to a flywheel of 80, 90, 110, or 120 per cent of the actual flywheel effect. Carrying through the solutions for such cases, it would be a simple matter to estimate the value of flywheel effect in limiting instability.

Again, the value of T_d might be changed by the same percentage, and thus not only would the effect of change in damping torque be evaluated, but also the relative value of increase in flywheel effect, or change in design so as to change the damping torque, would be known.

Another investigation of interest would be to determine the relative instability at different loads, so that the effect of an increase in load might be predetermined. For example, the stability of a typical five-step compressor installation at 0, 25, 50, 75, and 100 per cent full load might have been determined; but since only lack of time prevented the solution of this problem, it will probably be carried through as a later development.

With an exhaustive study of the problem of the stability of synchronous motors at variable torque loads so easily available, it may be that the integraph will provide a valuable source of data for the design of motors for this purpose. Since the results of any change can be predetermined, it becomes only a matter of determining the weight of loss in efficiency on one hand as against the increase in stability on the other, in order to arrive at the best design.

Thus the recording product integraph has shown its usefulness in one special field of electrical engineering, and it will doubtless become increasingly helpful in many other problems which have heretofore been solved only approximately, or not at all.

The author acknowledges the aid and co-operation extended to him by A. R. Stevenson, Jr., of the General Electric Company; and by F. G. Kear, then research assistant in the Electrical Engineering Department at Massachusetts Institute of Technology.

Correcting Power-factor with Capacitors on the Distribution Feeders of the Gray's Harbor Railway & Light Company

Feeder Capacitors Chosen in Preference to Synchronous Condensers—Operating Power-factor Greatly Improved—Installation Also Saves Energy, Increases System Capacity, and Improves Service—Investment and Operating Costs—Tests

By WILLIAM S. HILL
General Superintendent, Gray's Harbor Railway & Light Company
Aberdeen, Washington

> *To remove the cause of low power-factor is admittedly the ideal answer to this troublesome problem of operation. To impose a penalty on the consumers who are responsible for the condition is effective only as a disciplinary measure. To neutralize the cause is in many cases the most practical solution, one example of which is here described by Mr. Hill as he presented it to the Seattle Section of the A.I.E.E. last February.*—EDITOR

MOST power companies serving an industrial load are confronted with a low-power-factor situation. This condition is relieved to some extent in certain instances by the company imposing a penalty, or we might say giving a better rate to the consumer who maintains his power-factor above a certain minimum level. Low power-factor can be corrected or compensated for in several ways as follows: by selection of the proper kind and size of motors for the drives; by installation of capacitor or static condenser units to the individual motors where the character of load will not permit the obtaining of high power-factor; by use of capacitor units or synchronous condensers at the power station or substations; or the installation of capacitor units out on the feeder lines or on the secondary side of transformer banks of fairly large power consumers, etc.

The proper selection of motor is meant of course to include the synchronous motor where applicable. In recent years, synchronous motors have been designed to serve in lumber mills where heretofore only induction motors were applicable. The low-power-factor difficulties of the Gray's Harbor Railway & Light Company, however, could not be corrected by the substitution of synchronous motor drives in the various industrial plants. The owners had purchased other kinds of motors in years gone by and would be reluctant to go to the expense of changing over to synchronous motors to relieve a condition which apparently was not bothering them. We had to find a solution to the problem which existed on our system as a whole. Therefore the remarks contained herein are peculiarly pertinent to our own operating conditions, but they undoubtedly apply in a large degree to other systems as well.

At times there arises the question as to whether the consumer should be penalized for low power-factor. The enforcement of such a penalty might cause him to discontinue service, although a live power salesman can give him sufficient reasons to take advantage of the opportunity to improve the voltage, lighten the load on his wiring, reduce the cost of wiring (possibly changing of service outlets), increase the speed of his motor equipment, and as a result increase the output of the plant. On the other hand, when low power-factor is general over the entire system, it is advantageous to the power company to make an investment at its own expense. After studying out conditions we came to the conclusion that in the saving in generator, exciter, and line losses, the lower excitation required, with resulting increased capacity in generators, lines, transformer banks, etc., we had sufficient reasons to justify the capital expenditures necessary to effect the correction.

The Gray's Harbor Railway & Light Company, serving Aberdeen, Hoquiam, Cosmopolis, and surrounding territory in Gray's Harbor County, serves a considerable industrial load having a daily load factor of 65 per cent. A large proportion of the load consists of lumber mills and the allied industries. These mills cut fir, hemlock, spruce, cedar, etc., and in order to handle the heaviest load with comparative ease they select a motor which is in excess of the requirements for normal operation. Induction motors seem to be peculiarly adapted to this particular kind of work and their application under these conditions naturally results in low power-factor. Tests at the mills indicate a power-factor from 45 to 60 per cent. Our system is heavily encumbered with this kind of load. All of our three-phase four-wire 4150-volt feeders are equipped with three single-phase regulators, and in general supply lighting as well as power service, there being no exclusive power feeders. These feeders are all of 4/0 and 2/0 copper and radiate directly from our Electric Park plant over distances of 2¼ to 6 miles to the load centers.

A 5000-kw. turbine generator was installed at the Electric Park station in 1925 and became very heavily loaded in the fall of 1926 owing to greatly increased load and to a decrease in power-factor. The pointer on the power-factor indicating meter on the generator panel seemed to be stuck at 60 per cent (the lowest reading), for it never moved away from that point. Our exciter became overloaded, and we carried as high as 1200 amp. on the generator which has a full-load rating of 868 amp. at 80 per cent power-factor. Something had to be done. A study of the application of synchronous condensers and capacitors was made. A number of factors entered into the choice of each of these two types of equipment. Our calculations were based on 2000 kv-a. as the capacity necessary to bring the existing power-factor up to 80 per cent. With respect to first cost the synchronous condenser had the advantage, with a cost of approximately $5.00 per kv-a. as compared to $17.00 per kv-a. for capacitors. Based on 2000 kv-a., the total first costs would be $10,000 for the synchronous condenser and $34,000 for the capacitors, or $24,000 in favor of the former. However, in our particular case, the installation of a synchronous condenser would require an addition to our present power-house building which would tend to offset the advantage just mentioned.

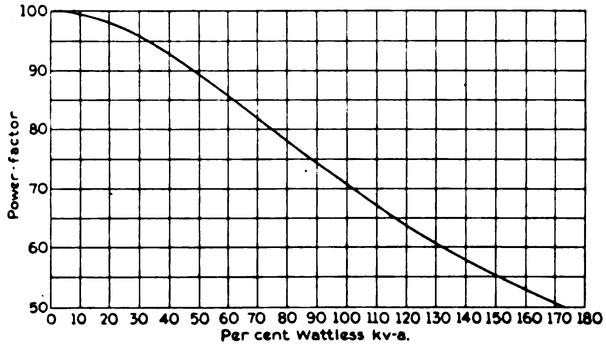


Fig. 1. Curve of Wattless Kv-a. for Any Power-factor in Per Cent of Kw-load

From the point of view of internal losses, the capacitor with losses of one quarter of one per cent is vastly superior to the synchronous condenser with losses of 3 to 4 per cent. Using 2000 kv-a. as a basis, the losses are 5 kw. and 78 kw. respectively, and for 24-hr. operation of the corrective equipment the annual losses would be 43,800 kw-hr. and 683,280 kw-hr. The difference, 639,480 kw-hr., if valued at 5 mills per kw-hr. would give an annual saving of $3,197 in favor of the capacitors.

Later in this article it will be shown that we have made an estimated saving of 500,000 kw-hr. annually in feeder line losses by installing capacitors, which saving could not be made by the synchronous condenser. If we value these kw-hr. also at 5 mills, which is exceedingly conservative for power at the consumer's premises, we have an additional saving of $2,500 in favor of the capacitors as compared to the synchronous condenser. The total annual saving would then amount to $5,697. This amount capitalized at 15 per cent is $38,000. In other words, based on savings in internal losses and line losses, we could afford to pay $38,000 more for the capacitors than for the synchronous condenser, whereas the difference in price as given in the foregoing was only $24,000. We therefore discarded the idea of utilizing a synchronous condenser. We also felt that capacitors located out on the lines would increase the capacity of the feeders in addition to lowering the load on the generator. A series of checks were made at industrial plants and we found that the power-factor averaged from 45 to 60 per cent. We finally decided to install six 300-kv-a. capacitor units, four of them rated 440 volts to be installed on the secondary side of the industrial transformer banks, and two rated at 4000 volts to be installed directly on 4000-volt feeder circuits where this corrective influence would be felt back to the power station. These latter two circuits have a great many small power installations scattered throughout their length.

We have been able to maintain a better voltage on the feeders since the higher power-factor on these circuits has increased the range of our regulators. We saved the installation of larger transformer banks for two consumers after the 440-volt capacitors were installed. We had to lower the taps on a bank of transformers in order to prevent excessive voltage on the motors in one industrial plant. This of course has increased the consumer's bill somewhat, but he has been compensated by securing a greater output as a result of the high voltage being maintained on his motors.

The installation of the capacitors is quite an easy matter. They can be placed on suitable racks on the poles or in a mill yard at a very reasonable cost. Our installation costs covered the pole racks, the raising of the steel racks and outdoor casings, the assembling and connecting up of the 5-kv-a. units which compose the completed 300-kv-a. capacitor, and also connections to the primary or secondary lines. Slightly more time was required for connecting up for 440-volt service because of the heavier wire used.

A typical 4150-volt installation on poles is shown as the cover illustration of this issue. As will be noted, the installation is very simple and neat in appearance.

In Fig. 2 is shown a typical installation of a 440-volt capacitor unit where space was found on the heavy timbers under the pole rack near the transformers in an industrial-plant yard. (A fence has since been constructed around this capacitor.)

The capacitors were installed and connected by September 30, 1927. Whereas in the fall of 1926 we were laborously carrying our load and were at times on the verge of losing some of it, in the fall of 1927 we carried an average of 1300 kw. additional load on this generator. We actually did carry 1600 kw. more on this generator during peak hours.

In order to make a comparison we selected a typical day in 1926 and the same day of the same week in 1927. This comparison may not give a true picture of actual conditions since we have steam turbines in two saw mills connected to the Electric Park plant through 13,200-volt lines and they may or may not have been absorbing more or less wattless kv-a. on one day or the other. However, a check of readings and operating conditions over a considerable period indicates that the tabulated values very closely represent the average conditions and are sufficiently accurate to illustrate the point in question. The readings were taken from our station log and are typical because no effort was made to school the operator to be quite accurate for the purpose of the test. Table I shows the conditions before and after the installation of the 1800 kv-a. of capacitors and the effect on the exciter and generator.

TABLE I

Date	D-c. Volts	D-c. Amp.	A-c. Amp. Per Terminal	Indicating Wattmeter	Kv-a.	P-f. Per Cent
11/23/26	108.1	335	845	3900	6060	64.4
11/22/27	85.8	289	667	3930	4780	82.1

GENERATOR RATING

5000 kw., 3 phase, 60 cycle, 4150 volts, 868 amp. per terminal, 6250 kv-a. at 80 per cent p-f.
Excitation voltage at full-load 80 per cent p-f., 96 volts.
Field current voltage at full load 80 per cent p-f., 330 amp.

These readings are the average of 16 hr. run on these two days, so for our purposes will represent fairly accurately the changed conditions and admit of our setting up some calculations which will show more clearly the actual results obtained. The calculations are also based on 16 hr. since our normal practice has been to shut down our plant at midnight and allow the load to be carried until 7 a.m. by some of our other turbines which are located in two sawmills. It will be observed that the field volts and amperes in 1926 with a kw. load of 3900 are in excess of those prescribed in the rating of the generator for full-load conditions, and that the kv-a. load on the generator was fairly close to maximum 80 per cent power-factor conditions when we had a power-factor of 64 per cent. Likewise the field volts and amperes, amperes per terminal of the generator in 1927 with a kw. load of 3930 are well below those prescribed for normal rating and permit an additional load to be carried. By calculating the amount of energy required for excitation in 1927 and comparing with that in 1926, from the average readings as shown, using as a basis 16 hours' operation per day and 300 days per year, we find that 54,800 kw-hr. are saved.

The values in this table have been plotted as a vector diagram in Fig. 3 which may give a clearer picture of the results obtained. It will be observed that under the original conditions with an energy load of 3930 kw., the original kv-a. was 6060 with 4620 lagging reactive kv-a. at 64.4 per cent p-f.; whereas the final kv-a. with an energy load of 3930 kw. was 4780 while we had only 2720 lagging reactive kv-a. at 82.1 per cent p-f. showing 1900 kv-a. corrective. Since we had installed 1800 kv-a. of corrective equipment, the calculations are reasonably accurate when considering that they were based on normal hourly readings by the station operator. Also, it must be remembered that the amount of corrective kv-a. which the capacitors furnish to the system varies directly as the square of the voltage applied, and, therefore, while the nominal rating of the capacitors installed is 1800 kv-a., the corrective kv-a. may vary considerably depending upon the voltage at the capacitors.

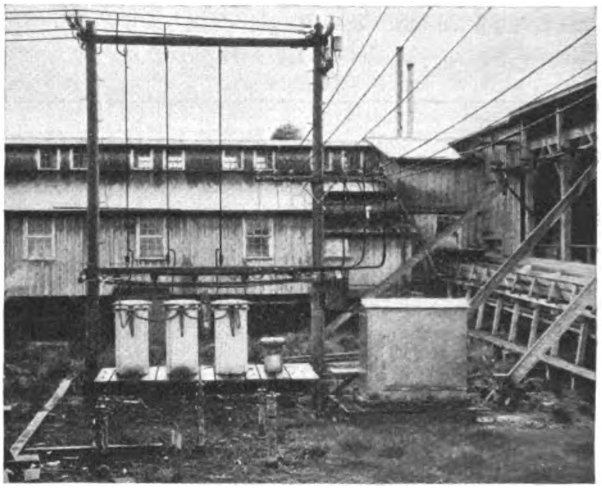


Fig. 2. Typical Installation of 440-volt 300-kv-a. Capacitor Unit Connected to the Secondary Side of a Transformer Bank

In calculating the I^2R loss in the 5000-kw. generator, using the average figures in Table I and the resistance of the generator as furnished by the manufacturer, we find the losses to be 36.629 kw. in 1926 as compared with 22.822 kw. in 1927, or a saving of 13.807 kw. On a basis of 16 hours' operation and 300 days a year, this saving amounts to 66,283 kw-hr. In calculating the I^2R loss in the exciter under the same conditions, we find the loss in 1926 to be 3.456 kw. and that in 1927 as 2.726 kw., or a saving of 0.730 kw. On the same basis of operation this amounts to 3,504 kw-hr. per year. The losses of the motor on this exciter were not taken into consideration.

Calculations of the five feeders to which these capacitors are connected, from observations of the current readings of the station recording instruments with and without the capacitors on the line, indicate a saving in I^2R loss of 500,000 kw-hr. per year.

A summary of the total savings is given in Table II.

TABLE II

5000-kw. Generator I^2R loss	66,003	kw-hr.
50 kw. Exciter I^2R loss	3,504	"
Decrease in excitation	54,801	"
Line losses	500,000	"
	624,588	"

Assuming a cost of $0.005 per kw-hr. at the busbars, this saving amounts to $3123. Since most of this saving results from a reduction in line losses, the unit cost of $0.005 per kw-hr. assumed is exceedingly conservative. The cost of the capacitors and the installation did not exceed $31,500. The savings in kw-hr. referred to in Table II represent a return of 10.1 per cent on the investment. We have not considered the saving in capital account necessary to increase the size of two of the industrial power transformer banks, and undoubtedly have not enumerated all of the other savings which have been obtained. Furthermore, we have been carrying on this generator an increased average load of 1300 kw. which might be said to have been reclaimed by the expenditure of $31,500, or $24.25 per kw. of capacity. Obtaining real station capacity at a cost of $24.25 per kw. is very attractive, For peak loads of several hours' duration we have carried 5100 kw. on this generator as compared to 3500 kw. the year before, or a net increase of 1600 kw. in capacity. On this basis the increased capacity cost $19.70 per kw.

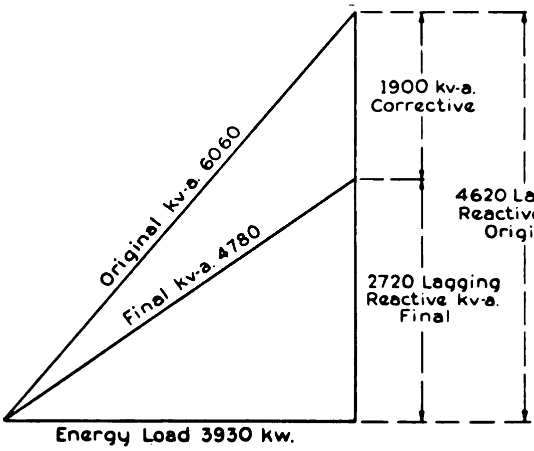


Fig. 3. Vector Diagram of Original and Final Kv-a. with Energy Load, Showing 64.4 Per Cent Power-factor Improved to 82.1 Per Cent

From another angle, the saving of $3125 represents 15 per cent on an investment of $20,800. Subtracting this amount from $31,500, the cost of the capacitors, we find that $10,700 of investment is not covered. We might say we have received 1300 kw. added generating capacity as well as the added feeder, transformer, and regulator capacity for this $10,700, or at a cost of $8.23 per kw., which is a most attractive figure. This undoubtedly is the best method of arriving at the ultimate capital investment required for this installation.

While it is true theoretically that we have added only to the electrical generating capacity of our equipment, still it must be admitted that since no more load could be placed on the generator the so-called excess boiler and turbine capacity was tied up and could not be released until we were in a position to handle more electrical load. If another turbine generator were available, it would have been possible to carry the added load on it. In reality, then, we actually released 1300 kw. of capacity in steam and electrical equipment which had been tied up by the low-power-factor condition.

It might be well to state that all of our energy is generated by steam and certain of the savings effected by our installation would be of relatively less importance on a hydroelectric system where plenty of water was available at all times. However where stored water is of importance, these savings would have much the same value as on our system.

As we stated previously, we made checks of the power-factor on the several 4150-volt feeders and followed them by tests at the consumers' premises on these feeders. From these data we were enabled to select the locations of the worst conditions and determined definitely the size of capacitors to be purchased and installed there.

After they were all in operation, a series of tests were made on each feeder at the power station with recording ammeter, and power-factor and recording wattmeter charts. Unfortunately, the graphic wattmeter was not received in an operating condition at the factory and a calibration curve could not be obtained for the charts obtained. However, we arranged to secure charts which show the effect on power-factor and amperes per phase with and without the capacitors on the line.

Fig. 4 shows the results obtained on feeder No. 3, where we have a 300-kv-a. capacitor connected directly to the 4150-volt lines. This illustration shows a reduction of 30 amp. per phase and an increase in power-factor from 70 to 95 per cent, taking the lowest check readings of the indicating power-factor meter, or a reduction of 215 kv-a. lagging reactive load on this feeder. The amount of reduction in lagging reactive load seems smaller than it should be, but this particular feeder servies a fluctuating sawmill load which undoubtedly accounts for any apparent discrepancy especially when taking readings over such a short period as a half hour.

Fig. 5 shows the results obtained on feeder No. 5 which covers a portion of the business district in Hoquiam but which also has considerable power load. A 300-kv-a. capacitor is connected to the 440-volt side of a bank of transformers where we were on the verge of increasing the capacity of the transformer bank because of excessive kv-a. load. The charts indicate a reduction of 30 amp. per phase and an increase in power-factor from 82 to 97 per cent, where again we take the lowest check readings of the indicating power-factor meter. This capacitor was cut out on a dark and cloudy day for only 15 minutes and the results shown are not as large as we would obtain on a clear day when a considerable number of the business houses would not have their stores fully lighted (high power-factor load). This illustration

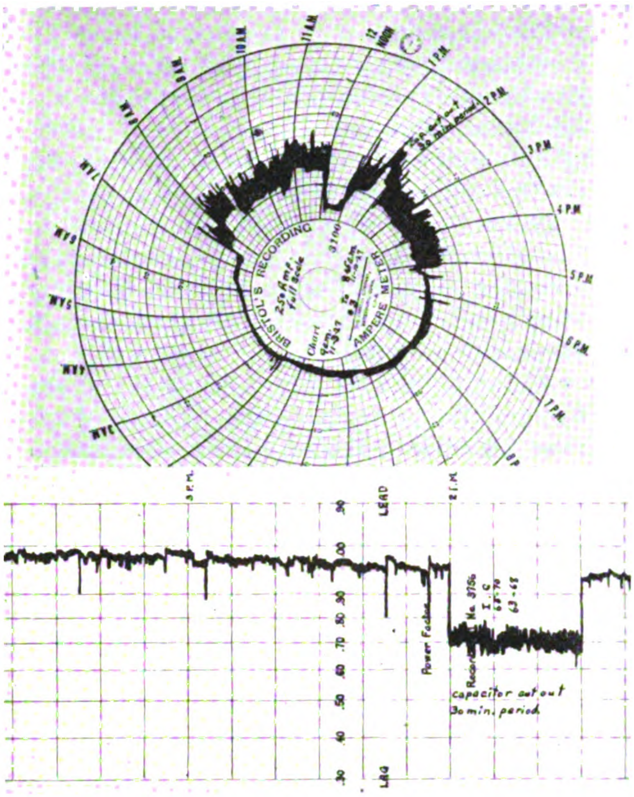


Fig. 4. Current and Power-factor Test with and without 300-kv-a. 4150-volt Capacitor on Feeder Serving a Fluctuating Sawmill Load

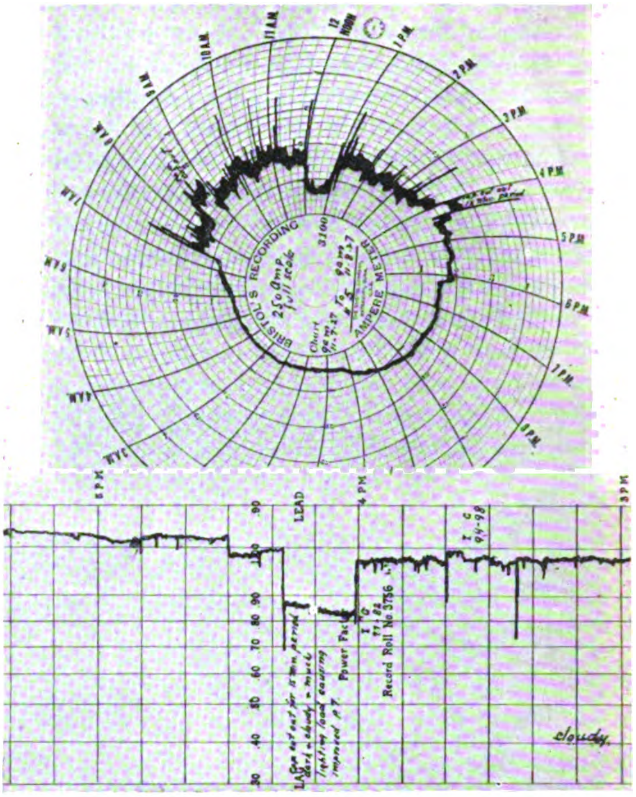


Fig. 5. Test Showing Reduction in Current and Improvement in Power-factor by a 300-kv-a. 440-volt Capacitor on a Combined Light and Power Feeder

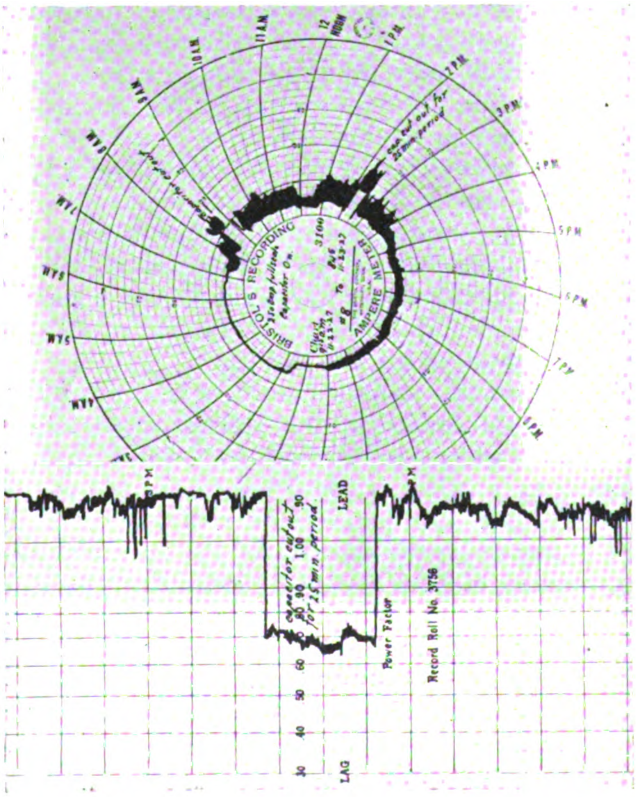


Fig. 6. Test with and without a 300-kv-a. 440-volt Capacitor on a Lightly Loaded Feeder

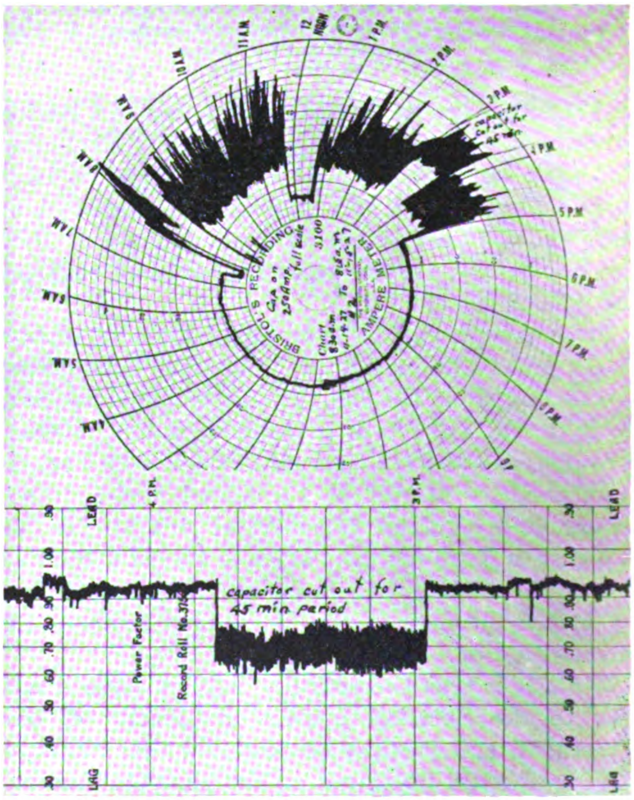


Fig. 7. Test on a Feeder Furnishing the Complete Power Requirements of a Sawmill

indicates a corresponding reduction of approximately 215 lagging reactive kv-a. on this feeder.

In Fig. 6 are shown the corresponding conditions for the long 4/0 feeder where the capacitor was installed on the 440-volt side of a consumer's bank of transformers. This feeder is rather lightly loaded. The reduction in amperes per phase was about 32 and the power-factor was increased from about 68 per cent lagging to better than 90 per cent leading. No serious consequence resulted from this change except high voltage on the power secondaries, which was afterward lowered by changing the taps on the consumer's transformers. This capacitor likewise corrected for approximately 220 lagging reactive kv-a.

The charts in Fig. 7 were taken on a feeder from which we supply the complete power requirements of a sawmill. The 300-kv-a. capacitor is located on the 440-volt side of the transformer bank. The amperes were reduced by over 60 with a correction in lagging reactive kv-a. load of 430. The power-factor was changed from an average of about 70 to 92 per cent. This of course materially reduced the bad condition on this feeder since on part of its route the circuit is through a cable under the Chehalis River, and the reduction of 60 amp. in 175 materially reduced the load on this cable. This reduction in amperes likewise reduced the load on the feeder voltage regulators to such an extent as to place them within operating range again and remove them from the maximum boost position. In installing capacitors on unregulated feeders, care should be taken to avoid the possibility of increasing the voltage to such an extent that, when the major portion of the load goes off, it will burn out the lamps of the few people who might be using the service during these hours. In our case, no danger of this character arose because we have automatic feeder voltage regulators on every feeder.

Summarizing, we have made kw-hr. savings amounting to a return of 10.1 per cent on the total investment, increased the peak capacity of our generator by 1600 kw. (or 45.7 per cent over conditions prevailing a year ago), increased the capacity of all of the feeders on which the capacitors are installed, as well as relieved the load on our feeder voltage regulators (with the result of increased range of the regulators), and increased the voltage to the consumer which means higher bills with a more satisfied consumer due to the better operating conditions resulting in increased output. Also, it is usually considered that the consumer should correct his own power-factor and be given a bonus for so doing or else be penalized for not doing so; but ours is an excellent example of the profit to be gained by the Central Station in making this correction on the consumer's premises. The figures used in calculating corrected lagging reactive kv-a. and the unit cost of $0.005 per kw-hr. for energy saved in line losses are certainly conservative but at the same time show sufficient justification for the installation.

Landmarks of Two Ages

The Santa Cristina Church, Built During the Seventh Century, and a Section of the 30,000-volt 50-cycle 3-phase Transmission Line of the Spanish Northern Railway Along Its Tortuous Route Between the Cobertoria and Pajares Substations

New Three-power Locomotive

A NEW battery-oil-electric locomotive, carrying its own power plant and the first of its kind ever built, was recently tested in New York by the New York Central Railroad for which it was constructed to serve in freight yards that are not completely electrified and where part of the time it is required to operate through city streets.

As this new locomotive will also be called upon to operate at times over tracks which are electrified, third-rail shoes and an overhead collector have been included. These permit the operation of the traction motors from external sources of supply when in electrified districts.

The new locomotive, which has a service weight of 128 tons, is a product of designs by the New York Central's electrical engineering department working in combination with the General Electric Company, the Ingersoll-Rand Company, and the Electric Storage Battery Company, and was built by the American Locomotive Company.

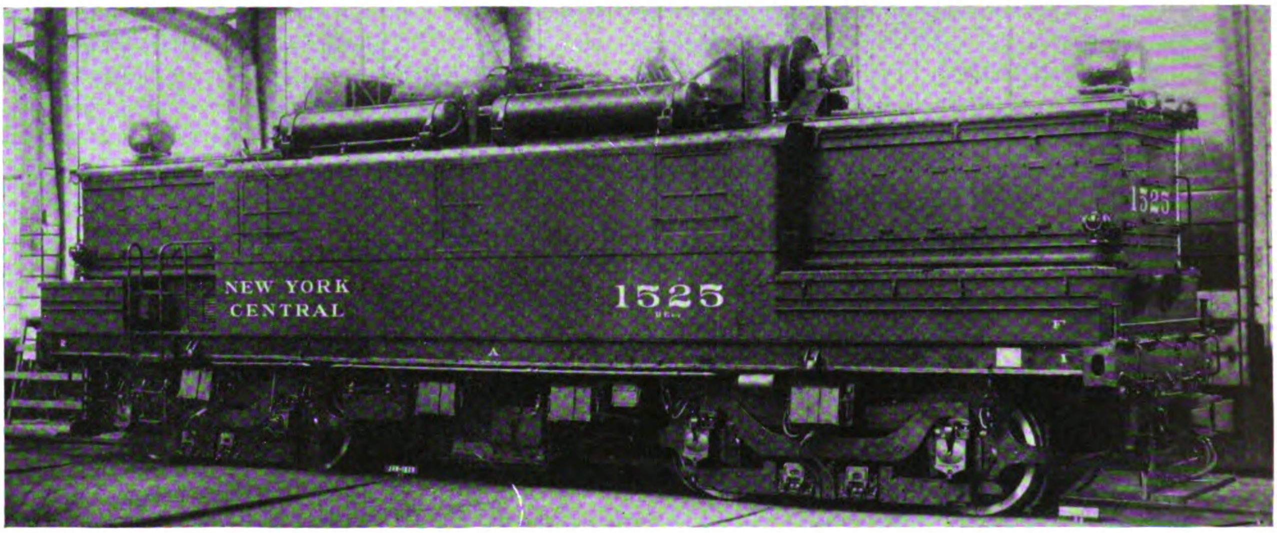
When the locomotive is operating in the non-electrified sections, the power is furnished by an Exide storage battery of 218 cells weighing 17 tons, one of the largest batteries ever used for locomotive service. The battery is charged by a 300-hp. Ingersoll-Rand oil engine that is governor controlled for constant speed operation and is connected directly to a 200-kw. General-Electric generator. This generator is so designed that, if it is operated at the same time that power is being supplied to the traction motors, it will divide the load with the storage battery in periods of heavy output without overloading the engine and will return automatically to charging the battery as soon as the load has decreased. The battery supplies the surges of power required in switching, when the locomotive must respond quickly.

The locomotive is 46 ft. 8 in. long over the couplers and is of the swivel-truck type with a cab in three sections, the batteries being carried in the end sections and the oil engine in the central section. The batteries are arranged in tiers. The central section of the cab, in addition to housing the power plant and control apparatus, has two operating compartments for the engineman. Each operating compartment is directly accessible from the outside and gives access to the power plant.

In the roof of the central compartment, directly above the oil engine, is a hatch to permit the removal of the engine if necessary. The locomotive is equipped with four General-Electric 600-volt single-geared traction motors each of 415 hp. The control is non-automatic, with individual electro-pneumatically operated contactors, and the locomotive may be operated from either end.

There are four available methods of operation: from the storage batteries alone; from the storage batteries and engine-generator together; from the third rail; and from the overhead collector. As the change-over from external to internal power supply, or vice versa, occurs automatically, indicating lights are provided in the enginemen's cab to show under which condition of operation the equipment is running.

Fuel tanks having a storage capacity of 200 gal. of oil are provided. These are sufficient to operate the engine at full load for about ten hours.



The New Battery-oil-electric Locomotive

The Photo-electric Cell

By **DR. L. R. KOLLER**
Research Laboratory, General Electric Company

THE photo-electric cell is a vacuum-tube device by means of which light can be made to control the flow of an electric current. It finds many applications in industry, both in recording changes in light intensity and in control work.

In our February issue the author described the application of the photo-electric cell to recording the ever-varying intensity of daylight. This application is one of the latest additions to the field of usefulness of the cell. The possibilities of other services are indeed fascinating, and a wide-spread knowledge of the characteristics of the cell will materially assist in their discovery.—EDITOR

The underlying principle of the cell was first made clear by Hallwachs in 1888, following some observations made by Horz a year earlier. Hallwachs found that if he charged a zinc plate to a negative potential (by connecting it to the negative terminal of a battery) and then exposed it to ultra-violet light, it gradually lost its charge; but it did not lose its charge when so exposed after being raised to a positive potential. Since the time of Hallwachs this phenomenon has been thoroughly investigated, and it has been found that the metals and many other substances as well exhibit it to a greater or lesser degree.

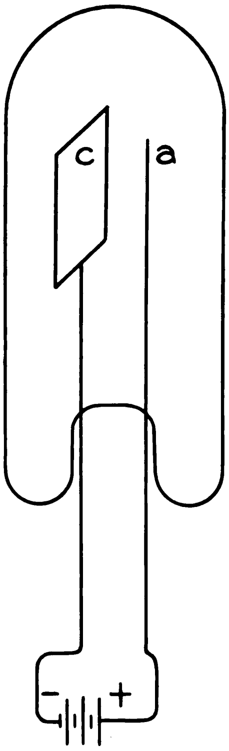


Fig. 1. Elementary Diagram of Photo-electric Cell and Its Electrical Connections

In terms of present-day physics, this phenomenon observed by Hallwachs is explained by saying that light energy is absorbed by the photo-sensitive surface and photo electrons are emitted. If the plate is charged negatively, these are enabled to escape from the surface.

The photo-electric cell is represented schematically in Fig. 1. Two electrodes, insulated from each other, are sealed in a well-evacuated glass bulb. The electrode *a* is of any suitable shape, usually a wire or a grid, and of a metal such as nickel. The other electrode *c* is made of some photo-sensitive material, such as the zinc of Hallwachs' experiments. The electrode *a* (called the anode) is connected to the positive terminal of a battery, and *c* (called the cathode) is connected to the negative. If no light is permitted to reach the bulb, no current can flow across the evacuated space. If, however, light is allowed to fall upon the cathode, it will cause the emission of electrons (photo electrons) from *c*, which will be drawn over to *a* due to its positive potential, and so a small current will flow in the circuit.

This type of cell is radically different from its forerunner, the selenium cell, which depends for its action on the fact that a thin layer of the element selenium undergoes marked changes in electrical resistance when the intensity of the light falling upon it is varied.

A common form of photo-electric cell is shown in section in Fig. 2. Instead of using a metal plate for the cathode or sensitive surface, the whole inside of the bulb wall, with the exception of a small opening for the entrance of the light, is covered with one of the alkali metals which happen to be particularly sensitive. Contact is made with this surface by a flush seal, shown at the left in Fig. 2.

In the most recent forms, the inside of the bulb wall is first silvered and the alkali metal then deposited on this silver surface.

A small amount of one of the inert gases, such as helium or argon, is sometimes introduced into the cell. The effect of this gas is to give an internal amplification. Some of the photo electrons which pass from the cathode to the anode will happen to collide with gas molecules, and in some instances the molecules will be ionized; *i.e.*, each will be broken up into a positive ion and an electron. The electron will continue towards the anode together with the original electron, and the positive ion will be drawn to the cathode. Thus in place of the original electron we now have two electrons travelling towards the anode and one positive ion travelling towards the cathode; three charges in place of one. The result will be a three-fold multiplication of the current. The internal amplification that can be obtained in this way depends upon the kind of gas used, the pressure, the construction of the cell, and the voltage applied between electrodes.

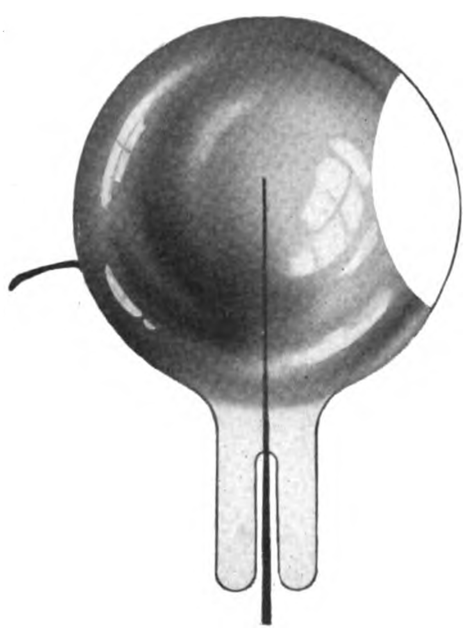


Fig. 2. Representation of the Cross-section of a Photo-electric Cell. The electrode in the center is the anode, the cathode is the coating on the interior of the wall to which electrical connection is made through the seal at the left, and at the right is the clear "window" to permit the entrance of light

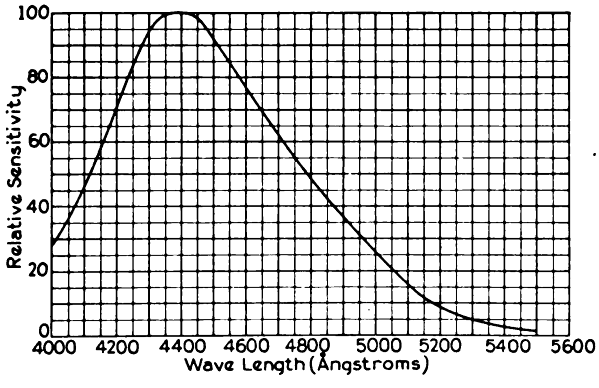


Fig. 3. Relative Sensitivity of the Photo-electric Cell to Lights of Different Color but of Equal Energy

The particular value of the photo-electric cell lies in the fact that the current which passes through it is directly proportional to the light flux falling upon it, so that the cell can be used as an accurate means of measuring quantity, or changes in quantity, of light. This remarkable fact is connected with the quantum theory of the nature of light. Light energy is not continuous and cannot be indefinitely subdivided, but exists in units of definite size called quanta. A quantum is the smallest possible unit of energy, just as an atom is the smallest particle of matter. Each quantum that is absorbed by the photo-sensitive surface causes the liberation of one electron. The mechanism of this exchange is by no means clearly understood, but the correctness of the fact is well established. The speed with which the electron leaves the surface of the metal depends upon the frequency of the light, according to a relation first expressed by Einstein in 1905. The higher the frequency (*i.e.*, the shorter the wavelength) of the incident light, the greater will be the velocity of the escaping electron. The velocities of the photo electrons, however, are small; and for most practical purposes need not be taken into consideration.

The statement that the current through a photoelectric cell is proportional to the quantity of light which falls upon it requires some modification, for photo-electrically active substances are not equally sensitive to all wavelengths, *i.e.*, to all colors of light. Take for example the case of potassium shown in Fig. 3.(1) This curve shows the relative photo currents passing through a cell when illuminated by lights of different colors but of equal energy. The curve shows that the response of the cell depends upon the color of the light used (as well as upon the quantity of the light). The maximum value of the curve lies in the blue portion of the spectrum, and it falls off rapidly towards the red. A curve of this same general type holds for most photo-electrically active substances.

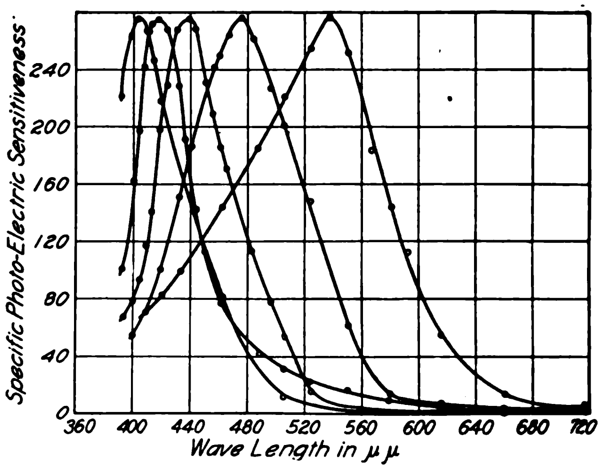


Fig. 4. Photo-sensitivity Curves of Various Alkali Metals, Drawn with Equal Maximum Ordinates. From left to right, these curves apply to Li, Na, K, Rb, and Cs

The point of maximum sensitivity falls in different parts of the spectrum for different materials. The wavelength at which maximum sensitivity occurs is characteristic of any given substance. The point where the curve touches the axis is known as the long-wave limit as the material is insensitive to wavelengths any longer than this. This point also varies for different substances but is characteristic of any one substance.

Wavelength-sensitivity curves for the alkali metals are shown in Fig. 4.(1) These curves show that in passing down the series of the alkali metals as they occur in the periodic table, the peak in the curves moves towards the right, *i.e.*, they become more red sensitive. This is a very important factor to note if the cell is to be used for any work involving color. It is a common practice in the manufacture of a number of types of photo-electric cells to treat the alkali metal with hydrogen and so form the alkali hydride. The hydrides are more sensitive than the untreated metals, and the maxima in the sensitivity curves show a shift towards the red.

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(1) From E. F. Seiler, *Astrophysical Journal*, 52, 129, 1920

In Fig. 5 is shown a visual sensitivity curve which gives the response of the human eye to lights of equal intensity but of different colors.(2) The curve shows that the eye is most sensitive to a wavelength of about 5550 which is a yellow-green color. A photo-electric cell made with potassium, however, would be most sensitive to blue light of wavelength 4400. Photo-electric cells, then, do not "see" colors in the same way as the human eye. Where measurements involving color are to be made, the cells must be corrected by means of absorbing screens or filters so that the sensitivity curve of the combination will more nearly approach that of the eye. From Fig. 4, it can be seen that of the alkali metals caesium most nearly approaches the visual sensitivity curve.

In the photometry of lamps by means of photo-electric cells, color sensitivity is of great importance. For example, a gas-filled and a vacuum lamp of the same candlepower according to visual photometric measurements will not appear of the same power to a photo-electric cell, because the gas-filled lamp operates at a higher temperature and so emits a larger proportion of its radiation in the blue end of the spectrum. Since all photo-electric cells are more sensitive to blue than to red radiation, the gas-filled lamp will appear the brighter to an uncorrected cell

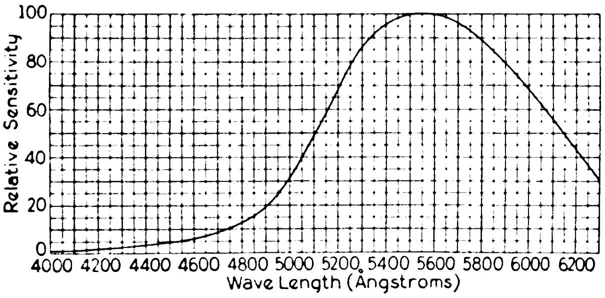

Fig. 5. Relative Sensitivity of the Human Eye to Lights of Different Color but of Equal Energy. Compare with the curve in Fig. 3

If the cell is to be used for making quantitative measurements, the amount of voltage to be applied to the anode is of importance. The number of electrons emitted from the photo-sensitive surface is dependent only on the incident light. The number which succeed in reaching the anode, which is the photo-electric current, depends upon the anode voltage. At low voltages only a few electrons will reach the anode. As the voltage is raised, a larger proportion will reach it; and finally a voltage will be attained sufficiently high to draw over all the electrons emitted. This is the saturation voltage, analogous to the saturation voltage of a hot-cathode vacuum tube. Higher voltages can produce no increase in current, because there are no more electrons to be drawn over. A volt-ampere curve of this type is shown in Fig. 6. At voltages below the saturation voltage, the current is not proportional to light intensity because only a part of the electrons reach the anode.

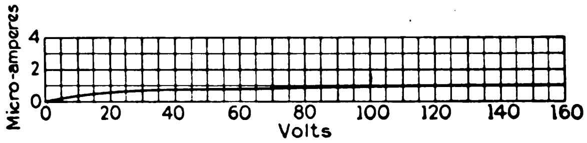

Fig. 6. Characteristic Volt-ampere Curve of a Vacuum-type Photo-electric Cell

In the case of a gas-filled cell, this curve is different in form. At low voltages the two curves are practically the same. As soon as the potential difference between the electrodes exceeds the ionizing voltage of the gas used, some of the gas molecules become ionized, which results in an increase in the current. This can be seen in Fig. 7, where the curve is gradually rising in the neighborhood of 20 volts. As the voltage is increased, more and more ions are produced, resulting in a still larger current as is shown by the more rapid rise of the curve until about at 160 volts enough ions are produced to result in a glow discharge. In this condition the discharge is self-sustaining and independent of the light source. To obtain the greatest sensitivity, the cell is used at a voltage slightly below the glow voltage. This subject has been investigated at considerable length by Cambell.(3)

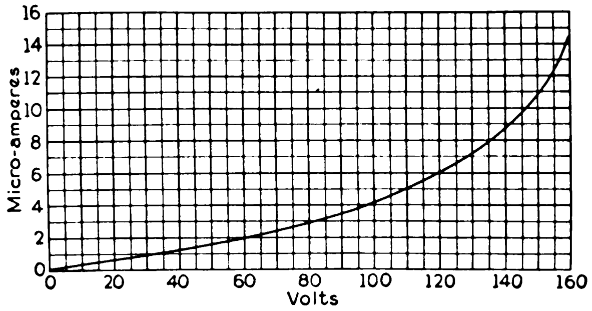

Fig. 7. This Curve, Compared with That in Fig. 6, Shows the Gain in Current by Adding to the Cell a Certain Amount of Gas for Ionization

For measuring ultra-violet radiation, substances such as magnesium and cadmium are used, as these are sensitive to ultra-violet but not to visible radiation. Ordinary glass is not transparent to the short ultra-violet waves, so for these purposes quartz must be used for the cell enclosure.

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(2) From *Bulletin of the Bureau of Standards*, 14, 167, 1918.

(3) *Philosophical Mag.*, April, 1927, p. 945.

Various Methods of Connecting Indicating Wattmeters

PART II

Connections for Three-phase Four-wire Circuits (Cont'd)—Measurements in Connection with a High-tension Line—Reactive Volt-ampere Meters

By **JOHN AUCHINCLOSS**
Switchgear Engineering Dept., Philadelphia Works, General Electric Company

It is interesting to note that the reversal of the current transformer secondary delta shown in Fig. 21* necessitates the reversal of current *Y* rather than current *X* at the meter in the reversed connection shown in Fig. 23. This is shown vectorially in Fig. 24. The voltage vectors e_1, e_2, and e_3 are first drawn at the instant corresponding to the arrows shown on the primary lines in Fig. 23. Assuming for the sake of simplicity that the primary currents i_1, i_2, and i_3 are in phase with the leg-voltages e_1, e_2, and e_3 respectively, they may be drawn as shown in Fig. 24 (b) and marked *a*, *b*, *c*, *d*, *e*, and *f* to correspond with similar markings shown in Fig. 23. The current transformer secondary delta is then constructed and superposed upon the line voltages as shown in Fig. 24 (a), observing in this construction that since the ends marked *a* and *d* are connected together at the transformers shown in Fig. 23, the same markings must be joined together in the vector diagram of the secondary delta. Similarly, points *c-f* and *e-b* must be connected to form the two remaining corners of the delta.

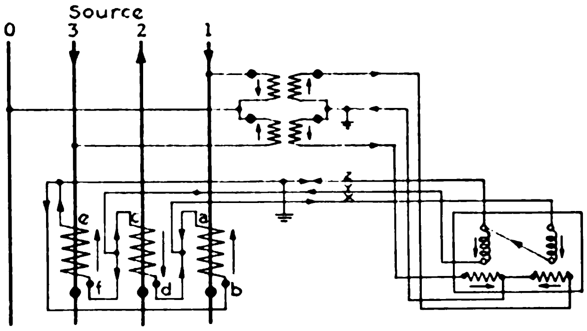

Fig. 23. The Two-coil Polyphase Wattmeter of Fig. 21 Shown with Reversed Connections in the Current Element and Transformer Delta

Comparing the delta formed in Fig. 24 (a) with that shown in Fig. 22, it will be observed that the two deltas are displaced 180 deg. from each other, while the voltage vectors e_1, e_2, and e_3 remain in the same position with respect to each other. Consequently, the arrows across the potential coils of the meter are the same in both Figs. 21 and 23, and to obtain a positive torque in the instrument it is necessary that the arrows across the current coils be also the same in both figures. To accomplish this the connections to these coils are made just the reverse in Fig. 23 of what they are in Fig. 21.

This is also shown on the vector diagram at Fig. 24 (a) in which it will be noted that the current *X*, which was 150 deg. displaced from the voltage e_1 in Fig. 22, has now a displacement of only 30 deg.; and since the cosine of 30 deg. is positive, the current *X* will give a positive torque with the voltage e_1. On the other hand, the current *Y* which was previously displaced only 30 deg. from its voltage e_3, is now displaced 150 deg. Since the cosine of 150 deg. is negative, the current *Y* should give a negative torque with the voltage e_3 and oppose the torque produced by the other instrument. To change the sign of the torque from negative to positive it is necessary to shift the phase displacement from 150 deg. to 30 deg. This may be accomplished in two ways, *viz.*, by reversing the connections to the current coil or by reversing the potential coil connections. The former method was chosen in Fig. 23.

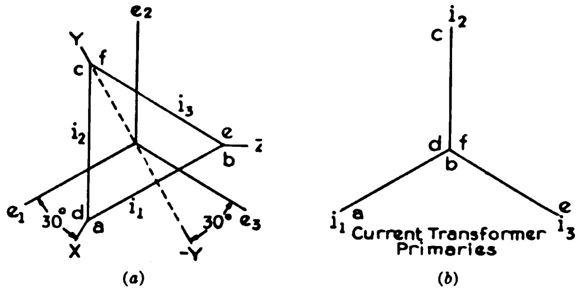

Fig. 24. Vector Diagrams for the Circuit Shown in Fig. 23

Should it be preferred to reverse the potential instead of the current coil the connections would be made as shown in Fig. 25, and the corresponding vector diagram that in Fig. 26. On balanced load at unity power-factor the torque as before is:

$$e_1 X \cos 30° + e_3 Y \cos 30°$$

while on unbalanced load it is:

$$e_1 i_1 \cos \alpha_1 + e_1 i_2 \cos (60° - \alpha_2) + e_3 i_2 \cos (60° + \alpha_2) + e_3 i_3 \cos \alpha_3$$

which with balanced voltages reduces to the familiar form

$$e\,(i_1 \cos \alpha_1 + i_2 \cos \alpha_2 + i_3 \cos \alpha_3)$$

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*Figs. 1 to 22 appeared in Part I of this article, published in the issue of May, 1928, p. 257.—Ed.

This expression is due to the fact that current X which operates with the voltage e_1 contains components of both i_1 and i_2, while the current Y which operates with the voltage e_3 contains components of i_3 and i_2. The components of i_2, therefore, produce a torque with both e_1 and e_3, while i_1 and i_3 operate only with their individual voltages e_1 and e_3, Fig. 24 (a).

The instrument, therefore, indicates correctly with a reversal of either the current or potential coil connections. It is obvious, of course, that *both* should not be reversed.

while, for the sake of economy, the potential transformers are located on the low-voltage side of the Y-Δ transformer bank. Almost invariably the Y-side is the high-voltage side and the Δ-side the low.

A study of the vector conditions of this arrangement as shown in Fig. 27 reveals that the voltages on the low-voltage side are in phase with the currents on the high-voltage side (at unity power-factor). The instrument having three current coils and two potential coils, which requires voltage and current in phase at unity power-factor, can therefore be used for this condition.

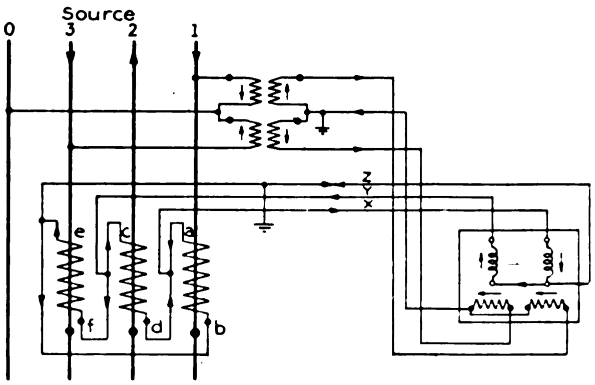

Fig. 25. Connections for Two-coil Polyphase Wattmeter When Potential Coil is Reversed

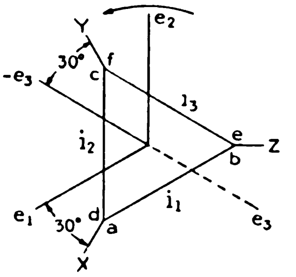

Fig. 26. Vector Diagram Showing Relationships of Current and Potential When Connections Given in Fig. 25 Are Used

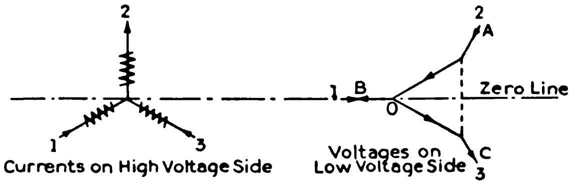

Fig. 27. Vector Diagram of the Conditions Encountered in the Usual Arrangement of a High-tension Line

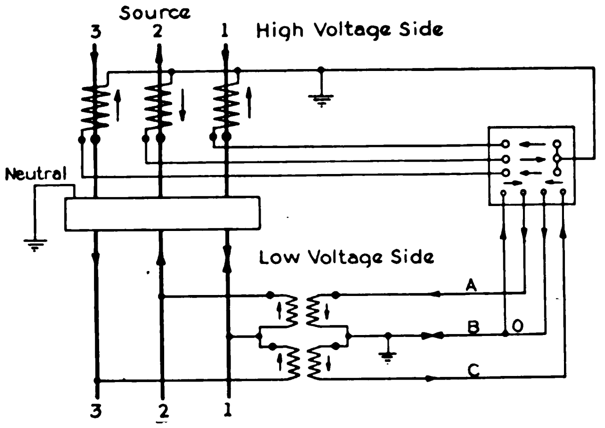

Fig. 28. Diagram of Connections for a Wattmeter in a High-tension Circuit Showing Relative Directions of Current Flow

A problem which commonly comes up for consideration involves a high-tension line on which it is desired to measure power. The neutral of the step-up transformer is grounded for protection only; *i.e.*, no power is expected to be carried over the grounded neutral. As a matter of fact, the neutral ground is so fully protected by delicate relaying equipment as to preclude the possibility of any such contingency. Consequently, the problem resolves itself into nothing more nor less than measuring the power flowing along a three-phase three-wire circuit, and any of the methods already described may be followed.

Occasionally, however, it is desired to measure power(3) on a high-tension line when the current transformers are located in the line for relay purposes

The connections for the instrument under these conditions together with arrows denoting the instantaneous direction of the currents through the various coils are shown in Fig. 28. It will be seen from these diagrams that at this particular instant the potential transformer secondary lead B has no current flowing in it owing to the fact that its primary vector lies on the zero line, while the secondary leads A and C have equal currents flowing over them in opposite directions and meeting at the point O. The latter is seen from the primary vector diagram (Fig. 27) in which the current A flows from phase 2 above the zero line to O and current C comes from O to phase 3, below the zero line.

Another possible method would be to use a standard two-element instrument in connection with the double auto-transformer shown in Fig. 29. Here the potential transformers are shown connected from 2 to 1 and from 1 to 3. This is done in order to obtain voltages having the phase 2-B and A-3 which are in the same position with regard to the currents in lines 1 and 3 as the voltages and currents already shown in Fig. 12.

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(3) Usually, this would be for indicating or graphic wattmeters only, but not for integrating meters upon which charges are billed, because the regulation of the transformer bank would affect the accuracy of the total watthours charged against the customer's account.

Obviously, if mid-taps on the secondaries of the potential transformers were used, the voltage applied to the instrument would be only 110 cos 30°, or 95 volts. This would necessitate special calibration of the meter scale. The purpose of the auto-transformer is to increase the applied voltage from 95 to 110; and consequently its winding must supply $\frac{110}{\cos 30^\circ}$, or 127 volts, with 63.5-volt taps, which will give 110 volts from 2 to B and from A to 3 for the instrument. No special calibration is now required and the meter is standard in every way. Superposing these voltages upon the current vectors we obtain a vector relation between voltage and current exactly like that shown in Fig. 5, except that in Fig. 29 they are shown at unity power-factor.

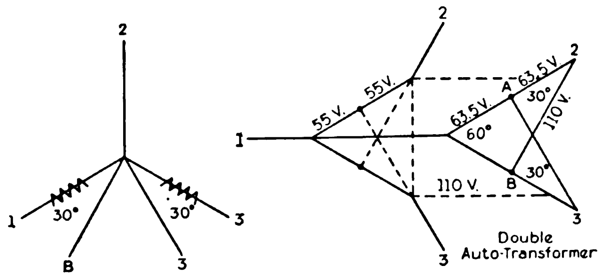


Fig. 29. Vector Diagram for Unity Power-factor Conditions in a Double Auto-transformer

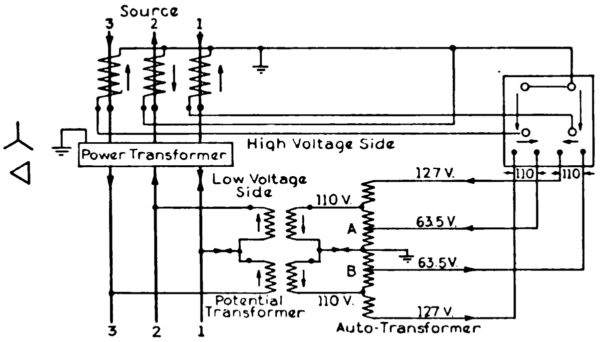


Fig. 30. Diagram of Connections and of Instantaneous Current Direction for a Wattmeter in a High-tension Circuit

In this case, three current transformers are shown, although only two are used. This is done because on a grounded neutral line three current transformers are usually employed for relay equipment.

It should be noted that at the instant chosen phase 1 is positive to phase 2 and phase 3 positive to phase 1. The current through the primaries of the potential transformer will consequently flow from phase 1 to phase 2 and from phase 3 to phase 1. On the secondaries these currents flow in the opposite direction.

Similarly, at the auto-transformers point B is positive to phase 2 and phase 3 positive to point A. The currents through the meter coils will therefore be from B to phase 2 and from phase 3 to point A as shown in Fig. 30. Since at this instant the vector of phase 1 lies on the zero line there is no current flowing out in this phase.

Referring to Fig. 30, it will be observed that the common point of the transformers is connected to phase 1. This is done merely for the sake of simplicity to obtain a vector figure exactly similar to that shown in Fig. 12. If the transformers have also to be used for synchronizing, however, it may be desirable to connect the common point to phase 2 with the secondary ground also on this phase. This may be accomplished with the connections shown in Fig. 31. The corresponding vector diagram is shown in Fig. 32. In the latter it will be noted that an instant has been chosen which is a few degrees earlier in the cycle than that shown in preceding diagrams. This has been done to obtain a definite current flow from phase 1 to point B, for it is obvious that at the instant chosen previously the voltage 1-B would lie on the zero line, and consequently there would be no current flowing across these points and no current in the corresponding potential coil of the instrument. It should also be noted that in Fig. 31, the left-hand element of the instrument is energized from the current transformer located in phase 2. This is necessary to obtain the correct 30-deg. displacement with the voltage A-3 (Fig. 32). It might be questioned

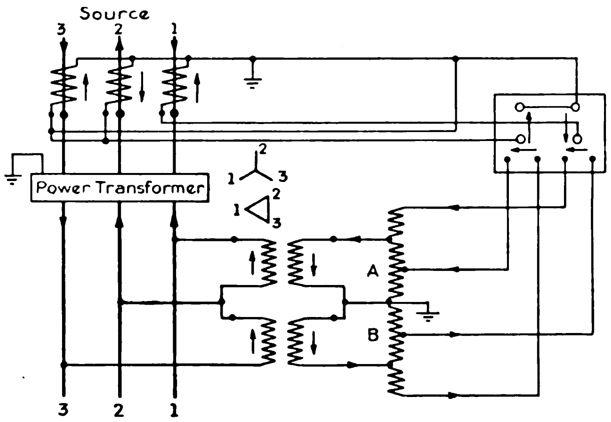


Fig. 31. Modification of the Connections Shown in Fig. 30 to Provide for Use of the Instrument Transformers for Synchronizing

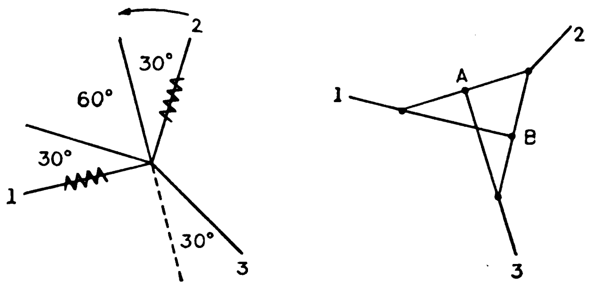


Fig. 32. Vector Diagram of Proper Conditions of Current and Voltage Derived from Connections Shown in Fig. 31

why A-3 could not be reversed at the potential coil, thereby obtaining a 30-deg. displacement with the current in phase 3. This reversal is shown dotted in Fig. 32. Such a connection, however, would be useless since the instrument would read correctly only at unity power-factor, for at say 30-deg. lag the currents and voltages would be exactly in phase; and instead of the instrument's reading 1.5 Ei, it would actually read $2Ei$, which is 33 per cent too high.

REACTIVE VOLT-AMPERE METERS

By impressing a voltage on the potential coil which is 90 deg. displaced from that ordinarily used in measuring watts, the polyphase instrument previously described can be used for measuring reactive volt-amperes. In fact, the same instrument is frequently used for reading both quantities, a throw-over switch in the potential circuit accomplishing the required changes in the connections to the potential coils. If the load is steady, simultaneous readings of these two values afford a ready insight into most other electrical conditions existing in the circuit. For example, from Fig. 33, where

E = voltage
I = current
X = quadrature or reactive component
R = in-phase or power component
α = displacement angle

it is evident that

$$ER = \text{watts} = W$$

$$EX = \text{reactive volt-amperes} = RVA$$

and

$$\sqrt{W^2+RVA^2} = \text{volt-amperes} = VA$$

$$\frac{W}{VA} = \cos\alpha = \text{power-factor}$$

$$\frac{RVA}{W} = \tan\alpha = \text{reactive factor}$$

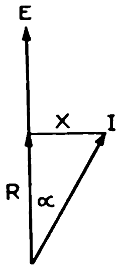


Fig. 33. The Relationship of Quantities Involved in Considering Reactive Power

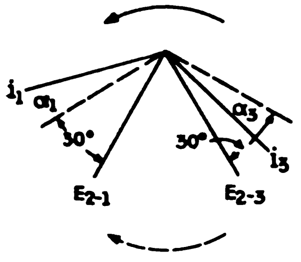


Fig. 34. A Diagram of Voltage and Current in a Polyphase Circuit Showing Independence of Instrument Action and Phase Rotation When Reading Watts

When polyphase instruments are used, the exact meaning of α in the foregoing formulas is rather obscure.

When reading watts on a polyphase circuit the phase sequence of the system has no influence on the action of the instrument since the torque is in the same direction, regardless of whether the current is leading or lagging the voltage across the potential coil. This will be seen upon reference to Fig. 34 which covers the general case. It is obvious that whether the phases rotate in the direction indicated by the full arrow or by the dotted arrow, the torque remains the same, *viz.*,

$$E_{2-1}\, i_1 \cos(30^\circ+\alpha_1) + E_{2-3}\, i_3 \cos(30^\circ-\alpha_3)$$

which is positive for either direction of phase rotation; *i.e.*, whether the power-factor is leading or lagging.

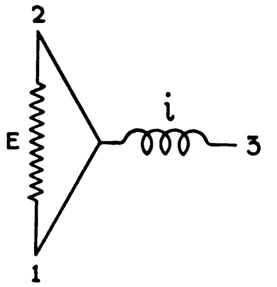


Fig. 35. Diagram Showing the Dependence of Lag or Lead Upon Phase Sequence in Measurement of Reactive Volt-amperes

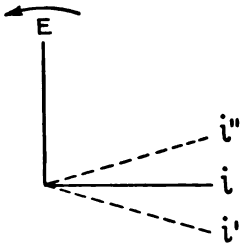


Fig. 36. Current and Potential Vectors Shown at a Normal Phase Displacement of 90 Deg. The departure of the current vector from this position may give either positive or negative readings

When measuring reactive volt-amperes by producing a torque proportional to the sine of the displacement angle, the direction of the torque for a leading current is the reverse of that for a lagging current. That the direction of this torque, as developed with a quadrature voltage, is dependent also upon phase sequence, will be evident from a study of Fig. 35. This diagram represents a single-phase instrument connected to a three-phase circuit, the potential coil being connected across the line voltage in quadrature with the line current (at unity power-factor) in which the current coil is connected.(4)

This connection may be represented vectorially as shown in Fig. 36. At unity power-factor, then, the torque is proportional to

$$Ei \cos 90^\circ = 0$$

regardless of the phase sequence of the system.

Should the phase sequence of the three-phase system be as indicated by the arrow in Fig. 36 (*i.e.*, with the current at unity power-factor 90 deg. behind the voltage) then the insertion of inductive reactance in the load will cause the current to fall back by the angle α to the position i' and the resulting torque will be

$$Ei \cos(90^\circ+\alpha)$$

which is negative.

If, however, the phase sequence were in the opposite direction with the same value of reactance in the load, or in other words, having the same lagging power-factor, the current must take up the position i'' and the torque is

---

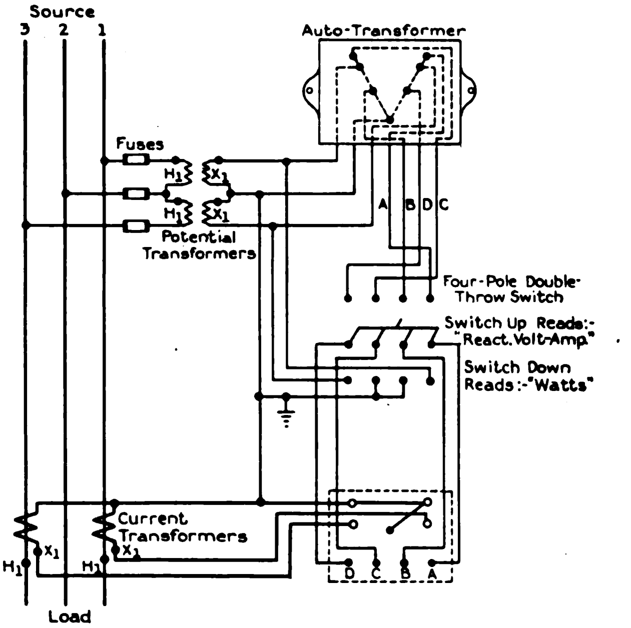
(4) For a more complete discussion of the theory of operation of these instruments, see "Measuring the Reactive Component," by John Auchincloss, GENERAL ELECTRIC REVIEW, April, 1924, p. 263.

$$Ei \cos (90° - \alpha)$$

which is positive.

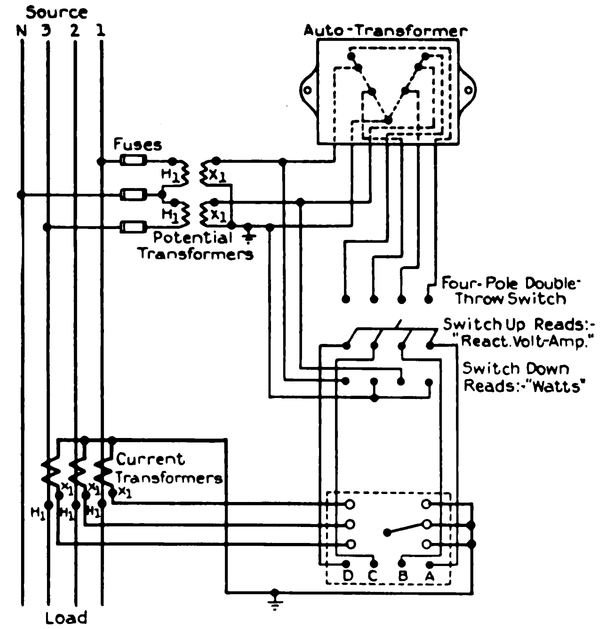
The direction of the torque and, consequently, the direction of deflection of the pointer therefore depend upon the sequence or order in which the phases rotate, so that if the meter were connected up without regard to phase sequence, it might, if equipped with a zero-center scale, indicate lead for lag and *vice versa.* Similar reasoning applies also to the polyphase instrument where the current of each element operates with a voltage other than the one which drives it through the winding.

When the conditions are such that the current is generally lagging or generally leading the instrument may be equipped with a full scale from zero in the left-hand corner to maximum in the right. If, however, as is usually the case when this instrument is used,



Note:- Connections Assume a Phase Sequence 1-2-3. If Instrument when Connected Shows Lag and Lead Reversed, Correct by Reversing "A" and "B" also "C" and "D" at Auto-Transformer

Fig. 37. Diagram of Connections for Obtaining Measurements of Both Watts and Reactive Volt-amperes in a Three-phase, Three-wire Circuit



Note:- Connections Assume a Phase Sequence 1-2-3. If Instrument when Connected Shows Lag and Lead Reversed, Correct by Reversing "A" and "B" also "C" and "D"

Fig. 38. The Diagram of Connections Corresponding to Fig. 37 in which Conditions Are Adapted to a Three-phase Four-wire Circuit

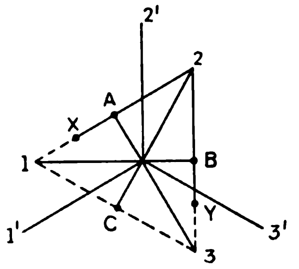


Fig. 39. A Composite Vector Diagram of Voltages and Currents Occasioned through the Use of Directional Relays Together with a Two-element Wattmeter and a Two-element Reactive Volt-ampere Meter

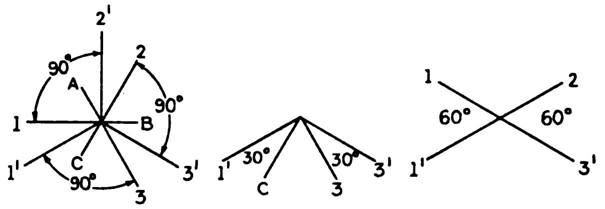


Fig. 40. Separate Vector Diagrams for Relays, Wattmeter, and Kv-a. Meter with 90-, 30-, and 60-deg. Displacement of Voltage and Current Respectively

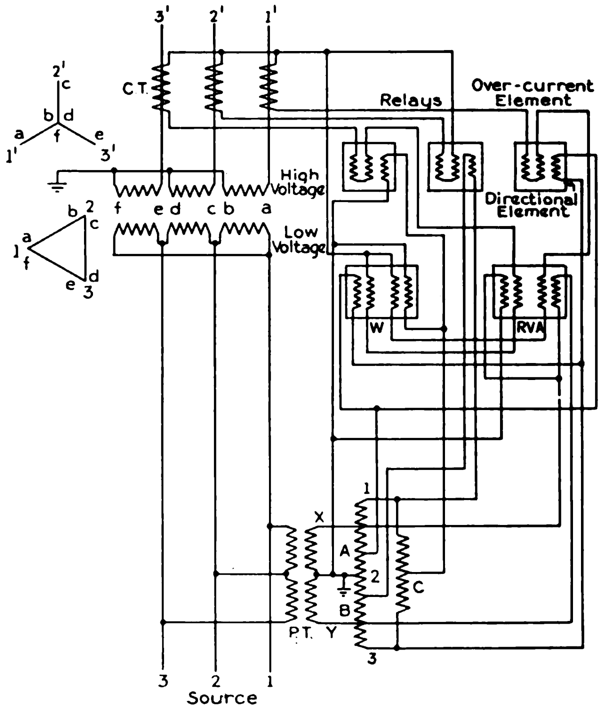


Fig. 41. Actual Diagram of Connections Involving Relays, meters and Kv-a. Meters for Measurements in a High-tension Circuit

the current may be either leading or lagging, the scale has a zero mark in the center and a maximum at both the right- and left-hand corners. The connections for using the instrument to read watts or reactive volt-amperes on a three-phase, three-wire circuit are shown in Fig. 37 and on a three-phase, four-wire circuit in Fig. 38.⁽⁴⁾

Another problem not very often encountered, but nevertheless interesting, requires that three directional relays, one two-element indicating wattmeter, and a two-element indicating reactive volt-ampere meter be connected to current transformers on the high-voltage side and potential transformers on the low-voltage side of a $Y\Delta$-connected bank of power transformers. In this combination the relays required in the current and potential coils of each element a displacement of 90 deg. in phase, in the wattmeter 30 deg., and in the reactive volt-ampere meter 60 deg. displacement.

To accomplish this, three of the auto-transformers previously described were connected as shown vectorially in Fig. 39. Two low-voltage potential transformers were connected in open delta (60°) to points X and Y of two of the auto-transformers, and a third auto-transformer connected across their extreme ends, 1 and 3 (127 volts) giving the relay voltages A-3, B-1 and C-2 (110 volts) in quadrature with the high-voltage currents $1'$, $2'$ and $3'$, respectively. Since the current coils of the wattmeter are connected in phase $1'$ and $3'$ the corresponding voltages must be A-3 and 2-C, while the voltages in quadrature with these required by the reactive volt-ampere meter are Y-X and X-2. It should be noted that the voltage 3-1 cannot be used in place of Y-X since that is 127 volts while the meter requires 110 volts. Fig. 39 shows a composite vector diagram of all these voltages and currents, while in Fig. 40 they are separated for each individual device. Fig. 41 shows the actual connection diagram derived from the two preceding illustrations.

(Concluded)

Induction Motor-generator Sets for Motion-picture Projection

Constant-potential multiple-arc induction motor-generator sets have been designed to convert 50- or 60-cycle alternating-current from power lines to direct current suitable for the motion-picture projectors of the arc type.

The generators may be driven by single-phase or polyphase squirrel-cage induction motors or by high-reactance squirrel-cage induction motors. These sets are built in a variety of sizes, their continuous capacities ranging from 1.6 to 30 kw. All of them are designed to carry 100 per cent overloads for three-minute periods occurring at 15-minute intervals. The continuous ratings range from 20 to 300 amp., and the overload capacity is provided to carry the extra load during periods when changing from one projection machine to another.

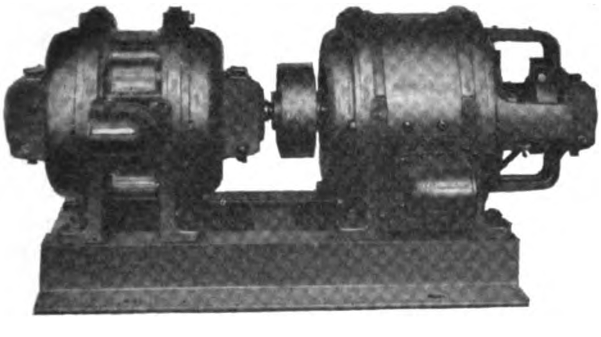


Fig. 1. Typical Motor-generator Set for Direct-current Supply to Motion-picture Projectors

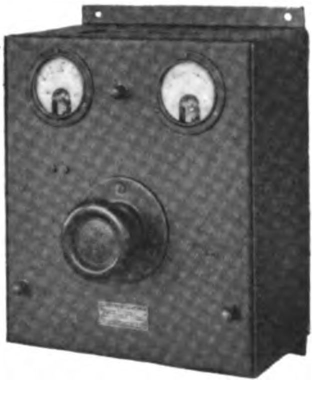


Fig. 2. Typical Generator Panel

The generators are compound wound and will hold the voltage constant within a very small range at all loads up to their maximum rated capacity. The motors are selected to carry the maximum overload without appreciable drop in speed. The sets which include high-reactance motors may be started on full voltage without exceeding the starting current permitted by the N.E.L.A. rules.

Suitable generator panels, arranged for wall mounting, include a direct-current ammeter, direct-current voltmeter, and a generator-field rheostat, all in a sheet-metal enclosing case. The side of this cabinet is removable to permit easy access to the back of the panel.

To secure stable operation of the arc, a ballast resistor must be used in series with each projector, spotlight, etc. This resistor includes a starting section, a permanent-resistance section, and a regulating section with seven regulating points. The proper regulating point is readily selected by means of a dial switch mounted on the front of the panel.

A List of Welded Structures

Prepared by
FRANK P. McKIBBEN
Consulting Engineer, Black Gap, Pennsylvania

THE following list of steel structures in which the parts are connected by welds shows that this process is no longer in the experimental stage, but is a means of fabrication so extensively used as to warrant the attention of all structural engineers and architects. Welding has also been widely adopted in joining structural shapes and plates to produce substitutes for large castings—substitutes which are stronger, more reliable, and more economical. If the accompanying list be any indication of the trend in structural engineering one must conclude that the architect and engineer in their endeavor to keep pace with the development of improvements in the building arts must at least investigate the possibility of welding as a means of fabricating certain types of steel bridges and steel buildings. In many cases it will be found that welding is better and cheaper than any other form of construction.

That responsible designers are taking this matter in hand is some assurance that its use will be directed along safe channels.

Bridges

(1). Single-track Pennsylvania R. R. Bridge over Susquehanna River at Havre de Grace (Md.), double-decked by Maryland State Roads Commission for use as a highway bridge.

(2). Single-track railroad bridge at East Pittsburgh Works of Westinghouse Co. Length of bridge, 62 ft. 4 in.

(3). Through Warren trusses with sub-divided panels for railroad bridge at Chicopee Falls (Mass.). Welded structure weighs 80 tons.

(4). Bridge connecting two buildings, General Electric Company, Schenectady (N. Y.). Built 1921. Span 35 ft.

(5). Welded columns used in bridge for carrying coal wagons.

(6). Welding steel reënforcing rods to existing steel arch bridge over Willamette River, Oregon City (Ore.). Approximately 50,000 welds.

(7). Reëforcing Chicago Great Western R. R. Bridge over Missouri River. Leavenworth (Kan.); originally built 1893; consists of one 440-ft. drawn span and two 330-ft. fixed spans. Electric welding by American Bridge Co. involved 225,000 lb. of structural steel, 7500 lin. ft. of welding and 2000 lb. of welding steel. Reënforcing done between February 28 and April 28, 1927, without interrupting trains.

(8). Princess Bridge, Melbourne (Australia); strengthened by welding.

Buildings

(9). Shop Building No. 1, West Philadelphia plant of General Electric Company. Approximately 1000 tons of beams, columns and trusses. Electrically welded, 1927-28. Shop 136½ ft. wide by 551 ft. long, with a wing. Trusses 58½ ft., 77 ft., and 78 ft. spans. Has bridge cranes and wall cranes.

(10). Shop building, 42 ft. long, 66 ft. wide, connecting existing Buildings 44 and 45 at Bridgeport (Conn.) plant of General Electric Company. Has four trusses of 64-ft. span supported on steel columns. Fabricated by American Bridge Co. Erected by Leak and Nelson Co., Bridgeport (Conn.). Built 1928.

(11). Shop Building No. 38 at Pittsfield (Mass.) plant of General Electric Company. Width, 60 ft. Length, 280 ft. Three stories. Fabricated and erected 1928 by American Bridge Co.

(12). Five-story building at Sharon (Pa.) for Westinghouse Electric & Manufacturing Co. Built 1926 by American Bridge Co. Live load 300 lb. per sq. ft., 790 tons of steel; 70 ft. wide, 220 ft. long, 80 ft. high.

(13). Main factory building of the Lincoln Electric Co., Cleveland (Ohio). Consists of three stories and a basement, and has vertical members welded; constructed in 1916. Building is 50 ft. by 130 ft.

(14). Mill building. Roof trusses; three 19-ft. spans. Brixton, England. Area of building 20,000 sq. ft.

(15). Mill building at Tottenham, London. Built for Galvanizing Equipment Co. by Welded Construction Co. Main truss span 40 ft. with two 15-ft. side spans.

(16). Mill building. One story, built by C. B. & Q. R. R. at Eola (Ill.).

(17). Warehouse for materials of Chicago District Plant of Mississippi Valley Structural Steel Co. at Melrose Park (Ill.). Trusses, columns, and crane columns are all welded. Crane, 20-ton capacity, span 84 ft. Dimensions of building, 118 ft. by 340 ft.

(18). Material shed building. Built 1920 by Electric Welding Co. of America in Brooklyn. Roof trusses, 40 ft. span; tested by Department of Buildings, City of New York. Has traveling crane.

(19.) One-story building, Derry (Pa.); built 1927 for Westinghouse Electric and Manufacturing Co. by Jones and Laughlin Steel Corp. Dimensions of building 150 ft. by 460 ft. with a wing; 336 tons of steel.

(20). Building at Youngstown (Ohio), built 1926 by Youngstown Welding Co. Dimensions 70 ft. by 220 ft.; carries traveling crane. All field erection welded; no bolts or rivets used.

(21). Building at St. Louis (Mo.) Plant of Mississippi Valley Structural Steel Company. Dimensions 30 ft. by 50 ft. All shop work welded; field work bolted.

(22). Garage building, Canton (Ohio), for Peerless Auto Sales Co. Two stories and basement. Building is 100 ft. by 150 ft.

(23). One-story garage at Dallas (Tex.) built by Dallas Power and Light Co. Two units, each 16 ft. by 140 ft.

(24). One-story building at Berkeley (Calif.). Built by Berkeley Construction Co. Dimensions, 50 ft. by 250 ft. Has two 10-ton cranes.

(25). One-story building. Built at Sharon (Pa.) for Westinghouse Electric and Manufacturing Co. in 1926. Building 40 ft. by 102 ft.; 27 tons of steel. Crane runway for one-ton crane.

(26). One-story building at East Pittsburgh Works of the Westinghouse Co. Built 1926 and 1927; 40 ft. by 100 ft.

(27). One-story garage at East Pittsburgh (Pa.) by Westinghouse Co. in 1926; 60 ft. by 90 ft.

(28). One-story steel frame building Newport News Shipbuilding and Dry Dock Co., Newport News (Va.). Building 63 ft. by 53 ft.; carries completely welded ten-ton fish belly crane.

(29). One-story building constructed in Detroit for Barnes Wire Fence Co. This structure was entirely field welded; no bolts or rivets were used.

(30). Two one-story buildings at Waukegan (Ill.). Built by General Boilers Co.; one 25 ft. by 85 ft.; the other 25 ft. by 50 ft.

(31). Steel frame of 25-story addition welded to original steel building of First National Bank Building in Detroit (Mich.), 1927.

(32). Fifteen-story steel frame addition to J. L. Hudson Co. Department Store, Detroit (Mich.); has steel frame of 12 lower stories welded to existing building of same owner.

(33). Steel frame of twelve-story addition welded to original steel building of People's Outfitting Co., Detroit (Mich.)

(34). Steel frame of four-story addition welded to original steel building of Detroit Trust Company.

(35). Welding employed in some connections for addition to Michigan Bell Telephone Building in Detroit.

(36). Addition to chemical laboratory of Westinghouse Co., at East Pittsburgh (Pa.).

(37). Two additions, each a one-story building, welded to original building of Chevrolet Motor Car Co., at Janesville (Wis.).

(38). Foundry building extension, 100 ft. by 54 ft. Main roof truss span, 35 ft. Built 1920 at Stoke-on-Trent, England, by Welded Construction Co. Carries traveling crane of 50-ft. span.

(39). Retort house, West Melbourne Works, Metropolitan Gas Co., Australia; 400 tons steel.

(40). Kiln and coolers at Saulte Sainte Marie Plant of Union Carbide Co. welded by Reeves Bros. Co., Alliance (Ohio). Kiln, 8 ft. by 125 ft.; three coolers 5 ft by 55 ft.

(41). Concentration plant for Ammonical Liquor, Australia.

(42). Additional floor welded in between original roof and original top floor of Crowley, Miller and Co. Department Store, Detroit.

(43). Steel floor at Hershey Chocolate Co., 24 ft. by 64 ft.

(44). Roof trusses over West Yard at Decatur (Ill.) Plant of Mississippi Valley Structural Steel Co. Span 84 ft.; both shop work and field splices welded.

(45). Roof trusses over West Yard at St. Louis (Mo.). Plant of Mississippi Valley Structural Steel Co. Span 78 ft.

(46). Roof trusses, oxy-acetylene welded at Cincinnati Prest-o-Lite plant in 1926. Spans 27 to 48 ft.

(47). Roof trusses. Tested at Buffalo by Union Carbide and Carbon Co. in 1926. Span, 40 ft.

(48). Welded trusses in building for Modern Power Device Co. at Port Washington (Wis.).

(49). Roof truss. Stables at Horse Show, Olympia, England. Built 1920. Span, 14 ft.; area of building, 9,000 sq. ft.

(50). Crane runway, Coplay Cement Mfg. Co., carries crane and 10-ton bucket. Seven plate-girder spans and steel A-frames.

(51). Brackets for crane runway, General Electric Company, Schenectady (N. Y.).

Cars, etc.

(52). Locomotive tender. Built for Boston & Albany R. R. at West Springfield (Mass.).

(53). Gondola freight car. Built for C. B. & Q. R. R. by American Car and Foundry Co. in 1921. Capacity 50 tons.

(54). Annealing car, General Electric Company, Schenectady (N. Y.). Built 1921.

Cranes, etc.

(55). Large boom for derrick. Built 1922; 90 ft. long.

(56). Ten-ton fish belly crane in foundry, Newport News Shipbuilding & Dry Dock Co., Newport News (Va.). January, 1927. Span, 50 ft. 5½ in.

Frames and Towers

(57). Steel frame for large electric sign, Radio Station WGY, General Electric Company, Schenectady (N. Y.).

(58). Steel frame for boxing area lights at Philadelphia Sesqui-centennial Exposition.

(59). Transmission line, tower, Belgium; 72 ft. high.

(60). Transformer tower built by General Electric Company, Schenectady (N. Y.). Built 1920. Carries traveling crane. Dimensions, 40 ft. by 40 ft. by 40 ft.; 34,339 lb. of steel.

(61). Radio tower at Peking, China, 115 ft. high.

(62). Tower in Cave, California.

Ships, etc.

(63). Ship *Fullagar*. Built in 1920 by Cammel Laird & Co., at Birkenhead, England. Length, 150 ft.

(64). U. S. Coast Guard Cutter *Northland* built in 1927 at Newport News Shipbuilding and Dry Dock Co.; rudder metal arc welded, and metal arc welding used extensively on decks, bulkheads, etc.

(65). Motorboat *Sea Hornet* built in 1920 by Kyle & Purdy, City Island, New York (N. Y.). Length, 58 ft.

(66). Tugboat, built in 1920 by Electric Welding Co., Gothenburg, Sweden. Length, 52 ft., 6 in.

(67). Tugboat built 1915 by Geary Boiler Works, Ashtabula (Ohio). Length, 42 ft.

(68). 1200-ton Oil Barge *Hisco*. Built 1917 by Tank Shipbuilding Co., Newburg (N. Y.).

(69). English Channel Barge *AC-1320*. Built 1917. Length 125 ft., 9 in.

(70). Oil Barge *Independent*. Built 1924 by Saxe Providence Boiler Works, Providence (R. I.). Length, 76 ft.

(71). Steel floating caisson for dry dock. Hikoshuma Drydock, Japan.

(72). Pontoon for 120-ton derrick with shell, deck, and two bulkheads welded. Built in 1925 at Norfolk Navy Yard.

(73). Pontoon for crane at Kobe, Japan; length, 65 ft.

(74). Seventeen 45-ton 172-ft. U. S. Navy battle towing targets built at Norfolk Navy Yard and Puget Sound Navy Yard between 1918 and 1927. Supersede 50-ton riveted targets previously used.

Tanks

(75). Oil storage tanks of 40 and 150 ft. diameter, respectively. Built 1920 for Oklahoma Producing and Refining Corporation by Electric Welding Co. of America.

(76). Two 1000-barrel oil storage tanks at Uniontown (Pa.); oxy-acetylene welded in 1924 by Linde Air Products Co.

(77). Tank used as creosoting still; oxy-acetylene welded at works of Reeves Bros. Co., Alliance (Ohio). This tank, which is 90 in. diameter by 90 ft. long and built of one-inch steel plates, operates at 200 lb. steam pressure and 24 in. vacuum; tested to three times normal steam pressure.

(78). Steel swimming pool, Standard Club, Chicago. Pool is 60 ft. long and 30 ft. wide.

(79). Steel swimming pool built 1924 by Norfolk Navy Yard for the Officers' Club. Capacity of tank, 100,000 gal.

(80). Gas holder for Metropolitan Gas Co., Melbourne, Australia; 600 tons steel; 3,000,000 cu. ft. capacity.

(81). Purifiers for Metropolitan Gas Co., Melbourne, Australia; 150 tons steel; 3,000,000 cu. ft. capacity.

(82). Gas holder at Columbus (Ohio) oxy-acetylene welded by Linde Air Products Co. in 1924; diameter, 50 ft.

(83). Gas holder at Salt Lake City (Utah) oxy-acetylene welded by Linde Air Products Co. in 1924; diameter, 50 ft.

(84). Gas holders at Albion (Mich.) built 1928 by Western Gas Construction Co. of Fort Wayne (Ind). Diameter, 74 ft.; max. height 113 ft. 7 in. Structure embodied 258 tons of steel, arc-welded; four tons of welding wire used in 21,414 linear ft. of welding.

(85). New steel roof on gas holder at Fitzroy, Australia. Built by Metropolitan Gas Co.

Tests

(86). General Electric Company shear, tensile and compressive tests, 1927.

(87). Westinghouse Electric Co. Tests made at Carnegie Institute of Technology.

(88). American Bridge Co. Tensile specimens, griders and columns were tested at Bureau of Standards.

(89). Metropolitan Gas Co., Australia. Tests on welded tubular columns in connection with 6-ton 50-ft. jib for derrick crane.

(90). Metropolitan Gas Co., Melbourne, Australia. Tensile tests, etc. at Melbourne University.

(91). Morgan Engineering Co. Shear tests.

(92). Ernest Humphreys, Jr. Shear Tests, 1921.

(93). Westinghouse Electric and Manufacturing Co. vibration and impact tests.

(94). Mississippi Valley Structural Steel Co. tests on garage roof of 49-ft. span.

(95). Bureau of Buildings, New York City, roof trusses tested at Brooklyn, New York.

(96). Coplay Cement Co. shock tests.

(97). University of Illinois, various tests.

(98). Lloyd's Bureau of Shipping.

(99). Union Carbide tests.

(100). American Bridge Co. roof trusses, 58 ft. 6 in. span, tested at Trenton Plant, December, 1927, for General Electric Co.

The Application of Oxygen and Hydrogen to Industrial Operations

PART V

ELECTRICAL APPARATUS USED IN THE PRODUCTION OF THE GASES

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Manager's Staff Light, Heat, and Power Dept.
Schenectady Works, General Electric Company

THE electrical equipment described in this article is operating in a plant that has the following annual gas producing capacity:

Oxygen	24 million cu. ft.
Nitrogen	35 million cu. ft.
Hydrogen	32 million cu. ft.

The connected electrical load when the plant is operating at normal capacity is approximately 750 kw., of which 500 kw. is direct current and 250 kw. is alternating current. The direct-current load results from the operation of electrolytically decomposing water to produce oxygen and hydrogen. The alternating-current load is the total occasioned by the drives of the following equipment: the compressors for delivering the gases to the shop supply system, the oxygen, nitrogen, and liquid-air producing unit, and the various blowers.

The value of the electrical apparatus represents approximately 50 per cent of the value of all the equipment installed; and the continued successful and economical operation of such a plant is vitally dependent upon the proper selection of the electrical equipment.

The load-factor is practically 100 per cent due to the continuous operation of the plant 24 hours a day and 7 days a week, with a slight reduction in load on Sundays.

The operating power is furnished to the plant switchboard at 550 volts, 60 cycles from an adjacent substation. From the switchboard the circuits are divided for three separate services, as follows: first, that for the operation of the various motor-driven compressors and auxiliary pumps and also of the electrically-heated nitrogen-purifying furnace; second, that for the electrolysis of water; and third, that for lighting and for the operation of the unit heater and ventilator motors.

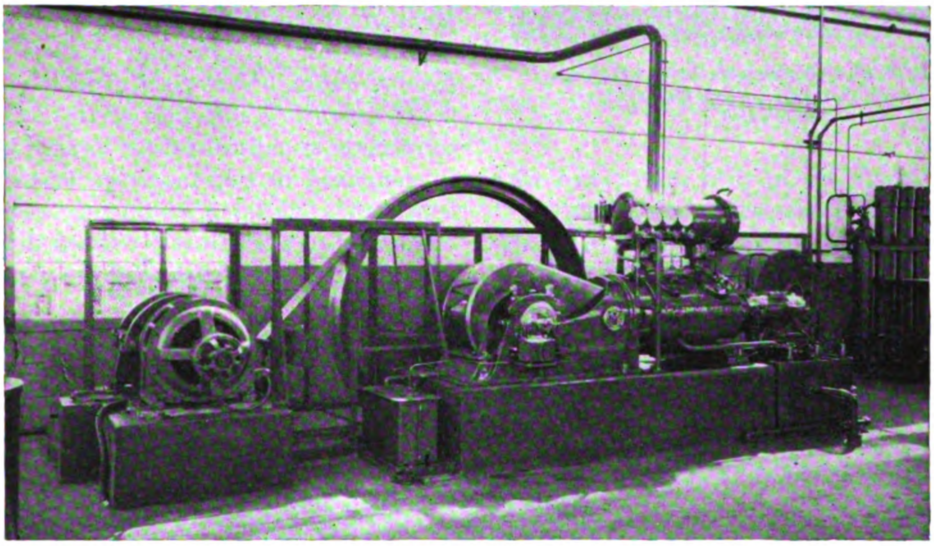


Fig. 27. The 75-hp. Synchronous Motor Drive of the Liquefaction-column Compressor

For the first requirement, a 550-volt three-phase power line extends from the main switchboard the length of the plant, each individual motor being connected to this line through a suitable fused safety switch. The motor driving the liquefaction column compressor is a 75-hp. synchronous motor with direct-connected exciter and is operated with a standard, automatic, reduced-voltage starting panel having push-button control. A synchronous motor was chosen for this duty in order to hold the compressor speed constant in spite of large variations in load. With this exception, squirrel-cage induction motors are used; the seven shop-line delivery compressors being equipped with motors that are capable of exerting high starting torque, a necessary requirement for automatic pressure control. As the largest of these motors is only 30 hp., the across-the-line starting system of control is employed, in connection with either a thermal-limit fused cutout and line switch or a contactor equipment with thermal relays. The latter type of control equipment is necessary in the case of the shop-line delivery compressor motors as they are started and stopped by pressure-operated control switches which are in turn interlocked with limit switches on the various gas-storage tanks.

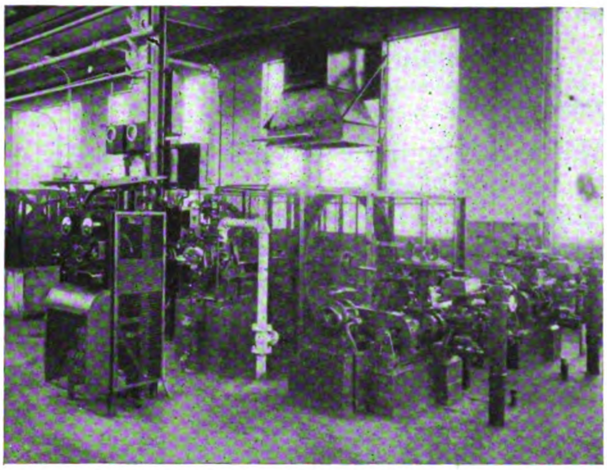


Fig. 28. One of the Shop-line Compressors and Two Hydrogen-nitrogen Mixers with Their Squirrel-cage Motor Drives and a Control Panel

A nitrogen purification furnace of 60 kv-a. is also operated from this circuit through a 550/110-volt, three-phase transformer and automatic temperature-control panel.

The electrolysis of water is accomplished in two banks of electrolytic cells of the "tank" type, each bank being rated 2000 amp., and consisting of 50 cells in series.

The voltage required to maintain the normal flow of current through this type of cell varies with the temperature and general condition of the cells—from 120 volts per bank of 50 cells at 25 deg. C. to 100 volts per bank when the temperature has risen to approximately 60 deg. C. Also, a 10 per cent variation in voltage from the normal value required at any given temperature will vary the current approximately 50 per cent. After being operated, these cells have a residual e.m.f. of approximately 75 volts per bank. Since any reversal of current through the cells would cause oxygen to be released in the hydrogen system, and vice versa, it is necessary that the voltage applied to each bank should always be of the correct polarity and preferably higher than the residual. At normal rating, each bank of cells will produce 31.6 cu. ft. of hydrogen and 15.8 cu. ft. of oxygen per clock hour, and the output of gas per kilowatt-hour of alternating-current power delivered to the conversion equipment is 7 cu. ft. for hydrogen and 3.5 cu. ft. for oxygen.

The cells are isolated from the remainder of the plant in a room that is provided with ample ventilating facilities. All switching is done in the adjoining dynamo room, a wire-glass window giving the switchboard operator a full view of the cells. The a-c.–d-c. conversion apparatus is also located in this room and is comprised of two motor-generator sets. Each set consists of a suitable 550-volt synchronous motor direct connected to a separately excited direct-current generator rated 180/300 kw., 900 r.p.m., 90/150 volts. A direct-connected exciter on each set furnishes excitation for both the motor and the generator.

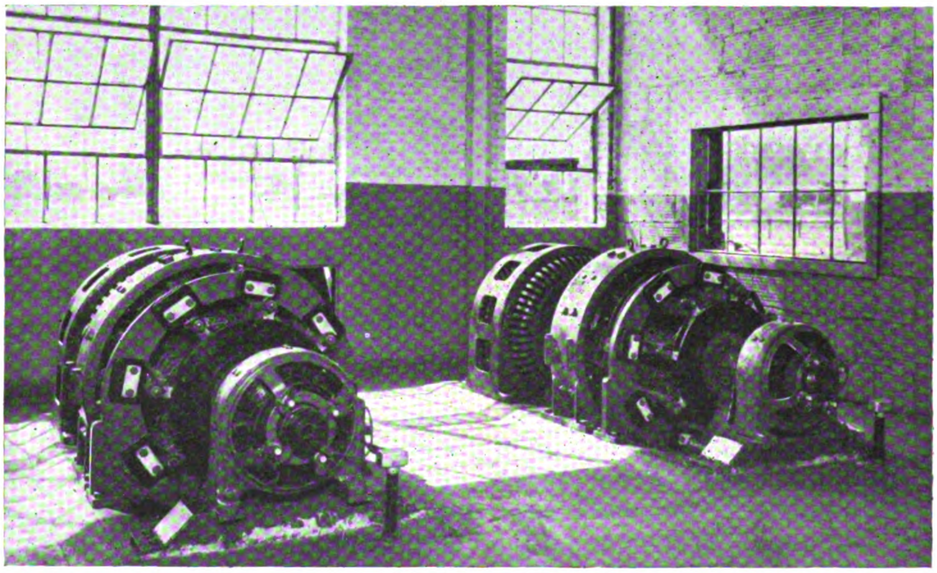


Fig. 29. The Two 300-kw. Motor-generator Sets, with Direct-connected Exciters, for Feeding the Electrolytic Cells (Another view is shown in the frontispiece)

Motor-generator sets were chosen instead of synchronous converters principally because of the voltage range desired. At the required rating, the initial cost of the two types of apparatus would have been about equal and it was regarded that the operating advantages of a motor-generator set more than offset the slightly higher efficiency of a synchronous converter.

The switchboard contains a panel for each generator and for each bank of electrolytic cells, generator No. 1 and cell bank No. 1 normally operating as a unit and separated from generator No. 2 and cell bank No. 2. In case of emergency, a bus-tie panel allows either generator to feed either bank of cells or both at reduced load.

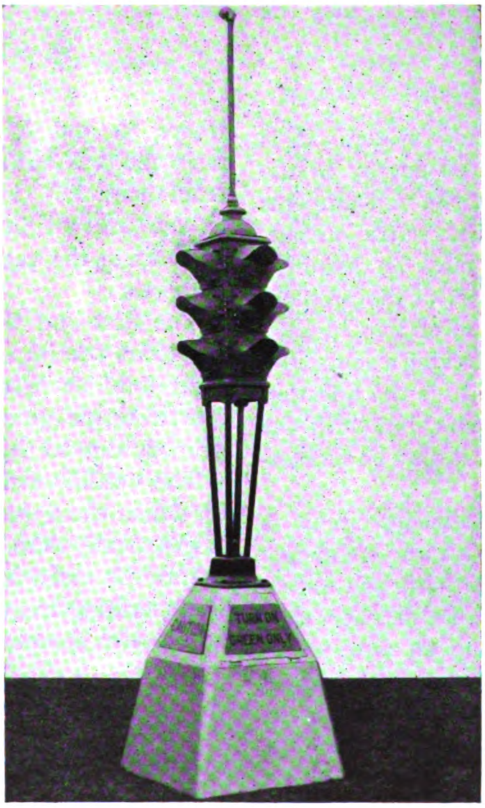
Each cell-bank panel consists of two single-pole overload circuit breakers equipped with undervoltage release coil that is designed to trip if the voltage falls below 85 volts and that is connected through a relay the contacts of which are closed only when potential of correct polarity is on the bus. Also the undervoltage trip of the right-hand breaker is so connected through an auxiliary switch that it cannot be reset unless the left-hand breaker is first closed. This makes it virtually impossible to close these breakers, or for them to remain closed, unless the bus voltage is higher than the residual voltage of the cell bank and is of the correct polarity. Moisture-proof push buttons located in the cell room permit the breakers on either cell-bank panel to be tripped from those points. The direct-current switches, breakers, conductors, etc. have a capacity for continuous operation at approximately 50 per cent overload.

All building lamps are mounted in vapor-proof globes and are fed from an auxiliary lighting transformer which in turn is supplied directly from the incoming power lines. An auxiliary 6-volt lighting system supplies five 6-volt, 50-c.p. lamps placed at important points throughout the building. This circuit is fed from a 6-volt storage battery that is kept charged from either of the cell-bank circuits, In case the normal lighting circuit fails at any time. the auxiliary lighting circuit lamps are automatically switched on. A special 550-volt three-phase circuit is also run directly from the incoming lines to feed five unit-heater blowers.

The undervoltage trip circuits of the generator, cell-bank, motor, and pump-line panels are all interlocked so that by pushing an emergency trip button, one of which is located by each principal exit, all power in the building except the lighting and unit-heater motors will be shut off.

(*To be continued*)

New Pedestal-type Traffic Signal



A New Pedestal-type Traffic Signal

A new pedestal-type traffic signal, using lights in groups of three each on all four sides, has been designed to stand in the center of street intersections, or for mounting on curbs. This signal differs principally from previous types in the design of the base and the length of the overhead fitting. Older types were mounted on a cast-iron base which, in turn, was mounted in concrete; consequently, in erecting it a form had to be made into which the concrete could be poured, and the signal base then fixed to the concrete. The new type incorporates a form for the concrete as part of its base, not only saving the time and labor of making a form but also making the signal and base an integral unit and therefore less liable to breakage.

An overhead fitting, used when the control and power circuits enter the signal from above, is a pipe through which these circuits are fed to the signal. Previously this was very short; but, with the new type, a four-foot fitting is available in addition to the standard 6-inch type. Use of this fitting, however, is optional, as the control and power circuits may enter the signal from underground in accordance with the usual practice.

Another improvement in design is in the four pipes which support the signal on its base. These have been shortened, bringing the signal indications more nearly into line of vision.

The new signal may be obtained also in an adjustable form for use at street intersections where the angle of intersection is other than 90 degrees. In the adjustable form, the signal faces are so arranged that they can be moved to any position, either greater or less than a right angle.

Vacuum Tubes as Oscillation Generators

PART V

VACUUM TUBES AS DRIVERS OF COUPLED OSCILLATING CIRCUITS—PRIMARY GRID COUPLED CIRCUITS

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Research Laboratory, General Electric Company

THE circuit in which the output of a vacuum tube oscillator is to be used seldom has the proper constants to permit it to be directly connected to the tube as the oscillating circuit. For this reason two circuits are often used with a coupling between them. This arrangement is flexible enough to enable the circuit fed by the tube to present the required values of apparent resistance and reactance. The necessary voltages and currents will then appear next to the tube; and the circuit in which the power is to be used will also have the correct current and voltage demanded by the output.

In introducing the flexibility of the coupled circuit arrangement, it is apparent that many design problems will be encountered. Moreover, it will be seen that these problems will be of a dual character because, first, it is necessary to show how a desired scheme of voltages and currents can be obtained in theory; and, second, it is necessary to show that in practice, the theoretical performance will be obtained, and not some other system of oscillations which may be possible with the same circuit.

Properties of Coupled Oscillating Circuits

In designing a coupled oscillating system, it is desirable to have considerable stored energy in the circuit next to the vacuum tube. If the volt-amperes in this circuit exceed 4π times the watts output, sinusoidal voltages and currents may be assumed throughout the system. The first part of the problem is then well in hand when the action of two coupled circuits driven by a source of power having a sinusoidal voltage is understood. At the same time the question of the stability of oscillations appears, but with the factors influencing it exposed so that it may be successfully attacked.

The general type of coupled oscillating circuit is shown in Fig. 35. Instead of having a radio-frequency transformer consisting of two coils with self-inductances L_1 and L_2 and mutual-inductance M, there may be one coil with several taps giving an auto-transformer arrangement or one coil common to both circuits and other coils arranged so that each affects one circuit only. In any case, the only difference is in the insulation between the circuits and between either circuit and ground. As far as the oscillations are concerned, one circuit may be just like another, for a given current in either circuit may be made to induce the same voltages in its own circuit and in that to which it is coupled regardless of the type of coupling used. In fact, the coupling arrangement may consist of a system of condensers without materially changing the problem. The scheme in Fig. 35 is, however, quite common in practice, and other circuits are conveniently described in the nomenclature belonging to it, so it will be made the subject of present consideration.

The analysis of a circuit of this type has already been made.(5)

Referring to Fig. 35, the "apparent complex impedance of the primary circuit" is given by:

$$Z_1' = R_1 + \frac{M^2\omega^2}{Z_2^2} R_2 + j\left(X_1 - \frac{M^2\omega^2}{Z_2^2} X_2\right) \qquad (1)$$

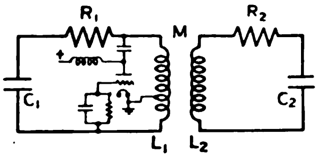

Fig. 35. The General Type of Coupled Oscillating Circuit

where R_1 and R_2 are the primary and secondary resistances, X_1 and X_2 are the primary and secondary reactances, Z_2 is the secondary impedance, M is the mutual inductance, and ω is the angular velocity of the vectors representing the currents and voltages.

The first and third terms on the right-hand side represent the impedance of the primary circuit in a form familiar in electrical theory. The second and fourth terms give the apparent impedance of the primary due to the presence of the secondary.

The oscillator will tend to operate at frequencies for which the apparent total reactance of the primary is practically zero, the slight deviations being of the nature discussed in Part IV. Let it be assumed that the operating frequencies are those for which Z_1' has no reactive component, or that

$$X_1 - \frac{M^2\omega^2}{Z_2^2} X_2 = 0 \qquad (2)$$

If it be attempted to solve this equation by substituting in the values for X_1, X_2, Z_2, and so on, a mass of algebraic expressions results which is practically impossible of solution and interpretation. A graphical solution of the problem, however, not only gives a clear picture of the nature of the phenomena involved, but also indicates the approximations to be made in order to get simple and accurate algebraic solutions of the problem under special conditions.

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(5) Pierce: "Electric Oscillations and Electric Waves," Chapter XI, pp. 156 to 159.

In Fig. 36 is shown the method employed. The reactance of the primary inductance, ωL_1, is proportional to the frequency and hence is represented by a straight line. The reactance of the condenser, $\frac{-1}{\omega C_1}$, is indicated by a hyperbolic curve and their sum gives a curve representing the true reactance of the primary. The point where it crosses the axis gives the conditions for zero reactance and represents the frequency at which the circuit would oscillate by itself. The apparent reactance of the primary due to the secondary is given by the expression $-\frac{M^2\omega^2}{Z_2^2}X_2$, and this is also plotted in Fig. 36. The sign of this term has been reversed in plotting it, hence, the intersections of this curve with that giving the reactance of the primary itself will show the conditions for which the apparent total reactance of the primary is zero. The curves may intersect only once, or three times, as shown. In the latter case, there are two frequencies at which stable oscillations can take place and one frequency for which the oscillations are unstable and require special means for their maintenance. The unstable frequency is the intermediate one, for any variation of the frequency results in a change occasioned by the secondary in the apparent reactance of the primary which is opposite in sign and greater in magnitude than the change occurring in the primary itself. The latter change is, of course, in the direction of stability; hence the total change is such that a variation in the frequency results in a change in reactance causing further change in the same direction. At the other two frequencies, both primary and secondary react to frequency variations in such a way that both produce changes in the apparent primary reactance opposing the variations.

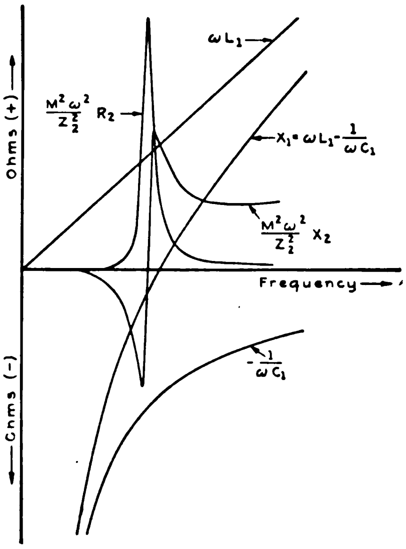

Fig. 36. A Graphical Representation of the Equation Applying to Coupled Circuits

The apparent resistance of the primary due to the secondary $\left(\frac{M^2\omega^2}{Z_2^2}R_2\right)$ peaks sharply at the resonant frequency of the secondary and is relatively very small at other frequencies. Hence, each of the three frequencies giving zero apparent primary reactance has a corresponding resistance different from the others. If no means are used to cause the oscillations to occur at any particular frequency, they will usually pick the stable frequency corresponding to the lowest resistance which, of course, represents the smallest secondary current and power output.

The curves shown in Fig. 36, which were calculated from an assumed set of values picked to show the general form of the curves, are far from representative of practice; in fact, they show an inoperative set of conditions because the primary inductive and capacity reactances are not great enough for proper flywheel effect. In practice, the interesting part of the curves occurs in a very narrow range of frequency as shown in Fig. 37. The primary reactance may be represented by a straight line over the range considered, and the reactance of either the primary inductance or capacity is far too large to be shown.

Single-frequency Conditions

In the manufacture of radio-frequency apparatus, the primary oscillating circuit is usually considered a part of the set to be supplied by the manufacturer while the secondary circuit is the user's antenna or other load. It is desirable that the connections between the two be limited in number and as simple as possible, for trouble at this point is certain to lead to misunderstandings. As a result, manufacturers are somewhat limited in the schemes which may be employed to insure operation at a certain frequency when other frequencies are also possible. The conditions, then, which result in there being only one frequency giving zero apparent primary reactance are of considerable practical importance. It is possible to fulfill the necessary conditions in two ways. The intersection of the two reactance curves may be made to fall in the "central" section of the curve showing the apparent primary reactance due to the secondary as at *A*, Fig. 38, or it may fall in either of the "end" sections as at *B* or *C*. In the latter case, the apparent primary resistance will be too low for any practical use because of the small secondary current. The useful case, then, is that shown as *A*. In order to have an intersection at this point without the other two, it is necessary that the slope of the curve giving the apparent primary reactance due to the secondary be less than the slope of the reactance curve of the primary itself. Taking the derivatives of the equations of both curves, there results[6]

$$\frac{d}{d\omega}\left(\frac{M^2\omega^2}{Z_2^2}X_2\right)=2\frac{M^2\omega^2}{Z_2^2}L_2$$

$$\frac{dX_1}{d\omega}=2\,L_1$$

and equating them for the limiting case gives

$$L_1=\frac{M^2\omega^2}{Z_2^2}L_2 \qquad (3)$$

As both circuits are tuned to resonance at the frequency of operation, the volt-amperes in the primary will be $I_1^2\omega L_1$ and in the secondary $I_2^2\omega L_2$. The secondary voltage will be $I_1\omega M$ and the current $\frac{I_1\omega M}{Z_2}$. Substituting this in the expression for the secondary volt-amperes and using also Equation (3) results in

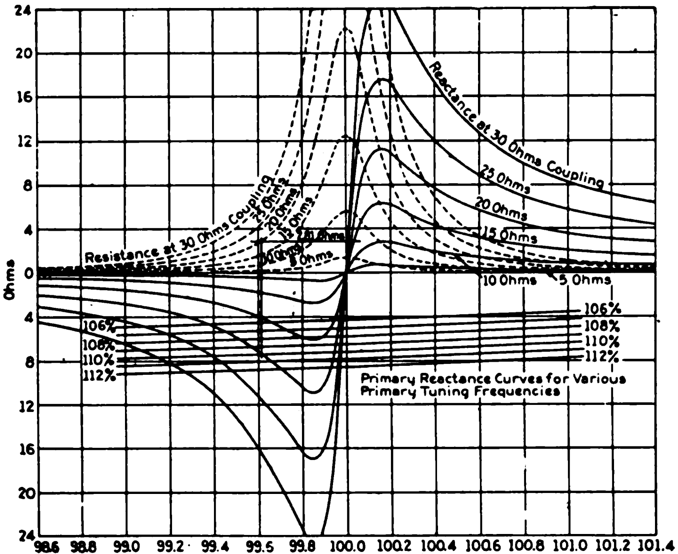

Fig. 37. Characteristics of a Large Multiple-tuned Antenna

Ohms coupling are at 100 per cent frequency (18,300 Cycles). Coupling is to one of six tuning coils.

Antenna capacity, 0.053 mfd.; resistance, 0.5 ohm.; reactance, 164.2 ohms at 100 per cent frequency.

Reactance of primary condenser, 35.2 ohms at 100 per cent frequency.

$$I_2^2\omega L_2=\frac{I_1^2\omega^2M^2}{Z_2^2}\omega L_2=I_1^2\omega L_1 \qquad (4)$$

Hence, satisfactory stable operation with only one possible frequency can be obtained if the volt-amperes in the primary circuit are equal to or exceed those in the secondary. All that is necessary is to tune both circuits separately to the desired frequency and provide the proper amount of coupling between them.

Separation of Resonant Frequencies

The curves in Fig. 37, calculated from an actual set of conditions met in practice, illustrate in what a small frequency range all three frequencies giving a zero value of apparent primary reactance are apt to lie. This makes possible a set of assumptions by means of which many useful calculations can be made.

Let it be assumed that both primary and secondary circuits are tuned separately to correspond to an angular velocity ω' and that the three frequencies are expressed in terms of variations from this of $\Delta\omega$. The frequency changes so little that a coupling of constant ohmic reactance will be assumed, that is, $M\omega$ will be taken as equal to T, a constant. Each inductance will change its reactance by an amount equal to $L\,(\Delta\omega)$ and for small values of $\Delta\omega$ the capacities will change their reactances by an equal amount in the opposite direction. Hence the reactances of primary and secondary will be given by $2\,L_1\,(\Delta\omega)$ and $2\,L_2\,(\Delta\omega)$ respectively. Substituting these values in Equation (2), it is seen that for zero apparent primary reactance

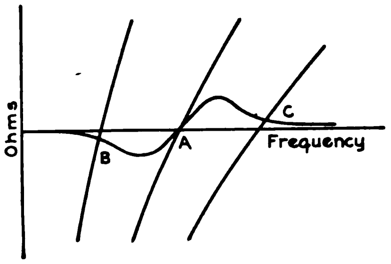

Fig. 38. A Graph Showing Conditions which Give Only One Possible Frequency of Oscillation

$$2\,L_1\,(\Delta\omega)-\frac{T^2\cdot 2\,L_2\,(\Delta\omega)}{4\,L_2^2\,(\Delta\omega)^2+R_2^2}=0$$

Solving for $\Delta\omega$ results in

$$\Delta\omega=0 \text{ and } \Delta\omega=\pm\tfrac{1}{2}\sqrt{\frac{T^2}{L_1L_2}-\frac{R_2^2}{L_2^2}} \qquad (5)$$

Two of these roots may be imaginary, in which case there will be only one wavelength. The necessary condition is

$$T< R_2\sqrt{\frac{L_1}{L_2}} \qquad (6)$$

Inspection shows Equations (6) and (3) to be equivalent, for under the assumed conditions *i.e.*, both circuits having zero reactance, Z_2 is equal to R_2 and $M\omega$ is equal to T.

Power Transferred at the Stable Frequencies

It is interesting to see the power transferred at the "side," or stable, frequencies. The secondary impedance will be

$$Z_2=\sqrt{4\,L_2^2\,(\Delta\omega)^2+R_2^2} \qquad (7)$$

which by substituting the value of $\Delta\omega$, as given by Equation (5), is reduced to

$$Z_2=T\sqrt{\frac{L_2}{L_1}} \qquad (8)$$

---

(6) For this case $X_2=0$ and, therefore, $\frac{dZ_2}{d\omega}=0$ leaving Z_2 to be treated as a constant. Also $\frac{d\,(\omega L_2)}{d\omega}=-\frac{d\left(\frac{1}{\omega C_2}\right)}{d\omega}$ giving $\frac{dX_2}{d\omega}=2\,L_2$.

The secondary impedance is, therefore, proportional to the coupling throughout the range in which the operation will not be at the resonant frequency of the two circuits and, if the primary current is assumed constant, the secondary current and output will remain constant and will not increase with an increase in the coupling. Hence, the available output is limited to that obtained with the largest coupling giving stable operation at the resonant frequency, as given by Equation (6). Throughout the range wherein only the resonant frequency of the separate circuits is obtained, the output is, of course, proportional to the square of the coupling.

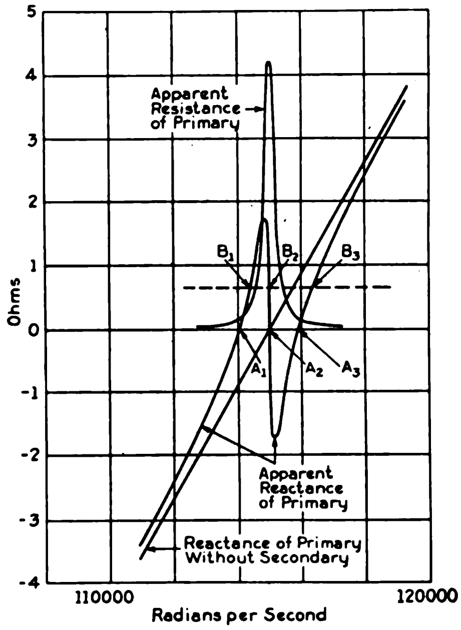

Fig. 39. **A Graph of Total Apparent Primary Reactance and Resistance**

In this particular case, however, there are limits imposed by the fact that both circuits are tuned to the same frequency. If the circuits are tuned to different frequencies, it is possible to transfer much more power. One "side" frequency is then nearer the secondary resonant frequency than before and the other is farther away. If it is possible to operate at the frequency nearer the secondary resonant frequency, the output will be increased. Figs. 36 and 37 both illustrate this point.

Minimum Frequency Difference with Stability

Instead of plotting the curves of primary reactance and apparent primary reactance due to the secondary separately, and with the sign of the latter reversed as in Figs. 36 and 37, it is possible to add the two and plot the resultant as in Fig. 39. The points A_1, A_2, and A_3, where this curve crosses the axis, then represent the three frequencies of oscillation. A change in the primary reactance may be taken care of by shifting the axis, when a series of points such as B_1, B_2, and B_3 will represent the frequencies giving zero reactance. The section of the curve lying between the maximum and minimum points is the unstable range. These points will not correspond with those obtained from the curve due to the effect of the secondary alone, but the error involved in assuming them the same may be neglected when the value of the secondary volt-amperes is large compared with that in the primary. In any case, the error is of a conservative nature if the unstable range is calculated from the effect of the secondary alone. With such an assumption, it is not difficult to set the approximate limits of the range of instability.

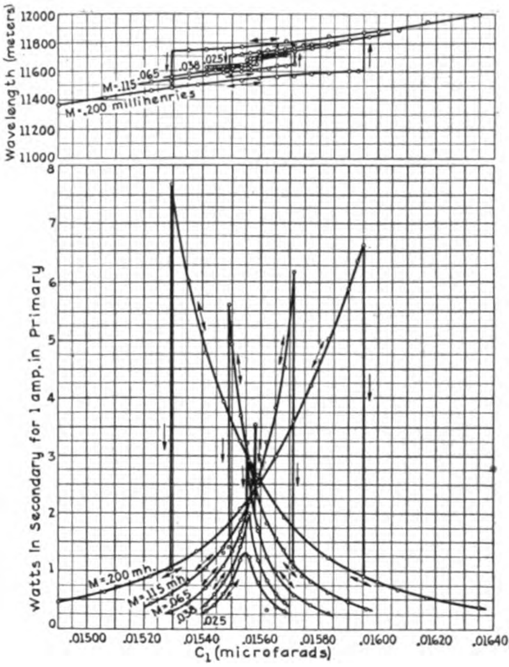

Fig. 40. **Resonance Curves for a Coupled Oscillating System with Grid of Pliotron Connected to Primary**

Let the coupling in ohms be represented by the transformer constant T as before, neglecting the slight change which occurs, and let the separation between the angular velocities corresponding to the maximum and minimum points and the resonant frequency of the secondary be denoted by ω_0. Also, let the secondary reactance be denoted by $2\ \omega_0 L_2$ using the same approximation as before. Then the apparent primary reactance due to the secondary will be, by Equation (2), $\dfrac{T^2\cdot(2\ L_2\omega_0)}{R_2^2+(2\ L_2\omega_0)^2}$ and the derivative of this with respect to ω_0 will give the desired points when equated to zero, *i.e.*, when

$$\frac{d}{d\omega_0}\left[\frac{T^2\cdot(2\ L_2\omega_0)}{R_2^4+(2\ L_2\omega_0)^2}\right]=0$$

The term $2\ L_2T^2$ is a constant and may be dropped; and after differentiation there results

$$(R_2^2+4\ L_2^2\omega_0^2)^{-1}-8\ L_2^2\omega_0^2\ (R_2^2+4\ L_2^2\omega_0^2)^{-2}=0$$

and this is satisfied when

$$\omega_0=\pm\frac{R_2}{2\ L_2} \tag{9}$$

To allow a safe margin for stability, a slightly larger value of ω_0 should be used, say

$$\omega_0 = \pm \frac{3\,R_2}{4\,L_2} \tag{10}$$

It will be seen from the considerations presented in this connection that high-frequency oscillator circuits must of necessity vary in their form according to the constants of the circuit to which they have to supply power. This is because of the manner in which the reactance in the load circuit can act on the oscillating circuit to change its behavior, an action which can be prevented or compensated for in various ways depending on the individual case. Complete oscillating systems of several forms will be dealt with in the sections immediately following.

PRIMARY GRID COUPLED CIRCUITS

It has just been shown that, if the secondary volt-amperes in a coupled oscillating system are less than those in the primary, the circuits can both be tuned to the same frequency with no possibility of the system oscillating in any but the intended manner. The secondary will cause an apparent resistance within the primary coil to which it is coupled, but this will be the sole effect. Hence, the primary circuit will be designed as though the load resistance were in a certain coil, and this coil will then be coupled with the secondary in such a manner that the induced secondary voltage will be sufficient to send the desired current through the secondary resistance.

If the secondary volt-amperes are of a value greater than is feasible to supply in the primary, it will be necessary to operate at one of the group of three frequencies giving zero as the apparent primary reactance, although this will be the stable frequency at which the tube will not normally operate because the power developed is much greater than on the other stable frequency. The curves shown in Fig. 40, which were obtained from test data, illustrate the difficulty involved.

The circuit in connection with which these curves were obtained is shown in Fig. 41. In the secondary circuit, the inductance consists of two coils in parallel with coupling to only one. This corresponds to the conditions met in large multiple-tuned antennas where coupling to only one out of five or six tuning coils is possible. The constants of these antennas are usually such, however, that the current division among the coils is not seriously disturbed by this irregularity, and the components of the impedance at the coupling point are obtained by dividing the watts and volt-amperes to be supplied by the square of the current in that coil. By proceeding along this line, it is found that the effect on the primary is substantially the same as though only one secondary inductance were used.

The primary circuit shown in Fig. 41 is of the Colpitts type with a small variable capacity in addition to the plate and grid condensers. The function of this is to allow the tuning of the primary circuit to be varied continuously over a range such that its resonant frequency may be made greater or less than the resonant frequency of the secondary by itself. With a coupling between the circuits so loose that, even though they be tuned to the same frequency, the primary volt-amperes will exceed those in the secondary, the characteristics are what might be expected. The secondary current rises to a sharp peak at the condenser setting giving its resonant frequency, and the secondary has such a slight effect on the primary as to pass almost unnoticed.

With a close coupling between the primary and secondary, the curves of circuit performance will be found to have a peculiar characteristic. Starting with a certain value of primary capacity, it is possible to change this and later duplicate the original conditions by returning to the original setting of the condenser provided a certain value is not passed. When this is done, there is an abrupt change in the nature of the oscillations which then follow an entirely different set of curves and refuse to conform to the original conditions upon again adjusting the condenser to the setting at which the transition took place. There is, however, another value of primary capacity at which a similar transition is made back to the original operating conditions. This results in a peculiar, looped set of curves which some investigators have termed "drag loops."

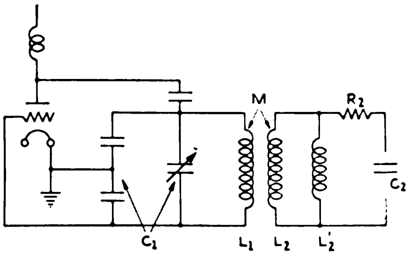

Fig. 41. The Diagram of the Circuit Used in Obtaining Data for the Curves in Fig. 40. The grid leak resistance and its choke are not indicated

The reason for the existence of such a phenomenon is not difficult to understand. Suppose that the primary tuning is so far removed from that of the secondary that oscillations of only one frequency can occur. As the primary frequency is made to approach that of the secondary, a point will be reached at which the primary reactance curve will have three intersections with the curve representing the effect due to the secondary. Two stable frequencies of oscillation are then possible, but the one obtained by the uniform change of the primary capacity will persist until the curve of primary reactance has passed through the values giving three intersections and is about to enter the other range where only one intersection can be obtained. At this point the prevailing oscillations become unstable and are replaced by those of the other stable frequency. Once this frequency is in control, the changes follow a second set of curves until, on the return of the primary frequency to the other limit giving three intersections of the reactance curves, the oscillations again become unstable and a transition occurs to the original set of curves.

If an attempt is made to start the circuit with the variable condenser set in the range for which two frequencies are possible, it is almost certain to pick the one for which the output is the smallest unless the outputs at the two frequencies do not differ widely. Thus, it will be seen that the real problem in primary-grid-coupled circuits is in starting them, and "keying" them when used for radio telegraph sets.

Simultaneous Operation at Two Frequencies

If the grid excitation of a tube is such that a continuous plate current is drawn, it is possible to maintain oscillations at several frequencies simultaneously. Assume that a sinusoidal variation in the grid potential produces a sinusoidal change in plate current. The plate voltage also will vary sinusoidally and the tube's performance may be analyzed in such a way that it can be considered to have a negative resistance to the alternating component of plate current. That is, the oscillating circuit has a sinusoidal voltage which is applied in proper magnitude and phase to both grid and plate, and, if a sinusoidal variation in plate current results, the value obtained by dividing the sinusoidal component of the plate voltage by the sinusoidal component of the plate current may be termed the negative resistance of the tube. For steady operating conditions, the negative resistance of the tube must be equal to the positive resistance of the circuit measured between the points to which it is connected. The negative resistance of a tube may be made fairly constant until oscillations of a certain magnitude are reached, when the assumed conditions will no longer hold and the negative resistance will increase. Thus the amplitude of the oscillations is fixed in practice; for there will be only one value for which tube negative resistance and circuit resistance are equal. Highly efficient circuits draw current for only a small part of the cycle, and hence the tube's negative resistance will be much higher under steady conditions than when the oscillations are building up at the start. This is largely due to the grid bias voltage which requires some time to build up, during which interval the plate current flows for a larger portion of the cycle. In such circuits one predominant set of oscillations will usually so increase the effective negative resistance of a tube for oscillations of any other frequency that they will be unable to get started. Circuits might easily be arranged, however, so that this would not be the case, and then oscillations would be simultaneously obtained at several different frequencies.

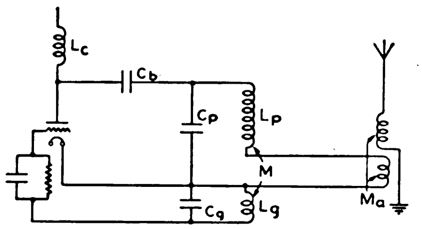

Fig. 42. A Diagram of Connections Used for Eliminating an Undesired Oscillation Frequency by Grid Phasing

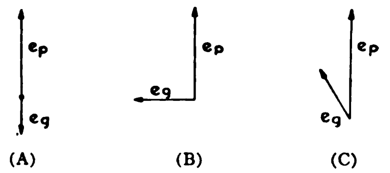

Fig. 43. Grid Excitation Phases Obtained with the Circuit Shown in Fig. 42

If multiply-periodic circuits are to operate at one frequency without oscillations at other possible frequencies, it is important that nothing be permitted to disturb the operation. Any disturbance may increase the chance of the undesirable oscillations to take hold by decreasing the effective negative resistance of the tube for this frequency, and, as undesirable oscillations usually occur with small output, the resistance does not have to decrease very much before the undesired oscillations become the ones favored by the tube. Hence, "keying" of radio sets must be more or less "smooth" if they are multiply-periodic. One scheme which is readily apparent is to change the tuning of the circuits by manipulating the key. This is not altogether desirable, however, because of the interference caused by the radiation of the "idle" wave.

Grid Keying

The simplest keying for a telegraph set is that in which the desired end is obtained by working in the grid circuit, because both power and voltage are relatively low. If control of the grid circuit could be made to change the strength of the oscillations without danger of shifting to undesirable frequencies, this would be very satisfactory. Low-power oscillations of the desired frequency could be maintained during the "off" periods and brought up to full strength during the "on" periods.

It has been suggested that the value of the grid leak resistance might be increased to reduce the amplitude of the oscillations. Without entering into an extended consideration of the oscillating characteristics of a tube under these conditions, it is apparent that oscillations will take place, for the commencement of oscillations takes place when there is no charge on the grid condenser and the grid resistance is, therefore, immaterial. The amplitude of the oscillations will be small, because the plate current is roughly proportional to the grid current, and the grid current is kept small by the high leak resistance. Since the oscillation amplitude is small, the grid bias must be small, as there would otherwise be no grid current to maintain the bias. An approximate determination of the tube loss can be made by assuming no oscillations at all, which is the limiting case. The plate potential is then the direct potential, and the grid is at ground potential. This condition eliminates all low-impedance tubes which are destroyed quickly by full impressed plate voltage with grid at ground potential. There are probably no high-voltage oscillator tubes made which can stand full plate voltage with grounded grid for any considerable time.

A second scheme would be to reduce the oscillation amplitude by external bias. This means is subject to the objection that an external bias circuit is not self-starting and oscillations reduced to a low value might cease altogether, due to some small change in impressed voltage. A better way of obtaining a bias would be to insert a resistance in the filament ground lead and use the voltage drop caused by the plate current.

Elimination of One Frequency by Grid Phasing

If the coupling is such as to give two oscillating frequencies, which are fairly well separated, the undesired frequency may be discriminated against by the adjustments of the circuit. One way of accomplishing this result is as follows: Referring to Fig. 42, the grid circuit is seen to be inductively coupled to the primary or plate circuit and is tuned to resonate at a frequency between the operating points of the main coupled circuit. For the lower frequency, then, the grid excitation is shown in Fig. 43(A), these phases being correct for oscillation. At the resonant frequency of the grid circuit, the vectors are as shown in Fig. 43(B). Grid and plate voltages are in quadrature, and oscillations are not maintained. At the second resonant point of the main circuit, the vectors are as shown in Fig. 43(C). Here the grid excitation actually opposes any oscillations at that frequency.

With such an arrangement, although the main circuit is doubly periodic, oscillations at the undesired frequency are opposed. It is of course necessary to have the grid coupling loose enough so as not to give rise to still further undesirable resonant points.

Enough has been said to show the problems connected with doubly-periodic circuits in which the grid is connected to the primary. Further interesting examples (7) might be given, but this type of circuit is too elaborate to find much use in practice so they will be omitted.

(To be continued)

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(7) "Vacuum Tubes as Power Oscillators," Part II, by D. C. Prince, *Proceedings of Institute of Radio Engineers*, August, 1923.

New Railway Signal Choke Coils

A new choke coil has been designed for location between the lightning arresters and the transformers on railway signal high-voltage power lines so that an incoming surge will first meet the arrester and then the choke coil. The functions of the choke coil are: (a) to hold back the lightning disturbance from the apparatus until the arrester has had time to discharge, and (b) to reduce the wave front of whatever part of the disturbance passes through the coil, thereby preventing serious rise in potential across the end turns of the transformers or other apparatus to be protected.

As the current requirements in railway-signal service are unusually small, it was found possible to utilize double-galvanized iron wire as the conductor in the coil without sacrificing efficiency. This wire is equally as effective as copper wire, it costs less, and yet is durable and weather-resisting. The coil consists of 24 turns of No. 6 wire mounted between treated maple supports for turn-to-turn bracing. The supports are secured to brass terminal blocks to which the ends of the coil are attached.

Since the coil weighs but 4⅜ lb., an insulator support is unnecessary and it is permissible to suspend the coil from the transmission line by means of the lead wire. This wire and the wire connecting the coil to the transformer are fastened to the respective ends of the coil by wrapping the wire around the grooved portion of the terminal block and inserting the end of the wire in the terminal hole where it is secured by set screws.

The new coil is designed for a maximum voltage of 25,000, and a maximum current of 25 amp.

Heaviside's Operational Calculus as Applied to Engineering and Physics

Part XXI: **Effect of Electric Charges Applied at Points of a Transmission Line or Cable—Green's Function.**
Part XXII: **Disconnecting a Cable and Discharging a Cable**

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Fig. 43 shows a point charge Q applied to a transmission wire. The problem is to find the potential difference between wire and ground after the charge has been applied.

Since we are dealing with "distributed" constants the general equations given in Part XV apply.
For the right-hand side:

$$V_1 = K_1 \cosh n\,(l-y) + K_2 \sinh n\,(l-y)$$

$$i_1 = \frac{Y}{n}\left[K_1 \sinh n\,(l-y) + K_2 \cosh n\,(l-y)\right].$$

Since for $y=l$, $i_1=0$ and therefore $K_2=0$

$$\therefore\ V_1 = K_1 \cosh n\,(l-y)$$

and

$$i_1 = \frac{Y}{n} K_1 \sinh n\,(l-y).$$

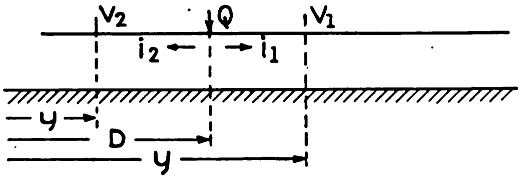

Fig. 43

For the left-hand side:

$$V_2 = K_3 \cosh ny + K_4 \sinh ny$$

$$i_2 = \frac{Y}{n}\left[K_3 \sinh ny + K_4 \cosh ny\right].$$

For $y=0$, $i_2=0$ and therefore $K_4=0$

$$\therefore\ V_2 = K_3 \cosh ny$$

and

$$i_2 = \frac{Y}{n} K_3 \sinh ny.$$

For $Y=D$, $i_1+i_2=i=p\,Q\,1$

pQ represents a current since $Q=\int idt = \frac{i}{p}$ and we have perfect right to put the unit symbol after pQ because the current must be impulsive. Before the charge is applied there is no current in the wire and, since the wire itself is no source of current, as much current flows out of a section as flows in, thus at any time after $t=0$ at point D, $i_1=-i_2$ or $i_1+i_2=0$.

At $t=0$, however, there is an inrush of current due to the application of the charge. This inrush which is $i=i_1+i_2$ is thus of impulsive nature; it is zero before and after $t=0$ and is infinite during an infinitely short time at $t=0$. Yet it is definite because $\int idt$ is Q; that is, $\frac{i}{p}=Q$.

Since $i_1+i_2=pQ\,1$,

$$\frac{Y}{n}\left[K_1 \sinh n\,(l-D) + K_3 \sinh nD\right] = pQ\,1.$$

This is satisfied if

$$K_1 = \frac{\cosh nD}{\sinh nl}\,p\,Q\,\frac{n}{Y}\,1$$

and

$$K_3 = \frac{\cosh n\,(l-D)}{\sinh nl}\,p\,Q\,\frac{n}{Y}\,1$$

$$\therefore\ V_1 = \frac{\cosh nD}{\sinh nl}\cosh n\,(l-y)\,\frac{npQ}{Y}\,1 \qquad (94)$$

$$V_2 = \frac{\cosh n\,(l-D)}{\sinh nl}\cosh ny\,\frac{npQ}{Y}\,1 \qquad (95)$$

In the case of a cable without leakage and inductance, $Y=pC$ and the equations become

$$V_1 = \frac{\cosh nD}{\sinh nl}\cosh n\,(l-y)\,\frac{nQ}{C}\,1 \qquad (96)$$

$$V_2 = \frac{\cosh n\,(l-D)}{\sinh nl}\cosh ny\,\frac{nQ}{C}\,1 \qquad (97)$$

Heaviside gives a large number of similar problems, all of which can readily be solved in a manner similar to that just indicated.

It may be instructive to solve one of these equations. Find for instance V_1, the potential on the right-hand side, in the case of an "ideal" cable.

Equation (96) rewritten with trigonometric relations is:

$$V_1 = \frac{\cos mD \cos m\,(l-y)}{j \sin ml}\,\frac{jm\,Q}{C}\,1, \text{ since } -m^2 = pCR$$

$$= \frac{m \cos mD \cos m\,(l-y)}{C \sin ml}\,Q\,1$$

$Z_{(p)}=0$ gives roots $ml=s\pi$ or $m=\frac{s\pi}{l}$

also $-m^2=pCR$

Thus $\frac{pdZ}{dp} = -\frac{s^2\pi^2}{CRl^2}$ and $l^2 \cos ml \frac{CR}{2s\pi} = (-1)^s \frac{s\pi}{2}$

$$Y_{(p)} = (-1)^s \cos mD \cos my \frac{s\pi}{l}, \quad \frac{Y_{(0)}}{Z_{(0)}} = \frac{Q}{Cl}$$

$$\therefore V_1 = \frac{Q}{Cl}\left[1 + 2\sum_{s=1} \cos my \quad \cos mD \; \epsilon^{-\frac{s^2\pi^2}{CRl^2}t}\right]$$

The equation for V_2 becomes identical with that of V_1 as can be seen from Equations (96) and (97). They are alike if we interchange y and D. The values of V_1 and V_2 differ however numerically since in the first case y is always larger than D, and in the second it is smaller.

Thus in this case

$$V = \frac{Q}{Cl}\left[1 + 2\sum_{s=1} \cos\frac{s\pi y}{l} \cos\frac{s\pi D}{l} \epsilon^{-\frac{s^2\pi^2}{CRl^2}t}\right] \quad (98)$$

This equation holds for both sides.

For certain reasons it is well to study two more cases where a charge is suddenly applied to a point in a transmission-line wire.

For this purpose we shall consider a wire which is grounded at both ends as in Fig. 44.
Here, the procedure is similar to that which has been described, and the following operational equations are readily obtained

$$V_1 = \frac{\sinh nD \;\; \sinh n\,(l-y)}{\sinh nl} \; \frac{npQ}{Y} \mathbf{1} \quad (99)$$

$$V_2 = \frac{\sinh n\,(l-D) \; \sinh ny}{\sinh nl} \; \frac{npQ}{Y} \mathbf{1} \quad (100)$$

When these equations are solved for the conditions of an "ideal" cable by the Expansion Theorem we find:

$$V_1 = \frac{2Q}{Cl} \sum_{s=1}^{s=\infty} \sin\frac{s\pi y}{l} \sin\frac{s\pi D}{l} \epsilon^{-\frac{s\pi}{CRl^2}t} \quad (101)$$

and V_2 identical with V_1 as far as the general equation is concerned. Obviously, however, the values of y in the first case are larger than D, and in the second are smaller.

In this case, then,

$$V = \frac{2Q}{Cl} \sum_{s=1}^{s=\infty} \sin\frac{s\pi y}{l} \sin\frac{s\pi D}{l} \epsilon^{-\frac{s^2\pi^2}{CRl^2}t} \quad (102)$$

This relation holds for the entire line.

A third similar case is shown in Fig. 45. The left-hand side is here grounded and the right-hand side open circuited.

Here the operational equations become

$$V_1 = \frac{\sinh nD \; \cosh n\,(l-y)}{\cosh nl} \; \frac{pnQ}{Y} \mathbf{1}$$

$$V_2 = \frac{\sinh ny \; \cosh n\,(l-D)}{\cosh nl} \; \frac{pnQ}{Y} \mathbf{1}$$

The two expressions are identical if y and D are interchanged.

The solution will again be the same for both sides, so that the potential at any point is

$$V = \frac{2Q}{Cl} \sum_{s=1}^{s=\infty} \sin\left(\frac{2s-1}{2}\frac{\pi}{l}D\right) \sin\left(\frac{2s-1}{2}\frac{\pi}{l}y\right) \epsilon^{-\frac{(2s-1)^2}{4}\frac{\pi^2}{CRl^2}t} \quad (103)$$

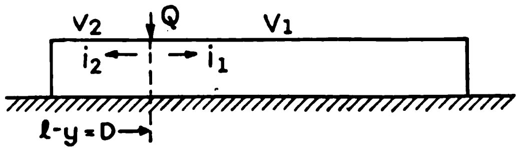

Fig. 44

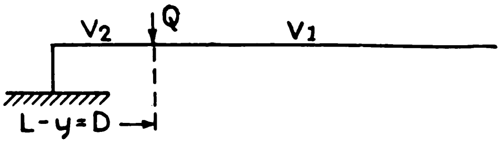

Fig. 45

Fig. 46

In all three cases there is no potential on the line at $t=0$ except at the point $y=D$ where it is infinity because a finite charge is given to a point of infinitely small capacity.

A graph indicating the *physical significance* of these equations when they are used to express arbitrary functions is given in Fig. 46. They are very useful when for instance we are dealing with space distributions of charges, and they are similar to Duhamel's integral when time distributions are involved. It is to be noted therefore that these equations represent an impulsive function similar to $p\,\mathbf{1}$ with the difference that the variable is y instead of t.

When we integrate $p\,\mathbf{1}$ we get back to the unit function, that is, $\frac{1}{p}\,p\,\mathbf{1} = \mathbf{1}$. So here the space total is $\mathbf{1}\,\frac{Q}{C}$

The three Equations (98), (102), and (103) when $\frac{Q}{C}$ is taken as unity are special types of Green's functions which are exceedingly useful to express arbitrary functions in trigonometric series.(5) We shall denote them by G.

Thus

$$G=\frac{1}{l}\left[1+2\sum_{s=1}^{s=\infty}\cos\frac{s\pi}{l}y\,\cos\frac{s\pi}{l}D\right]$$

$$=\frac{2}{l}\sum_{s=1}^{s=\infty}\sin\frac{s\pi y}{l}\,\sin\frac{s\pi D}{l}$$

$$=\frac{2}{l}\sum_{s=1}^{s=\infty}\sin\frac{2s-1}{2}\,\frac{\pi}{l}y\,\sin\frac{2s-1}{2}\,\frac{\pi}{l}D$$

Every Fourier's series involves one form of these. An infinite number can be worked out by assuming different terminal arrangements.

Each Green's function is an impulsive function.

If we multiply a continuous function of D say $f(D)$ by a Green's function, which is an impulsive function that exists only at $D=y$, the product is zero except at that point where it is infinite. But, if we take the space total of the product $G\,f(D)$, the result is $f(y)$ because G exists only at $D=y$ and its total is 1.

Thus $\int G\,f(D)\,dD=f(y)$ if the limits include the point $D=y$. If not, the result is zero.

PART XXII: DISCONNECTING A CABLE AND DISCHARGING A CABLE

Disconnecting a Cable

Before the switch is opened, the current is, in this example, Fig. 47, assumed to have reached its steady state, that is, $I=\dfrac{E}{Rl}$ in all parts of the cable. After the switch is opened, the current at the switch is zero.

If, therefore, we determine the expression for some peculiar e.m.f. impressed at the generator such that at the generator the current is constant, equaling $-I$ at all times after closing the switch, then the sum of the original current and the current caused by this peculiar e.m.f. will obviously be the current in any part of the cable.

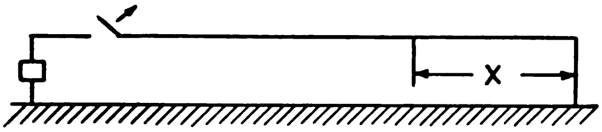

Fig. 47

It has been shown that with the receiving end of a cable grounded

$$e=e_g\frac{\sinh nx}{\sinh nl}$$

and

$$i=\frac{Y}{n}e_g\frac{\cosh nx}{\sinh nl}$$

$$\therefore\ i_g=e_g\frac{Y\cosh nl}{n\sinh nl}$$

The peculiar e.m.f. which makes $i_g=-I\mathbf{1}$ is then

$$e_g=-\frac{n\sinh nl}{Y\cosh nl}I\mathbf{1}$$

and thus the current in any part of the cable due to this e.m.f. is:

$$i_1=-\frac{Y\cosh nx}{n\sinh nl}\,\frac{n}{Y}\,\frac{\sinh nl}{\cosh nl}I\mathbf{1}=-\frac{\cosh nx}{\cosh nl}I\mathbf{1}\qquad(104)$$

The total current is then

$$i=i_1+I=I\left[1-\frac{\cosh nx}{\cosh nl}\mathbf{1}\right]$$

$$=\frac{E}{Rl}\left[1-\frac{\cosh nx}{\cosh nl}\mathbf{1}\right]$$

At the end of the line $x=0$

$$\therefore\ i_r=\frac{E}{Rl}\left[1-\frac{1}{\cosh nl}\mathbf{1}\right]$$

$$i_1=-\frac{I}{\cosh nl}\mathbf{1}=-\frac{Y_{(p)}\,I}{Z_{(p)}}\mathbf{1}$$

Problem

Find the current at the grounded end of an "ideal" cable.

Refer to Equation (104) for $x=0$

$$i_1=-\frac{I}{\cosh nl}\mathbf{1}$$

$$Y_{(p)}=1$$

$$Z_{(p)}=\cosh nl=\cos ml$$

where $$-m^2=n^2=pCR$$

$$p\frac{dZ}{dp}=p\frac{dZ}{dm}\frac{dm}{dp}=-\frac{ml}{2}\sin ml$$

$$\frac{Y_{(o)}}{Z_{(o)}}=\frac{1}{\cos 0}=1$$

$$\therefore\ i_1=-\left[I+\sum\frac{2I\,\epsilon^{pt}}{-ml\sin ml}\right]$$

$$Z_{(p)}=0\text{ gives }\cos ml=0\quad\therefore\ ml=\frac{\pi}{2},\ \frac{3\pi}{2},\ \cdots\frac{k\pi}{2}$$

or

$$ml=\frac{2s+1}{2}\pi$$

$$\therefore\ p=-\left(\frac{2s+1}{2}\right)^2\frac{\pi^2}{CRl^2}$$

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(5) Heaviside gives a most interesting discussion on this subject in his treatise on the Electro-magnetic Theory, Vol. II, p. 99.

An analogue between pure mathematics and the operational mathematics of Heaviside by means of the theory of H functions was prepared by Dr. J. J. Smith and appeared in the *Journal of the Franklin Institute*, Nov., 1925.

$$\therefore i_1 = -\left[\dot{I} - \sum_{s=o}^{s=\infty} \frac{2I\,\epsilon^{pt}}{\frac{2s+1}{2}\pi \sin \frac{2s+1}{2}\pi}\right]$$

$$\therefore i = I + i_1 = \frac{4\mathrm{E}}{Rl\pi} \sum_{s=o}^{s=\infty} \frac{\cos s\pi}{2s+1}\,\epsilon^{-\alpha t} \qquad (105)$$

where $$\alpha = \left(\frac{2s+1}{2}\right)^2 \frac{\pi^2}{CRl^2}$$

At $t=0$ the current is $\frac{E}{Rl}$. Equation (105) gives

$$i = \frac{4E}{Rl\pi}\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots\right]$$

Thus $\frac{4}{\pi}\left[1 - \frac{1}{3} + \frac{1}{5} \cdots\right]$ must be unity, a relation which is well known.

Discharging a Charged Cable

Referring now to Fig. 48, switch a is closed from $t=0$ to $t=t_1$, at which time switch b is closed. Find the current in the cable. Short circuiting the battery means that the impressed e.m.f. is zero at time t_1. This again is equivalent to leaving the original battery on, but applying a battery of potential $-E\mathbf{1}$ at time $t=t_1$. The total current is then the sum of the currents due to these two e.m.f.'s.

Current due to $E\mathbf{1} = i_1 = \frac{Y}{n} E \frac{\sinh nx}{\cosh nl}\mathbf{1} = E\frac{Y_{(p)}}{Z_{(p)}}\mathbf{1}$, as is shown in Equation (56).

This is readily solved by the Expansion Theorem, which gives $i_1 = \frac{2E}{Rl}\sum_{0}^{\infty}(-1)^s \sin\left(\frac{2s+1}{2}\pi\frac{x}{l}\right)\epsilon^{\beta t}$

When e.m.f. $-E\mathbf{1}$ is applied at $t=t_1$, the current is obviously

$$i_2 = -\frac{2E}{Rl}\sum_{0}^{\infty}(-1)^s \sin\left(\frac{2s+1}{2}\pi\frac{x}{l}\right)\epsilon^{\beta(t-t_1)}$$

$$\therefore i = \frac{2E}{Rl}\left[\sum_{0}^{\infty}(-1)^s \sin\left(\frac{2s+1}{2}\pi\frac{x}{l}\right)\epsilon^{\beta t} - \sum_{0}^{\infty}(-1)^s \sin\left(\frac{2s+1}{2}\pi\frac{x}{l}\right)\epsilon^{\beta(t-t_1)}\right]$$

where

$$\beta = \left(\frac{2s+1}{2}\right)^2 \frac{\pi^2}{RCl^2}.$$

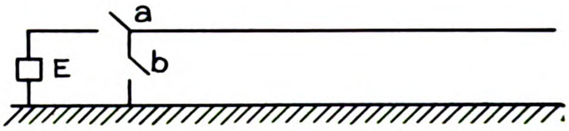

Fig. 48

The corresponding solution, if the cable were connected to earth at the receiving end, is easily found to be

$$i = \frac{2E}{RL}\left[\sum_{1}^{\infty}(-1)^s \cos\left(s\pi\frac{x}{l}\right)\epsilon^{\alpha t} - \sum_{1}^{\infty}(-1)^s\, c\cos\left(s\pi\frac{x}{l}\right)\epsilon^{\alpha(t-t_1)}\right]$$

where

$$\alpha = \frac{s^2\pi^2}{CRl^2}.$$

A similar problem has been worked out at the end of Part XV.

(To be continued)

Lighting for Alighting

A Night View of the Madison (Wis.) Airport. Illumination is produced by seven 120-deg. airport floodlights in connection with seventeen boundary lights and one 24-in. rotating airport beacon

LIBRARY SECTION

Condensed references to some of the more important recent articles in the technical press, and to new books of interest to the industry, as selected by the General Electric Main Library.

Arc Welding

Examples of Arc Welded Steel Construction. Gilbert D. Fish.
Am. Weld. Soc. Jour., Mar., 1928; v. 7, pp. 10-19.

Welds of Extreme Ductility Produced by "Electronic Tornado Process." A. F. Davis.
Am. Weld. Soc. Jour., Mar., 1928; v. 7, pp. 49-52.

Cars, Gas-Electric

What the Gas-Electric Car Means to the Railroads. W. R. Stinemetz.
Rwy. Age, Mar. 31, 1928; v. 84, pp. 753-755.
("An outline of limitations and possibilities, supported by performance records.")

Dielectric Phenomena

Effect of Curved Boundaries on the Distribution of Electrical Stress Round Conductors. J. D. Cockcroft.
I. E. E. Jour., Apr., 1928; v. 66, pp. 385-409.
("An attempt at an exact mathematical investigation of the electrical stress distribution round certain systems of conductors which are met with in practice.)

Electric Drive—Steel Mills

Cold-Rolling of Strip Steel. H. C. Uhl.
Iron & St. Engr., Apr., 1928; v. 5, pp. 171-177.
(Explains the machinery and processes used, including the electric drive equipment.)

Electric Heating, Industrial

Study of the Requirements of Electrically Heated Melting Pots for Soft Metals. P. H. Clark and A. N. Otis.
Ind. Engng., Apr., 1928; v. 86, pp. 177-180, 188.

Electric Lighting—Aviation

Airport Lighting Makes Schedule Flying Possible at Night.
Elec. Rec., Apr., 1928; v. 43, pp. 427-429.

Excess Voltage

Transmission Line Surges and Their Effect on Operation. Joseph H. Cox.
Elec. Lt. & Pr., Apr., 1928; v. 6, pp. 24-27.
(Presents "a study of voltage surges by means of the klydonograph." Serial.)

Governors

Fluid Governors for Prime Movers. Henry F. Schmidt.
Elec. Jour., Apr., 1928; v. 25, pp. 168-171.
(Serial.)

Hydroelectric Plants

Heating Trash Rack Bars Electrically. C. R. Reid.
Elec. News, Mar. 15, 1928; v. 37, pp. 29-32.
(Describes an electric heating method for combating frazil ice at hydroelectric plants.)

Ionic Theory

A Few Remarks on the Problem of the Recombination of Positive Ions and Electrons. J. Franck.
Franklin Inst. Jour., Apr., 1928; v. 205, pp. 473-479.

Lightning Protection

Effect of Grounded Neutral on the Efficiency of Lightning Protection Equipment. K. B. McEachron.
Iron & St. Engr., Apr., 1928; v. 5, pp. 160-163.

Radio Engineering—Measurements and Testing

Bridge Method for the Measurement of Inter-Electrode Admittance in Vacuum Tubes. E. T. Hoch.
Inst. Radio Engrs. Proc., Apr., 1928; v. 16, pp. 487-493.

Direct-Capacity Bridge for Vacuum-Tube Measurements. Lincoln Walsh.
Inst. Radio Engrs. Proc., Apr., 1928; v. 16, pp. 482-486.

Ship Propulsion, Electric

U. S. Aircraft Carriers.
Mech. Engng., Apr., 1928; v. 50, pp. 280-284.
(A condensed account of the electric drive equipment of the *Lexington* and the *Saratoga*.)

Telephony, Automatic

Principles of Relay Timing in Connection With Automatic Telephone Circuits. T. H. Turney.
I. E. E. Jour., Apr., 1928; v. 66, pp. 341-384.
(An extensive paper on the theory involved.)

Waste Heat

Steam Generation; IX. Waste Heat Boilers, Their Application in the Carbonizing Industry.
Wld. Power, Apr., 1928; v. 9, pp. 229-238.

NEW BOOKS

(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW.)

Die Messwandler; Ihre Theorie und Praxis. I. Goldstein. 166 pp., 1928, Berlin, Julius Springer.
(This little volume deals with the instrument transformer, a subject given but scant treatment in the general works on transformers and on electrical measuring instruments. The bulk of the book, denoted as Part A, treats of the current transformer in four chapters. Chapter I devotes 55 pages to the fundamental principles involved; Chapter II considers the various types of current transformers now in use, discussing their design, operating characteristics, protective schemes, etc.; Chapter III discusses the accuracy and errors of current transformers under various operating conditions; while Chapter IV briefly indicates methods of testing. The remaining portion of the volume, denoted as Part B, gives a 27-page discussion of the potential transformer. A short list of selected references to literature on the subject completes the work. The book should be of use not only to designing and operating engineers, but in the testing laboratory as well.)

Electric Rectifiers and Valves. A. Güntherschulze. (Trans. & rev. by N. A. de Bruyne.) 212 pp., 1928, N. Y., John Wiley & Sons.

Electric Winders; a Manual on the Design, Construction, Application and Operation of Winding Engines and Mine Hoists. H. H. Broughton. 402 pp., 1928, N. Y., D. Van Nostrand Co.
(This English text is "intended primarily for the engineering staffs of coal and metalliferous mines" and for "those interested in the design, construction and operation of electric winders and hoists," according to the author's words in his "Preface." Its 400 large-size pages are replete with data and constants of assistance to designers, although it does not pretend to be a complete treatise on design. The 23 chapters consider such topics as the mechanical and electrical features of mine hoists, the rating of the electrical parts, the various kinds of hoist drums and mine shafts encountered, the use of flywheels, the Ward Leonard and the three-phase systems, the power supply, hoist cables and their accessories, gearing, brakes, and control equipment. The volume is copiously illustrated with a total of 288 diagrams, graphs, and photographs of equipment, and 64 tables of data are provided.)

Elements of Machine Design. James D. Hoffman and Lynn A. Scipio. 327 pp., 1928, Bost., Ginn & Co.

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GENERAL

GENERAL ELECTRIC COMPANY, SCHENECTADY, N. Y.

GENERAL ELECTRIC REVIEW

SCHENECTADY, NEW YORK

Member Audit Bureau of Circulations

Manager, M. P. RICE   *Editor*, J. R. HEWETT   *Associate Editor*, E. C. SANDERS

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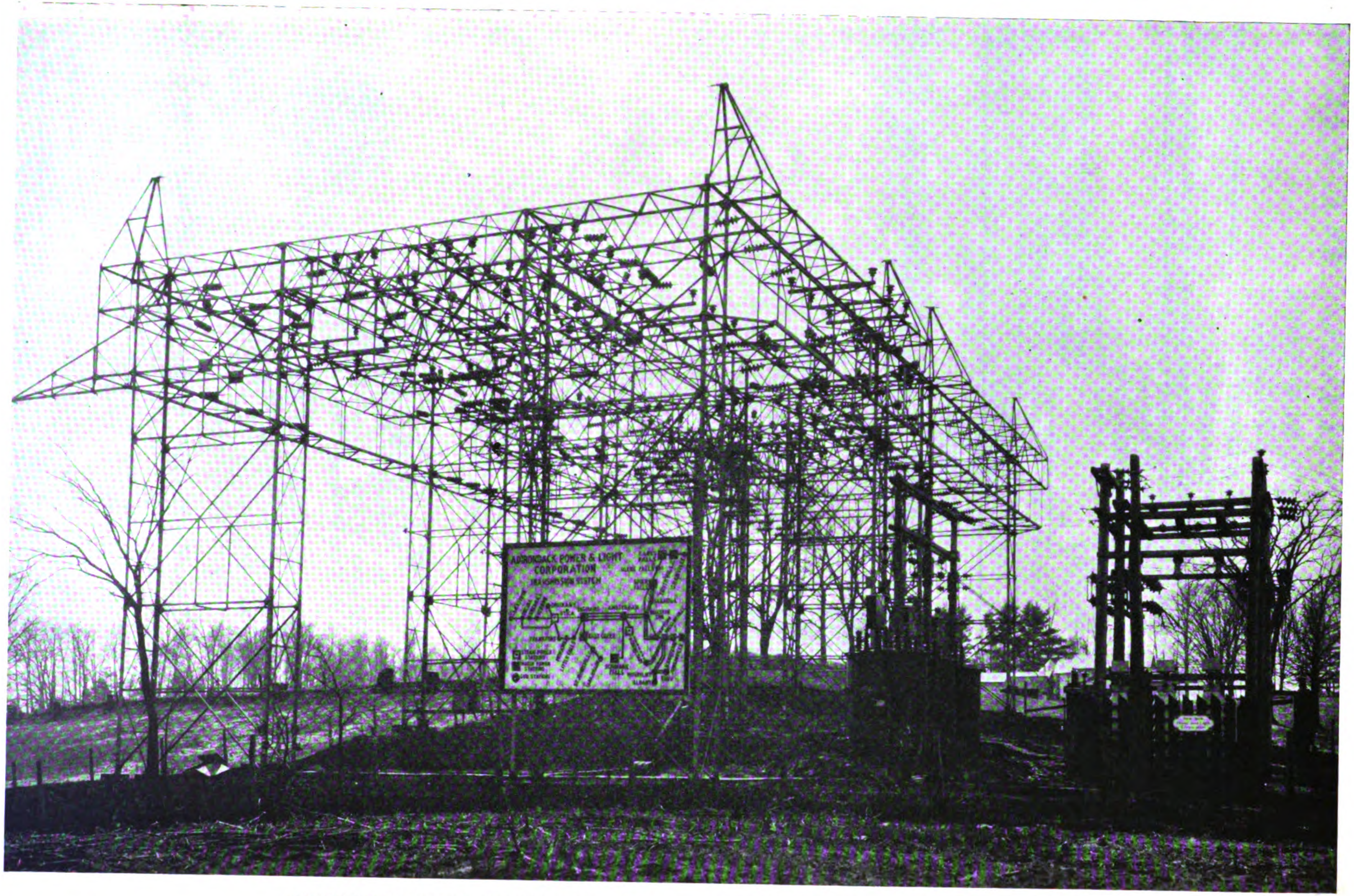

SERIES CAPACITORS AT THE BALLSTON SUBSTATION, NEW YORK POWER & LIGHT CORPORATION

The capacitor installation consists of three 60-cycle 415-kv-a. 3170-volt single-phase capacitors with individual protective equipments. Each unit is connected in series with a transmission conductor, and the line current through it produces a voltage effect that compensates for the inductive reactance in the line. (*See p. 432*)

GENERAL ELECTRIC REVIEW

Vol. 31 August, 1928 No. 8

AND NOW THE SERIES CAPACITOR

From interesting phenomena to practical working devices has been the history of electrical engineering. The electric generator and motor, the arc and the incandescent lamp, the transformer, and radio were at one time but little more than interesting playthings. With the passage of time, these toy-like devices were developed into full-fledged machines to furnish new and more useful means for converting nature's energy into power for operating railroads and factories, for lighting homes and transmitting intelligence across the ocean and far beyond.

Now, our transmission lines reach all urban centers of population. Our executives and engineers have kept pace with the ever-increasing demand for power; but within the next few years these same engineers and executives will be taxed harder than ever to meet demands for still more power. Already, they have become faced with the prospect of encountering certain economic and technical limitations in transmission and distribution. Is the public to be limited in the extended use of this commodity? Is the farmer to be told that he cannot be supplied with this energy, except at a premium price? Fortunately, we now see our way clear to removing these limitations as the result of another laboratory plaything having been made into a commercial machine. The tuned circuit, familiar to radio enthusiasts and only yesterday thought to be a thing to be avoided in a commercial transmission line, is today utilized to broaden the field of power-transmission engineering.

Such a circuit, including a capacitor in series with a transmission line, is carrying thousands of kilowatts and has been doing so for the past five months. The installation of the series capacitor equipment is described by Mr. Shelton in this issue of the REVIEW. Together with its associated protective equipment, the capacitor has fulfilled all expectations. The combined unit has gone through lightning storms and functioned perfectly. There it stands, now beckoning to engineers for more difficult work to do.

A series capacitor can be selected that will compensate for the inductive reactance of the line and transformers in the transmission circuit in which it is connected. In this manner, the overall voltage characteristics can be made to approximate those which would exist if resistance only were present. Figuratively, the series capacitor eliminates the element of distance in power transmission and gives the same voltage condition at the receiving end of the line as if the generator were directly connected to the receiver bus, except of course for the effect of line resistance. The magnitude of the line current is not changed. Thus, by utilizing a series capacitor, approximately the characteristics of direct-current transmission can be obtained and the advantages of the alternating-current system retained.

For short lines, the series capacitor can be located at any point in the line with the same net overall results. For long lines, where charging current is a factor at light loads, the capacitor can be divided into two sections with one unit installed at the receiving end of the line and the other at the generating end; thus the charging current of the line through the capacitor at the generating end gives a drop in voltage equivalent to the rise in voltage over the inductance of the line. In case there are branch circuits tapped from the line, the total capacitance may be split up into sections located just ahead of each tap-off point to hold the potential at these points at a pre-determined value.

Furthermore, the stability of a long transmission line can be materially increased by the installation of a series capacitor because its compensation of the inductive reactance enables a greater load to be carried over the line before the load limit is reached.

The application of series-capacitor equipment to overhead lines of low voltage is equally attractive. Heretofore, very little improvement in voltage regulation was obtained by increasing the size of conductors beyond a certain limit, because the controlling factor was the inductive reactance of the line. With the addition of a series capacitor, however, the copper may be increased in size until the economic limit of copper is reached. Thus it is possible to use low-cost line construction in supplying large rural sections with electric power.

No other commercial piece of apparatus is known which has such low losses as the capacitor; approximately 0.2 of one per cent of the kv-a. rating.

All of the knowledge gained over a period of many years in the manufacture of capacitors for power-factor correction has been utilized in the design and manufacture of series-capacitor elements. This is likewise true of the protective equipment. Therefore the series capacitor and its protective equipment, although a new device, can be expected to give satisfactory service without having to pass through an extended period of years for trial installations.

T. A. E. BELT

Cathode-ray Oscillographs and Their Uses

Principle of the Cathode-ray Oscillograph—Fundamental Types—Characteristics—Description of Instrument—Examples of Application

By EVERETT S. LEE
General Engineering Laboratory, General Electric Company

> *How long is an instant? We have been accustomed to think of it as a time so short that it could not be measured—and we can still continue to do so but some of the "instants" of yesterday are now found to be of microseconds duration and readily capable of measurement by means of the cathode-ray oscillograph.*—EDITOR

THE recent revival in the use of cathode-ray oscillographs assures continued progress in the solution of those engineering problems in which the phenomena vary with time at a rate beyond the range of the electromagnetic oscillograph. The early instruments, of 25 years ago, were of the high-voltage cold-cathode type and were used by only a few investigators—notably Prof. Harris J. Ryan in his early studies of wave forms and in his later measurements of power in high-tension circuits. The complications and limitations of these early instruments stimulated the development of a simpler instrument, which resulted in the production of the low-voltage hot-cathode type with a resulting increase in use and interest in cathode-ray oscillographs, though the field of usefulness of this type is largely restricted to the recording of periodic recurring phenomena. The more recent extension of instrument design by A. Dufour to include the recording of transient phenomena by the use of the high-voltage cold-cathode type has again greatly enlarged the field of usefulness of cathode-ray oscillographs and increased the interest in them. Continued advance is being made in familiarizing engineers with the inherent advantages and limitations of these instruments, in designing instruments which can be moved from the laboratory to the work in factory or field, and in simplifying the operating technique. In view of this situation, it is the purpose of this article to discuss the nature of cathode-ray oscillographs in their present status, to describe a newly available instrument of the high-voltage cold-cathode type for laboratory and field use, and to consider the use and application of these instruments.

General

The cathode-ray oscillograph is an oscillograph in which the moving element is a beam of cathode rays the deflection of the beam being produced by the force exerted between it and a directly applied electrostatic or electromagnetic field proportional to the phenomenon being recorded. Since the deflecting fields may be directly applied to the beam, the only moving parts to introduce inertia effects are the particles of the beam itself, which are electrons and for oscillographic purposes are of negligible mass. Fundamentally, therefore, the cathode-ray beam is an ideal oscillographic moving element.

The physical laws governing the action of cathode rays are quite well known from the results of extensive study and research, and the application of this knowledge to instrument design has been thorough. (1) As is usually the case, however, design considerations prohibit all the desirable features from being combined into one instrument, and thus there is opportunity to design several types of cathode-ray oscillographs depending upon which factors are utilized and which are sacrificed. It is because of this situation that there are available at present instruments of three general types, such as the low-voltage hot-cathode type of the Western Electric Company, (2) the medium-voltage hot-cathode type of Wood, (3) the high-voltage cold-cathode type of Dufour, (4) and the high-voltage cold-cathode type of Rogowski. (5) To these may now be added the high-voltage cold-cathode type of the General Electric Company, the first of this type to be produced in this country.

Nature of the Cathode-ray Oscillograph

In discussing the nature of the cathode-ray oscillograph it may be well to keep in mind the elements of the more familiar electromagnetic oscillograph in which a beam of light is made to move over a screen or photographic film according to the motion of a current-carrying element in a magnetic field, which element carries current proportional to the phenomenon being recorded. Because of the inertia of the current-carrying element, the electromagnetic oscillograph is limited in range to recording phenomena which do not vary at a rate more than the order of 10,000 cycles per second. In the cathode-ray oscillograph, the cathode-ray beam is made to move over a screen or photographic plate or film according to the force exerted between it and a directly applied electrostatic or electromagnetic field proportional to the phenomenon being recorded. Therefore, in the cathode-ray oscillograph there are no mechanical moving parts having inertia to limit the response of the beam to the phenomenon being observed, so that the upper limit of movement of the beam available for recording purposes is measured in millions of cycles per second. This inherent advantage of the cathode-ray oscillograph allows it to be applied to phenomena of very short time duration, such as a few millionths of a second.

---

(1) MacGregor, Morris, J. T., and Mines, R., "Measurements in Electrical Engineering by Means of Cathode Rays," *Journal of the I.E.E.*, vol. 63, pp. 1056-1107, November 1925.

This is an excellent article on the theory of cathode rays and their application to oscillographs.

(2) Johnson, J. B., "A Low Voltage Cathode-Ray Oscillograph," *Bell System Technical Journal*, vol. 1, pp. 142-151, November 1922.

(3) Wood, A. B., "The Cathode-Ray Oscillograph," *Journal of the I.E.E.*, vol. 63, pp. 1046-1055, November 1925.

(4) Dufour, A., *L'Onde Electricque*, vol. 1, pp. 638 and 699, 1922; vol. 2, p. 19, 1923.

(5) Rogowski, W., Flegler, E. and Tamm, R., "A New Design of Cathode-Ray Oscillograph," *Archiv Fur Elektrotechnik*, vol. 18, pp. 513-524, September 21, 1927.

In the electromagnetic oscillograph each current-carrying element is limited to recording the effect of one quantity varying with time. In the cathode-ray oscillograph the motion of the beam may be produced by the simultaneous application of more than one deflecting field in different planes, each proportional to a single quantity varying with time, the oscillograph thus operating as a cyclograph. This is another inherent advantage of the cathode-ray oscillograph as it provides for obtaining important related phenomena such as volt-ampere characteristics.

To obtain these advantages requires means for producing a cathode-ray beam, means for applying the deflecting fields to the beam, and means for recording the motion of the beam. In practice, these vary in extent from the simpler instrument of the low-voltage hot-cathode type, limited largely to the recording of recurring periodic phenomena, to the more complicated instrument of the high-voltage cold-cathode type suitable for recording both recurring periodic phenomena and transient phenomena.

The Cathode Rays

The cathode rays are produced in a tube into which are sealed a metal cathode and a metal anode, these parts being common to all types of cathode-ray oscillographs. When a difference of potential is maintained between the cathode and the anode, the cathode being at a higher potential than the anode, there is a conduction of electricity through the air between them; and when the air pressure is reduced to a value of the order of 10 microns, the resulting conduction has been given the name of cathode rays. These rays consist of negatively charged particles and the particles are known as electrons. The conditions in the tube at this time are characterized (in a glass tube) by a bluish-green fluorescence appearing on the walls.

It has been shown ⁽⁶⁾ that cathode rays travel in straight lines, emerge normally from the cathode, are deflected by electrostatic and magnetic fields, can exert mechanical pressure, can convey very considerable amounts of kinetic energy, and can penetrate small thicknesses of matter. The cathode rays cause fluorescence in a screen covered with a suitable salt, such as calcium tungstate, and react upon photographic films so that upon development the record of the path of the rays over the film is revealed. The velocity of propagation of the cathode rays is a function of the potential difference between the cathode and anode.

Design Considerations

In utilizing the properties of the cathode rays, the designer is forced to choose between deflection sensitivity and photographic sensitivity. The greater the velocity of the cathode rays, the greater the photographic sensitivity but the less the deflection sensitivity, and vice versa. This condition has resulted in the design of the two types of instruments, the low-voltage type with low photographic sensitivity and high deflection sensitivity, and the high-voltage type with high photographic sensitivity and low deflection sensitivity. The low or medium voltage instruments are of the hot-cathode type. The high-voltage instruments are of the cold-cathode type.

The Hot-cathode Type

In the hot-cathode type of cathode-ray oscillograph, negative ions are independently introduced by a heated filament at the cathode to neutralize the accumulation of positive ions at the cathode thus permitting the cathode rays to be produced with a relatively low potential difference between cathode and anode, such as 300 to 3000 volts. The deflection sensitivity is thereby increased, though the photographic sensitivity is decreased. Because of the decreased photographic sensitivity an instrument designed for the lower of these voltages is suitable only for observing or photographing periodic recurring phenomena, where the trace may be maintained a sufficient length of time for photography, unless augmented by external devices, such as the transient visualizer with which transients at lower frequency have been observed and photographed. ⁽⁷⁾

The Cold-cathode Type

In the cold-cathode type of cathode-ray oscillograph, the cathode rays are produced only by the potential difference between the cathode and the anode. The potential difference required at the low value of air pressure necessary for the formation of cathode rays is quite high, such as 30 to 60 kv., this voltage being consumed largely at the cathode to overcome the positive ions attracted to it. This high voltage produces a high velocity of electrons which means that relatively greater forces are required to deflect the beam, thus decreasing the deflection sensitivity of the oscillograph. The photographic sensitivity is greatly increased, however, which is the design factor gained at the expense of deflection sensitivity. High photographic sensitivity is of great importance in studying many problems in engineering,

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(6) Crowther, J. A., *Ions, Electrons, and Ionizing Radiations*, Fourth Edition, 1924, pp. 86-90.

(7) Turner, H. M., "The Transient Visualizer," *Transactions of the A.I.E.E.*, vol. XLIII, pp. 805-813, June 1924.

for it allows transient phenomena to be observed and photographed. This is really the feature of the high-voltage cold-cathode type of instrument; and, in the present state of the art, the sacrifice of deflection sensitivity for photographic sensitivity is justified.

Sensitivity

Wood has shown (3) that the electrostatic sensitivity is inversely proportional to the potential difference between cathode and anode; the magnetic sensitivity is inversely proportional to the square root of the potential difference between cathode and anode; and the photographic sensitivity is proportional to the square of the potential difference between cathode and anode. Relatively then, for different values of potential difference between cathode and anode, these sensitivities rank as shown in Table I.

TABLE I

Voltage between Cathode and Anode	Electrostatic Sensitivity	Magnetic Sensitivity	Photographic Sensitivity
300	100	10	10^{-4}
3000	10	3.16	10^{-2}
30,000	1	1	1
60,000	0.5	0.25	4

These relations do not apply exactly to the different instruments designed to operate at these voltages, but they do give an idea as to the order of the relations. Unfortunately, electrostatic and magnetic sensitivity have to be sacrificed for good photographic sensitivity. Herein is an opportunity for advance in design.

The conceptions of sensitivity may be made clearer by considering the cathode-ray beam as a stream of electrons produced by a difference in potential and directed at a photographic film. In the course of their straight-line travel toward the photographic film, the electrons pass through an electrostatic or an electromagnetic field at right angles to their travel which causes them to be deflected, thus producing a trace on the photographic film. The amount of deflection for a given field strength depends upon the length of time the electrons are under the action of the field, hence the deflection sensitivity is greater the lower the velocity of the electrons, and vice versa. The trace on the photographic film depends upon the energy liberated by the electrons striking the film, hence the greater the velocity of electrons the greater the energy liberated and the shorter the time required to produce the trace, *i.e.*, the greater the photographic sensitivity.

Air Pressure

The air pressure in the cathode-ray tube for the production of cathode rays as a gaseous discharge must be of the order of 10 microns. Two methods are available for obtaining and maintaining this pressure. The simplest is to exhaust the cathode-ray tube to the desired pressure and then seal it permanently. This precludes the use of photographic films inside the tube to record the movement of the cathode-ray beam, and limits the tube to uses where the movement of the cathode-ray beam can be observed visually on a fluorescent screen or, if the phenomenon is periodically recurring, to where the recurring trace on the fluorescent screen can be photographed by means of an external camera. This is the means employed in the present low-voltage instruments.

Although cathode rays have been projected through a window in the tube, (8) unfortunately there has yet been no way successfully developed whereby the projected beam can be utilized for measurement purposes. The difficulty is due to the considerable dispersion of the beam after leaving the metal window, and to the interference to the beam caused by the window support. Herein is also opportunity for advance in design.

The second method of exhaust is to provide an exhaust pump and to evacuate the tube, at the time of use, to the desired air pressure as shown by an air-pressure gauge. While this is an inconvenience, it is the price which has to be paid for being able to insert a photographic film into the tube and obtain on it a record of the movement of the cathode-ray beam. This is the means employed with the present medium-voltage instruments and high-voltage instruments.

Deflecting Force

The deflecting force to produce motion of the cathode-ray beam is produced by the action between the cathode-ray beam and either an electrostatic field between metal plates, proportional to the phenomenon to be observed, or an electromagnetic field between poles, proportional to the phenomenon to be observed, and through which field the cathode-ray beam passes. The deflecting field is applied to that portion of the beam which is projected beyond the anode, through a small hole in it, and which portion is but a small fraction of the whole beam between cathode and anode. This provides a fine beam for tracing a clearly defined line on the fluorescent viewing screen or the photographic plate or film.

Recording

The trace of the cathode-ray beam may be observed on a fluorescent screen, or recorded on a stationary or moving photographic film. Various combinations of motions are available by the action of the electrostatic or magnetic fields on the cathode-ray beam in different planes, and in conjunction with the motion of the film when a moving film is used. The trace of the beam on a fluorescent screen or stationary film is a point; the action of a field upon the beam to move it in one plane produces a line trace; the simultaneous action of two fields upon the beam in two directions will produce a line trace corresponding to the resultant action of the two fields upon the beam.

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(8) Coolidge, W. D., "The Production of High-Voltage Cathode Rays Outside of Generator Tube," *Journal of Franklin Institute*, vol. 202, p. 693, December 1926.

Means for producing records on a screen on a linear time axis have been described.(9) A linear time-axis can also be obtained with a moving film in the usual manner.

For stationary films it is possible to sweep the beam rapidly across the film, superimposing the phenomenon to be observed during such transit. Or, the beam may be subjected to a periodically varying field of known frequency and be swept across the plate, during which transit the phenomenon to be observed is superimposed upon this motion. The resulting trace is the resultant of these motions and the record must therefore be analyzed with proper consideration of the conditions of its formation.(19) The flexibility of the cathode-ray oscillograph in these respects provides opportunity for much individual ingenuity in obtaining useful records.

Timing

As in all oscillographic work, the problem of timing the phenomenon to take place so that it will be observed or recorded is present with the cathode-ray oscillograph. For recurring periodic phenomena this problem is comparatively simple; for transient phenomenon it is usually quite complicated and in some cases has to be left entirely to chance. Since the duration of many phenomena to be recorded is of the order of a few microseconds, the several necessary operations have to be performed accurately and quickly. Both mechanical means and electrical means have been utilized in solving this problem.(10) With the high-voltage cold-cathode type of instrument, the complexity of the problem is increased, since the cathode-ray beam can be maintained for only a fraction of a second because of the excessive heat liberated at the anode and of the gas evolved in the tube at the surface of the cathode which affects the electron stream. To a large extent the usefulness of the cathode-ray oscillograph is limited by the problems incident to timing so that study in this regard is imperative. In this connection there also enters the problem of connecting the cathode-ray oscillograph deflection plates properly into the circuit, which requires that the operator understand the reactions occuring in the circuits during the short time intervals involved.

Résumé

From the foregoing considerations it is seen that the term cathode-ray oscillograph covers three general instrument types of various designs, which range in simplicity from the low-voltage type confined largely to the study of recurring phenomena, to those of the medium-voltage, and high-voltage type suitable for recording very fast transient phenomena and which of necessity are more complicated and require a higher technique for satisfactory application. It was realized, however, that many fields of usefulness existed for the latter type of instrument if arranged for convenient operation with the necessary elements and auxiliary equipment in a form sufficiently portable to allow its being transported out of the laboratory to the work in the factory and field; and it was with this realization that such an instrument has been made available.

GENERAL ELECTRIC CATHODE-RAY OSCILLOGRAPH

This oscillograph has been designed to be self-contained and relatively easily portable so that it may be taken to the desired work and operated in daylight, and thus increase the field of usefulness of the high-voltage type of cathode-ray oscillograph in the solution of engineering problems.

The oscillograph with all auxiliary equipment complete is mounted on two portable trucks. The main unit is shown in Fig. 1. The cathode tube is mounted in the electrostatically shielded dark-room *D* and emerges therefrom in the form of a conical metal bell containing the photographic film holder supported on the head *A*. The indicating vacuum gauge *I* of the thermocouple type is mounted directly under the conical bell in sight of the operator. On the bottom of the truck are mounted the transformer *E* for exciting the cathode-ray tube, the oscillator *F* for the timing wave, and the control panel *H*.

The two units are shown in Fig. 2, this view showing the reverse side of the main unit and exposing to view the mercury-condensation pump *G*. On the second unit are mounted the rough vacuum pump *K* with its driving motor *M*, the synchronous switch *N*, and the water-circulating pump *L*.

Fig. 3 shows the cathode-ray tube *C* with deflection tube *B*, and the shielded dark-room box opened for inspection. Fig. 4 shows the roll-film daylight-loading photographic film holder mounted on the vacuum bell door removed from the vacuum bell.

The two units together weigh about 700 lb., and require a floor space about 6 ft. by 10 ft. The maximum power supply required is 4 kw. at 110 volts, 60 cycles.

Frequency Range

The upper limit of frequency is not definitely known; it is probably several million cycles per second Satisfactory records have been obtained at frequencies up to one million cycles. Superimposed oscillations are observable at frequencies many times higher than this. The time rate of change which can be recorded is of the order of a small fraction of a microsecond.

---

(9) Bedell, F. and H. J. Reich, "The Oscilloscope: A Stabilized Cathode-ray Oscillograph with Linear Time-Axis," *Journal of the A.I.E.E.*, vol. 46, pp. 563-567.

(10) Harrington, G. F. and A. M. Opsahl, "Technique of Dufour Cathode-ray Oscillograph for the Study of Short-Time Phenomena." *The Electric Journal*, vol. 24, pp. 384-388, August 1927.

Sensitivity

The voltage deflection sensitivity is 200 volts (crest value) per centimeter deflection when the cathode-ray beam is produced by a difference in potential of 60,000 volts (crest value) between anode and cathode. The maximum deflection on each side of zero is 5 cm. Voltages up to 1000 volts (crest value) may be impressed directly on the electrostatic deflecting plates, and voltages above 1000 volts (crest value) may be recorded through the use of suitable resistance or capacitance multipliers.

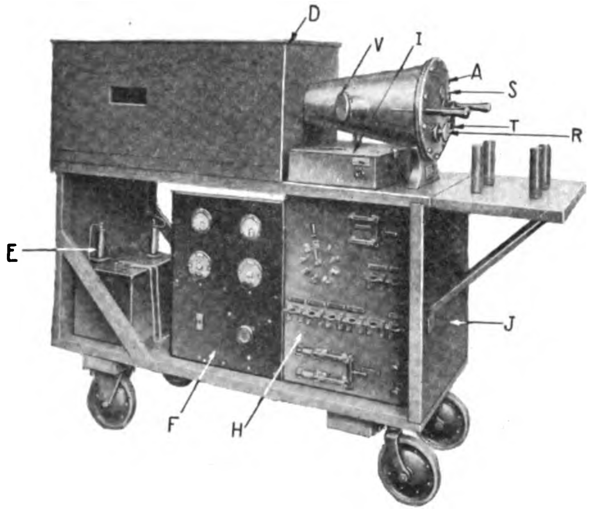

Fig. 1. General Electric High-voltage Cathode-ray Oscillograph. Main unit showing operating side

A: Vacuum bell
D: Shielded housing
E: Tube transformer
F: Timing oscillator
H: Control panel
I: Vacuum gauge (thermocouple type)
J: Movable table
R: Film winding knob
S: Viewing window for film-exposure number
T: Shutter operating handle
V: Viewing window

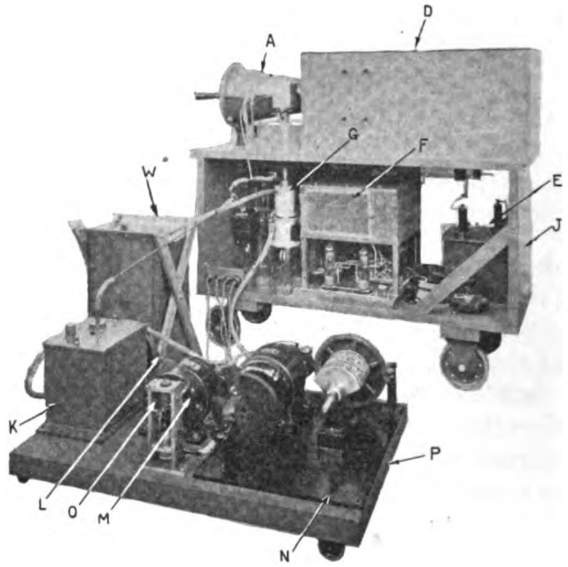

Fig. 2. Complete High-voltage Cathode-ray Oscillograph. Reverse side of main unit, and auxiliary truck on which are mounted vacuum pump and synchronous switch

A: Vacuum bell
D: Shielded housing
E: High-voltage transformer for exciting cathode-ray tube
F: Timing oscillator
G: Mercury condensation pump
K: Rotary oil-sealed vacuum pump
L: Water-circulating pump
M: Driving motor for rotary vacuum pump *K*
N: Synchronous switch
O: Relay for the synchronous switch
W: Water-cooling reservoir

The electromagnetic deflection sensitivity is 120 ampere-turns (crest value) per centimeter deflection when the cathode-ray beam is produced by a difference in potential of 60,000 volts (crest value) between anode and cathode. Coils permit the measuring of currents from a few milliamperes (crest value) to several hundred amperes, and shunts extend the range to several thousand amperes. These values are for fixed deflection plates, but greater sensitivity can be obtained for special cases by using movable deflection plates. The use of special high-speed films also allows of increasing the sensitivity.

Cathode Tube and Deflection Tube

The cathode tube *C*, Fig. 3, is of glass and contains the cold cathode and the anode. The cathode tube is joined to the glass deflection tube *B* which contains the deflecting plates. These tubes are each easily interchangeable, the cathode tube for renewal, and the deflection tube for different combinations of deflecting plates which may be necessary for the different kinds of recording.

The cathode tube and deflecting tube are surrounded by a housing *D*, Fig. 3, which acts as an electrostatic shield and dark-room. A lead glass window in the housing permits inspection of the tube without fear of burns from x-rays.

The cathode tube is excited from the transformer *E*, Fig. 1. Voltage is applied to the tube for only a short time interval at the peak of the negative half-cycle of the voltage wave. This is controlled by the synchronous switch *N*, Fig. 2.

Exhaust System

The tubes and vacuum bell are exhausted to the required air pressure by two vacuum pumps *G* and *K*, Fig. 2. Pump *K* is for moderate vacuum and pump *G* is for high vacuum. These pumps can be relied upon to exhaust the tubes and vacuum bell to the required working pressure within ten minutes. Pump *G* requires cooling water having a temperature below 20 deg. which may be supplied from a tap if available, or if not available from a collapsible cooling reservoir *W* and water-circulating pump *L*.

This exhaust system may seem extensive, but it has been found necessary if the time required to exhaust the tube and vacuum bell after insertion of the film is to be short. A duration of ten minutes is considered satisfactory since there are usually other things to be done; a longer time than this, however, is considered inconvenient.

The air pressure is measured by means of a direct reading vacuum gauge *I*, Fig. 1, of the thermocouple type, located so as to be easily read by the operator.

Recording

The record is obtained on a photographic film of the six-exposure type which can be loaded in daylight into the photographic film holder on the head of the bell, Fig. 4. The film is moved by the knurled head *R*, Fig. 1, operating through a sylphon seal. The fluorescent screen in front of the film holder, Fig. 4, is operated by the handle *T*. The phenomenon may be observed on the fluorescent screen through the viewing window *V*, Fig. 1, before being photographed.

A moving film for recording purposes can also be used. This film is 21 in. long and is used for the lower range of frequencies.

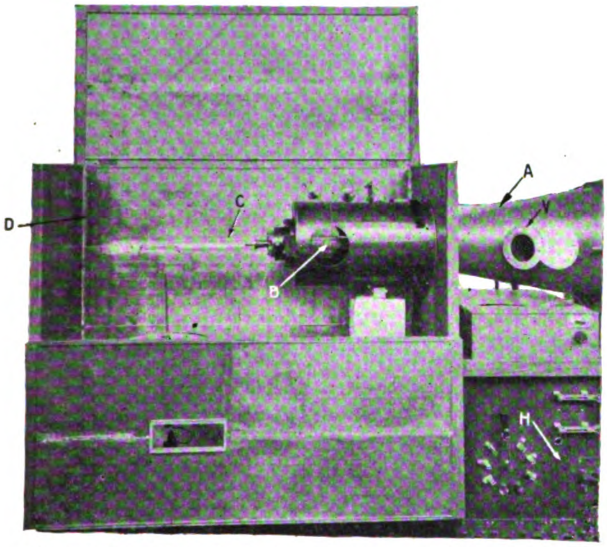

Fig. 3. High-voltage Cathode-ray Oscillograph. Shielded housing opened to show cathode tube and deflection tube

A: Vacuum bell
B: Detection tube
C: Cathode tube
D: Shielded housing
H: Control panel
V: Viewing window

Timing

A timing oscillator *F*, Figs. 1 and 2, is included for timing, having a range of frequencies from 20,000 to 1,000,000 cycles.

A synchronous switch *N*, Fig. 2, is employed to mechanically impress the phenomenon upon the deflecting plates after the cathode-ray beam has been established. This switch is driven by a synchronous motor and is provided with the necessary contacts for the usual type of work met in the use of this instrument.

The timing problem requires considerable study on the part of the operator. The operation of the cathode-ray oscillograph of the cold-cathode high-voltage type requires that the cathode-ray beam be established and swept across the screen or film, and that during such transit the phenomenon exert its effect upon the beam. Such sequence of events must occur correctly in order in the short time available; hence the switching problems are not simple. Nor can any single switching scheme or device be applied with uniform success to the recording of all phenomena. There have thus resulted several schemes for such work all of which become useful for the purpose for which they are designed. To provide a universally applicable scheme is not feasible.

Portability

Because this instrument has been designed for use in the field or in the factory as well as in the laboratory, all the parts have been substantially constructed and all required auxiliary equipment has been included. The mounting provides for portability, and for operation in daylight. The only external power required is a maximum of 4 kw. at 110 volts, 60 cycles. Maximum speed of operation has been made possible through quick-acting vacuum pumps. In these respects the development is considered unique.

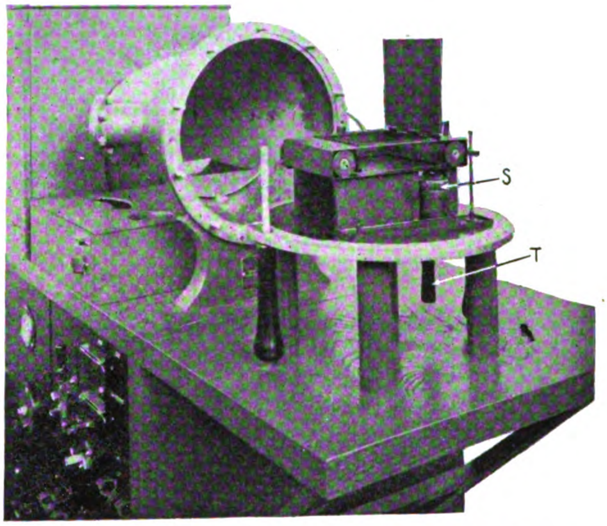

Fig. 4. High-voltage Cathode-ray Oscillograph. Vacuum bell door removed, showing photographic film holder

S: Viewing window for film-exposure number
T: Shutter operating handle

APPLICATIONS AND USES OF CATHODE-RAY OSCILLOGRAPHS

The complete list of possible applications of cathode-ray oscillographs is a long one. The low-voltage type in the field of recurring phenomena is useful:

(*a*). For determining amplitude or extent of a varying quantity such as current, voltage, sound, or mechanical vibration. (2)

(*b*). For frequency measurements. (11)

(*c*). For determining the relations between two quantities in the same circuit each varying with time. (12) (13)

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(11) Rasmussen, F. J., "Frequency Measurements with the Cathode-ray Oscillograph," *Transactions of the A.I.E.E.*, vol. XLV, pp. 1256, November 1926.

(12) Lloyd, W. L., Jr., and E. C. Starr, "Methods Used in Investigating Corona Loss by Means of the Cathode-ray Oscillograph," *A.I.E.E.* Summer Convention, 1927.

(13) Ryan, Harris J., "The Cathode-ray Alternating Current Wave Indicator," *Transactions of the A.I.E.E.*, vol. XXII, pp. 539-548, July 1903 and "A Power Diagram Indicator for High-Tension Circuits," *Transactions of the A.I.E.E.*, vol. XXX, pp. 1089-1113, April 1911.

The high-voltage type is applicable to the fields of recurring phenomena and also to the recording of transient phenomena for such applications as:

(*a*). Study of time-voltage characteristics of spark gaps (14) and insulation.

(*b*). Studies of transmission-line disturbances. (15) (16)

(*c*). Lightning arrester studies. (17)

(*d*). Studies of conditions in circuits at radio frequencies.

(*e*). Telephone protection problems.

(*f*). Mechanical vibration tests.

(*g*). Recording of pressure in guns, gasoline engines, etc.

(*h*). Magneto or spark-coil discharge investigations.

(*i*). Oil circuit breaker and switching phenomena.

(*j*). Determination of wave-form for calibration purposes. (18) (19)

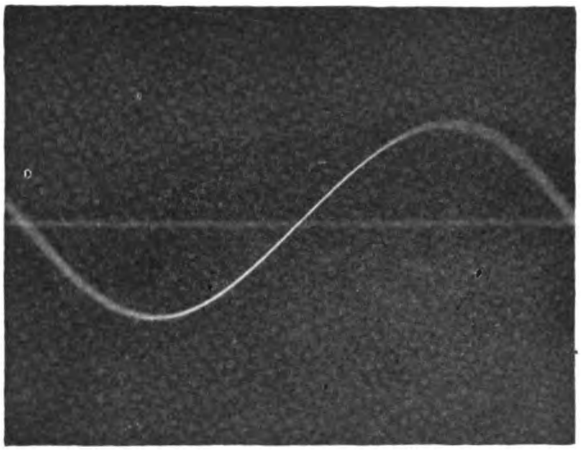

Fig. 5. Cathode-ray Oscillogram of a Voltage Wave at a Frequency of 20,000 Cycles Per Second

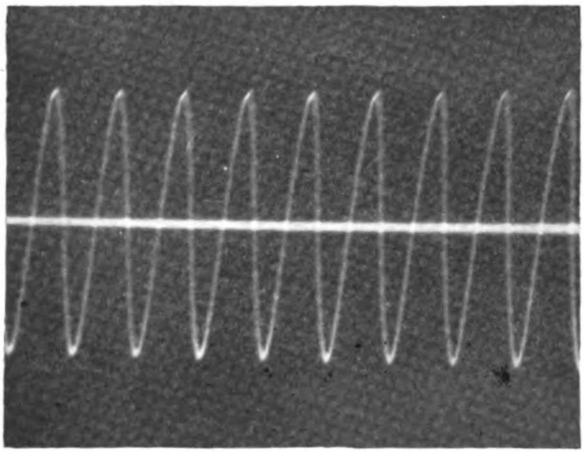

Fig. 6. Cathode-ray Oscillogram of a Voltage Wave at a Frequency of 100,000 Cycles Per Second

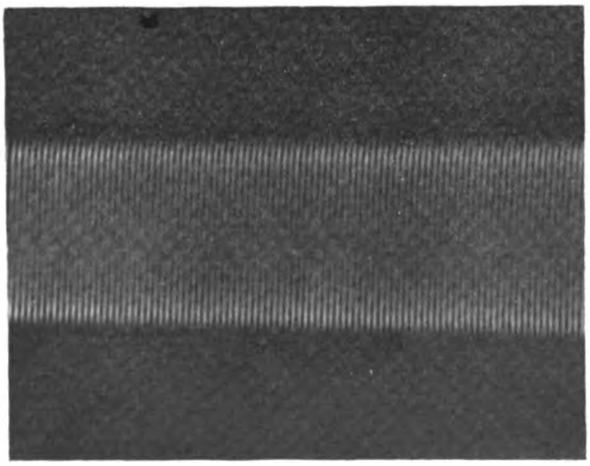

Fig. 7. Cathode-ray Oscillogram of a Voltage Wave at a Frequency of 1,000,000 Cycles Per Second

These are suggested applications, some of which have already been made with success, others of which have not been tried. In extending the use of the cathode-ray oscillograph to these, the technique will also be extended and other applications will appear. As representative of some of these applications, and to explain in somewhat more detail the operation and use of the high-voltage cold-cathode type of cathode-ray oscillograph described, a few oscillograms taken with this instrument will be considered.

Periodic Phenomena

Fig. 5 shows one cycle of a voltage wave at a frequency of 20,000 cycles. Such an oscillogram is obtained by focusing the cathode-ray beam off the edge of the film. The synchronous switch is next put in operation to produce the beam, to superimpose the phenomena upon the beam and then to sweep the beam across the film. The timing may be obtained by a second exposure with the beam acted upon by a voltage at known frequency, or this may be recorded on another film. Fig. 6 shows the oscillogram of a voltage wave at a frequency of 100,000 cycles; and Fig. 7 at a frequency of 1,000,000 cycles. These are probably the simplest oscillograms to obtain, but they are nevertheless quite important and cannot be obtained in any other way.

Fig. 8 shows an oscillogram taken in power vacuum-tube studies at 50,000 cycles, with the plate voltage at 14,000 volts. The upper curve shows the plate voltage and the lower curve shows the grid voltage each with respect to ground potential shown as the straight line. This record was obtained by making three transits of the beam across the film. The phase relations are not shown correctly in this case, though means for doing this can doubtless be arranged.

Fig. 9 shows a plate current of about 2 amp. (crest value) in a power vacuum tube at a frequency of

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(14) McEachron, K. B., and E. J. Wade, "Study of Time Lag of the Needle Gap," *Transactions of the A.I.E.E.*, vol. XLIV, pp. 832-842, June 1925.

(15) Norinder, H., "Experimental Investigation of the Shape of Lightning Discharges," *Elekto.*, vol. 55, pp. 139-141, August 1, 1925, and "Investigation of Traveling Waves with Cathode-ray Oscillograph," *Elekto.*, vol. 55, pp. 152-157, September 5, 1927.

(16) Rogowski, W., E. Flegler, and R. Tamm, "Uber Wanderwelle Und Durchschlag," *Archir Fur Elecktrotechnik*, vol. 18, pp. 475-512, June 27, 1927.

(17) McEachron, K. B. (Discussion), *Transactions of the A.I.E.E.*, vol. XLV, p. 175, February 1926.

(18) McEachron, K. B., "Measurements of Transients by the Lichtenberg Figures," *Transactions of the A.I.E.E.*, vol. XLV, p. 712, May 1926.

(19) Lee, E. S. and C. M. Faust, "The Measurement of Surge Voltages on Transmission Lines Due to Lightning," *Journal of the A.I.E.E.*, vol. XLVI, pp. 149-158, February 1927.

50,000 cycles. This oscillogram is obtained in the same way as those previously described except that the phenomenon is applied to the beam through an electromagnetic field between magnet poles.

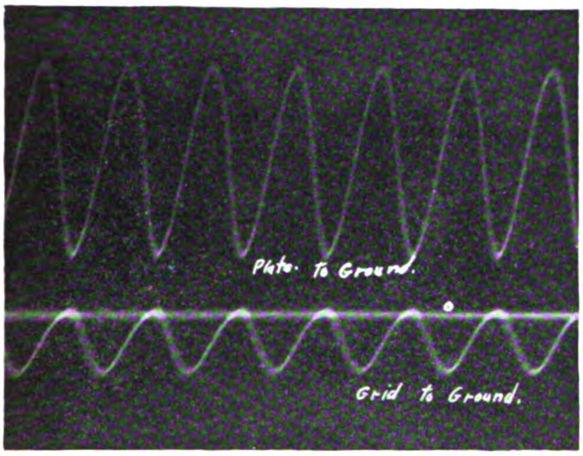

Fig. 8. Cathode-ray Oscillogram Showing Plate-voltage Wave Form and Grid-voltage Wave Form of a Power Vacuum Tube at 50,000 Cycles Per Second

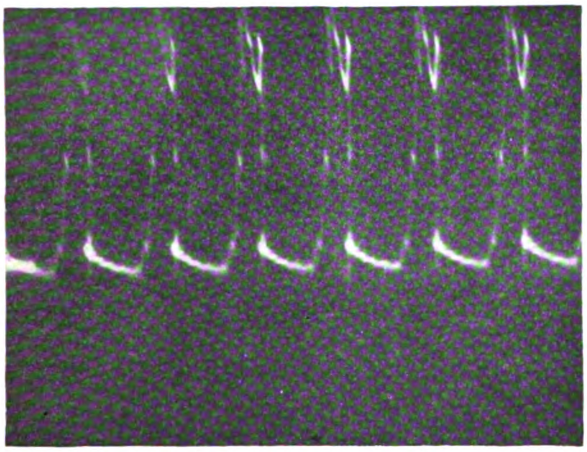

Fig. 9. Cathode-ray Oscillogram Showing Plate-current Wave Form in a Power Vacuum Tube at 50,000 Cycles Per Second

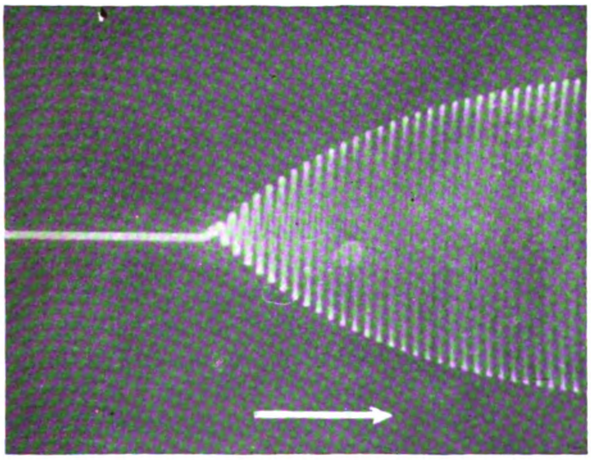

Fig. 10. Cathode-ray Oscillogram of Current Building Up in a Resonant Circuit at a Frequency at 1,000,000 Cycles Per Second

Transient Phenomena

Oscillograms of transient phenomena can be taken in many forms such as have been previously presented. (10) (14) (20)

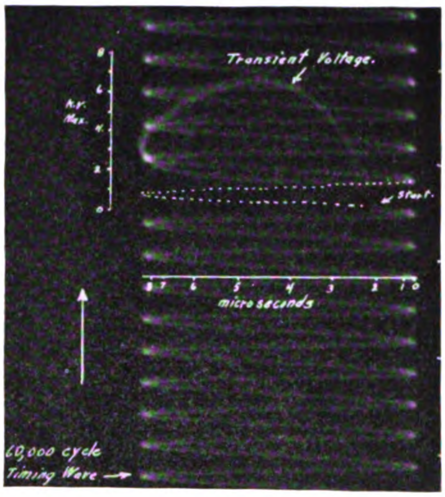

Fig. 11. Cathode-ray Oscillogram Showing a Transient Superimposed on a Timing Wave of 50,000 Cycles Per Second

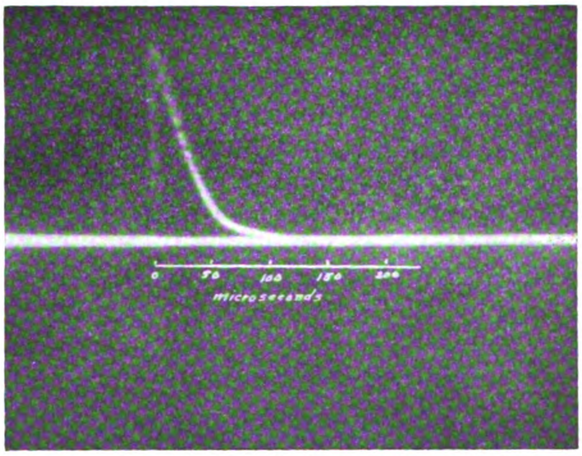

Fig. 12. Cathode-ray Oscillogram of a Capacitor Discharge

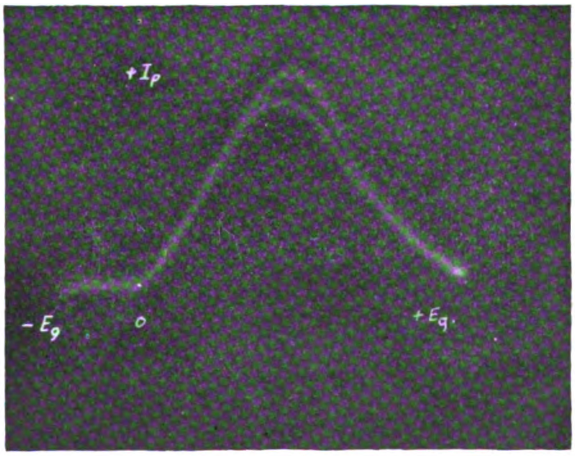

Fig. 13. Cathode-ray Oscillogram Showing Variation of Plate Current with Grid Voltage in a Power Vacuum Tube

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(20) Committee on Protective Devices (Report), *Transactions of the A.I.E.E.*, vol. XLV, pp. 885-887, June 1926.

Fig. 10 shows the current building up in a resonant circuit at 1,000,000 cycles. In this oscillogram the synchronous switch had to superimpose the current phenomenon upon the beam during the transit of the beam across the film, which transit took place in about 50 microseconds.

Fig. 11 shows a type of oscillogram in which the cathode-ray beam is produced in focus off the edge of the film, a voltage at a known frequency is applied to the timing deflecting plates, the beam is swept across the film, and during such transit the phenomena is superimposed upon the beam. The duration of the transient is about 0.9 cycle of a 50,000-cycle timing wave or 18 microseconds. The trace of the transient on the film is the resultant of three forces and the true form of the transient with time has to be resolved from the resultant trace.(19) Such a record may be somewhat confusing though it has the advantage of a longer zero axis on the same size film.

Fig. 12 shows an oscillogram of a transient without the timing wave. This shows a capacitor discharge, the whole time duration of which is about 100 microseconds. The first rush occurred within 4 microseconds.

Cyclograms

Fig. 13 shows a cyclogram showing the variation of plate current with grid voltage in a power vacuum tube, the voltage being in the order of 2000 volts at a frequency of 60 cycles. Such records are of great value and their usefulness can be extended to other phenomena, such as volt-ampere characteristics of insulation, lightning arresters,(17) and the like.

CONCLUSIONS

It has been shown that:

(1). Cathode-ray oscillographs are available in several types, such as:

(*a*). The low-voltage hot-cathode type, the simplest type to operate but restricted largely to the recording of periodic recurring phenomena.
(*b*). The medium-voltage hot-cathode type.
(*c*). The high-voltage cold-cathode type suitable for recording recurring phenomena and transient phenomena of short duration, such as for fractions of a microsecond or for a few microseconds.

(2). The high-voltage cold-cathode type of cathode-ray oscillograph need no longer be considered an instrument that must be located permanently in the laboratory, because it may now be moved to the work in the factory or field.

(3). The auxiliary equipment required for use with the high-voltage cold-cathode type of cathode-ray oscillograph for recording transient phenomena though extensive is justified.

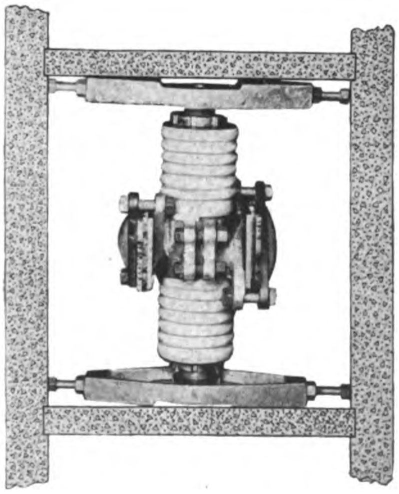
(4). The technique of operation for the high-voltage cold-cathode type is still relatively complex requiring that the operator be able to master the switching problems involved.

(5). The relative portability of newer instruments as compared with older permits the application of the high-voltage cold-cathode type of cathode-ray oscillograph to a wide variety of situations where previously impracticable.

New Connection Support is Easily Installed

A new double, extra-strong bus or connection support requires no setting of bolts in concrete or drilling for expansion bolts, but is simply mechanically expanded against the sides, top, and bottom of the compartment or enclosure in which it is installed. It consists of an ordinary cylindrical bushing with expansion elements at either end and a connection clamp over the middle.

The expansion element is a pipe nipple with a stationary flange and a movable lock nut.



Side View of Connection Support

Vertical expansion is accomplished by unscrewing the lock nut against the bushing until the assembled unit bears rigidly against the top and bottom shelves. The stationary flange is provided with a screw and lock nut at either end by means of which it is expanded horizontally and firmly held against the sides of the enclosure.

No lead or cement is used with this support, and it is shipped knocked down to be assembled and lined up in place.

Short-cut Methods of Calculating Stresses in Bus Structures

Selection of Proper Supports—Spacing and Arrangement of Buses and Main Connections—Calculation of Stresses Simplified by Conveniently Arranged Data

By W. SPECHT
Switchgear Department, Philadelphia Works, General Electric Company

WITH the steady increase of station capacities and the constant trend toward interlinking generating stations into one large system, it became necessary to pay particular attention to methods of arranging and supporting buses and connections that might carry maximum short-circuit currents, the latter in some cases reaching the magnitude of a hundred thousand amperes and more.

The electromagnetic force acting upon the insulators that support the conductors is expressed in lb. per insulator by the formula

$$F = \frac{5.4 \times 10^{-7} \times k \times I^2 \times L}{S}$$

and in the past it was thought that this force was all that had to be conisdered.

In this formula

I is the initial r.m.s. value of the total current in amperes in an unsymmetrical short circuit.

k is the shape correction factor (See Chart II.)

L is the length of the bus-bar span between center of insulators, in feet.

S is the spacing between centers of phases, in inches.

Extensive tests have shown that the conductors as well as their supports vibrate due to the application of sudden forces.⁽¹⁾ The frequency of the test conductors ranged from 8 to 150 cycles and that of the supports from 30 to 1700 cycles. These figures show that it is quite possible that one or both frequencies may resonate with the usual system frequencies of 25 to 60 cycles, and thus increase the stresses imposed on the supports by the electromagnetic force.

The actual support stresses may be readily calculated by applying the methods described in this article and making use of the data that have been arranged in convenient form.

In Figs. 1 to 4 are represented the four most usual arrangements of buses. The sketches are of single-phase buses because these are basic; and the calculation of stresses will be based on single-phase short circuits only, as such are the most numerous and usually the most severe. It is not possible to foretell with great exactness which arrangement will be best unless a thorough analysis is made of the forces acting on the supports in the lateral and longitudinal directions. In Figs. 2 and 3 the longitudinal forces may be neglected if the bar is 3 in. or more in width and the spans not longer than 6 ft. 0 in. In Figs. 1 and 4, however, the longitudinal pull has to be taken into consideration because the deflection of the bus bar in the direction of the electromagnetic force may be sufficiently great to make the curved path quite a little longer than the straight line between the insulators. Most of the copper used for buses is not over ¼ in. thick and consequently does not offer much stiffness to bending across the longer axis.

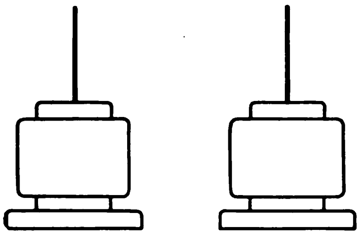


Fig. 1

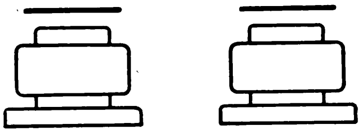


Fig. 2

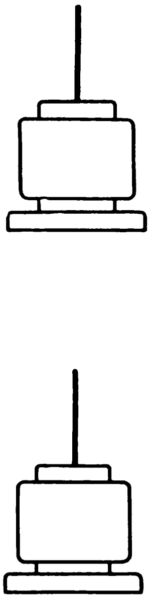


Fig. 3

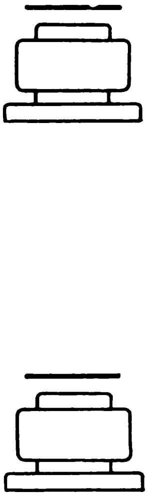


Fig. 4

Figs. 1 to 4. Cross-sectional Diagrams of the Usual Horizontal and Vertical Arrangements of Single-phase Buses and Supports

The opinion has been commonly held that a strong insulator, which is a stiff support, is much better than a weaker and more flexible support. Unfortunately, the strength of an insulator and its stiffness

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(1) O. R. Schurig and M. F. Sayre made the investigations and calculations, and reported them in detail in the *Journal of the A.I.E.E.*, Feb., 1925.

are not in a straight-line proportion. The strength does not increase as rapidly as the stiffness; and therefore similar forces acting on two different insulators, for example, one twice as stiff as the other, will impose on the former insulator a stress more than two times as great as the stress on the latter.

An ideal support is one which has high mechanical strength and a low frequency of vibration. One way to produce such a support would be to mount a strong insulator on a flexible base, like a steel plate, whose mechanical damping will be relatively large. From time to time suggestions have been advanced whereby the longitudinal stresses may be overcome. One of these suggestions was to hold the buses between rollers, at the insulator clamps, thus permitting a free expansion of the copper in a longitudinal direction. This method is impractical in the majority of cases because, in station arrangements where the buses are in compartments, the branch connections can be taken off only at the insulator clamps and therefore an absolutely solid joint is required. Should conditions permit the use of this method, the stress on the insulator will be minimized; but if the electromagnetic force is strong enough to exceed the elastic limit of copper it will bend the bar permanently, thereby reducing the ground clearances or actually grounding the system. If the deformation is great enough and the number of spans is large, it may happen also that the buses will slip out of the end insulators. Another suggestion that has frequently been made is to use spring washers at the clamps or to put spacers in the clamps, so that the bus will not be clamped tightly to the insulator. The inherent weakness of these latter constructions is the same as that of the first, and in addition they may introduce a slapping action which will hasten the destruction of the insulator on account of a hammer blow introduced by the sudden application of a large force in the direction of the clearance between the clamps.

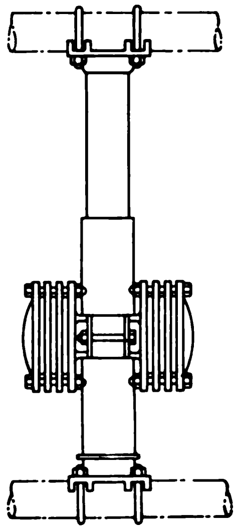


Fig. 5. Insulated Rod Type of Busbar Support for 3500-volt Service

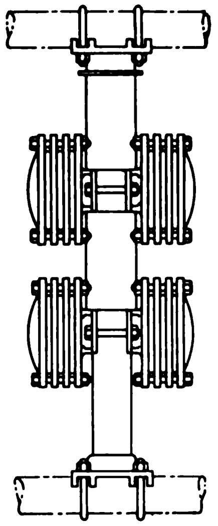


Fig. 6. The Same Type of Support as Shown in Fig. 5

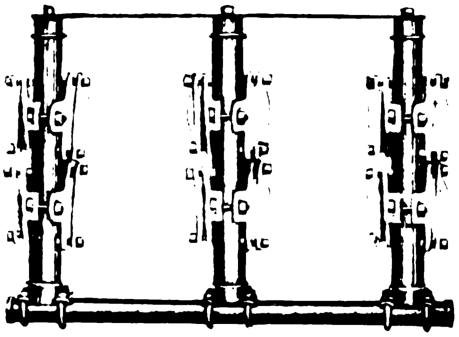


Fig. 7. Insulated Rod Type of 3500-volt Supports Arranged Side by Side

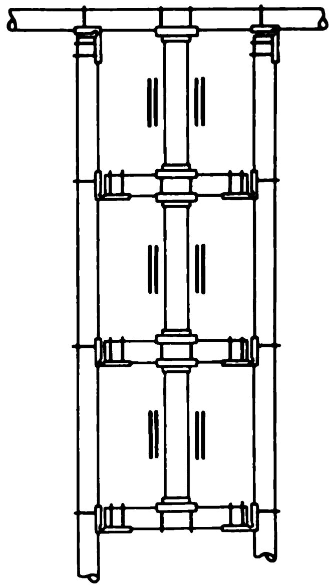


Fig. 8. Insulated Rod Type Supports Arranged One Above Another

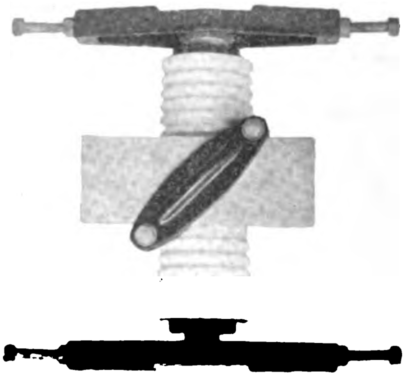


Fig. 9. Insulated Rod Type Support for 15,000-volt Service

The practical method of reducing the effects of longitudinal stress is to place the buses on edge with respect to the direction of the electromagnetic force, preferably as shown in Fig. 3, because the supports in this case are subjected to tension or compression,

CHART I
ELECTROMAGNETIC FORCE ON BUSBARS CAUSED BY SHORT-CIRCUIT CURRENT

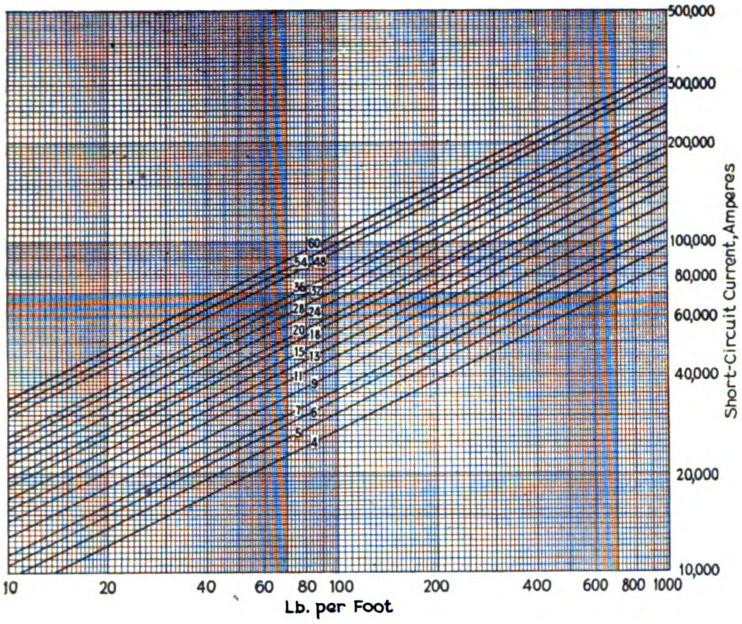
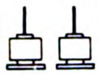
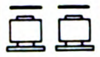
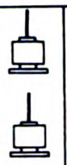
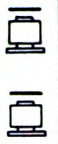


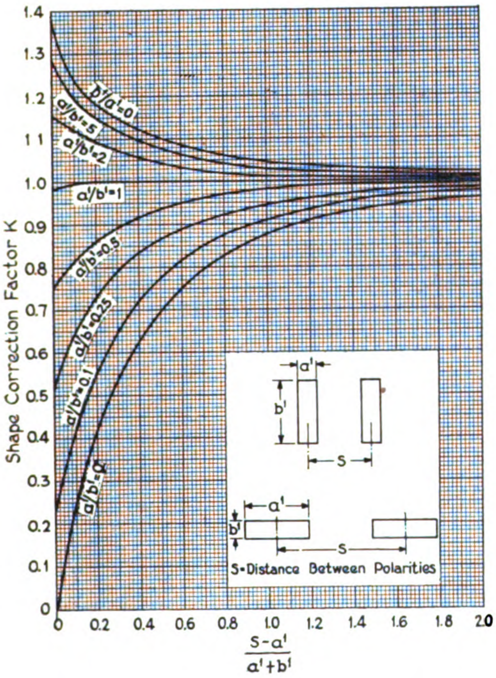
CHART III
VALUES OF STRESS FACTOR

Frequency	Span				
60 Cycles	24"	See Note(a)	See Note(b)	3.0	5.0
	36"	1.8 – 2.6	See Note(b)	3.3	2.0
	48"	1.4	See Note(b)	3.3	1.4
	54"	1.3	See Note(b)	3.6	1.3
	62"	1.2	See Note(b)	5.6	1.2
25 Cycles	24"	2.8 – 3.2	2.8 – 3.4	3.0	3.0
	36"	3.5 – 4.1	2.8 – 3.4	3.0	3.3
	48"	2.5	2.8 – 3.4	3.0	2.3
	54"	2.1	2.8 – 3.4	3.0	1.8
	62"	1.8	2.8 – 3.4	3.0	1.5

NOTE (*a*): For short spans the stress factor varies so much (from 2.3 to 5.7) as to render special calculation necessary in all such cases. This Note may also serve as a warning against the introduction of intermediate supports on long spans, as such an arrangement may produce resonance.

NOTE (*b*): It is impossible to use one stress factor for this condition on account of various values for different insulators and special calculation must be made.

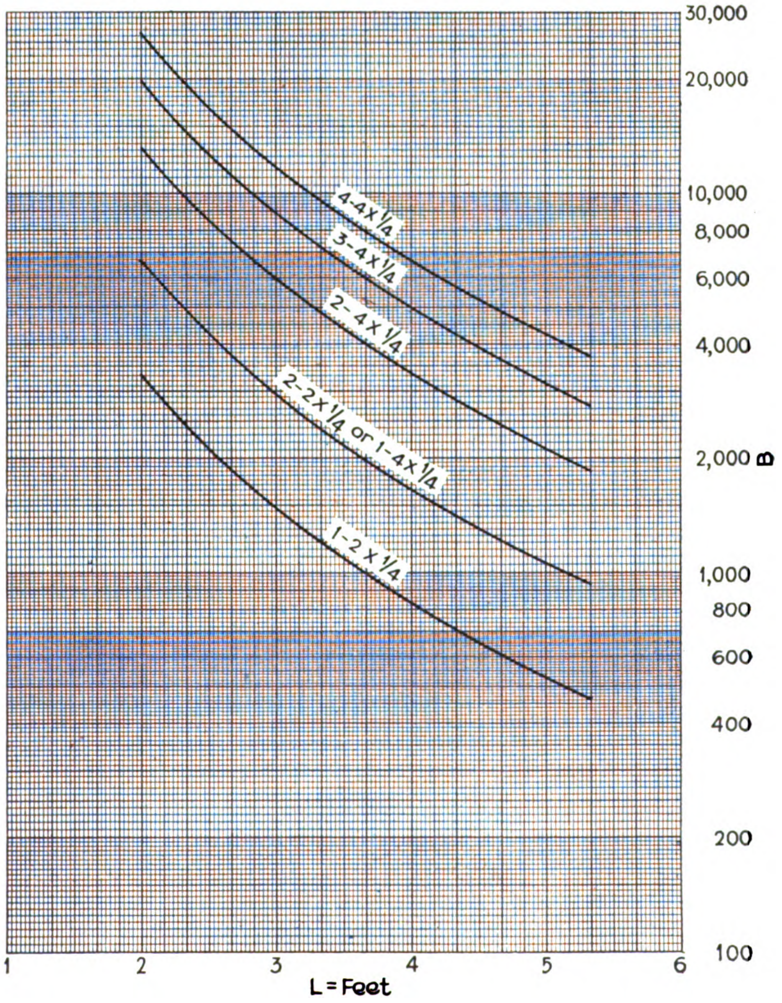
CHART II
ELECTROMAGNETIC FORCE CORRECTION FACTOR FOR BUSBAR SHAPE



and not to bending as in Fig. 2. (The cantilever strength of the ordinary insulator is always less than the strength in tension or compression).

If, due to the harmonic frequencies, it is impossible to use these arrangements and a flexible base will not change the frequencies sufficiently to bring them out of resonance, the arrangement shown in Fig. 4 may

CHART IV
VALUES OF FACTOR *B* FOR DIFFERENT NUMBER AND SIZE OF BUSBAR LAMINATIONS



be used, separating the phases to reduce the stresses and placing in the buses flexible connectors made of braided copper. These connectors should be inserted at intervals of 4 or 5 spans, preferably between insulators of a short span because a short span will offer a greater resistance against the deformation from lateral forces. Incidentally, these flexible connectors will also help to reduce the insulator stress that arises from linear expansion and contraction of bus copper due to temperature changes. If the total length of the bus in one straight run does not exceed 50 ft., the elongation due to the usual variations of temperature is not sufficient to warrant the use of expansion joints.

CHART V
STIFFNESS FACTOR *S* OF TYPICAL INSULATORS

Moderate Duty		Heavy Duty		Extra Heavy Duty	
7,500 Volts	15,000 Volts	7,500 Volts	15,000 Volts	7,500 Volts	15,000 Volts
7,000	5,100	14,000	10,500	11,500	9,000

CHART VI
VALUES OF FACTOR g_1

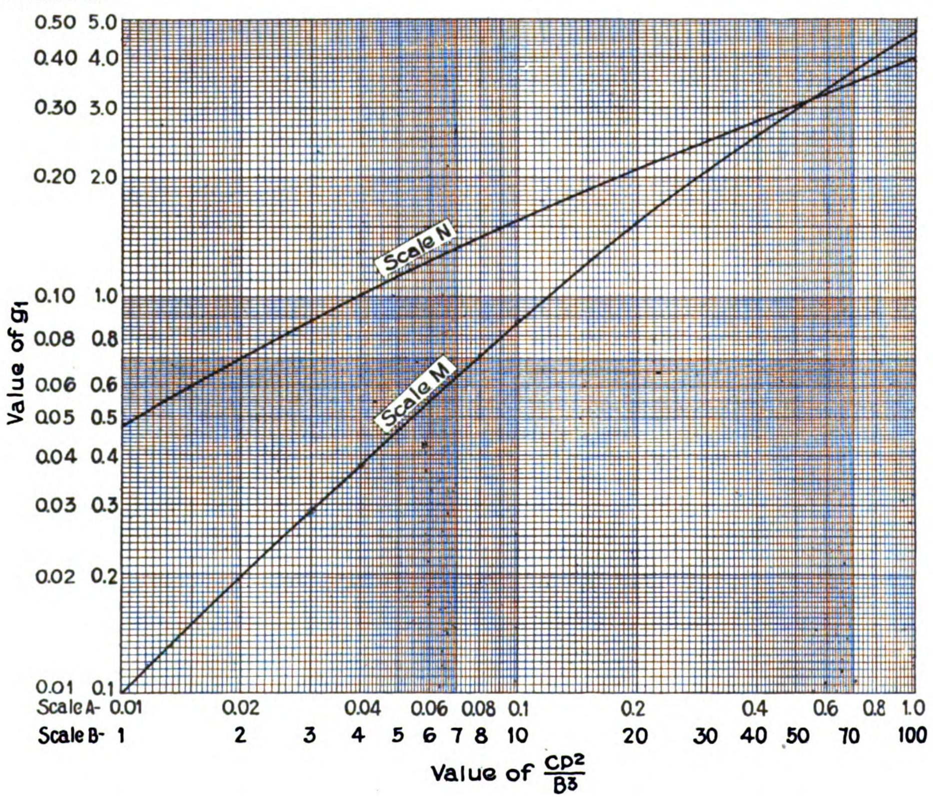


CHART VII
VALUES OF FACTOR *q* FOR DIFFERENT NUMBER OF SPANS

Number of Spans	*q*
1	¾
2	1
3	1½
4	2
6	2½
8	2½
10	2¾

Where none of the foregoing methods will suffice to reduce the stresses to a safe limit, special supports as shown in Figs. 5 or 6 may be used. However, it must be noted that the connection in such an arrangement must be taken off at a point other than the supporting insulator. Some types of these insulators may be placed one beside the other (Fig. 7), or one above the other (Fig. 8), since their strength is very great in either direction. Fig. 9 shows an insulator for voltages above 3500 volts of a construction similar to those in Figs. 5 and 6.

After arriving at a preliminary selection of the bus arrangement, the stresses to which the insulators will be subjected during a short-circuit should be checked. A practical method for their approximation will now be described.

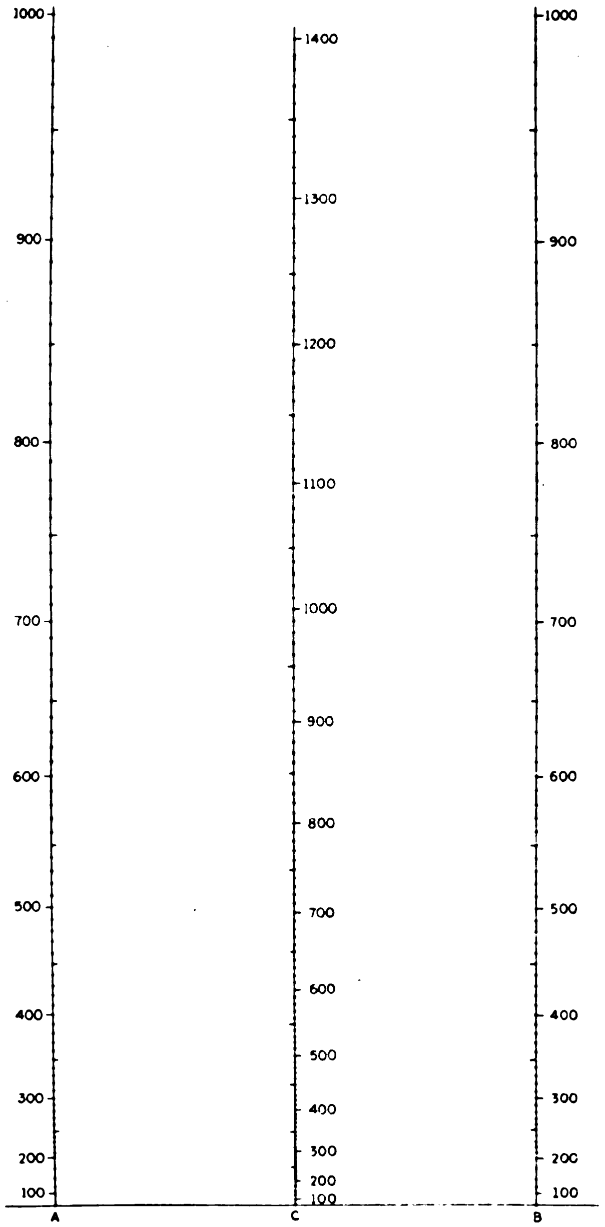
CALCULATION OF MAXIMUM SUPPORT STRESS

Lateral Stresses

From a knowledge of the initial r.m.s. value of the total short-circuit current, and the spacing between the phase centers, the electromagnetic force in pounds per foot exerted on the buses may be readily determined by referring to Chart I. In this chart the horizontal line representing the short-circuit current value intersects the oblique line corresponding to the bus spacing in inches. Beneath this point of intersection, the electromagnetic force in pounds per foot may be read on the horizontal scale.

CHART VIII

COMBINED LATERAL AND LONGITUDINAL SUPPORT STRESS



To find the total lateral force P upon the insulator, multiply the value just found in Chart I by the length in feet between the supports, by the shape correction factor from Chart II, and the by stress factor p selected from Chart III in conformity to the arrangement of supports used.

Chart III contains the commonly used spans; others may be interpolated with an accuracy sufficient for all practical purposes.

Stress Factor

The stress factor p may be determined from Chart III for those cases where special calculation is not necessary.

Longitudinal Stresses

Having ascertained the lateral support stress P, one must determine the factors B and C from the formulas

$$B=\frac{26\times 500\ A}{L^2}\ \text{and}\ C=\frac{6\times 37\ L\times S\times A}{17\times 2A+(12LS\times 10^{-6})}$$

before the longitudinal force F' on a two-span bus structure can be figured from

$$F'\ (F'+B)^2=CP^2$$

In these formulas

A = the total cross-sectional area of the bus at the place where the longitudinal force is to be determined.

L = the length of bus-bar span between centers of insulators, in feet.

S = stiffness of insulator in the direction of the longitudinal forces, in pounds per inch.

This procedure however can be simplified to quite an extent if we take into consideration only such laminations as are used in the majority of cases. Chart IV gives values for one $2\times\frac{1}{4}$, two $2\times\frac{1}{4}$, and one to four $4\times\frac{1}{4}$ laminations.

Furthermore, with sufficient accuracy for all practical purposes, the factor C may be taken as

$$C=0.37L\times S$$

Selecting now the value of S from Chart V, this factor can be determined. Find now the value of

$$\frac{CP^2}{B^3}$$

If this value is less than 1, then use the horizontal Scale A in Chart VI, follow it vertically until it intersects Curve M and the horizontal line will give the factor g_1 on vertical Scale A, with which B must be multiplied to obtain F'

$$F'=g_1\ B\ \text{lb.}$$

Should the values of $\frac{CP^2}{B^3}$ be between 1 and 100 then use the horizontal and vertical Scales B and Curve N in Chart VI. This value of F' is the longitudinal force on the end support of a bus structure consisting of two spans only; for more than two spans, multiply F' by a factor q from Chart VII to obtain F, the force acting on the end insulator.

The combined lateral and longitudinal support stress will be found for the bus arrangement shown in Fig. 1 from Chart VIII. Locate P and F on Scales A

and *B* and draw a straight line between these points; the intersection of this straight line with Scale *C* gives the resultant support stress. Comparing this stress with the breaking stress of the insulator in question, given in Chart IX, it will be seen whether the structure is safe. A good practice is to consider it unsafe unless there is a safety margin of 100 per cent.

For bus structure arranged as in Fig. 4, Chart VIII cannot be used because the stresses in bending and tension are not directly comparable. Therefore proceed as follows:

Multiply the lateral stress *P* by the ratio of the breaking stress for bending to the breaking stress in tension (taking these values from Chart IX) and add the longitudinal force *F*. Compare the result with the breaking stress in bending for the insulator in question as given in Chart IX; and if there is less than 100 per cent safety margin, the stresses should be reduced by any of the methods previously described.

CHART IX

BREAKING STRESSES FOR PORCELAIN-POST INSULATOR BUSBAR SUPPORTS

Tension .... 3,500 Lb.
Compression .... 25,000 Lb.
Moderate Duty
7500 Volts

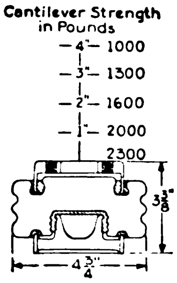
Tension .... 5,000 Lb.
Compression .... 25,000 Lb.
Heavy Duty
7500 Volts

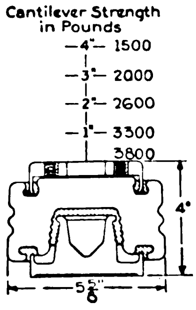
Tension .... 7,000 Lb.
Compression .... 50,000 Lb.
Extra-heavy Duty
7500 Volts

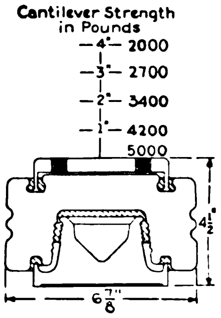
Tension .... 3,500 Lb.
Compression .... 25,000 Lb.
Moderate Duty
15,000 Volts

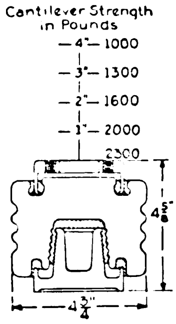
Tension .... 5,000 Lb.
Compression .... 30,000 Lb.
Heavy Duty
15,000 Volts

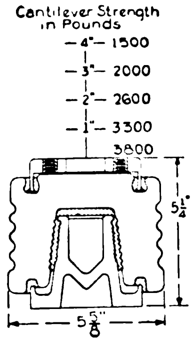
Tension .... 7,000 Lb.
Compression .... 55,000 Lb.
Extra-heavy Duty
15,000 Volts

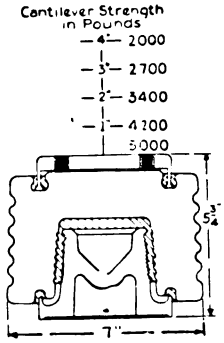












Red Brick Building Floodlighted

That buildings need not be of white or light-colored material in order to be floodlighted has just been demonstrated by the illumination of a group of red-brick factory buildings comprising the Johnson and Johnson plant at New Brunswick, N. J.

Because red is a very poor reflector of white light, the floodlighting units were equipped with red lenses. The effect is unusual and spectacular, and attracts the attention of persons traveling on nearby highways and on the main line of the Pennsylvania Railroad between New York and Washington.

When the matter of floodlighting this plant was first suggested, tests were made in the Illuminating Engineering Laboratory at Schenectady with projectors having various colored lenses. White light prominently revealed the mortar between the bricks, but when red was substituted the effect was of a soft spread of light which could be seen at a cosiderable distance.

Modern Equipment for Mass Transportation

Analysis of the Traffic Problem—Types of Trolley Cars—Determination of Motor Capacity—Transportation by Buses—Comparative Costs of Bus and Trolley Operation

By **J. C. THIRLWALL**

Railway Engineering Department, General Electric Company

EVER since when, ages ago, man began to show civilized tendencies, the transportation problem has been in evidence. The various early forms of animal-drawn wheeled vehicles with their limitations of capacity and speed underwent comparatively little improvement during the centuries preceding the modern age. The first material change came with the development of the steam locomotive, early in the nineteenth century, making possible mass transportation as we know it today, which in turn is the basis of the industrial civilization in which we live.

Modern steam or electric trains carry groups of hundreds of passengers at average speeds of forty to fifty miles per hour. Electric subway trains carry individual loads of fifteen hundred to two thousand people at average speeds of sixteen to eighteen miles per hour. Single trolley cars move one hundred to one hundred and twenty-five persons through congested city traffic at nine to ten miles per hour, and in many cases operate on headways of fifteen to twenty seconds.

Mass transportation has arisen largely through the difference in the demands of the present day and of the requirements of long ago, chiefly in connection with suburban traffic. What such mass transportation means can be illustrated. Boston proper has a population of 800,000; the daily passenger movement from adjacent communities into Boston, by steam cars, trolleys, buses, and automobiles is approximately the same number; and the Boston Elevated, which operates nearly all of the rapid transit and surface line mileage, handles an average of 1,100,000 people on each week day. The city of Philadelphia has a population of 2,000,000, and the Philadelphia Rapid Transit Company carries daily 2,750,000 passengers; and 300,000 others are carried in private automobiles.

Of all the agencies serving the traveling public, particularly in the cities, the electric railway occupies the preëminent position and is, as yet, absolutely indispensable to the very life and growth of an urban population. Even in spite of the tremendous growth of the use of private automobiles and of gas-propelled buses, the electric railways still carry in the larger cities 80 to 90 per cent of the riding public.

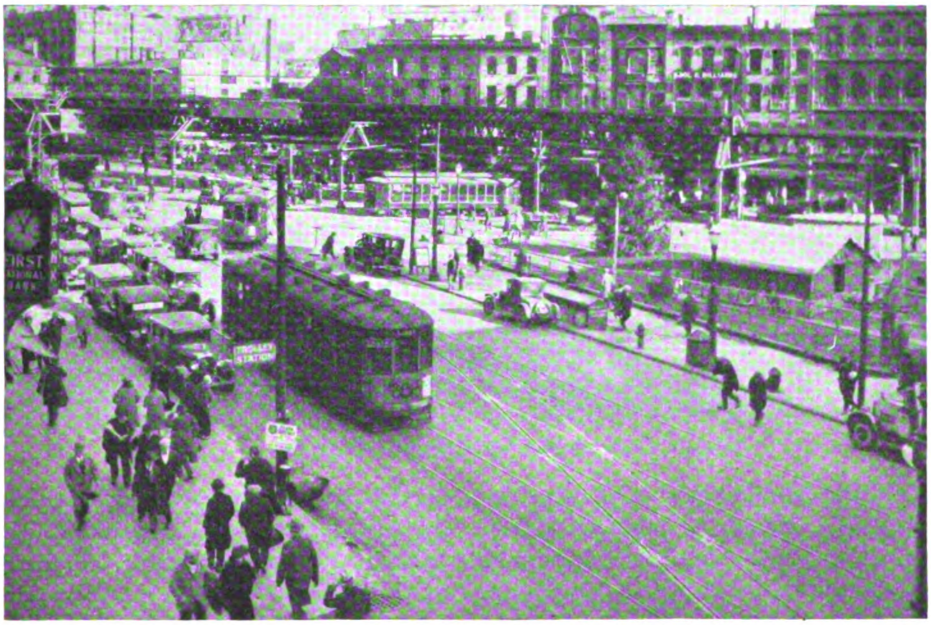


Fig. 1. Surface Cars in a Typical Traffic Scene

Moreover, the traffic is very largely concentrated within a period of an hour and a half in the morning and likewise during one hour in the evening. It is the ability to handle the peak loads of the home-going rush that really governs the size and type of the vehicle and of its equipment. It is no uncommon condition on surface-car routes in the larger cities to have cars carrying loads of 125 passengers operating on thirty-second headways, taking 15,000 people away from the business district between five and six p.m. In the New York subways, ten-car trains operating on ninety-second headways carry 60,000 passengers per hour in each direction from the business area. However, it is more usual in small or medium-size cities to select the rolling stock to meet the conditions of individual lines on which 400 to 800 people must be carried home during the evening rush hour.

From the standpoint of economical operation, large cars on long headways would meet the latter condition admirably. For instance, a line 2.5 miles long would require thirty minutes for a round trip. Two cars, each carrying 50 seated and 70 standing passengers, operating on fifteen-minute headways could carry away 480 people per hour; three cars, on ten-minute headways, could accommodate 720 people. But riders do not like to wait ten to fifteen minutes for a street car. If such long headways are maintained, many will use their autos and many more will walk.

To satisfy this demand, smaller units on shorter headways are quite widely used. These are usually single-truck cars, seating 29 and capable of carrying, without undue crowding, about 26 additional passengers, standing. Five such cars on six-minute headways could carry home 550 people per hour; seven on 4.5-minute headways, could accommodate 715. These cars weigh about 15,000 lb, each; the 50-passenger car weighs about 34,000 lb. Three of the latter weigh very nearly as much as seven of the former; thus the energy consumption would be almost the same in either case. Maintenance of equipment and of tracks is closely proportional to the total ton-miles moved, so there would be little to choose here. The only increase in operating cost, therefore, through the use of the smaller cars is the increased wage of the operators or motormen (assuming that both types are one-man operated). Four extra men are required for the small cars. Their wages would amount to about $2.50 per hour. If forty more passengers (less than 6 per cent of the total) at seven cents each are attracted by the more frequent service the operating company has justified its selection of equipment. In many cases, a reduction of 50 per cent in headways operated has resulted in 25 to 30 per cent increase in riding.

The so-called Birney Safety cars were originally designed to provide a service cheap enough to permit the maintenance of a five-cent fare. They were, twelve years ago, far lighter per seat than anything on rails. These cars are about 35 ft. long, seating 29 passengers if double-end and 35 if single-end. A single truck with 8½-ft. wheelbase is used on which are mounted two 25-hp. motors, usually of either the type having ball bearings on the armature shaft, or that having sleeve bearings. The usual form of platform controller is used; but other special control devices known as safety features were first introduced on this type of car. These consist of a special "dead man's handle" for the controller so interlocked with the air-brake system that if the car operator loses conscious control of the car and releases his downward pressure on the control handle, the brakes are applied in full emergency, sand is applied to the rail to assist in making a quick stop, and the doors, which are normally held closed by air pressure, are balanced so that they can be pushed open by a passenger. These cars were designed to be operated by one man, so the ordinary door control is put into the brake valve and the doors themselves are so arranged that they cannot be opened, save in emergency, until the brakes have been fully applied and the car brought to rest.

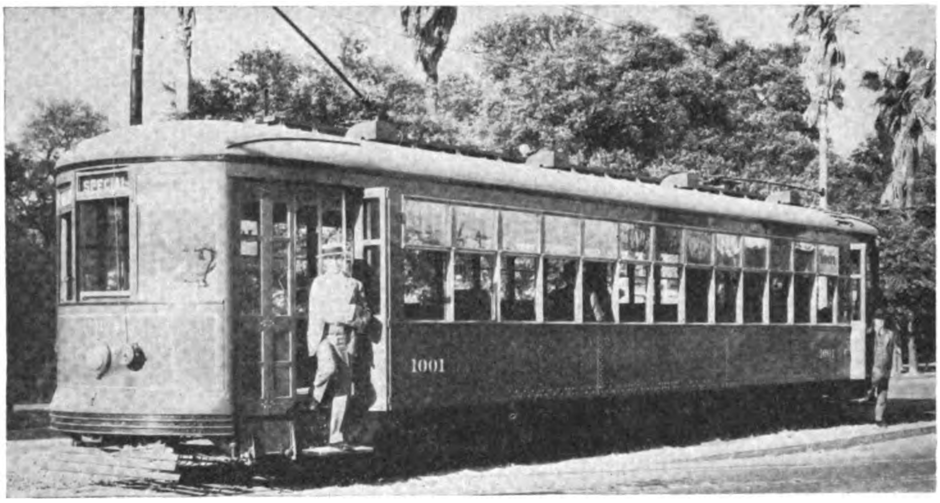


Fig. 2. A New Low-level Steel Safety Car

The more usual type of city car has four motors on double trucks, and has a body seating 44 to 52 people. Such cars, equipped but without load, will weigh from 30,000 to 36,000 lb. The motors in the lighter types will be of 25-hp. rating and, in the heavier, of 35-hp. rating at 600 volts. They ordinarily use platform controllers; but, where train operation is considered desirable, as in Chicago or Oakland (Calif.), either light-weight pneumatic or magnetic type remote control is used. Both types employ groups of contactors, carried beneath the car body. In the former type, the contactors are brought in and dropped out by a cam shaft rotated by an air engine, the engine being controlled by magnet valves energized from a master controller on the platform. In the latter, each contactor is solenoid operated, the solenoids being energized from a master controller. On such cars air brakes are always used. A typical compressor provided in a large number of instances has a capacity of 15 cu. ft. of air per minute, and operates against a pressure of 90 lb. per sq. in. Its distinctive feature is the mounting of the gear in the center of the crankshaft, and a single bearing for the motor armature. Oil is carried up from the bottom of the crank case by the gear and thrown into a splash pan at the top, which carries it to every bearing and allows it to flow back into the reservoir.

Rapid transit cars on the elevated and subway lines of our largest cities are always equipped with two motors and with multiple-unit control, and a large-capacity multiple-unit contactor-type control, usually cam operated, of the previously mentioned type. These cars are ordinarily much larger than surface cars, but because of side doors their seating capacity is not much greater. For instance, the Interborough cars in New York City seat only 52, but have a standing capacity of over 100. The Brooklyn-Manhattan Transit Subway cars, however, accommodate 74 seated, and about 150 standing, passengers. Most elevated cars in New York, Brooklyn, Boston and Chicago are comparable to the Interborough type. Subway cars in Boston and Philadelphia approximate the Brooklyn-Manhattan Transit size. The former weigh about 76,000 lb. and use two 120-hp. motors; the latter weigh about 94,000 lb. and use two 160-hp. motors.

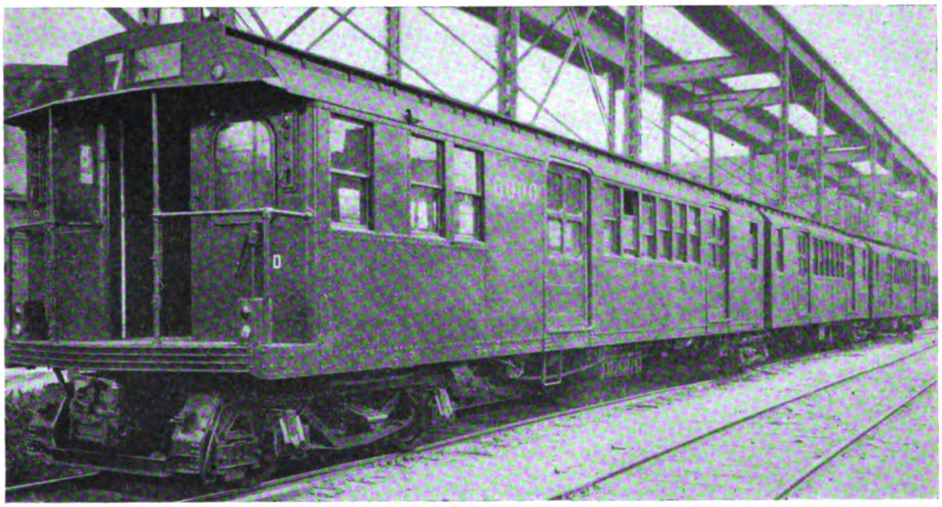


Fig. 3. A Three-unit Articulated Car Used for Subway Service

The most recent design for subway service is the articulated type adopted by the Brooklyn-Manhattan Transit Corporation. These are three body units, mounted on four trucks, two of which are beneath adjacent platforms. Each of these units seats 160 passengers and has a combined seating and standing capacity of 559 passengers. Three units are commonly coupled together in rush hours, with a total capacity per train of 1677 people.

The weight of such a unit, without load, is 172,000 lb. Four 195-hp. motors and two pneumatic cam controllers are used on each three-body unit. A compressor having a capacity of 35 cu. ft. per min. supplies air. By this construction, the weight per passenger handled is reduced from approximately 475 lb., as on the Interborough type, to 310 lb. The riding qualities of these units are excellent, and a reduced number of trucks, motors and controllers have to be maintained. It is probable that these articulated units will be widely used in this class of service.

It seems well to analyze at this point the motor capacity required for these classes of service. Surface cars on city lines are usually geared for 25 to 27 mi. per hr. maximum speed at 550 to 575 volts (the average usually obtained at the cars, though 600 to 625 volts are carried at the substation). Now it only takes about 3 amp. per ton to maintain such speeds against a friction of 25 lb. per ton, which would be the equivalent of about 2 hp. per ton after allowing for the motor efficiency; but the weights must be accelerated to the free running speed and as rapidly as passenger comfort will permit. So, while only 20 to 25 lb. tractive effort per ton may be required to overcome rolling and windage friction, 150 to 200 lb. additional tractive effort per ton may be required for the initial acceleration. This requires motors rating 5 to 7 hp. per ton and capable of developing 35 to 40 per cent more power for brief periods at each acceleration. As an illustration, let us take the standard surface cars used by the Boston Elevated Railway System. These weigh 31,000 lb. empty, and with average load about 17 tons. They use four motors which have an hourly rating of 37 amp. at 600 volts, or 25 hp. The total rated capacity (100 hp.) is thus approximately 6 hp. per ton. These motors will stand accelerating currents of 50 to 55 amp. giving an output for a few seconds at each start of 35 to 37 hp., or 8.5 hp. per ton.

Similarly, the Detroit Municipal Railway operates 425 modern cars seating 52 passengers and weighing with average load about 21 tons. They are each equipped with four motors, rated 51.5 amp. at 600 volts, or 35 hp. The rated total (140 hp.) is 6.7 hp. per ton. The motors, however, will stand 75 amp. accelerating current, which corresponds to 9.7 hp. per ton.

The Brooklyn-Manhattan Transit articulated subway units, weighing with average load about 100 tons and carrying 780 rated hp. of motors have even more power per ton of weight; but such equipment must have not only the ability to accelerate nearly as rapidly as a surface car, or at about 1.75 mi. per hr. per sec., but must have free running speeds of about 45 mi. per hr. Such equipments, with few exceptions, use tapped field control, whereby they accelerate initially on full field, but finally on a weakened field, giving an effect equivalent to a two-speed transmission in an automobile. This permits the use of a greater gear reduction and consequently obtains a higher tractive effort than would be practical on full field operation. The complications of field control, however, have made its use on the usual trolley car impractical and it is rarely used except for high-speed equipments.

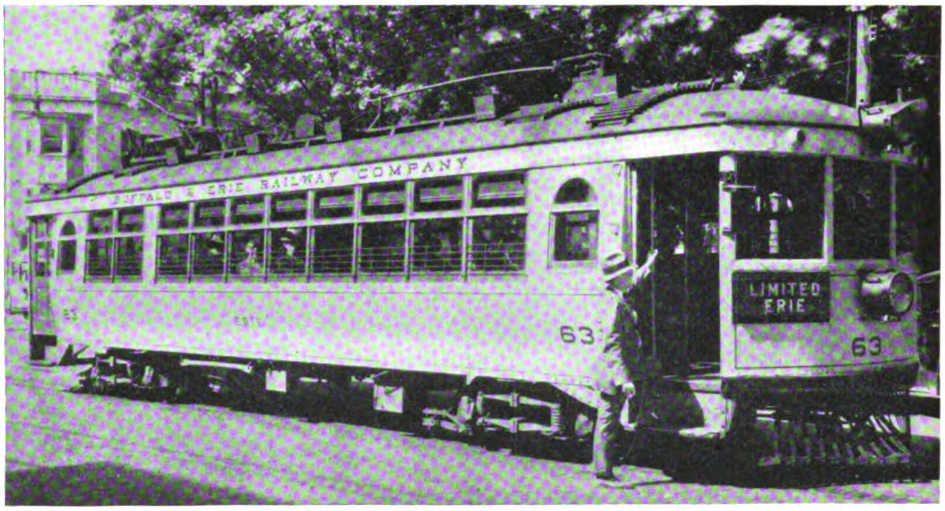


Fig. 4. A Light-weight Double-truck Car for Interurban Traffic

Interurban cars, also, since they frequently run at 45 to 60 mi. per hr., require ample horsepower ratings per ton, although the rate of acceleration in service requiring infrequent stops is less important than where stops are close together. For instance, a number of four-motor tapped-field equipments have been applied to cars, weighing with average load about 21 tons, that will make 45 to 47 mi. per hr. at 600 volts. The rating of the motors of this type used on light weight cars is 40 hp., so these cars have about 7.6 hp. per ton. Others, weighing about 27 tons, carry four 60-hp. motors, or nearly 9 hp. per ton Still others, weighing about 40 tons, use four 90-hp. motors, giving also 9 hp. per ton. The two latter types are geared for 55 to 60 mi. per hr. and seat 56 passengers. The former seats only 44.

The most recent type of vehicle used in mass transportation is the rubber-tired, self-propelled bus. Its efficacy for such service on city streets, as compared to the trolley car, is the subject of much argument. However, they are being used by railway companies in increasing numbers and on more and more heavy traffic routes. For instance, Philadelphia has some 600 on city runs; Newark (N. J.), over 300; Boston 150, etc. The double-deck buses used on Fifth Avenue in New York City are familiar examples, also.

The most commonly used, however, are single-deck buses seating 29 to 33, and capable of carrying in decent comfort a maximum of perhaps 55 to 60. They are propelled by engines that develop about 80 to 90 hp. but which, owing to auxiliary apparatus losses in the charger, pump, fan, etc., transmit not over 65 to 75 hp. to the drive shaft. Such buses weigh when empty 13,000 to 15,000 lb. and with average load about 8 to 9 tons; thus their maximum horsepower per ton is appreciably less than that available for acceleration on the rail car, and the friction of a bus is decidedly higher. Moreover, time is lost in mechanical transmission in the process of gear shifting and in accelerating the engine after each gear shift to its speed of maximum output. The acceleration of the bus is normally much lower than that of a trolley car, and its schedule speed, despite its ability to maneuver through traffic is usually less than the rail car.

However, the development of electric transmission for such vehicles has materially improved their schedule speeds in city service; and some 1800 buses are already equipped with electric drive. The standard equipment for these buses is a direct-current generator direct coupled to the engine and two motors of railway type each connected by a drive shaft to separate gears on the rear axle. The generator is shunt wound, with a teaser field that is connected across the battery for a few seconds at each acceleration. Its output at 225 volts is 295 amp. or 66.5 kw. The rating of each motor is 225 volts and 150 amp., or 33.7 kw. The equipment is capable of transmitting to the drive shaft the output of an engine actually delivering 90 hp.; and the engine builders are increasing their output until they begin to approach this figure. A small controller is used having a forward, reverse, and neutral position, with an additional contact for electric braking, at which point the motors are reversed and put in series with a rheostat.

Such an electric equipment adds about 2000 lb. to the weight of the bus, and about 20 per cent to its initial cost; but by eliminating gear and clutch movements, and by permitting the engine to operate at its point of maximum output while power is required, it makes possible faster schedules, reduces mechanical shocks and torsional strains on the engine and chassis, and gives to the passenger a smoother and less noisy ride. These advantages have led the Public Service Transportation Company of New Jersey to repeatedly add to their equipment to the extent of acquiring a total of 900 equipments during the past three years; the Mitten Management to buy nearly 700 for Philadelphia and Buffalo, and the Capital District Transportation Company to place about 70 of them in service in Albany and Troy.

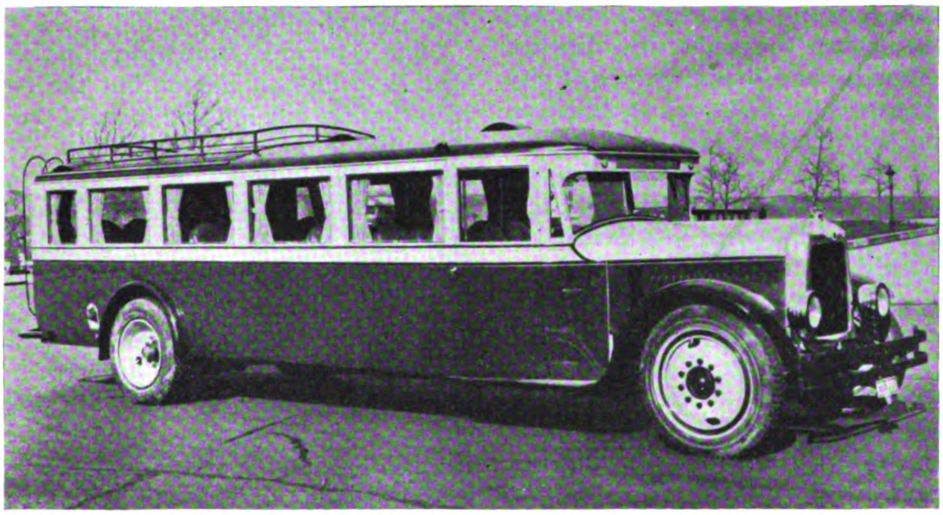


Fig. 5. A Typical Gas-electric Bus

But, assuming that a bus can make as good a schedule as a street car, its smaller capacity and higher operating costs are handicaps to its success. Assume a five-mile route through a large city, extending 2.5 miles in each direction from the business center, making a round trip distance of ten miles, and assume that either rail cars or buses can make the round trip in one hour. If traffic is reasonably heavy, it will require five minute headways for rail cars carrying 100 people each during the peak hours, or 12 cars. This means that 1200 people per hour will be taken in each direction from the business district during the period, say, from 4:30 p.m. to 6:00 p.m. Since the buses carry only 60 each, they would have to operate on three-minute headways, and 20 would be required to take away the same volume of traffic. The twelve street cars will make 120 car miles per hour; the 20 buses, 200 bus miles per hour. The relative cost per hour of operation will be as follows:

	Cars	Buses
Maintenance of Way and Structure	$4.80	$ 0.40
Maintenance of Equipment	3.60	12.00
Crew Expense	7.20	12.00
Power	4.20	6.00
Total	$19.80	$30.40
Increased costs per hour, peak (4 hours)		$10.60
Increased costs per hour, normal (14 hours)		5.30
Increased costs per day,		116.50
Increased cost per year, approximately		42,000

Twenty-three buses, including spares, would cost about $275,000, and the cost of their garage would add $55,000, making a total investment of $330,000. The depreciation and other fixed charges on this would be about $53,000 per year.

Thirteen rail cars would cost $180,000. Tracks and line at $110,000 per route mile, including paving, would cost $550,000. Power and substations of adequate capacity would add $100,000. Shops would cost $40,000. The total investment would be $870,000, or more than double what the bus installation would require. The depreciation rate however, would be decidedly less. The buses themselves constitute the major part of the investment and certainly have not more than ten years life. Street cars have at least twenty years of useful life; and thousands are in service much longer. Average adequate depreciation on electric railway property is only 3.2 per cent. The total fixed charges on the entire investment should not exceed $113,000 per year. This is $60,000 more than on the buses as against a saving of $42,000 in operating cost; so the buses would appear more economical if a new line were to be built and equipped to handle this volume of traffic.

On the other hand, if the rails are already laid, the overhead lines in place, power and substations installed, and the question is simply whether to purchase new rail cars or to substitute buses, both first cost and operating cost will decidedly favor the continuance of rail operations.

Reflection of Light by Terra Cotta Surfaces

Knowledge of the Fundamentals of Reflection Essential to Wise Choice of Material—Daylight and Floodlight Effects—Results of an Extensive Test—Conclusions as to Types of Surface

By **FRANK BENFORD**
Research Laboratory, General Electric Company

THE rapidly increasing use of high-quality brick, artificial stone, and particularly of terra cotta for the façades of buildings of the better class makes the subject of the reflection of light by these materials of considerable interest. A large number of recent buildings are of sufficient civic importance or architectural worth to warrant their being floodlighted so as to stand out against the night sky. America is developing a distinctive and imposing type of city architecture that is particularly adapted to American needs, and it is no easy burden that is laid on the illuminating engineer when he is asked to coöperate in maintaining the beauty of a façade at night, and perhaps to add a touch of fairlyand magic that the light of day seldom gives.

Even in the present transient state of architecture, while the step-back building is in its first stages of development, and floodlighting is the exception rather than the rule, the night appearance of our larger cities has been transformed. For the time being we are inclined to be content with a floodlighting installation that covers the façades at all points, and the details of the engineering do not painfully thrust themselves upon us. This happy condition is largely a touch of that pleasing ignorance that leads to bliss, but this also is transient. As the architect and engineer develop skill in the applications of materials and light to this problem, there will grow up a technique and a critical point of view that will not let us be content with certain conditions that are now acceptable on account of their novelty. Therefore, there need be no apologies made here for going into some of the simpler fundamentals of floodlighting as applied to terra-cotta façades. These fundamentals are well established facts in other arts, and they, along with a vast array of derived data, will soon be the engineering foundations upon which will nightly rise glowing forms of beauty that will charm and delight beyond the "castles in Spain" of our childhood fancy.

One happy result of more attention to façade illumination will be the impression made on the airplane passenger who sees our cities first from the air rather than from a Pullman window. Certainly the chance to make a good first appearance will not be lost, and the backyard view that now greets the curious eye of the stranger will no longer require apologetic explanations. American architecture is rapidly breaking away from ancient traditions and developing along sound engineering and climatic lines, and the increasing simplicity and honesty of construction, which are after all the very soul of architectural beauty, are a challenge to the best skill of the illuminating engineer.

ILLUMINATION FUNDAMENTALS

Two Types of Reflection

Light is reflected from opaque surfaces in two ways that have distinct names, but the two processes of reflection are but limiting cases of the same phenomenon. In the materials in which we are interested the two are so combined as to be almost inseparable, and we can speak of them only by the general names of the "specular" component and the "diffuse" component. The most familiar example of specular reflection is met in the glass mirror of daily use. Here, the incident light is reflected regularly so that an image can be seen in the mirror. Many surfaces such as unsilvered glass, varnish, or a vitreous glaze possess this property of specular reflection; and it is of great importance in the case of the glaze applied to terra cotta. Freshly fallen snow is the most familiar example of a mat surface; and when the snow becomes coated with a thin coating of ice we have one of nature's combinations of a mat surface glazed with a specular surface.

Specular Reflection

The amount of light reflected from a specular surface varies with the angle of incidence, and in the case of some of the more common optical materials this phenomenon is known with great accuracy. Considering only the light reflected from the first surface of a piece of glass having a refractive index of 1.52, the amount reflected at normal incidence is 4 per cent; at 30 deg. incidence it is 5 per cent; at 60 deg. incidence it is 9 per cent; and at 90 deg. or grazing incidence it is 100 per cent. A piece of plane window glass, if receiving light within 2 deg. of grazing is as good a mirror as the best silvered mirror used in the usual manner, and this feature of almost perfect specular reflection at grazing incidence is common to almost all surfaces, many of which we ordinarily do not look upon as specular or glazed.

The application of glazing to a mat surface leads to a composite surface that resembles two separate and independent surfaces, each reflecting a portion of the incident light. The light reflected from the first surface of the glaze may produce a glare spot that locally almost conceals the mat reflection, although the latter may account for 95 per cent of the total light. It is nearly impossible to visually estimate the strength of the two components of a composite surface, and recourse must be had to photometric methods, under known conditions that duplicate to a large degree the actual conditions of installation.

Diffuse Reflection

The other type of surface, the diffuse reflector, is best exemplified by blotting paper. Equal amounts of light falling from any direction upon the blotter cause it to appear equally bright when viewed from any direction. This statement is not altogether exact, for even a blotter may be used as a crude mirror that will give a specular image at near grazing incidence. There are a number of surfaces that are often referred to as "perfect" diffusers. By "perfect" is meant the degree of diffusion, not the fraction of light that is returned. Thus, magnesium carbonate will reflect 98 per cent of the incident light by a process of almost perfect diffusion, and lampblack is an almost perfect diffuser that returns only a few per cent of the incident light. In illuminating engineering there is a constant call for diffusing surfaces that are not only perfect diffusers but also good reflectors; but it may be that the architect wishes a different combination, such as good diffusion and moderate reflection, or high reflection and little diffusion.

Reflection Factors

The "reflection factor" of a surface measures the fraction of incident light that is reflected, but it does not specify the manner in which reflection takes place. A blotter and a silvered mirror may have identical reflection factors although the manners of reflection are at opposite poles. It follows that the architect cannot define his reflecting surface by merely giving the coefficient of reflection. The *manner* must also be specified in some way, and this is extremely difficult to do because the extent and manner of reflection varies with the way in which the illuminating light falls upon the surface. No general rules are possible and recourse must be had to laying down specific conditions under which the surface will be used, and then duplicating more or less completely these conditions in the photometric analysis of the surface.

Daylight and Floodlight

The daytime appearance of highly glazed (specular) or good mat (diffusing) surfaces are related in a rather definite manner but not in a definite degree. Let us assume a façade to be faced with a highly glazed terra cotta having a coefficient of reflection of 0.70 and another façade to have a good mat surface, and also a coefficient of 0.70. Let us view these two façades under a bright overcast sky. Each façade receives equal amounts of light but the glazed surface may appear twice as bright as the mat surface. In the glazed surface we would see a small section of the dome of the sky by direct reflection, but the brightness of the mat surface would depend on the sky brightness from horizon to zenith, *i.e.*, one-half of the dome of the sky. But let us assume that we wish to make a visual comparison between specimens of the two surfaces, and the two specimens are laid on a horizontal surface under the same type of sky. The glazed surface will reflect, as before, a local spot of sky, but the mat surface will reflect light from the entire dome of the sky and hence will be twice as bright as in the first test (and as in actual service), and of course the visual comparison will be in error to this extent. Under illumination coming from the entire dome of a uniform sky the two surfaces will appear identically bright, but this is not a usual condition of illumination for façades. In general the specular surface as used will have a two to one advantage in brightness over the mat surface; and if we depend upon reflected light for the daytime illumination of the floors of city canyons, the specular surface is greatly to be desired.

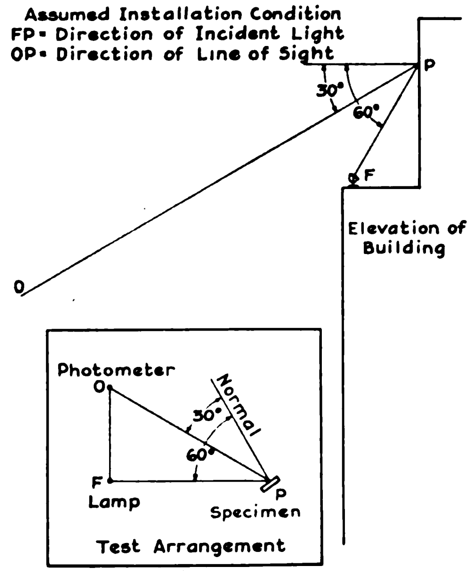

Assumed Installation Condition
FP= Direction of Incident Light
OP= Direction of Line of Sight
30°
60°
P
F
O
Elevation of Building
Photometer
O
Normal
30°
60°
F
Lamp
P
Specimen
Test Arrangement

Fig. 1. Conditions for Determining the Effective Coefficient of Reflection Under Floodlighting

The optical conditions of floodlighting are essentially different from the optical conditions under natural light. In general, daylight has a strong downward slant, while floodlight has an even stronger upward slant. The choice of reflecting materials for effective service under both types of illumination is therefore a matter that requires careful study of the optical properties of the material under consideration.

A large number of specimens of terra cotta have been tested in some detail for their optical properties under the optical conditions illustrated in Fig. 1. The specimen is assumed to be installed on the façade of a step-back building, with the light incident at 60 deg. below the horizontal, the line of sight of the observer being 30 deg. below the horizontal. These conditions are of frequent occurrence and may be taken as typical of a large number of floodlighting installations; but if the installation conditions vary greatly from these test conditions the following data will have to be modified, often to a considerable extent. A good diffuser is obviously to be preferred for floodlighting, and as a specular surface has much to recommend if for daylight service it is evident that a mat surface covered with a transparent glaze is one of the combinations that seems to be a fair compromise between the opposing requirements of daylighting and floodlighting.

Surface Texture

The geometric character of the surface, *i.e.*, the presence of ribs or raised figures has a vital influence on the reflecting properties of the surface; and later some test figures will be given to show the relative effect of having the ribs horizontal or vertical on the face of the building. The rates of depreciation of light reflection by the two surfaces will differ because the horizontal ribs will more readily collect dust, and here is a task in photometry that may have a bearing on future design. If the rate of depreciation is high, the vertical rib may on the average taken over a period of years be the superior light-reflecting surface. Later, one possible solution to this problem will be pointed out.

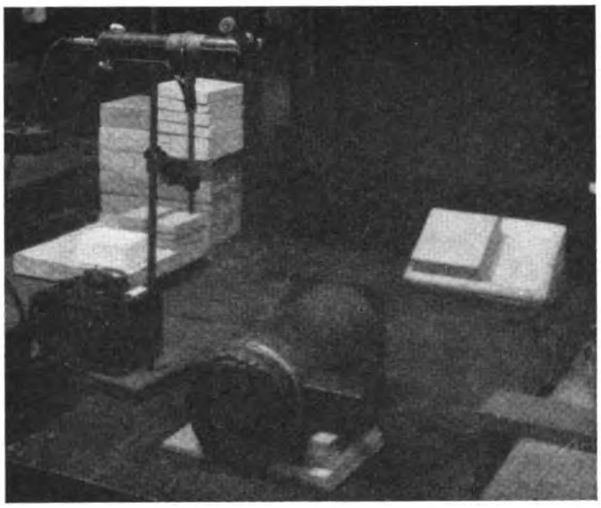


Fig. 2. Arrangement for Testing the Coefficient o. Reflection of Terra Cotta

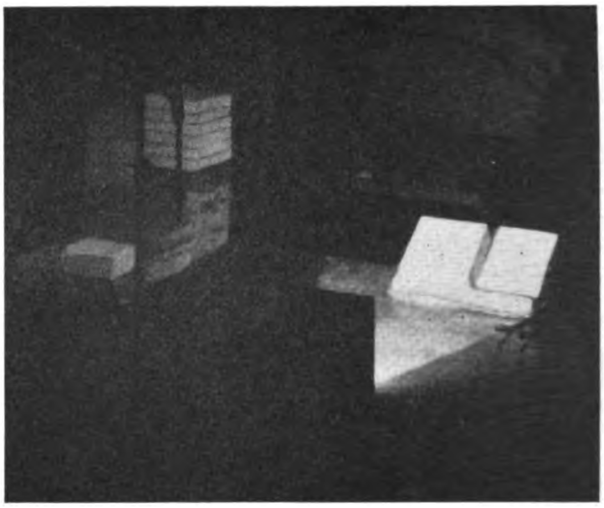


Fig. 3. Illumination of Specimen of Terra Cotta for Testing Coefficient of Reflection

TESTS

Various photometric tests have been made on a group of thirty-eight specimens of terra cotta; and an analysis of three of these tests brings out several important features about the action of these different specimens under different types of illumination.

One set of tests was of the total reflection under diffuse illumination. This corresponds to the reflection of diffused light received from the entire dome of the sky. These measurements were made with a reflectometer that adds the reflected light regardless of its direction of incidence or its manner of reflection. The same group of samples was given a second test under illumination striking at 60 deg. from the normal, and the brightness of the surface was photometered from an angle of 30 deg. from the normal. The two sets of data were arranged in the order of the coefficients as determined by the first test, and also in the order of brightness as determined by the second test. If the two measurements lead to the same order of samples then the two sets of data should plot along a straight line. In Fig. 4 the points are plotted, and the deviation from the diagonal line (or line of identical order) is of such an extreme degree as to indicate a complete lack of identity between the two sets of readings. We can therefore say with absolute assurance that the only certain way to judge surfaces of this character for effectiveness under given conditions is to test under identical physical and optical conditions. Any variations in the relations between sample, light, and observer will lead to erroneous impressions and mistaken opinions.

Horizontal and Vertical Ribs

A group of twelve tooled, or dragged, surfaces were photometered for brightness under the foregoing angular conditions with the ribs first horizontal and then vertical. The gain by placing the ribs horizontal ranges from 0.03 to 0.15, or from six to thirty per cent. Here is evidently a factor that is worth studying for the sake of economy. There are several other factors that should be considered in connection with tooled or dragged surfaces. A horizontal rib collects dirt on its upper surface, but the lower surface is the effective reflecting surface and the depreciation for floodlight incident from below will differ greatly from the depreciation for daylight incident from above. Such a surface would tend to maintain its night appearance while it gradually darkens in daylight.

There is an optical illusion in the appearance of a horizontal rib that is of interest and importance. Under illumination that crosses the rib, one side of the rib is bright and the other side is dark or even in absolute shadow. Such a surface, when viewed from a distance that permits the ribs to be recognized, will give an impression of greater brightness than will a flat surface of equal average brightness. This is because we judge the brightness of a surface largely by the brightness of its more prominent features and tend to ignore the less prominent features, which in this case are the shadows of the ribs.

It is possible that the same error of judgment will occur in the case of a façade broken by cornices and ledges when viewed from a great distance, and here may be a great aid in illuminating surfaces that are in a difficult position to reach with projected light.

Texture and Glaze of Surfaces

The great importance of the texture of a terra cotta surface as distinguished from its coefficient of reflection, or degree of glazing, can best be illustrated by photographs of various specimens under different conditions of illumination. It should be kept in mind that we are dealing with light, and a sand grain in the terra cotta that is only a thousandth of an inch square is just as effective in scattering the light that strikes it as is a rib that is a quarter-inch wide. Therefore, any surface that in its unglazed state feels rough to the touch is a rough surface optically, and at several hundred feet it may visually react to illumination in the same way as a deeply pitted surface.

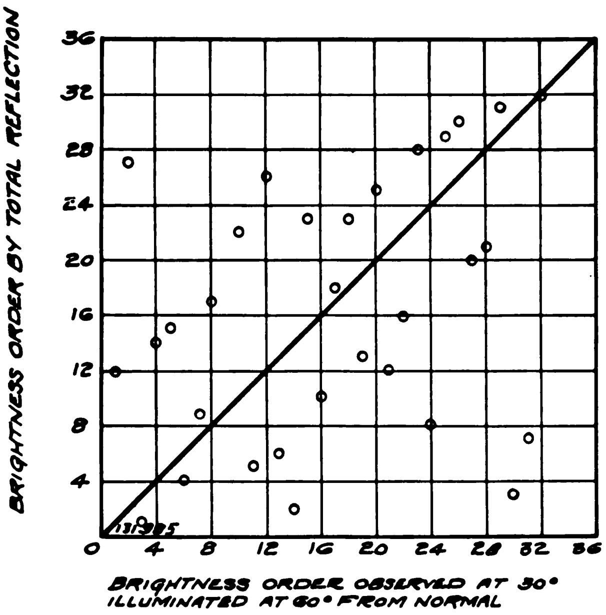


Fig. 4. Comparison of Total Reflection and Effective Reflection

In order to illustrate this and other facts the illumination of the specimens was arranged as follows: On the left panel of each group of three photographs, Figs. 6 to 44, the light is a beam coming from 60 deg. below the normal with the line of sight 30 deg. below the normal, as in the previous tests. The center panel is illuminated by a combination of direct and diffused light that corresponds to daylight when the sunshine penetrates a lightly overcast sky. The third, or right-hand panel, is illuminated by light incident normally, as occurs when floodlights are installed across the street and on a level with the illuminated surface. We are concerned here mostly with the left and center panels as representing usual floodlight and daylight conditions, respectively.

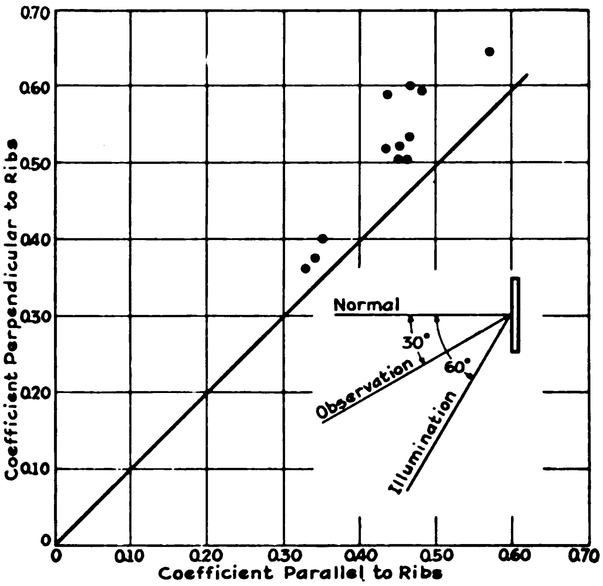


Fig. 5. Influence of Direction of Ribs on Effective Coefficient

The groups of photographs are arranged in the order of brightness under floodlighting conditions as determined by photometric measurement. It will be observed that the photographs do not follow this order any too faithfully. There are too many independent steps in the exposing, developing, and printing to allow photography to be an accurate process for comparing different surfaces, but the relation between the three panels in a group should be fairly accurate on account of all three going through the photographic process simultaneously.

The color of the sample also upsets the photographic comparison, and the gold samples are given a subnormal value due to the low sensitivity of the photographic plate to yellow light.

The first group (Fig. 6) is a simultaneous photograph of white blotting paper under the three types of illumination. This was taken to demonstrate the substantial equality of the illuminations received on the three sections.

In the photographs of the building material, the most noticeable feature is the erratic way in which the left-hand and center panels are related. In general, the rough and unglazed surfaces are relatively brighter under floodlight, and the smooth and glazed surfaces are darker. There are some seeming contradictions to this rule, notably the surface of the specimen in Fig. 7, which is a smooth white surface with a mat glaze.

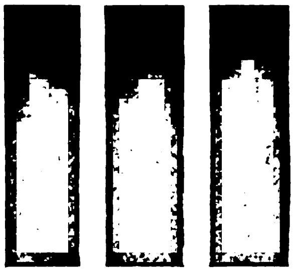


Fig. 6. Blotting Paper

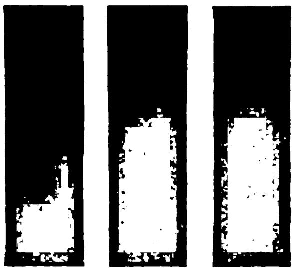


Fig. 7. White, mat glaze

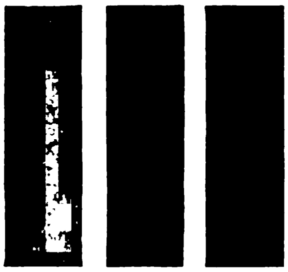


Fig. 8. Cream White, special mottled glaze

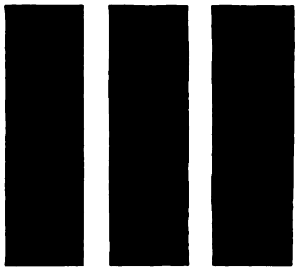


Fig. 9. Cream, rough-ribbed mat glazed

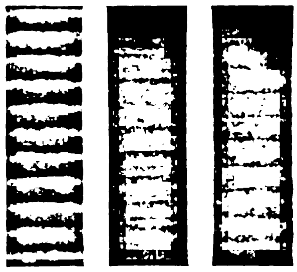


Fig. 10. Cream, 5-line tooled mat glaze

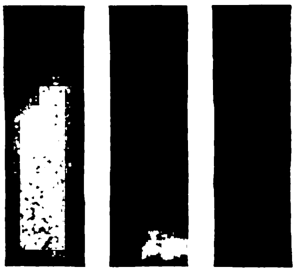


Fig. 11. White, lustrous glaze

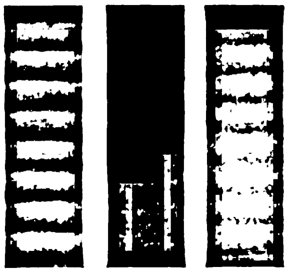


Fig. 12. Cream, 4.5-line tooled, mat glaze

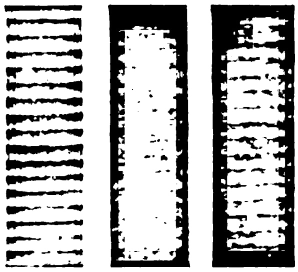


Fig. 13. Cream, 8-line tooled, mat glaze

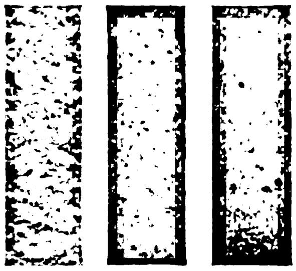


Fig. 14. Buff, rough surface, unglazed

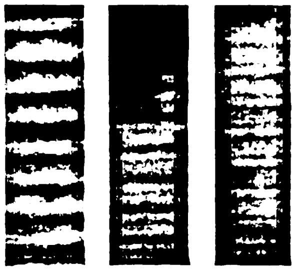


Fig. 15. Cream, 4-line tooled, mat glaze

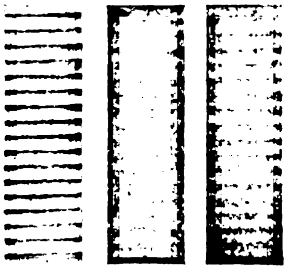


Fig. 16. Cream, 8-line tooled, mat glaze

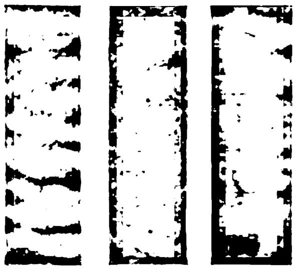


Fig. 17. Cream, irregular drag, mat glaze

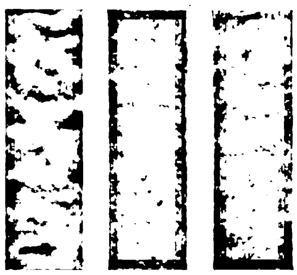


Fig. 18. Cream, special pitted surface, mat glaze

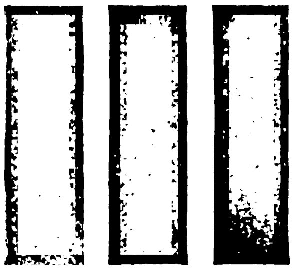


Fig. 19. Buff, smooth texture, unglazed

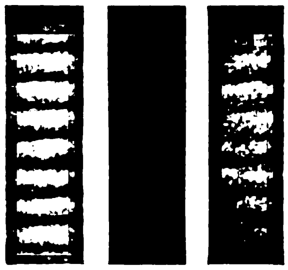


Fig. 20. Cream, 4-line, complex tooled, mat glaze

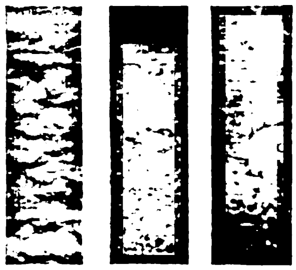


Fig. 21. Buff, medium irregular texture, unglazed

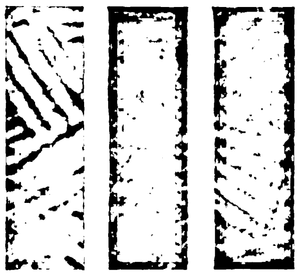


Fig. 22. Cream, irregular basket weave, mat glaze

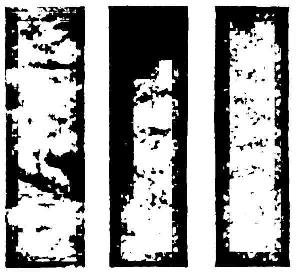


Fig. 23. Buff, extra rough texture, unglazed

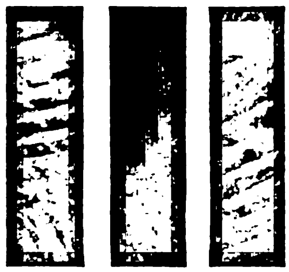


Fig. 24. Buff, irregular criss-cross combing, unglazed

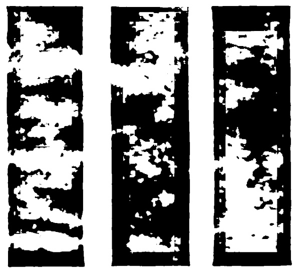


Fig. 25. Buff and White, mottled, rough texture, mat glaze

Fig. 6. Photographs of Blotting Paper as a Standard of Comparison **Figs. 7 to 25. Photographs of Specimens of Terra Cotta Having Different Surfaces**

In the left-hand panels of three, the photographs were made by light from 60 deg. below the normal. In center panels, by diffused incident light. In right-hand panels, by light incident along the normal.

Fig. 26. Buff, irregular combing, unglazed

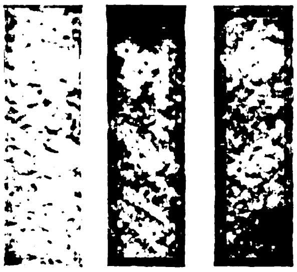


Fig. 27. Light Gray, rough texture, unglazed

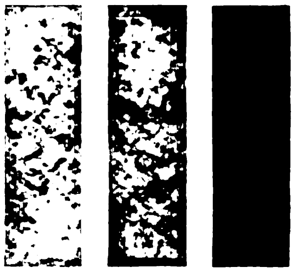


Fig. 28. Mottled Dark Cream, rough texture, unglazed

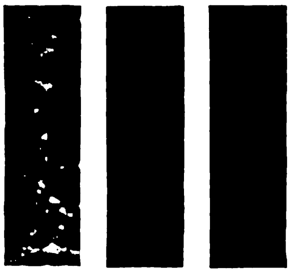


Fig. 29. Light granite, rough texture, unglazed

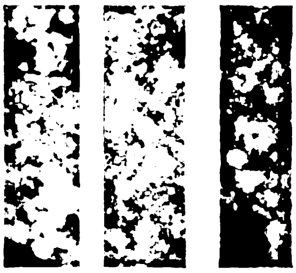


Fig. 30. Conglomerate, Buff and White, rough texture, glazed

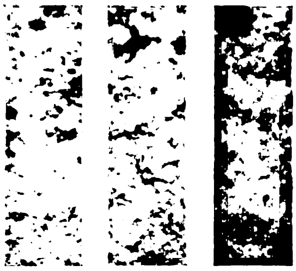


Fig. 31. Conglomerate, White and Green, rough texture, glazed

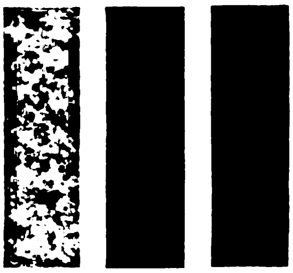


Fig. 32. Light Granite, rough texture, unglazed

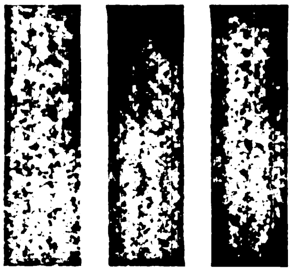


Fig. 33. Light Gray, 6-line tooled, smooth unglazed

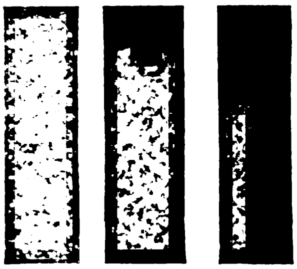


Fig. 34. Limestone, plain texture, unglazed

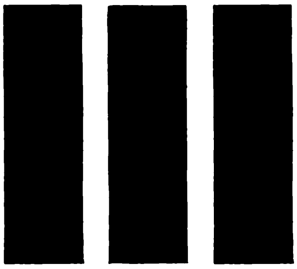


Fig. 35. Light Gray Limestone, 8-line tooled, unglazed

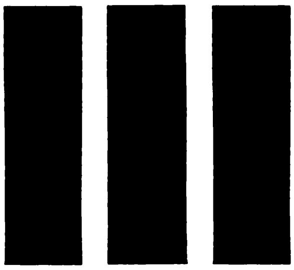


Fig. 36. Light Gray Limestone, 6 and 8-line tooled, unglazed

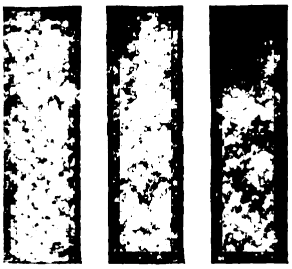


Fig. 37. Buff, White and Gray, conglomerate, unglazed

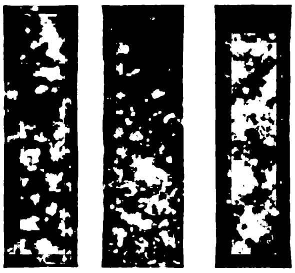


Fig. 38. Light Granite, conglomerate, lustrous finish

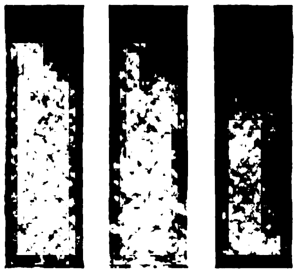


Fig. 39. Buff Mottled, smooth texture, mat glaze

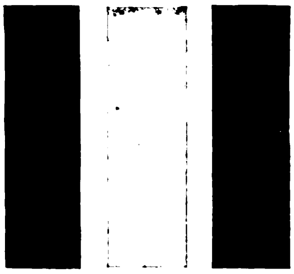


Fig. 40. Gold, smooth texture mat glaze

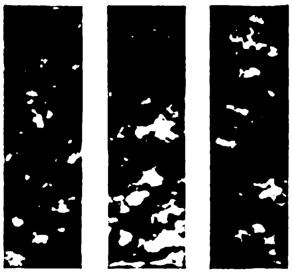


Fig. 41. Dark Granite, lustrous glaze

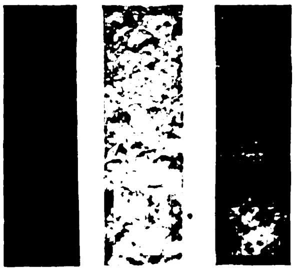


Fig. 42. "Nugget Gold," mottled, rough texture, combination mat and lustrous glaze

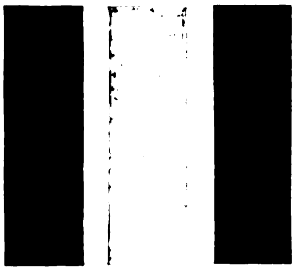


Fig. 43. Gold, smooth texture, lustrous glaze

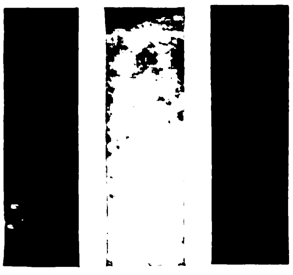


Fig. 44. Gold, smooth texture, lustrous glaze

Figs. 26 to 44. Photographs of Other Specimens of Terra Cotta Taken in the Same Manner as Those in Figs. 7 to 25

In the left-hand panels of three, the photographs were made by light from 60 deg. below the normal. In center panels, by diffused incident light. In right-hand panels, by light incident along the normal.

An examination of the surface with a low-power microscope will show that the reflecting surface is granular, and is optically as rough as the coarsest ribs. The smoothness is in the glaze, which fills in between the grains and presents an almost plane surface. The specimen in Fig. 44, which is gold plated, has an optically smooth surface and the effectiveness of this surface under floodlight is practically zero.

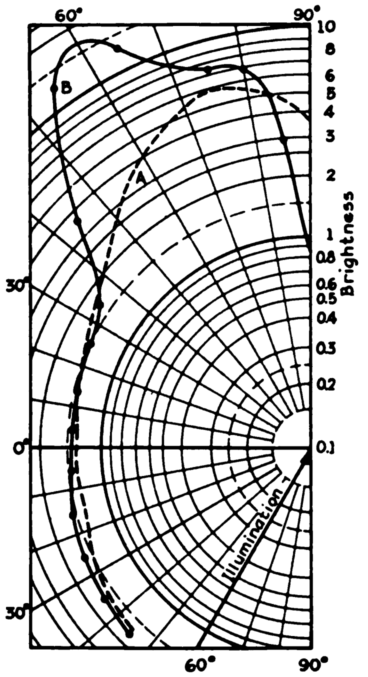


Fig. 45. Relative Brightness in Plane of Incidence with Illumination at 60 Deg.

Curve *A*: Material of Specimen in Fig. 8.
Curve *B:* Material of Specimen in Fig. 7.

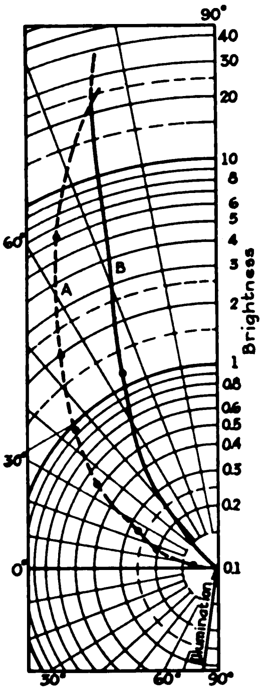


Fig. 47. Relative Brightness in Plane of Incidence with Illumination at 80 Deg.

Curve *A:* Material of Specimen in Fig. 8.
Curve *B:* Material of Specimen in Fig. 7.

The second striking feature is that the surfaces, as viewed under daylight (center panels) are strongly influenced by the degree of glaze. The photographically brighter surfaces are uniformly described as having a "lustrous glaze" and the degree of brightness is almost independent of the character of the surface beneath the glaze. Thus the specimens in Figs. 11 and 38, although at opposite ends of the scale of

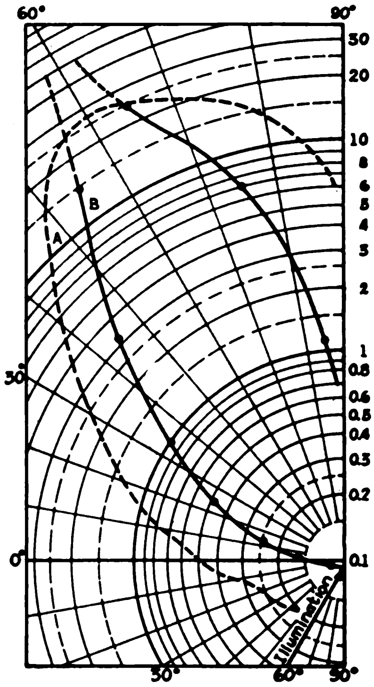


Fig. 46. Relative Brightness in Plane of Incidence with Illumination at 60 Deg.

Curve *A:* Material of Specimen in Fig. 42.
Curve *B:* Material of Specimen in Fig. 43.

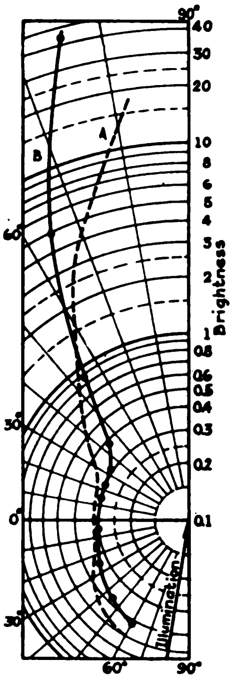


Fig. 48. Relative Brightness in Plane of Incidence with Illumination at 80 Deg.

Curve *A:* Material of Specimen in Fig. 42.
Curve *B:* Material of Specimen in Fig. 43.

floodlight reflection would, under daylight, be almost equally bright. Here the small percentage of total light that is concentrated into a glare spot is the dominating factor in determining its daylight appearance.

The character of the glare spots in the center panel reveal the true nature of the optical surface. The specimen in Fig. 7, having a soft or mat glaze over a mat surface, is wholly free from glare spots. The specimen in Fig. 11, with a lustrous glaze over the same surface, has a sharp glare spot at the bottom of the panel. This spot is from the surface of the glaze and not from the granular backing.

A general comparison of all the photographs brings out quite strongly the floodlighting superiority of rough surfaces over smooth surfaces, of unglazed surfaces over glazed surfaces, and furthermore the presence of glare spots from the sun is seen to be a serious defect of the highly glazed surface.

Several specimens, Figs. 33 and 35, were too small to be photographed with the ribs running horizontally, and that in Fig. 35 makes an interesting contrast with the one in Fig. 36 which is greatly similar in many respects but was photographed with the ribs horizontal.

A final study of the beneficial effect of roughening the surface is given in Figs. 45 to 48. The arrows toward the center of the web indicate the direction in which the light was incident. The photometric analysis was made by observing the brightness of the specimen at various angles in the plane of the incident light. The observed variations in brightness were so great that it was necessary to use a logarithmic scale in order to make the curves legible at all points. Surface *A* in Fig. 45 is greatly inferior to surface *B* in coefficient of total reflection, but on account of the greater roughness of *A* combined with a lesser degree of glaze, surface *A* is almost equal to *B* for floodlighting; and as the incident light approaches grazing incidence the rough surface becomes markedly superior, as shown in the lower quadrant in Fig. 47. If the ceramic finishes (surfaces beneath the glaze) were identical, there would result a clear superiority for the rough and thinly glazed surfaces. A considerable part of the light from *B* is concentrated in forming a glare spot in the direction of the upward extension of curve *B*, Fig. 45, and this light is obviously wasted under floodlight, although as previously pointed out it may be desirable in the daytime.

In the case of gold surfaces, Fig. 46, the increase due to changing from a smooth texture to a rough texture is of the order of three to one; and in addition the surface takes on a texture that is much more interesting and pleasing than the flat uniformity of the smooth surface.

Figs. 47 and 48 illustrate the changed brightness of the four samples when the incident light is at 80 deg., or within 10 deg. of the grazing direction. The useful downward light is greatly weakened and in the case of the white samples the rough surface is slightly superior instead of being slightly inferior as it was under light incident at 60 deg. from the normal. The gold samples show the same change, the rough surface being six times as effective as the smooth surface, and this change illustrates the increasing importance of roughening the surfaces as the light is incident at near the grazing angle.

The photographs and test data here given merely indicate the complexity of the problems that the architect and engineer must face. A close study of the factors involved will lead to a more careful selection of building materials and, it is hoped, to a thorough consideration of the task of the illuminating engineer who all too often is not remembered until the building is finished. His best efforts are then often of little benefit to either his peace of mind or his reputation. Earlier coöperation between architect and engineer will make them each wonder how he ever did business without the aid of the other, and every man with a seeing pair of eyes will call them blessed.

Gas-electric Buses for Boston

The increasing interest being manifested in electric drive for gasoline buses is evidenced by the decision of the Boston Elevated Railway Company to put 28 such equipments in service in that city. This decision was reached after a thorough trial of three of the electric vehicles for a period of a year.

The 28 buses will be built by three different companies. Ten will be manufactured by the Twin Coach Company of Kent (O.); eleven will be manufactured by the American Car and Foundry Motors Company of New York; and seven will be built by the Versare Corporation of Albany (N. Y.); the electrical equipment in each case being of General Electric make.

The ten Twin Coach buses will each be equipped with two electric generators for supplying power to two driving motors, and the seven Versare buses will each have one electric generator to furnish power to two driving motors. Similar details for the remaining eleven buses have not yet been determined.

The Series Capacitor Installation at Ballston, N. Y.

Three-phase 1245-kv-a. Rating—Connected in Loop Circuit—Increases Load-carrying Capacity of Line—Improves Voltage Regulation—Equipment and Operation

By E. K. SHELTON
Lightning Arrester Engineering Department, Pittsfield Works
General Electric Company

On March 2, 1928, there was placed in commercial service on the lines of the New York Power and Light Corporation at Ballston (N. Y.) the first installation of its kind in the world; a series capacitor for the compensation of transmission-line reactance. With the closing of the control switches in the substation, three single-phase 415-kv-a. capacitors were cut into the 33,000-volt 3-phase circuit running from Ballston to Amsterdam. The immediate effect was an increase of the load current in this circuit from 33 to 67 amp., and a very appreciable increase in the voltage on the distribution buses at Ballston. These actions illustrate one of the possibilities of the series capacitor: *viz.*, the control of load distribution over loop circuits. The Ballston-Amsterdam 33-kv. circuit forms a part of one side of a loop system between the Spier Falls generating station and Amsterdam. With the compensation of the inductive reactance in the Ballston-Amsterdam circuit, the total impedance in that side of the loop is reduced and a corresponding increase made in the load-carrying capacity of that line.

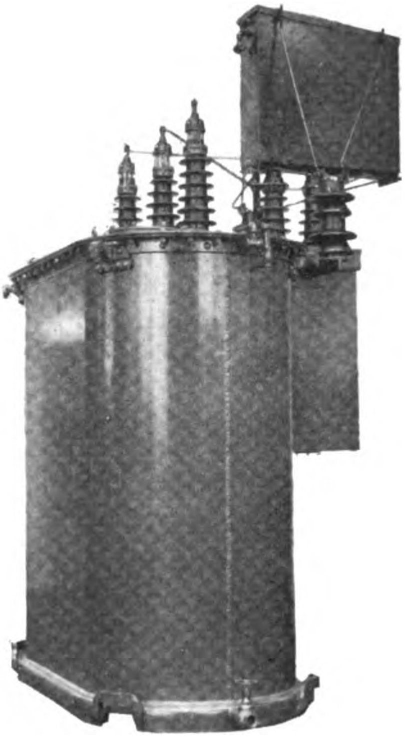


Fig. 1. A Single-phase Assembly of One of the Three Outdoor Series Capacitors

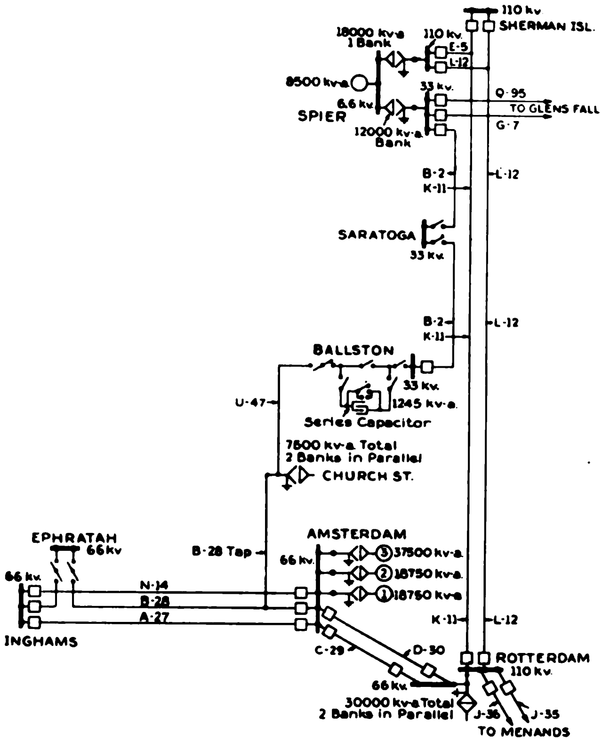


Fig. 2. One-line Diagram of the Connections of the Series-capacitor Installation in the System of the New York Power and Light Corporation

The use of the series capacitor to compensate for transmission-line reactance is a new advance in electrical engineering, for the practical application of such a capacitor has presented problems that, until recently, had been unsolved. The solution, as represented by the Ballston installation, includes several interesting engineering achievements. The fundamental principle involved is the compensation of the inductive-reactance effect of the line by the introduction of capacitive reactance in series. This eliminates the line inductive reactance as a factor in voltage regulation and load limitation, and leaves only the factor of line resistance. In other words, with 100 per cent compensation of the inductive reactance of the line, the line as a unit is tuned to the fundamental 60-cycle frequency. Therefore, as far as the line alone is concerned, the current during a short-circuit period would be limited only by the ohmic resistance of the line. In actual operation, however, this excessive flow of current does not take place, for in a transmission system there is other inductive reactance (generators, transformers, connecting transmission lines, etc.) besides the line reactance that is compensated for by the series capacitor. Thus the short-circuit current is limited by the *system* reactance. Short-circuit currents may nevertheless build up to ten or twenty times normal load value, and if permitted to pass through the series capacitor would cause ten to twenty times the normal voltage drop across the capacitor. Obviously, means must be provided for protecting this unit from such high-voltage conditions. The development of an effective and reliable protective equipment for this purpose has required much research and numerous laboratory tests.

In the Ballston installation, the line reactance is 24 ohms to neutral. To compensate for this reactance, a capacitance of 110 mf. per phase is required. Each single-phase capacitor unit is rated 415 kv-a., 3170 volts, 132 amp., at full load, 60 cycles; and is insulated from ground for the line potential of 33 kv. Fig. 1 shows a complete single-phase assembly of a series capacitor on the factory floor. The installation of these units in the outdoor substation at Ballston is shown at about the center of the frontispiece photograph of this issue. A close-up view of them appears in the cover illustration. The location of this series-capacitor installation, and its connections to the adjacent system of the New York Power & Light Corporation, are indicated on the single-line diagram in Fig. 2.

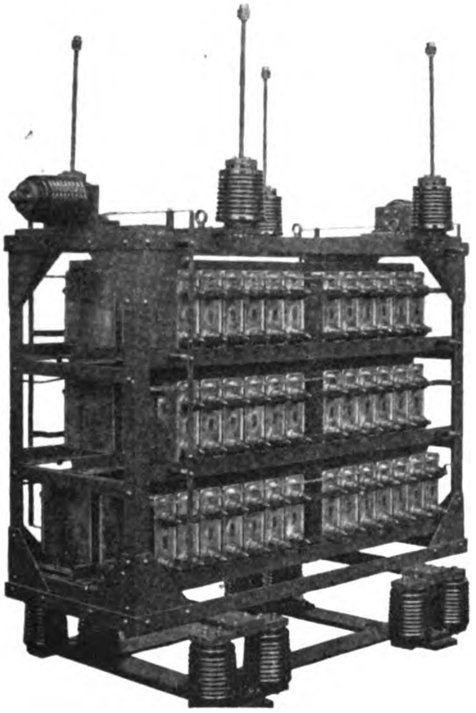


Fig. 3. Internal Assembly of One of the 415-kv-a. 3170-volt Single-phase Series Capacitors Mounted on Insulating Rack for 33-kv. Line Potential

The series-capacitor assembly of the required microfarad rating is built up of units similar in appearance and detail to the capacitors ordinarily used in shunt connection on industrial power circuits for power-factor correction. The same oil-treated paper dielectric is employed, and the unit assemblies are contained in hermetically sealed cases. The group of units for the 415-kv-a. assembly is rigidly mounted on an angle-iron rack structure which is in turn, as shown in Fig. 3, mounted on an insulating rack to provide the insulation required to ground for the 33-kv. line potential. The rack and capacitor cases are electrically connected to one terminal of the capacitor bank in order to maintain a definite potential on the structure. The whole assembly is immersed in oil in an elliptical steel tank. The oil is required only as insulating medium because the loss in the capacitor is almost negligible (amounting to only 750 watts at full load rating for each single-phase capacitor).

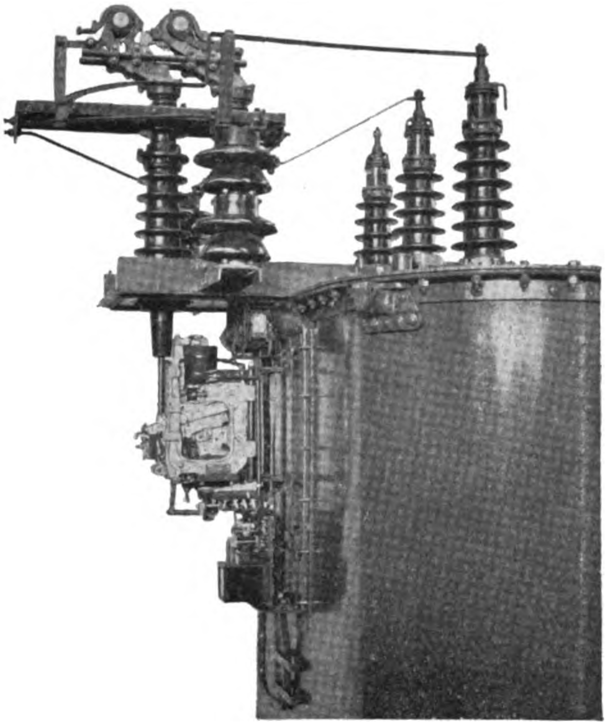


Fig. 4. Close-up View of Capacitor Protective Equipment

To visualize the capacitor in another way, each single-phase assembly represents an area of 6200 sq. ft. of dielectric, or a bulk of approximately 800 lb. of paper through which the power load on this circuit is transmitted.

The protective equipment is designed to take care of two possible abnormal conditions: first, a short-circuit of the line on the load side, which would throw line voltage across the capacitors; and second, a dielectric failure in the capacitor itself, which would shunt the load current through the short-circuited section. To provide this two-fold protection a combination of a special spark-gap and a shunting contactor, actuated by a high-speed mechanism, has been developed and tested under conditions of short-circuit current many times that which can be obtained under actual service operation. Fig. 4 shows the protective equipment, with housings removed, mounted on the tank and cover of the capacitor assembly. The spark-gap is formed of tungsten-faced hemispherical electrodes to give fast operation and to avoid pitting and burning of the gap incident to the severe arcs caused by short-circuit currents. This sphere-gap, combined with a powerful magnetic blowout and with shunting contacts, is mounted on insulators on an extension of the tank cover. The high-speed mechanism with the control relays is supported on the side of the tank directly below. The mechanism operates the contacts by means of an insulated rod carried up through one of the gap-supporting insulators.

The sphere-gap is adjustable, and for this installation is set to break down at 150 per cent of normal full-load voltage drop across the capacitor. When, due to short-circuit or overload, the voltage across the capacitor rises to this value, the gap arcs over forming a shunt path for the short-circuit line current thus preventing further rise of voltage across the capacitor. Through control relays, operated from current transformers in the line and gap circuits, the overload current trips the high-speed mechanism which then closes the shunting contactor and extinguishes the arc at the sphere-gap. The mechanism operates extremely fast. It completes its performance within 0.01 sec. In addition to its closing function, the high-speed mechanism is adjusted to open the shunting contactor and cut the capacitor back into service when the line current has decreased to approximately normal full-load value.

Thus the protective equipment automatically short circuits the capacitor at a predetermined voltage, introducing the line reactance with its current limiting effect, and then cuts the capacitor into service again when the circuit conditions become normal.

To protect against the possibility of a dielectric failure in the capacitor, the high-speed mechanism is designed to trip automatically and close the shunting contactor. This is accomplished by means of an impedance relay which has normally balanced current and potential windings energized from current and potential transformers connected across the capacitor. Since current and voltage increase in direct proportion with respect to the capacitor, the balance is maintained at all loads. Should a failure of dielectric occur, the potential winding of the relay is de-energized and the relay closes its contacts, tripping the high-speed mechanism and short circuiting the capacitor. In this operation, all three units are short circuited out of service and are locked out until a relay is reset by hand.

The current and potential transformers required for the operation of the control circuits are supported from the tank cover over the capacitor assembly. The third bushing on the cover is employed to bring out the gap circuit through a current transformer which is separate from that carrying the normal load.

This protective equipment was thoroughly tested at the factory for the complete range of short-circuit-current values up to 10,000 amp. and operated with entire satisfaction at all values. The same protective scheme can be applied to series capacitors for either higher or lower circuit voltages. Other possible methods of protection have been considered and tested, but they lack the effectiveness and reliability of the one which has been here described. The service results of this first installation have continued the satisfactory record made at the factory.

With this practical demonstration of the ability of the series capacitor to compensate for transmission-line reactance, and its possibilities from an economic standpoint, it is seen that a device of fundamental importance has been made available to transmission engineers. With the elimination of line reactance, alternating-current transmission takes on the advantages of direct-current transmission. Where line reactance limits the load that can be carried over a given circuit, the application of a series capacitor may easily double the load capacity of the circuit. The service possibilities of the series capacitor now having been made available, alternating-current power transmission has entered a new era. Great interest in the development of the series capacitor has been evidenced by the larger utilities, and the operation of the Ballston installation is being closely observed by transmission engineers. This pioneer installation will be followed by others as soon as the usefulness of the series capacitor is fully appreciated. Thus the capacitor is rapidly expanding from its original limited field of application to a broad field of service in power transmission and distribution.

Single-stage Centrifugal Air Compressors for Cupola Blowing

A new type of cupola-blowing equipment has been designed for the purpose of improving the melting operation in foundries by automatically insuring the supply of a constant *weight* of air to the cupola, irrespective of atmospheric conditions and the resistance to the passage of air through the cupola.

Instead of operating the cupola by supplying a constant *volume* of air, or air at a constant *pressure*, the new equipment supplies a constant *weight* of air or oxygen. The reasons for this scheme of operation are as follows: A definite weight of oxygen is required to burn a certain weight of coke. Where operation is based on a constant volume of air, instead of on a constant weight, the weight of oxygen blown into the cupola in cold weather is excessive (because the air is then heavy) and this results in oxidized or burned iron. On the other hand, in warm weather, when the air is lighter, an insufficient weight of oxygen results in slow melting and spongy iron.

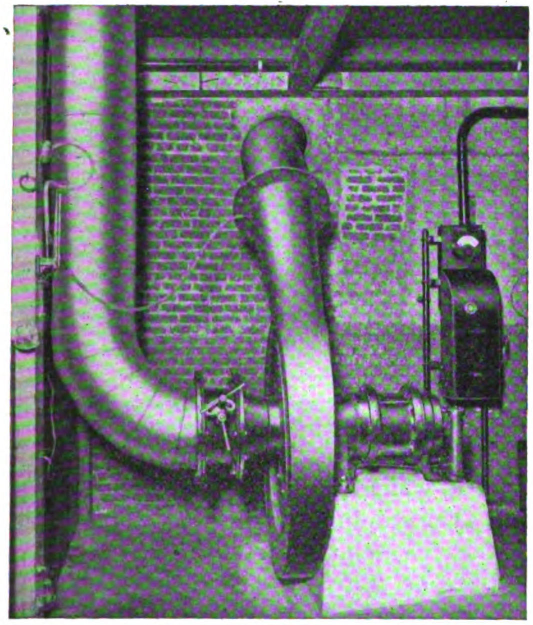


Fig. 1. Typical Installation of a Single-stage Centrifugal Air Compressor for Discharging a Constant Weight of Air Per Minute to a Foundry Cupola

The new equipment will be manufactured with either hand or automatic control, and can be set to give the proper weight of oxygen to meet varying operating conditions. A meter indicates the amount of air flowing. By slight adjustments of the mechanism, the amount of air can be kept constant and the oxygen delivered will remain constant regardless of atmospheric changes.

The automatic gate and control panel can be set to obtain the best operating conditions. The operator has then only to start and stop the compressor and, since there are no wearing parts to affect the volume or pressure of the compressor, there will be no changes in the weight of oxygen supplied to the cupola for a given setting.

Constant-speed motors are used with all sets, except when direct-current motors are required. As a result, the weight of air flowing into the cupola is not controlled by variable speed, but by opening and closing the blast gate. This increases or decreases the resistance in the pipe line so that the total resistance through the pipe line and cupola is just sufficient to allow the proper weight of air to flow. When the resistance through the cupola increases, the blast gate is automatically opened wider and the total resistance is kept the same. Likewise, when the resistance in the cupola decreases, the blast gate partially closes so that the total resistance is kept the same and the weight of air flowing remains constant.

The equipment has totally enclosed bearings provided with forced lubrication, thereby minimizing the possibility of wear from the dust prevalent in a foundry. A gear-type oil pump, with a gravity feed from an oil reservoir located in the base of the main bearing housing, provides the lubrication to all bearings. This pump is also totally enclosed and is immersed at all times in a bath of oil.

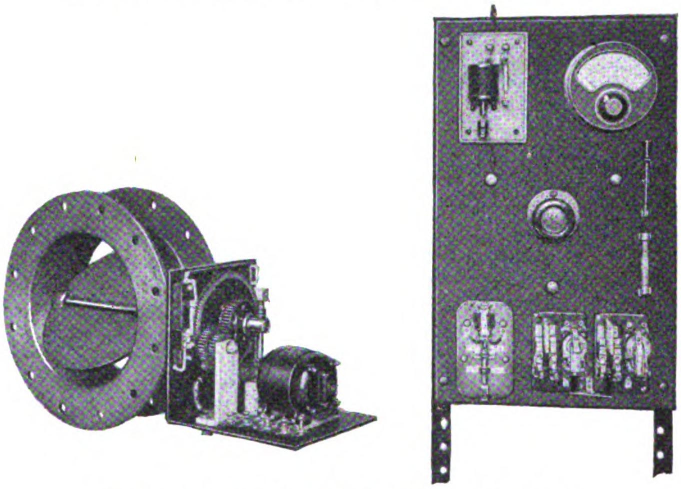


Fig. 2. Automatic Gate and Control Panel Used in the Equipment Shown in Fig. 1

One-piece shaft construction eliminates all couplings, belts, or gears; and the entire equipment is compact, requires but small floor space and a simple foundation. The casing can be swung to any of eight different positions to simplify installation. All parts are readily accessible, and complete inspection can be made in a few minutes without the necessity of removing the pipes.

All single-stage units are equipped with 230-volt, direct-current motors, or 2- or 3-phase, 60-cycle, 220-, 440- or 550-volt motors. Units that are larger than 58 hp. in rating may be driven by 2200-volt alternating-current motors, and steam turbines can also be used for the various sizes.

Synchronous Converter Armature Windings

Lap Windings—Wave Windings—Number of Phases and Poles—Taps on A-c. End—Relationships

By B. L. ROBERTSON

Engineering General Department, General Electric Company

MOST textbooks and handbooks treating of synchronous converters do not go much into the details of the actual connections of the slip-ring leads to the armature winding, nor mention the possible numbers of commutator bars which may be used. They assume that the student is already thoroughly familiar with armature windings from his previous work in direct-current machines, which assumption in many instances is not correct. This article is written for the purpose of clarifying some of the details of the construction of converter windings about which the student of synchronous machines often inquires. A treatment of wave windings in converters, which are uncommon and on which little information can be found, is also included.

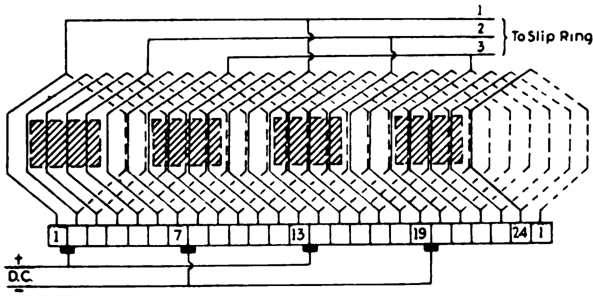


Fig. 1. Lap or Multiple Winding of 24 Full-pitch Coils in Double Layer for a Four-pole Converter

The armature of a synchronous converter is primarily that of a direct-current generator, but the winding is also tapped at the end opposite the commutator and leads are brought out to slip-rings. The number of leads depends upon the number of phases and poles, and upon the type of winding. The armature may be lap or wave wound, each type presenting different problems in connecting the machine to the alternating-current supply. Lap windings are the most common and are simpler to analyze. Wave-wound converters are rarely used in practice, but their study is quite interesting.

Definitions

It is perhaps well to describe first the two types of armature windings—*lap* and *wave*—and to define *phases* as applied to converters. Although these terms are probably clearly understood, to restate their definitions at this point will aid the discussion which follows.

Lap, or multiple, windings are those in which the two ends of a coil are fastened to adjacent commutator bars. Fig. 1 shows such a winding in the developed form for a four-pole machine. It is double layer (two coil sides per slot), the full lines representing the conductors in the top of the slot and the dotted lines those in the bottom. The total number of commutator bars, or coils, assuming a simple winding, may be even or odd. It will be observed that there are four parallel paths through the armature between the supply lines.

Lap-wound multipolar machines consist essentially of a number of bipolar units which operate in parallel electrically but which physically are connected in a closed circle. The winding is tapped at similar points and it can easily be seen that in multiple-wound armatures there are as many paths between brushes as there are poles. Therefore, in order to utilize all of the conductors, the number of brush sets required is also equal to the number of poles.

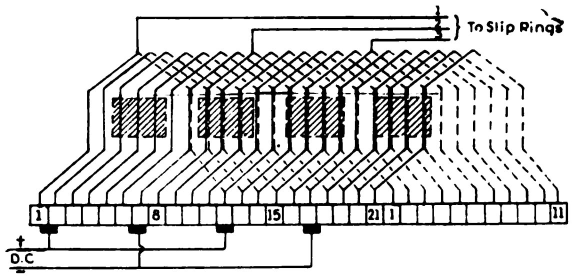


Fig. 2. Wave or Series Retrogressive Winding of 21 Coils in Double Layer for a Four-pole Converter

Wave, or series, windings have the coil ends fastened to commutator bars approximately two pole pitches apart, as shown in Fig. 2. The number of coils or commutator bars for a simple wave-wound machine may be even or odd, depending upon the number of poles. It is determined from the following expression:

$$n = k\frac{p}{2} \pm 1 \tag{1}$$

where

n = number of coils or bars
k = an integer
p = poles

A two-pole winding may be considered electrically as either lap or wave. In the first case the coils would have the closed diamond shape, and in the second case they would of be the "frog-leg" type. A two-pole wave winding then, which is nothing more than a lap winding, may have an even or odd number of commutator bars. This is readily observed if a few simple cases are sketched.

Wave windings of more than two poles are not, in general, repetitions of the two-pole case. This is due to the fact that, in traversing the armature periphery once, the coil end must close on either of the commutator bars which are adjacent to that from which the start was made. This condition is fixed by the ± 1 in Equation (1). Therefore, the total number of coils must not be divisible by the number of pairs of poles. This leads to the result that a machine having an even number of pairs of poles must have an odd number of coils and cannot have an even number, while a machine with an odd number of pairs of poles may have either an even or odd number of coils. These facts are combined in Equation (1).

Consideration of commutation requires that the sides of a coil have nearly the same position under adjacent poles. That is, a coil must have very little voltage generated within it while being commutated, and hence its sides must lie in the neutral zones as shown in Figs. 1 and 2. At this position the coil encloses maximum flux. The condition of a coil spanning exactly one pole pitch, however, is seldom found for most windings are of fractional pitch.

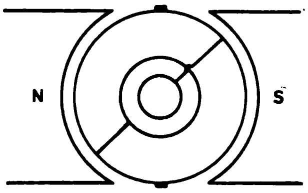


Fig. 3. Single-phase

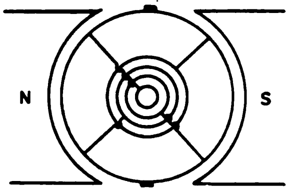


Fig. 4. Two- or quarter-phase

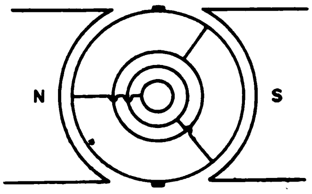


Fig. 5. Three-phase

Figs. 3 to 5. Slip-ring Connections for Two-pole Lap-wound Converters

Fractional-pitch windings are those in which the coil sides do not lie in slots 180 electrical degrees apart. Short pitching is the usual practice with lap windings, the object being to decrease the amount of copper required for end connections. In such a winding the induced voltage in a coil is necessarily slightly less, and the commutation zone is widened, thus decreasing the leakage reactance or electrical inertia of the coils. These factors promote good commutation. There is no advantage in short pitching single-turn wave windings, but there is a gain in short pitching coils which have more than one turn.

A point of note, perhaps mostly of academic interest, concerns the meaning of phases and arises particularly in reference to converters. A phase is the section or sections of an armature winding between the connections to two slip-rings electrically adjacent in time phase. This definition is based on internal considerations of the armature and is at variance with the accepted designation which considers only the number of phases in the alternating-current system to which the converter is connected.

In accordance with the foregoing definition, the single-phase converter is really a two-phase machine, the two phases being in parallel. Furthermore, as viewed in this light, the quarter-phase converter is strictly four-phase and not two-phase. Two-, three-, and four-ring converters would then be spoken of as two-phase, three-phase, and four-phase machines. Treated thus, it would not be necessary to refer to the exceptions so often appearing in reference to two-ring and four-ring converters, their analyses then falling directly in step with that of all multi-ring machines.

This particular point appears later in Table I for wave-wound converters, in which it is seen that the three-phase case logically belongs between those of the single-phase and two-phase cases.

As a corollary to the definition that has been given, may be stated that the time-phase angle α between voltages of adjacent phases is

$$\alpha = \frac{360^\circ}{r}$$

where

r = number of slip-rings or phases.

Lap-wound Converters

The lap winding may be treated in either of two ways: one dealing with a certain number of phases and a varying number of poles; and the other assuming a definite number of poles with different phase operation. The two-pole case, which is the simplest, incurs no difficulties with any number of phases except that the number of coils, or commutator bars, should be divisible by the number of slip-rings. The armatures of two-pole converters have no equipotential points and the entire winding must be traversed before returning to the potential of the starting point. Such a winding can therefore have only one circuit per phase and one connection per slip-ring. Figs. 3, 4, and 5 show the diagrammatic connections for one-, two- or quarter-, and three-phase converters respectively. The same scheme holds for an n-ring machine.

Any direct-current commutating machine must be furnished with equal voltages between all adjacent segments, equally displaced in time phase. This constitutes a closed symmetrical polygon of voltages; and the armature coils of the converter, looking at them from the alternating-current end, are grouped in a rather different manner from those of the alternator or synchronous motor. Fig. 6 shows the phase groups of a three-phase two-pole full-pitch eighteen-coil synchronous motor, the numbers given corresponding to the phase in which that coil side is connected. For the converter armature, the phase distribution is given in Fig. 7, this arrangement following from similarity to the developed winding in Fig. 1. The phase rotation for both the synchronous motor and the converter is in the order 1, 2, 3, and the m.m.f. waves thus move at the same speed. It will be noted that the distribution factor for the converter is somewhat less than that for the motor. In this case, the copper of the motor is being used 15 per cent more efficiently than in the converter.

Consider now the single-phase machine having more than two poles. Since a multipolar lap winding has as many equipotential points as it has pairs of poles, there will be as many brush sets as there are poles, and similarly as many slip-ring connections. Each phase contains $\frac{p}{2}$ parallel circuits and all slip-rings thus have one lead per pair of poles, any two consecutive leads to the same ring being 360 electrical degrees apart. Referring to Fig. 8, it is seen that the single-phase two-pole case is given in triplicate, and six leads are brought out from the armature, three to each slip-ring. They are symmetrically placed so that all of those connected to one ring are at corresponding positions with reference to the field poles.

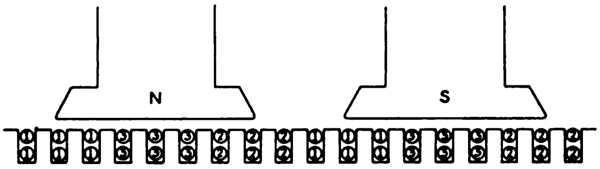


Fig. 6. Phase Belts of an 18-coil Two-pole Synchronous Motor

Likewise, for the three-phase six-pole converter, Fig. 9, three leads connected to the armature at equidistant points are fastened to the same slip-ring, the total number of leads being nine. Any two consecutive connections to the armature are 120 electrical degrees apart and in traversing a pair of poles we pass through the three phases and return to the slip-ring from which we started. It can thus be said that for an n-ring converter the total number of taps to the armature will be equal to the product of the number of pairs of poles and the number of slip-rings, two adjacent taps to any one ring being two pole pitches apart.

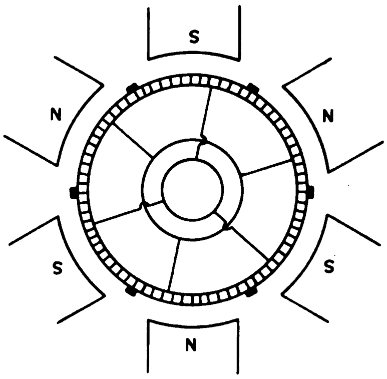


Fig. 8. Slip-ring Connections for a Six-pole Single-phase Converter

The number of commutator bars in a lap-wound converter should be divisible by the product of the number of pairs of poles and the number of phases. This can be expressed as

$$\frac{2b}{pr} = \text{an integer}$$

where

b = total number of commutator bars.

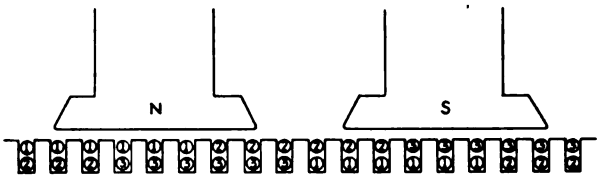


Fig. 7. Phase Belts of an 18-coil Two-pole Synchronous Converter

As far as the generator end of the converter is concerned there may be any number of coils or bars, but from the slip-ring end other conditions must be considered. As an illustration, a four-pole three-phase converter might have 96 commutator bars. This means 48 bars per pair of poles and the number of coils in each phase group would be 16. The winding is balanced. If, however, 100 bars were used in this machine, 50 would be allotted to each pair of poles. Two phase groups would carry 17 coils and the third would contain 16 coils, an unbalance being the result. Although this unbalance might be slight, operating conditions would hardly justify such a winding.

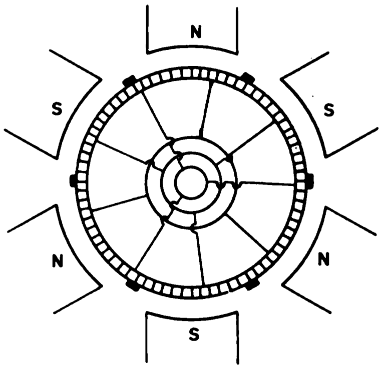


Fig. 9. Slip-ring Connections for a Six-pole Three-phase Converter

Wave-wound Converters

As has been mentioned, wave-wound converters are not common and appear only as small machines. These converters are of interest, however, because their armature connections are different from those of the lap-wound machine. The wave-wound armature for the two-pole converter is the same as the lap-wound armature, because in either case the coil ends are fastened to adjacent commutator bars. Therefore the schematic diagram for the one-, two-, and three-phase wave-wound machines may be represented by Figs. 3. 4, and 5 respectively. The number of bars must be even for one and two phases, but may be even or odd for three phases. To obtain a balanced winding it may be said that the total number of bars must be divisible by the number of slip-rings.

For the wave winding, we have seen that the connections between the direct-current lines and the armature were multiples of those for the two-pole case, the factor being the pairs of poles. This was done to aid commutation, but was not required. However, this idea of increasing the connections to the armature cannot be carried over to the alternating-current end of the converter, with the addition of extra slip-ring taps, in an endeavor to improve the current distribution. A multipolar wave winding must be traversed before returning to the potential of the starting point. Therefore, like the bipolar winding, there can be only one circuit per phase and only one tap may be made between any ring and the armature regardless of the number of poles.

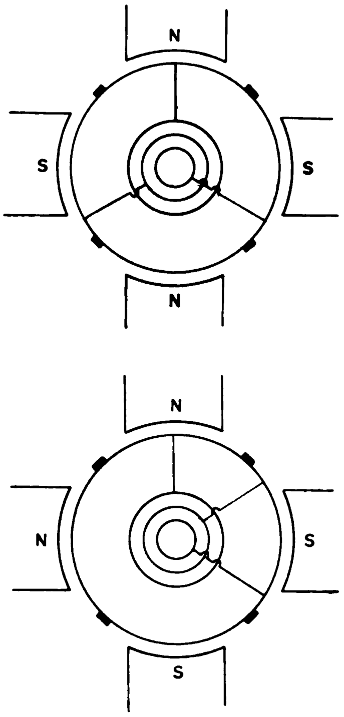


Fig. 10. Alternative Methods of Connecting Slip Rings to the Wave-wound Armature of a Four-pole Three-phase Converter

Referring to Figs. 1 and 2 it is seen that, for the lap winding, two symmetrically placed taps are brought out to each ring in order to utilize all of the armature conductors, while for the wave winding all of the coils are in use with only one tap per ring. In the latter case if more than one tap were connected to any ring these taps would be across coils which, at full voltage generation within the coils, would cause large short-circuit currents to flow. In any case, these taps could not be symmetrically placed because the total number of coils is not divisible by the pairs of poles.

Wave-wound converters having more than two poles are quite limited as to choice in the number of phases, because of the possible numbers of coils or bars which may be used for balanced windings. Table I shows why the wave winding is so rarely met.

TABLE I

POSSIBLE VALUES OF *n* FOR WAVE-WOUND CONVERTERS

n = number of coils or bars
x = any integer, including zero

Poles	1 Phase 2 Taps	3 Phase 3 Taps	2 Phase 4 Taps	6 Phase 6Taps
2	$2x$	$\frac{3}{2}\pm\frac{3}{2}+3x$	$2\pm2+4x$	$3\pm3+6x$
4	——	$3\pm0+6x$	——	——
6	$2x$	——	$6\pm2+12x$	——
8	——	$6\pm3+12x$	—— ——	——
10	$2x$	$\frac{15}{2}\pm\frac{3}{2}+15x$	$10\pm6+20x$	$15\pm9+30x$
12	——	——	——	——
14	$2x$	$\frac{21}{2}\pm\frac{9}{2}+21x$	$14\pm6+28x$	$21\pm15+42x$
16	——	$12\pm3+24x$	——	——
18	$2x$	——	$18\pm10+36x$	——
20	——	$15\pm6+30x$	——	——
22	$2x$	$\frac{33}{2}\pm\frac{9}{2}+33x$	$22\pm10+44x$	$33\pm21+66x$
24	——	——	——	——

The values of n were derived from the expression,

$$n = k\frac{p}{r} \pm 1.$$

The poles p were first assumed, and k was then varied to obtain values of n divisible by r, the number of rings. In order to show the progression of n as p is increased, the tabulation has been made in the particular manner given.

The slip-ring connections to the armature winding may be made in either manner shown in Fig. 10. In both cases the leads are 120 electrical degrees apart.

For the sake of convenience, the illustrations used in this article have been of the simple winding composed of two coil sides per slot. The general case, however, must include more than this number, and in usual design a slot contains four, six, or eight coil sides. Under such conditions, the proper number of coils per phase may require that one coil every certain distance along the armature surface be omitted in order that a balanced winding result. This necessity is then fulfilled by leaving the coils inert. They are termed *dummy* coils and act only as a filler in the slot. There are often two dummy coils per phase, depending upon the number of slots and the coil sides per slot.

Low-voltage A-c. Networks

Part V: SECONDARY CABLES

By D. K. BLAKE

Central Station Engineering Department, General Electric Company

THE primary cables from the station or substation to the network area are usually three-conductor except where 2300- or 4000-volt circuits utilize the existing single-conductor cables. The mains and branches are usually of single-conductor cable because of the single-phase network transformers. The conductor sizes are usually tapered in two or three steps as illustrated by the practice of one company which uses 4/0 three-conductor cable for feeders, 2/0 single-conductor for mains and No. 2 single-conductor for the branches from the mains to the transformers.

One company has standardized on three-phase transformers; and 26,000-volt 3-conductor cable is used throughout. After the feeder cables enter the network area they are looped through the oil-filled potheads mounted on the transformers. This practice simplifies the vault installation by eliminating T-joints and thereby reducing joint troubles.

The secondary cables are single-conductor except in a few cases where three-conductor cable was previously used in the radial system. One company is also making use of some Edison tube which has been in operation for years on the direct-current system. The single-conductor cable is more suitable for splicing the numerous consumers' services than is the three-conductor cable. When considering three-conductor *vs.* single-conductor cables, it is necessary to take into consideration the relative current-carrying capacity and voltage regulation. These factors are covered in the following contribution by F. H. Buller:

"The current carrying capacity of a lead-sheathed cable depends on the following factors:

(1). Whether the cable is a single or three-conductor cable.

(2). The spacing and arrangement of the cables.

(3). Whether the cable sheaths are bonded together or grounded, or both, at more than one point.

(4). The copper and duct temperatures.

"Throughout this consideration a copper temperature of 80 deg. C. and a duct temperature of 40 deg. C. are assumed. It is assumed also that the cable sheaths are bonded together every few hundred feet, as this is the usual condition of operation.

"The effect of spacing and arrangement of cables is shown in Table IX. The figures are percentages based on the three-conductor cable as 100 per cent. The spacing between adjacent conductors was assumed to be four inches.

"The effect of the presence of primary circuits on secondary circuits in the same duct bank, with a given ambient condition in the secondary duct, was found to be very small. A reduction of two per cent in secondary current carrying capacity or an increase of four per cent in secondary losses for a given current should cover this effect. On the other hand, the effect of the secondary currents on the primary circuit is quite marked. Table X indicates the magnitude of this effect. The second arrangement is obviously the better. Table X is based on the following assumptions: Nine 500,000-cir. mil cables, insulated for 600 volts, spaced four inches apart. Primary current is assumed to be in phase with the secondary current; secondary current assumed to be five times the primary current. The effect of primary on secondary will decrease as the ratio of secondary to primary current increases; but the effect of secondary on primary will increase as this ratio increases. If three cables of each circuit are placed in one duct, or three-conductor cables are used, the effect of one circuit on another will be negligible.

TABLE IX

CURRENT-CARRYING CAPACITY OF LOW-VOLTAGE CABLES

Cable Size	Three-conductor Cable	SINGLE-CONDUCTOR		
		One Duct	Separate Ducts	
			***	** *
0000 B.&S.	100	114	131	132
350,000 cir. mil	100	111	125	127
500,000 cir. mil	100	112	123	125

TABLE X

EFFECT OF SECONDARY CURRENT ON PRIMARY CIRCUIT

Arrangement	Average Increase in Total Losses All Circuits	Decrease in Current Carrying Capacity (based on hottest cable)
S_1 S_2 S_3 P_1 P_2 P_3 P'_3 P'_2 P'_1	+10%	+10%
P_1 P_2 P_3 S_3 S_2 S_1 P'_1 P'_2 P'_3	−4%	+9%

"The effect of spacing and arrangement upon the inductive reactance of cables is indicated in Table XI. The figures in Table XI are percentages, based on the inductive reactance of a three-conductor cable as 100 per cent. The reactance of the three-conductor cable is small, being of the same order as its resistance. One circuit only is assumed. The foregoing figures represent the average reactance of the circuit as compared with that of a three-conductor cable. When a flat or right-angled spacing is used, the reactance of the outside cables will be greater than that of the middle one. The magnitude of this variation is indicated in Table XII, based on the reactance of one cable of a three-phase circuit with 4-in. equilateral spacing which is taken as 100 per cent."

TABLE XI

EFFECT OF SPACING AND ARRANGEMENT ON INDUCTIVE REACTANCE OF CABLES

Cable Size	Three-conductor Cable	SINGLE-CONDUCTOR CARLE		
		One Duct	Separate Ducts •••	 •• •
0,000 B.&S.	100	117	315	297
350,000 cir. mil	100	115	290	275
500,000 cir. mil	100	115	283	266

It is the usual practice to install the three single-conductor cables in the same duct. It is obvious from the tables that the additional carrying capacity obtained by single-conductor cables is limited to cases where the transformer spacing is short enough to make heating and not voltage regulation the governing factor. This distance will have to be very short to justify the placing of cables in separate ducts, for the gain in carrying capacity is very small compared with the increase in reactance. The cables from the transformer vault to the street mains may well be considered a case where separate ducts would be justified.

TABLE XII

UNBALANCE OF REACTANCE IN SINGLE CONDUCTOR CABLES

Cable Size	Flat Spacing A B C			Right-angled Spacing B A C		
	A	B	C	A	B	C
0000 B.&S.	115	100	115	107	100	107
350,000 cir. mil	117	100	117	108	100	108
500,000 cir. mil	118	100	118	108	100	108

A few network systems use junction boxes with fuses at the intersection of the street mains. Aside from these it is the usual practice to splice all cables solid, without junction boxes or fuses. This practice is based on the belief that faults developing in the secondary cables will burn clear and not reëstablish. This belief is based on the experience of some operators of d-c. Edison systems who have made a practice of burning clear faults that develop on the street mains. It seems that an a-c. arc ought to be easier to clear at the same voltage than a d-c. arc because there is added the reactance, and the voltage passes through zero.

Kehoe(4) gives a 6-volt resistance drop per 100 ft. as sufficient to melt solder at joints and terminals, and 24 volts per 100 ft. to melt copper in the degree necessary to start an arc. Some engineers consider it advisable to consider only the case of a solid copper-to-copper short and provide transformer capacity at suitable intervals to give 24 volts resistance drop. If the neutral is installed near enough to be short-circuited with a phase conductor, the transformer spacing for a 120/208-volt network will be

$$\frac{120}{6}\times 100 = 2000 \text{ ft.}$$

for the 6-volt drop per 100 ft. and

$$\frac{120}{24}\times 100 = 500 \text{ ft.}$$

for the 24-volt drop per 100 ft. If the neutral is separate from the phase cables then these values are 73 per cent greater, or 3460 ft. and 867 ft. respectively. These values must be decreased to compensate for the impedance drop through the transformers.

There are two problems involved in the practice of burning off cable faults. The first problem is to get enough current to generate the heat required to start an arc at the point of failure. The second is to reduce the voltage across the arc to a point where the arc is unstable and will go out. Therefore, the condition for establishing an arc is just opposite to that for extinguishing the arc.

Single-conductor cables are preferable from the standpoint of clearing faults that develop in the cables. The faults will first develop from copper to lead. The lead, having a much lower melting point (about one-third) than copper, will burn back much faster than the copper. This will result in quickly increasing the arc length to such a point that voltages of the order of 115 volts cannot maintain the arc. It is believed that in the great majority of cases the arc will clear quickly enough to avoid failure in the other cables in the same duct. However, cases are known where the heat in the arc was sufficient to burn the insulation rapidly enough to maintain the arc from copper to lead for a distance of several feet.

When arcs occur from copper to copper they may readily burn clear; but, on the other hand, they may not burn clear until a large amount of damage is done. Experience on Edison systems indicates that the difficulty of burning off copper-to-copper faults without serious consequences increases in a greater proportion as the cable size increases. There are several explanations as to why the larger cables are more difficult to burn clear. The larger cables require more energy to start the arc in case of solid short circuits; hence there is more energy in the arc to make it stable. The larger cables contribute more molten metal, metal vapors, and gases from the larger amount of insulation. Moreover, they occupy a larger percentage of duct space and consequently make it more difficult for the metal, vapors, and gases to escape. The presence of these vapors and gases around the arc has a tendency to make an arc stable by lowering the voltage required to maintain it.

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(4) Underground Alternating-current Network," by A. H. Kehoe, *A.I.E.E. Journal*, June 1924.

One investigator(5) studying how to make a welding arc stable claims that there are three important factors governing the stability of an arc. "Temperature is the first and the most important factor. The second factor lies in the nature of the material of the cathode. . . . The third factor affecting the thermonic emission from the cathode is the gas surrounding the cathode. . . . The gas that has the most marked effect is hydrogen. . . . Even a trace of hydrogen is sufficient to increase the thermionic emission. . . . The carbon monoxide presents an interesting subject for study because in spite of a very high heat of dissociation and a comparatively high ionizing potential, it is the gas in which the welding arc can be maintained with the greatest ease."

The factors of "temperature" and "gas" are the ones of interest in considering the effect of cable size on the ability to burn off cable faults. The larger the cable the higher is the temperature of the arc because of the added arc energy; and the increased restrictions of the duct prevent sufficient air circulation to cool the arc. The larger the cable the more gases are available from the insulation and the easier they are confined around the arc. The paper insulation saturated with oil would seem to be a worse offender than rubber. It is obvious that hydrogen and carbon are present in either and that the heat might produce them in sufficient quantities to have an appreciable influence. It may be questionable as to whether carbon dioxide or carbon monoxide is first formed, but Alexander claims that "carbon dioxide is dissociated in the arc into oxygen and carbon monoxide."

Another factor tending to maintain arcs on larger cables is the higher arc voltage available because of the lower cable impedance. One engineer explains this by means of the curves shown in Fig. 27. The curved line gives the volt-ampere characteristic of an arc with a fixed spacing of electrodes. This spacing will not vary appreciably with different sizes of conductors as long as they are in the duct. The spacing will not vary while burning because both conductors burn back at the same rate. The curve shows that with a certain current flowing a certain voltage is required to maintain the arc. If, for example, a 500,000-cir. mil cable is used, the current flowing causes a drop (e') through the resistance of the cable, giving the voltage (e) across the arc. This voltage (e) is greater than required to maintain the arc at that current, so the current increases and the voltage decreases until a stable point is reached and the arc is sustained. If now a 250,000-cir. mil cable is used, the resistance is doubled and the current in the arc causes a large drop in the resistance of the cable which is sufficient to lower the voltage across the arc below that necessary to maintain the arc. In the case of a-c. arcs the reactance is an appreciable factor and causes a smaller variation between cable sizes. It is probable that the characteristic curves of these arcs come in between the voltampere curves of the 500,000-cir. mil and 250,000-cir. mil cables so that the latter would always clear. Experience indicates very little difficulty in clearing cables of the order of 250,000-cir. mil size. This explanation applies only to faults that occur in the duct at an appreciable distance from a feeding point, and to cases where the mains continue to burn after the transformer cables or Edison feeder have burned clear of the network mains.

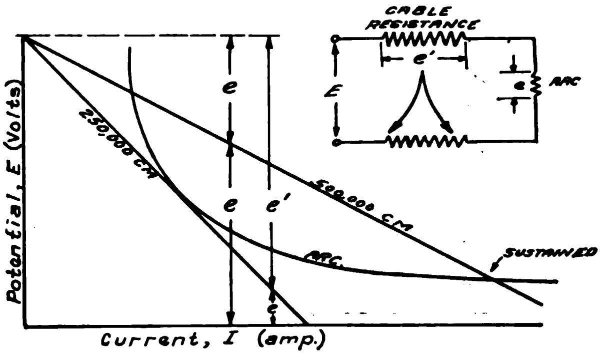

Fig. 27. The Volt-ampere Characteristics of Cables and Arcs

It is obvious that the practice of depending on cable faults to burn off imposes severe duty on the transformers, network protectors, and cables feeding current into the arc. It seems that cable faults occurring in the ducts can be expected, in some cases at least, to burn back to the manhole and that this will take quite a long time—perhaps as long as thirty minutes. Therefore, it is essential that thorough consideration be given every possible scheme or practice that will avoid copper-to-copper shorts and that will tend to make the arc unstable if it should be established. If this is not done there is danger that a fault burning an appreciable quantity of cable before it clears will strain the other equipment and weaken it to such an extent that other faults will soon develop. Thus it is possible to defeat the very purpose of the network by producing a condition wherein failures may occur on two or more transformers connected to separate feeders.

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(5) "Stability of the Welding Arc," by P. Alexander, *A.I.E.E. Journal*, February 1928, p. 109.

A few companies install junction boxes with fuses in the network mains. This seems to be the most effective way of preventing arcs of long duration. On the other hand, experience indicates that great care must be used in selecting and installing fuses; otherwise, they will blow when they should not or fail to blow when they should. If small fuses are used in order to be sure they will blow when arcs occur then there is danger that loose connections, corrosion, and overheating (by overloads or previous short circuits) will so alter their characteristics that they will blow on much lower values and cause a wholesale blowing of fuses under overload or fault conditions. It is very difficult to restore service on a network after a wholesale blowing of fuses. To avoid this, the tendency has been to use fuses having a high blowing point believing that, while these will take a long time to blow, they will finally blow before the arc travels too far. A case is known where this practice resulted in the burning of about 200 ft. of cable without the fuse blowing and the arc finally cleared at the junction box. Most engineers do not use junction boxes with fuses because they believe the uncertainty of fuse performance does not warrant the added investment and vault space required for the boxes.

It seems that the fuses in the transformer secondary circuit may be depended upon to go a long way in preventing cable arcs of long duration. When a cable arc develops the adjacent fuses would blow and the additional cable impedance in series with the arc would reduce the arc voltage below the stable point. Referring again to Fig. 27, it will be seen that the cable volt-ampere characteristic would fall below the arc volt-ampere characteristic so that the arc cannot be sustained. The blown fuses would thus serve as a fault locater. The fuses in any transformers near the fault that did not blow should be inspected and replaced if they show signs of overheating or if there is any question about their condition. Of course, there is some uncertainty with the fuses in this case as in the other; but there is no additional expense or space involved, for fuses are usually provided on the automatic network protector now generally used. Furthermore, the network protectors receive frequent inspection and a more accurate check on fuses is feasible.

Single-conductor cable in separate ducts, and careful separation in the vault would considerably reduce the possibility of a copper-to-copper short circuit. The increase in reactance and the value of duct space, however, discourages this practice. The uninsulated neutral cable should not be placed in the same duct with the phase cables, for a copper-to-lead fault is almost sure to touch the neutral and start a copper-to-copper arc. On new construction the bare neutral may be imbedded in the concrete near the duct. The cables should also be bonded promptly after installation and before operation is begun, otherwise if a fault should develop there is danger of destroying much of the cable by the circulating current in the lead sheath flowing through the high-resistance lead-to-lead contacts. The heat generated at these points may be sufficient to melt the lead and burn the insulation. On new construction, some consideration should be given to three or four small ducts for the secondary cables. If this is not feasible then a very large duct, say a 6-in., should be considered. Such a duct may be large enough to relieve the gases and vapors from the presence of the arc. Large ducts will also permit the electromagnetic forces of short-circuit currents to force the conductors apart and extinguish the arc. Another practice to consider is the possibility of using two sets of small conductors for mains; say two 250,000-cir. mil cables instead of one 500,000-cir. mil. The cost of course, will be somewhat increased; but in the case of wide streets the practice of running one set down one side of the street and the other set down the other side may help to reduce the difference by the saving in the length of service laterals. Besides, two sets of mains will give a better system for load division and motor starting because of the reduction in reactance.

(To be continued)

World's Largest Paper Machine

A paper machine which will not only be the biggest in the world but which will also break all previous production records is to be installed at the Fort William (Ont.), mill of the Backus-Brooks Company in Canada.

This machine, made by Bagley & Sewall, will be 304 in. in width and will have a speed range from 625 to 1500 ft. per min. It will be equipped with the General Electric regulator-type drive, manufactured by the Canadian General Electric Company. Power will be supplied by a 1000-kw. synchronous motor-generator set.

The new machine will be the second to be installed at the mill. The first, now being installed, is a 264-in. machine equipped with the same type of drive.

Heaviside's Operational Calculus as Applied to Engineering and Physics

***Part XXIII:* Fractional Differentiation and Integration**
***Part XXIV:* Indefinitely Long Transmission Line or Cable**
***Part XXV:* Problems Involving Convergent and Asymptotic Series**

By DR. ERNST JULIUS BERG
Consulting Engineer, General Electric Company
Professor of Electrical Engineering, Union College

The second volume of Heaviside's "Electromagnetic Theory" contains a great deal of suggestive matter in reference to the interpretation of divergent series and fractional derivatives. Much of it is very difficult to understand, and indeed some of his results are given with a statement that "this is not as simple as it looks."

Heaviside illustrates his use of operational methods by many examples and the final solutions are, as far as the writer has checked them, always correct. But the reasons suggested are not always clear: often they are far from clear.

The difficulties arise in connection with the asymptotic solution when the operational equation contains fractional powers of p.

In some problems the series, obtained by division in rising powers of p, gives the complete solution; in others, certain terms have to be added. This matter is discussed in the next part of this serial.

Heaviside arrived at the following relation:

$$p^{\frac{1}{2}} 1 = \frac{1}{\sqrt{\pi t}} 1 \tag{106}$$

from two different points of view.

1st: From the solution of certain heat problems worked out by Fourier, which will be shown later.

2nd: By generalizing the formula for the derivatives of any powers of t.

It is: $$\frac{d}{dt}\left(\frac{t^n}{\underline{|n}}\right) = \frac{t^{n-1}}{\underline{|n-1}} = p' \frac{t^n}{\underline{|n}} = \frac{t^{n-1}}{\underline{|n-1}}$$

or, more generally $$p^m \frac{t^n}{\underline{|n}} 1 = \frac{t^{n-m}}{\underline{|n-m}} 1 \tag{107}$$

There is no difficulty about this as long as m and n are positive integers and $n \geqq m$. When $m > n$ we get the factorial of a negative integer which is minus infinity and the result is zero.

Heaviside felt that Equation (107) in his system should hold even when m and n are fractions and proceeded to study them.

The unit function would then correspond to $\frac{t^0}{\underline{|0}}$ or we would write $\frac{t^0}{\underline{|0}} 1$. This is rather remarkable in a way since $\frac{t^n}{\underline{|n}}$ as n approaches zero is a peculiar graph. It is a cylindrical surface for all positive and negative values of t, which surface collapses to a point at $t=0$. Therefore the positive real part would look like 1, the "unit function."

But to return to $p^{\frac{1}{2}} 1 = \frac{1}{\sqrt{\pi t}}$ as Heaviside writes it, or $p^{\frac{1}{2}} 1 = \frac{1}{\sqrt{\pi t}} 1$ as we prefer to write it:

If the graph corresponding to $\frac{1}{\sqrt{\pi t}}$ is plotted for various values of t, we find that it starts at infinity and decreases to zero. When this chart is multiplied by the unit function it is in no way modified. Thus $p^n \left(\frac{1}{\sqrt{\pi t}} 1\right)$ is obtained by differentiating $\frac{1}{\sqrt{\pi t}}$ and multiplying the result by the unit function.

The question remains, however, whether Equation (106) is true. It can be tested in various ways.

We may see whether
$$p^{\frac{1}{2}} (p^{\frac{1}{2}} 1) = p\, 1 \tag{108}$$
or whether
$$p^{-\frac{1}{2}} (p^{\frac{1}{2}} 1) = 1 \tag{109}$$
or whether
$$p^{\frac{1}{2}} (p^{\frac{1}{2}} \sin \omega t) = \omega \cos \omega t \text{ etc.} \tag{110}$$

Consider Equation (106) and refer to Equation (107). If $m=\frac{1}{2}$ and $n=0$ we get $= p^{\frac{1}{2}} \frac{t^0}{\underline{|0}} = \frac{t^{-\frac{1}{2}}}{\underline{|-\frac{1}{2}}}$.

$\underline{|-\frac{1}{2}} = \sqrt{\pi}$ as will be shown later.

Thus $p^{\frac{1}{2}} \frac{t^0}{\underline{|0}} = \frac{1}{\sqrt{\pi t}}$. This is certainly similar to Equation (106) and would lead us to suspect that

$$p^{\frac{1}{2}} \left(\frac{t^0}{\underline{|0}} 1\right) = \frac{1}{\sqrt{\pi t}} 1 \tag{111}$$

Try now to verify Equation (108) by the use of Equations (107) and (111). The left-hand side gives

$$p^{\frac{1}{2}} p^{\frac{1}{2}} \left(\frac{t^0}{\underline{|0}} 1\right) = p \left(\frac{t^0}{\underline{|0}} 1\right)$$

The right-hand side gives

$$p^{\frac{1}{2}} \left(\frac{t^{-\frac{1}{2}}}{\underline{|-\frac{1}{2}}} 1\right) = \frac{t^{-1}}{\underline{|-1}} 1$$

if we consider the unit function as an ordinary constant.

In Part I was considered the possibility that $p\,1$ could be represented by $\dfrac{t^{-1}}{\lfloor\!\underline{-1}}\,1$

Consider next Equation (109).

$$p^{-\frac{1}{2}}\,p^{\frac{1}{2}}\left(\frac{t^0}{\lfloor\!\underline{0}}\,1\right)=p^{\frac{1}{2}}\,\frac{t^{-\frac{1}{2}}}{\lfloor\!\underline{-\tfrac{1}{2}}}\,1$$

The left-hand side gives $\dfrac{t^0}{\lfloor\!\underline{0}}\,1$

The right-hand side gives also $\dfrac{t^0}{\lfloor\!\underline{0}}\,1$ when we again consider the unit function as a multiplying factor.

Finally consider Equation (110).

$$p^{\frac{1}{2}}\sin\omega t\,1=p^{\frac{1}{2}}\left(\omega t-\frac{\omega^3t^3}{\lfloor\!\underline{3}}+\cdots\right)1$$

$$=\left(\omega\frac{t^{\frac{1}{2}}}{\lfloor\!\underline{\tfrac{1}{2}}}-\frac{\omega^3t^{\frac{5}{2}}}{\lfloor\!\underline{\tfrac{5}{2}}}+\cdots\cdot\right)1.$$

This can be shown to be a converging series and can also be written in the following more convenient form which will be treated in Part XXVI.

$$p^{\frac{1}{2}}\left(p^{\frac{1}{2}}\sin\omega t\,1\right)=\left(\frac{\omega t^0}{\lfloor\!\underline{0}}-\frac{\omega^3t^2}{\lfloor\!\underline{2}}+\cdots\right)1=\omega\cos\omega t\,1$$

Incidentally we may note that fractional derivatives are *transients*. It is seen that fractional differentiation and integration introduce factorial of fractions.

The so-called Π function (pronounced pie function) is the generalized factorial. Its value can be obtained from the gamma function or directly by series expansion

$$\Pi(x)=\Gamma(x+1)=\lim_{n=\infty}\frac{1\cdot2\cdot3\cdots\cdot(n-1)n}{(x+1)(x+2)\cdots\cdot(x+n)}n^x \quad (112)$$

The most important values for our purposes are the following:

x	0	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
$\Pi(x)=\lfloor\!\underline{x}=$	1	$\dfrac{\sqrt{\pi}\,1}{2}$	$\dfrac{\sqrt{\pi}\,1\cdot3}{2\cdot2}$	$\dfrac{\sqrt{\pi}\,1\cdot3\cdot5}{2\cdot2\cdot2}$

x	0	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{5}{2}$	$-\frac{7}{2}$
$\Pi(x)=\lfloor\!\underline{x}=$	1	$+\sqrt{\pi}$	$\dfrac{-2\sqrt{\pi}}{1}$	$\dfrac{+2\cdot2\sqrt{\pi}}{1\cdot3}$	$\dfrac{-2\cdot2\cdot2\sqrt{\pi}}{1\cdot3\cdot5}$

Since a number of problems involve fractional derivatives of the unit function, the following tabulation will prove a convenience.

$$p^{\frac{1}{2}}\,1=\frac{t^{-\frac{1}{2}}}{\sqrt{\pi}}\,1 \qquad p^{-\frac{1}{2}}\,1=\frac{2\,t^{\frac{1}{2}}}{\sqrt{\pi}}\,1$$

$$p^{\frac{3}{2}}\,1=\frac{-t^{-\frac{3}{2}}}{2\sqrt{\pi}}\,1 \qquad p^{-\frac{3}{2}}\,1=\frac{2\cdot2\,t^{\frac{3}{2}}}{1\cdot3\sqrt{\pi}}\,1$$

$$p^{\frac{5}{2}}\,1=\frac{1\cdot3\,t^{-\frac{5}{2}}}{2\cdot2\sqrt{\pi}}\,1 \qquad p^{-\frac{5}{2}}\,1=\frac{2\cdot2\cdot2\,t^{\frac{5}{2}}}{1\cdot3\cdot5\sqrt{\pi}}\,1$$

$$p^{\frac{7}{2}}\,1=\frac{-1\cdot3\cdot5\,t^{\frac{7}{2}}}{2\cdot2\cdot2\sqrt{\pi}}\,1 \qquad p^{-\frac{7}{2}}\,1=\frac{2\ \ 2\ \ 2\ \ 2\,t^{\frac{7}{2}}}{1\cdot3\cdot5\cdot7\sqrt{\pi}}$$

Before leaving this interesting subject it may be well to call attention to another feature connected with fractional differentiation.

Take, for instance, successive derivatives of $\dfrac{t}{\lfloor\!\underline{1}}$.

$$\frac{d}{dt}\frac{t}{\lfloor\!\underline{1}}=\frac{t^0}{\lfloor\!\underline{0}}$$

$$\frac{d^2}{dt^2}\frac{t}{\lfloor\!\underline{1}}=\frac{t^{-1}}{\lfloor\!\underline{-1}}$$

$$\frac{d^3}{dt^3}\frac{t}{\lfloor\!\underline{1}}=\frac{t^{-2}}{\lfloor\!\underline{-2}}$$

$$\frac{d^n}{dt^n}\frac{t}{\lfloor\!\underline{1}}=\frac{t^{-n}}{\lfloor\!\underline{-n}}$$

The last three expressions are usually written as zero since the factorial of a negative integer is ∞. However, when operated upon with $p^{\frac{1}{2}}$ they give $\dfrac{t^{-1\frac{1}{2}}}{\lfloor\!\underline{-1\tfrac{1}{2}}}$, $\dfrac{t^{-2\frac{1}{2}}}{\lfloor\!\underline{-2\tfrac{1}{2}}}$, and $\dfrac{t^{-n-\frac{1}{2}}}{\lfloor\!\underline{-n-\tfrac{1}{2}}}$, respectively. All three are definite and different. There are zeros and zeros.

PART XXIV: INDEFINITELY LONG TRANSMISSION LINE OR CABLE

Consider a transmission line that is of infinite length, or at any rate so long that whether the line is open or closed at the far end no potential difference exists. This is a situation which very nearly applies in very long telegraph cables.

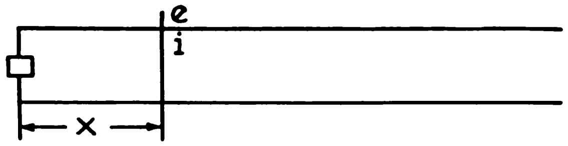

Fig. 49

In these problems it is obviously best to count the distance x from the sending end rather than from the receiving end as was done in Part XV.

Referring to Fig. 49,

$$\frac{\partial e}{\partial x}=-i\,(R+pL)$$

$$\frac{\partial i}{\partial x}=-e\,(G+pC)$$

Thus

$$\frac{\partial^2 e}{\partial x^2} = YZe = n^2 e$$

$$\therefore e = A\epsilon^{nx} + B\epsilon^{-nx}$$

For $x = \infty$, $e = 0$, therefore $A = 0$ and $e = B\epsilon^{-nx}$; for $x = 0$, $e = E$, therefore $B = E$ and

$$e = E\epsilon^{-nx}\,1 \qquad (113)$$

$$i = \frac{Y}{n}E\epsilon^{-nx}\,1 \qquad (114)$$

Therefore $\frac{e}{i}$, the resistance operator on unit function, is $= \frac{n}{Y}\epsilon^{-nx}$ at any point of the line.

In the case of an "ideal" cable, $Y = pC$, $Z = R$, and

$$\frac{n}{Y} = \frac{\sqrt{pCR}}{pC} = \left(\frac{R}{pC}\right)^{\frac{1}{2}} \qquad (115)$$

The current at the battery is thus

$$i_0 = \left(\frac{pC}{R}\right)^{\frac{1}{2}} E\,1$$

and

$$\frac{i}{i_0} = \frac{\frac{Y}{n}E\epsilon^{-nx}}{\frac{Y}{n}E} = \epsilon^{-nx}$$

Thus

$$i = E\epsilon^{-nx}\left(\frac{pC}{R}\right)^{\frac{1}{2}} 1 \qquad (116)$$

A very similar problem in heat conduction through an infinitely long bar had many years previously been worked out with conventional mathematics involving Fourier's series. The differential equation relating to the temperature is:

$$\frac{d\theta}{dt} = h^2\frac{d^2\theta}{dx^2} \qquad (117)$$

where h^2 is termed by Kelvin the diffusivity and by Maxwell the thermometric conductivity.

The rate of heat transferred through unit area is

$$W = -K\frac{d\theta}{dx} \qquad (118)$$

where K is the thermal conductivity of the substance.

When these equations are solved, subject to the condition that at the origin $x = 0$, the temperature is kept constant at θ_0 degrees, the rate of flow of heat at the origin becomes

$$W_0 = \frac{K\theta_0}{h\sqrt{\pi t}} \qquad (119)$$

Heaviside compared this problem with that given in the foregoing case of the infinite "ideal cable," when

$$\frac{\partial e}{\partial x} = -iR \qquad (120)$$

and

$$\frac{\partial i}{\partial x} = -Cpe = -C\frac{de}{dt}$$

which two equations combined give:

$$\frac{\partial^2 e}{dx^2} = RC\frac{de}{dt} \text{ or } \frac{de}{dt} = \frac{1}{RC}\frac{d^2 e}{dx^2} \qquad (121)$$

Comparing Equations (117) and (121) we note that $h^2 = \frac{1}{RC}$. Comparing Equations (118) and (120) we note that $K = \frac{1}{R}$ and $W = i$ and $\theta_0 = E\,1$ and

$$\frac{K}{h} = \sqrt{\frac{C}{R}} = \left(\frac{C}{R}\right)^{\frac{1}{2}}.$$

The operational solution of the current given above was

$$i = \left(\frac{pC}{R}\right)^{\frac{1}{2}} E\,1 = \left(\frac{C}{R}\right)^{\frac{1}{2}} Ep^{\frac{1}{2}}\,1$$

or at the origin

$$i = \left(\frac{C}{R}\right)^{\frac{1}{2}} Ep^{\frac{1}{2}}\,1 \text{ for } x = 0$$

or substituting heat constants

$$W_0 = \frac{K}{h}Q_0 p^{\frac{1}{2}}\,1$$

but it is shown in Equation (119) to be

$$W_0 = \frac{K}{h}\frac{Q_0}{\sqrt{\pi t}}.$$

Thus Heaviside concludes that

$$p^{\frac{1}{2}}\,1 = \frac{1}{\sqrt{\pi t}} = (\pi t)^{-\frac{1}{2}}.$$

Therefore, the current at the battery is:

$$i_0 = \sqrt{\frac{C}{R\pi t}}\,E.$$

The solution for the current when an alternator is connected to the cable is given in Part XXV.

We will now proceed to solve Equation (116).

$$i = E\sqrt{\frac{C}{R}}\;p^{\frac{1}{2}}\left[1 - nx + \frac{n^2x^2}{|\underline{2}} - \cdots\cdots\right]1$$

Substituting for $n=\sqrt{pCR}$ we get:

$$i=E\left[p^{\frac{1}{2}}\sqrt{\frac{C}{R}}-pCx+\frac{p^{\frac{3}{2}}C^{\frac{3}{2}}R^{\frac{1}{2}}x^2}{\underline{|2}}\right.$$

$$\left.-\frac{p^2C^2R\,x^3}{\underline{|3}}+\frac{p^{\frac{5}{2}}C^{\frac{5}{2}}R^{\frac{3}{2}}x^4}{\underline{|4}}+\cdots\right]1$$

The terms involving even powers in p are zero because $p\,1$, $p^2\,1$, etc. are zero, therefore

$$i=E\sqrt{\frac{C}{\pi Rt}}\left[1-\frac{CRx^2}{2\,\underline{|2t}}\right.$$

$$\left.+\frac{1\cdot 3\ C^2R^2x^4}{2\cdot 2\,\underline{|4}\,t^2}-\frac{1\cdot 3\cdot 5\ C^3R^3x^6}{2\cdot 2\cdot 2\,\underline{|6}\,t^4}+\cdots\cdots\right]$$

$$\text{Let}\quad y=\frac{CRx^2}{4t}$$

Then

$$i=E\sqrt{\frac{C}{\pi Rt}}\left[1-y+\frac{y^2}{\underline{|2}}-\frac{y^3}{\underline{|3}}+\cdots\right]=E\sqrt{\frac{C}{\pi Rt}}\,\epsilon^{-\frac{CRx^2}{4t}}$$

This problem is of interest because the solution is a convergent series although t appears in the denominators. The reason is that the development of ϵ^{-nx} introduces factorials.

PART XXV: PROBLEMS INVOLVING CONVERGENT AND ASYMPTOTIC SERIES

In his second volume on "Electro-magnetic Theory," Heaviside gives on page 42 the solution of a problem involving an infinitely long "ideal" cable connected to a battery through a coil. On page 40 he gives a similar problem where instead of a coil he inserts a condenser. In both cases he obtains operational solutions involving fractional exponents of p. He proceeds to expand them in powers of $\frac{1}{p}$ and in powers of p. The expansions in powers of $\frac{1}{p}$ give convergent solutions and are conveniently used for small values of t. The asymptotic solution—if complete—is suitable for large values of t, and is always subject to a certain error which however can be determined.

In the second problem the asymptotic expansion is the complete expansion; in the first problem an additional term is added and no satisfactory reason can be found in his three volumes on "Electro-magnetic Theory" or in his two volumes of "Electrical Papers." To be sure, on page 159 of his first volume of "Electrical Papers" he discusses a similar problem and this page gives a clue at least to his reason.

J. R. Carson has shown that the true convergent and asymptotic solution can be obtained by the use of definite integrals. Unfortunately; however, these integrals are frequently difficult to evaluate. It would obviously be of great help if an operational method could be found.

In connection with the study of operational calculus at Union College, and with the assistance of Dr. J. J. Smith and Messrs. S. J. Haefner and E. W. Hamlin, we have found several methods which are applicable to problems involving only one variable t; in other words, to problems of infinite lines where the condition at the generator $(x=0)$ is desired. Undoubtedly, investigations along these lines have been made by others but the writer is familiar only with the work of W. O. Pennell, which covers the same field as ours and gives identically the same results.

This field pertains to operators which are of the form:

$$\frac{\varphi(p)}{\psi(p)}$$

where $\psi(p)$ is of a form which can be rationalized to contain only integral powers of p, and $\varphi(p)$ contains only integral and fractional powers of p.

The operational solutions for infinite lines where the condition at the generator is desired are of the form:

$$h(p)=\frac{Y(p)}{1+(ap)^{\frac{n}{2}}}\,1$$

Rule

Apply the Expansion Theorem in the usual way, that is, find the roots, find $p\frac{dZ}{dp}$, etc., and the real part of the result is the "additional term." There is just one thing to be careful about and that is to choose the right roots.

$\sqrt[3]{1}$ has three roots $1\underline{|0}$ and $1\underline{\left|\pm\frac{2\pi}{3}\right.}$

$\sqrt[3]{-1}$ has three roots $1\underline{|\pi}$ and $1\underline{\left|\pm\frac{\pi}{3}\right.}$

$\sqrt[5]{1}$ has five roots $1\underline{|0}$ and $1\underline{\left|\pm\frac{2\pi}{5}\right.}$ and $1\underline{\left|\pm\frac{4\pi}{5}\right.}$

$\sqrt[5]{-1}$ has five roots $1\underline{|\pi}$ and $1\underline{\left|\pm\frac{\pi}{5}\right.}$ and $1\underline{\left|\pm\frac{3\pi}{5}\right.}$

The method given above seems to be the simplest of several which can be used and is discussed in detail in the Feb., 1928, issue of the *Journal of the Franklin Institute*.

The solution of a number of characteristic problems will now be demonstrated.

Problem

An infinitely long "ideal" cable is connected to an alternator of voltage $e=E\sin\omega t$ at $t=0$. Find the equation for the current at the generator.

It was shown in Part XXIV that the operational solution when unit e.m.f. was supplied was

$$i=E\sqrt{\frac{C}{R}}\,p^{\frac{1}{2}}\,1$$

The operational expression for $\sin\omega t$ is $\dfrac{p\omega}{p^2+\omega^2}\,1$

Thus $$i=E\sqrt{\frac{C}{R}}\,\omega\,\frac{p^{\frac{3}{2}}}{p^2+\omega^2}\,1$$

Convergent Solutions

Since

$$\frac{1}{p^2+\omega^2}=\frac{1}{p^2}-\frac{\omega^2}{p^4}+\frac{\omega^4}{p^6}-\cdots$$

$$i=E\,\omega\sqrt{\frac{C}{R}}\,[p^{-\frac{1}{2}}-\omega^2 p^{-\frac{5}{2}}+\omega^4 p^{-\frac{9}{2}}\cdots]\,1$$

$$=2E\,\omega\sqrt{\frac{Ct}{R\pi}}\left[1-\frac{(2\omega t)^2}{1\cdot3\cdot5}+\frac{(2\omega t)^4}{1\cdot3\cdot5\cdot7\cdot9}-\cdots\right] \quad (122)$$

or we could obtain the solution by "shifting"

$$i=E\sqrt{\frac{C}{R}}\,p^{\frac{1}{2}}\,\frac{\epsilon^{j\omega t}-\epsilon^{-j\omega t}}{2j}\,1$$

Note that

$$p^{\frac{1}{2}}\,\epsilon^{j\omega t}\,1=\epsilon^{j\omega t}\,(p+j\omega)^{\frac{1}{2}}=\epsilon^{j\omega t}\,[p^{\frac{1}{2}}+\tfrac{1}{2}\,p^{-\frac{1}{2}}\,j\omega-\cdots]\,1$$

and

$$p^{\frac{1}{2}}\,\epsilon^{-j\omega t}\,1=\epsilon^{-j\omega t}\,(p-j\omega)^{\frac{1}{2}}=\epsilon^{-j\omega t}\,[p^{\frac{1}{2}}-\tfrac{1}{2}\,p^{-\frac{1}{2}}\,j\omega+\cdots]\,1$$

The result becomes

$$i=E\sqrt{\frac{C}{R}}\,[A\sin\omega t+B\cos\omega t]$$

where $$A=\frac{1}{\sqrt{\pi}}\left[t^{-\frac{1}{2}}+\frac{t^{\frac{3}{2}}\,\omega^2}{3\,\underline{|2}}-\frac{t^{\frac{7}{2}}\,\omega^4}{7\,\underline{|4}}+\cdots\right]$$

and $$B=\frac{1}{\sqrt{\pi}}\left[\frac{\omega t^{\frac{1}{2}}}{\underline{|1}}-\frac{\omega^3\,t^{\frac{5}{2}}}{5\,\underline{|3}}+\frac{\omega^5\,t^{\frac{9}{2}}}{9\,\underline{|5}}\right]$$

Note that A and B converge rather rapidly. As t becomes larger and larger, A and B become more nearly equal and in the limit $t=\infty$, $A=B=\sqrt{\dfrac{\omega}{2}}$

Thus the final current is:

$$i_p=E\sqrt{\frac{C\omega}{2R}}\,(\sin\omega t+\cos\omega t)$$

It is of interest to note that there are several ways to get the convergent solution. We might have expressed

$$\sin\omega t=\omega t-\frac{\omega^3 t^3}{\underline{|3}}+\cdots$$

When these terms are operated on by $p^{\frac{1}{2}}$ we get

$$i=E\sqrt{\frac{C\omega}{\pi R}}\left[2\sqrt{\omega t}-\frac{2\cdot2\cdot2}{1\cdot3\cdot5}\,(\omega t)^{\frac{5}{2}}+\frac{2^5\,(\omega t)^{\frac{9}{2}}}{1\cdot3\cdot5\cdot7\cdot9}-\cdots\right]$$

Finally by referring to the table of miscellaneous formulas and our comments on the use of Duhamel's integral we get:

$$i=\omega\sqrt{\frac{C}{\pi R}}\int_{u=0}^{u=t}\frac{\cos\omega u}{\sqrt{t-u}}\,du$$

We have given four different solutions, all giving the same result and all suitable for small values of t. Incidentally it is seen that $p^{\frac{1}{2}}\sin\omega t$ is a transient which eventually becomes $\sqrt{\omega}\,\sin(\omega t+45°)$.

It is also of interest to note that if vector representation is used, in which case $p=j\omega$, we get $p^{\frac{1}{2}}\sin\omega t=p^{\frac{1}{2}}\,(j\omega)=\sqrt{\omega}\,\underline{|45°}$ which is a vector of size $\sqrt{\omega}$ which leads the zero vector by 45 deg. All this agrees with what should be expected. There is a real significance in fractional differentiation.

Asymptotic Expansion

$$i=E\sqrt{\frac{C}{R}}\,\frac{p^{\frac{3}{2}}\,\omega}{p^2+\omega^2}\,1$$

$$=E\sqrt{\frac{C}{R}}\,p^{\frac{3}{2}}\,\omega\left[\frac{1}{\omega^2}-\frac{p^2}{\omega^4}+\frac{p^4}{\omega^6}-\cdots\right]1 \quad (123)$$

$$=E\sqrt{\frac{C}{R}}\,\omega\left[\frac{p^{\frac{3}{2}}}{\omega^2}-\frac{p^{\frac{7}{2}}}{\omega^4}+\frac{p^{\frac{11}{2}}}{\omega^6}-\cdots\right]1$$

$$=-E\sqrt{\frac{C}{R\pi t}}\left(\frac{1}{2\omega t}-\frac{1\cdot3\cdot5}{(2\omega t)^3}+\frac{1\cdot3\cdot5\cdot7\cdot9}{(2\omega t)^5}\cdots\right) \quad (124)$$

Equation (124) is not the complete solution because in this case there is an "additional term."

$$y=\frac{p^{\frac{3}{2}}}{p^2+\omega^2}$$

has a pair of imaginary roots $p_1=+j\omega=\omega\,\underline{|90°}$ and $p_2=-j\omega=\omega\,\underline{|-90°}$ which are obtained in the usual way by making $Z_{(p)}=0$

$$p\frac{dZ}{dp}=2p^2$$

$$\frac{Y_{(p)}}{p\dfrac{dZ}{dp}}=\frac{p^{\frac{3}{2}}}{2p^2}=\frac{1}{2p}$$

$$\therefore \frac{Y_{(p)}}{p\frac{dZ}{dp}} \text{ for } p=p_1=\frac{1}{2\omega}\underline{|-45^\circ} = \frac{1}{2\omega}(\cos 45^\circ - j\sin 45^\circ)$$

$$=\frac{1}{2\sqrt{2\omega}}(1-j)$$

Similarly

$$\frac{Y_{(p)}}{p\frac{dZ}{dp}} \text{ for } p=p_2=\frac{1}{2\sqrt{2\omega}}(1+j)$$

$$\therefore y=\frac{1}{2\sqrt{2\omega}}(1-j)\,\epsilon^{j\omega t}+(1+j)\,\epsilon^{-j\omega t}$$

$$=\frac{1}{\sqrt{2\omega}}(\cos\omega t+\sin\omega t)$$

and the complete asymptotic solution is:

$$i=E\sqrt{\frac{C\omega}{2R}}(\cos\omega t+\sin\omega t)$$

$$-E\sqrt{\frac{C}{R\pi t}}\left(\frac{1}{2\omega t}-\frac{1\cdot 3\cdot 5}{(2\omega t)^3}+\cdots\right)$$

Problem

A condenser C_1 is connected between the battery and an infinitely long "ideal" cable as shown in Fig. 50. Find the potential difference at the right-hand side of the condenser.

It has been shown that the impedance operator which replaces the cable is $\left(\frac{R}{pC}\right)^{\frac{1}{2}}$

Thus the total impedance at the battery is

$$\left(\frac{R}{pC}\right)^{\frac{1}{2}}+\frac{1}{pC_1}$$

therefore the current at the battery is

$$i=\frac{E}{\left(\frac{R}{pC}\right)^{\frac{1}{2}}+\frac{1}{pC_1}}\mathbf{1}$$

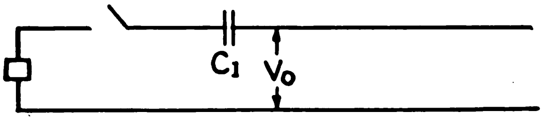

Fig. 50

The voltage consumed by the condenser is $\frac{i}{pC_1}$

Thus

$$V_0=E\mathbf{1}-\frac{E}{\left(\frac{R}{pC}\right)^{\frac{1}{2}}+\frac{1}{pC_1}}\frac{1}{pC_1}\mathbf{1}$$

This can be brought to the following form

$$V_0=E\frac{h^{\frac{1}{2}}p^{\frac{1}{2}}}{h^{\frac{1}{2}}p^{\frac{1}{2}}+1}\mathbf{1} \quad \text{where } h=\frac{RC_1^2}{C}$$

Convergent Solution

$$V_0=E\,h^{\frac{1}{2}}p^{\frac{1}{2}}\left[\frac{1}{h^{\frac{1}{2}}p^{\frac{1}{2}}}-\frac{1}{hp}+\frac{1}{h^{\frac{3}{2}}p^{\frac{3}{2}}}-\cdots\right]\mathbf{1}$$

$$=E\left[1-\frac{1}{h^{\frac{1}{2}}p^{\frac{1}{2}}}+\frac{1}{hp}-\frac{1}{h^{\frac{3}{2}}p^{\frac{3}{2}}}+\cdots\right]\mathbf{1}$$

Here we have one series of even powers in $\frac{1}{p}$

This part sums up to E $\epsilon^{\frac{1}{h}t}$

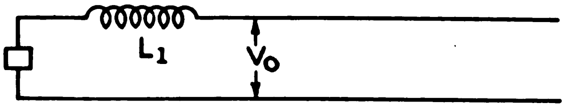

Fig. 51

and in addition we have a series in fractional powers of $\frac{1}{p}$

The latter becomes

$$-2E\left(\frac{t}{h\pi}\right)^{\frac{1}{2}}\left[1+\frac{2t}{1\cdot 3h}+\frac{1}{1\cdot 3\cdot 5}\left(\frac{2t}{h}\right)^2+\cdots\right.$$

Thus the convergent solution is:

$$V_0=E\left\{\epsilon^{\frac{t}{h}}-2\left(\frac{t}{h\pi}\right)^{\frac{1}{2}}\left[1+\frac{2t}{h}\cdot\frac{1}{1\cdot 3}\right.\right.$$

$$\left.\left.+\left(\frac{2t}{h}\right)^2\frac{1}{1\cdot 3\cdot 5}+\cdots\right]\right\}$$

Aympytotic Expansion

$$V_0=E\,h^{\frac{1}{2}}p^{\frac{1}{2}}\left[1-h^{\frac{1}{2}}p^{\frac{1}{2}}+hp-h^{\frac{3}{2}}p^{\frac{3}{2}}+h^2p^2-\cdots\right]\mathbf{1}$$

$$=E\left[h^{\frac{1}{2}}p^{\frac{1}{2}}-hp+h^{\frac{3}{2}}p^{\frac{3}{2}}-h^2p^2+\cdots\right]\mathbf{1}$$

The even powers in p contribute nothing, thus

$$V_0=E\left[h^{\frac{1}{2}}p^{\frac{1}{2}}+h^{\frac{3}{2}}p^{\frac{3}{2}}+\cdots\right]\mathbf{1}$$

$$=E\left(\frac{h}{t\pi}\right)^{\frac{1}{2}}\left[1-\frac{1\cdot h}{2t}+1\cdot 3\left(\frac{h}{2t}\right)^2-1\cdot 3\cdot 5\left(\frac{h}{2t}\right)^3+\cdots\right]$$

The question is whether this is the complete asymptotic solution.

In this case it is the complete solution, because when the denominator is rationalized we get

$$V_0=E\frac{h^{\frac{1}{2}}p^{\frac{1}{2}}}{h^{\frac{1}{2}}p^{\frac{1}{2}}+1}=E\frac{h^{\frac{1}{2}}p^{\frac{1}{2}}(h^{\frac{1}{2}}p^{\frac{1}{2}}-1)}{hp-1}$$

$Z_{(p)}=0$ gives $p=\frac{1}{h}$ which is real. When this is used the numerator becomes zero, thus $V_0^1=0$.

Problem

Consider next a similar problem of an infinitely long "ideal" cable where the condenser is replaced by a coil of inductance L_1, as in Fig. 51.

It is readily found that

$$V_0=\frac{E}{1+g^{\frac32}p^{\frac32}}\mathit{1} \qquad \text{where } g=\frac{C^{\frac12}L_1}{R^{\frac12}}$$

Convergent Solution

$$V_0=E\left[\frac{1}{g^{\frac32}p^{\frac32}}-\frac{1}{g^3p^3}+\frac{1}{g^{\frac92}p^{\frac92}}-\cdots\right]\mathit{1}$$

$$=E\frac{4\pi}{3}\left(\frac{t}{g\pi}\right)^{\frac32}\left[1+\left(\frac{2t}{g}\right)^3\frac{1}{5\cdot7\cdot9}\right.$$

$$\left.+\left(\frac{2t}{g}\right)^6\frac{1}{5\cdot7\cdot9\cdot11\cdot13\cdot15}\cdots\right]$$

$$-E\left[\left(\frac{t}{g}\right)^3\frac{1}{\underline{|3}}+\left(\frac{t}{g}\right)^6\frac{1}{\underline{|6}}+\cdots\right]$$

This solution is complete but does not show directly that there is an oscillation.
This can be brought out by rewriting the operational equation with rationalized denominator. It is:

$$V_0=\frac{E}{1+g^{\frac32}p^{\frac32}}\mathit{1}=\frac{E(1-g^{\frac32}p^{\frac32})}{1-g^3p^3}\mathit{1}=E\,(A+B)$$

where

$$A=\frac{1}{1-g^3p^3} \text{ and } B=-\frac{g^{\frac32}p^{\frac32}}{1-g^3p^3}$$

A is solved by the Expansion Theorem.

$Z_{(p)}=0$ gives three roots

$$\frac{1}{g} \text{ and } \frac{1}{g}\underline{|\pm120°}$$

The result is

$$\mathrm{A}=E\left[1-\frac{1}{3}\epsilon^{\frac{t}{g}}-\frac{2}{3}\epsilon^{-\frac{t}{2g}}\cos\frac{\sqrt{3}}{2g}t\right]$$

B when expanded in powers of $\frac{1}{p}$ gives the same result as we obtained in the expansion given above. It is

$$B=\frac{4\pi}{3}\left(\frac{t}{g\pi}\right)^{\frac32}\left[1+\left(\frac{2t}{g}\right)^3\frac{1}{5\cdot7\cdot9}+\cdots\right]$$

It is interesting to note that

$$\left(\frac{t}{g}\right)^3\frac{1}{\underline{|3}}+\left(\frac{t}{g}\right)^6\frac{1}{\underline{|6}}+\cdots=-\left(1-\frac{1}{3}\epsilon^{\frac{t}{g}}-\frac{2}{3}\epsilon^{-\frac{t}{2g}}\cos\frac{\sqrt{3}}{2g}+\right)$$

Asymptotic Expansion

$$V_0=E\,[1-g^{\frac32}p^{\frac32}+g^3p^3-g^{\frac92}p^{\frac92}+\cdots]\,\mathit{1}$$

The even powers in p contribute nothing.
Thus

$$V_0=E\left[1+\frac{\pi}{2}\left(\frac{g}{2\pi t}\right)^{\frac32}1-1\cdot3\cdot5\cdot7\left(\frac{g}{2t}\right)^3\right.$$

$$\left.+1\cdot3\cdot5\cdot7\cdot9\cdot11\cdot13\left(\frac{g}{2t}\right)^6+\cdots\right]$$

To find the "additional term"

$$V_0'=E\frac{(1-g^{\frac32}p^{\frac32})}{1-g^3p^3}$$

$$p\frac{dZ}{dp}=-3\,g^3p^3 \quad \text{which for } p^3=\frac{1}{g^3} \text{ gives } -3.$$

$$1-g^{\frac32}p^{\frac32} \text{ for } p=\frac{1}{g}=0$$

$$1-g^{\frac32}p^{\frac32} \text{ for } p=\frac{1}{g}\,\underline{|+120}=1-1\,\underline{|180}$$

$$1-g^{\frac32}p^{\frac32} \text{ for } p=\frac{1}{g}\,\underline{|-120}=1-1\,\underline{|-180}$$

$$\therefore\ V_0'=\frac{1+1}{-3}\epsilon^{(-\alpha+j\beta)\,t}+\frac{1+1}{-3}\epsilon^{(-\alpha-j\beta)\,t}$$

$$=-\frac{4}{3}\epsilon^{-\alpha t}\cos\beta t. \quad \text{where } \alpha=\frac{1}{2g} \text{ and } \beta=\frac{\sqrt{3}}{2g}$$

The complete asymptotic solution is then

The asymptotic expansion V_0+V_0'.

Problem

An infinitely long line of constants R, L, and C is connected to a storage battery of voltage E. Find the current at the battery.

The operational solution for any infinitely long line has been shown to be

$$i=\frac{Y}{n}E\,\mathit{1}=\frac{pC}{\sqrt{pC\,(R+pL)}}E\,\mathit{1}=\left(\frac{pC}{R+pL}\right)^{\frac12}E\,\mathit{1}$$

Let $\frac{R}{2L}=\alpha$ then $i=\frac{C^{\frac12}}{L}\,\frac{p^{\frac12}}{(p+2\alpha)^{\frac12}}E\,\mathit{1}$

Convergent Solution

$$i=\frac{C^{\frac12}}{L}\frac{E}{\left(1+\frac{2\alpha}{p}\right)^{\frac12}}\mathit{1}=E\frac{C^{\frac12}}{L}\left[1-\frac{2\alpha}{2p}+\frac{1\cdot3}{2\cdot4}\left(\frac{2\alpha}{p}\right)^2+\cdots\right]\mathit{1}$$

$$=E\frac{C^{\frac12}}{L}\left[1-\frac{\alpha}{p}+\frac{1\cdot3}{\underline{|2}}\left(\frac{\alpha}{p}\right)^2-\frac{1\cdot3\cdot5}{\underline{|3}}\left(\frac{\alpha}{p}\right)^2+\cdots\right]\mathit{1}$$

$$=E\frac{C^{\frac12}}{L}\left[1-\alpha t+\frac{1\cdot3}{(\underline{|2})^2}(\alpha t)^2-\frac{1\cdot3\cdot5}{(\underline{|3})^2}(\alpha t)^3+\cdots\right]$$

This is a convergent series suitable for small values of αt.

Asymptotic Expansion

$$E\frac{C^{\frac12}}{L(2\alpha)^{\frac12}}\left[p^{\frac12}-\frac{1}{2}\,\frac{p^{\frac32}}{2\alpha}+\frac{1\cdot3}{2\cdot4}\,\frac{p^{\frac52}}{(2\alpha)^2}-\cdots\right]\mathit{1}$$

$$=E\frac{C^{\frac12}}{L}\,\frac{1}{\sqrt{2\alpha\pi t}}\left[1+\frac{1^2}{8\alpha t}+\frac{1^2\cdot3^2}{\underline{|2}(8\alpha t)^2}+\frac{1^2\cdot3^2\cdot5^2}{\underline{|3}(8\alpha t)^3}+\cdots\right]$$

This is the complete solution. The "additional term" is zero as can easily be seen.

Characteristics of Asymptotic Series Illustrated Numerically in This Series

Assign certain values to the independent variable *t*, in this case perhaps most conveniently to $8\alpha t$. Let it be for instance 8.

Then the series becomes

$$1+\frac{1}{8}+\frac{9}{2.64}+\frac{225}{6.512}+\cdots$$

$$=1+0.125+0.0351+0.0735+\cdots.$$

Note that the successive terms are smaller up to the third term. After that they increase.

Heaviside says, and other mathematicians agree with his statement, "Seek the place where the smallest term occurs, then the error where the series is summed up to this term is less than the smallest term."

In this case the sum is 1.1601 with an error less than 0.0351.

The larger the value of $8\alpha t$, the smaller is the percentage error.

(To be continued)

New Welder-tractor Combination

A new combination of electric arc welder and Fordson tractor embodies improvements consisting of the substitution of a new type welding equipment and the addition of head and tail lights and a protective cover.

The principal equipment consists of a standard Fordson tractor, belt-connected to the new type 25-volt 300-amp. (one-hour rating) 1750-r.p.m. ball-bearing generator. This unit is mounted directly on the tractor and is protected by metal canopy and canvas side curtains. Other equipment includes governor, power take-off, muffler, waterproofed pulleys on the engine and the generator, belt and belt tightener, disk-type industrial rubber-tired wheels front and rear, extension frame, off-set crank, control panel and reactor, head and tail lights, battery and charging control. Spoke-type light industrial wheels front and rear, and standard Fordson farm wheels front and rear may be used in place of those named.

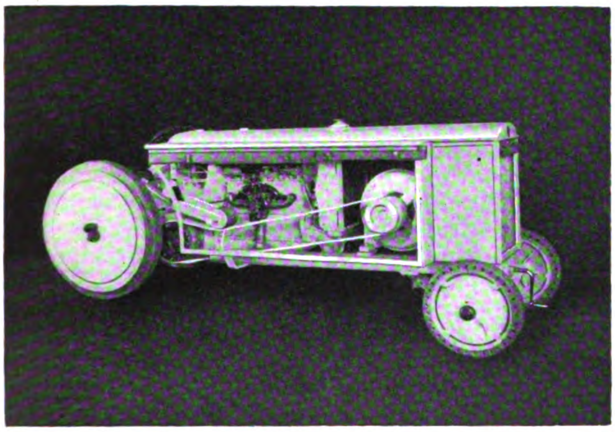

Fig. 1. Side View of the Welder-tractor Showing Internal-combustion Motor Belt Connected to Welding Generator

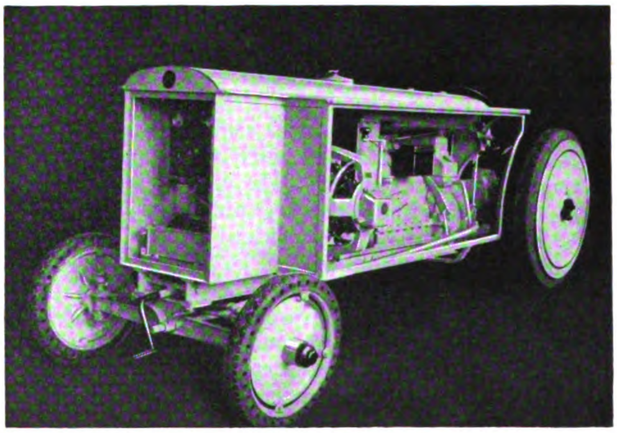

Fig. 2. View of the Welder-tractor from the Other Side Showing Controller and Coil Box

The overall length of the complete unit is 12 ft.; the height four feet eight inches; the width five feet two inches, and the net weight is approximately 4900 lb.

Among other applications, this equipment is particularly useful to welding contractors, boiler shops, marine repair shops, structural contractors, oil-well drilling companies, oil refineries, pipe-line owners, and tank builders. The outfit has been particularly designed to provide such activities with a complete, portable welding machine suitable for hard, continuous use. The utility of the tractor as a hauling device has not been impaired. For field use, the unit will haul equipment and tools to the job and then supply welding current for the work. For welding pipe lines, and for use along railroad rights-of-way, extra-long welding leads are not necessary because the equipment is so easily moved at a moment's notice.

The battery, which supplies current for the lights and ignition, is charged while the welding generator is operating. It is provided with ammeter and automatic cutout.

Three Honorary Vice Presidents and Two Vice Presidents Elected by General Electric Company

THE board of directors of the General Electric Company at its meeting July 6th conferred upon Vice President Jesse R. Lovejoy, Vice President George F. Morrison, and Burton G. Tremaine, all members of the board of directors of the Company, the title of Honorary Vice Presidents in recognition of their long and valuable services.

They are to remain as directors of the Company and retain their association with the several departments of the Company's activities with which they have been so closely identified.

At this same meeting, T. W. Frech, manager of the Incandescent Lamp Department of the Company, was elected to the position of vice president in charge of the Incandescent Lamp Department, and Dr. W. R. Whitney, director of the Research Laboratory, was elected a vice president of the Company and director of research.

Jesse R. Lovejoy has been a vice president of the General Electric Company since 1907, and was elected a director in 1922. He began his electrical career with the Thomson-Houston Company in 1886. In his 42 years of service, he has been identified with many phases of the Company's business, since its organization in 1892. Commercial work commanded his greatest attention and he effected sales organizations and departments for promotion of the Company's business throughout the country and in foreign fields.

As a member of the American Institute of Electrical Engineers, the American Association for the Advancement of Science, the Franklin Institute and association with the technical staff of the Company, he has kept in close touch with the progress of the electrical art and industry. He is also a member of many national business organizations and social clubs. More recently, as president of the G. E. Employees Securities Corporation, he has given attention to the Company's plan to encourage thrift and savings by its many thousand employees and to the profitable investment of such savings for their benefit. His election as honorary vice president is a mark of distinction for service rendered by him to the Company.

George F. Morrison, an Edison Pioneer and a member of many commercial and engineering organizations, became connected with the Edison Lamp Works, Harrison, New Jersey, at the age of 15, continuing in the employ of the General Electric Company for 45 years. During that period he had many positions of responsibility in connection with the development, manufacture and sale of incandescent lamps. He was elected vice president of the General Electric Company in 1916 and has been a director since 1922. His election as honorary vice president is a fitting testimonial of his life's work in the incandescent lamp industry throughout the world.

Burton G. Tremaine became connected with the Fostoria Incandescent Lamp Company of Fostoria, Ohio, in 1898. In May, 1901, the Fostoria Incandescent Lamp Company was combined with the Sunbeam Incandescent Lamp Company of Desplaines, Illinois, which was then managed by F. S. Terry; and this was the beginning of the lifelong friendship and business association of Terry and Tremaine, as well as the beginning of the National Electric Lamp Company.

Under the managership of Messrs. Terry and Tremaine, the National Electric Lamp Company grew rapidly, and in 1911 became the National Lamp Division of the General Electric Company. Messrs. Terry and Tremaine continued to manage this division until it was consolidated with the Edison Lamp Works to become the Incandescent Lamp Department of the General Electric Company.

In 1923, Mr. Tremaine was made a director of the General Electric Company, which office he continues to hold. He is director of several public utility and industrial companies, and a member of many clubs. His new title is conferred on him in recognition of his long and valuable service to the Company.

T. W. Frech entered the employ of the National Lamp Works as an assistant to the late F. S. Terry, in 1901. From 1901 to 1913, Mr. Frech was connected with various lamp factories of the National Lamp Works and the General Electric Company. During this period he worked as manager of several factories.

In 1913 he left the General Electric Company to become vice president and general manager of the Peerless Motor Car Company of Cleveland, but returned to the General Electric Company in November, 1921, to become manager of its National Lamp Works.

In 1925, Mr. Frech was appointed associate manager, with W. R. Burrows, of the Incandescent Lamp Department. In 1927, Mr. Burrows was elected a vice president of the Company, and left the Lamp Department. As a result of this change, Mr. Frech became manager of the Incandescent Lamp Department, which was his position when he was made a vice president of the Company.

Dr. Willis R. Whitney, a graduate of Massachusetts Institute of Technology, received his degree of Ph. D. from Leipzig in 1896. He received the honorary degrees of Sc. D. from Union, Ch. D. from the University of Pittsburgh, Sc. D. from Syracuse University, and Sc. D. from the University of Michigan. He held the following positions at the Institute of Technology, following his graduation: Assistant instructor Sanitary Chemistry 1892-94, instructor Theoretical Chemistry and Proximate Analysis 1898-1901, assistant professor Theoretical Chemistry 1901-04, non-resident associate professor Theoretical Chemistry 1904-08, and non-resident professor Chemical Research 1908.

Since 1900, Dr. Whitney has been director of the Research Laboratory at Schenectady. Among his early activities, Dr. Whitney, in conjunction with Professor A. A. Noyes, successfully developed a recovery process for alcohol and ether from collodion which insured the commercial practicability of the present photographic paper. His most notable achievement has been the creation and development of the Company's great Research Laboratory at Schenectady.

Dr. Whitney has written a number of papers on Colloids, Corrosion of Iron, Alloys, Chemistry of Light, etc., and his translation of Le Blanc's Textbook of Electrochemistry is very well known. He has received many medals for his accomplishments, and has been honored by a number of societies by honorary and active membership. His election as vice president is an honor highly merited.

LIBRARY SECTION

Condensed references to some of the more important recent articles in the technical press, and to new books of interest to the industry, as selected by the General Electric Main Library.

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Cars, Gas-Electric
Rail-Cars and Their Operation. Walter C. Sanders.
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(On the design, performance, operating costs, etc., of gasoline and gas-electric rail cars.)

Commutation
Commutation Control of Booster Converters. L. A. Vaupre.
Power Pl. Engng., May 1, 1928; v. 32, pp. 510-514.

Electric Cables—Testing
Testing High Tension Cables.
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(Abstract of a paper by Martin Höchstadter.)

Electric Controllers
Choosing the Controller to Suit the Motor's Load. W. H. Costello.
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Energy from Dynamic Braking Cuts Heating Costs. A. W. Baumgarten.
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(Describes instruments and methods.)

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Thermal Method of Standardizing Dielectric Power Loss Measuring Equipment. J. A. Scott and others.
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Some Aspects of Pacific Coast Interconnections. P. M. Downing.
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Lux Fire-Extinguishing Installation; Barking Power Station.
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(Appears also in Bell System Technical Journal, Apr., 1928, pp. 168-186.)

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Recent Developments in Low Power and Broadcasting Transmitters. I. F. Byrnes.
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Easy Method for Testing Welds for Ductility. W. B. Miller.
Am. Weld. Soc. Jour., Apr., 1928; v. 7, pp. 47-51.

NEW BOOKS

(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW)

A.C. Commutator Motor. C. W. Olliver. 281 pp., 1927, N. Y., D. Van Nostrand Co.

("The gradual standardization of A.C. networks makes the use of A.C. motors essential in the greater number of industrial applications. On the other hand, the increasing demand for efficient speed regulation and a growing necessity for a high power-factor has led to the use of converting apparatus and D.C. motors, an expensive and inefficient process." So says the author of this British book on the a-c. commutator motor. He goes on to say: "There is, for some reason which is difficult to appreciate, a strong prejudice against the A.C. commutator motor which affords an almost perfect solution to the problem, not perhaps so much by itself as when combined with the more usual induction motor. It is the object of this book to attempt to break down any such prejudice and show that the future lies with such combinations as will be described." The author treats his subject from two angles, the analytical side and the practical application of such machines. In the early chapters he discusses theoretically the subject of commutation in d-c. and a-c. machines, the theory of single-phase and repulsion motors, the polyphase commutator motor, cascaded sets, compensated motors, and phase advancers. This comprises over half the book. The second part deals with the use of the a-c. commutator motor for the drive of cranes and hoists, textile mills, paper mills, pumping sets, mining machinery, machine tools, etc. Separate chapters are devoted to single-phase traction and to rolling mill drives. The book is illustrated by means of charts, diagrams, and full-page photographs of installations, the great majority of the examples being of European machinery and practice.)

Probability and Its Engineering Uses. Thornton C. Fry. 476 pp., 1928, N. Y., D. Van Nostrand Co.

(The theory of probabilities is familiar in a general way, to the engineering student, as a sort of mathematical exercise. He associates it with such simple problems as the chance of picking a red ball out of a bag of mixed red and white balls, and similar calculations. Probability is of course, utilized by insurance companies in actuarial calculations, but the book at hand demonstrates its value in engineering applications. To familiarize the reader with the theory, the author first discusses permutations, combinations, factorials, binomial theorem, etc., illustrating the application of the associated laws and methods by means of simple problems such as "the chance of throwing an ace exactly once in six throws of a die." Bernoulli's Theorem, Maxwell's equation, the Poisson Law, the Gram-Charlier Series, and the Multinominal Law are some of those discussed mathematically and by illustration. Then, leading to the engineering applications, the author applies probability theory to such problems as telephone traffic and the probable flux across a surface, and then derives the equation which forms the mathematical basis for the entire Kinetic Theory of Gases. The volume is an outgrowth of a set of notes used for classes at the Bell Telephone Laboratories, and which were later revised for a course at Massachusetts Institute of Technology.)

Transmission Line Engineering. W. W. Lewis. 361 pp. 1928, N. Y., McGraw-Hill Book Co., Inc.

(This book deals with the more important electrical problems which confront the engineer who is engaged in the calculation and design of transmission lines. By means of the formulas and very complete tables, the engineer is enabled to determine the constants of a transmission line, its loss, regulation and efficiency. Short circuit currents, both three-phase and single-phase, may be calculated. Chapters are devoted to the subject of triple harmonics, autotransformers and telephone interference. Disturbances on transmission lines, and the best methods of protection are fully treated. The subject of power limit or system stability is outlined and recommendations are given for grounding systems for generating stations and substations. Numerous examples are given to illustrate the theory, also references to the pertinent literature. There are 200 illustrations and a full index.)

World of Atoms. Arthur Haas. (Tr. by Horace S. Uhler) 139 pp., 1928, N. Y., D. Van Nostrand Co.

(A compact volume consisting of ten related, non-mathematical lectures presented at the University of Vienna. "They were intended to present to the lay public the achievements of modern atomic physics in as brief and yet thorough a manner as possible, and at the same time in an easily understandable form." The author goes on to say, however, that, "The lectures can lay claim to the characterization 'popular' only in the sense that every lay reader who has at his command a sufficiently general education, can begin the reading of the book and continue to the end, without great difficulty and with comprehension, insofar as his interest in the subject lends him the necessary patience and perseverance." The lectures consist of: I, Matter and Electricity; II, The Building-Stones of Atoms; III, Light-Quanta; IV, Spectra and Energy Levels; V, The Elements; VI, The Atom as a Planetary System; VII, Molecules; VIII, Radioactivity; IX, Transformations of the Elements; X, Wave Mechanics of the Atom. Although mathematics has been intentionally omitted from the lectures, the author presents a few pages of supplementary notes giving such formulas as may be sought by those readers who desire to pursue that phase of the subject. The tenth lecture, "Wave Mechanics of the Atom," was prepared especially for this American edition, and, according to the translator, it "constitutes the pièce de résistance of the intellectual feast.")

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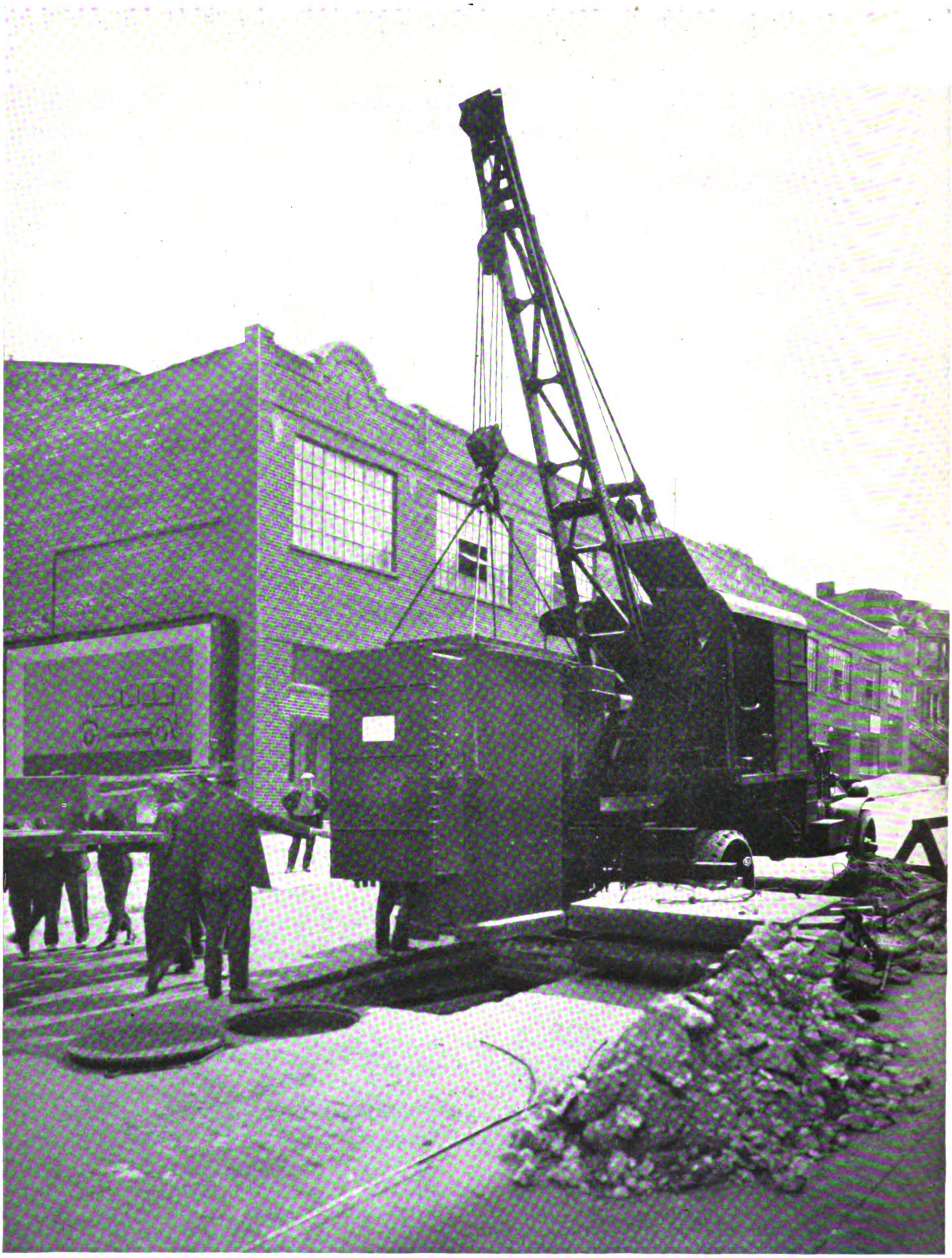


POWER AND PROTECTION FOR A-C. NETWORKS

A three-phase 60-cycle 500-kv-a. 26,400-125/216 Y-volt transformer with protective equipment (total weight 10 tons) being lowered into a vault for operation on the a-c. distribution system of the Brooklyn Edison Company. Transformer equipment for such service will be described in a forthcoming issue in Mr. Blake's serial which this month treats of the protective equipment

(See p. 480)

GENERAL ELECTRIC REVIEW

Vol. 31 September, 1928 No. 9

EYES

In the long, but grand, tale of evolution one of the greatest steps upward must have been when Nature gave the first primitive eye to a living thing. The evolution of the eye is a wonderful story, but fairly well understood and there seems little reason to doubt that the first eye was nothing more complicated than a lens-shaped drop of water on a sore spot.

Nature gradually worked out her problem of eyes till we see different species endowed with many types of eyes each evolved and developed to meet the particular needs of the individual species. The human eye, some consider Nature's masterpiece in eyes; others are not so complimentary. However man did marvelous work in the study of Nature and her laws with the unaided eye. But man's greatest advance in the study of Nature came when he added other eyes to those with which he was originally endowed. We can hardly imagine the modern scientific world without the microscope and the telescope—those two direct aids to human vision; and our more modern and more wonderful achievements in the realms of physical research would have been quite impossible without our newer eyes—the spectroscope and x-ray diffraction apparatus.

It is hard to trace accurately the origin of the microscope and telescope. Some historians claim that a convex lens of rock-crystal found in the ruins of the palace of Nimrud was for use as a microscope, but there appears little evidence to prove the contention. Others say that Roger Bacon, who died in 1294, ground lenses for use as microscopes. We *do* know that old Anthony Leeuwenhoek, the janitor of the city hall of Delft, prompted by his innate genius and stimulated by his indefatigable energy and perseverance, did grind wonderful lenses and discovered new worlds upon new worlds. He found life in myriad places where man had no suspicion that life existed. To him every tiny drop of water was a new world teeming with new and strange forms of life. He can almost be said to have discovered the kingdom of microbes. What great things tiny lenses have done since the days of Leeuwenhoek! Old Leeuwenhoek, who was born in 1632 and and who died in 1723, has aptly been called the "Father of the Microscope," and it was indeed his pioneer work that enabled Spallanzani, the ever lovable Louis Pasteur, Koch, Roux, Behring, old Elie Metchnikoff, Theobald Smith, Bruce, Ross and Grassi, Reid and Paul Ehrlich, with many others, to rid both man and beast, not only partly, but largely, of the curse of infectious disease.

When we add to these triumphs what the microscope has done in the realms of chemistry, physics, botany, biology and a list of other "ologys" too long to mention, we can begin to realize what our new eye for seeing into the kingdom of the minute has done for the advancement of mankind.

The origin of the telescope, like that of the microscope, is more or less obscure. We all know the story of the little boy who, holding one spectacle lens in front of another while looking at the moon, exclaimed, "Papa, the moon is in the room." Probably that was the first telescope.

However the first man who *knew* how to make and use telescopes was our old friend, of Inquisition fame, Galileo Galilei. His famous sixty nights spent in discovering and studying the satellites of Jupiter with his new "plaything" put a new complexion on man's knowledge of the earth, proved the theories of Copernicus and were a potent factor in sounding the death knell of the medieval period and ushering in the Renaissance.

Galileo's telescope with an opening of only 2¼ inches in diameter had an area 81 times as great as the fully expanded iris diaphragm of the human eye and could therefore gather 81 times as much light. Herschel's 18-inch telescope could gather as much light as 5000 human eyes added together. A 60-inch telescope can gather 57,600 times the light a human eye can and the King of telescopes, the great Hooker telescope of Mount Wilson, with its 100-inch lens can gather as much light as 160,000 human eyes combined.

It is these wonderful instruments, with their companions in usefulness, the spectroscope and photographic plate, that have extended man's knowledge into limitless space and turned the siderial universe into an experimental research laboratory for the use and advancement of man.

This new eye of man's for peering into the realm of the infinitely great has enabled man to discover new planets, both major and minor, new stars literally beyond number, new spiral nebulae and new universes beyond universes.

The spectrograph, although perhaps not so directly a new eye, is, nevertheless, a new eye with which man has learned more of the secrets of Nature than by any other means. Its history and development is fascinating, but, in all its ramifications, is too long to deal with here. But its earliest beginning is interesting to record.

When that immortal genius, Sir Isaac Newton, closed the shutters of his room and, leaving a thin slit through which a beam of sunlight streamed, placed a wedge of glass (in modern language, a prism) in the path of the beam, and when he saw the beam of light spread out on the wall of his room in all the glorious colors of the spectrum, it was then that he exclaimed, "Behold the Specter!"—Thus was spectroscopy, the most magnificent of our modern sciences, born, christened and ushered into the world.

The spectrum has always seemed to us the Holy of Holies of science, because from it we can learn more of the secrets of the Creator than by any other means. It does not matter whether we point our spectrograph at an incandescent element in our Research Laboratory here on earth or at the Creator's laboratories in the remotest stars or furthest spiral nebulae, they write their "signatures" for us and tell us who (or which) they are. The spectrograph is man's first great analytical eye.

The x-ray diffraction apparatus, or shall we call it an x-ray spectroscope, is the most wonderful analytical eye that has yet been devised by man. It is our new eye for peering into the realms of the infinitely small and with it we can determine distances of less than one hundredth millionth of a centimeter. To find its origin we must go back to the year 1896 when Röntgen discovered x-rays, a new form of light of extremely small wavelength. But it was not until 1912 when Laue with his brilliant inspiration, work and assumptions, found a method of actually determining the distances between the different atomic planes of crystals that the science of x-ray spectroscopy was born. Sir William Bragg and many others have since made brilliant contributions to the new science.

In brief, every solid substance is composed of atoms arranged in a manner characteristic of that particular substance. The arrangement of the atoms in the specimen is determined by measuring the angles at which the x light is diffracted in passing through the crystals. It is possible to calculate the wavelength if the interatomic spacing is known or to calculate the spacing if the wavelength is known. So with the x-ray diffraction apparatus the exact position of every atom in a crystal is determinable for, by filtering, a known wavelength of x light can be obtained.

It is most important for the scientist to be able to determine the position of the atoms in a solid body as it is the arrangement of the atoms that largely determines the properties of the substance. And here is just where our new eye has left the old-fashioned chemist so far behind. A solid body consisting of just the same atoms may have different properties—say one piece may be brittle and the other ductile, but chemical analysis reveals no cause for the difference. Our new eye tells the physicist why one is brittle and the other ductile—the atoms are *arranged* differently.

Since November, 1924, we have been running a series of articles in our columns by Dr. Wheeler P. Davey, dealing with the different methods of using our new eye in the determination of atomic arrangements and of atomic energies in the crystal state.

We feel that this series of articles will be a valuable contribution to the literature of this most useful and up-to-date subject.

When we realize that a good microscope can see nothing smaller than an object as large as 2000 molecules combined, and that with our new x-ray eye we can determine the spacings between the individual atoms in the unit cell we know what a powerful instrument of research has been put into the hands of the scientist.

The four new eyes we have mentioned here are all the products of the greatest eye we have, the mind's eye. These are only a beginning—the possibilities of the future are infinite! J. R. H.

Carrier-current Control of Street-lighting Circuits

Systems Before Carrier Current—Search for Ideal System Led to Carrier Current—Its Development and Operation Under Varied Service Conditions

By F. M. SPAUGH and H. E. ALLEN
Street Lighting Department Radio Engineering Department
General Electric Company

ABOUT the year 1893 the first station-type constant-current transformer was built. This was designed primarily for installation in attended stations and took care of the needs very competently at that time because the area to be lighted was relatively small and cables could be taken directly from this attended station to the street-lighting units. Such a system involved the use of a control panel in the station. This system gave assurance of careful supervision for the control was in the hands of the station operator. It also utilized vacant space in the station which at that time was always available.

Owing to the rapid growth of cities, the increasing popularity of better street lighting, and the development of larger series lamps, it became evident that the generating and distribution capacity of the central station would have to be greatly increased to meet the demand of the increasing house distribution and street-lighting loads. The average station had not been designed to accommodate such an increase in station equipment and it was evident that new stations would have to be built or existing apparatus would have to be removed from the stations in the downtown districts. Not only were the stations crowded but the underground ducts were filled to capacity with outgoing copper. In the majority of cases it was either uneconomical or impossible to enlarge a station, because of the cost or unavailability of the adjacent property. Long before the situation had become acute, the farsighted engineers of the various manufacturing companies and public utilities had been searching for some means of de-centralizing the street-lighting distribution equipment. This resulted in the development of the first pole-type constant-current transformer in 1914. Such a transformer was entirely automatic in operation and could be connected directly to the primary supply by means of an ordinary switch or any method of remote control.

With this de-centralization of the street-lighting circuit, it was necessary to turn the circuits on and off at dusk and dawn. One of the first methods was to send out an operator to perform this operation manually. The control apparatus consisted of an oil switch (Fig. 1) mounted on a pole with the transformer. The switch was operated by means of chains or ropes. This means of control was rather expensive as it required sending a man around every night and every morning to switch the separate circuits on and off. Also, the reliability of the operation depended upon the trustworthiness of the person doing the work. This form of control was satisfactory for the needs of the time, but as people began to realize the advantages of street lighting and the importance of maintaining proper schedules, they demanded better lighting and more uniform schedules. This required that the central station adopt some means of remote control.

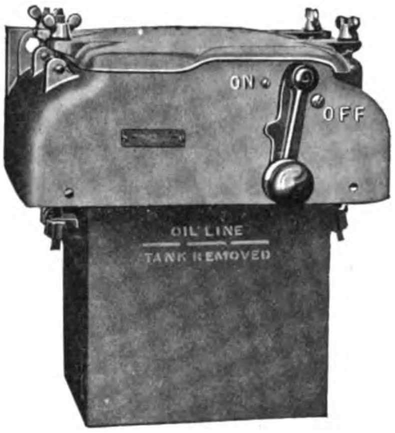


Fig. 1. The Type of Pole-mounted Oil-circuit Breaker Used for Manually Controlling a Pole-type Constant-current Transformer. In this service the breaker is arranged for operation from the ground by chains or ropes

In response to these demands, some central stations decided to run a primary loop around the town to take care of street-lighting circuits only. To this primary distribution system the constant-current transformers were directly connected. The control was placed in the hands of the station operator, who had merely to close the breaker on this primary loop to turn on the lights. In this way very accurate schedules were maintained. This method, however, had the disadvantage of being expensive because a primary distribution loop was tied up for part-time service; whereas if some other form of remote control were available, this feeder could have been used to furnish power by day as well as night.

The first attempt to meet the requirements of controlling these constant-current transformer circuits without using an individual feeder was by means of a clock mechanism which actuated a set of contacts.

Such a time-switch combination (Fig. 2) fulfilled the requirements to a limited extent. It was inexpensive; all that was required was the time switch itself. One objection to such a scheme was its inaccuracy, because a time switch could not be set closer than 10 to 15 min., and in addition the operation was dependent upon a relatively delicate clock mechanism. The main objection, however, was that time switches had to be wound and reset once a week. This resulted in a high labor charge.

The next development in the control of street-lighting circuits was a contactor actuated by an operating coil. The coils were designed for operation on either a series or a multiple circuit. These so-called controllers (Fig. 3) when arranged with a series operating coil made possible the use of such control schemes as cascade or series systems. In the cascade system (Fig. 4), the operating coil of a controller is connected in series with the nearest constant-current circuit. When the first circuit is energized, it in turn energizes the operating coil of the second controller which picks up the contactor; this action closes the primary to the constant-current transformer. In the second circuit there is also a similar combination. Thus circuits can be cascaded to any desired number. The series system (Fig. 5) is very similar in that the operating coil of the controller is in series with a series loop around the business part of town. Thus it constitutes a series pilot circuit. Such schemes have a relatively low installation cost as the only apparatus necessary for control is the controller itself. However, such systems have the disadvantage that any outage of any series circuit means the outage of all beyond that point; and in the case of a series control system, it means that the outage of the series loop in turn results in the outage of all the circuits operated from it. Therefore the use of cascade or series-control systems is limited.

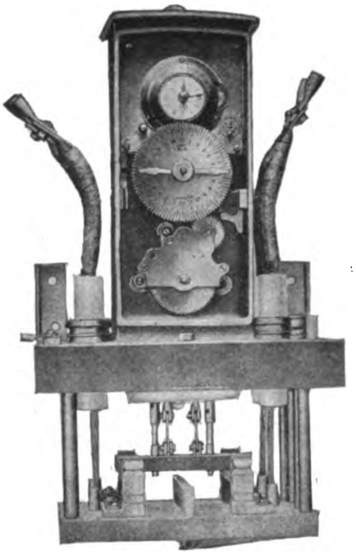


Fig. 2. Hand-wound Time Switch for Controlling a Pole-type Constant-current Transformer

Controllers that are similar except in having a shunt operating coil may be used in a pilot-wire control system. This system involves running a pilot wire (Fig. 6) from the central station to the various control points, or running a pilot-wire loop around the city to take care of all of the street-lighting circuits. A number of central stations are now using a three-phase four-wire distribution system having a grounded neutral. When this is done, the grounded neutral can be used for a return of the pilot circuit and it is necessary only to run one pilot wire, thus reducing the installation cost. This system is very reliable and has proved very satisfactory almost everywhere that it has been tried. As the control is in the hands of the station operator, it assures an accurate adherence to the control schedule; but, on the other hand, such a system is relatively expensive because of the high installation and maintenance costs of the very long pilot wire.

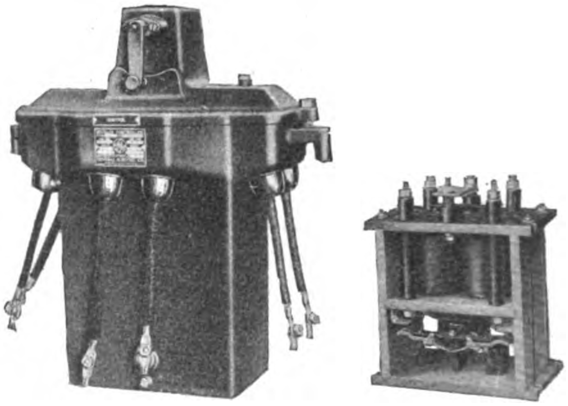


Fig. 3. Controller with Series Operating Coil for Use on Series or Cascade Systems

In view of the shortcomings and limitations of the various available control schemes, specifications were drawn up for an ideal type of control, and in these the following points had to be considered:

(1). The control must be in the hands of a station operator so that reliable schedules may be maintained and so that in case of foggy or dark days the lights may be controlled ahead of schedule.

(2). Minimum station space must be required by any apparatus necessary to control the various circuits.

(3). No additional wires should be necessary for controlling the various street-lighting circuits; and if a copper path is required, it must be the most reliable path available.

(4). The system must be entirely flexible.

(5). The system must be economical; *i.e.*, the installation and maintenance expense must be well within reason.

(6). The utmost reliability is required.

Control Problem

The foregoing general requirements for an ideal control system indicated that the power conductors themselves should be used for transmitting the control energy. Carrier-current control at once suggested itself as the ideal method by means of which these requirements could be fulfilled.

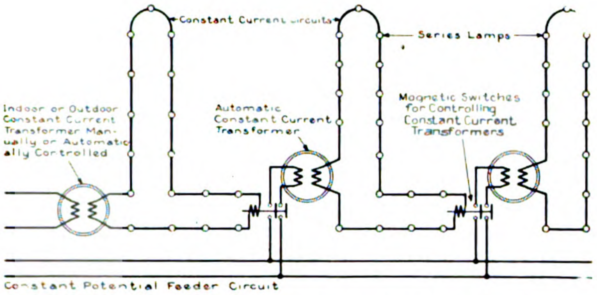

Constant Current Circuits
Series Lamps
Indoor or Outdoor Constant Current Transformer Manually or Automatically Controlled
Automatic Constant Current Transformer
Magnetic Switches for Controlling Constant Current Transformers
Constant Potential Feeder Circuit

Fig. 4. Diagram of Connections of Cascade System of Street-lighting Control

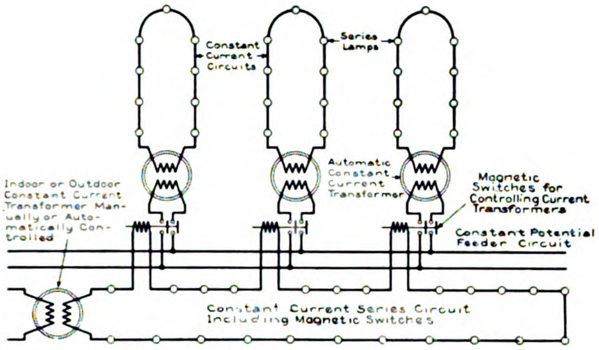

Constant Current Circuits
Series Lamps
Automatic Constant Current Transformer
Indoor or Outdoor Constant Current Transformer Manually or Automatically Controlled
Magnetic Switches for Controlling Current Transformers
Constant Potential Feeder Circuit
Constant Current Series Circuit Including Magnetic Switches

Fig. 5. Diagram of Connections of Series System of Street-lightng Control

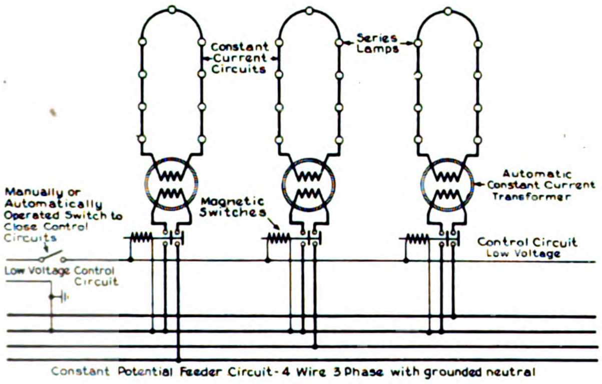

Constant Current Circuits
Series Lamps
Automatic Constant Current Transformer
Manually or Automatically Operated Switch to Close Control Circuits
Magnetic Switches
Control Circuit Low Voltage
Low Voltage Control Circuit
Constant Potential Feeder Circuit - 4 Wire 3 Phase with grounded neutral

Fig. 6. Diagram of Connections of Pilot-wire System of Street-lighting Control

In general, the operation of carrier-current control equipment is as follows: The carrier-current control energy is superimposed upon an existing power line or metallic circuit at the station end, thus energizing this line with the carrier power. At the point at which remote equipment is to be controlled (Fig. 7) a receiving apparatus is installed which will be actuated by these carrier-control impulses. The electrical control circuit will then be either closed or opened depending upon the type of control impulse impressed upon the receiving equipment.

This control energy had to be impressed upon the feeders and transmitted over these feeders without causing any reaction with the regular feeder service. Capacitive coupling to the power lines was obviously the proper means to adopt. These capacitors have a high reactance to the commercial frequency but readily pass the control-frequency energy, thus interfering in no way with the normal functioning of the line.

Control Frequency

Carrier energy can be transmitted over practically any metallic circuit but the same carrier frequency cannot always be used for the best results over different circuits. It was necessary to make a detailed study of commercial distribution feeders, connected apparatus, etc., in order that control frequencies could be selected which would enable the use of the minimum amount of control energy to perform all possible control problems with a high degree of service.

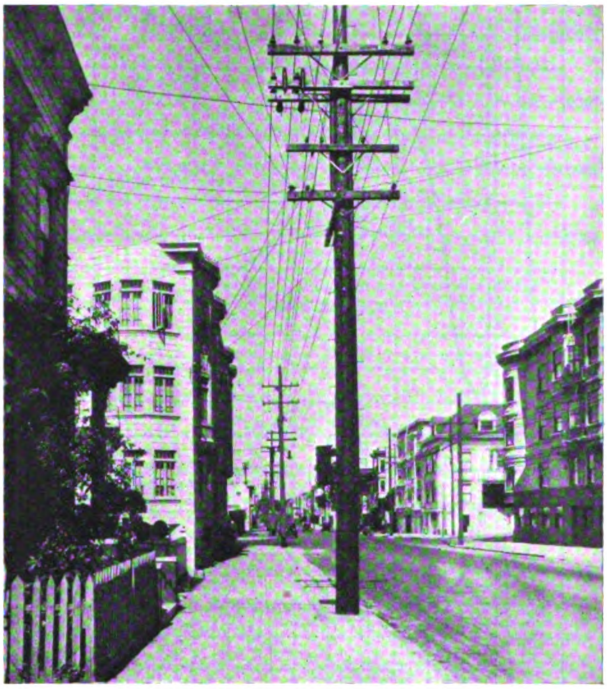


Fig. 7. An Installation of Carrier-current Control on an Aerial Street-lighting Circuit. The coupling-capacitor and drainage units are shown mounted on a cross-arm and the receiver on the left-hand side of the pole a short distance from the ground

In order that the carrier-blocking effect from the feeder voltage regulators might be determined, these were studied as an independent unit. Fig. 8 shows curves of voltage regulator impedance against carrier frequency. These curves show that the impedance of this regulator for the maximum load curves plotted is in excess of 700 ohms at 35, 40, and 45 kc. with the regulator coils in various positions. These three frequencies are the ones used for carrier-current control as explained later. The impedance of the regulator is seen to decrease with the higher primary loads, showing that the regulators would have their minimum carrier-frequency choking effect at the highest feeder loads.

These curves also show that the carrier energy can be impressed upon a feeder upon the line side of the voltage regulators with assurance that only a small amount of this energy will leak back into the substation bus.

The impedance of a primary feeder will be dependent partially upon the impedance of the connected transformers. Fig. 9 shows a curve of primary impedance against carrier frequency for several types of distribution transformers with the secondary winding of the transformer short circuited. Fig. 10 gives the same type of curves for these transformers with an open circuit upon the secondary terminals. These curves show the variation of the transformer primary impedance with respect to a change of transformer load and change of carrier frequency.

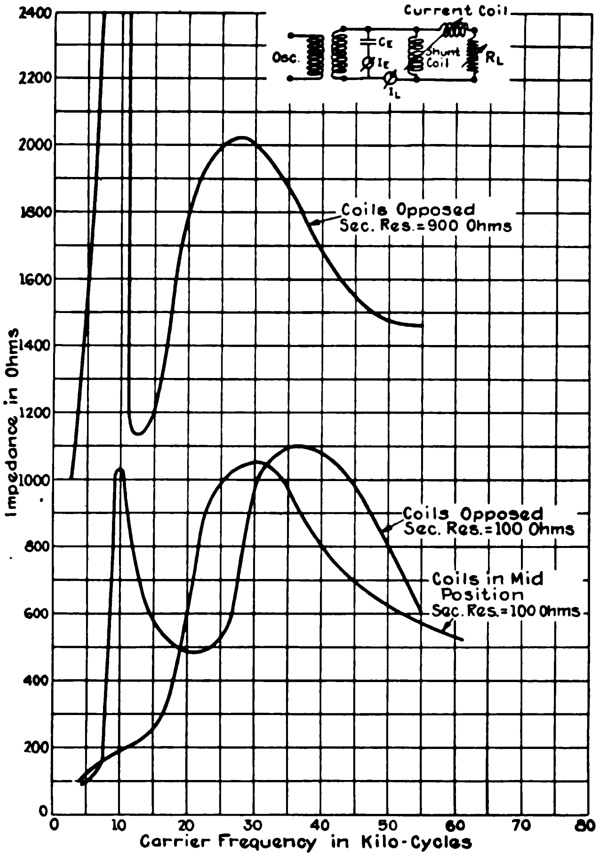


Fig. 8. Impedance Curves of Feeder-voltage Regulators with Respect to Different Frequencies of Carrier Current

With the impedance characteristics of feeder-voltage regulators and distribution transformers at hand, the next step was to determine the carrier-frequency characteristics of several primary feeders. Measurements were made on a number of 4000 and 2300-volt primary feeders which were supplying commercial load. These measurements in most cases were taken upon the line side of the voltage regulators. The measured characteristics of one feeder are as shown in Fig. 11. Power transmission tests at carrier frequencies were also made upon this same feeder at a point approximately six miles distant from the transmitter, which was located at the substation. From the substation end, 8700 ft. of this feeder consisted of a three-conductor underground cable and the remaining ten miles consisted of a four-wire aerial line. The curve in Fig. 12 shows that the maximum energy was transmitted between the two points at 40 kc.

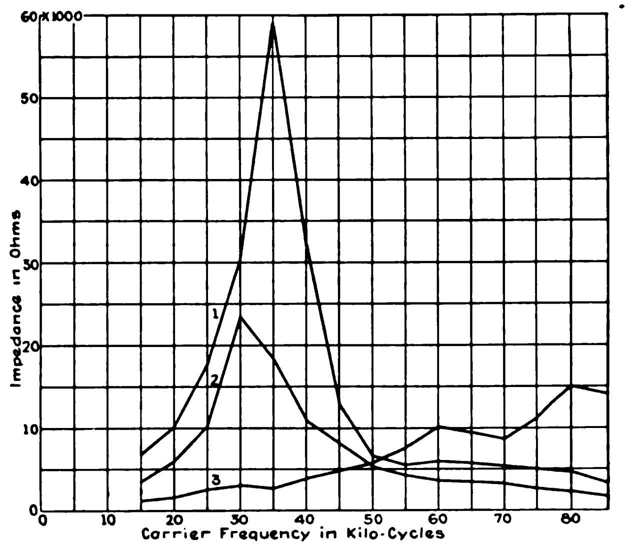


Fig. 9. Impedance Curves of Distribution Transformers, Taken on the High-voltage Winding with the Low-voltage Winding Short Circuited

Curve *1:* 3-kv-a. transformer
Curve *2:* 7½-kv-a. transformer
Curve *3:* 15-kv-a. transformer

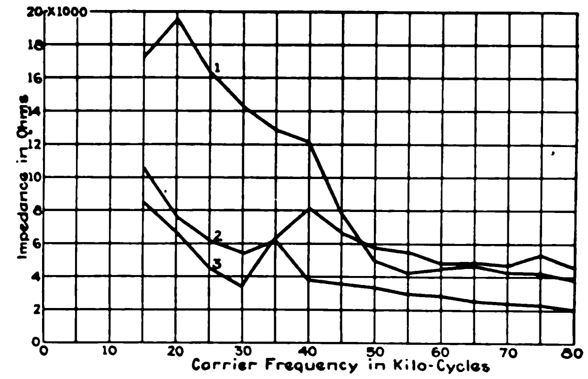


Fig. 10. Distribution Transformer Impedance Curves Taken on the High-voltage Winding with the Low-voltage Winding Open Circuited. (Compare with Fig. 9)

Curve *1:* 3-kv-a. transformer
Curve *2:* 7½-kv-a. transformer
Curve *3:* 15-kv-a. transformer

From analyses of these and other similar impedance measurements, and a detailed study and measurement of a great many distribution systems of this same general type, it was determined that 40 kc. would be the best average frequency to use with 35 and 45 kc. as alternative frequencies.

Control Energy Required

After arriving at these conclusions as to the proper control frequency to use, and with a knowledge of the voltage distribution along the feeder, it was necessary to determine the amount of energy at the carrier frequency which would satisfactorily energize and perform control operations on feeders of these types.

In order to determine the amount of transmitter energy required, it was necessary to develop the carrier-control receiver first. This receiver had to be immune from operation and damage by power-line surges, voltages induced by lightning strokes, etc., but had to operate upon a very low carrier-control voltage that represented no appreciable amount of control energy.

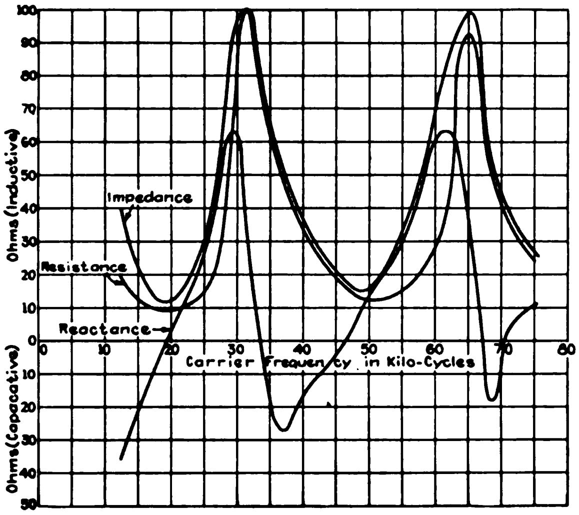


Fig. 11. Carrier-frequency Characteristics of One Phase of a 4150-volt Distribution Feeder Approximately 12 Miles Long

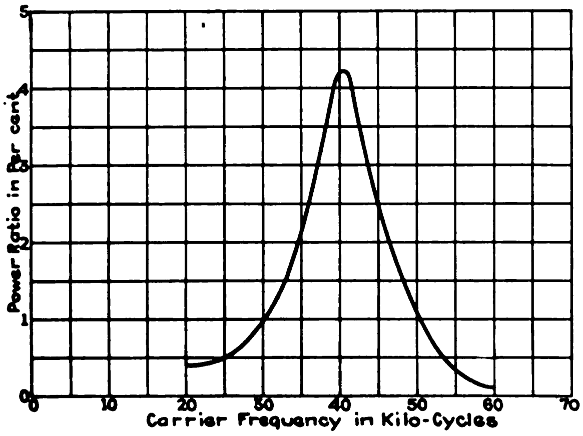


Fig. 12. Per Cent of Transmitted Power Received in a Coupled and Tuned Circuit (with 50,000-ohm load) Over One Phase of a 6-mile Section of a 4150-volt Feeder; Initial 8700 ft. Underground Cable, Remainder Aerial Line

The carrier-control receiver finally developed utilized a single, type PT-210 pliotron tube supplied with alternating-current plate, grid, and filament voltages, and used as a biased detector, the connection diagram of which is shown in Fig. 13. The filament of the tube is energized continuously at a reduced filament rating. The standby plate current of the vacuum tube is zero, thereby requiring no plate energy except during the operating cycle of the receiver. The minimum expected life of this type of tube run under these conditions is about one year. Vacuum tubes are, however, still in service under similar conditions after a period of over three years.

No receiver tuning circuit is necessary. A fixed carrier-frequency input transformer is used which has an inherent resonance peak covering the three selected control frequencies, thereby eliminating all need for delicately adjusted input circuits. The receiver input circuit has a very high impedance at the control frequency. This means that the actual carrier energy required for receiver operation is of a negligible value.

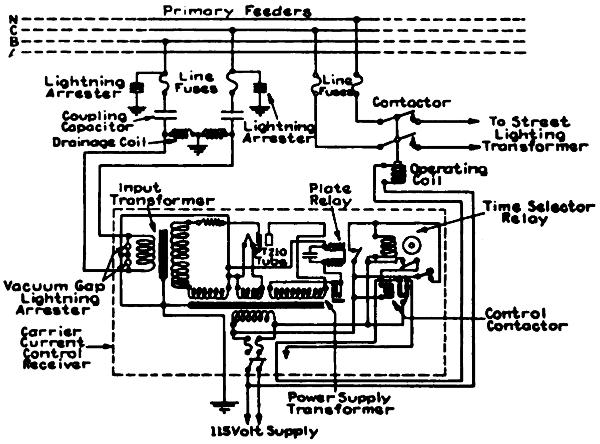


Fig. 13. Diagram of Connections of Carrier-current Control Receiver

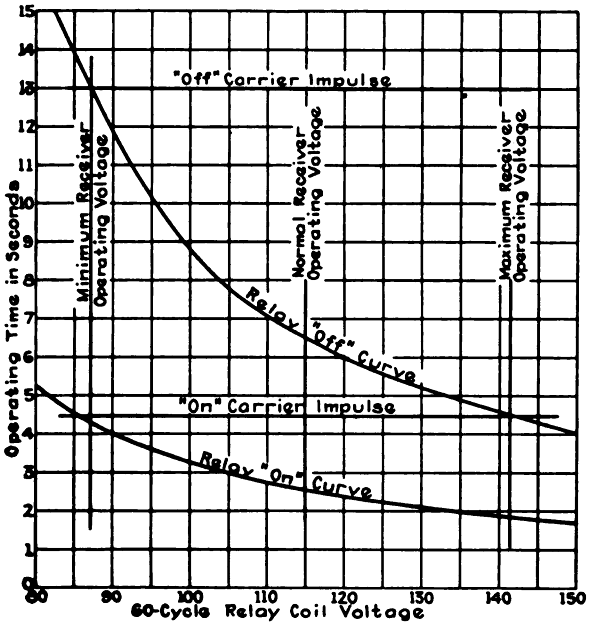


Fig. 14. Characteristic Curves of Time-selector Relay

The receiver is coupled to the carrier-energized phase of the transmission line through high-voltage coupling capacitors as shown in Fig. 13. The coupling-capacitor reactance is high at the power frequency, so there is in effect no ground placed upon the distribution system by this capacitor. The capacitor, however, offers a very low reactance path to the carrier frequencies and thus allows the carrier energy to pass freely into the control receiver. A drainage coil having a low reactance at the commercial frequency is mounted in the capacitor case and is connected between ground and the low side of the capacitor, thus being utilized to hold the 60-cycle voltage at that point to practically a zero value. The carrier-frequency voltage, however, is maintained at practically its former value due to the very high impedance of this drainage coil at the carrier frequencies. Vacuum-gap lightning arresters are used in the receiver to protect it against high-voltage surges.

In order to obtain selectivity, *e.g.*, "on" or "off" operation of the controlled equipment, a time-selector relay was developed, utilizing the inductive-disk relay principle. This relay contains a small mercury-tube switch which either opens or closes a control circuit as desired. A short carrier impulse will close these control contacts and an impulse approximately two and one half times as long will open them. Time-voltage curves of this relay are shown in Fig. 14. The "on" carrier impulse is of 4½ sec. duration and as shown in the curves, the relay coil voltage may drop to 86 volts but the relay will still function "on" properly. The "off" carrier impulse is of 13 sec. duration which for correct "off" operation gives 87 volts as the minimum operating potential. The maximum operating voltage is at the point where the relay "off" operating curve and the "on" carrier impulse line cut each other, *viz.*, 141 volts. The normal operating voltage of the equipment is 115 volts. These curves show that the operating voltage may vary 23 per cent plus or minus and the time-selector relay will still operate satisfactorily.

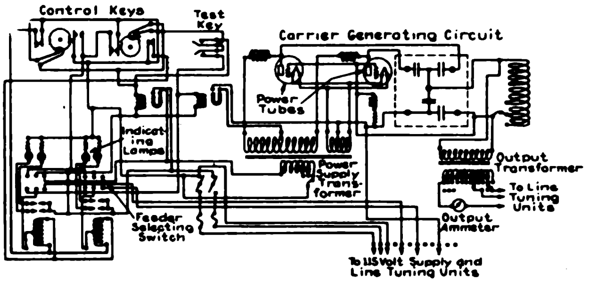


Fig. 15. Diagram of Connections of Carrier-current Control Transmitter

Transmitting Equipment

The receiver development having been completed, and with impedance curves of the distribution line and connected apparatus available, analyses showed that 100 watts of carrier-frequency control energy would be sufficient to control equipment reliably over the majority of distribution feeders.

The carrier generator developed is of a simple full-wave self-rectifying design and utilizes an alternating-current voltage directly for the power supply, no motor-generator set being necessary. The oscillator has a normal power output of 100 watts. The carrier generator is coupled to the distribution circuit through an output transformer by means of which the distribution-line impedance and the oscillator impedance are matched so as to effect a maximum transfer of energy into the line. The complete transmitter circuit used is as shown in Fig. 15.

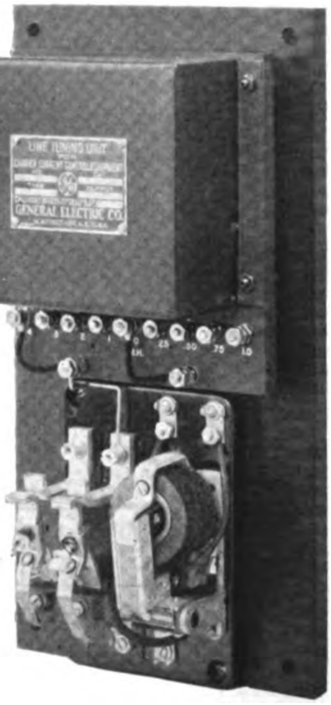


Fig. 16. Tuning Unit for Connecting and Resonating the Transmitter to the Feeder Circuit

A line-tuning unit (Fig. 16) is used to connect and resonate the transmitter to the particular feeder upon which coupling equipment has been installed. The relay which is mounted on the lower part of the panel acts as the means of connecting the carrier-generator and line-tuning equipment to the line-coupling capacitors. The tuning coil which is located at the top portion of this panel is used as a means of tuning the coupling capacitors, transmission line, and the output transformer output winding, to resonance with the carrier generator.

A keying mechanism which is driven by a coil spring is utilized to start the transmitter, connect it to the desired feeder, send out the controlling carrier impulse, and then disconnect and shut down the equipment and put it in readiness for the next operation. One such key is used for the "on" operation and another for the "off" operation, the only difference being in the length of the impulses they send out.

A feeder selecting and indicating group is mounted upon the transmitter for each individual circuit over which equipment is to be controlled. This assembly includes a red and a green indicating lamp which indicate to the operator the last carrier impulse sent out over the particular circuit in question. The number of feeders or circuits which may be energized from one transmitter may be increased by merely adding additional transmitter-panel-selecting and indicating groups and additional feeder coupling and tuning equipment.

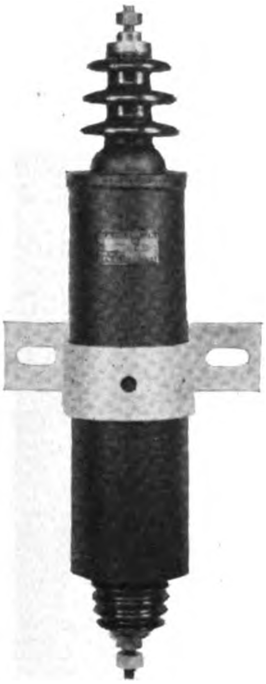


Fig. 17. A Typical Carrier-current Coupling Capacitor

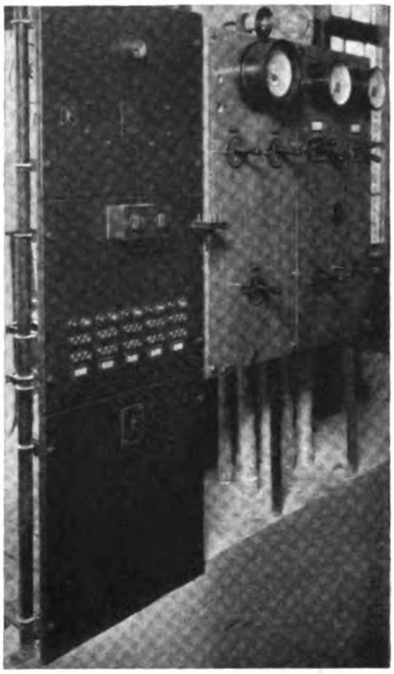


Fig. 19. Transmitter Panel (at the left) Equipped for Carrier-current Control of Five Separate Street-lighting Feeders

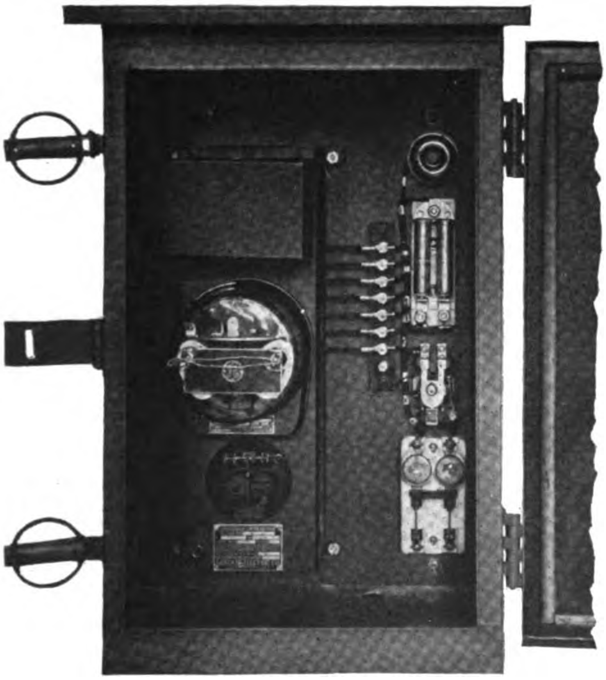


Fig. 18. Receiver Unit for the Carrier-current Control of Street-lighting Circuits

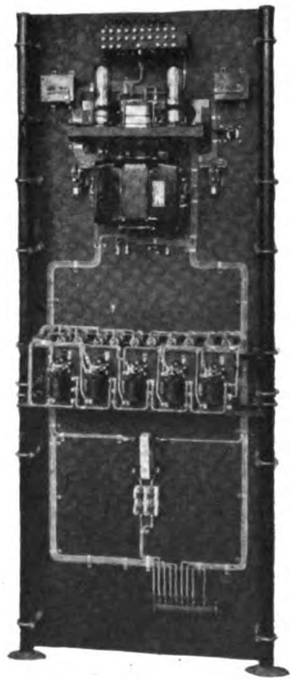


Fig. 20. Rear View of Transmitter Panel Showing Carrier-current Generator and Feeder-selecting and Indicating Units

The transmitter is coupled to the transmission line through high-voltage coupling capacitors very similar to those shown in Fig. 17 except that their capacitance is five times as great. A drainage coil is also included in the capacitor case.

Commercial Design of Receiver

The final form of receiving equipment is shown in Fig. 18. The left-hand unit in the receiver is detachable and upon it is mounted practically all of the essential receiver parts. The time-selector relay is mounted in the center of the unit and the plate circuit relay can be seen through the round opening in the lower part. The box at the top contains both the carrier-frequency and the power-supply transformers. The receiver tube is mounted in back of this unit. The power-supply apparatus, control contactor, and lightning arresters are mounted upon the receiver panel proper.

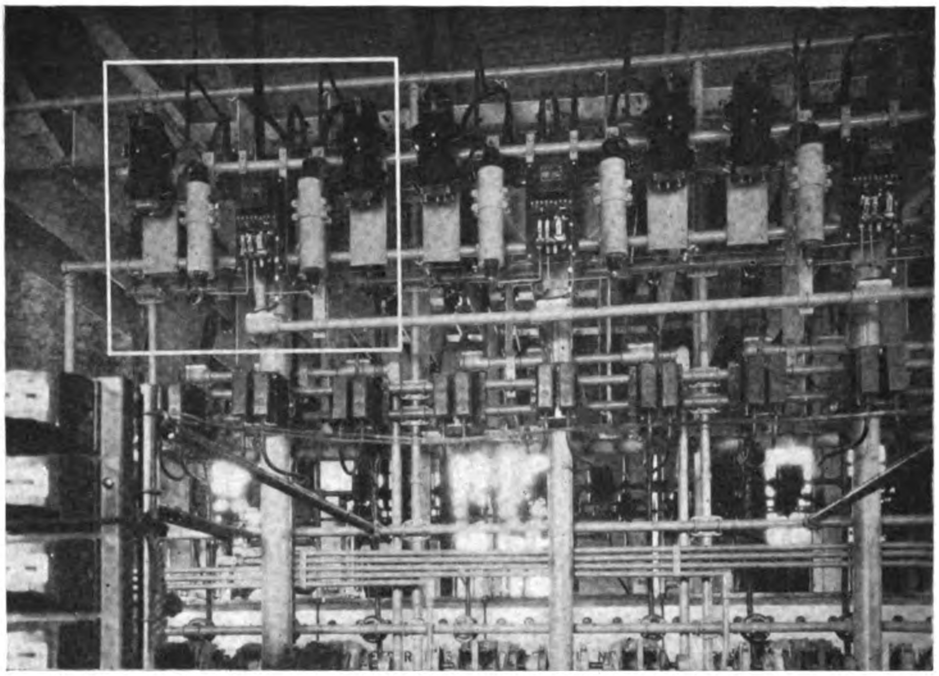


Fig. 21. An Installation of Coupling Capacitors and Line-tuning Equipments. The equipment included within the white rectangle is necessary for one feeder

Transmitter

A front view of the commercial form of transmitting equipment is shown in Fig. 19, and a rear view in Fig. 20. The panel is of standard switchboard dimensions. All equipment mounted on the front of the panel is utilized for actuating control circuits and is at a low potential. The carrier-current generator itself is mounted on the back at the top (Fig. 20). The feeder selecting and indicating groups are mounted on the center section of the panel. The control keys are located at the top of the center panel on the front, there being two keys, one for the "on" operation and the other for the "off" operation.

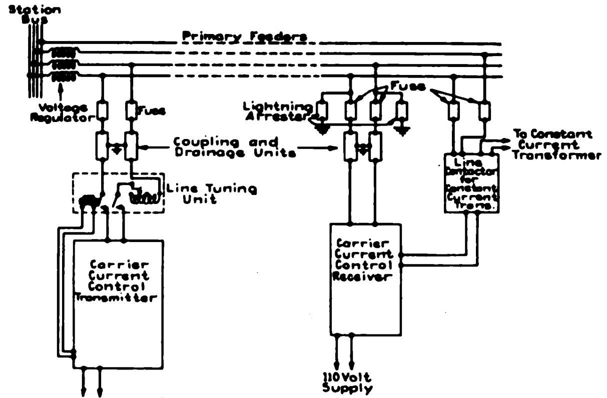

Station Bus
Primary Feeders
Voltage Regulator
Fuse
Lightning Arresters
Fuse
Coupling and Drainage Units
To Constant Current Transformer
Line Contactor for Constant Current Trans.
Line Tuning Unit
Carrier Current Control Transmitter
Carrier Current Control Receiver
110 Volt Supply
110 Volt Supply

Fig. 22. Schematic Diagram of Connections for a Complete Carrier-current Control Street-lighting Installation

Application of Carrier-current Control Equipment

A commercial installation of carrier-current receiver equipment is shown in Fig. 7.

In the carrier-current transmitter installation in Fig. 19, the panel is shown mounted adjacent to two standard arc-type street-lighting control panels. The compactness of the transmitter equipment is further emphasized by the fact that it is equipped to control five distribution feeders and could be equipped to control fourteen without increase in panel size.

Fig. 21 shows a view of line-coupling equipment on three 4000-volt feeders. The two line-coupling capacitors are connected to the two phase wires through two expulsion-type fuses. The line-tuning unit is mounted between these four units.

Usually each feeder has its own individual induction voltage regulator. In such a case the carrier energy is supplied to the feeder from the line side of the regulators. Fig. 22 is a schematic wiring diagram of a complete carrier-current control installation.

Special Applications

It is not uncommon in present-day practice to regulate several feeders simultaneously by one set of regulators. When bus regulation is used, it is possible to energize this bus with carrier energy and actuate control receivers installed upon all of the feeders leaving this bus. The actual amount of carrier-control energy required to energize the system will depend upon the number and type of feeders leaving the common bus and upon the capacity of each.

There are also instances where booster transformers are installed out upon a primary feeder. In case it is desired to provide control at a point beyond these transformers, it may be necessary to install carrier by-passing equipment around the transformer windings.

It will be found at times that there is no actual metallic circuit between the point at which the transmitter is to be located and the proposed location of the street-lighting control equipment. In case the feeders upon which the two sets of equipment are to be located are in close proximity to each other, carrier by-passing equipment can be installed between the two sets of lines. This consists of two high-voltage condensers which offer a high impedance to the 60-cycle power current but which offer a ready path to the carrier energy between the lines.

There are several specific conditions which if present would necessitate the use of by-passing equipment. For example, disconnecting switches when open might interrupt the continuity of the carrier transmission circuit. Another occasion for by-passing would be where the transmitter was located at a manually operated substation that feeds a particular section with an adjacent section fed from an unattended substation. In this case, the lines would somewhere be in very close proximity to each other at which point the by-passing equipment could be installed.

Also, it may be necessary to perform special switching operations on the same transmission system to which the carrier-current street-lighting control equipment is connected. Under such a condition it would be necessary to be able to control one type of equipment without interfering with the other. This may be done by utilizing a code-selector type of receiver at these special control points instead of the time-selector type generally used for street-lighting circuit control. By using the code method, the operation of one type of receiver will not cause the other to function. All time-selector type receivers upon the same circuit will actuate their respective switches simultaneously. A number of code-selector receivers may be installed upon the system, each having a different code combination from the others, with the result that each can be operated at will without affecting any other control circuit.

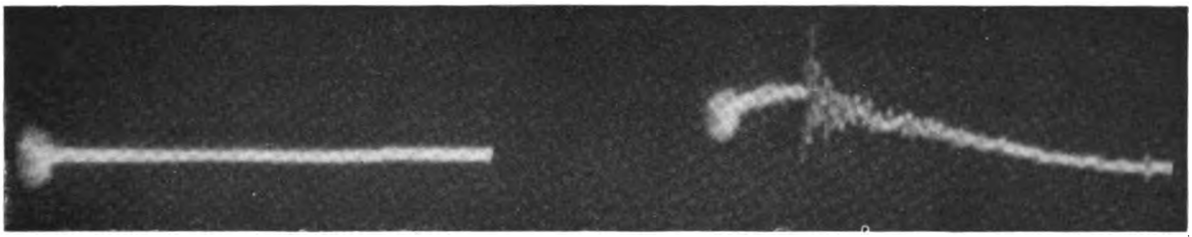
Street-lighting circuits are now being installed which have a combination of both "all-night" and "midnight" street lamps. The midnight lamps are turned "off" at about midnight, whereas the all-night lamps remain in service until morning. It is possible by means of a time-selector carrier-current control receiver of a triple-function type to turn the street-lighting circuits of both types "on" simultaneously, and to turn them "off" separately at midnight and morning.

Results Accomplished

Carrier-current developmental equipment for controlling street-lighting circuits has been in actual service in Schenectady (N. Y.) for the past three years and also at Rochester (N. Y.) and Bayonne (N. J.) for approximately two years. In addition, there are now commercial installations at Glens Falls (N. Y.), San Francisco (Calif.), and at Detroit (Mich.). The actual service records of all these equipments have demonstrated beyond question the practicability of this method of control.

It can be seen that the requirements as previously laid down for an ideal control system have been fulfilled by the application of carrier current to this problem. Apparatus formerly used primarily for radio communication purposes has been successfully applied to the remote control of street-lighting and other circuits. The application of this type of equipment, vacuum tubes, etc., to problems of the public utilities has a great potential field and only a beginning has so far been made.

Cathode-ray Oscillogram of Lightning Surge



First record of the voltage of a lightning surge on a transmission line obtained in this country by the cathode-ray oscillograph. The oscillograph employed was so arranged that it was set into operation automatically by the surge, thus obtaining the record with certainty rather than by chance.

The record was made on the 220-kv. Wallenpaupack-Siegfried transmission line of the Pennsylvania Power and Light Co. at 12:45 p.m., July 27, 1928. The oscillogram at the left is of the base line; that at the right shows the voltage surge, the record being 40 millionths of a second in duration. (The oscillograph used was described in the August, 1928, issue of the Review.)

High-speed Circuit Breakers, with Special Reference to A-c. Type

Advantages of High Speed—Magnetic Operating Mechanism—Air-break Circuit Breaker—Current-rupturing Tests—Selective Operation

By J. W. McNAIRY

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General Electric Company

> *In our July, 1926, issue we printed the author's A.I.E.E. paper describing direct-current high-speed circuit breakers in railway service, and below we publish his description of alternating-current high-speed breakers as he presented it to the recent Summer Convention of the A.I.E.E. at Denver, except for the omission of a mathematical section. The subject of high-speed breakers is comprehensively covered in this pair of articles.*—EDITOR

HIGH-SPEED circuit breakers were first applied on direct-current railway systems for protecting commutating apparatus against the effects of short-circuits or flashovers. Later development led to their application to all the power circuits of this type of system. A paper describing the principal features of a complete breaker installation of the direct-current type on the terminal electrification of the Illinois Central R.R. was presented at the June 20, 1926 meeting of the A.I.E.E.⁽¹⁾ Reference was made at that time to the development work being carried on in connection with alternating-current applications.

The higher voltages of the trolley network of the usual alternating-current system makes high-speed operation of circuit breakers more difficult; and progress therefore has not been as rapid as with the direct-current type. Some applications of alternating-current breakers to railway feeders designed to operate considerably faster than the usual oil breaker have been made in the past, but speeds approaching that of the direct-current breaker have only recently been attained.

Both oil and air-break types of alternating-current breakers having an operating speed comparable to that of the direct-current type have recently been developed, and commercial designs suitable for voltages up to 12,000 volts alternating-current are now available.

The application of an air circuit breaker to this type of service marks the first application of breakers of the air type to moderately high alternating-current voltages.

Advantages of High-speed Operation on A-c. Railway Circuits

Short-circuits are necessarily more frequent on railway systems than on other types of power distribution networks, because of the nature of the service. High-speed operation of the protecting breakers for single-phase alternating-current systems reduces the short-circuit stresses on transformers and generator windings and limits the burning of insulators, conductors, or parts of motive equipment caused by arcs resulting from these short-circuits, in the same manner as in direct-current applications. Breakers of the high-speed type are frequently effective in preventing the burning off of trolley wires in case of an insulator arc over, thereby minimizing this cause of service delays.

One of the most important problems in connection with alternating-current railway electrification is that of inductive interference with adjacent communication and signal circuits under short-circuit conditions. The lower impedance of the grounded track-return circuit results in a maximum current in case of a failure in the trolley circuit, and therefore the ground current is usually limited only by the impedance of these metallic conductors. The railway circuit differs in this respect from the usual power circuits where there is no grounded metallic return circuit. Induced voltages in adjacent low-voltage circuits are therefore higher, on the average, and appear more frequently than in regions where ordinary power distribution networks are located.

It is expected that the application of high-speed breakers to alternating-current railway systems will greatly reduce this interference by reducing to a minimum the duration of induced voltages in the parallel circuits. The reduction in the duration of the induced voltage minimizes the duty on protective devices in the communication circuits; and it is expected that the false operation of relays, bells, and other devices, resulting from this induced voltage, will be eliminated to a great extent. This is, perhaps, one of the greatest benefits to be derived from the application of this type of circuit breaker to the alternating-current railway feeder.

Speed of Operation

The total time required for the operation of the usual alternating-current circuit breaker may be divided as follows: (1) time required for relay operation,

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⁽¹⁾ *Transactions of A.I.E.E.*, Vol. XLV, p. 962; also GENERAL ELECTRIC REVIEW, July, 1926, p. 490.

(2) time required for the separation of the contacts, and (3) time required for extinguishing the arc.

In some forms of breakers, (particularly those described in this article) relays have been eliminated and the circuit breaker is operated directly by the short-circuit current.

The speed of operation of the mechanism is more or less definitely fixed by the mechanics of the device, and the separation of the contacts occurs at a definite time after the trip point is exceeded.

For alternating-current applications, the ideal breaker is one which opens the circuit at the normal zero point of the current wave, thereby avoiding undesirable voltage transients. Since the separation of the contacts may occur at any instantaneous current value, depending upon the magnitude of the short-circuit current and the displacement of this current wave, the arc-extinguishing system should be so porportioned that the current will not be forced down too rapidly before a normal zero point is reached.

This action is usually not difficult to secure because at zero current no energy is delivered to the arc and it is most easily extinguished at this point. It is possible, however, to design high-speed breakers having too powerful a blowout or current-rupturing system so that when the first short-circuit current loop approaches a cycle in duration, and the contacts open when the line current is near the peak of the wave, the circuit may be opened too rapidly and undesirable transient voltages may result.

It is sometimes assumed that most failures on a single-phase 12,000-volt system occur at the peak of the voltage wave and, therefore, that the usual short-circuit is symmetrical with a duration of a normal half cycle for the first current loop. A great many short-circuits however are produced mechanically, the conductors coming together at any time so that the short-circuit is established at any point on the voltage wave. Records obtained during experimental investigations using contacts in both air and oil for applying short-circuits at 12,000 volts show transients having all degrees of displacement of the current wave.

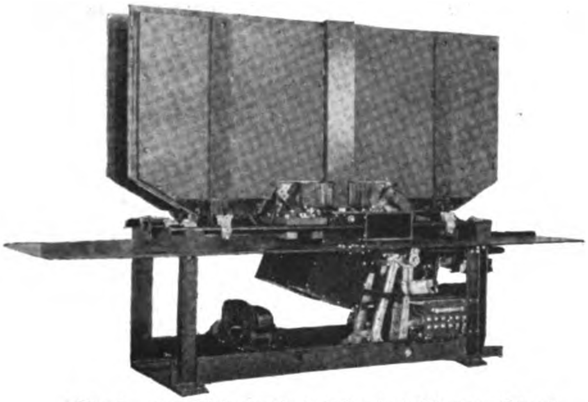

Fig. 1. A-c. High-speed, 12,000-volt, 1500-amp., Circuit Breaker without Outdoor Housing

The duration of the first loop of current is therefore frequently less than one-half of a normal cycle. The magnitude of the current in such a first loop is not a fair indication of the severity of the short-circuit, and the breaker does not receive the full tripping current or current through the blowout system until after the second current loop is reached.

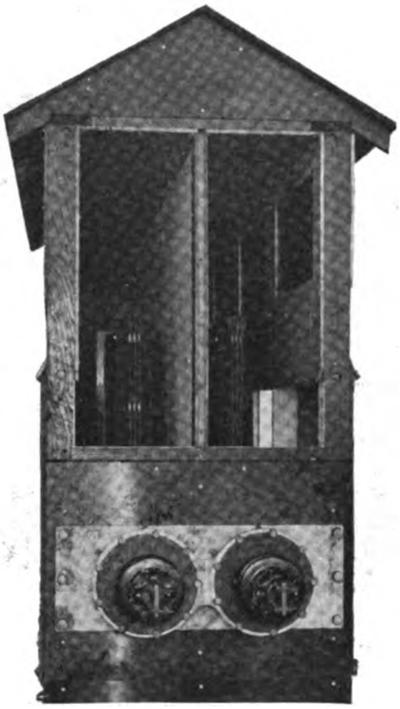

Fig. 2. High-speed Air Circuit Breaker in Outdoor House with One End Removed to Show Location of Circuit Breaker. Location of high-voltage terminals also shown

The design of a breaker to limit all short-circuits to a single loop of current is therefore difficult and, if successful, the relatively powerful current-rupturing system might reduce the current at a dangerous rate when the first loop approaches a cycle in duration.

It thus appears that the most logical design of an alternating-current breaker is one capable of opening severe short-circuits at the first zero after a current loop, having a duration of one-half a normal cycle or longer.

Oil *vs.* Air-type Breakers

One method of obtaining high-speed operation is to greatly increase the mechanical speed of the conventional type of oil breaker, so that sufficient arc length for the interruption is obtained in the specified time. This means the exertion of relatively great operating forces, either from powerful springs or solenoids, with a corresponding increase in the mechanical difficulties of starting or stopping the mechanism.

One of the most effective means of rapidly increasing the arc length is by means of a magnetic blowout. By properly directing the magnetic forces exerted on an arc stream drawn between the contacts of a circuit breaker, the arc can be lengthened at almost any desired rate. The contacts of the breaker therefore need only be separated sufficiently to withstand the voltage as the circuit is opened by the collapse of the arc stream. By applying a magnetic blowout to an oil breaker, the necessary contact separation at the instant the circuit is interrupted can be greatly reduced and a mechanism moving at moderate speeds can be utilized. The speed of operation and rupturing capacity of developmental switches of this type have met expectations. Elimination of oil, however, is always desirable where possible. This is particularly true for railway applications.

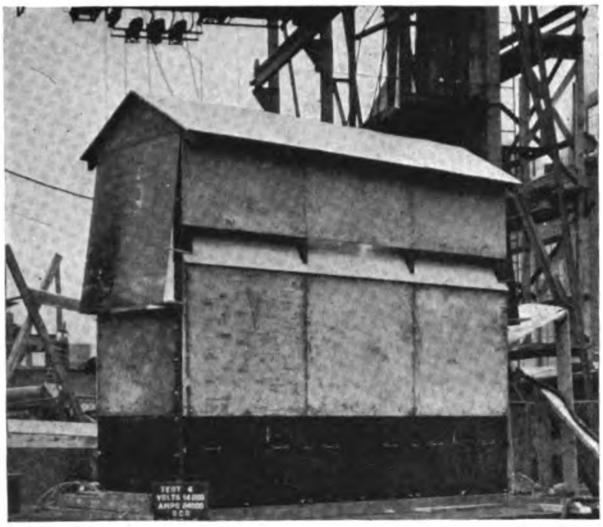

Fig. 3. 12,000-volt 1500-amp. High-speed Air Circuit Breaker Complete with Outdoor Protective Housing Opening 24,000 amp. (r.m.s.) at 14,000 Volts

The feeder network of important sections of electrified steam-railway systems, where continuity of power supply is imperative, requires a relatively large number of breakers for proper sectionalizing. On a four-track system of this type, the 12,000-volt breakers may average as high as two per mile and the maintenance of such a large number is obviously an important item in the operating expense.

The exposure of current-carrying conductors of the railway network to the wear and tear of current collectors, in addition to conditions common to all aerial conductors, results in frequent short-circuits. The majority of these failures are to ground, which in railway circuits means a line-to-line short-circuit, the current being limited by the circuit reactance only. Railway circuit breakers are therefore frequently called upon to open severe short-circuits.

If a given circuit breaker opens a circuit repeatedly without attention, the oil deteriorates to such an extent as to endanger the reliability of the breaker. Repeated short-circuits are most likely to occur during bad weather conditions when filtration or changing the oil is most difficult, particularly in outdoor installations. The air breaker is therefore much to be preferred for this type of service, assuming that it is the equivalent of the oil breaker in all other respects.

The direct-current air-break circuit breaker has demonstrated its ability to withstand repeated short circuits without deterioration or objectionable damage to its contacts. Alternating-current air-break breakers of the same general type have, during test investigations, withstood 20 or more maximum short-circuits repeated at two-minute intervals without requiring attention and without deterioration of the current-rupturing ability. Breakers of this type, Fig. 1, are therefore on a par with the direct-current type in this respect.

On the other hand, the air circuit breaker is not ideal in all respects. For instance:

(1). The air breaker proper is not suitable for outdoor mounting, and a protective housing is necessary. An air breaker complete with self-contained housing suitable for outdoor installations is shown in Fig. 2. Fig. 3 was taken by leaving the photographic plate exposed while the breaker was opening a 24,000-amp., 12,000-volt short-circuit, the oscillogram of which is shown in Fig. 7.

(2). The operation of the circuit breaker on severe short-circuits makes somewhat more noise than the oil type.

(3). The application of the air breaker is at present limited to moderately high alternating-current voltage circuits.

Both types of breakers will therefore undoubtedly find a field of application on alternating-current railway-feeder circuits.

Magnetic Operating Mechanism

Practically all direct-current high-speed breakers are of the magnetic-tripping type, most designs making use of a flux-shifting principle wherein the flux is shifted from a holding armature to a magnetic bypass circuit to release the breaker. The principle of operation of the flux-shifting type is well known and has been described repeatedly in publications.(2) Briefly, it consists of a holding magnet energized by a direct-current coil across the poles of which a holding

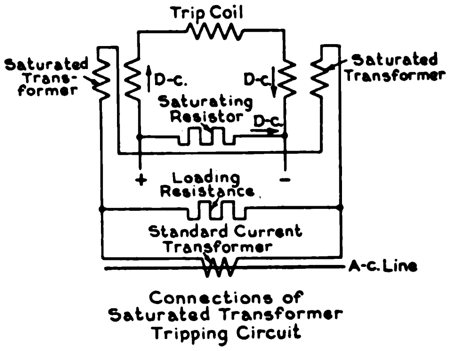

Fig. 4. Connections of the Tripping Circuit for the Magnetic Type of A-c. High-speed Circuit Breaker

armature is bridged. Between the poles a tripping coil is introduced in such a way that when a current of proper polarity is applied the flux is shifted instantaneously from the armature to the core of the tripping coil, thereby releasing the breaker. This type of mechanism, when tripped under short-circuit conditions by the line current, furnishes a practically instantaneous release by eliminating tripping relays, which is a very effective arrangement when maximum speed is desired. The mechanism has been successfully applied to both oil and air-type 12,000-volt alternating-current breakers.

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(2) Tritle, U. S. Re-issue Patent 15,441. "New Type of High-speed Circuit Breaker," by J. F. Tritle, GENERAL ELECTRIC REVIEW, April, 1920, p. 286.

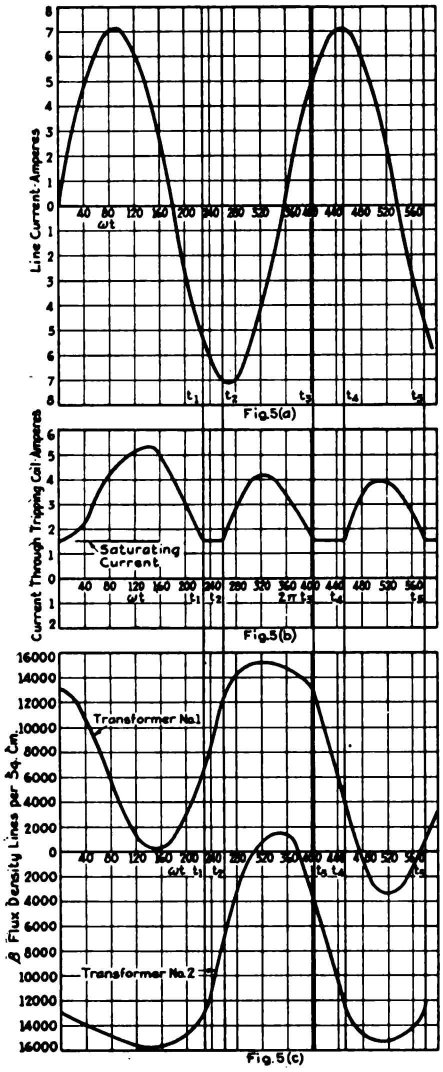

Fig. 5. Calculated Current and Flux Curves for Saturated Tripping Transformers Used in Connection with the Magnetic Type of Mechanism of the A-c. High-speed Circuit Breakers

Some means of providing a unidirectional current in the tripping coil of this type of mechanism is essential for applications to alternating-current circuits. The saturated-current-transformer type of trip circuit, as shown in Fig. 4, was adopted for this purpose. The tripping circuit is usually designed for a current of 5 amp. so that standard current transformers can be used in the main power circuit.

The operation of this trip circuit may perhaps be better understood by first considering the phenomena responsible for the so-called "starting current" when voltage of proper polarity is suddenly applied to a transformer, the core of which is magnetized by residual flux. The starting current for the transformer is the result of lowered reactance because of over-saturation of the core, the available range in flux in one direction having been reduced because of the initial residual flux.

The transformers used in the tripping circuit are of normal current-transformer design. When there is no alternating-current through the windings, the flux in the cores of these transformers is maintained near the saturation point by direct current through one of the windings of each transformer, this current being the same as that used for the holding coil of the circuit breaker.

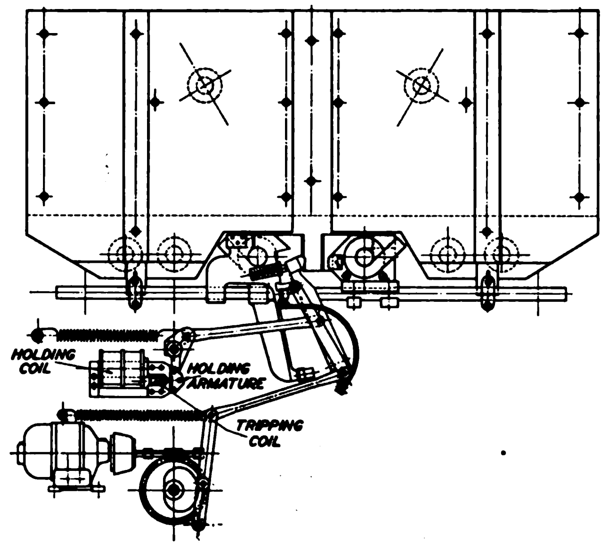

Fig. 6. Operating Mechanism of the Air-break 12,000-volt A-c. High-speed Circuit Breaker

The connections are such that when current of given direction is passed through the primary winding of both transformers, such as during the first half cycle of an alternating-current short-circuit, it exerts a magnetomotive force to increase the flux in the core of one transformer and to decrease the flux of the other transformer simultaneously.

Since the cores of both transformers are initially saturated in opposite directions, the decreasing flux in one transformer generates a considerable secondary voltage, whereas the transformer in which the flux is increasing generates only a slight secondary voltage because of saturation in the core. The secondary voltages of the two transformers are connected in opposition around the tripping-coil circuit and the resultant unbalanced voltage circulates current around the circuit formed by the trip coil, transformers, and resistors as shown by the current curve in Fig. 5 (*b*).

No matter what the direction of the primary current, one transformer will be active on the first half cycle, the polarities being such that the current impulse through the trip coil is always in the same direction. The phenomenon is transient and the unidirectional current is not maintained under steady-state conditions.

This arrangement does not provide a steady overload trip point so that standard type of overcurrent relays are usually included for this purpose. The high-speed trip functions only in case of a short-circuit or heavy overload that results in a suddenly increasing line current.

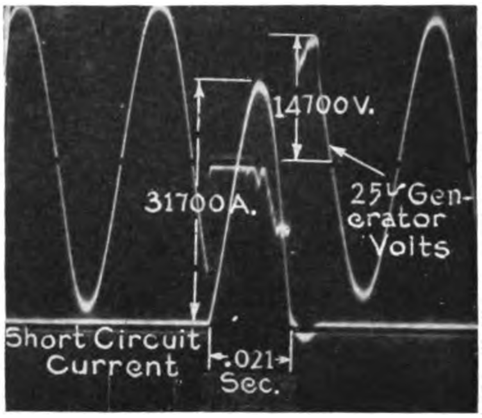

Fig. 7. Oscillogram of "OCO" Short-circuit Test at 14,000 Volts, 24,000 Amp. (r.m.s.) on 12,000-volt Air-Type High-speed Circuit Breaker

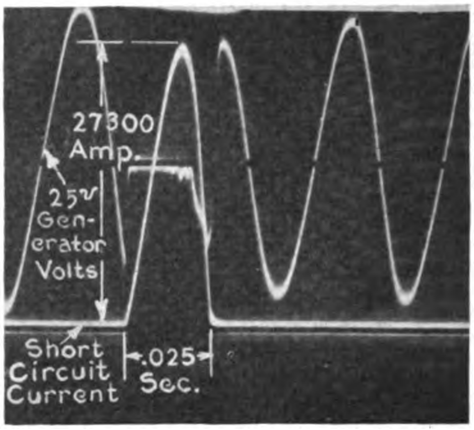

Fig. 8. Oscillogram of "OCO" Short-circuit Test at 12,000 Volts, 22,000 Amp. (r.m.s.) on 12,000-volt Air Circuit Breaker

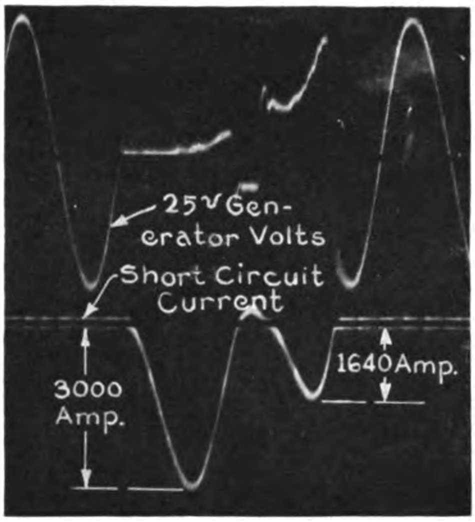

Fig. 9. Oscillogram of 12,000-volt, 2700-amp (r.m.s.) Short-circuit Test on Air Circuit Breaker

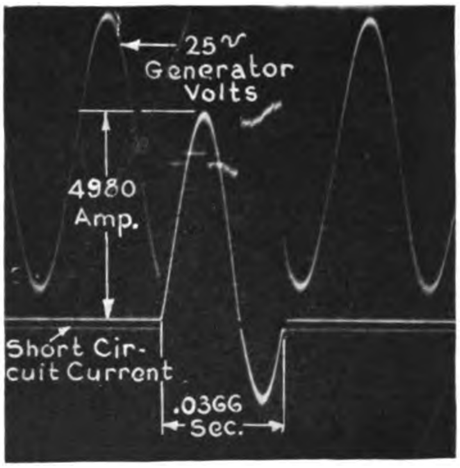

Fig. 10. Oscillogram of a 3700-amp. (r.m.s.) 12,000-volt Short-circuit Test on an Air-break Circuit Breaker

Any of the usual types of relay protective systems may be used in conjunction with the high-speed trip.

Description of the Air-break Circuit Breaker

Photographs of a 12,000-volt, 1500-amp. single-phase air-break circuit breaker are shown in Figs. 1 and 2. A drawing of the principal parts of the mechanism is shown in Fig. 6. The armature, which is held in the closed position by a holding coil and released by a wire-wound trip coil inserted between the poles of the holding magnet, is shown connected to the main high-voltage contact arm through insulated pull rods.

The high-voltage contact arms are operated entirely through suitable insulating pull rods, the mechanism being grounded. The circuit breaker as a whole consists of two sets of contacts and arc chutes which are connected in series, the moving contact arms being connected together by a suitable bus so that they form in effect a bridging contact similar to that used in oil circuit breakers.

There are two sets of springs: the main springs, operating directly on the armature for opening at high speed; and a second set attached to the pull rods at the bottom of the arm exert the contact pressure.

The mechanism is inherently trip free, the operation being as follows:

Resetting is accomplished by means of a motor-operated cam. The first movement of the cam operates the lever attached to the pull rods at the bottom of the contact arms, rotating these arms about the top bumper as a fulcrum. The holding armature is pulled to the closed position by the main pull rods, attached between the center of the contact arms and the bell cranks on the armature shaft. This armature is latched as the peak of the cam is reached without reducing the gap between contacts. Further movement of the cam allows the bottom springs to rotate the contact arm to the closed position around the top pull rods as a center.

The bottom of the arms, when opening on short circuits, is held approximately stationary by the weight of the parts attached at this point. In the event the breaker is closed on a short circuit, the mechanism therefore operates the contacts at full speed regardless of the fact that the rollers have not yet left the cam surface.

The high-voltage circuit through the breaker is practically direct, entering on one side through series blowout coils, one end of which is connected to a stationary contact. The circuit is completed through the moving contact and arm and another set of series blowout coils from which the connection is made to the second contact mechanism, a duplicate of the first.

When the contacts separate under short-circuit conditions, the arc is drawn on the arcing horns and detached blowout coils are cut in successively as the arc travels along these horns. The general arrangement of the blowout coils, arc suppressor plates, arcing horns etc., is the same as that used on standard designs of 3000-volt direct-current breakers, with the exception that a greater number of blowout coils are used and the arc chutes are designed to permit a considerably longer arc.

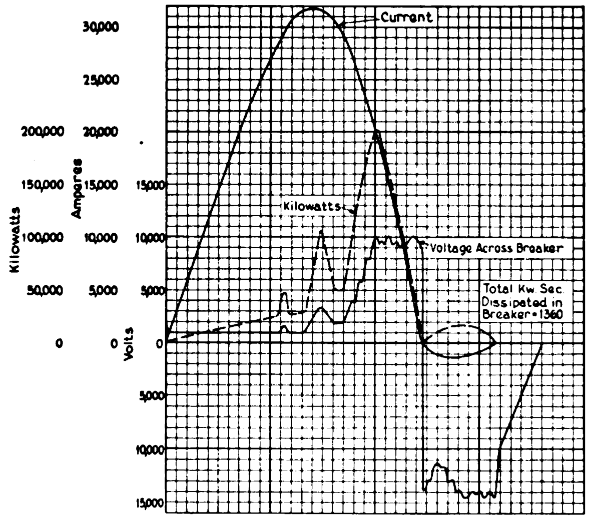

Fig. 11. Curves Taken from Oscillogram in Fig. 7 Showing Kilowatts Dissipated in the Arc Chute of Air Circuit Breaker During a Short-circuit Test at 14,000 Volts

A new feature which is introduced in this breaker consists of high-resistance arcing horns made from a special material and designed to absorb a considerable voltage as the arc is forced along them under the action of blowout coils. These arcing horns are effective in reducing the energy in the arc stream and in reducing the flame and noise of the breaker when opening under short-circuit conditions.

Some typical oscillograms obtained during short-circuit tests, including tests at 14,000 volts, single phase, 25 cycles are shown in Figs. 7 to 10 inclusive. The curves in Fig. 11, were plotted to show the current, voltage, and power relations during a short-circuit test of 14,000 volts. It is interesting to note that the energy is liberated in the arc chute at a maximum of 200,000 kw.

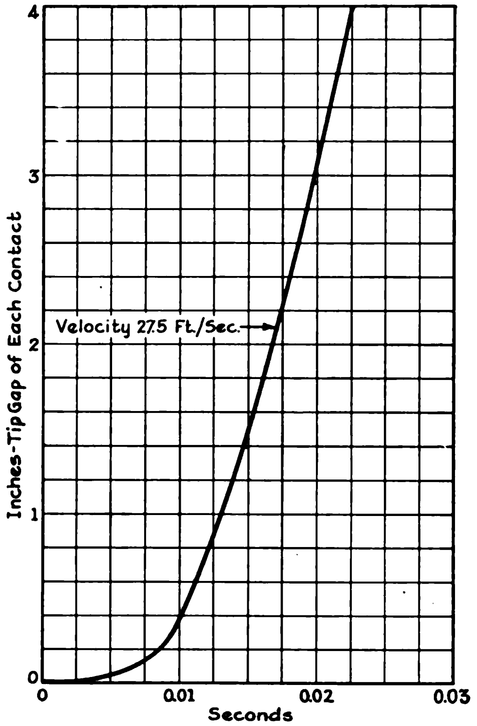

Fig. 12. Mechanical Speed of 12,000-volt Air Circuit Breaker

The speed of operation of the mechanism taken with a mechanical speed recorder is shown by the curve in Fig. 12. It will be noted that a total tip gap of more than 6 in. (two contacts in series) is reached in slightly less than the time required for a normal half cycle (of 25 cycles) after the trip point is exceeded.

Current-rupturing Tests on Air Breaker

This breaker has successfully opened a current of 24,000 amp. (r.m.s.) at 14,000 volts repeatedly. A series of twenty "OCO" tests at 12,000 volts, 22,000 amp. (r.m.s.) were made at two-minute intervals without examination or attention of any kind to the circuit breaker. A single contact and arc chute of this type of breaker, equivalent to one half the standard 12,000-volt breaker has opened a maximum current of 41,000 amp. (r.m..s.) at 7000 volts repeatedly, both the tests at 14,000 and 7,000 volts being the maximum available at the time the tests were conducted.

Selective Operation of the High-speed A-c. Breaker

Selective operation is essential for high-speed alternating-current breakers applied to railway feeders. Under all short-circuit conditions, the high-speed feeder breakers should isolate defective feeder sections between substations without disturbing the power flow to adjacent inter-connected sections.

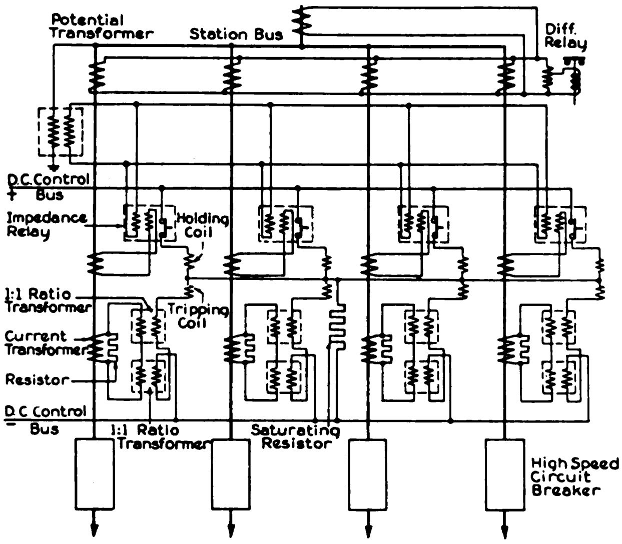

Fig. 13. Differential Circuit for Selective Operation of Four Magnetic-type High-speed Circuit Breakers Applied to a 12,000-volt Trolley Feeder Network

The connections shown in Fig. 13 are utilized for obtaining selective operation where the unidirectional-current impulse type of trip is used. Advantage is taken of the unidirectional-trip circuit in a differential connection. The tripping circuits of all breakers are connected to a common bus in such a way that the trip current circulating through any trip coil must return either through the trip coils of parallel breakers or the saturating resistor. When the current is increased simultaneously through all breakers feeding the feeders in a given direction from the substation, the tripping current for all of these breakers must pass through the saturating resistor, because equal voltages are generated by the trip circuits of each breaker. The current required for tripping all of the circuit breakers under this condition is much higher than when the current is increased through one breaker only. By selecting a proper value for the saturating resistor, the current necessary for operation with a simultaneous current increase through all breakers can be made higher than the exchange current over the trolleys between substations in case of internal short-circuit in one substation.

If the current is suddenly increased in one breaker only, by a short-circuit on one feeder, tripping current is supplied to the breaker on the faulty feeder, this current returning through the coils of the remaining breakers and assisting to hold them closed. The tripping current for the breaker involved is considerable less than when all breakers carry current, because of the lower impedance of the return circuit for the tripping current.

A simplified diagram showing breakers located in a typical feeder network is shown in Fig. 14.

The operation of the breakers under typical short-circuit conditions, with connections as shown in Fig. 13 is as follows:

(1). *Feeder Short-circuits*

In the event of a short-circuit on a feeder between substations, the breaker supplying this feeder carries a current considerably in excess of that of the parallel breakers, for the parallel breakers carry only the exchange current between substations. Tripping current is supplied to the trip coil of this breaker, operating it, some of this current returning through the trip coils of the parallel breakers in reverse direction for tripping and thus assisting in holding them closed.

In the event of a feeder short-circuit directly in front of one substation, all of the breakers in the distant substation carry equal currents until after the breaker in the nearby substation opens. The short-circuit current is removed from all but one of the breakers at the distant station by the operation of the nearby breaker that directly feeds the short-circuit. The remaining distant breaker is the one that is connected to the faulty feeder. The current through this breaker is suddenly increased because of the removal of the heavy short-circuit current from the high-voltage transmission line and this breaker is operated, clearing the remaining short-circuit.

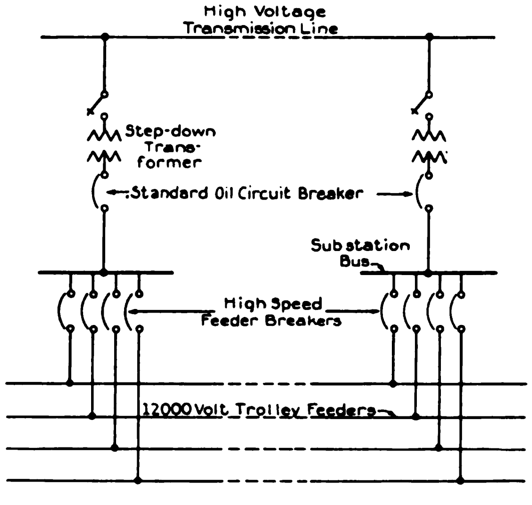
High Voltage Transmission Line
Step-down Trans-former
Standard Oil Circuit Breaker
Substation Bus
High Speed Feeder Breakers
12000 Volt Trolley Feeders

Fig. 14. Simplified Diagram of a Typical High-speed Breaker Installation on a 12,000-volt Railway Feeder Network

(2). *Substation-bus Short-circuit*

In the event a short-circuit occurs on the bus at a substation, all breakers interconnecting the two substations carry equal exchange currents. These breakers are therefore not operated by the high-speed trip, and the bus is isolated by the operation of a standard type of differential relay which opens all of the breakers connected to this bus. The feeders between substations are fed from the distant station and are not de-energized.

(*3*). In the event a short-circuit occurs in the step-down transformer or in the high-tension line, the feeder breakers between the faulty substation and adjacent substation carry equal exchange currents and therefore do not receive a sufficiently high tripping current to operate. The faulty transformer or high-voltage line is disconnected from the bus by the operation of a standard differential relay around the transformer or by reverse-power relays in case of a high-voltage-line short-circuit, both of which operate a circuit breaker on the low-voltage side of the transformer.

Conclusion

The development of both air-break and oil-break 12,000-volt alternating-current high-speed circuit breakers having an operating speed comparable to that of the direct-current type makes possible the same degree of selectivity and protection on alternating-current electrified systems as has been obtained for a number of years on equivalent direct-current systems.

It is expected that this type of breaker will be particularly beneficial in minimizing inductive interference in signal and communication circuits.

Mammoth Tramway to Span Western Canyon

A MAMMOTH electric tramway, by far the largest of its type to be built in this country, has been recently erected by the Michigan-California Lumber Company at Camino (Calif.). The tramway will be used to carry lumber a distance of a half mile across a canyon, and it is expected that the installation will be very spectacular in operation because of the heavy weights, high speeds, and distances involved.

A single tram carrier, carrying a rail car loaded with a maximum of 24,000 lb. of sawed lumber (a maximum gross weight of 29 tons) will pass between two loading terminals on opposite sides of the canyon. The distance between terminals is approximately 2700 ft., the depth of the canyon is 1200 ft., and the maximum speed of operation will be 1800 ft. per min., or a little over 20 miles per hour. The carrier will make a round trip in a little over six minutes, including the 200 sec. required for loading and unloading. The equipment will be capable of handling this cycle continuously. Four steel cables, each two inches in diameter, will support the tram carrier which will run on 32 wheels. The traction rope will be endless and will be one inch in diameter.

The electrical equipment consists, principally, of two 225-hp. wound-rotor intermittent-duty hoist motors, operating mechanically and electrically in parallel, and controlled through suitable primary and secondary magnetic-contactor panels. A single manually-operated master controller will govern both motors. The two motors are geared to two large sheaves, each eight feet in diameter, which will engage the traction rope. Each motor is equipped with a motor-mounted brake. Momentarily, the two motors will be able to exert a combined output of 800 hp. and, for short periods, most of this capacity will be required.

Stopping the carrier at either terminal will normally be accomplished automatically, in four graded steps, by means of a geared limit switch. After being so stopped at either end, the carrier will be brought to the final position. This will require accurate spotting in order to roll off the loaded rail car, and the final spotting will be done by an operator located at each terminal, using an "inching" push-button.

The motors and power plant are located at one side of the canyon, and the carrier will be started and accelerated by the operator handling the main master switch located on the motor or power side of the canyon. The necessity for inching the load to its final position on either side of the canyon by operators located on either side, 2700 ft. apart, made it necessary to provide for the transfer of the control back and forth across the canyon, with a red and green signal light system and proper interlocking of the control circuits for safety reasons.

The entire equipment will operate under both overhauling and motoring loads at different times during the cycle of operation. For example, as the 29-ton loaded carrier descends into the sag of the supporting cables—approximately 135 ft. at the middle of the span—the motors will act as generators for the purpose of limiting the speed. At other times, when ascending into the terminals, maximum motoring power will be required. For this reason, special provision is made in the control equipment to prevent high speeds under overhauling loads. With electric power connected, the motors cannot, after starting, be operated with secondary resistance in the circuit for, were they so operated, they would gain excessive speed. Solenoid brakes are provided to act, should the power fail, when the 29-ton loaded carrier is about to descend into the sag or when otherwise needed.

The tramway will also be used, on occasion, as a man carrier, as it will often be necessary to transport men across the canyon during operations.

Characteristics of Photo-electric Tubes

Relation Between Light and Current—Small and Large Tubes of Vacuum and Gas-filled Types Investigated—Characteristic Curves of Light, Current, Sensitivity, and Gas Pressure

By DR. L. R. KOLLER and H. A. BREEDING
Research Laboratory, Illuminating Engineering Laboratory
General Electric Company

THE most useful property of the photo-electric tube is the definite relation that exists between the intensity of light falling upon the tube and the amount of current that a constant voltage will force through the tube. (1) Elster and Geitel, Lenard, Ladenburg, Hermann, Richtmyer, and many others have performed experiments leading to the conclusion that the relation is almost linear. Others, including Ives, Dushman and Karrer, Hughes, Richardson and Compton, and Kunz have concluded that the relation is not linear, or at least is so only for special types of construction or under special conditions.

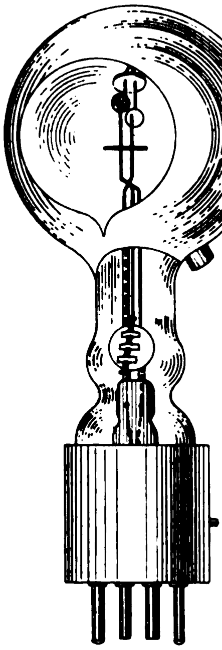

Fig. 1. Contour of the Photo-electric Tubes Investigated
(Except the one shown in Fig. 8)

The results presented in this article show that within certain limits of voltage and illumination intensity the relation is a linear one for the types of tubes studied. The tubes were set up in a dark room on a photometer bench and the light intensity varied by changing the position of the test lamp. The tube was placed in the position ordinarily occupied by the photometer head. A diaphragm with a circular opening one inch in diameter was placed in front of the tube. The test lamps employed were of the 120-volt gas-filled flood-lighting type and were of 250- and 1000-watt rating. Under these conditions, the inverse-square law could be assumed since the distance between lamp and tube was large compared with the dimensions of the filament and the diaphragm. The distance between lamp and tube was varied from eighteen to four feet. The range of intensities ran from 1 to 400 foot-candles, and the voltages applied to the tubes ranged from 22½ to 206 volts.

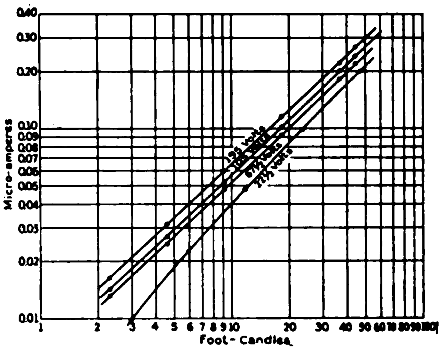

Fig. 2. Light-current Curves for Several Values of Voltage Applied to the Small Vacuum Tube

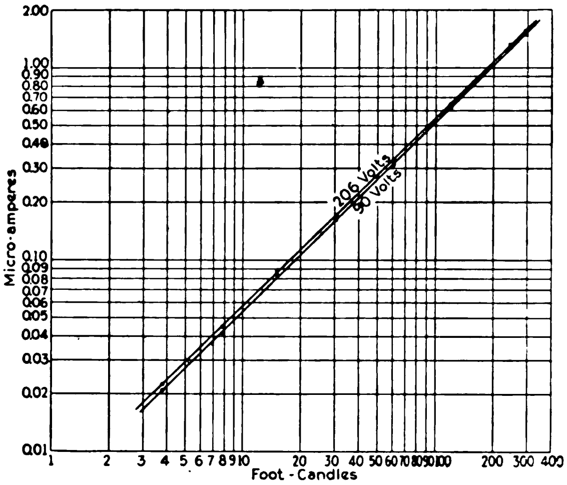

Fig. 3. Characteristics of the Small Vacuum Tube for a Range of Light Intensities Higher Than That Represented in Fig. 2

Figs. 2 to 6 show the light-current relation for four tubes of the type of construction shown in Fig. 1. These tubes are spherical glass bulbs with silvered inside surface upon which a monatomic film of caesium is deposited, the caesium and silver serving as the cathode.* The anode is a pair of wires projecting into the bulb to its center. Two of the tubes were 2½ in. in diameter with a 1½-in. window. The anode leads carried at their tip a small nickel disk ⅛ in. in diameter. Two other tubes were 7 in. in diameter with 6-in. windows and anode disk ½ in. in diameter. One large tube and one small tube were high-vacuum tubes. The other two were filled with argon at pressures of 0.020 and 0.040 mm. respectively.

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(1) A general description of the construction of the photo-electric tube, and an explanation of its performance with respect to the color of the incident light, the particular alkali metal coating used, and the type of tube (whether vacuum or gas-filled) were contained in the article, "The Photo-electric Cell," by L. R. Koller, GENERAL ELECTRIC REVIEW, July, 1928, p. 373.

*A paper on the photo-electric behavior of monatomic caesium films will be published soon by Mr. Kenneth T. Bainbridge, of Princeton University.

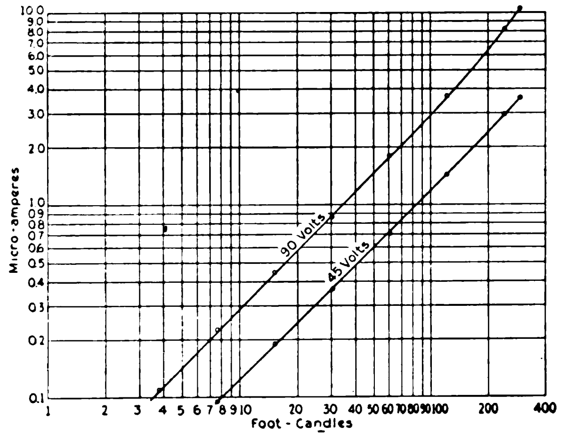


Fig. 4. Light-current Curves for a Gas-filled Tube That is Otherwise Identical to the One Concerned in Figs. 2 and 3

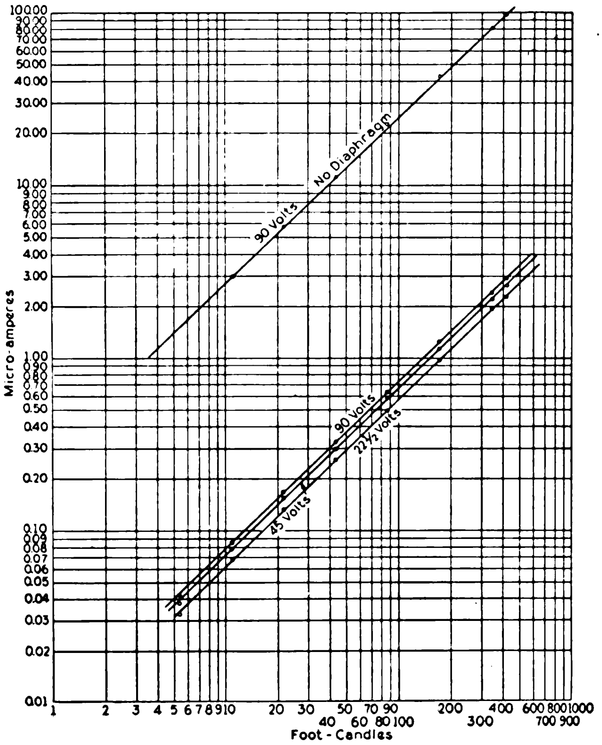


Fig. 5. Characteristic Curves of the Large Vacuum Tube

Consideration will first be given to the data for the small vacuum tube, as shown in Fig. 2. For the three higher voltages the curves are linear and all have practically the same slope. The 22½-volt curve departs markedly from a straight line at the lower intensities, where the slope becomes steeper. At the higher intensities, the slope again approaches the normal value. Fig. 3 shows the characteristics of the same tube at light intensities up to 300 foot-candles. The curves remain linear with the same slope and do not indicate the approach of a saturation condition at the high intensities.

Fig. 4 shows similar curves for a tube of identical construction but containing argon at a pressure of about 0.040 mm. Under the same conditions of illumination and voltage, this tube permits five times as great a current flow due to the ions produced by the collision of the gas molecules and the photo electrons.

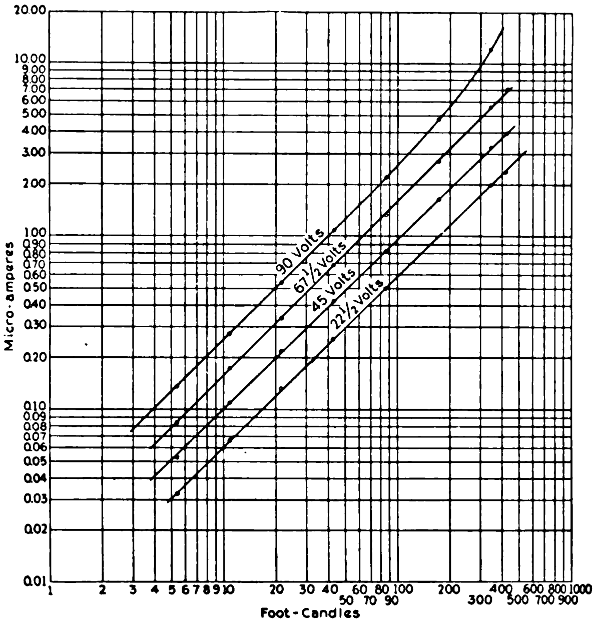


Fig. 6. Light-current Curves for a Gas-filled Tube That is Otherwise Identical to the One Concerned in Fig. 5

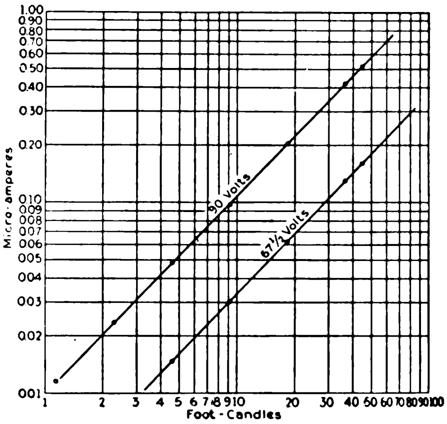


Fig. 7. Characteristic Curves of a Small Gas-filled Tube of the Construction Shown in Fig. 8

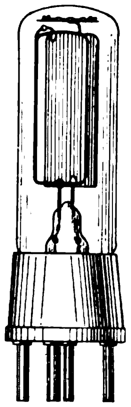


Fig. 8. Parallel-plane Type of Photo-electric Tube Construction

At low voltages the relation is linear over the range tested, but at 90 volts the curve rises more rapidly for illuminations beyond approximately 50 foot-candles.

Fig. 5 shows the results of similar measurements made on the vacuum seven-inch cell using the same size diaphragm and no diaphragm. In the tests when the diaphram was used, the sensitivity is slightly higher than in the case of the small vacuum tube but the curve is linear over the same range. The difference in sensitivity is probably due to the fact that the number of internal reflections was not the same in both cases as the light was introduced into the large tube somewhat off center to avoid striking the anode. Without the diaphragm the curve is still linear, and the slope remains unchanged. The current is larger in the ratio 33.7 to 1, in fair agreement with the relative areas of the diaphragm and the tube window which were in the ratio of about 1 to 36. The edge of the tube window is not sharply defined, making a precise calculation difficult.

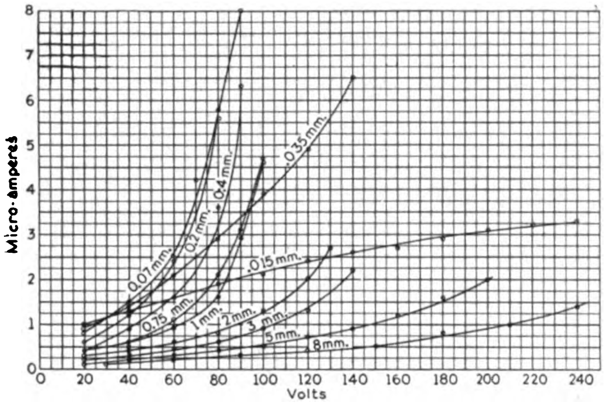


Fig. 9. Voltage-current Curves of a Gas-type Cell with Different Gas Pressures

Fig. 6 shows the light-current relations for the large tube which is of the same construction but which contains argon at a pressure of 0.020 mm. These curves show that the gas produces a three-fold increase in sensitivity. They are linear over the same wide range of intensities, except where the 90-volt curve begins to curve upward at intensities above 100 foot-candles.

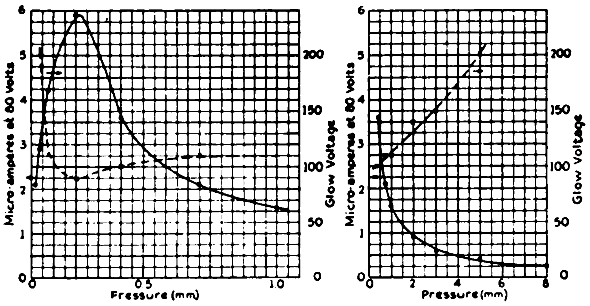


Fig. 10. Pressure-sensitivity Curves (in full) and Pressure-glow-voltage Curves (dotted) of a Gas-type Tube

Fig. 7 shows the relation for a small gas-filled tube of the parallel-plane type of construction that is shown in Fig. 8. The cathode is a flat silver plate ⅝ in. by 1¾ in. coated with a monatomic film of caesium, while the anode is a wire loop of the same dimensions parallel to the plate and about ¼ in. from it. The measurements were not carried above 45 foot-candles, but within that range the current was very nearly proportional to the illumination.

If a law of direct proportionality were followed, the slopes of the lines would be unity. There is a slight, but not systematic, deviation from this value. The relation can best be expressed by the formula

$$i = KI^n$$

where i = photo current in micro-amperes
K = a constant
n = a constant
I = illumination intensity in foot-candles.

The values of n for the curves in Figs. 2 to 6 are given in Table I. The average value differs from unity by an amount well within the limits of experimental error, showing that for tubes of this type operated at the voltages given and using a small diaphragm the photo current is proportional to the light intensity.

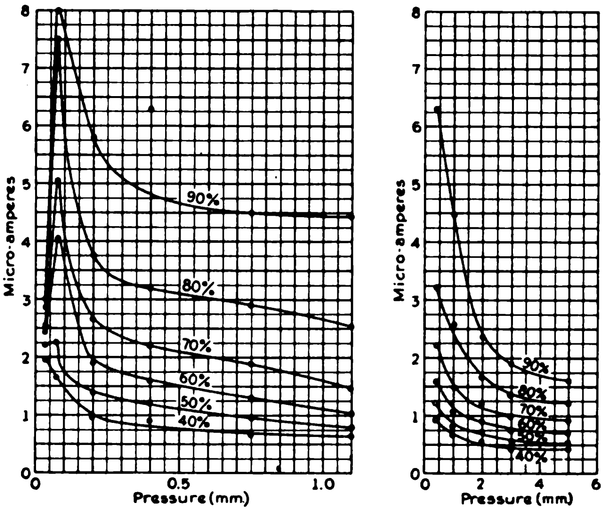


Fig. 11. Pressure-sensitivity Curves at Various Percentages of Glow-voltage

Fig. 9 shows the effect of variations in gas pressure on the sensitivity of a caesium tube under constant illuminating conditions. The tube was of the 2½-in. spherical type; and the various pressures of argon were introduced while the tube was still on the pumping system. It was illuminated by a 60-watt vacuum

TABLE I

Fig.	Tube	Volts Applied	n
2	Small, Vacuum	195 105 67.5	0.933 0.949 0.944
3	Small, Vacuum	206 90	1.01 0.95
4	Small, Gas-filled	90 45	1.02 0.99
5	Large, Vacuum	90 22.5	1.06 1.10
6	Large, Gas-filled	67.5 22.5	1.01 0.982
8	Parallel-plate, Gas-filled	90 45	1.04 1.04
			Average 1.005

lamp operating on 120 volts at a distance of 6 in. from the tube. A diaphragm with a one-in. opening was placed directly in front of the tube. A volt-ampere curve was taken for each pressure. From Fig. 9 it can be seen that as the pressure is increased the curve is first displaced upward (indicating higher sensitivity) and the slope becomes steeper. The maximum sensitivity is reached at a pressure of about 0.075 mm. As the pressure is further increased, the sensitivity and the slope of the curves decrease again. The pressure giving the highest sensitivity is not always the most desirable for operation on account of the steep slope and consequent instability of the tube. The glow voltage under these conditions passes through a minimum at 0.25 mm.

Fig. 10 gives the relation between glow voltage and pressure, and sensitivity (at a fixed voltage) and pressure.

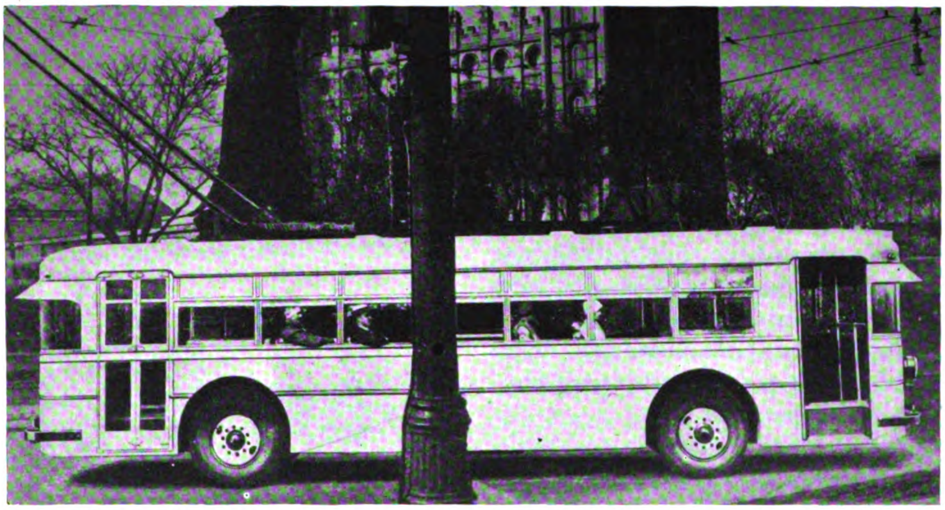
In Fig. 11 sensitivity at a definite percentage of glow voltage is shown as a function of pressure.

Interest Revives in Trackless Trolleys

A REVIVAL of interest in the trackless trolley bus is foreseen in the decision of the Manila Electric Company of Manila (P. I.) to add to its rolling stock eight trolley buses of a new design. These have been built for that city by the Twin Coach Company of Kent, Ohio.

One of the principal factors contributing to the present interest is the new street-car type of body now being introduced by bus builders. Another important element is the fact that, in some cities where tracks are being removed, the overhead construction can easily be adapted to the operation of trolley buses. The improvement in body design is traced to the popularity of gas-electric drive. The standard design of gas-electric bus can be converted to a modern trolley bus by eliminating the gasoline engine, generator, and 250-volt motors ordinarily employed and substituting an overhead collector, foot-operated control and two 600-volt motors. Proponents of the trackless trolley bus point out that electric power is much cheaper than gasoline, thus giving a distinct advantage to the former where definite routes are already established.

The trolley buses for Manila are the first of the new design. The drive used consists of two 35-hp., 600-volt motors and multiple-unit foot-operated control in connection with 11-to-1 worm gearing and 38-in. tires.



One of the New Trackless Trolley Buses

The Manila Electric Company has been operating trolley buses for a number of years and is convinced of their desirability, and the reports of other cities using this type of vehicle (Rochester and Cohoes, New York) indicate that the trolley bus is most economical and reliable in operation.

The trolley bus has the advantage of being able to maneuver eight or nine feet on either side of the overhead trolley, thus allowing it to go around obstructions or to pull up toward the curbing. This tends also to reduce accidents.

Low-voltage A-c. Networks

PART VI: PROTECTION

By D. K. BLAKE

Central Station Engineering Department, General Electric Company

THE early form of a-c. network systems in America consisted of a few single-phase transformers connected to a common secondary grid of single-phase three-wire 115/230-volt mains. The transformers of any given network were connected to one particular feeder. The protective equipment consisted of the usual primary fuse cutout with the addition of a fuse in the secondary circuit. The secondary fuses were sometimes connected in the transformer secondary, sometimes in the secondary mains and sometimes as a combination of both methods, as shown by (a), (b), and (c), respectively, in Fig. 28. This practice has been abandoned by most companies because of the difficulties and uncertainties of fuse operations. It is a very difficult matter to obtain selective operation of fuses. Thus, when a fault occurred a large proportion or even all of the fuses would blow. It was then necessary (especially in case (a) in Fig. 28) to replace a large percentage of the fuses simultaneously, otherwise the overload would blow an individual fuse as soon as it was inserted.

One company claims successful operation of the method of fusing shown in Fig. 28 (c) with the transformer secondary fuses designed to blow at 150 per cent load; and the fuses in the secondary mains blow at 200 per cent of the rating of the smallest adjacent transformer. It is probable that most difficulty with fuse protection is due to the attempt to fuse for overload values instead of short-circuit values. Some cases are cited where the fuses for short-circuit conditions give reliable performance.

Several years ago an effort was made to obtain selective operation by means of a fuse-type network protector illustrated diagramatically in Fig. 29. The coils *A*, *B*, *C*, *D*, and *E* are wound on the same iron core. The arrows indicate the flow of current with the transformer carrying load. The ampere-turns of *B* and *C* are equal and their sum is equal and opposite to the ampere-turns of *A*. They, therefore, have no magnetic effect on the iron core. In coils *D* and *E* the current enters at the center and divides equally. Since the turns are wound in the same direction in each coil, the ampere-turns of one half neutralize the ampere-turns of the other half. Under normal conditions, the core is not excited and no voltage is generated in coils *D* and *E*. If a breakdown or short circuit should occur in the transformer the current through *B* and *C* will be reversed. Under this condition the ampere-turns of *A*, *B*, and *C* act in the same direction and produce a magnetic flux in the iron core. The flux in the core induces a voltage in coils *D* and *E*. The voltages across these coils are short circuited through the fuse links. The large currents which circulate through *1-2-3* and *4-5-6* blow the fuses almost instantaneously, thereby disconnecting the secondary of the transformer from the network. The primary fuses blow practically simultaneously with the secondary fuses. The cost of this device prevented its extensive use.

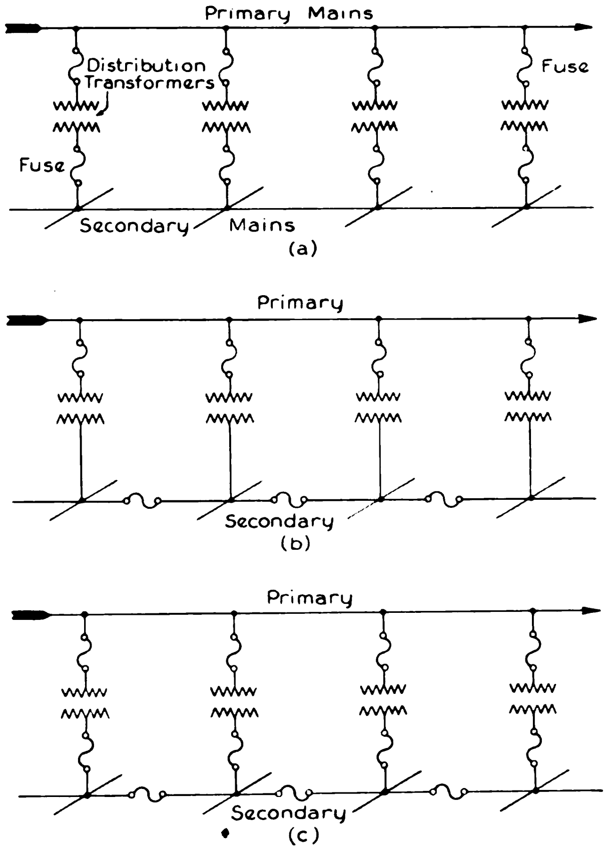


Fig. 28. Methods of Connecting Fuse Cutouts for the Protection of A-c. Network Systems

The foregoing methods of protection were limited to networks supplied from one primary feeder. They would be further unsuitable for two or more feeders because of the added selectivity required and the large number of fuses to be replaced after a primary feeder fault.

The present application of low-voltage a-c. network systems in high load density areas requires two or more primary feeders so that no interruption to service is incurred when a primary feeder fails. It is therefore necessary to consider the application of schemes utilizing automatic circuit breakers in combination with suitable relay equipment. The protective schemes now in use may be divided broadly into those having the principal protective equipment in the primary feeders and those having the principal protective equipment in the secondary side of the transformers. In both types the primary cables and distribution transformers are protected in combination. These protective schemes may again be divided into those having loop primary feeders and those having radial primary feeders.

The loop primary feeders utilize the well-known method of sectionalizing the feeder by means of automatic oil circuit breakers. Two such systems use pilot-wire protection between breakers and another uses reverse-power relays in combination with overload relays. It is well known that the overload verse-power relay equipment is limited to cases where the number of sections or breakers in a loop are small because of the high overload relay settings necessary for selectivity. The pilot-wire protection permits the use of any number of sections or breakers and is, therefore, particularly suitable for high-voltage circuits which have to be long in order to pick up sufficient load.

One large system uses a high-voltage breaker for each transformer and cable section, as shown in Fig. 30. The oil circuit breaker and transformer are located in the same vault. The connection of the protective circuit is shown in Fig. 31. Relays are not used; but instead the a-c. trip coil is direct connected to the pilot-wire circuit. When conditions are normal, the load current sets up in the pilot wire a current which circulates between the pilot wire and current transformer secondaries, but none circulates through the trip coils. If a fault occur in the distribution transformer or in the high-voltage cable, current will flow into the fault from the adjacent sections and from the secondary network. Under these conditions the current set up in the pilot-wire circuit does not circulate as before but divides between the two trip coils and operates the breaker. The fuses in the transformer secondary are depended upon to clear the transformer from the network. Selectivity between fuses is expected to be good in this case because they are selected for short-circuit values; and the supply from four directions reduces the current in adjacent transformer fuses to a safe value for selectivity. In this scheme a single primary fault involves only one cable section and one transformer so that fuse replacement is not difficult or objectionable.

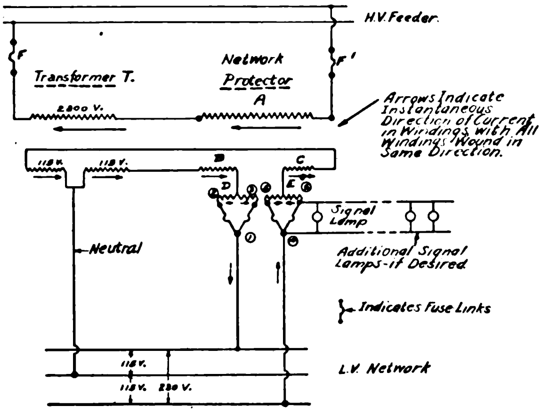

H.V. Feeder.
F
F'
Transformer T.
Network Protector A
2300 V.
Arrows Indicate Instantaneous Direction of Current in Windings with All Windings Wound in Same Direction.
115 V.
115 V.
B
C
D
E
Signal Lamp
Neutral
Additional Signal Lamps-if Desired
Indicates Fuse Links
115 V.
115 V.
230 V.
L.V. Network

Fig. 29. Diagram of a Circuit Providing Selective Operation of a Fuse-type A-c. Network Protector

The pilot-wire protective system requires particular attention to the ratio of current transformers, size of pilot wire and the volt-ampere characteristic of the a-c. trip coils. In order to balance the secondary currents in the pilot-wire circuit at a particular load, the ratio of the current transformers *B* must be selected to be equal to the ratio of the current transformers *A* times the ratio of the distribution transformer. Suppose the ratio of *A* is 60:1 (300/5 amp.) and the distribution transformers 10:1 (2300/230 v.); then current transformer *B* must be 600:1 (3000/5 amp.). If the transformers have a 13,800-volt primary then the secondary current transformers must be 3600:1 (18,000/5 amp.). If the two secondary leads of a single-phase transformer can be looped through the opening of a bar-type current transformer, the ratio may be cut in half *i.e.*, 1800:1 (9000/5 amp.).

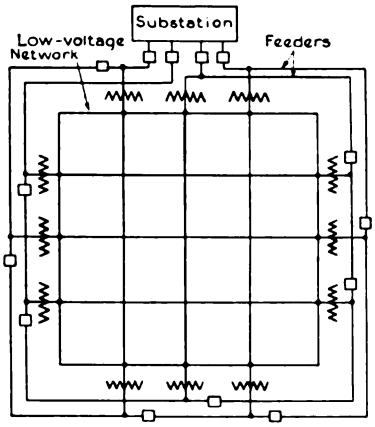


Fig. 30. Network Protection on a Large System Through the Use of High-voltage Circuit Breakers

The current transformers must also have similar characteristics throughout the operating range. The reason for this is illustrated in Fig. 32, which shows the curves of two current transformers with secondary current as ordinates plotted against primary current abscissa. At x value of primary current the secondary current of transformer No. 1 is represented as a, and that of transformer No. 2 as b. The unbalance current due to the difference in characteristics will be $a-b$ and will divide and flow through the trip coils or relays. The trip coils or relays, therefore, must be set high enough to prevent operation on such short-circuit currents flowing through sections not in trouble.

Trip coils or relays require a certain volt-ampere value to operate the breaker latch or to close contacts. Suppose a circuit breaker is used that requires a trip coil of 80 volt-amperes rating to trip the latch and that the coil is wound for five amperes trip. Then the required voltage is 16 volts. Now if a short circuit develops in one section, the current in the pilot wires of the other sections will produce a voltage drop which is applied across the trip coils. If the impedance of the pilot wire is sufficient to give a 16-volt drop, five amperes will flow through the trip coils and trip the breakers on the sections not in trouble. If the trip coils are wound for two amperes trip value, 40 volts impedance drop would be permissible in the pilot wires. It is evident, therefore, that the design of a pilot-wire protective system requires the coördination of current transformers, pilot wires, and trip coils or relays.

The type of protection generally used consists of radial-type primary feeders with air circuit breakers connecting the transformers to the network in the manner shown in Fig. 33. No high-voltage protective equipment is used. This permits feeder voltages of any value to be used without requiring the development of high-voltage subway equipment. The cables and transformers are the only high-voltage equipment except the grounding and disconnecting switches sometimes installed as a part of the transformer. The chief value of a disconnecting switch is in providing ease of phasing out the circuits during installation and repair work. The 2300-volt or 4000-volt radial circuits installed previous to the forming of the network retain the oil fuse cutouts, but with a copper strap replacing the fuse links. These cutouts are used for disconnecting purposes. The reduction to a minimum of high-voltage equipment no doubt accounts for the popularity of this protective scheme.

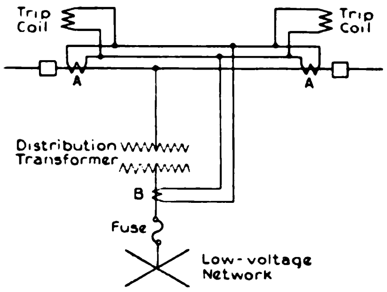


Fig. 31. The Pilot-wire Protective Circuit Used in Connection with the Circuit Breakers of the System Shown in Fig. 30

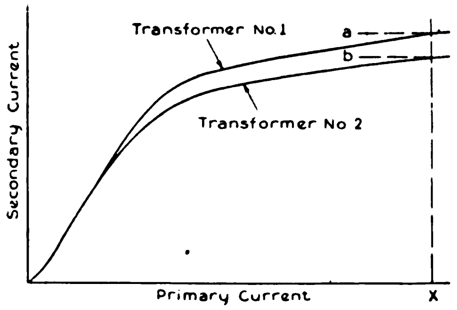


Fig. 32. The Relationships of Primary and Secondary Currents in Current Transformers which if Not Properly Selected May Occasion Erratic Operation of Protective Equipment

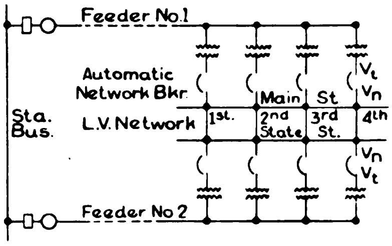


Fig. 33. Diagram Showing the Use of Automatic A-c. Network Air Circuit Breakers on the Low-voltage Side of Transformers

The air circuit breaker is electrically operated and controlled by the relay equipment. The combination of relay equipment and circuit breaker is referred to as an automatic network protector. The device is made automatically reclosing to eliminate the necessity of going to so many transformer locations to restore the feeder and transformers to service. Some are also likely to be overlooked if they were manually operated.

The protective function of the relay is to open the protector when a fault occurs in the distribution transformers or primary cables. The circuit is cleared from the system in the usual way by the oil circuit breakers at the substation. The network relays are of the reverse-power type and are applied to trip only in the event of energy flowing from the network to the transformer. The relays used for short-circuit protection are used also to function as remote-control relays when it is desired to switch a feeder into or out of service. This is accomplished by designing the reverse-power relay so that it will trip on reverse energy values lower than the exciting currents of the distribution transformer. Thus, if the feeder oil circuit breaker of feeder No. 1 in Fig. 33 is opened, then all of the transformers of that feeder will be disconnected from the network by the transformer exciting currents operating the relays to trip the protector. Similarly, when the feeder breaker is closed the same relays will function as reclosing relays. Because of the fact that the relays will operate on such low values as the transformer exciting current, it is necessary that the protector be prevented from reclosing unless the transformer voltage be higher than the network voltage, and have the proper phase relation. If this were not done the protector would trip and reclose periodically (an operation called "pumping") until the transformer voltage should become higher than the network voltage. The relays control the reclosing by measuring the voltage across the contacts of the protector when it is open. The voltage existing across the contacts of the open protector is the vector difference of the transformer and network voltages. When this voltage is in the proper direction the relay recloses the protector, for the transformer voltage will then be high enough and in the proper phase relation. Thus, the network relays not only give short-circuit protection but will also trip the protectors when the feeder breaker opens and will reclose the protectors when the feeder breaker is closed.

Some engineers consider the feature of tripping on transformer exciting current undesirable because of the frequent operations caused by elevator loads feeding back during light-load periods. They intend to use higher reverse-power settings such as 25 per cent to 100 per cent full load. When this is done the reclosing may be accomplished without such an accurate check on voltage conditions.

(To be continued)

Industrial Electric Heating

PART VI

THE SINGLE-PHASE ARC FURNACE

By N. R. STANSEL

Industrial Engineering Department, General Electric Company

THE single-phase arc furnace is in wide use for melting in the non-ferrous metal industry. Gillett's(16) estimate (as of February 1927) gives 111,300,000 kw-hr. as the annual consumption of energy by this type of furnace in that industry in the United States and Canada.

Several forms of the single-phase arc furnace have been developed and placed in service. However, the only form of construction which has found extensive use is illustrated in Fig. 84. This construction consists of a horizontal, cylindrical steel shell with a refractory lining and with horizontal carbon electrodes along the axis of the cylinder. The arc is formed between the electrodes and above the charge of metal. Thus the heat is developed within the chamber, a primary merit of electric heat.

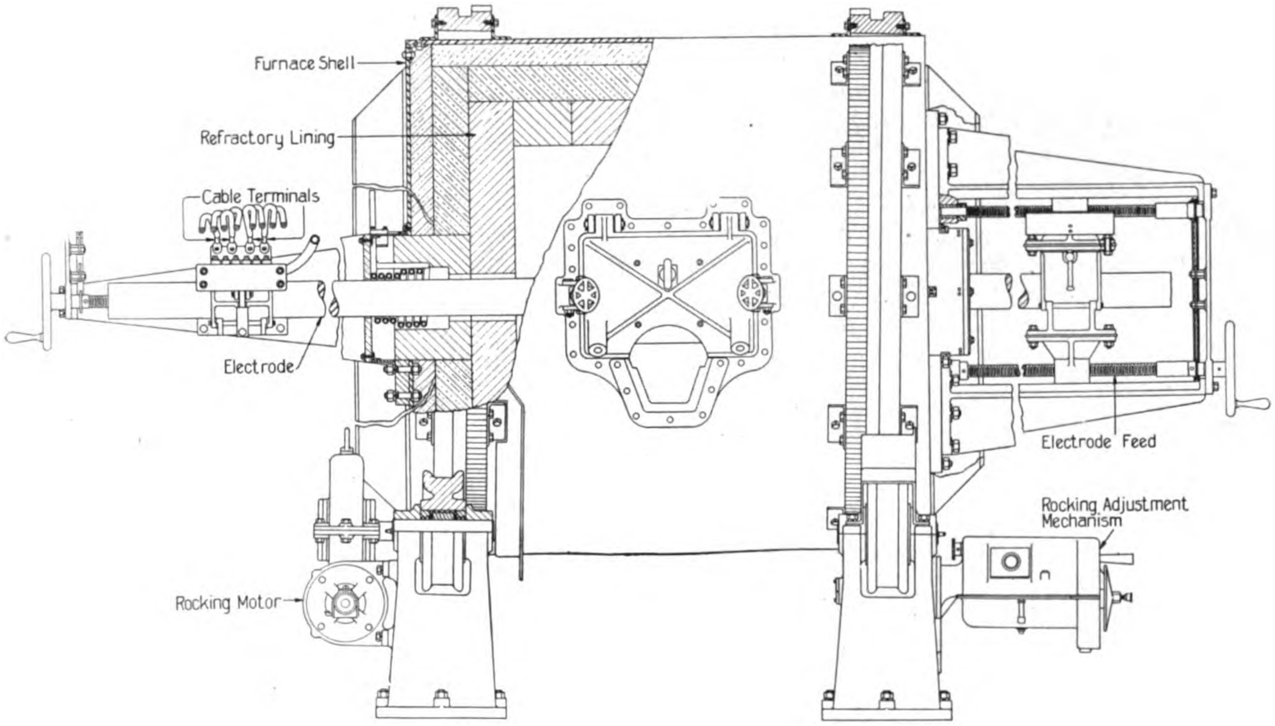


Fig. 84. Sectional View of a Single-phase Arc Furnace

With the exception of the atomic-hydrogen arc used for welding, the carbon arc is our hottest arc. Various investigators give different values as the temperature of the carbon arc. The value given in Table XXIV and the temperatures of some metal arcs in this table were selected by Compton.(30) The arc makes possible the concentration of a large amount of power in a small space. Part of the heat developed by the arc is transferred by radiation directly to the charge below and the remainder to the furnace walls from which part of the energy is re-radiated to the charge. While the average temperature of the furnace is below the temperature of the arc, the temperature gradient to the charge is

TABLE XXIV

Electrodes	Arc Temperatures
Carbon	3413 deg. C. absolute
Tungsten	3000 deg. C. absolute
Iron	2430 deg. C. absolute
Nickel	2365 deg. C. absolute

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(16) Twenty-five years of Non-ferrous Electrothermics," *Trans. Amer. Electochem. Soc.*, April, 1927.

(30) Compton: "The Electric Arc," *A.I.E.E. Journal*, Vol. 46 (1927), p. 1192.

The following table facilitates the location of material hitherto published in this serial:

Part	*Issue*	*Figs.*	*Equations*	*Examples*	*Tables*	*Footnotes*
Part I	Oct. 1927, p. 488	1 to 23	(1) to (8b)	1 to 3	I to III	(1) and (2)
Part II	Nov. 1927, p. 551	24 to 43	(9) and (10)	4 to 6	IV to VII	(3) to (13)
Part III	Dec. 1927, p. 600	44 to 57	(11) and (12)	8 to 11	VIII to IX	(14) and (15)
Part IV	Mar. 1928, p. 125	58 to 73	(13) and (14)	......	X to XX	(16) to (26)
Part V	Apr. 1928, p. 204	74 to 83	(15) to (21)	12 and 13	XXI to XXIII	(27) to (29)

high. The transfer of heat to the charge is at a rate proportional to the difference of the fourth powers of the absolute temperatures of the hot and cold surfaces.(31) The photograph of a 60-cycle furnace arc shown as the cover illustration of this issue of the REVIEW furnishes an interesting study of the arc as a source of heat inside a closed chamber. This photograph was taken immediately after a charge of brass had been poured. The current in the arc was 2500 amp., and the arc voltage 100 volts. In addition to the transfer of heat by radiation, there is a transfer of heat to the charge by conduction from the parts of the furnace walls in contact with the charge; however, the greater part of the total heat flow to the charge is by radiation. This gives to the arc furnace a high melting rate, an essential feature for the economical use of electric energy in this class of service.

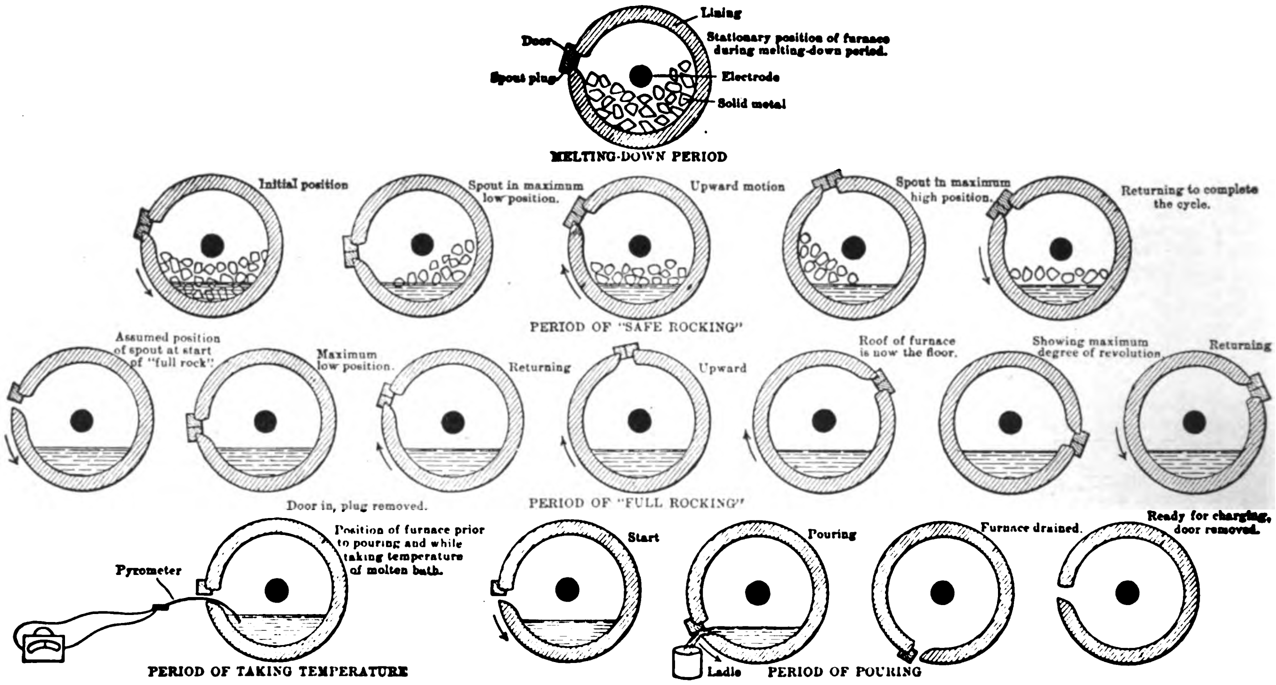
Lining
Stationary position of furnace during melting-down period.
Door
Spout plug
Electrode
Solid metal
MELTING-DOWN PERIOD
Initial position
Spout in maximum low position.
Upward motion
Spout in maximum high position.
Returning to complete the cycle.
PERIOD OF "SAFE ROCKING"
Assumed position of spout at start of "full rock".
Maximum low position.
Returning
Upward
Roof of furnace is now the floor.
Showing maximum degree of revolution.
Returning
PERIOD OF "FULL ROCKING"
Door in, plug removed.
Position of furnace prior to pouring and while taking temperature of molten bath.
Pyrometer
PERIOD OF TAKING TEMPERATURE
Start
Pouring
Furnace drained.
Ready for charging, door removed.
Ladle
PERIOD OF POURING

Fig. 85. Diagrams Showing Positions of a Rocking-type Arc Furnace During Loading, Melting, and Pouring

The diagrams in Fig. 85(32) illustrate the different stages of operation of the form of arc furnace shown in Fig. 84. This furnace is arranged to be rocked after the melting of the metal has begun. As previously stated,(33) the stirring of an alloy during the melting process is essential for the production of a homogenous metal. In the arc furnace, there are no currents induced in the charge and hence no electrodynamic action to stir the molten metal. The rocking action serves this purpose. In a form of arc furnace that is similar to the rocking furnace, the furnace shell is completely rotated.

The heat received at the surface of the metal must be conducted to the interior of the charge as fast as received or the surface will be overheated and excessive volatilization of the metal will take place. The mechanical stirring action continually exposes a fresh surface of metal to the radiations and aids conduction in carrying the heat into the mass. Equalization of temperature is thus maintained and overheating prevented. The motion of the furnace shell also serves to wash the interior surfaces of the furnace walls, cools the refractory lining, and aids in the heat transfer to the metal.

The operation of the arc furnace is simple, and the operator need give but little if any attention to the arc because of a combination of many factors which influence the arc. It may be of interest to discuss to some extent some of the conditions which determine the operation of the alternating-current arc between solid carbon electrodes.

Considering only a half cycle, the alternating-current arc is fundamentally a direct-current arc. The essential difference between the two arcs lies in the character of the voltage between the electrodes. In the direct-current arc, the voltage and current are continuous. In the alternating-current arc the current is started at the beginning of each half cycle by means of a hot cathode, and the instantaneous values of current vary during the half cycle. Corresponding to the reversals of the current at the end of each half cycle, the electrode tips alternate as the cathodes of the arc. Using effective values, the volt-ampere characteristic of the alternating-current arc, represented by the curve in Fig. 86, is similar to the volt-ampere characteristic of the direct-current arc; *i.e.*, the characteristic is negative, an increase of current being accompanied by a decrease in the arc voltage. (Thus, when operated from a constant-voltage circuit, an arc is inherently unstable.)

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(31)See Equation (8) in Part I.

(32)Reproduced from "Melting Brass in a Rocking Electric Furnace," *Bulletin No. 171*, The Bureau of Mines.

(33)On p. 127 of Part IV of this serial (Melting Non-ferrous Metals).

A certain instantaneous value of voltage is required to start the current across the gap between the electrodes at the beginning of each half cycle. This value depends upon the cathode temperature and the electrical conductivity of the atmosphere within the gap. The higher these two values, the lower will be the required starting voltage. There is thus a delay in the starting of the current at the beginning of the half cycle, as seen in the oscillogram in Fig. 87. The length of this period of zero current varies with the atmosphere of the arc. If the gap between the electrodes contains an atmosphere which is a comparatively good conductor, such as metal vapor, the current may start practically simultaneously with the voltage as is seen in the oscillograms in Fig. 88. The oscillograms in both Figs. 87 and 88 are of a 2500-amp. 100-volt 60-cycle arc in a brass-melting furnace.

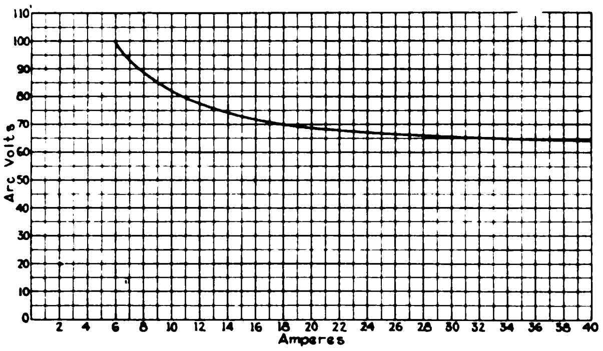

Fig. 86. Volt-ampere Characteristic of an Alternating-current Arc

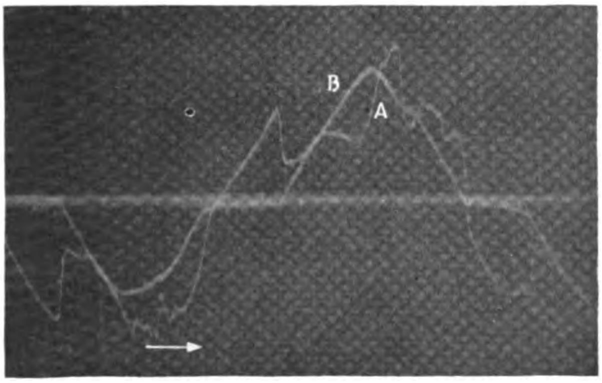

Fig. 87. Oscillogram of an Alternating-current Arc

A: Voltage *B:* Current

The atmosphere surrounding the arc in a melting furnace varies continually, and during the greater part of the time the metal vapors within the furnace determine this atmosphere. An influence that disturbs the arc is the movement of gases from the chamber to the outside atmosphere. During operation, a pressure is established within the furnace chamber and as the chamber is rocked a varying flow of gases passes the stationary arc. Thus the arc atmosphere is changing continually during the rocking operation. An effect of a variation in the arc atmosphere is shown in Fig. 89. The records show effective values.

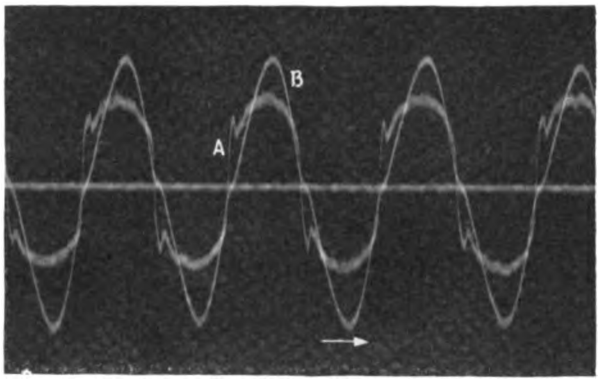

Fig. 88 (a)

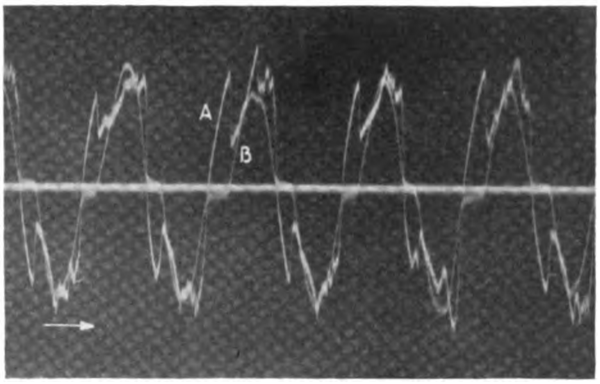

Fig. 88 (b)

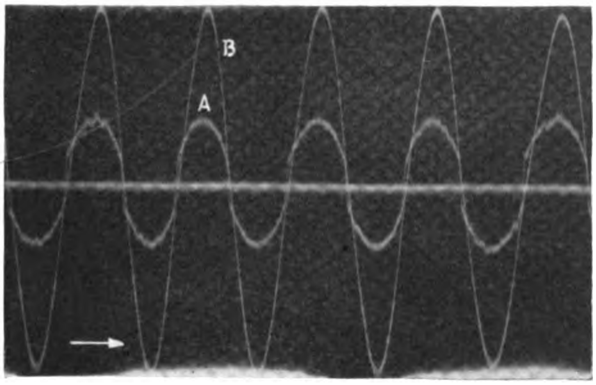

Fig. 88 (c)

Fig. 88. Oscillograms of the Arc in a Single-phase Arc Furnace While Melting Brass

A: Voltage *B:* Current

In this case the current was held constant at 35 amp. The variation in the arc atmosphere was caused by the dispersion of the gases driven off from the tips of the electrodes by the heat of the arc. At the beginning of the period the quantity of gas in the gap between the electrodes was a maximum. The gas was slowly dispersed by convective movement and as the gas disappeared the arc voltage rose, reaching a constant value when the normal arc atmosphere was attained. The watts in the arc likewise increased as shown in the corresponding chart.

The arc distorts the voltage wave and to some extent the current wave, as will be noted from Figs. 87 and 88. The extent of this action varies widely.

Theoretically there is no rectification of current by an alternating-current arc between similar carbon electrodes. However, the tips of the electrodes become dissimilar to a more or less extent owing to the erratic burning of the arc and to replacement of the electrodes so that there is some rectification. However, the direct-current component in the circuit at any time is comparatively small and is of low voltage so that the direct-current watts are not sufficient to influence the operation of the arc.

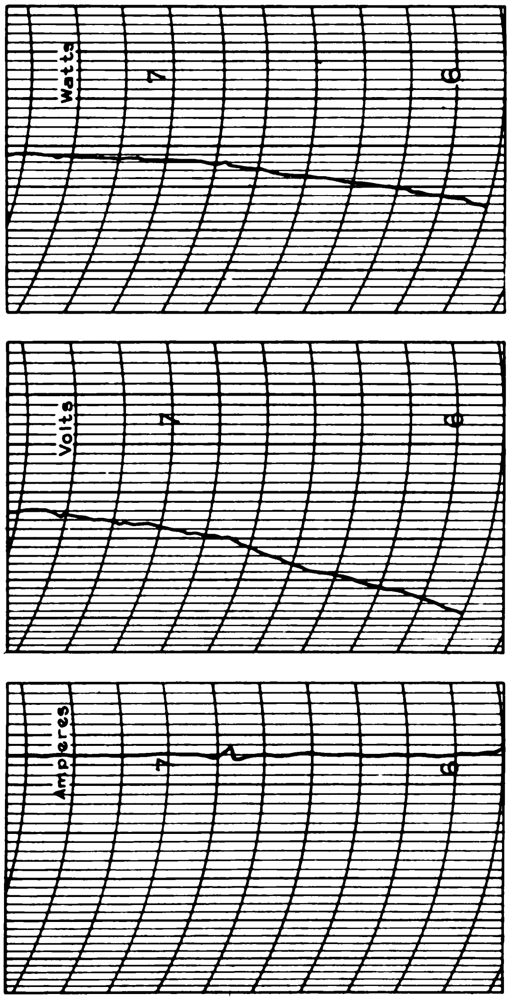

Fig. 89. Effect of the Variation of Arc Atmosphere on the Arc Voltage and Watts. (60-cycle arc.) Current held constant at 35 amp. Arc voltage at start 38, at end 56 (read from right to left . Watts at start 1375, at end 1400

The ratio of the watts in the alternating-current arc to the corresponding volt-amperes (*i.e.*, the power-factor of the arc) is less than unity, and in any particular case has a value dependent upon the length of the period of zero current at the beginning of the half cycle and upon the shape of the wave forms. There is of course a continuous variation in the values of the arc power-factor, but it is difficult to segregate the effects of the various factors which have an influence upon it. Fig. 90 shows that the power-factor varies with the watts in the arc, particularly at low currents. The effect of frequency upon the arc power-factor, as given by Duddell and Marchant,(34) is shown in Fig. 91. This similar effect accompanying an increase in the watts in the arc and an increase in the frequency appears to be directly related to the cathode-temperature effect, *i.e.*, to the zero-current period of the half cycle. It should be noted that the arc power-factor is not a matter of phase displacement as represented by the cosine of the angle of lag in circuits containing reactance. The sum of all the influences which affect the arc power-factor result in an average value that is generally above 85 per cent for non-ferrous metal-melting furnaces.

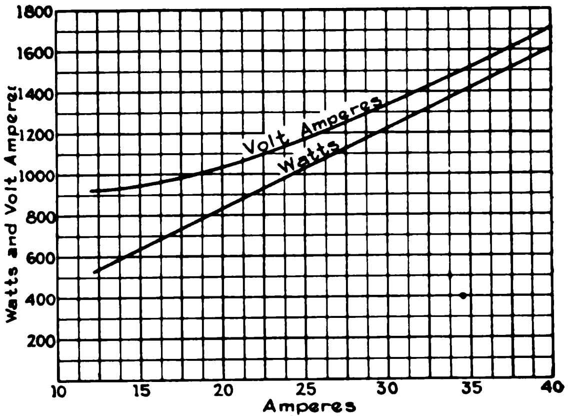

Fig. 90. Change in Arc Power-factor with Change in Watts in the Arc

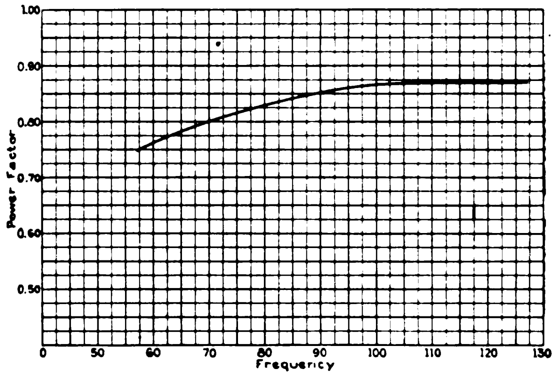

Fig. 91. Effect of Frequency Upon the Arc Power-factor. Arc between solid carbons in open air. Current held constant

The predominating influence upon the operation of the arc is the arc atmosphere, and the most active element in this atmosphere is oxygen. This element affects the arc adversely and is a limiting factor in arc operation. The effect of oxygen is more pronounced in the alternating-current arc than in the direct-current arc. In the latter, the cathode area is in continuous operation and thus largely inaccessible to oxygen; whereas in the alternating-current arc, the arc is extinguished at the end of each cycle so that there is opportunity twice during each cycle for oxygen to act upon the entire cathode area. While the atmosphere in the chamber of a metal-melting furnace is largely of metal vapors, the presence of air (oxygen) cannot be entirely eliminated.

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(34) 'Experiments on Alternating-current Arcs," *Journal of the Institution of Electrical Engineers*, vol. 28, p. 1.

A number of analyses of the gases in an arc furnace during the melting of different charges of a copper-zinc alloy of uniform composition showed the oxygen content in the chamber to range from 8 to 20 per cent for the different charges. In each case the amount of oxygen present in the chamber remained practically constant during the melting period. This variation in the amount of oxygen present, with all other conditions remaining the same, probably accounts for much of the variableness observed in the operation of the alternating-current arc from charge to charge.

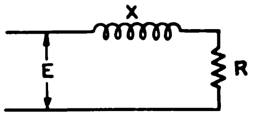


Fig. 92. A Circuit Containing a Resistance *R* and a Reactance *X*

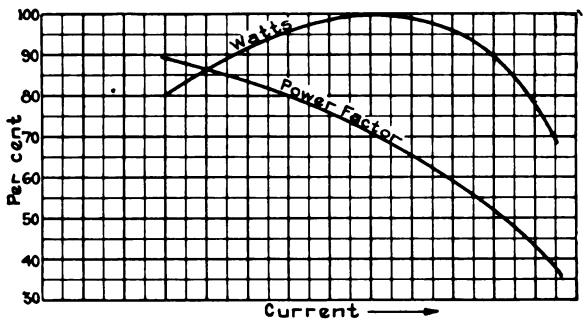


Fig. 93. Characteristic of the Circuit in Fig. 92, Assuming Sine Wave Forms

The alternating-current arc, when operating from a constant-voltage circuit, is rendered stable (*i.e.*, is maintained) by a reactance in series with the arc. The higher this reactance the more steady the current. The extent to which the current fluctuates is termed the degree of stability. In the use of an arc for lighting service a comparatively steady-burning arc is desired. In arc-furnace operation, as far as the furnace alone is concerned, the variations of the current are of little consequence so long as the arc does not go out and the power input is maintained. However, the voltage regulation of the power system depends upon the extent of the current fluctuations in the system, and the variations of the current in the arc circuit should be kept within limits which will not cause the voltage regulation of the power system to exceed the limits necessary to be observed in any particular case. This is a local condition for each installation, and no standard limit for the permissible current variation in the arc-furnace circuit can be established. If, for example, the arc furnace is supplied by an individual feeder from a substation, there is much more latitude in current regulation than would be the case if the furnace operated from a network to which other apparatus is connected. Also, if the kv-a. capacity of the furnace is small compared to the total load on the power system in the immediate neighborhood of the furnace installation, the current variations in the furnace circuit may be negligible with respect to the voltage regulation of the power system. Again, if a number of arc furnaces in the same plant are in operation at the same time, the permissible individual current variation is greater than for a single furnace.

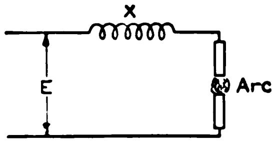


Fig. 94 (a)

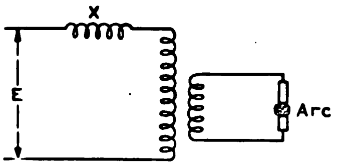


Fig. 94 (b)

Fig. 94. Single-phase Arc-furnace Circuit without and with a Supply Transformer

The relations between current, watts, and power-factor in an alternating-current circuit made up of a resistance and a reactance, Fig. 92, and operated at a constant voltage *E*, are shown by the curves in Fig. 93. Sine wave forms are assumed. In such a circuit the maximum power occurs when the power-factor is 0.707. These curves are not directly applicable to the arc-furnace circuits in Fig. 94 because the curves show the total watts in the circuit whereas in the operation of an arc we are concerned primarily with the power in the arc and need to know the current value which gives maximum power in the arc rather than the value which gives maximum power in the circuit. The power in the arc is the total power in the circuit minus the electrical and magnetic losses in the circuit. The maximum power in the arc occurs at some lower value of current than gives the maximum power in the circuit, *i.e.*, at some value of circuit power-factor higher than 0.707. Also we do not have sine wave forms in the arc circuit. These relations are illustrated in Fig. 95, which is a record of the operation of a small arc furnace. If the wattmeter is connected in the circuit close to the terminals of the furnace, the difference between the currents for maximum total power and maximum power in the arc will be small. This being the case it is sufficient to determine the maximum power obtainable in the furnace circuit for a given installation, and then operate at this power or at some lower value (obtained with a lower value of current) corresponding to the melting rate desired. As there are two values of current—one high and one low—which give any one particular value of power in the arc, the more efficient operation for a given melting rate is obtained of course by the use of the low value of current. For a given value of reactance in series with the arc, the more stable operation is obtained with the high value of current.

The relation between the arc length and the watts in the arc can for practical purposes be represented by a straight line, as illustrated in Fig. 96. Assuming a constant supply voltage, the voltage available for the arc with a given current varies inversely with the amount of reactance in series with the arc. Thus, increasing the reactance to reduce the current fluctuations reduces the maximum power values. Hence the series reactance, X in Fig. 94, is usually adjusted for a value between the value of the power in the arc (*i.e.*, the melting rate) and the voltage regulation of the power system. Under average conditions, the total reactive drop in single-phase arc-furnace installations varies between 35 and 60 per cent at normal current. Some value within this range usually gives a satisfactory melting rate and a satisfactory electrical load with respect to voltage regulation and overall power-factor. The total reactive drop includes the reactive drop of the conductors plus that of the separate reactance X, Fig. 94. In some cases the reactance of the conductors is an appreciable part of the total reactance. The curve in Fig. 97 shows the per cent voltage drop corresponding to the per cent reactive drop.

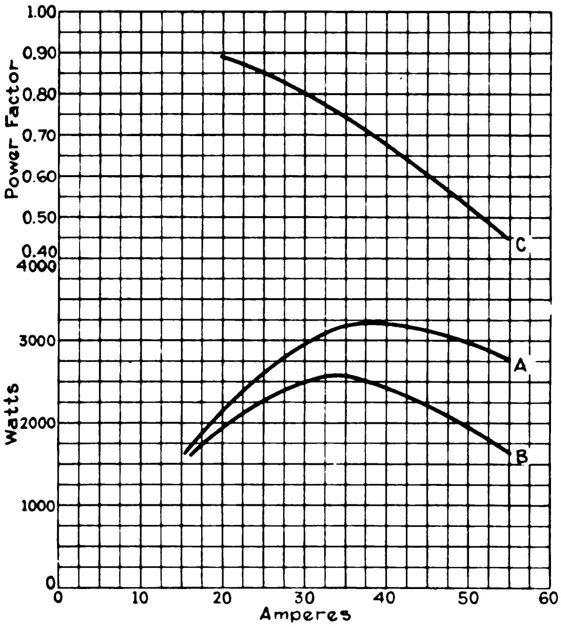


Fig. 95. Characteristic of the Arc Furnace Circuit

Curve *A*: Total watts
Curve *B*: Watts in the arc
Curve *C*: Power-factor of the circuit

The arc voltage is determined by several factors. It is desirable to use a comparatively low voltage for furnace installations on account of the insulation of the conductors and electrodes and for safety reasons. On the other hand, a low voltage means a high current value with corresponding heavy conductors and a high reactance drop in these leads. Another factor is the character of the atmosphere. While as previously mentioned the atmosphere of a metal-melting furnace is largely metal vapor, some oxygen is present. The greater the contact this element has with the cathode areas, the shorter will be the length of the arc (*i.e.*, the lower the arc voltage) which can be maintained. The arc voltages in use are generally within the range of 70 to 100 volts. Assuming a given voltage across the arc, Fig. 94, at a given current the corresponding open-circuit line voltage is determined by the impedance outside of the arc. For example, if the reactance value selected gives 50 per cent reactive drop at the given current, and

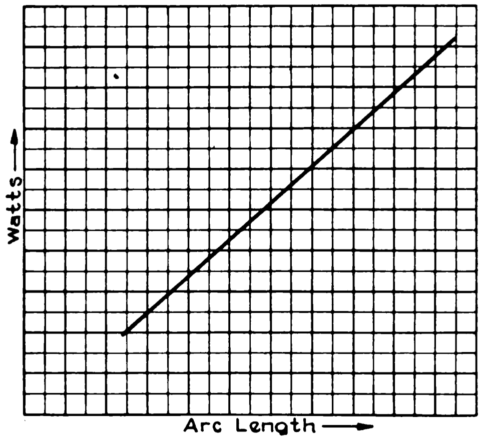


Fig. 96. Relation of the Watts and Arc Length with Constant Current

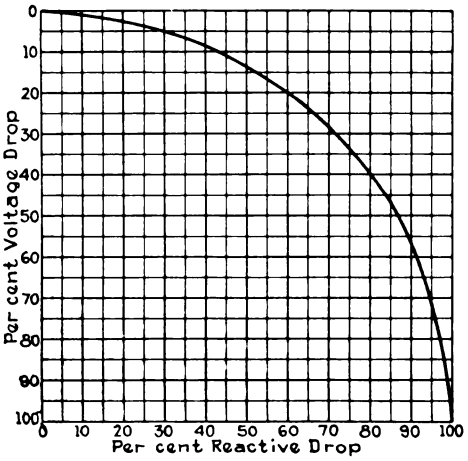
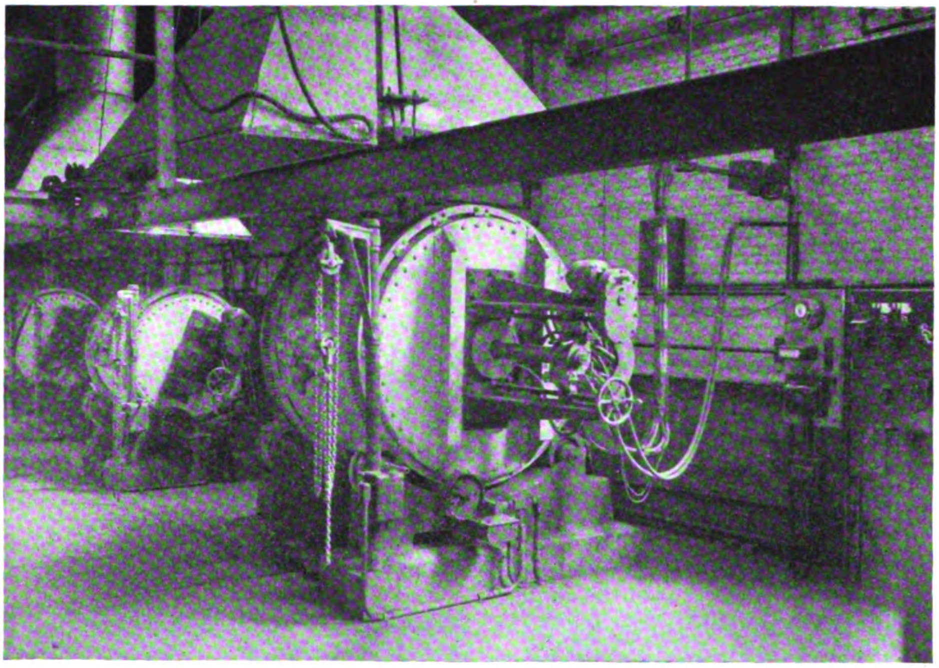


Fig. 97. Per Cent Voltage Drop Corresponding to Per Cent Reactive Drop. Sine wave forms assumed

the arc voltage is to be 80 volts, the open-circuit voltage from Fig. 97 is $\frac{80}{1.00-0.13} = 92$ volts plus the ohmic drop in the conductors, in the flexible leads to the furnace, and in the electrodes. For an average installation with conductors of moderate length, this totals 95 to 100 volts. A transformer is usually required to step down the supply voltage to that required for the furnace circuit and the separate reactance *X* is then as a rule placed in the high-voltage side of the circuit as shown in Fig. 94. With such arrangement, an individual transformer is required for each furnace and the reactance of the transformer is included in the total reactance of the circuit.



(*Courtesy Detroit Electric Furnace Co.*)

Fig. 98. A Typical Single-phase Arc Furnace Installation Including One 2000-lb. and Two 750-lb. Rocking-type Units

Obviously, the size of a melting furnace is limited by a number of practical considerations some of which are peculiar to each industry. First, we have the limitation of the size of single-phase loads on a power system. This of course is relative only, but in general it is desirable to keep individual single-phase loads below 500 kv-a., and more often below 300 kv-a. Second, we have the general principle that the shorter the period of melting, and the shorter the length of time that the metal is held in the molten state, the less the heat loss per charge, *i.e.*, the more efficient the operation. This prescribes the selection of the size of furnace, *i.e.*, holding capacity, and a melting rate with reference to the rate at which the metal can be poured such that the entire contents of the furnace can be poured as soon as the pouring temperature is attained. The ideal method is continuous melting and continuous pouring. This is approached if the plant is served by two or more furnaces so operated that molten metal is more or less continuously available. Third, in any plant, space arrangement and material handling facilities fix a limit as to the largest furnace unit practicable. Production conditions also vary. Hence, on the whole, two or more furnaces are likely to be better suited to economic operation than a single large furnace. Two or more furnaces of different sizes may represent the best combination. The arc furnace is very flexible as to size and can be adapted to any method of operation and size of plant. This is represented by the available range of standard sizes of non-ferrous-alloy melting furnaces. For illustration, the list of standard furnaces of one manufacturer is given in Table XXV. The most popular size in this table is the furnace which has a holding capacity of approximately 250 lb.

TABLE XXV

STANDARD SIZES OF SINGLE-PHASE ARC FURNACES FOR MELTING NON-FERROUS ALLOYS

Electrode Diameters In.	Nominal Holding Capacity Lb.
3	200–250
3	300–350
4	750–1000
4	1000–1250
5	1500–1800
5	1800–2500

The size of the electrical equipment for a given arc-furnace installation is determined in the same way as for any other electric melting furnace, *i.e.*, it is based upon the weight of the charge, heat absorption of the metal, the rate of melting, and the heat loss.(35) As our knowledge of the heat absorption of alloys is limited, it is more convenient to base such estimates upon the results obtained in practice. A value with the range of 250 to 300 kw-hr. per ton (which includes all losses) is representative of the electrical energy required for melting the various copper-zinc alloys. The value in any particular case depends, of course, upon the composition of the alloy, impurities in the charge (particularly water), size of furnace, pouring temperature, and method of operation. The average length of the melting period which suits most plant operations, regardless of the size of the furnace unit, is 30 min.

Example 14. Assume an alloy high in copper content and 300 kw-hr. per ton as the energy value. The weight of the charge is 800 lb. The melting period is 30 min.; the arc voltage 80 volts. The overall power-factor is 80 per cent. The kv-a. load is

$$P=\frac{800\times300}{2000\times0.5\times0.80}$$

$$=300 \text{ kv-a.}$$

As the entire contents of the arc furnace is poured at each melt, this type of furnace is quite suitable for melting miscellaneous alloys which are often required in manufacturing plants. If an exact analysis is required in each case it is sometimes desirable to run a wash heat between charges of different analyses. The arc furnace is ready for use as soon as loaded and can be used intermittently or continuously as may be required. The first charge in a cold furnace will require a longer melting period than subsequent charges on account of the heat absorption of the refractory lining. The charge may consist of any class of material available, ingots, scrap of all kinds such as borings, turnings, "mattresses," "cabbages," etc., or any mixture of new metal and old. It is desirable to limit the impurities in the charge but this is a matter of care in metallurgy and not a limitation of the furnace. Usually the furnace is loaded as shown in the diagrams in Fig. 85., *i.e.*, so that the charge does not short circuit the electrodes. However, in the loading of loose scrap, such as "mattress" material, the space-factor of the charge is low and the chamber is often filled completely. This of course short circuits the electrodes and care must be taken not to overload the electrical apparatus.

The operation of the arc furnace consists of loading, melting, and pouring. Thus the load on the elecrical equipment is not continuous. For a melting period of 30 min. the ratio of melting time to total time for continuous operation will range from 0.50 to 0.80 depending upon the methods of loading and pouring. Mechanical loading equipment is often used with the larger furnaces.

The use of round, solid graphite electrodes is standard practice for the single-phase arc furnace. The diameters of electrodes for the standard sizes of furnaces are given in Table XXV. The current density in the electrodes varies from 300 amp. per sq. in. in the small diameter electrodes to 200 amp. per sq. in. in the 4-in. diameter electrode. The electrode consumption in non-ferrous-alloy melting ranges from 3 to 5 lb. per ton of metal melted.

Various types of refractory linings for single-phase arc furnaces are in use. The refractory problem here is much the same as with other furnaces, and will be referred to later in this serial in a part that treats of "Refractories for Electric Furnaces."

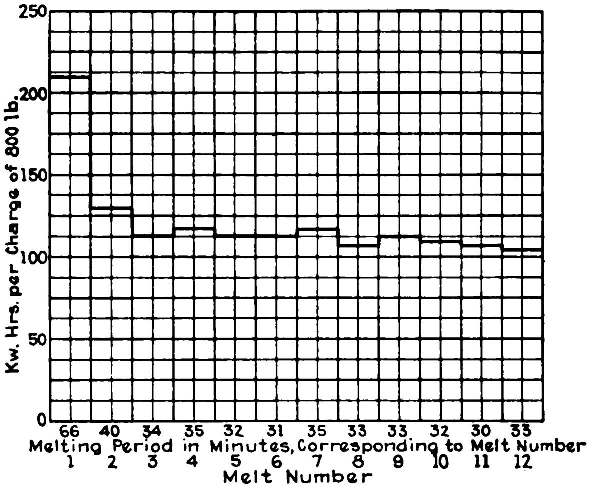


Fig. 99. Chart from the Log of the Operation of a Single-phase Arc Furnace

The control of the pouring temperature with the arc furnace is the same as with other electric melting furnaces, *i.e.*, by the control of the amount of energy put into the furnace. For this purpose the standard equipment of the furnace includes a watthour meter. Having once determined the kilowatt-hours needed to bring a given charge to the desired pouring temperature, the same energy input is used for subsequent charges of the same kind and weight.

The electrical equipment of the single-phase arc furnace consists of a transformer, a reactor, a circuit breaker in the high-voltage side of the transformer, an indicating wattmeter, a watthour meter, and a motor for giving mechanical motion to the furnace. An automatic electrode feeder is frequently added to the larger furnaces. With the small furnaces the electrodes are operated by hand. Details of this electrical equipment will be described later in this serial in a part entitled, "Electrical Apparatus for Electric Heating Equipment."

A typical installation of single-phase arc furnaces is shown in Fig. 98.

The chart in Fig. 99 is from the log of a day's run of a single-phase arc furnace melting an alloy of the composition 80 Cu, 10 Pb, 8 Zn, 2 Sn. The data are as follows:

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(35)See Part III of this serial (Melting Non-ferrous Metals)

Melts per day	12
Weight of charge	800 lb.
Total alloy melted	9600 lb.
Average pouring temperature	2183 deg. F.
Total time in operation	9 hr. 14 min.
Total melting time	7 hr. 12 min.
Average melting period	36 min.
Ratio melting time to total time	0.77
Total kw-hr.	1461
Kw-hr. per ton	304

Referring again to Fig. 99, it will be noted that the first charge of the day required 211 kw-hr. and a melting time of 66 min. During the second melt the furnace reached the steady state condition of heat flow and succeeding melts required an average melting period of 36 min. Disregarding the first two melts of the day we have for continuous operation:

Total time for 10 melts	8 hr. 37 min.
Total melting time	5 hr. 28 min.
Average melting period	32.8 min.
Ratio of melting time to total time	0.635
Total kw-hr.	1120
Kw-hr. per ton	280

The difference between the sum of the kilowatt-hours for the first two melts, and twice the average kilowatt-hours per melt for the remainder of the day represents the heat stored in the walls which is lost overnight. The stored heat of the furnace performs a useful function in maintaining the temperature of the metal while the arc is off during pouring.

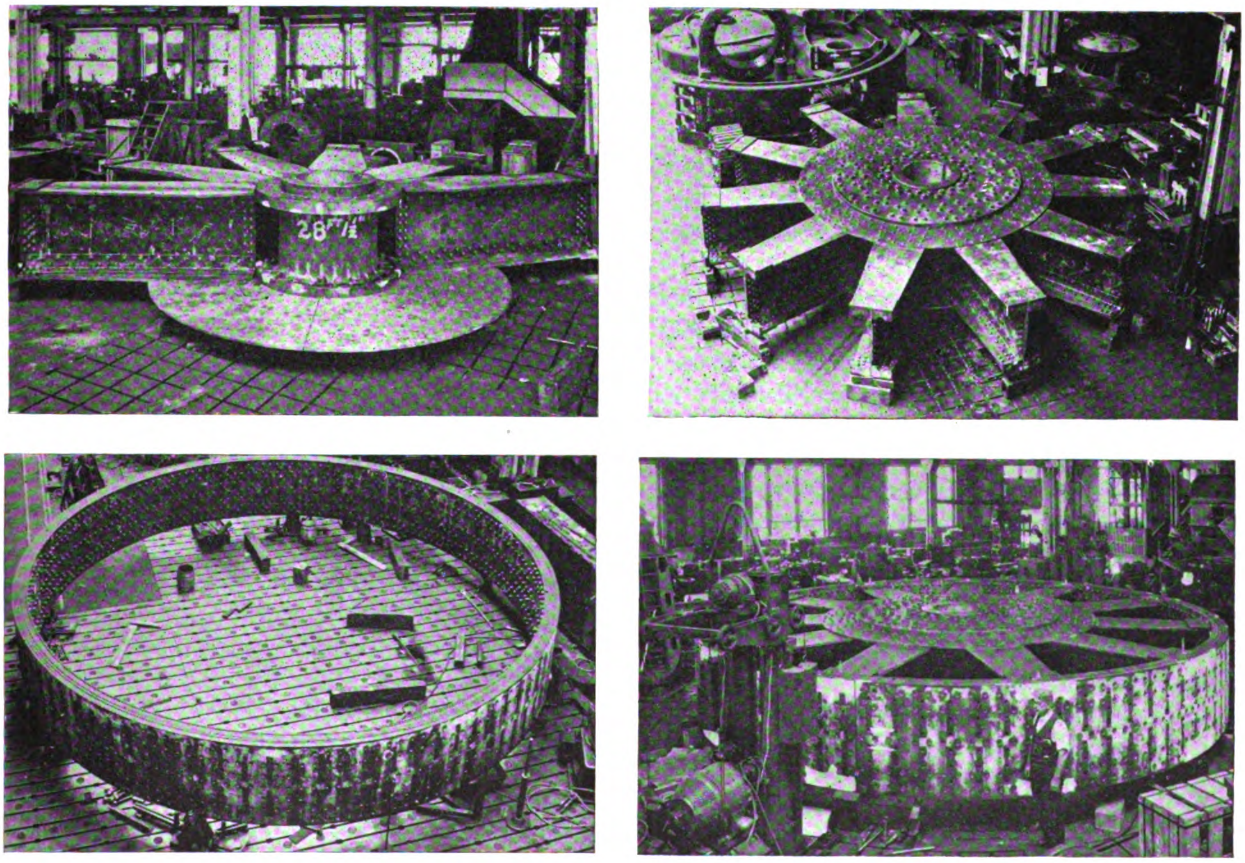
The cost of melting metal in an arc furnace includes the items for the electrical energy, the labor, the refractory linings which are renewed from time to time, the electrodes, and the maintenance. These individual costs vary with the conditions of operation, metal melted, rate for power, etc., according to local conditions. A manufacturer of single-phase arc furnaces gives the data in Table XXVI as representative operating and cost figures.

TABLE XXVI

Alloy	85 Cu, 5 Sn, 5 Pb, 5 Zn
Pouring temperature	2150 to 2200 deg. F.
Operation	8 hr. per day
Hand charging	
Average weight of charge	350 lb.
Average melting period	28 min.
Melts per day	13
Production	4550 lb.
Power, 298 kw-hr. per ton, at 2¢	$5.96
Lining cost (life 1200 melts)	0.36
Electrode cost (4½ lb. per ton)	1.04
Labor, one operator (at 55¢ per hour)	1.94
Ladle heating	0.20
Maintenance	0.25
Metal loss, 6/10 per cent at 11¢ per lb.	1.32
Total cost per ton melted	$11.07

(To be continued)

Rotor for Conowingo Generator During Course of Fabrication



Automatic Dynamometer Control Panels

A FULLY automatic control equipment is now available for use with the dynamometers employed in testing internal-combustion engines. Although much of this testing is of the nature of research work, and thus requires a very close, accurate control of both armature and field, it has been found that the usual manual type of control can be superseded to advantage by the automatic type.

The new control equipment consists of two panels, one a control panel and the other a master panel. This arrangement permits mounting the master panel on the dynamometer base, providing complete control of the dynamometer at the point where the testing is being done. The automatic control panel, with its motor-operated rheostats and absorption resistor, is mounted at any suitable remote location.

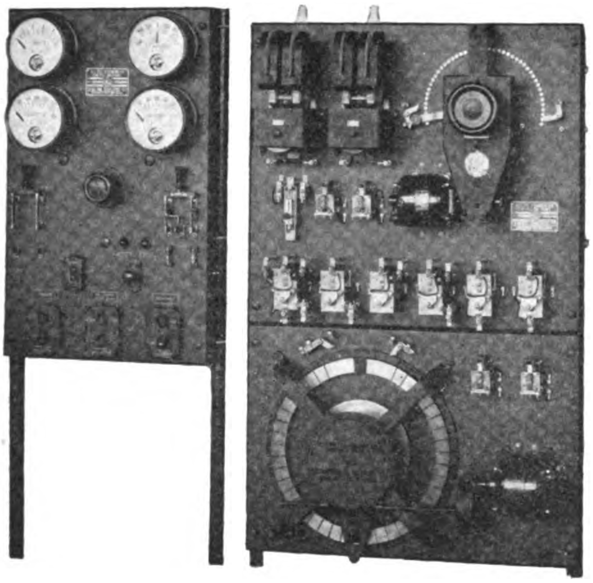


Fig. 1. Fully-automatic Control Panel and Master Panel for 150-hp. Dynamometer

The automatic control panel has a motor-operated armature dial switch and a motor-operated field dial switch. The control is designed to allow an inrush current of 2000 amp., which is sometimes necessary to start a "stiff" engine. The armature dial switch is of a screw type, having a large number of staggered steps arranged for armature regulation up to the maximum rated current when the dynamometer is operated either as a motor or a generator. When acting as a motor, the armature and field switches operate in proper sequence in either direction. They return automatically to their normal position when the dynamometer is stopped.

On overloads, the line contactor opens and a relay shuts off the ignition of the engine to prevent it from racing when the load is removed. As practically all dynamometers are separately excited, another relay shuts off the ignition in case of failure of the dynamometer-field supply. When running the dynamometer as a generator, the output is usually pumped back into the line but if desired it can be dissipated in the load resistor by opening the line contactor and closing the load resistor contactor by means of a push-button on the master panel.

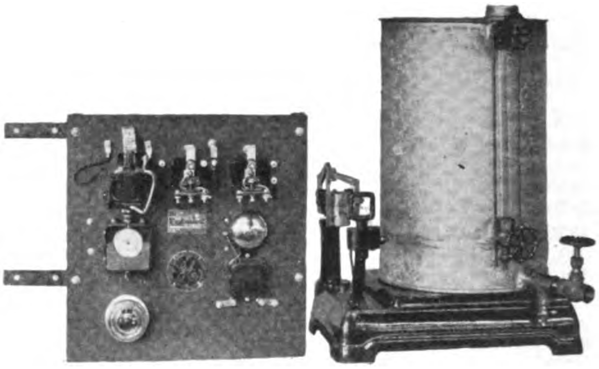


Fig. 2. Fuel-metering Panel with Contact-making Scale and Fuel Tank

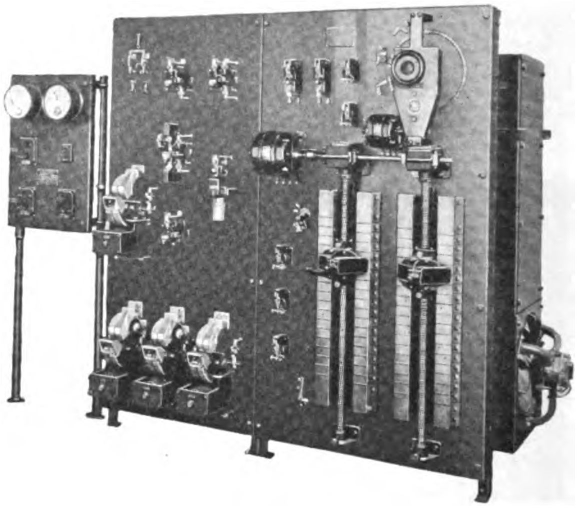


Fig. 3. Fully-automatic Control Panel and Master Panel for 300/400-hp. Dynamometer

The master panel is equipped with a field-reversing switch and several push-buttons, thus providing for the starting and stopping of the apparatus as well as the control of the motor-operated armature and field resistors on the automatic panel. A vernier field rheostat, also mounted on the master panel, provides fine adjustment after the motor-operated field rheostat has brought the speed or load approximately to the value desired. A voltmeter and ammeter in the line provide complete indication of voltage and current under all operating conditions.

The Application of Oxygen and Hydrogen to Industrial Operations

PART VI

HYDROGEN, NITROGEN AND THE ELECTRIC FURNACE

By F. P. WILSON, JR.
Manager's Staff, Schenectady Works, General Electric Company

IN a previous article [4] some of the uses and characteristics of hydrogen were explained; and now a brief description will be given of some of the features that must be considered in using hydrogen and hydrogen-nitrogen mixtures in electric furnaces.

Modern equipment makes possible the production of hydrogen of almost 100 per cent purity, which means freedom from oxygen. This is in contrast with one of the production difficulties encountered in the past, which was the production of hydrogen without a dangerous content of oxygen.

The degree of safety obtained in the applications of hydrogen to electric brazing and annealing furnaces is vitally dependent upon the technique employed in its handling. The use of city gas in baking ovens is a similar application with respect to hazards. Modern baking ovens are large and of heavy construction, and in many principles of design are not unlike the furnaces built for hydrogen brazing and annealing. In the past, the ventilation of such baking ovens has been so regulated as to remove only enough of the products of combustion to prevent the burners from becoming extinguished. The danger with such ovens occurs when unburned gas accumulates with air in the top of the oven, forming an explosive mixture which may be ignited by burner flames or lighting devices. The accumulation of illuminating gas may result from leaky valves or a temporary drop in gas pressure extinguishing the flame.

Industrial annealing, brazing, or oxide-reducing furnaces containing hydrogen or hydrogen-nitrogen atmospheres are electrically heated. The function of the hydrogen or hydrogen-nitrogen mixtures is not to furnish heat but to produce a non-oxidizing or a reducing atmosphere in the furnace. No combustion of hydrogen normally occurs inside the furnace because no air is admitted. This is in contrast with gas-combustion furnaces, in which air is either fed to the furnace or mixed with the fuel gas and in which the proportion of air to fuel is intentionally correct for combustion and therefore correct also for an explosion if normal combustion is interrupted or incomplete. In the case of the electric brazing and annealing furnace, hydrogen or hydrogen-nitrogen mixtures flow through the furnace and exhaust from it at a vent where the gas is intentionally ignited. This vent is visible at all times and is a ready and convenient indication of any variation from normal gas flow. In the event of the use of a non-combustible mixture of nitrogen and hydrogen, the flow indication is obtained from flow meters. Because air is not admitted to the furnace and the source of heat is electrical, the hazards with this type of furnace are considerably less than those encountered in similar types of furnaces that depend upon the combustion of city gas and air for their source of heat.

After the initial starting of an electric furnace that contains a reducing-gas atmosphere, the major requirement in the interests of safety is to maintain a normal flow of pure gas through it. If the power supply fails, the internal cooling will cause some contraction in the gas volume but a slightly increased gas flow will afford adequate compensation.

If proper care is exercised in the design, installation and operation of electric furnaces utilizing reducing-gas atmospheres, the factor of safety obtained will be equal to or greater than that obtained with a similar furnace of the combustion type.

One characteristic of hydrogen which makes it differ (although only in degree) from other fuels is its extreme lightness. This characteristic introduces the necessity of certain details of design in electric furnaces designed for hydrogen atmospheres.

The point of hydrogen admission should be at the highest point in the furnace, and the dome of the furnace should be so constructed that a high point exists. This feature prevents the formation of gas pockets.

The exhaust from the furnace should be at the lowest possible point to insure a complete sweeping of the furnace at all times. Admitting a gas as light as hydrogen at the top of a domed furnace and permitting it to exhaust at the lowest point precludes the possibility of air being present as long as the hydrogen flow is maintained in the proper direction. Breathing, which is due to temperature variation in the furnace, can be prevented by proper regulation of the hydrogen flow. Outlet pipes at the top of the furnace must be eliminated, or a chimney effect will be produced by the light hydrogen drawing air in around the furnace seal or through the vent.

Although with proper technique the explosion possibility may be remote, the furnace design usually provides for some convenient method of relieving any excessive rise of pressure inside the furnace.

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(4) Part I of this serial, Nov., 1927, p. 544.

There are no mysteries involved in the use of hydrogen in electric furnaces, and whenever accidents have occurred their cause is traceable to the violation of some known law or principle either in furnace design or operation.

A general consideration with all types of gas furnaces is that the rate of rise of pressure in gaseous explosions varies inversely as the initial temperature. This indicates that the greatest care must be exercised during the operation of a furnace until its temperature is above the ignition point of the gas being used. Not only would the rate of rise of pressure be less but, if the temperature is above the ignition point, any accumulation would be prevented by the combustion of the gas and air as rapidly as they mixed.

The predominating characteristic of hydrogen which necessitates any variation in handling technique, with respect to the other common combustible gases, is its extreme lightness. Assuming that the furnace design incorporates features to properly compensate for this lightness, then the operation of an electric furnace having a hydrogen atmosphere will be identical with the procedure employed for other combustible gases lighter than air.

The primary consideration with these furnaces, as with bakery ovens or other fuel-gas furnaces, is to *avoid an accumulation of gas and air*. Whether the gas be hydrogen, methane, carbon monoxide, or city gas, if accumulation takes place with air in the correct proportion and ignition occurs in such a mixture, an explosion is inevitable.

Of course there are various gas characteristic factors involved which vary the intensity of such an explosion. The characteristics which, paradoxically as it may seem, make hydrogen more adaptable to the shifting type of furnace, are its low ignition temperature, high flame temperature, and high rate of flame travel. All of these tend to reduce the possibility of quenching the flame which burns across the furnace mouth during the shift. If this flame were extinguished, an accumulation of hydrogen and air would naturally result.

The following factors are involved in combustion, explosion, and detonation:

Composition of fuel
Composition of secondary gas
Composition of gaseous mixture
Dilution
Compression
Turbulence
Method of ignition
Temperature of ignition
Tube composition
Tube texture
Closed and open-end tubes
Tube length
Tube diameter
Change in tube diameter.

From the foregoing it is obvious that there are many factors other than the gas being used which determine whether the reaction will be combustion, explosion, or detonation.

Fuels which are unstable and break down easily at high temperatures will detonate easily, although the detonation wave can also be initiated in mixtures of hydrogen and air.

Detonation may be controlled through retarding the rate of combustion of a fuel by adding chemicals to the mixture which serve to increase its specific heat and prevent excessive temperatures being reached, or by the use of diluents.

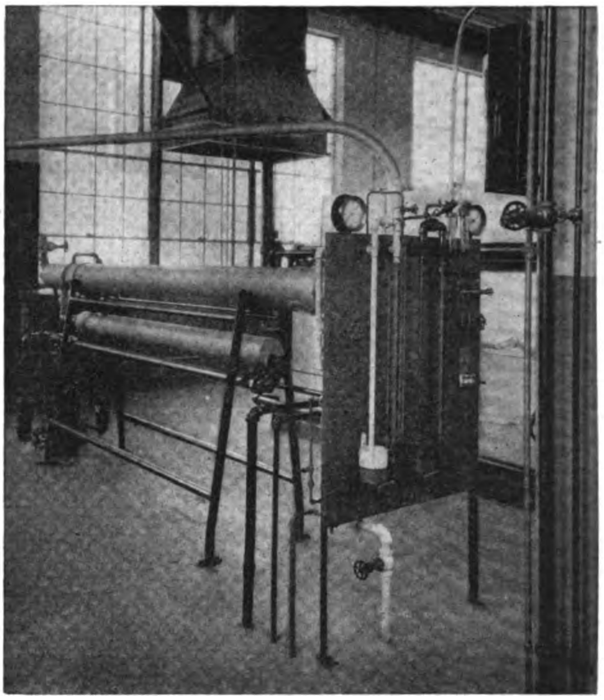

Fig. 30. Nitrogen Burner for Producing an Industrial Supply of This Gas by Using Hydrogen to Burn the Oxygen Out of Air

Excessive temperatures cause thermal decomposition of the fuel, and a selective burning of the constituents develops a high density in the gases ahead of the flame front and prevents complete combustion of the fuel.

Mason and Whalen show that the rate of reaction between a combustible gas and oxygen increases with the concentration of whichever has the greater stopping power for radiant energy and that this stopping power depends on the density of the gas. This is demonstrated in the case of methane, where the lowest ignition temperatures are obtained when there is excess oxygen. With other hydrocarbons there is a diminution in ignition temperature as the proportion of combustible gas is increased.

It is common experience that an increase in temperature invariably increases the rate of chemical action. Because combustion is a chemical reaction, it is apparently reasonable to expect that the rate of gaseous combustion must be increased by the increase in temperature. This is true where gases are passed through a hot zone that serves to ignite the succeeding layers of gas, as is the case in gas furnaces of the combustion type, surface combustion, ammonia oxidation, and similar processes. But in combustion in a closed vessel, the rate of flame travel depends on how rapidly each layer of gas will ignite the adjacent inherent layer.

In spite of modern equipment and highly developed technique for the handling of hydrogen in electric furnaces, there remain hazards which are inherent in the use of this gas in furnaces of comparatively large cubical contents.

The difficulty in arresting flame propagation throughout a hydrogen system introduces the necessity of water-flashback arresters which in turn saturate the gas and add to the drying difficulties and operating cost.

These factors, in combination with other economic considerations, have been largely responsible for retarding the more general development of this field.

Recent work, however, has indicated that a non-explosive mixture of hydrogen and nitrogen can be substituted for hydrogen in many electric-furnace applications with minor modifications in the process and should lend impetus to a more general adoption of this type of furnace for various operations.

In addition to the freedom from detonation which such a nitrogen-hydrogen mixture assures, its use results in economies that in many instances reduce the gas costs 50 per cent bcause of the lower cost of nitrogen production.

The degree of success obtained with gas-filled electric-brazing and annealing furnaces depends upon the following factors. The extent to which they apply varies slightly with the percentage of hydrogen and nitrogen used, but the attainment of maximum results is vitally dependent upon these conditions regardless of the gas medium selected:

(1). Proper selection of gas cycles and percentage of gas mixtures.
(2). Maintenance of constant gas purity, flow, and dryness.
(3). Condition of work placed in furnace:
    (*a*). Mechanical fit of joints to be brazed.
    (*b*). Cleanliness of work.
(4). Technique of furnace operation.
(5). Dryness of furnace atmosphere.
(6). Furnace design and its adaptability to the work.

Mixtures of hydrogen and nitrogen can be obtained by several methods, the two most preferable of which seem to be the combustion of hydrogen and air in an apparatus such as shown in Fig. 30, or the liquefaction and fractionation of air to produce nitrogen, with a subsequent mixing with hydrogen produced electrolytically. The apparatus shown in Figs. 19 and 23(6) permit the use of the latter process. In many instances where the hydrogen load for the operation of a burner, Fig. 30, is insufficient to warrant an electrolytic plant, it might prove advisable to obtain the gas by the dissociation of gaseous ammonia.

As is commonly known, the addition of a diluent to any reactive mixture generally decreases the rate of reaction. Thus, the addition of nitrogen to a hydrogen-air mixture would greatly reduce the explosibility of such a mixture. The use of a non-detonating mixture gives an added factor of safety to compensate for the personal equation and deviation from the standard code of operating procedure.

The effect of diluents on the velocity of the detonation wave was accurately determined by Dixon in detonating explosive mixtures containing hydrogen, methane, ethylene, and other fuels. In every case the addition of a diluent decreased the velocity of the detonation wave. It was found that the detonation wave was easily propagated in mixtures of hydrogen and air.

According to Dixon, the amount of nitrogen per unit volume of reagent necessary to prevent propagation of the detonation waves varies as the velocity of the detonation wave propagated in similar mixtures free from the nitrogen diluent. The effect of nitrogen on the initiation of the detonation wave is shown in Table XI.

TABLE XI

Fuel Mixtures	VELOCITY	
	(*Berthelot*) Meters/sec.	(*Dixon*) Meters/sec.
$2\ H_2+O_2$	2821	2810
$2\ H_2+O_2+7\ N_2$	(Wave not initiated)	

The fact that nitrogen used as a diluent in explosive mixtures decreases the velocity of flame travel and the rate of rise of pressure in all types of gaseous explosions indicates that the velocity of flame travel and rate of rise of pressure are similarly affected by diluents.

(*To be continued*)

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(6) These illustrations appeared in Part III of this serial, April, 1928, p. 197.

A Study of Crystal Structure and Its Applications

PART XI

THE DIFFRACTION OF LIGHT BY CRYSTALS

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IN Part X* of this serial, certain properties of crystals were calculated. The nature of these properties was such that it was necessary not only to know the arrangement of atoms in the crystal but also to make definite assumptions as to the nature of the forces between electrons and protons and the spatial arrangement of electrons in the atom. In spite of the apparently crude assumptions used, the calculated results were very satisfactory; and reasons were given for believing that the results of calculations from somewhat more elegant assumptions would not have been greatly different. It is the purpose of the present chapter to illustrate a second type of calculation in which each atom is taken as a unit with very little reference to the architecture of the atom itself.

The calculation chosen is that of W. L. Bragg (132) on the optical constants of calcite and aragonite. These minerals are both Ca CO_3, but their crystal structures are different. (133) (134) It is well known that both of these crystals are birefringent, *i.e.*, the degree with which they can bend a beam of polarized light depends upon the orientation of the crystal with respect to the plane of polarization of the light. It is generally assumed in Physical Chemistry that the optical properties of a compound may be calculated if only we know the optical properties of each of its components. This is approximately true in the case of isotropic substances, *e.g.*, jellies, solutions and crystals belonging to the cubic system. Bragg makes the additional assumption that in all other crystalline solids, the crystal structure must also be taken into account, for in a crystal the arrangement of atoms markedly affects the forces acting between them.

The customary procedure in the case of isotropic substances is to picture the electric vector in a beam of polarized light as pulling the electrons of the various atoms in one direction and pushing the corresponding protons in the opposite direction. In this way each atom is thought of as becoming an "electric doublet," *i.e.*, it acts like a tiny rod with a + charge at one end and a − charge at the other. The change in the strength of the electrostatic field (electric polarization) in a definite small volume, caused by the doublets thus formed, is then calculated by considering the effect of such doublets arranged in a hit-or-miss fashion in the isotropic substance. Such a calculation may be expected to apply to the case of a substance crystallizing in the cubic system, for all three axial directions are necessarily equivalent. It will not apply, however, even approximately to substances crystallizing in other systems, and for these the configuration of atoms in the crystal has to be taken into account. Bragg has made the necessary calculations for calcite and aragonite, taking into account the arrangement of the atoms in the crystals, and (within the degree of approximation to which the assumptions hold for isotropic substances) these calculations lead at once to the correct values for the indices of refraction of the crystals.

Index of Refraction of Isotropic Media

It is reasonable to assume that if a substance is placed between two charged electrodes so that a constant electrostatic field is imposed on it, all the electrons in the atoms of that substance will be pulled over slightly towards the positive electrode and all the protons will similarly be shifted over toward the negative electrode. Let s be the distance which is now assumed to exist between the center of gravity of the negative charges and the center of gravity of the positive charges in a given atom. This distance will be proportional to the strength (E') of the electrostatic field in which the atom finds itself, and to the charge (e) on each electron and proton. It will depend, too, upon the magnitude of the electrostatic forces between the electrons and protons in the atom. It will be assumed that this restoring force is independent of the orientation of the atom and this force will be

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(132) W. L. Bragg, *Proc. Roy. Soc. 105*, 370, (1924)., *Ibid. 106*, 346 (1925).
(133) W. L. Bragg, *Proc. Roy. Soc. A 89*, 248, (1914).
R. W. G. Wyckoff, *Amer. Jour. Sci. 50*, 317, (1920).
W. L. Bragg, *Proc. Roy. Soc. 105*, 16, (1924).
R. W. G. Wyckoff, *Amer. Jour. Sci. 9*, 145, (1925).
(134) W. H. Bragg, *Phil. Trans. Roy. Soc. A, 215*, 253, (1915).

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*References made to material hitherto published in this series of articles may be readily located from the following table:

Part	*Issue*	*Figs.*	*Equations*	*Tables*	*Footnotes*
Part I	Nov., 1924; p. 742	1 and 2	1 to 16	I and II	1 to 21
Part II	Dec., 1924; p. 795	3 to 10	17 to 32	III	22 to 27
Part III	Feb., 1925; p. 129	11 to 22	33 to 37	IV	28 to 42
Part IV	Apr., 1925; p. 258	23 to 28	38 to 41	None	43 to 48
Part V	May, 1925; p. 342	29 to 35	None	None	49
Part VI	Aug., 1925; p. 286	36 to 51	42	V to VII	50 to 58
Part VII	Oct., 1925; p. 721	52 to 59d	43 to 47	VIII and IX	59 to 65
Part VIII	Feb., 1926; p. 118	60 to 67	48	X to XIII	66 to 82
Part IX	Apr., 1926; p. 274	68 and 69	49 to 56	XIV to XXVI	83 to 117
Part X	Aug., 1926; p. 580	70 and 71	57 to 66	XXVII to XXXI	118 to 131

included, for any given kind of atom, in the factor of proportionality. We may therefore write

$$s = E'e\lambda \tag{67}$$

where λ is characteristic of the atom under consideration. The factor λ may be defined as the displacement per unit force on the atom. If Equation (67) is written

$$E'e = \frac{s_1}{\lambda_1} = \frac{s_2}{\lambda_2} \tag{68}$$

the expression $\frac{s}{\lambda}$ may be interpreted as representing the strength of the doublet.

Let $\bar{E}$ be the average value of the strength of the electrostatic field throughout the whole substance which has been placed between the electrodes. Since the dielectric constant of a vacuum is by definition unity, the increase in the field strength caused by the presence of the material substance is $(K-1)\bar{E}$ where K is the dielectric constant of the substance. This change in the field strength is called the "polarization per unit volume," and is designated by P. Assume that in each cubic centimeter of the substance there are N_1 atoms having a displacement constant λ_1, and therefore an electric momenent s_1e; N_2 atoms of displacement constant λ_2 and electric moment s_2e; etc. Then

$$\begin{aligned}(K-1)\bar{E} = P &= N_1\, s_1\, e + N_2\, s_2\, e + \cdots\cdots \\ &= N_1 \frac{s_1}{\lambda_1} \lambda_1 e + N_2 \frac{s_2}{\lambda_2} \lambda_2 e + \cdots\cdot \end{aligned} \tag{69}$$

Now consider some single atom located at a point A, for example. There will be some doublets so close to it that they may be regarded as making individual contributions to the strength of the electrostatic field at A. Other doublets, more remote, may be regarded as being so far from this point that they act with their neighbors to give the effect of a substantially uniform electrostatic field. The average field strength ($\bar{E}$) which acts on any individual atom represents the strength of the field imposed by the charged electrodes plus the effect of these more remote doublets. The additional effect of the doublets which are near the atom at A may be determined as follows: Let a sphere be drawn with A as a center and with a radius large enough to reach to the more remote doublets mentioned above, and let all the doublets inside this sphere be removed. On one side of A the surface of the sphere acts as though it were covered with a continuous coating of positive electricity; on the other side of A it will act as though it had a continuous coating of negative electricity. It may be shown that the field due to these charged halves of the sphere is $\frac{1}{3}P$. If the substance between the electrodes is isotropic, the average effect of the doublets inside such a sphere is zero. The net effect is that the total electric field acting at the point A is no longer represented by Equation (68) but by

$$E'e = \frac{s_1}{\lambda_1} = \frac{s_2}{\lambda_2} = \cdots\cdots = (\bar{E} + \tfrac{1}{3}P)\, e \tag{70}$$

Equation (69) must therefore be altered to read

$$\begin{aligned} P &= (\bar{E}+\tfrac{1}{3}P)\, N_1e^2\lambda_1 + (\bar{E}+\tfrac{1}{3}P)\, N_2e^2\lambda_2 + \cdots\cdot \\ &= (\bar{E}+\tfrac{1}{3}P) \sum Ne^2\lambda \qquad (71) \\ &= E' \sum Ne^2\lambda \end{aligned}$$

or

$$\begin{aligned} K-1 = \frac{P}{\bar{E}} &= \frac{P}{E'-\tfrac{1}{3}P} \\ &= \frac{E' \cdot \sum Ne^2\lambda}{E'(1-\tfrac{1}{3}\sum Ne^2\lambda)} \qquad (72) \\ &= \frac{\sum Ne^2\lambda}{1-\tfrac{1}{3}\sum Ne^2\lambda} \end{aligned}$$

Equation (72) will still hold true if the field across the electrodes is an alternating field, such as would be produced by the electric vector of a light wave, except that near all resonance frequencies the value of λ will depend upon the frequency of the field. Equation (72) may be made more useful for our purposes by means of the well known principle that, if the dielectric constant of a substance is measured for fields of the same frequency as the light which we happen to use, then that dielectric constant is inversely proportional to the square of the velocity of light in that substance. Remembering that the ratio of the velocity of light in a vacuum to the velocity in a given substance is the index of refraction of that substance we have,

$$\frac{K_{\text{substance}}}{K_{\text{vacuum}}} = \frac{(\text{Velocity of light in vacuum})^2}{(\text{Velocity of light in substance})^2} = n^2$$

Since the dielectric constant of a vacuum is, by definition, unity, we can replace K in Equation (72) by n^2, giving

$$n^2 - 1 = \frac{\sum Ne^2\lambda}{1-\tfrac{1}{3}\sum Ne^2\lambda} \tag{73}$$

If M is the molecular weight of the substance and ρ is its density then the quantity $\frac{n^2-1}{n^2+1} \cdot \frac{M}{\rho}$ is approximately a constant which is substantially independent of the physical state of the substance, provided it is isotropic. It is called the "molecular refractivity" of the substance and is denoted by R. From Equation (73) we have at once

$$\frac{M}{\rho} \cdot \frac{n^2-1}{n^2+1} = R = \tfrac{1}{3} \frac{M}{\rho} \cdot \sum N e^2 \lambda \qquad (74)$$

Now N_1, N_2, etc., represent the number of atoms of kinds (1), (2), etc., per cubic centimeter. $N_1 \frac{M}{\rho}$, $N_2 \frac{M}{\rho}$, etc., are therefore the number of atoms per gram molecule, and may be represented by $a\, N_0$, $b\, N_0$, etc., where N_0 is the Avogadro number. In terms of N_0, Equation (74) becomes

$$\begin{aligned} R &= a\,(\tfrac{1}{3} N_0\, e^2 \lambda_1) + b\,(\tfrac{1}{3} N_0\, e^2 \lambda_2) + \cdots\cdot \\ &= a\, A_1 + b\, A_2 + \cdots \qquad (75) \end{aligned}$$

where A_1, A_2, etc., are the atomic refractivities. On the basis of their optical properties in isotropic substances, it is possible to assign values for the refractivities for the atoms and ions of many elements, and the molecular refractivity of a compound is approximately the sum of the atomic (or ionic) refractivities of its constituents, provided only that the substance is isotropic.

Index of Refraction of Non-isotropic Media

So far we have assumed that the immediate neighbors of any given atom were situated around it in an entirely haphazard fashion. Although not so arranged in the case of crystals of cubic symmetry, the mathematics of cubic crystals is such as to lead to essentially the same result. This is not the case for non-cubic crystals. The net electrostatic field at a point due to the more closely situated atoms is no longer zero. It must be calculated in terms of the relative positions of these atoms (or ions) and the distance between them as given by crystal analysis.

Consider an atom situated at the origin of coördinates, and let one of the doublets in its immediate neighborhood have coördinates x, y, z, so that it is at a distance

$$r = \sqrt{x^2 + y^2 + z^2}$$

from the origin. The x-component of the field which it sets up at the origin is, in Heaviside units,

$$\frac{\mu_x}{4\pi r^3} \cdot \frac{3x^2 - r^2}{r^2} + \frac{\mu_y}{4\pi r^3} \cdot \frac{3xy}{r^2} + \frac{\mu_z}{4\pi r^3} \cdot \frac{3xz}{r^2} \qquad (76)$$

where μ_x, μ_y, and μ_z are its components of polarization. Fields of this sort from each doublet in the neighborhood are superimposed on the field which we considered in the case of isotropic substances, so that the simple form of Equation (70) must be replaced by a group of equations of the form

$$\begin{aligned} \frac{s_1}{\lambda_1} &= (\bar{E} + \tfrac{1}{3} P)\, e + (a_1 s_1 e + a_2 s_2 e + \cdots\cdot\cdot)e \\ \frac{s_2}{\lambda_2} &= (\bar{E} + \tfrac{1}{3} P)\, e + (b_1 s_1 e + b_2 s_2 e + \cdots)\, e \qquad (77) \end{aligned}$$

etc.

The new terms $a_n s_n e$ give the changes to the field which have been contributed by doublets of strength $s_n e$. All doublets of a given kind situated at a distance r from the origin of coördinates are included in the same term. For instance, if for the doublets of strength $s_1 e$ the components of polarization are $\mu_x = s_1 e$, $\mu_y = 0$, $\mu_z = 0$; then Equation (76) gives the value of a_1 as

$$a_1 = \sum_{r_0}^{r_1} \frac{1}{4\pi r^3} \cdot \frac{3x^2 - r^2}{r^2} \qquad (78)$$

In general, μ_y and μ_z will not be zero, but the error introduced will be small and can be allowed for in numerical calculations if desired.

The equations illustrated by Equation (77) may be written in the form

$$\frac{s_1}{\lambda_1} = (\bar{E} + \tfrac{1}{3} P)\, e + \left(\frac{s_1}{\lambda_1} a_1 e^2 \lambda_1 + \frac{s_2}{\lambda_2} a_2 e^2 \lambda_2 + \cdots\cdot \right) e \qquad (77a)$$

Calculations based on values for a_1, a_2, etc., given by Equation (78), and on values for λ_1, λ_2, etc., given by atomic refractivities, give values for the second parenthesis which are small (± 0.2 for calcite and aragonite) in comparison with $(\bar{E} + \tfrac{1}{3} P)$, so they may be represented by a corrective factor thus:

$$\begin{aligned} \frac{s_1}{\lambda_1} &= C_1\, (\bar{E} + \tfrac{1}{3} P)\, e \\ \frac{s_2}{\lambda_2} &= C_2\, (\bar{E} + \tfrac{1}{3} P)\, e \qquad (79) \end{aligned}$$

etc.

For non-isotropic media, Equation (71) therefore becomes

$$P = C_1\, (\bar{E} + \tfrac{1}{3} P)\, N_1 e^2 \lambda_1 + C_2\, (\bar{E} + \tfrac{1}{3} P)\, N_2 e^2 \lambda_2 + \cdots\cdot$$

and Equation (73) becomes

$$n^2 - 1 = \frac{C_1 N_1 e^2 \lambda_1 + C_2 N_2 e^2 \lambda_2 + \cdots}{1 - \tfrac{1}{3}\,(C_1 N_1 e^2 \lambda_1 + C_2 N_2 e^2 \lambda_2 \cdots\cdot)} \qquad (80)$$

We are now ready to consider calcite and aragonite and to compare their indices of refraction as calculated by Equation (80) with the results of experiment.

Structure and Optical Properties of Calcite and Aragonite

The rhombohedron of calcite shown in Fig. 72 (previously reproduced as Fig. 35) may be regarded as having a distorted Na Cl structure. When a calcite crystal, bounded by its cleavage planes, is placed so that its short body-diagonal is vertical, the structure may be thought of as being made up of three interpenetrating simple triangular lattices. The arrangement of atoms, looking down vertically upon such a crystal, is shown in Fig. 73. The CO_3^{--} group is equally spaced from each of three Ca^{++} ions below and three Ca^{++} ions above. The oxygens lie midway between one of the lower Ca^{++} and one of the upper Ca^{++}.

Although aragonite belongs to the orthorhombic system, the positions of the Ca^{++} and CO_3^{--} are very nearly what they would have been if the structure had been triangular close-packed. The arrangement of atoms, looking along the Z-axis of the prism is shown in Fig. 74. Here again the CO_3^{--} group is equally spaced from each of three Ca^{++} ions below and three Ca^{++} ions above, but now each oxygen lies midway between three Ca^{++} ions.

When the electric vector of a polarized light wave is parallel to the planes of CO_3^{--} in calcite, the index of refraction (ω) is 1.658; when it is perpendicular to these planes the index of refraction (ϵ) is 1.486. If aragonite had true hexagonal symmetry it, too, would have two indices of refraction, one (ω) with the electric vector perpendicular to the CO_3^{--} planes, (*i.e.*, parallel to the Z-axis) and a second (ϵ) with the electric vector parallel to the CO_3^{--} planes (*i.e.*, in the plane of the X- and Y-axes). But since the symmetry of aragonite is only pseudo-hexagonal, and is actually that of the orthorhombic system, it has three indices of refraction. These are: $\alpha, = 1.530$ when the electric vector is parallel to the pseudo-hexagonal Z-axis; $\beta, = 1.681$ when it is parallel to the pseudo-hexagonal X-axis; and $\gamma, = 1.686$ when the electric vector is parallel to the pseudo-hexagonal Y-axis. Aragonite, therefore, is not far from being a uniaxial crystal with $\omega = 1.530$, and $\epsilon = 1.683$.

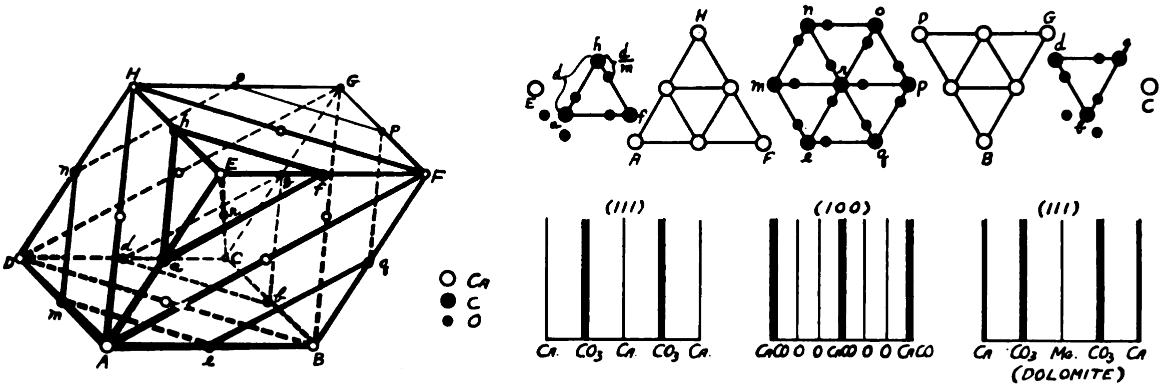

Fig. 72. Structure of Calcite (*Bragg*) (*This is Fig. 35 repeated*)

Values found in the literature(135) for the ionic refraction of calcium, carbon, and oxygen indicate that the calcium ions ($I_{Ca^{++}} = 1.99$) are responsible for only about 15 per cent of the total refractivity of calcite. Assuming that the molecular volume of $CaCO_3$ in calcite(136) is

$$\frac{M}{\rho} = 36.13$$

the expression $Ne^2\lambda$ in Equation (80) becomes (see Equation (75)

$$Ne^2\lambda = 3\,\frac{M}{\rho}\,I_{Ca^{++}} = 0.165 \qquad (81)$$

for the calcium ions in calcite. The corresponding molecular volume in aragonite is 34.01, giving a value for $Ne^2\lambda$ of 0.175. The rest of the refractivity is to be accounted for by the carbon and oxygen. It remains to apportion the remainder between the carbon and the oxygen. Wasastjerna has shown(135) that ions and molecules of the type RO_3 have nearly identical refractivities. For instance $R_{Al_2O_3} = 10.6$; $R_{SO_3^{--}} = 10.8$; $R_{NO_3^-} = 10.4$; $R_{CO_3^{--}} = 11.1$. It is evident, therefore, that most of the refractivity is due to the three oxygens and that the element to which they are directly combined in the crystal has but little effect. This fits in very well with the theories published by Lewis, Langmuir, Kossel, and others, in which they picture each of the three oxygens as having two extra electrons so that for our purposes they may be considered as acting like negatively charged ions. In the case of the CO_3^{--} radical, this leaves the carbon with only its positive nucleus and the two "K" electrons. The effect of such a C^{++++} should be very small in comparison with that of the three O^{--} ions. On this basis the refractivity of O^{--} must be a little less than $\frac{11.1}{3} = 3.7$. If, instead, we use the value from NO_3^{--} it must be a little less than $\frac{10.4}{3} = 3.5$. Bragg assumes that the allowance for C^{++++} should be such as to give 3.30 for the refractivity of O^{--}. For oxygen, the value of $Ne^2\lambda$ in Equation (80) is, then, $3\,\frac{3 \times 3.30}{36.13} = 0.822$ in calcite, and $3\,\frac{3 \times 3.30}{34.01} = 0.873$ in aragonite where N is, as in Equation (69), the number of atoms (in this case oxygen ions) per cubic centimeter.

Confining our attention for the present to calcite, we now have the following values ready to substitute in Equation (80);

for Ca^{++}, $N_1e^2\lambda_1 = 0.165$

for O^{--}, $N_2e^2\lambda_2 = 0.822$

for C^{++++}, $N_3e^2\lambda_3 = 0$

(*i.e.*, it is considered to be negligible in Equation (80) or an equation of this type).

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(135) J. A. Wasastjerna, *Soc. Scient. Fenn. Comm. Phys. Math.*, *1*, 37 (1923).

(136) This value is the one recorded by Groth and used by Bragg. The actual density of calcite is 2.71, which gives a molecular volume of 36.93, and a value for $Ne^2\lambda$ for Ca^{++} of 0.162. The final value of n (calculations of the first approximation) on this basis is 1.459 as compared with Bragg's value of 1.468. Since the difference is of the same order of magnitude as that caused by the uncertainty in $I_{O^{--}}$, there is little point in trying to correct Groth's value for the molecular volume. In any case, it would hardly affect the values for the *differences* in the corresponding indices for calcite and aragonite. As Bragg has pointed out, it is these *differences* that are most important.

These are all calculated on the basis of the number of ions per cubic centimeter because our original equations, such as Equation (69), were based on a volume of one cubic centimeter.

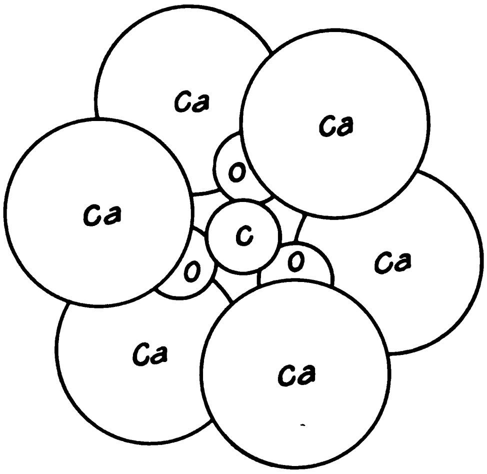

Fig. 73. Arrangement of Atoms in Calcite Looking Along the Short Diagonal of the Rhombohedron (*Bragg*)

It remains to find values for the coefficients C_1 and C_2. Since the refractivity of Ca^{++} is small, there will be but little error introduced into the final answer by calling C_1 unity. The value assigned to C_2 for the oxygen terms will depend upon how far we wish to carry our approximations in finding the radius of the sphere mentioned in the derivation of Equations (70) and (77). The roughest possible approximation, short of blindly calling C_2 equal to unity so as to give the sphere a radius of zero, is to consider a sphere which is so small as to take in only the three O^{--} ions of a single CO_3^{--} group; *i.e.*, r in Equation (78) equals x. If in Fig. 75 the electric vector is perpendicular to the plane of the CO_3^{--} group, then the polarization of each O^{--} is given by Equations (77a) and (78) in terms of $\frac{S_2}{\lambda_2}$. In Equation (78) r is 2.25×10^{-8} cm., and since the X-axis is in the direction of the electric vector of the light wave, $x=0$. In Equation (77a), therefore, $a_2=-1$ for each of the two oxygen ions at the circumference of the sphere. The negative sign indicates that the polarization represented by this term opposes that of the $(\bar{E}+\frac{1}{3}P)\,e$ term. Equation (78) now becomes

$$\frac{s}{\lambda_2}=(\bar{E}+\tfrac{1}{3}P)\,e+\frac{s_2}{\lambda_2}\,a_2\,e^2\,\lambda_2$$

$$=(\bar{E}+\tfrac{1}{3}P)\,e-\frac{s_2}{\lambda_2}\cdot\frac{2}{4\pi r_0^3}\cdot e^2\,\lambda_2 \qquad (82)$$

$$=\left(\frac{1}{1+\dfrac{e^2\lambda_2}{2\pi r_0^3}}\right)(\bar{E}+\tfrac{1}{3}P)\,e$$

The factor $\dfrac{1}{1+\dfrac{e^2\lambda^2}{2\pi r_0^3}}$ is the C_2 of Equations (79) and (80). The numerical value of this factor may be found by an equation like (81),

$$N_2\,e^2\,\lambda_2=3\,\frac{\rho}{M}\,I_{O^{--}}=3.30\times3\,\frac{\rho}{M}$$

since

$$\frac{M}{\rho}\,N_2=N_0=\text{Avogadro's constant}=6.062\times10^{23}$$

and since $\frac{1}{3}\,N_0\,e^2\,\lambda_2=3.30$

we have $e^2\,\lambda_2=1.63\times10^{-23}$

and $\dfrac{e^2\,\lambda_2}{2\pi r_0^3}=\dfrac{1.63\times10^{-23}}{2\pi\,(2.25)^3\times10^{-24}}=0.228$

C_2 is therefore $\dfrac{1}{1.228}=0.815$

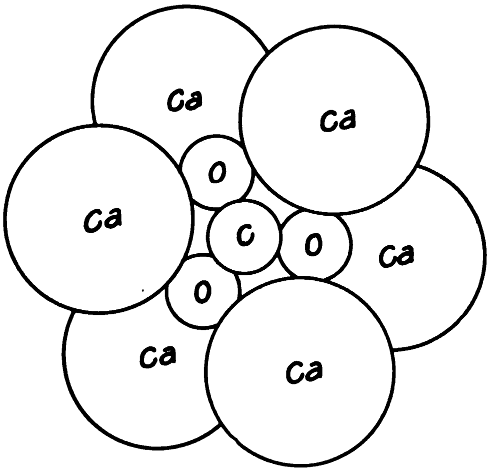

Fig. 74. Arrangement of Atoms in Aragonite Looking Along the Z-axis (*Bragg*)

We are now ready to substitute all our numerical values in Equation (80). Such a substitution gives

$$n^2-1=\frac{0.165+(0.815\times0.822)}{1-\frac{1}{3}\,\{0.165+(0.815\times0.822)\}}=1.155 \qquad (83)$$

or $n=1.468$

Since the electric vector was assumed in Fig. 75 to be perpendicular to the plane of the CO_3^{--} group, n will be the index of refraction for the extraordinary ray. The experimental value is 1.486. In view of the crude assumptions made in the calculations, the agreement is very satisfactory.

In Fig. 76 the electric vector lies in the plane of the CO_3^{--} group and is in the direction of a line joining the carbon to one of the oxygens. This fixes the X-axis. The two oxygen ions on the circumference of the sphere have coördinates

$x=1.95\times10^{-8}$ cm.
$y=1.125\times10^{-8}$ cm.
$z=0$

There will now be three $\frac{s_2}{\lambda_2}$ equations corresponding to the three sets of arrows in Fig. 76. We will assume that the elastic properties of O^{--} ions are the same along all three coördinates so that λ_2 will be the same in all three equations. The three s_2 terms, however, will be different as the field strengths differ. Thus

$$\frac{s_{2(a)}}{\lambda_2}=(\bar{E}+\tfrac{1}{3}P)\,e+2\cdot\frac{1}{4\pi r_0^3}\left(\frac{3\times(1.95)^2-(2.25)^2}{(2.25)^2}s_{2(b)}e\right.$$
$$\left.+\frac{3\times(1.125)\times 1.95}{(2.25)^2}\;s_{2(c)}\,e\right)e$$
$$=\bar{E}(+\tfrac{1}{3}P)\,e+\frac{1}{4\pi r_0^3}\,(2.50\;s_{2(b)}\,e+2.60\;s_{2(c)}\,e)\,e$$

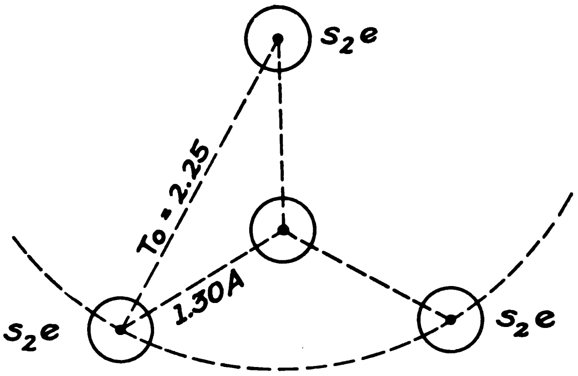

Fig. 75. Diagram of CO_3^{--} Group. The electric vector is perpendicular to the plane of the diagram (*Bragg*)

$$\frac{s_{2(b)}}{\lambda_2}=(\bar{E}+\tfrac{1}{3}P)\,e+\frac{1}{4\pi r_0^3}\,(1.25\;s_{2(a)}\,e-s_{2(b)}\,e)\,e$$

$$\frac{s_{2(c)}}{\lambda_2}=\frac{1}{4\pi r_0^3}\,(1.30\;s_{2(a)}\,e-2\;s_{2(c)}\,c)\,e$$

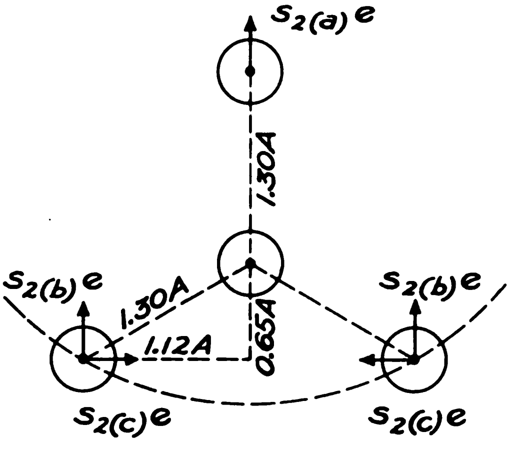

Fig. 76. Diagram of CO_3^{--} Group. The electric vector is parallel to the line joining the carbon to one of the oxygens (*Bragg*)

Solving these three simultaneous equations we have

$$\frac{s_{2(a)}}{\lambda_2}=1.36\;(\bar{E}+\tfrac{1}{3}P)\,e$$

$$\frac{s_{2(b)}}{\lambda_2}=1.07\;(\bar{E}+\tfrac{1}{3}P)\,e$$

$$\frac{s_{2(c)}}{\lambda_2}=0.103\;(\bar{E}+\tfrac{1}{3}P)\,e$$

The mean polarization in the direction of the electric vector of the light wave is

$$\frac{(s_{2(a)}+2\,s_{2(b)})\,e}{3\,\lambda_2}=1.17\;(\bar{E}+\tfrac{1}{3}P)\,e$$

A corresponding calculation for Fig. 77 leads to the same value for the coefficient of $\bar{E}\;(+\tfrac{1}{3}P)\,e$. This is as it should be, because of the symmetry of the CO^{--} group.

We are now ready to substitute the following values in Equation (80):

for Ca^{++}, $N_1e^2\lambda_1=0.165$; $C_1=1.00$

for O^{--}, $N_2e^2\lambda_2=0.822$; $C_2=1.17$

for C^{++++}, $N_3e^2\lambda_3=0$ (considered to be negligible).

This gives a calculated value for n in calcite of 1.676. Since the electric vector of the light wave is in the plane of the CO_3^{--} group, this should correspond to the index of refraction for the "ordinary" ray. The test value is 1.658. Here again the agreement is good when the rough approximations on which the calculations were based are considered.

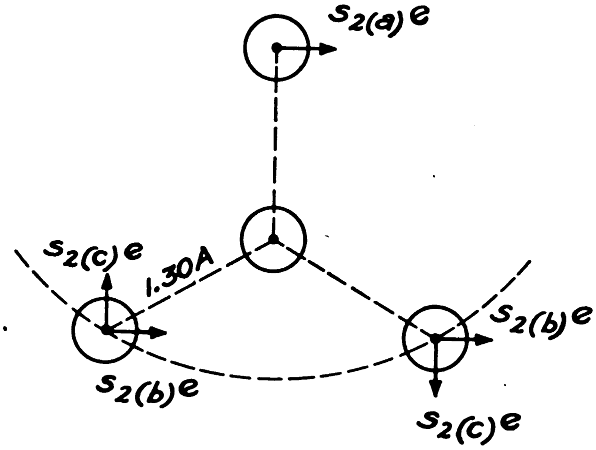

Fig. 77. Diagram of CO_3^{--} Group. The electric vector is parallel to the line joining two oxygen ions (*Bragg*)

These calculations may be refined by using a sphere of larger radius around each oxygen so as to include a larger number of neighboring ions. Such a computation ought, strictly, to take into account the effect of O^{--} on Ca^{++}, Ca^{++} on O^{--}, Ca^{++} on Ca^{++}, and O^{--} on O^{--}. If the radius of the sphere is increased from 2.25×10^{-8} cm. to 6.00×10^{-8} cm., each sphere in calcite would include 42 neighboring ions. In aragonite it would include 50. The uncertainty in the value for the ionic refractivity of oxygen makes it seem unwarranted to go through such a calculation without introducing certain simplifying assumptions. Bragg points out three assumptions of this sort, *viz.*:

(1) Components of polarization not in the direction of the field are negligible except in the case of the two nearest oxygen ions. This assumption is justified by the previously determined value for $\frac{s_{2(c)}}{\lambda_2}$ in calcite.

(2) The effect of Ca^{++} on O^{--} is negligible. Bragg's computations show that this assumption affects the accuracy only in and beyond the third decimal place.

(3) For Ca^{++} the constant C_1 is unity.

In Equation (82) the two oxygen ions on the surface of the sphere of radius 2.25×10^{-8} cm. introduced a term $-\frac{s_2}{\lambda_2}\cdot\frac{2}{4\pi r_0^3}\cdot e^2\lambda_2$. If the radius of the sphere is increased to 4.50×10^{-8} cm., additional ions are brought in whose effect is of the opposite sign, thus decreasing the value of the negative term. If the radius of the sphere is further increased to 5.5×10^{-8} cm., the additional ions add a negative quantity to the previous result. If the sphere is increased to a radius of 6.0×10^{-8} cm., a small positive quantity is added. In other words, increasing the radius of the sphere adds alternately positive and negative quantities which rapidly decrease in amount with increasing radius. For a radius of 6.0×10^{-8} cm., Equation (82) becomes

$$\frac{s_2}{\lambda_2}=(E+\tfrac{1}{3}P)\,e-\frac{1.52}{4\pi r_0^3}e^2\lambda_2 \qquad (84)$$

The variations in the second term of Equation (84) caused by further slight increases in the radius of the sphere are of the order of 10 to 15 per cent so that the variations in the calculated indices of refraction are of the order of one or two in the second decimal place. Using a radius of 6.0×10^{-8} cm., we have the following values for C_2 in Equation (80):

Calcite

(1) Polarization perpendicular to the plane of the CO_3^{--} group, $C_2=0.847$.

(2) Polarization parallel to the plane of the CO_3^{--} group, $C_2=1.100$.

Aragonite

(1) Polarization perpendicular to the plane of the CO_3^{--} group (*i.e.*, parallel to the Z-axis, and corresponding to the α index of refraction), $C_2=0.874$.

(2) Polarization parallel to the X-axis, and corresponding to the β index of refraction, $C_2=1.118$.

(3) Polarization parallel to the Y-axis, and corresponding to the γ index of refraction, $C_2=1.100$.

The corresponding refractive indices are given in Table XXXII. The agreement between the calculated and observed values is considerably better for the larger radius of the sphere, and is clearly within the limit of error mentioned above. It is to be expected that a further increase would iron out most of the remaining differences, and even make the calculated value of γ larger than that of β.

Optical Properties of Other Crystals

In the light of the preceding calculations it is possible to make some interesting inferences as to interatomic spacings in certain crystals. For instance, by assuming a succession of values for the distance between the O^{--} ions in calcite it is possible to compare the calculated values of the indices of refraction with the values determined by test. It is more convenient to compare the values for the "ionic birefringence" of O^{--}, which are found as follows: The ordinary "molecular birefringence" is found as the difference, $R_\omega-R_\epsilon$, between the molecular refractivity (R_ω) for the ordinary refractive index and the corresponding molecular refractivity (R_ϵ) for the extraordinary refractive index. The expression for molecular refractivity is given in Equation (74).

TABLE XXXII

Comparison Between Calculated and Observed Indices of Refraction for Calcite and Aragonite

	CALCULATED		OBSERVED
	$r=2.25$ A.	$r=6.00$ A.	
Calcite	1.468	1.488	1.486
	1.676	1.631	1.658
Aragonite	1.503	1.538	1.530
	1.730	1.694	1.681
	1.730	1.680	1.686
	1.730	1.687	1.683

The ionic refractivities, R_ω' and R_ϵ', of O^{--} are found by subtracting the ionic refractivity of the metal ion from $R_\omega-R_\epsilon$. This gives the ionic birefringence of O^{--} as $R_\omega'-R_\epsilon'$. If we assume that the metal ion contributes about equally to both R_ω and R_ϵ, then $R_\omega'-R_\epsilon'$ is approximately equal to $R_\omega-R_\epsilon$.

TABLE XXXIII

Dependence of Ionic Birefringence Upon Interionic Distances

d 10^{-8} cm.	R_ω'	R_ϵ'	$R_\omega'-R_\epsilon'$ $(=R_\omega-R_\epsilon)$
1.00	13.55	6.75	6.80
1.10	12.21	7.48	4.73
1.20	11.33	8.01	3.32
1.30	10.89	8.39	2.50

Table XXXIII gives Bragg's values for $R_\omega'-R_\epsilon'$ for different spacings between O^{--} ions in calcite. The value given by tests is 2.94. It is evident that the distance between O^{--} ions is between 1.20 and 1.30×10^{-8} cm.; interpolation gives $1.24^5\times10^{-8}$ cm. From x-ray diffraction data, W. H. Bragg(134) estimated this coefficient as between 1.24 and 1.34. Wyckoff, using the Laue method, estimated it (138) as being close to 1.24. This illustration shows the possibilities in using optical data to supplement the ordinary data of x-ray diffraction in determining the structure of crystals. For instance $NaNO_3$ has the same sort of structure as calcite,(139) but the molecular

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(137) W. L. Bragg. *Proc. Roy. Soc. 106*, 346, (1925).

(138) R. W. G. Wyckoff, *Amer. Jour. Sci. 50*, 317 (1920).

(139) W. L. Bragg. *Proc. Roy. Soc. 89*, 468, (1914); R. W. G. Wyckoff, *Phys. Rev. 16*, 149, (1920).

birefringence is greater in the ratio of 4.83 to 2.94. Table XXXIII would seem to indicate that this means that the O^{--} ions are closer together in the NO_3^- group than they are in the CO_3^{--} group. W. L. Bragg has made calculations similar to those in Table XXXIII for $NaNO_3$ using Wasastjerna's value of 0.75 for the ionic refractivity of Na^+. His calculations show that the interionic distance between O^{--} ions of a single NO_3^- group is 1.09×10^{-8} cm. This is consistent with the current pictures of the ionic structure of NO_3^- and CO_3^{--}, for the three O^{--} ions should should be more strongly attracted by N^{+++++} than by C^{++++}.

Similar calculations on the birefringence of Al_2O_3 lead Bragg to assign an interionic spacing of 1.45—1.48×10^{-8} cm. to the O^{--} ions. This is to be compared with 1.45×10^{-8} cm. as indicated by x-ray diffraction.

Conclusions

It is in the very nature of things that our pictures of atoms, ions, electrons, etc. must be only approximations to reality, for these pictures are all founded upon circumstantial evidence. If, however, such a picture based on one line of evidence is found to be consistent with still other evidence of an entirely different sort, we are encouraged to rely more strongly on the picture. The success of the calculations on compressibilities and photo-electric effects which were described in the previous chapter, coupled with the success of the calculations on the optical properties of crystals described in this chapter, lead us to believe that the pictures on which the calculations are based represent a fairly close approximation to reality. It is hard not to "believe in" electrons, protons, atoms, and ions when their calculated properties are in such close quantitative agreement with the known properties of the material world around us.

(To be continued)

Science and Invention

THE neglect of research in fundamental science in this country—testimony given in the utterances of public men and in the press seem to indicate that it is being neglected—may be attributable to the fact that we are prone to give all the credit for the development of modern wonders to the inventors of such things as the airplane and the telephone, and none whatever to the men who discovered the principles and laws upon which these things are based. In thinking of the airplane, for example, the average man thinks of the Wright brothers and never gives a thought to Langley and the other pioneers in the principles of aerial flight. The same is true of the radio; many inventors have appeared in this field since DeForrest introduced his telephone, but all of the fundamental work was done by Clerk Maxwell and other investigators and researchers, who, perhaps, were little concerned with the possibility of talking great distances through the air.

Inventors, as a rule, are the men who take the principles and laws discovered by scientists and apply them in the production of some new instrument or machine that is useful to man. Their work is important, but it is not fundamental. The problem of applying principles often calls for great genius, but the work of discovering principles is more important, because it is more fundamental, because it makes the work of the inventor possible.

Man has always been more or less adept at applying his knowledge to useful ends. Progress, in the modern sense of the word, did not begin in the world until man began his search to discover the principles and laws that control the material world. Since he began to uncover these laws and principles, the advance of invention has been carried on at a tremendous rate—one after another, man has developed steam, electricity, the internal combustion engine, and the water turbine. These engines are the fundamentals of power and the source of most of the energy that we use in the operation of other inventions. The telephone and telegraph, the airplane, the locomotive and the automobile, all are possible because we have easily controlled sources of power with which to operate them. But even with full control of power, there could have been no progress but for the discoveries of the principles and laws that controlled such apparently diverse things as lubrication and the ductility of metals.

Nearly every mechanical development that we have devised is based in one way or another upon the laws and principles discovered by Galileo, Newton and other pioneers in the field of pure science. The discovery of natural laws and principles must precede all invention. As wonderful as the progress based on the early discoveries of science has been, there is reason to believe that our knowledge of fundamental laws and principles is as nothing compared with those that remain to be discovered and that when discovered will be applied in the development of new inventions more remarkable than any we have yet known.

Unless we search for these undiscovered laws and principles, we risk the danger of moving around a circle, just as the world had moved for thousands of years before science showed it the way to break the circle.

—*From Monthly Letter No. 106 of The American Exchange-Pacific National Bank, New York City.*

Heaviside's Operational Calculus as Applied to Engineering and Physics

Part XXVI: **Difficulties of Interpreting Power Series Developments**
Part XXVII: **Linear Flow of Heat**
Part XXVIII: **List of Operators and Formulas**

By DR. ERNST JULIUS BERG
Consulting Engineer, General Electric Company
Professor of Electrical Engineering, Union College

It is not always possible to see the physical meaning of a power series. It may give a far more complete answer to a problem than is suspected by a superficial glance.

As an illustration, consider an alternating e.m.f. $E \sin \omega t$ impressed on an inductive circuit.

The operational solution is then, as set forth in Part VI,

$$i = \frac{p\omega}{(p^2+\omega^2)\,(r+pL)} E\,1$$

If this is expanded by "algebrizing" we get

$$i = \frac{\omega E}{L}\left[\frac{1}{p^2} - \frac{\alpha}{p^3} + \frac{\alpha^2-\omega^2}{p^4} - \frac{\alpha\,(\alpha^2-\omega^2)}{p^5} + \frac{\omega^4-\alpha^2\,\omega^2+\alpha^2}{p^6}\cdots\right]1$$

$$= \frac{E\omega}{L}\left[\frac{t^2}{\underline{|2}} - \frac{t^3\alpha}{\underline{|3}} + \frac{t^4}{\underline{|4}}(\alpha^2-\omega^2) - \frac{t^5}{\underline{|5}}\alpha\,(\alpha^2-\omega^2) + \frac{t^6}{\underline{|6}}(\omega^4-\alpha^2\omega^2+\alpha^2)\cdots\right] \qquad (125)$$

It is not readily seen nor generally known that this series gives

$$i = \frac{E}{Z}\left[\sin\,(\omega t-\beta) + \epsilon^{-\alpha t}\sin\beta\right] \qquad (126)$$

which is the proper result not only for the permanent term but for the transient.

In this case $\alpha = \frac{r}{L}$ and $\tan\beta = \frac{\omega L}{r} = \frac{\omega}{\alpha}$

To prove it, expand Equation (126) in a power series.

PART XXVII: LINEAR FLOW OF HEAT

The differential equations pertaining to heat problems in one dimension are identical to those of the "ideal" cable if the heat losses due to convection and radiation are neglected.

Let Fig. 52 represent a section of a conducting wall one part of which is for instance metal, the other part cork; and let us assume that a definite amount of heat is flowing to each unit area per unit time. Find the temperature at any point.

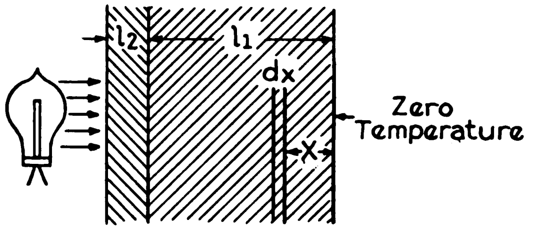


Fig. 52

Let V be the temperature above zero at any point in the material.

Let i be the amount of heat given to each unit area per unit time.

Let R be the heat resistance of the material; it is the temperature rise per unit length when unit heat flows through the material and steady condition has been reached. Thus Ri is the temperature drop per unit length.

Let C be the heat capacity per unit volume; it is the number of units of heat which are required to raise the temperature one degree. The temperature increases as x increases. The rate of increase is $\frac{\partial V}{\partial x}$, thus the difference in temperature over a distance dx is $\frac{\partial V}{\partial x}dx$ and this difference as has been stated is equal to $iRdx$.

$$\therefore i = \frac{1}{R}\frac{\partial V}{\partial x} \qquad (127)$$

There is a difference in flow of heat on each side of the element dx due to the heat absorbed.

This difference is $\frac{\partial i}{\partial x}\partial x$

But heat capacity is by definition

$$C = \frac{\text{total heat absorbed}}{\text{temperature rise}}$$

$$= \frac{\text{heat absorbed per unit time} \times \text{time}}{\text{temperature difference}}$$

Thus $$Cdx = \frac{\frac{\partial i}{\partial x}dx\,dt}{dV} \text{ or } \frac{\partial i}{\partial x} = C\frac{dV}{dt} \qquad (128)$$

From Equation (127) $\frac{\partial i}{\partial x} = \frac{1}{R}\frac{\partial^2 V}{\partial x^2}$

$$\therefore \frac{\partial^2 V}{\partial x^2} = CR\frac{dV}{dt} \tag{129}$$

$$\therefore \frac{d^2 V}{dx^2} = p\,CRV = n^2 V \tag{130}$$

$\therefore$ Thus from Part XV

$$V = K_1 \cosh nx + K_2 \sinh nx$$

$$i = \frac{Y}{n}\left[K_1 \sinh nx + K_2 \cosh n\, nx\right]$$

where $Y = pC$ and $n^2 = pCR$

If for $x=0$ (outside wall) $V=0$

then K_1 must be zero

$\therefore V = K_2 \sinh nx$

and $i = \frac{Y}{n} K_2 \cosh nx$

From an electrical point of view we have really two ideal cables of different constants n_1 and n_2. These cables are connected in series and the far end is short circuited (since for $x=0$, $V=0$). At the generator a fixed current $I\,\mathbf{1}$ is supplied and the problem is to find the voltage at any point in the system.

Referring to Part XIX, we substitute for the right-hand cable an impedance z_2 placed at the end of the first cable.

This impedance is:

$$z_2 = \frac{V}{i} = \frac{n_1 \sinh n_1 l_1}{Y_1 \cosh n_1 l_1} \tag{131}$$

We have then a single cable of length l_2 with an impedance z_2 at the end of the line.

The general equations for this line are:

$$V = K_1 \cosh n_2 x + K_2 \sinh n_2 x$$

$$i = \frac{Y}{n_2}\left[K_1 \sinh n_2 x + K_2 \cosh n_2 x\right]$$

In this case x is measured from the right-hand edge of the heavier shaded material towards the left.

For $x=0$ $\quad V = i_2 z_2 \quad \therefore i_2 z_2 = (K_1 + 0) = K_1$

therefore $\quad V = i_2 z_2\left[\cosh n_2 x + K_2 \cosh n_2 x\right]$

$$i = \frac{Y_2}{n_2}\left[i_2 z_2 \sinh n_2 x + K_2 \cosh n_2 x\right]$$

For $x=0$ $\quad i = i_2$

Thus $\quad i_2 = \frac{Y_2}{n_2} K_2$ or $K_2 = \frac{n_2 i_2}{Y_2}$

For $x = l_2$ $\quad i = I\,\mathbf{1}$

$$\therefore I\,\mathbf{1} = \frac{Y_2}{n_2}\left[i_2 z_2 \sinh n_2 l_2 + \frac{n_2 i_2}{Y_2} \cosh n_2 l_2\right]$$

$$\therefore i_2 = \frac{I n_2}{Y_2\left[z_2 \sinh n_2 l_2 + \frac{n_2}{Y_2}\cosh n_2 l_2\right]}\mathbf{1}$$

Thus the voltage at the junction between the two materials is

$$i_2 z_2 = \frac{I z_2 n_2}{Y_2\left[z_2 \sinh n_2 l_2 + \frac{n_2}{Y_2}\cosh n_2 l_2\right]}\mathbf{1}$$

and the voltage at any place x from the end of the short-circuited cable is from Part XV

$$V = \frac{I z_2 n_2}{Y_2\left[z_2 \sinh n_2 l_2 + \frac{n_2}{Y_2}\cosh n_2 l_2\right]} \times \frac{\sinh n_1 x}{\sinh n_1 l_1}\mathbf{1}$$

After substituting the values for z_2, Z_1 and Z_2 we get

$$V = \frac{I R_1 R_2 \sin m_1 x}{R_2 m_1 \cos m_1 l_1 \cos m_2 l_2 - R_1 m_2 \sin m_1 l_1 \sin m_2 l_2}\mathbf{1}$$

$$Z_{(p)} = 0 \text{ gives } \tan m_1 l_1 \tan m_2 l_2 = \sqrt{\frac{C_1 R_2}{C_2 R_1}}$$

The solution is very cumbersome and will not be attempted.

To introduce fractional differentiation another and simpler problem will be considered. Assume the wall in Fig. 52 has no metal lining, so that we may deal with the conduction of heat by one substance only.

From the general equations for a short-circuited "ideal" cable of finite length we have

$$e = e_g \frac{\sinh nx}{\sinh nl}$$

and $$i = e_g \frac{V}{n}\frac{\cosh nx}{\sinh nl}$$

Let again the flow of heat applied at $x = l$ be $I\,\mathbf{1}$

Then $e_g = \frac{n}{Y}\frac{\sinh nl}{\cosh nl} I\,\mathbf{1}$ or since $\frac{Z}{n} = \frac{n}{Y}$

$$e = \frac{R \sin ml \sin mx}{m \cos ml \sin ml} I\,\mathbf{1} = \frac{R \sin mx}{m \cos ml} I\,\mathbf{1} \tag{132}$$

The first question is whether the Expansion Theorem can be used. Is $p=0$ a root? If it were, then as p approaches zero Equation (132) should approach infinity. That is not the case. As p approaches zero, Equation (132) becomes IRx. Therefore the Expansion Theorem can be used.

$$Z_{(p)} = 0 \text{ gives } \cos ml = 0 \text{ or } ml = \frac{2S-1}{2}\pi$$

$\frac{dZ}{dp}$ becomes, after some transformations, $\frac{m^2 l}{2}(-1)^s$

Since $\cos ml = 0$ and $\sin ml = (-1)^s$

$$\frac{Y_{(0)}}{Z_{(0)}} = IRx$$

$$\therefore e = IR\left[x + \sum_{s=1}^{s=\infty} (-1)^s \frac{8}{\pi^2} l \frac{\sin mx}{(2s-1)^2}\epsilon^{-\frac{m^2}{CR}t}\right] \tag{133}$$

At the source of heat $x = l$

and $$e = IRl\left[1 - \frac{8}{\pi^2}\sum_{s=1}^{s=\infty}\frac{\epsilon^{-\frac{m^2}{CR}t}}{(2s-1)^2}\right]$$

where $$m = \frac{2s-1}{2}\,\frac{\pi}{l}$$

The same problem will now be considered from the point of view of an infinitely thick wall or an infinitely long "ideal" cable.

Referring to Part XXIV and measuring y from the heated side:

$$e = A\epsilon^{-ny} \qquad i = \frac{Y}{n}A\epsilon^{-ny}$$

For $y = 0$ $\quad i = I\mathbf{1}$ $\quad \therefore A = \frac{n}{Y}I\mathbf{1}$

and $$e = \frac{n}{Y}\epsilon^{-ny}I\mathbf{1} = \frac{R}{n}\epsilon^{-ny}I\mathbf{1}$$

$$= \frac{RI}{n}\left(1 - ny + \frac{n^2y^2}{\underline{|2}} - \cdots\cdots\right)\mathbf{1}$$

Since $n = \sqrt{pCR}$ we get

$$e = RI\left[\frac{p^{-\frac{1}{2}}}{\sqrt{CR}} - y + \frac{\sqrt{CR}\,y^2}{\underline{|2}}p^{\frac{1}{2}} - \frac{CR\,y^3}{\underline{|3}}p + \cdots\cdots\right]\mathbf{1}$$

$$\text{or } e = I\sqrt{\frac{R}{\pi C}}\left[2t^{\frac{1}{2}} - y\sqrt{CR} + \frac{CR\,y^2}{\underline{|2}}t^{-\frac{1}{2}} - \frac{C^2R^2y^4}{2\,\underline{|4}}t^{-\frac{3}{2}}\cdots\cdots\right]$$

At the heated surface ($y = 0$)

$$e_0 = I\,\frac{\sqrt{4\,Rt}}{\pi C}$$

Using the c g s. system and cork

$$R = \backsim 10{,}000 \text{ and } C = \backsim 0.07$$

1 watt per sq. in. corresponds to $\frac{0.256}{1000}$ gram cal. per sec. per cm.^{2}

PART XXVIII: LIST OF OPERATORS AND FORMULAS

Equivalent Operators

(1). $\frac{p}{p+\alpha}\mathbf{1} = \epsilon^{-\alpha t}$

(2). $\frac{p}{p-\alpha}\mathbf{1} = \epsilon^{\alpha t}$

(3). $\frac{1}{p+\alpha}\mathbf{1} = \frac{1}{\alpha}(1 - \epsilon^{-\alpha t})$

(4). $\frac{1}{p(p+\alpha)}\mathbf{1} = \frac{t}{\alpha} - \frac{1}{\alpha^2} + \frac{\epsilon^{-\alpha t}}{\alpha^2}$

(5). $\frac{p^2}{(p+\alpha)(p+\beta)}\mathbf{1} = \frac{1}{\alpha-\beta}\left[\alpha\epsilon^{-\alpha t} - \beta\epsilon^{-\beta t}\right]$

(6). $\frac{p}{(p+\alpha)(p+\beta)}\mathbf{1} = \frac{1}{\alpha-\beta}\left[\epsilon^{-\beta t} - \epsilon^{-\alpha t}\right]$

If $\omega_0^2 > \alpha^2$, $\omega^2 = \omega_0^2 - \alpha^2$ and $\tan\phi = \frac{\omega}{\alpha}$,

Then

(7). $\frac{p^2}{p^2+2\alpha p+\omega_0^2}\mathbf{1} = -\frac{\omega_0}{\omega}\epsilon^{-\alpha t}\sin(\omega t - \phi)$

(8). $\frac{p}{p^2+2\alpha p+\omega_0^2}\mathbf{1} = \frac{\epsilon^{-\alpha t}}{\omega}\sin\omega t$

(9). $\frac{1}{p^2+2\alpha p+\omega_0^2}\mathbf{1} = \frac{1}{\omega_0^2}\left[1 - \frac{\omega_0}{\omega}\epsilon^{-\alpha t}\sin(\omega t + \phi)\right]$

If $\alpha^2 > \omega_0^2$ and if $(-m)$ and $(-n)$ are the two roots of the equation $p^2 + 2\alpha p + \omega_0^2 = 0$, then

(10). $\frac{p^2}{p^2+2\alpha p+w_0^2}\mathbf{1} = \frac{1}{n-m}\left[n\epsilon^{-nt} - m\epsilon^{-mt}\right]$

(11). $\frac{p}{p^2+2\alpha p+w_0^2}\mathbf{1} = \frac{1}{n-m}\left[\epsilon^{-mt} - \epsilon^{-nt}\right]$

(12). $\frac{1}{p^2+2\alpha p+\omega_0^2}\mathbf{1} = \frac{1}{\omega_0^2} - \frac{1}{n-m}\left[\frac{\epsilon^{-mt}}{m} - \frac{\epsilon^{-nt}}{n}\right]$

If $\alpha^2 = \omega_0^2$, then, (two equal roots.)

(13). $\frac{p^2}{p^2+2\alpha p+\omega_0^2}\mathbf{1} = \epsilon^{-\alpha t}(1 - \alpha t) = \frac{p^2}{(p+\alpha)^2}\mathbf{1}$

(14). $\frac{p}{p^2+2\alpha p+\omega_0^2}\mathbf{1} = t\epsilon^{-\alpha t} = \frac{p}{(p+\alpha)^2}\mathbf{1}$

(15). $\frac{1}{p^2+2\alpha p+\omega_0^2}\mathbf{1} = \frac{1}{\omega_0^2}\left[1 - \epsilon^{-\alpha t}(1 + \alpha t)\right] = \frac{1}{(p+\alpha)^2}\mathbf{1}$

(16). $\frac{p}{(p-\alpha)^n}\mathbf{1} = \epsilon^{\alpha t}\frac{t^{n-1}}{\underline{|n-1}}$

(17). $\frac{1}{p^n}\mathbf{1} = \frac{t^n}{\underline{|n}}$

(18). $\sin wt = \frac{\omega p}{p^2+\omega^2}\mathbf{1}$

(19). $\cos\omega t = \frac{p^2}{p^2+\omega^2}\mathbf{1}$

(20). $\sinh\omega t = \frac{\omega p}{p^2-\omega^2}\mathbf{1}$

(21). $\cosh\omega t = \frac{p^2}{p^2-\omega^2}\mathbf{1}$

(22). $\epsilon^{-\beta t}\sin\omega t = \frac{p\omega}{(p+\beta)^2+\omega^2}\mathbf{1}$

(23). $\epsilon^{-\beta t}\cos\omega t = \frac{p(p+\beta)}{(p+\beta)^2+\omega^2}\mathbf{1}$

(24). $\sin(\omega t \pm \phi) = \frac{p\omega\cos\phi \pm p^2\sin\phi}{p^2+\omega^2}\mathbf{1}$

(25). $$\cos(\omega t \pm \phi) = \frac{p^2 \cos\phi \mp \omega\, p \sin\phi}{p^2+\omega^2}\, 1$$

(26). $$\epsilon^{-\beta t}\sin(\omega t \pm \phi) = \frac{p\omega \cos\phi \pm p\,(p+\beta)\sin\phi}{(p+\beta)^2+\omega^2}\, 1$$

(27). $$\epsilon^{-\beta t}\cos(\omega t \pm \phi) = \frac{p\,(p+\beta)\cos\phi \mp \omega p \sin\phi}{(p+\beta)^2+\omega^2}\, 1$$

(28). $$\left(\frac{p}{p+2\alpha}\right)^{\frac{1}{2}} 1 = \epsilon^{-\alpha t}\frac{p}{(p^2-\alpha^2)^{\frac{1}{2}}}\, 1 = \epsilon^{-\alpha t} J_0(i\alpha t)$$

(29). $$\frac{p}{(p^2+\alpha^2)^{\frac{1}{2}}}\, 1 = J_0(\alpha t)$$

(30). $$\frac{p}{(p^2+\omega^2)\,(p+\alpha)} = \frac{t^2}{\lfloor\!\underline{2}} - \frac{t^3}{\lfloor\!\underline{3}} + \frac{t^4}{\lfloor\!\underline{4}}(\alpha^2-\omega^2)$$
$$-\frac{t^5}{\lfloor\!\underline{5}}\alpha(\alpha^2-\omega^2) + \frac{t^6}{\lfloor\!\underline{6}}(\omega^4-\alpha^2\omega^2+\alpha^2)$$
$$= \frac{1}{\omega\sqrt{\alpha^2+\omega^2}}\left[\sin(\omega t-\beta) + \epsilon^{-\alpha t}\sin\beta\right]$$
$$\sin\beta = \sin\left(\tan^{-1}\frac{\omega}{\alpha}\right) = \frac{\omega}{\sqrt{\alpha^2+\omega^2}}$$

Relations Involving Hyperbolic and Allied Functions

(31). $$\epsilon^{\pm x} = 1 \pm \frac{x^2}{\lfloor\!\underline{2}} \pm \frac{x^3}{\lfloor\!\underline{3}} \pm \cdots\cdots\cdots = \cosh x \pm \sinh x$$

(32). $$\epsilon^{\pm jx} = 1 \pm jx - \frac{x^2}{\lfloor\!\underline{2}} \mp \frac{jx^3}{\lfloor\!\underline{3}} + \frac{x^4}{\lfloor\!\underline{4}}\cdots = \cos x \pm j\sin x$$

(33). $$\sinh x = x + \frac{x^3}{\lfloor\!\underline{3}} + \frac{x^5}{\lfloor\!\underline{5}} + \cdots\cdots = \frac{\epsilon^{x}-\epsilon^{-x}}{2}$$

(34). $$\cosh x = 1 + \frac{x^2}{\lfloor\!\underline{2}} + \frac{x^4}{\lfloor\!\underline{4}} + \cdots\cdots = \frac{\epsilon^{jx}+\epsilon^{-jx}}{2}$$

(35). $$\sin x = x - \frac{x^3}{\lfloor\!\underline{3}} + \frac{x^5}{\lfloor\!\underline{5}} - \cdots\cdots\cdots = \frac{\epsilon^{jx}-\epsilon^{-jx}}{2j}$$

(36). $$\cos x = 1 - \frac{x^2}{\lfloor\!\underline{2}} + \frac{x^4}{\lfloor\!\underline{4}}\cdots\cdots\cdots = \frac{\epsilon^{jx}+\epsilon^{-jx}}{2}$$

(37). $\sinh jx = j\sin x$

(38). $\cosh jx = \cos x$

(39). $\sin jx = j\sinh x$

(40). $\cos jx = \cosh x$

(41). $\sinh(-x) = -\sinh x$

(42). $\cosh(-x) = \cosh x$

(43). $\cosh^2 x - \sinh^2 x = 1$

(44). $$\frac{d}{dx}\sinh ax = a\cosh x$$

(45). $$\frac{d}{dx}\cosh ax = \alpha\sinh x$$

(46). $$\int \sinh ax dx = \frac{1}{a}\cosh x$$

(47). $$\int \cosh ax dx = \frac{1}{a}\sinh x$$

(48). $$\sinh x = x\left(1+\frac{x^2}{\pi^2}\right)\left(1+\frac{x^2}{\pi^2 2^2}\right)\left(1+\frac{x^2}{\pi^2 3^2}\right)$$
$$= x\prod_{1}^{\infty}\left(1+\frac{x^2}{\pi^2 s^2}\right)$$

(49). $$\cosh x = \left(1+\frac{4x^2}{\pi^2}\right)\left(1+\frac{4x^2}{\pi^2 3^2}\right)\left(1+\frac{4x^2}{\pi^2 5^2}\right)$$
$$= \prod_{1}^{\infty}\left[1+\frac{4x^2}{\pi^2(2s-1)^2}\right]$$

(50). $$\sin x = x\left[1-\left(\frac{x}{\pi}\right)^2\right]\left[1-\left(\frac{x}{2\pi}\right)^2\right]\left[1-\left(\frac{x}{3\pi}\right)^2\right]$$
$$= x\prod_{1}^{\infty}\left[1-\left(\frac{x}{s\pi}\right)^2\right]$$

(51). $$\cos x = \left[1-\left(\frac{2x}{\pi}\right)^2\right]\left[1-\left(\frac{2x}{3\pi}\right)^2\right]\left[1-\left(\frac{2x}{5\pi}\right)^2\right]$$
$$= \prod_{1}^{\infty}\left\{1-\left[\frac{2x}{\pi(2s-1)}\right]^2\right\}$$

(52). $$\sinh(x \pm jy) = \sinh x\cos y \pm j\cosh x\sin y$$
$$= \sqrt{\sinh^2 x + \sin^2 y}\;\underline{|\phi_1}$$

(53). $$\cosh(x \pm jy) = \cosh x\cos y \pm j\sinh x\sin y$$
$$= \sqrt{\cosh^2 x - \sin^2 y}\;\underline{|\phi_2}$$

(54). $$\sin(x \pm jy) = \sin x\cosh y \pm j\cos x\sinh y$$
$$= \sqrt{\sin^2 x + \sinh^2 y}\;\underline{|\phi_3}$$

(55). $$\cos(x \pm jy) = \cos x\cosh y \mp j\sin x\sinh y$$
$$= \sqrt{\cos^2 x + \sinh^2 y}\;\underline{|\phi_4}$$

Where $\tan\phi_1 = \pm\coth x\tan y$

$\tan\phi_2 = \pm\tanh x\tan y$

$\tan\phi_3 = \pm\tanh y\cot x$

$\tan\phi_4 = \pm\tanh y\tan x$

Factorials and Miscellaneous Formulas

$x =$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{5}{2}$
$\lfloor\!\underline{x} =$	$\sqrt{\pi}\frac{1}{2}$	$\sqrt{\pi}\frac{1}{2}\frac{3}{2}$	$\sqrt{\pi}\frac{1}{2}\frac{3}{2}\frac{5}{2}$	$+\sqrt{\pi}$	$+2\sqrt{\pi}$	$+\frac{2}{1}\frac{2}{3}\sqrt{\pi}$

(56). $$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{\lfloor\!\underline{2}}x^{n-2}y^2$$
$$+\frac{n(n-1)(n-2)}{\lfloor\!\underline{3}}x^{n-3}y^3 + \cdots\cdots$$

$$\left.\begin{array}{ll}
(57). & \sqrt{1\pm x}=1\pm\frac{1}{2}x-\frac{1}{2}\frac{1}{4}x^2\pm\frac{1}{2}\frac{1}{4}\frac{3}{6}x^3-\cdots \\[2ex]
(58). & \frac{1}{\sqrt{1\pm x}}=1\mp\frac{1}{2}x+\frac{1}{2}\frac{3}{4}x^2\mp\frac{1}{2}\frac{3}{4}\frac{5}{6}x^3+\cdots \\[2ex]
(59). & \frac{1}{1\pm x}=1\mp x+x^2\mp x^3+\cdots\cdots\cdots\cdots \\[2ex]
(60). & \frac{1}{1\pm x^2}=1\mp 2x+3x^2\mp 4x^3+\cdots\cdots\cdots\cdots
\end{array}\right\} x^2<1$$

$$\left.\begin{array}{ll}
(61). & \frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}\cdots\cdots\cdots\cdots \\[2ex]
(62). & \frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\cdots\cdots \\[2ex]
(63). & \frac{\pi^2}{6}=1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\cdots\cdots \\[2ex]
(64). & \frac{\pi^2}{12}=1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots\cdots
\end{array}\right\} \text{By Fourier's Series}$$

Expansion Theorem

$$\frac{Y_{(p)}}{Z_{(p)}}1=\frac{Y_{(o)}}{Z_{(o)}}+\sum_{p_1,\,p_2\ldots}\frac{Y_{(p)}\,\epsilon^{pt}}{p\dfrac{dZ}{dp}}$$

$$\frac{1}{H_{(p)}}=\frac{1}{H_{(o)}}+\sum\frac{\epsilon^{pt}}{p\,H'_{(p)}}$$

"Shifting" Theorem

$$f(p)\,(u\,\epsilon^{\alpha t})=\epsilon^{\alpha t}\left[f\,(p+\alpha)\,u\right]$$

(Where *u* is a function of *t*.)

Duhamel's Theorem

$$f(t)=E_{(o)}\,\phi\,(t)+\int_o^t\phi\,(t-T)\,\frac{\partial\,E(T)}{\partial\,(T)}\,dT$$

$$=E_{(t)}\,\phi\,(o)+\int_o^t E_{(T)}\,\frac{\partial\,\phi\,(t-T)}{\partial\,t}\,dT$$

$$=\frac{d}{dt}\int_o^t E_{(T)}\,\phi\,(t-T)\,dT$$

$$=\frac{d}{dt}\int_o^t E\,(t-T)\,\phi\,(T)\,dT$$

$$=\int_o^t\phi\,(T)\,\frac{\partial\,E\,(t-T)}{\partial\,t}\,dT+E_{(o)}\,\phi\,(t)$$

$$=E_{(t)}\,\phi_{(o)}+\int_o^t E\,(t-T)\,\frac{\partial\,\phi\,(T)}{\partial\,(T)}\,d\,(T)$$

$$\epsilon^{-\alpha t}\int_o^t\epsilon^{\alpha t}\sin\,(\omega T+\delta)\,dT$$

$$=\frac{\sin\,(\omega t+\delta-\phi)-\epsilon^{-\alpha t}\sin\,(\delta-\phi)}{\sqrt{\alpha^2+\omega^2}}$$

$$=\frac{-\cos\,(\omega t+\delta+\psi)+\epsilon^{-\alpha t}\cos\,(\delta+\psi)}{\sqrt{\alpha^2+\omega^2}}$$

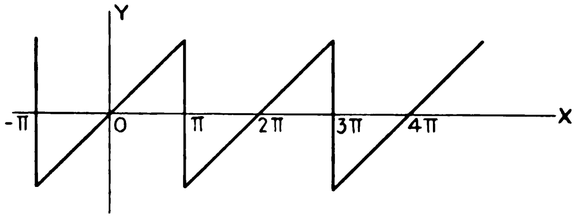
$$\epsilon^{-\alpha t}\int_o^t\epsilon^{\alpha T}\cos\,(\omega T+\delta)dT$$

$$=\frac{\cos\,(\omega t+\delta-\phi)-\epsilon^{-\alpha t}\cos\,(\delta-\phi)}{\sqrt{\alpha^2+\omega^2}}$$

$$=\frac{\sin\,(\omega t+\delta+\psi)-\epsilon^{\alpha t}\sin\,(\delta+\psi)}{\sqrt{\alpha^2+\omega^2}}$$

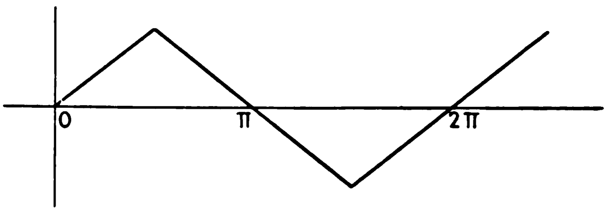
$$\left(\tan\phi=\frac{\omega}{\alpha}=\cot\psi\right)$$

Wave Shapes Expressed by Means of Fourier's Series



$$y=2\left(\sin x-\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x-\frac{1}{4}\sin 4x+\cdots\right)$$

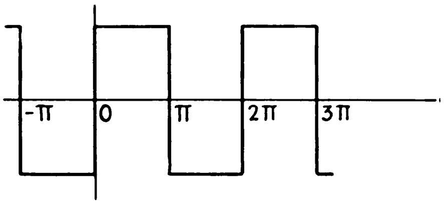
$y=x$ for $\pi>x>-\pi$



$$y=\sin x-\frac{1}{3^2}\sin 3x+\frac{1}{5^2}\sin 5x\cdots\cdots$$

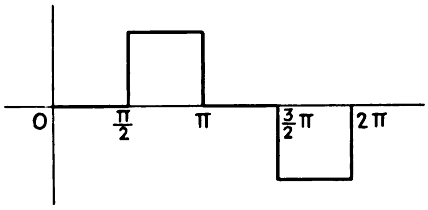
$$y=\frac{1}{4}\pi x,\ 0<x<\frac{\pi}{2}$$

$$y=\frac{1}{4}\pi\,(\pi-x),\ \frac{\pi}{2}<x<\pi$$



$$y = 4\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\cdots\right)$$

$$y = \pi,\ 0 < x < \pi$$



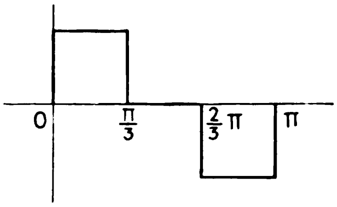
$$y = \sin x - \sin 2x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x$$

$$-\frac{1}{3}\sin 6x + \frac{1}{7}\sin 7x \cdots\cdots$$

$$= \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\cdots$$

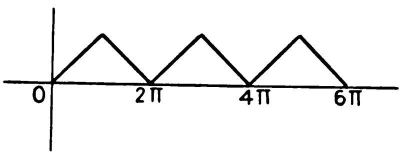
$$= \left(\sin 2x + \frac{1}{3}\sin 6x + \frac{1}{5}\sin 10x + \cdots\cdots\right)$$

$$y = 0,\ 0 < x < \frac{\pi}{2} \qquad y = \frac{\pi}{2},\ \frac{\pi}{2} < x < \pi$$



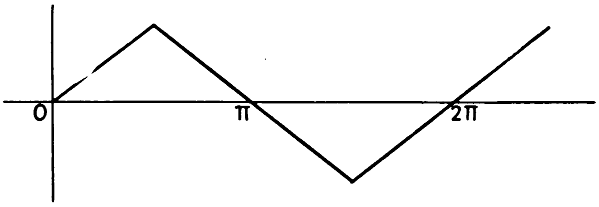
$$y = \sin 2x + \frac{1}{3}\sin 6x + \frac{1}{5}\sin 10x + \cdots\cdots$$

$$y = \frac{\pi}{3},\ 0 < x < \frac{\pi}{3};\ y = 0,\ \frac{\pi}{3} < x < \frac{2}{3}\pi;\ y = -\frac{\pi}{3},\ \frac{2}{3}\pi < x < \pi$$



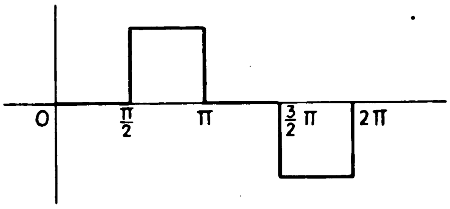
$$y = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \cdots\cdots\right)$$

$$y = x,\ 0 < x < \pi;\ y = -x,\ -\pi < x < 0$$



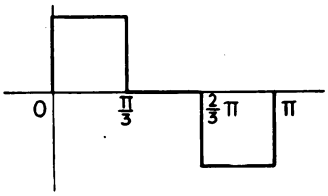
$$y = \frac{\pi^2}{16} - 2\left[\frac{1}{2^2}\cos 2x + \frac{1}{6^2}\cos 6x + \frac{1}{10^2}\cos 10x + \cdots\right]$$

$$y = \frac{1}{4}\pi x,\ 0 < x < \frac{\pi}{2};\ y = \frac{1}{4}\pi(\pi - x),\ \frac{\pi}{2} < x < \pi$$



$$y = \frac{\pi}{4} - \left[\cos x - \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x \cdots\cdots\cdots\right]$$

$$y = 0,\ 0 < x < \frac{\pi}{2};\ y = \frac{\pi}{2},\ \frac{\pi}{2} < x < \pi$$



$$y = \frac{2}{3}\sqrt{3}\left[\cos x - \frac{1}{5}\cos 5x + \frac{1}{7}\cos 7x - \frac{1}{11}\cos 11x + \cdots\right]$$

$$y = \frac{\pi}{3},\ 0 < x < \frac{\pi}{3};\ y = 0,\ \frac{\pi}{3} < x < \frac{2}{3}\pi;\ y = -\frac{\pi}{3},\ \frac{2}{3}\pi < x < \pi$$

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.00	1.0	0.25	0.78	0.80	0.449	1.8	0.165
0.02	0.98	0.30	0.741	0.85	0.427	2.0	0.135
0.04	0.96	0.35	0.705	0.90	0.407	2.5	0.084
0.06	0.942	0.40	0.67	0.95	0.387	3.0	0.05
0.08	0.923	0.45	0.638	1.0	0.368	4.0	0.018
0.10	0.905	0.50	0.607	1.1	0.333	5.0	0.0067
0.12	0.887	0.55	0.577	1.2	0.301	6.0	0.0025
0.14	0.870	0.60	0.549	1.3	0.273	7.0	0.0009
0.16	0.852	0.65	0.522	1.4	0.247	8.0	0.00034
0.18	0.835	0.70	0.497	1.5	0.202	9.0	0.00012
0.20	0.819	0.75	0.472	1.8	0.165	10.0	0.00004

A very complete bibliography can be found in "Electric Circuit Theory and Operational Calculus," by John R. Carson (McGraw-Hill & Co.).

(*Concluded*)

Arc-weld Rail Bonds

ARC-WELD rail bonds are becoming more generally used because the metallic arc-welding process enables this type of bond to be applied to any part of the rail without special equipment and the contact thus obtained between bond and rail is not subject to electrolytic action or corrosion; moreover, the union has both great mechanical strength and low electrical resistance. The welded contacts also discourage theft of the bond.

In some types of arc-weld bonds, steel terminals are electrically welded to the copper conductor during the process of manufacture. This forms a union between the two metals which is not affected by the heat of the arc during the later process of welding the bond to the rail. The joining of the copper conductor to the steel shell at the factory leaves but the familiar steel-to-steel weld to be made in the field.

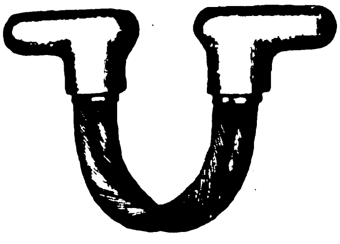


Fig. 1. Arc-welded Bond for Attaching to the Rail Head

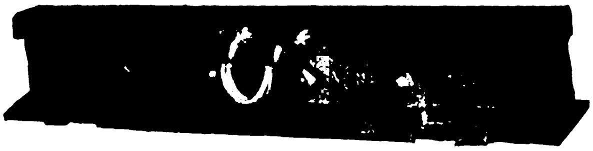


Fig. 2. The Same Type of Bond as Shown in Fig. 1 After Being Welded to a Rail Joint

In the arc-weld bonds shown in Figs. 1 to 6 the construction is fundamentally the same; they differ only in the shape of their terminals, which are formed for application to various parts of the rail. The terminals are of liberal design and provide a contact area corresponding to approximately six times the normal current capacity of the bond.

A recent improvement in the arc-weld bond is the addition of a copper sleeve that is placed around the copper conductor at the point where the conductor emerges from the steel terminal. This sleeve, by deadening vibration, prolongs the life of the conductor strands.

The metallic-electrode welding process enables the bonds to be applied with any form of arc-welding apparatus—motor-generator, dynamotor, or any form of ballast-resistor set—and the electrode may be either positive or negative. To obtain best results, the spot on the rail to which the bond terminal is to be attached should be thoroughly cleaned of any dirt, rust, or annealing scale by means of a file or steel-wire brush. After preparing the rail, the bond terminal should be clamped or otherwise held in position long enough to "tack" it fast with the arc. The usual practice of welding can then be followed. In applying arc-weld bonds, it has been found advisable to employ a short arc and a current of about 130 amp. at a welding voltage of 20 volts, this being the average voltage required for successful welding with the metallic electrode. Steel electrodes of either $\frac{3}{16}$- or $\frac{5}{32}$-in. diameter are used for the work. The approximate amount of welding wire required for installing arc-weld bonds may be estimated from Table I.

The bond shown in Figs. 1 and 2 may be used in either the single- or double-conductor form. Both are equipped with L-shaped terminals and are designed for application to the head of the rail. They are suitable for either single or double bonding. The standard overall length of bond up to 4/0 capacity is 7½ in.

The bond shown in Fig. 3 is especially designed for installation around the splice bar close to the web of the rail with the terminals welded to the rail base. The overall length of the bonds of 1/0 and 2/0 capacity is 8 in. greater than the splice bars, and 9 in. greater for those of 3/0 and 4/0 capacity. The design of the terminal employed makes this bond suitable for application to the head of the rail when Weber joints are used. The overall length of the 3/0 and 4/0 bonds in this case is 10½ in.

The bond in Fig. 4 is the same as that in Fig. 3 except that a sharpened hook is added as part of the terminal. This hook is thin and sharp at the end, and can be driven under the rail base on the top of a tie in case there is one there. The hook keeps the terminal near the edge of the rail base and is of great advantage in holding the bond in position while welding. The overall length of the bonds of 1/0 and 2/0 capacity is 7 in. greater than the splice bars, and 8 in. greater for those of 3/0 and 4/0 capacity.

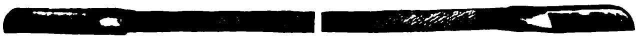
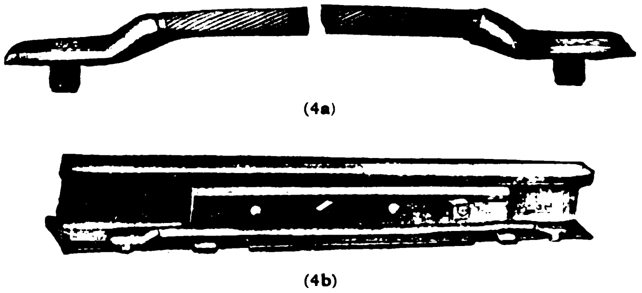


Fig. 3. Bond for Installation Around the Splice Bar with Its Steel Terminals Arc Welded to the Rail Base



(4a)

(4b)

Fig. 4. Bond Similar in General to That in Fig. 3 but with the Addition of Sharpened Hooks

Other forms of bonds, Fig. 5, are designed for application around the splice bar and for welding to the top of the rail base. In one type the terminals are semicircular in section, and by placing the flat side against the rail base the welding can be carried along both sides as well as the end of the terminal. In another type the terminals are flattened and tapered to a thin edge at the ends, thus providing a wide welding surface. The welding is carried from the rail up over the tops of the terminals. This feature is advantageous for rails having a narrow base. The overall length of either type in 1/0 and 2/0 capacity is 7 in. greater than the splice bars, and 8 in. greater for those of 3/0 and 4/0 capacity.

Another form of bond, Fig. 6, has been developed for all cross bonding. The entire bond is located below the rail base so that all parts of the bond are thoroughly concealed and protected from derailed mine cars or dragging equipment. The usual overall length is 6 in. greater than the track gauge.

Improved arc welding methods now make possible the attaching of a steel-terminal bond to a rail without first having the copper conductor welded into the steel terminal during the process of manufacture, as has heretofore been considered essential. The welding in the field is accomplished by a flux-coated copper electrode of either $\frac{3}{16}$- or $\frac{1}{4}$-in. diameter.

TABLE I

Approximate Amount of Welding Wire Required for Installing Arc-weld Bonds

Type of Bond Illustrated	Capacity	Lb. of Wire Per 100 Bonds
Figs. 1, 2, and 6.........	2/0 4/0	10 18
Figs. 3, 4, and 5.........	2/0 4/0	14 22
Fig. 7..................	2/0 4/0	28 28
Fig. 8..................	2/0 4/0	30 40

The application of these bonds requires no installation accessories. The terminal design permits free manipulation of the arc and easy welding. The copper sleeve, already described in connection with the other types of bonds, is also incorporated as a part of this type.

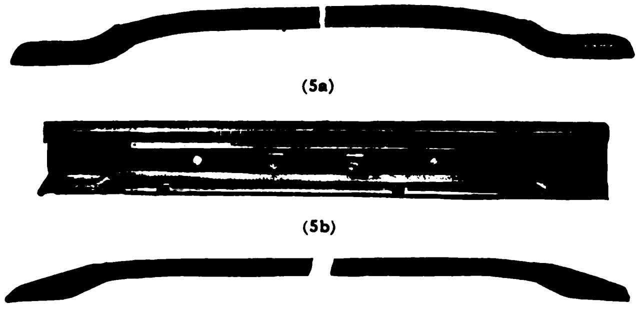

(5a)
(5b)
(5c)

Fig. 5. Semicircular-terminal Bond, Its Location When Installed, and a Tapered-terminal Bond

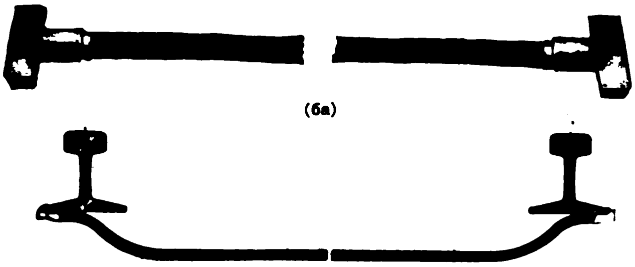

(6a)
(6b)

Fig. 6. Steel-terminal Arc-weld Cross Bond and Its Location When Installed

When installing these bonds, the place on the rail where the bond terminal is to be set should be cleaned of all rust or scale. The terminal should then be clamped or held in position and the arc started at the end and carried along the bottom toward the ends of the copper strands that project through the terminal. The attaching metal should be washed up over the copper strands, care being taken that all strands are welded in one mass. The operator should hold his electrode at such an angle that, during the greater part of the time, the arc is directed toward the rail inasmuch as the rail, being the larger mass of metal, will require more heat to insure proper penetration.

This type of bond can be installed with any standard arc-welding apparatus at a current of 180 amp. A convenient equipment is a resistor outfit of 200 or 300-amp. capacity, using 400/650 volts for railway work or 200/275 volts for mine track.

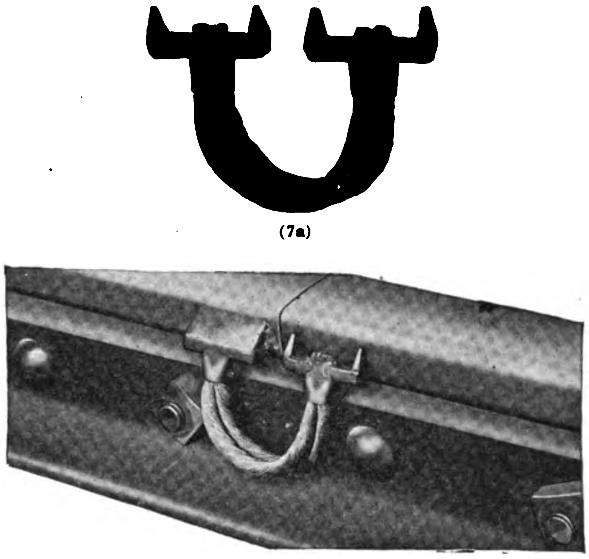

(7a)
(7b)

Fig. 7. Bond with Skeleton Terminals Designed to be Built up to Final Form When Being Arc Welded to the Rail Head

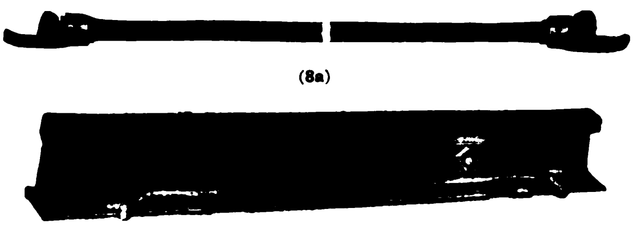

(8a)
(8b)

Fig. 8. Bond Similar to That in Fig. 7 in That the Welding of the Conductor to the Terminal to the Rail is Accomplished in One Operation

Fig. 7 shows a double-conductor bond for application to the rail head. In this position there is ample space for filler metal between the rail and the strands that project through the terminals; also, the bond is tilted to clear the splice bar. The terminals then slope downward from the top of the rail and are thus protected from possible damage. This bond is commonly used in 7- or $7\frac{1}{2}$-in. lengths for 4/0 capacity.

Fig. 8 illustrates a new form of bond for use in the base of the rail around the splice bar. The terminals are provided with hooks to go under the rail and are of great advantage in holding the bond in position while welding. These hooks are thin and pointed at the end, and can be driven under the rail even though there is a tie at the point of application. In 1/0 and 2/0 capacities, the overall length of the bond is 7 in. greater than the splice bar. For the 3/0 and 4/0 capacity bonds, the overall length is 8 in. greater than the splice bar.

LIBRARY SECTION

Condensed references to some of the more important recent articles in the technical press, and to new books of interest to the industry, as selected by the General Electric Main Library.

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A.I.E.E. Jour., June, 1928; v. 47, pp. 401-406.

Coal Handling

Fuel Handling and Ash Disposal; Methods and Equipment Used on the Philadelphia Electric Company System. J. R. McCausland.
Mech. Engng., June, 1928; v. 50, pp. 454-458.

Daylight

Daylight, Artificial Light and Artificial Daylight; Their Merits and Drawbacks. J. S. Dow.
Illum. Engr., June, 1928; v. 21, pp. 167-174.

Electric Arc

Theory of the Electric Arc. Joseph Slepian.
W. Soc. Engrs. Jour., Apr., 1928; v. 33, pp. 173-183.
(Discusses the nature of the electric arc and its behavior in air, in vacuo, and under pressure.)

Electric Distribution

Low-Cost Rural Construction. George W. Kable.
Elec. Wld., May 19, 1928; v. 91, pp. 1015-1017.
(Methods and costs for rural distribution lines.)

Electric Drive—Steel Mills

Selection of Main Drive Motors for Strip and Skelp Mills. A. F. Kenyon.
Iron & St. Engr., June, 1928; v. 5, pp. 248-255.

Selection of Motors for Main Drives of Merchant, Bar and Rod Mills. C. B. Houston.
Iron & St. Engr., June, 1928; v. 5, pp. 218-229.

Electric Heating, Industrial

Industrial Electric Heating, with Details as to a Variety of Applications. Odin Wilhelmy.
Elec. Lt. & Pr., June, 1928; v. 6, pp. 65-66, 116-117.
(Serial.)

Electric Motors

Characteristics That Determine the Selection of Motors. Theo. Schou and C. P. Robinson.
Elec. Mfg., June, 1928; v. 1, pp. 128-131.
("No. 1 of a series discussing motors of different types.")

Electric Transmission

Progress and Problems from Interconnection in Southeastern States. W. E. Mitchell.
A.I.E.E. Jour., June, 1928; v. 47, pp. 415-419.
(Abridgment.)

Elevators, Electric

Fundamentals of Modern Elevator Services. Bassett Jones.
Power, May 29, 1928; v. 67, pp 966-968.

Excess Voltage

Surge-Voltage Investigations on the 140-kv. System of the Consumers Power Company During 1927. J. G. Hemstreet and J. R. Eaton.
A.I.E.E. Jour., June, 1928; v. 47, pp. 441-444.
(Abridgment.)

Hydroelectric Development

Power from Pumped Water. E. J. Amberg.
Elec. Wld., May 12, 1928; v. 91, pp. 959-965.
(Abstract of a paper before the A.I.E.E. describing the Rocky River development of the Connecticut Light & Power Co. Water is pumped to a storage reservoir for later use.)

Lightning

Some Electrophysical Conditions Determining Lightning Surges. Harald Norinder.
Franklin Inst. Jour., June, 1928; v. 205, pp. 747-765.
(A study of the nature of lightning surges in transmission systems.)

Locomotives, Steam Turbine

2000 H.P. Ljungström Turbine Locomotive.
Int. Rwy. Cong. Assoc. Bul., May, 1928; v. 10, pp. 429-441.

Nozzles

Fifth Report of the Steam-Nozzles Research Committee.
I.M.E. Proc., No. 1, 1928; pp. 31-121.
(Presents the results of extensive tests.)

Power Costs

Discussion of the Elements Entering into the Cost of Producing Power. W. B. Skinkle.
Iron & St. Engr., June, 1928; v. 5, pp. 278-290.

Power Plants, Electric

Dividing Load Economically Among Power Plants by Use of the Kilowatt—Killowatt-Hour Curve. A. Wilstam.
A.I.E.E. Jour., June, 1928; v. 47, pp. 430-432.

Radio Engineering—Detection

Detection with the Four-Electrode Tube. J. R. Nelson.
Inst. Radio Engrs. Proc., June, 1928; v. 16, pp. 822-839.

Radio Engineering—Transmission, Directional

Beam Transmission of Ultra Short Waves. Hidetsugu Yagi.
Inst. Radio Engrs. Proc., June, 1928; v. 16, pp. 715-741.

Radio Engineering—Transmission, Short-Wave

Short Wave Radio Communication. C. W. Horn.
W. Soc. Engrs. Jour., Apr., 1928; v. 33, pp. 193-200.

Steam Turbines

Turbine Blading Problems. S. M. Robinson.
Am. Soc. Nav. Engrs. Jour., May, 1928; v. 40, pp. 223-228.

Switches and Switchgear, Automatic

Automatic Switching of Incoming Lines and Transformers Supplying Power to A-C. Substations. A. E. Anderson.
A.I.E.E. Jour., June, 1928; v. 47, pp. 423-426.
(Abridgment.)

Transient Phenomena

Quantitative Mechanical Analysis of Power System Transient Disturbances. R. C. Bergvall and P. H. Robinson.
A.I.E.E. Jour., June, 1928; v. 47, pp. 419-423.
(Motion pictures are taken of mechanical models in order to estimate the transients that may be expected on a 220-kv. line. Abridgment.)

Vacuum Tubes

Four-Element Tube Characteristics as Affecting Efficiency. David C. Prince.
Inst. Radio Engrs. Proc., June, 1928; v. 16, pp. 805-821.

NEW BOOKS

(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW*)*

Abrasive Handbook. Fred B. Jacobs. 547 pp., 1928, Cleveland, Penton Pub. Co.

Diesel Engine Design. Ed. 3. H. F. P. Purday. 360 pp., 1928, Lond., Constable & Co.

Die Symbolische Methode zur Losung von Wechselstromaufgaben. Ed. 2, enl. Hugo Ring. 80 pp., 1928, Berlin, Julius Springer.

(In this 80-page German booklet the author explains the nature and characteristics of the complex quantity and shows how it may be applied to the solution of various alternating-current problems. It is primarily addressed to the practical operating or designing engineer, and the problems selected for discussion are such as would be met in actual practice. In each case the author first states the problem and then gives its complete solution by means of complex numbers. This little work is in its second enlarged and revised edition, the first edition having appeared in 1921.)

Erdstrome: Grundlagen der Erdschluss- und Erdungsfragen. Franz Ollendorff. 260 pp., 1928, Berlin, Julius Springer.

(This German volume presents an extensive mathematical study of the earth currents resulting from the operation of our various electrical installations and systems, especially those in which the earth is included in the circuit as a return conductor. These have long been known as "vagabond" or "stray" currents, and their effects and behavior have been studied to some extent, but it has remained to the author to give us a complete summary of their behavior and to explain the theory of their calculation. His methods and observations are based on experiences and investigations at the Siemens-Schuckert Works, Germany. A five-page bibliography cites additional literature for each topic treated by the author.)

Hilfsbuch für die Elektrotechnik; Swachstromausgabe. Ed. 10, rev. Karl Strecker (Editor). 1137 pp., 1928, Berlin, Julius Springer.

(Originally this well-known German handbook was published in one volume, but in its present revision it has been divided into two volumes covering, respectively, the fields of heavy-current engineering and weak-current engineering. The present volume pertains to weak-current engineering. It is edited by a prominent German engineer and educator who has had the co-operation of numerous German specialists. It contains sections one, two and four of the complete handbook, section three forming the other volume and dealing with heavy-current engineering. Section one of the volume at hand comprises 103 pages and gives the principles of mechanics, heat, electricity and magnetism, and includes the standard mathematical tables and formulas. Section two presents a 147-page review of methods of electrical measurement. Section four is the most extensive, being divided into seven parts which deal with such subjects as sources of power and types of current used in electrical communication, theory and construction of overhead and underground telegraph lines, telegraph systems and apparatus, telephony in all its phases, signaling and alarm systems, electric clocks, and radio engineering. An 18-page subject index completes the volume.)

Intermediate Electricity and Magnetism. R. A. Houston. 170 pp., 1928, N. Y., Longmans, Green & Co.

(An elementary text prepared for the students of English schools. The author states that he has "endeavored to make the treatment as clear and attractive as possible," and that he has "added a chapter on Electrical Oscillations and Waves which seems to be required as a result of the development of broadcasting, although this subject is not usually included in books of this scope." The volume includes consideration of the common electrical and magnetic phenomena, electric measurements, and the like, and includes a chaper on "Cathode Rays, X-rays, and Radioactivity," as well as the one on "Electrical Oscillations and Waves." A short list of problems accompanies each chapter, and a compilation of typical English school examination questions on the subject matter covered brings the volume to a close.)

Isolierte Leitungen und Kabel. Ed. 3, rev. Richard Apt. 235 pp., 1928, Berlin, Julius Springer.

(A discussion, interpretation and elaboration, in German, of the standards for insulated wire and cable issued by the German "Verband Deutscher Elektrotechniker." The publication of this third edition was made necessary by the recent changes in the standards themselves, as well as by the more stringent performance requirements set for electrical conductors in general. The ever-increasing demands of the electrical industry for conducting materials which will operate with a high degree of safety under the various conditions imposed are in many instances quite different from those of a few years ago. This booklet discusses those requirements in their relation to the German electrical standardization rules.)

Lehrbuch der Elektrodynamik. Vol. 2: Makroscopische Elektrodynamik der Materiellen Korper. J. Frenkel. 505 pp., 1928, Berlin, Julius Springer.

(The first volume of this advanced treatise, in German, on the mathematical theory of electricity and magnetism was published in 1926 and has been previously noted in these columns. This second volume treats exclusively the subject of "macroscopic" electrodynamics, the first part dealing with the fundamentals of macroscopic theory, and the second part with some general problems of macroscopic theory. A third volume is promised for the near future.)

Leitfaden der Lichttechnik für Unterricht und Praxis. W. Voege. 80 pp., 1928, Berlin, Julius Springer.

(This little volume is an outgrowth of a course of lectures, which the author gave in Hamburg, primarily for the benefit of lighting installation men and other practical workers in the lighting field. It discusses briefly, and in an elementary way, the fundamentals and units involved, measurement of light sources and illumination, the laws of physics underlying illumination practice, sources of light, various lamps and lighting units, and the principles of design of outdoor and indoor illumination. A three-page table of constants and a brief subject index are provided.)

Matter, Electricity, Energy. Walter Gerlach (Tr. by Francis J. Fuchs.) 427 pp., 1928, N. Y., D. Van Nostrand Co.

(The sub-title of this work, "Principles of Modern Atomistics and Experimental Results of Atomic Investigation," gives the reader a more definite conception of what it covers. It has been translated into English from the second German edition. The preface to the first German edition states that this work differs from previous works on the subject in the manner in which the subject is treated. The intention is to avoid detailed and exhaustive treatment of the development of the atomic theory and of its applications to inorganic and organic chemistry and to mineralogy and astronomy. It is written to appeal more directly to the chemist, the mineralogist, the technical physicist, the engineer, and to all students of natural science. The author says that no special technical knowledge of any sort is presupposed, "except on occasion." The book consists of a series of interdependent lectures, each a unit in itself. This enables the reader to select for his study such subjects as appeal to him, without the necessity of reading all which precedes. The treatment is not intended to be exhaustive but, rather, to stimulate the reader and thoroughly ground him in the more modern views of atomic theory. Some subjects treated in the thirty chapters are: Atomic Rays and Atomic Problems Solved by Their Means; The Magneton; Spectral Emissions and the Periodic System of Elements; The Photoelectric Effect; The Physical Basis of Photochemistry; Electron-Affinity; Radiation Measurements; and Atomism and Macrocosm.)

Principles of Factory Organization and Management. Ralph C. Davis. 449 pp., 1928, N. Y., Harper & Bros.

Principles of Mechanical Refrigeration. Ed. 2. H. J. Macintire. 317 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Scientific Purchasing. Edward T. Gushée and L. F. Boffey 196 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

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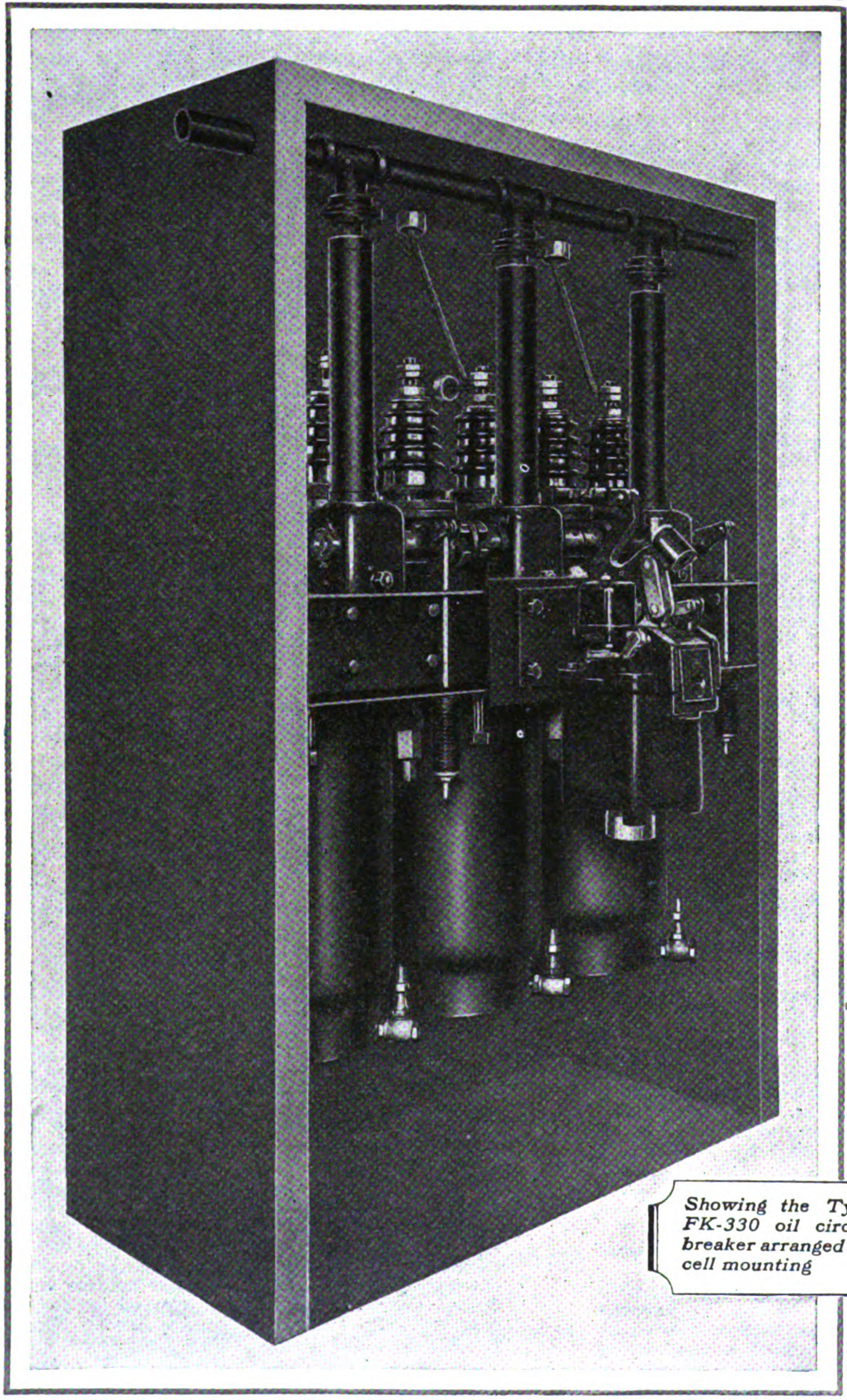
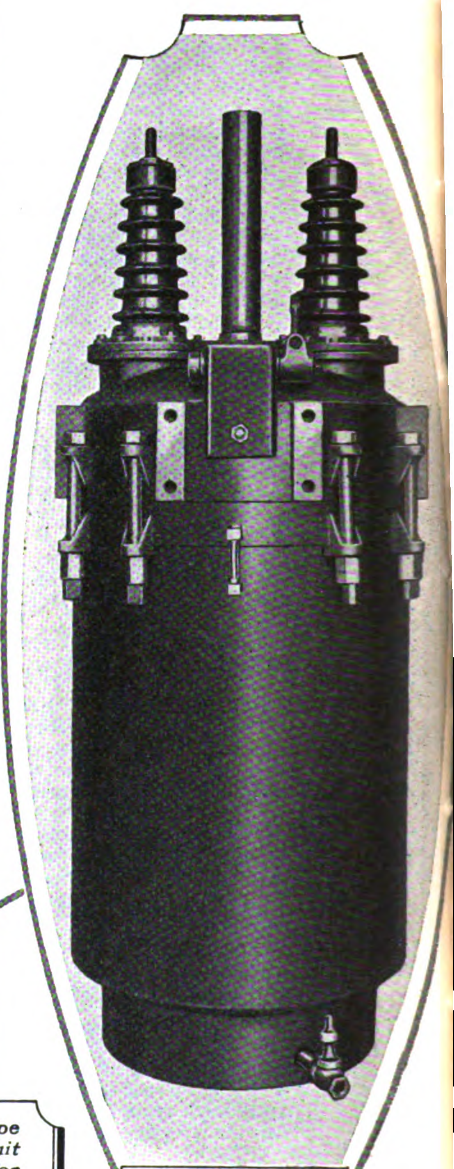


Showing the Type FK-330 oil circuit breaker arranged for cell mounting



Type FHK-330 oil circuit breaker unit

GENERAL

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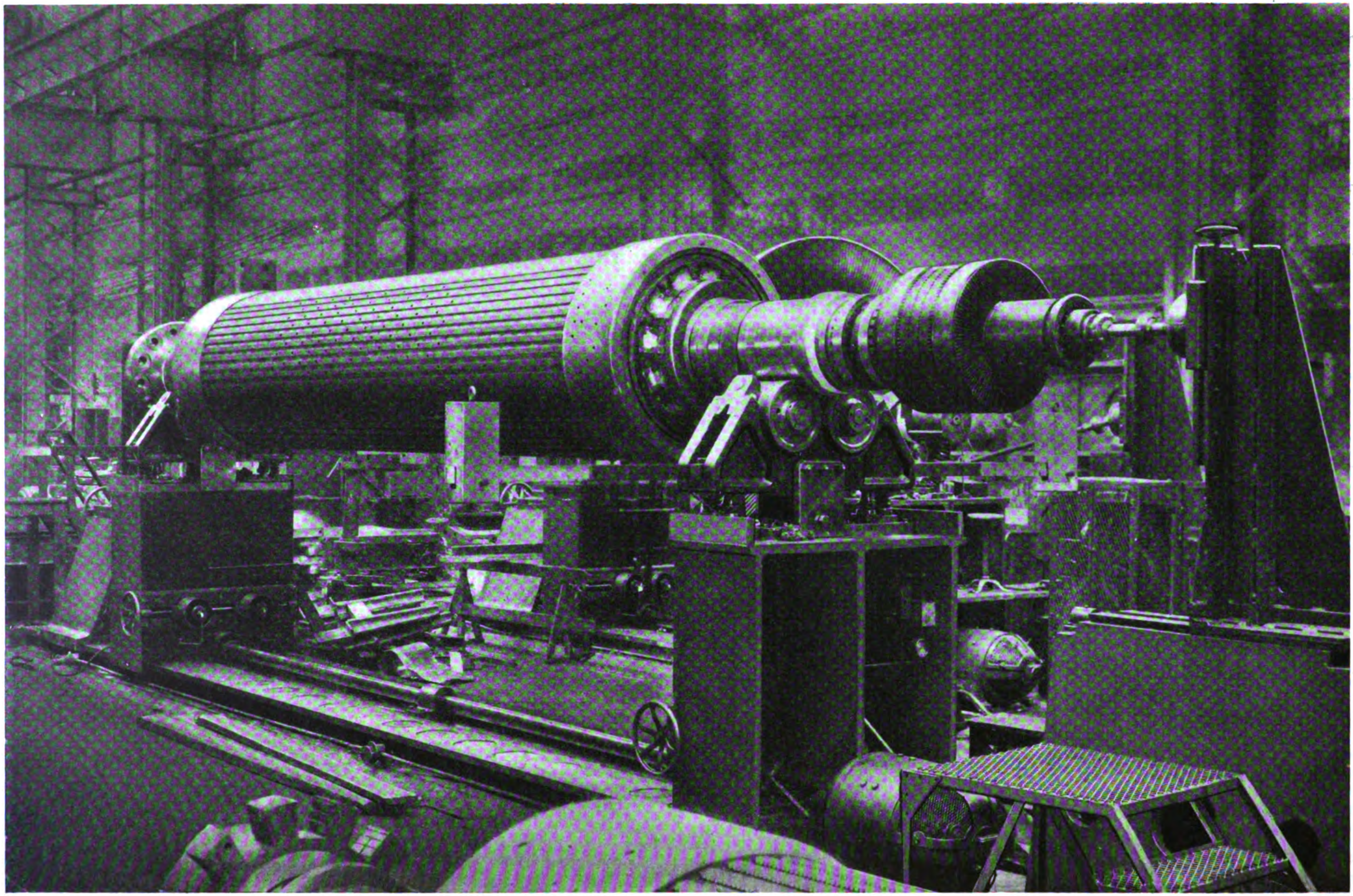

MASSES REVOLVING AT HIGH SPEED REQUIRE PRECISE BALANCING

The 60-ton revolving field of a 62,500-kv-a. 1800-r.p.m. alternator being dynamically balanced according to the principles explained by S. H. Weaver in an article in this issue (p. 542)

GENERAL ELECTRIC REVIEW

Vol. 31 October, 1928 No. 10

SENSES

How much do we know about our senses?

How many senses have we? A few years ago we should have said without hesitation, five; and recited, if we remembered them all—seeing, hearing, smelling, tasting, and feeling. This is all we learned about our senses at school, but now our modern physiologist doubles this number and adds the sense of heat, the sense of cold, the muscle sense, the sense of equilibrium, and the sense of pain.

Our physiologist assures us that the sense of heat and of cold are separate senses and that we have specialized nerves for conveying each sensation to the brain. If one set of these nerves is destroyed we can feel heat but not cold and vice versa. But unlike our senses of seeing, hearing, smelling, tasting, and our sense of balance, our senses of heat and cold have no well defined end-organs, but consist of innumerable nerve-fibers distributed in the skin all over our bodies. In this respect they resemble the sense of touch. He also tells us that our sense of balance *has* well defined end-organs in our inner ears, one on each side of the head, and if these are damaged we cannot walk straight and if damaged or diseased in birds they lose their power of flight.

Many books, even libraries of books, have been written on each of our senses and our nervous system, about their evolution, the most fascinating of studies, their physiology (habit and function) and their morphology (form and structure), but there is still lots more to be learned.

These senses and their end-organs, *e.g.*, eyes, ears, etc., are all part of our nervous system, which, among a host of other functions, has highly specialized nerves for connecting these end-organs, or our instruments for perceiving objects, sounds, tastes, smells, etc., with our brain—where dwells the seat of human consciousness.

So far as is known it is through the medium of our senses and our sense organs that we have our only contact with the outside world.

It would appear that our central nervous system is in some way related to our consciousness and constitutes the physical basis for the association of ideas and that it is in our nervous system that we store memories and thoughts and register experiences for future use. Also, in some way we do not *at present* understand, our very personalities are inseparably bound up with our nervous systems.We are either nice or nasty, witty or dull, quick or slow, useful or useless, according to the physical condition of our nerves.

J. Arthur Thomson, in "The Control of Life," says: "Some people believe that our inmost self uses the nervous system as a musician uses a piano." But here we enter into that age-long discussion which was revived at last year's meeting of the British Association by Sir Arthur Keith—that brilliant physiologist, morphologist, and embryologist, who for many years has been Hunterian Professor and Conservator of the Museum at the Royal College of Surgeons, London—taking the negative view; and this year again before the same Association by Sir William Bragg—the director of the Royal Institute of Great Britain—who took the affirmative view, "has man a soul?" This, we confess, is too difficult a subject for us, so we leave it where it has stood throughout the ages and simply quote Tennyson in "The Ancient Sage":

> "Thou canst not prove that thou are body alone,
> Nor canst thou prove that thou are spirit alone,
> Nor canst thou prove that thou are both in one:
>
> * * * * *
>
> For nothing worth proving can be proven
> Nor yet disproven."

We started thinking on the above subject after reading an article by Mr. H. M. Hobart, published in this issue, entitled, "Things We Don't Think Of," and some expressions he uses, such as, "If we were better read," "thought of almost everything," and "if we had better minds or better trained minds." And we started wondering what it was we had to train better.

We are always optimistic, especially of the future. We always believe in progress, we believe the world gets better and more worth living in every year. Then we remembered or half remembered one of those wonderful sayings of the wise old Confucius—which ran something as follows:

> "You sow a thought and reap a habit;
> You sow a habit and reap a character;
> You sow a character and reap your eternal destiny."

We came to the conclusion that we certainly had some things that ought to be better trained that would make us think more correctly, and if our senses and sense organs form the only contact we have with our environment and the outer world, they should be as highly trained as possible.

We suspect that when we went to school we did many things to train our senses, but we certainly were not trained to use our senses as such.

In our modern kindergartens and Montessori schools children are trained to use their senses, both individually and collectively. They are taught the use of their eyes in distinguishing colors and they learn the names of colors (we never did); they are taught to distinguish forms; they are taught sounds; to distinguish different smells and different tastes, and specially they are taught to develop their sense of touch. They are taught their alphabet and how to form their letters in writing by constantly running their tiny fingers over letters cut out of sandpaper, so that long before a pencil is put into their small hands they *can* write, and as soon as given a pencil they *do* write. They are also taught to develop their sense of balance by being made to walk along a straight line on the floor. And it all seems like play!

After our children and grandchildren have been trained in this direction for some generations—and it will require many generations to pass before the true result is known, because heredity plays many tricks on us—won't they be better fitted to live in a more complicated world? Because the world is getting more complicated all the time. Won't many things that require conscious effort for us become a matter of habit to them? Judging correctly of color, sounds, forms, dimensions, and a hundred and one things *that we can't think of now*, because they will be new things and we don't know them. If instincts are inherited habits, as we are taught, how many other things will be performed subconsciously by our better trained grandchildren that now require conscious mental effort for us?

After a few generations of radio and motor cars, etc., won't there be many instincts inherited by our great-great-grandchildren that we did not inherit?

We are children of the past—our grandchildren will be children of the present and future. Won't they make better engineers and scientists than we? It seems to us that the answer is largely in the affirmative.

Surely evolution hasn't stopped. In spite of the fact that man has himself assumed the proud title of Supreme Mammal we must remember that all our senses have taken ages upon ages of evolution to enable us to live better in our particular environment; to enable us to procure food—to multiply and to protect ourselves from innumerable enemies.

In many respects our sense organs are less highly developed now than they were years ago when the struggle for existence was greater. Man is neither complete nor perfect. Will new modes of life lead to the development of other highly specialized groups of nerve cells in the brain and new end-organs in the future? Why not a radio sense; a sense for detecting and even possibly sending radio messages? A sense for avoiding cars in our over-crowded streets? Evolution is still going on, and there is plenty of room in our skulls for more senses. The possibilities of the man of the future are infinite.

We have not all the same mental equipment and most of the work of the world will always have to be done by just ordinary people. The better trained their senses are, the better it will be for the world.

The great new work of the world, the great discoveries and the great inventions, will most likely always be done by those who possess that divine spark of genius. But sometimes we are likely to overestimate the divine spark. We like to remember Thomas Edison's receipt for invention—"One-tenth inspiration and nine-tenths perspiration!"

It is always well for all types of minds—mediocre minds and genius alike—to remember one piece of advice given by Mr. Hobart; to use the accumulated experiences of others as well as our own. Each one of us is a most wonderful organization in himself and we should remember that the firm, company, or organization for which we are working is also a very highly organized concern. If we want to do our best work we must use all our own senses and the accumulated knowledge of all those highly specialized individuals that go to make up our organization.

But when all is said and done the original workers can never become perfect. There will *always* be *things they don't think of; things they can't think of*, because their problems are *new*. The man who sets out to do something original will always make mistakes or things that look like mistakes. It is just the overcoming of these difficulties or these mistakes that makes so pleasant the life of the pioneer. Our minds may be mediocre or highly trained; but in the final analysis it is character and character alone that determines whether we succeed or fail.

There are many chapters in the following two lines:

> "Two men stood looking through the bars,
> One saw the mud, the other saw the stars."

Thank the Lord there is one thing that can't be standardized, and that is man! J. R. H.

Things We Don't Think Of

By **H. M. HOBART, M. Inst. C. E.**

Consulting Engineer, General Electric Company

> *The subject matter of our present article was delivered as an address before Mr. Doherty's class of the Advanced Course in Engineering at Schenectady. There is so much that is timely and useful in Mr. Hobart's remarks that we reproduce them almost in full for the benefit of our readers.*
>
> —EDITOR

MR. DOHERTY has asked me to take up with you today some subject the discussion of which will help you in your work as engineers. I have chosen as my subject, "Things We Don't Think Of," with a view to achieving this purpose.

It is a good thing to be confident of success if it does not lead us to underestimate the difficulties of the task on which we are engaged. Over confidence in one's self often leads to carelessness and to disappointing results. The kinds of difficulties which we don't think of and to which I shall be calling your attention today are those of which engineers have little or no warning. We are assigned the task of designing some sort of a thing, a transformer or a switch or a brush-holder or an oscillograph or a structure to be arc welded. We take into account everything that occurs to us, we make careful calculations, and we prepare a specification to be followed by the shop. In due course the manufacturing is begun. In a very large percentage of cases where there is any new feature about the thing, and unless we are exceptionally marvellous engineers or have been born under a very lucky star, the shop people will be pointing out necessary or desirable changes in our specifications. When we consider their suggestions fairly we shall often be compelled to admit that they are good ones. We should frankly accept the blame for not having consulted them prior to completing our design and specifications. This is one point; the manufacturing department may and usually will think of important things that we engineers have not thought of. It is entirely sensible for the engineer to obtain suggestions and help from shop foremen and workmen and from any other source, but if he does not do this till after he has declared his design completed and ready for manufacture, it very properly reflects upon him.

But as to this first mistake of not making use of all the best knowledge and experience available, while it is a very serious mistake indeed, it is entirely and easily within our power always to avoid it. However, things often appear so simple and trivial that we are tempted to conclude that the step would be superfluous. In my opinion it never or very rarely is superfluous. The best informed engineer can only know a little bit out of all that is known in engineering and manufacturing and he should realize that there are multitudes of shop methods of which he never will even have heard. In a small concern, or when engaged on work away from the factory, the engineer may be obliged to rely almost entirely upon the small store of knowledge and experience that he himself in his short life has been able to acquire. But in a large organization it seems to me that it is inexcusable not to supplement in all cases any small knowledge of our own with advice and suggestions from the very best specialists in all sorts of details. This advice and these suggestions are ours for the mere asking. Others will often remind us of important things which we ourselves did not think of.

Let us, however, assume that we have only completed our design after availing ourselves of the very best advice in everything we can think of relating to it. The Factory has built the thing and it is ready to test. It looks so fine and we have become so confident of its success that we expect that the test will be a mere formality and that the test results will agree with the calculations. It will be nothing unusual to be disappointed and still not be at fault. An investigation may finally show that some material employed was faulty, and had not been properly inspected, or that, without saying anything to anybody, someone had provided material which had been represented as "just as good" or even better than that specified.

But in other cases in which the thing will not work in the desired way it will be because there is some factor or factors which the engineer has not taken into consideration. In any except the very simplest machines there generally are such factors although sometimes the engineer could not reasonably be expected to even suspect them.

Unless we have complete knowledge of every factor affecting a result we cannot possibly predetermine that result exactly. If with incomplete knowledge of all the factors, the result we obtain is exactly that which we calculated, then we have made errors in our calculations. Since in all designs which present enough complications to be really interesting, there are always factors about which we have incomplete knowledge, we ought never to take pride in having the test results agree exactly with our calculations. Some engineers develop an uncanny instinct which guides them to calculate just enough wrong to make the result come out right. They are deceiving themselves if they conclude that their calculations were right.

As already suggested, when we test our machine we are very apt to learn of something which we hadn't thought of. Sometimes no living being had ever thought of it. In other cases, if we had been better read, or more linguistically skilled, we should have been warned by a report of the same kind of thing in some technical paper published perhaps in America, but perhaps only in some distant country and in some foreign language. Or, when we reflect on the test results and when we have cleared our mind by discussing the results with others, there may gradually dawn on us the knowledge of just what has happened. We then say to ourselves "Why, of course, how simple, why didn't I think of it before?" Or we may say, "this is almost the same sort of occurrence which Smith told me about a month ago in connection with the thing he was building at such and such a place. Indeed, now I come to think of it, it couldn't be any other way, it must be that way for this reason, etc."

But in another case the test at the factory may indicate that the thing is just what is wanted in every way. These factory tests may have seemed to be very thorough and even needlessly severe. Nevertheless, after it has been installed, there may be some service condition under which the thing fails. It may be that we could not possibly have known of this service condition or ought to have been warned about it by the purchaser. But on the other hand we may have known all about it but did not appreciate that it had any significance. The breakdown may occur immediately after installation, which is bad enough, or it may be of a slowly developing nature and may not be indicated by anything which can be noticed, until after a year or two of service, when the breakdown will occur. This latter is much the worst kind especially in view of modern industrial conditions.

To see how much more serious a breakdown of such a nature is today you have only to remember that a generation ago our machines were much more the product of manual labor and their parts were not so absolutely interchangeable as nowadays. Also, they were then made in lots of ten or one hundred. But now we are equipped with expensive special machine tools, and with them make vast quantities of the thing, all absolutely identical in every respect. In former times a weakness developing in a year or two, while very serious, involved only tens or a few hundreds of the article and the necessary changes in the design were made by workmen with simple tools and did not involve scrapping or altering expensive special machine tools. But under present-day conditions where the market for a single thing is relatively enormous, a fault might enter into tens of thousands of the product before detection and the new constructions necessitated might involve either new expensive manufacturing machinery or costly changes in the original manufacturing machinery.

You will now more keenly realize that nowadays, even far more than in the past, it is vitally important before beginning to manufacture to have designs as perfect as is humanly possible in every conceivable particular. The products must with certainty withstand, for a proper term of years, all the service conditions to which they will be subjected.

I have no desire to minimize the value to us of mathematics as a splendid tool for use in designing things properly and for use in helping us to think and to effectively investigate everything which is likely to occur. There are, however, also other important ways of increasing the probability that we shall have thought of almost everything before it is too late. Just now I advisedly said "thought of almost everything." Our endeavor should be to think of *everything* but we shan't. We shall forget things we ought to have remembered and there will be other things that not we alone, but no one else, anywhere, will ever have thought of. If we review the history of engineering we shall find many interesting instances of vastly important things that engineers did not think of. I just said "things they *did* not think of." We shall generally find that, like yourselves, they were very earnest and exceptionally intelligent men striving to do their very best. So I believe I would be right in saying "things they *could* not think of." We can do no more than our very best with the intellects with which we have been provided. Doing our very best with them includes training them effectively to use not only mathematical tools but logical methods of thinking. Logical thinking is probably improved by the study of mathematics. If we diligently study mathematics for many years and then turn to some activity in which we do not use the mathematics, I do not believe that there has been any waste of time or effort. For in the new activity we can profitably employ our ability in more logical reasoning than would have been possible for us without the mental discipline of our mathematical studies. In later life you may give up your gymnastic exercises, your football, rowing, and tennis, but the time spent on them, aside from the pleasure derived, will not have been wasted, as you will have a physically more effective body and that most important of all assets, *good health*.

This is beginning to sound too much like a sermon, so I will turn to the next section of my talk which will, I hope, be more interesting. In it, I mean to illustrate more concretely what I have in mind by the title of my talk, "Things We Don't Think Of," and cite a few examples of incidents in engineering which will, I believe, satisfy you that the best engineers who have ever lived have failed to think of things, some of which, after they have been thought of, seem very obvious. Some thirty years ago, a steam turbine was employed to drive a boat. The boat was 100 ft. long. The steam turbine was of 2000 hp. and was designed to run at 3000 r.p.m. After preliminary trials with a single screw propeller, the shaft was fitted with three propellers. At the shaft speed of 3000 r.p.m., the vessel's speed was only 18 knots, whereas the intended speed was 32 knots. The failure to attain the higher speed was due to a phenomenon known as cavitation which had already been encountered by naval architects but which had not been much studied. Insufficient allowance had been made for it in this first marine steam turbine application. The discrepancy in the result will appear less remarkable when I mention that most ship propellers usually run at such low speeds as 80 to 150 r.p.m., and to take correctly into account all the things which would happen at *twenty times* the speed corresponding to available data, was quite difficult. Several reconstructions were successively undertaken, and finally, using three turbines and three shafts instead of only one, and with three propellers on each shaft, each propeller having only two-thirds the diameter of the original design, and with a shaft speed of only 2000 r.p.m., the required speed of 32 knots was obtained. At a later date, a still more satisfactory result was obtained by replacing the nine propellers with three, each of greater diameter and pitch. This pioneer work was done by an engineer who is very generally recognized to be one of the best, if not the very best, of our time and who, amongst many great engineering accomplishments in many fields, has been foremost in developing the steam turbine for ship propulsion. But in this instance the importance of a known phenomenon had at first been very seriously underestimated. I have not cited this instance as being a conspicuous or unusual one. It is simply one amongst several which come to my mind. It is, we must regret to believe, but we must believe, because it is so, the usual instead of the unusual thing in pioneer engineering.

In connection with the example we have described, it is interesting to call attention to the commendable course of this engineer in publishing accounts of his successive steps from the first inadequate equipment to the final exceedingly successful one. His course in so doing was in accordance with one of the finest traditions of the engineering profession. Were engineers to publish descriptions of only their finally perfected machines, and were not to admit that the final result was attained only after several less satisfactory machines had been built, much valuable knowledge about things we are liable not to think of would be lost to the profession.

The most important object which Mr. Doherty has in view in the training opportunities which he is providing to the members of this class is to increase to the greatest possible extent the likelihood of having taken adequately into account in advance of construction as many as it is humanly possible to foresee of all the phenomena which will affect the result. Literally many millions of dollars a year would be saved by a large company if all its manufacturing problems could be completely solved prior to the manufacturing stage of regular production. Of course this ideal will never be attained, but a great improvement in that direction should be possible.

I remember that some 36 years ago, I with several others was engaged in measuring the core loss of transformers. Up to a certain date, all our results had been consistent. Suddenly and without warning we found ourselves in the midst of trouble. Part of the time the core losses would come out as we expected but the rest of the time they were some ten per cent low. Worst of all, the core loss on the same transformer, measured on another day, would be ten per cent different. We suspected that our measuring instruments might have gone wrong, but calibration vindicated them. Then one day we discovered that instead of the alternator with surface windings (then popularly known as the "pancake" type) which had always formerly been the source of supply, a machine of a then new type, with the windings in slots instead of on the surface, and with one slot per pole, was sometimes being used instead and that the lower core loss was obtained with that machine. Then we measured the wave shapes at no load and with full load (which was an elaborate process in those pre-oscillograph days and required a Thomson quadrant electrometer with sulphuric acid in a bowl to keep it dry), and found in the slot-wound machine a ten per cent lower peak voltage for a given root-mean-square voltage. I dare say that if we had had better minds or better trained minds, we should have been less surprised at these experiences.

Some twenty-five years ago there fell to my lot the task of investigating some very old patents describing interpole machines. The interpole construction was not successfully (or at any rate, widely) used until 1900 or thereabouts. But it had been described by many pioneer inventors, in patents issued a decade or more earlier than 1900. My recollection is that my search disclosed that amongst these early inventors of the interpole were several of those tremendously able men who did so much during the last two decades of the nineteenth century to develop dynamo-electric machinery. Nevertheless most of them appear to have overlooked for many years an absolutely essential point, *viz.*, that in order to have the interpole beneficial instead of actually harmful, the ampere-turns on the interpole must exceed the ampere-turns per pole on the armature. Otherwise the resultant flux is in just the wrong direction and makes matters worse instead of better.

This, though perfectly obvious to us now, was a thing most of them didn't think of, and when their machines were tested they did not get any improvement in commutation, but the reverse, and, not seeing why, the idea was given up for ten or fifteen years. When, about 1900, it came to be better understood what was the matter and the interpoles were designed more nearly correct, a tremendous advance was made in commutating machinery. To be exact, it is desirable here to mention that a few early workers on types of machines with distributed compensating windings had fully realized the point I have mentioned.

It was about twenty-three years ago, when I was working in Europe, that an American firm ordered some large waterwheel-driven polyphase generators from a European firm. I was retained to inspect these machines and was given strict instructions to insist on complete fulfillment of the specifications in all particulars. One clause in the specifications required a sudden short-circuit test and I was advised that it had developed in America that the instantaneous current was upwards of some twenty times the current at rated load. Probably if I had been better read I would not have needed to be advised of this. I imparted my newly acquired knowledge to the responsible European engineers who assured me that it was not so; that my American friends were wrong; because they reasoned, if it *were* so, their very thorough researches and studies would have led them to its discovery. However, they were entirely willing to make tests to convince me. Of course the result was to demonstrate that notwithstanding their belief in their infallibility (all of us are sometimes led into error by over-confidence in ourselves), this large momentary current *did* flow at the instant of short-circuit. I well remember the alacrity with which they then produced mathematical demonstrations of all aspects of the phenomenon. But you will note that, as is probably usual, the mathematical treatment *followed* instead of *preceding* the observation of the phenomenon. But we want to learn to really predict phenomena either by means of mathematics or of logical deduction, before making experiments. As an instance of the slowness with which knowledge sometimes travels, you may be interested when I tell you that it was about four years later that the stator windings of a turbine-generator were wrecked in a certain important generating station. I was called in to ascertain the cause and of course (particularly because of the previous experience) was at once able to do so. No one in that operating concern had happened to ever hear of this phenomenon and I do not think my explanation of the cause would have been accepted had it not been that by that time I could produce corroboratory evidence in the form of engineering papers published in the technical press. It was of course by that time quite common knowledge, but had happened to be still overlooked by my client. My report was acknowledged in a very complimentary manner and I was in the way of acquiring quite an opinion of myself. But pride cometh before a fall! My advice was next asked about some trouble they were having with a complicated control system and I was completely unable to make them any useful suggestion.

As an example of unexpected trouble developing in service a year or more after installation (and thus not being disclosed in the factory tests, made before shipment), I may mention the ageing of transformer iron. That was first observed by Mordey and Partridge about 1894. Transformers tested all right at the factory and showed low core loss and low temperature rise. After months or years of service, overheating was observed, often before failure of the transformer. Transformers were tested after being received back at the factory and were found to have core losses 30 per cent and even sometimes as much as 60 per cent higher than when they were new. It took years of experimenting with various alloyed steels before we were able to make transformers whose core loss would remain constant throughout the transformer's life and would also be of satisfactorily low amount.

The unexpected is constantly happening. In arc welding, brittleness in welds was ascribed to the presence of oxides and nitrides. It was concluded that if the welds were made in an atmosphere of hydrogen, so that oxygen and nitrogen could not reach the molten metal, the welds would be more ductile. To a great extent this expected result was obtained, but in the course of the experiments it was found that a mixture of 25 per cent of hydrogen and 75 per cent of nitrogen was quite as good as 100 per cent hydrogen; indeed the welding arc was more stable and required less voltage to hold it. So it was discovered that nitrogen, instead of being harmful, was, in large percentages, entirely permissible and desirable and that brittleness had been chiefly due (so far as the surrounding medium was of influence) to the presence of oxygen.

Indeed the exclusion of oxygen led at first to another trouble, that of porous welds. It has been found by Mr. Peter Alexander in his investigations in the Thomson Research Laboratory that when welding in air, that is to say, in an atmosphere rich in oxygen, the process of oxidation at the surface of the molten metal keeps the outer layers hot for a longer time and enables any gases in the metal to escape from the interior so that a non-porous weld is more readily obtained. But if you exclude oxygen by welding in a hydrogen atmosphere, the surface solidifies more quickly and gases are trapped under the surface in the body of the metal. There are means of overcoming this difficulty, but I am mentioning it as another instance of things which were not thought of until after they had happened.

A few years ago some experiments were made in which a very small commutator motor was enclosed in a hydrogen atmosphere with the reasonable expectation that in the absence of any oxygen there would not be any sparking at the brushes. But the difficulty was encountered that the commutator quickly became coated with a layer of carbon dust worn off from the brushes. In the hydrogen atmosphere there was no oxygen to burn up the carbon dust and carry the product away as a gas. That consequence was not thought of until after the event.

About 1912, an engineer with whom I am very well acquainted undertook to modify the design of an induction motor in such a way as to be able to greatly increase the rating of a certain frame size. He managed to find room for larger stator conductors and used a fewer number of conductors and more flux, increasing the magnetic densities. In order to not impair the efficiency, he wanted to increase the cross-section of the squirrel-cage rotor conductors as much as he could without too seriously encroaching upon the starting torque. To find room, he made the rotor slots very deep. He was quite concerned lest he should have made the power-factor too bad by this step of using deep rotor slots, and his anxiety in that respect was well-founded. But as to the starting torque, he was surprised and pleased to find that it was very much higher than he had expected or could at first explain. But when he put his mind on it he soon realized that it was due to the skin effect (or in this case, more properly, "deep-slot" effect) of these deep conductors at the instant of starting, when the secondary currents are of full line periodicity. He believed he had made a splendid discovery and he built some experimental motors in which he purposely exaggerated this effect and obtained high starting torque without sacrificing efficiency, but with slightly impaired power-factor. He called it the "deep-slot" effect. But in this conservative world a new proposal is rarely given an especially cordial welcome even in engineering matters. The disadvantage of lower power-factor was said to be too great. But now, many years later, this principle is being used quite a good deal for special kinds of motors. If the time has arrived when it can be recognized as a useful method, it is interesting to realize that although it has since been mathematically investigated by many people, it was originally a discovery made in test.

At that time Mr. F. Kierstead, who is a very good mathematician, predicted from his mathematical investigations, and before building a model, that two deep bars of a given total depth would develop more of this effect than a single deep bar. Some said he was all wrong; that it was absurd to suggest that sub-division could *increase* the "deep-slot" effect. But tests showed him to be right. With some proportions and arrangements, you increase the "deep-slot" effect by sub-division. With other proportions, the reverse is true. This prediction of Kierstead is one of the rare cases where the mathematical demonstration *preceded* the experimental demonstration. Here was a case where a gifted young man *did* think of something that would happen and which had never been known of before, and worked it out mathematically *before* it happened.

I must not tax your patience by giving you more examples of the proposition that it is very unusual, if ever, that in any engineering project *all* the things that happen, are foreseen. While occasionally the unforeseen happenings lead to beneficent discoveries, generally they are serious faults which are put right only at very great expense which could have been avoided if we had had the ability to foresee and avoid them. But as already stated and as has been illustrated by several examples, they are often things that nobody ever had thought of or could reasonably have been expected to think of.

The fact that things we don't think of are of such frequent occurrence, ultimately brings many engineers, as they grow older, into a state of mind where they will no longer make any change, however slight, in the things for which they are responsible, if they can possibly avoid it. In many great factories you find things being built in accordance with antiquated designs. You wonder how the responsible people can have so little enterprise as to go on year after year building things in these out-of-date ways. Often it is because they have no longer the courage or energy to face the probability of having things happen which they will not have thought of. This over-caution is much worse than being too confident. But the best way is continually to try to make things better than ever, and in so doing, do our very best to anticipate and provide for everything we possibly can. With the splendid facilities and specialized knowledge now available in every branch of engineering, we ought to be able to make progress toward better and better products, with a minimum of mistakes.

I have tried to lay stress on the duty which we each have, to obtain all the advice and help we can from others who have special knowledge and experience which we do not have ourselves. But this has been from the standpoint that one of us has the responsibility for making something, and he, keeping the responsibility, obtains the advice of others.

We shall in the near future need to do more and more things collectively and this will require us to collectively endeavor to plan things so well, in advance of executing them, that we shall obtain the most for the least expenditure. Many kinds of collective effort are undertaken by our professional societies. Joining our professional society is a step which many of us do not think of. Or if we think of it, we do not think of the aspect that becoming a member is part of the duty which we owe to our profession.

But, although, in my opinion, membership in our engineering society is one of our professional duties I am certain that it is much more than that; it is our professional privilege. The professional society confers on its members advantages which no earnest engineer can afford to forego. Collective effort on a subject (if it does not lead to each leaving the earnest effort to be made by the others) ought to result in decreasing the number of things which are not thought of.

Amongst the many examples of attempts on the part of engineers to contribute to engineering progress by collective action in committees of their professional societies, the American Engineering Council is an organization to which several professional engineering societies belong. Our own professional society, the American Institute of Electrical Engineers, is one of the member societies.

Through collective action in American Engineering Council, engineers are endeavoring to assist in contributing to the establishment of wise national policies in matters in which engineering and scientific knowledge and experience are involved. It is excellent for engineers to improve a machine or a process, but a small group of them can accomplish this. A relatively small group can also build and equip a generating station in which the very latest developments are incorporated and which gives the greatest result for the least cost. They can go further and construct transmission lines and distribution systems. But by this time, if not before, the stage is approached at which, excellent as the designs and constructions may be, harm may be occasioned unless the reasonable interests of other groups are taken into consideration. A transmission line crosses into another territory; ultimately it becomes economical to tie it in with some other system; if no national standards as regards voltage and periodicity and phase have been observed, expensive reconstructions have to be undertaken.

The investment in this country in electrical installations and material amounts to many billions of dollars. The precise figure need not be fixed upon for the present purpose. Let us take it at x billions of dollars. Statistics of the growth of the electrical industry have shown that the total investment is doubled every seven years or thereabouts. Then seven years from now, the investment will be $2x$ billions, in fourteen years $4x$ billions and in less than 30 years, $8x$ billions, provided much saturation does not set in. Consequently the present investment is only a matter of one-eighth of what it will probably be 30 years hence. Up to the present there has not been paid to standardization any remotely sufficient amount of attention. Each undertaking comprises features which a relatively small group of individuals agreed upon. But for the remaining seven-eighths of the total electrical engineering construction program for the next thirty years, it is absolutely essential to reasonable economy that individual opinions shall yield to collective opinions. The longer this is deferred the more expensive will be the reconstructions which will have to be undertaken if any approach to a coördinated and economical nationally-best system is to be obtained.

Moreover this electrical program must be coördinated with the very large needs for development in related engineering fields. While recommendations from engineering societies are of value, the local, state, and federal coördination of all these projects into a comprehensive policy will be essential for the maximum good of the nation at this stage of the country's development.

In the next decade literally billions of dollars will be spent in this country for hydroelectric power systems, for building gigantic dams and reservoirs, for dredging rivers and constructing canals for navigation, for reclamation, for irrigation, for flood control, for reforestation, for water supplies for communities, for bridges so that railways and highways may cross rivers, and for related purposes. It has been pointed out by a distinguished engineer that if the American people as a whole are to have the maximum return for these billions of dollars of expenditure, it should be made in accordance with a national policy in the development of water resources. Conversely, by establishing and following such a national policy, probably the work can be done for far less than will be spent if we continue the process of seeking out only the richest portions; and if we continue to permit their uncoördinated development without consideration of all the consequences from the standpoint of ultimate *National* economics.

The policy should be studied with regard not alone to irrigation, or to transportation, or to power development, or to flood control or to water supply to communities, since the achievement of any one of these objects alone, will often not only be prohibitively expensive, but may actually be detrimental to the later achievement of the others. But a plan developed in the first instance, to ultimately serve the best interests of *all* these objects, and with the costs distributed over *all* these objects, will often show the cost of each to be much lower than if provision were made for the attainment of one object alone, and without any, or with inadequate consideration, of the other objects.

Most of America's immense natural resources are still undeveloped. Indeed some of the greatest of them could not even have been recognized as being natural resources of vast value until they were made possible by the modern development of engineering methods in dam construction, reservoir construction, improved hydraulic machinery, and in high-voltage insulators, transformers and long-distance high-powered transmission lines.

If a national policy of water resources development should be established, an important part of it should relate to the adoption of a series of standards for the material to be used and the procedure to be followed in carrying out the projects. These standards should represent not only the best engineering and manufacturing knowledge, but also should involve the minimum suitable number of sizes and varieties of everything needed and a Standard Quality Specification for each size and variety. With the reduced number of sizes, each size and variety will be manufactured in much larger quantities and consequently at lower cost. The quantity of material to be kept in stores for replacements will be much less (particularly if *interchangeability* features are included in the program), and other similar obvious economies will be effected. There must be established and adopted and employed, a series of standard voltages, standard frequencies, standard phase arrangements, standard transformation ratios, standard transformer taps, etc., for generators, step-up transformers, high-voltage transmission lines, substations, step-down transformers, distribution circuits, distribution transformers and material required by the ultimate consumers, such as motors, lamps, electrically-driven tools, heating devices, domestic appliances, farm appliances, etc.

In the last eight years a great impetus has been given to such standardization activities by the projects established at the instigation of the Division of Simplified Practice of the Department of Commerce. The actual steps are taken by the industries concerned in each case. But the inspiration and a certain amount of guidance have been supplied by the Department of Commerce. The task is enormous, but an excellent foundation has been laid. I am fairly familiar with standardization work in other countries and in my opinion there is, with one possible exception, no other country which has made such a good beginning with this exceedingly important "simplified practice" portion of standardization.

National policies for the economical development of their natural resources in coal and water are being given much study by France, Italy, Russia, Germany, England, Japan and other countries and within a very few years these policies will be in the stage of being vigorously put into effect. Indeed in some instances this stage already has arrived. The extent to which American machinery and designs will be acceptable in these foreign enterprises, will be greatly increased if a broad coöperative attitude in support of a policy of quality standardization and simplification of product is adopted by the manufacturers in this country.

These, then, are some of the things which some of us probably have not sufficiently thought of in engineering matters of local and small scale and of broad and large scale.

All these matters, the small and the large, are important in the realization of the engineer's oft-quoted objective as set forth in the statement of The Institution of Civil Engineers, that it is "a Society established for the general advancement of Mechanical Science and more particularly for promoting the acquisition of that species of knowledge which constitutes the profession of a Civil Engineer, being the art of directing the great sources of power in Nature for the use and convenience of Man."

Recent Applications of the Electric Micrometer

I. Water Pressure Tests in a Hydroelectric Station.
II. Balance and Vibration Tests on Rotors of Large Steam Turbines.
III. Compression Indicator Diagrams of an Electric Household Refrigerator Unit.

By **A. V. MERSHON**
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THE manufacture of large steam turbines, large waterwheel turbines and other apparatus has increased the need for new and reliable testing instruments in order to determine improvements in design and operation. In large machines the forces acting are relatively large, therefore they must be controlled or directed in such a way as to stabilize conditions. The mass of a large steam turbine rotor, for example, must be carefully balanced in order to obtain smooth running. Where there is considerable mass in the rotor of a steam turbine, and where the peripheral speed is high, the balance must be very nearly perfect so that large unbalanced forces will not exist. A good balance would correspond to only a few thousandths of an inch radial motion in the rotor of a steam turbine.

In such cases the electric micrometer has been used to measure shaft movements in the order of 0.0001 in. or greater. The electric micrometer, furthermore, will measure rapid changes in water pressures in inaccessible parts of waterwheel turbines. Mechanical devices will not follow changes in pressure that are as rapid as one hundredth of a second. In another instance, the electric micrometer was used on a household refrigerator unit to measure the pressure changes that take place during one stroke of the piston operating in a sealed vessel. An indicator card was obtained showing the changes in pressure over a period of one-thirtieth of a second, the time required for one complete cycle.

A photograph of a newly designed electric micrometer is shown in Fig. 1. This instrument is portable and is of the following dimensions: 19 in. long, 13 in. wide, and 9 in. high. It is provided with terminals for connection to a small 500-cycle generator and a standard portable oscillograph which are used as auxiliaries.

The operation of the micrometer⑴ is, briefly, as follows: A dummy coil connected to terminals so marked on the micrometer is set at a fixed air gap. A test coil connected to another set of terminals is provided with an air gap varying according to the nature of the vibrations in a particular piece of apparatus. The magnetic changes in the test unit produce deflections on the oscillograph vibrator in proportion to the test variations. The change in the air gap can be calibrated in thousandths of an inch for obtaining a measure of mechanical vibrations and in pounds per square inch for recording transient pressures.

I. WATER PRESSURE TESTS IN A HYDRO-ELECTRIC STATION

Hydraulic research tests were made on a large waterwheel turbine to determine water pressures or water hammer. The reason for making these tests was to determine the pressure variations in the draft tube under the waterwheel where certain conditions exist which cause rapid wearing away of the material.

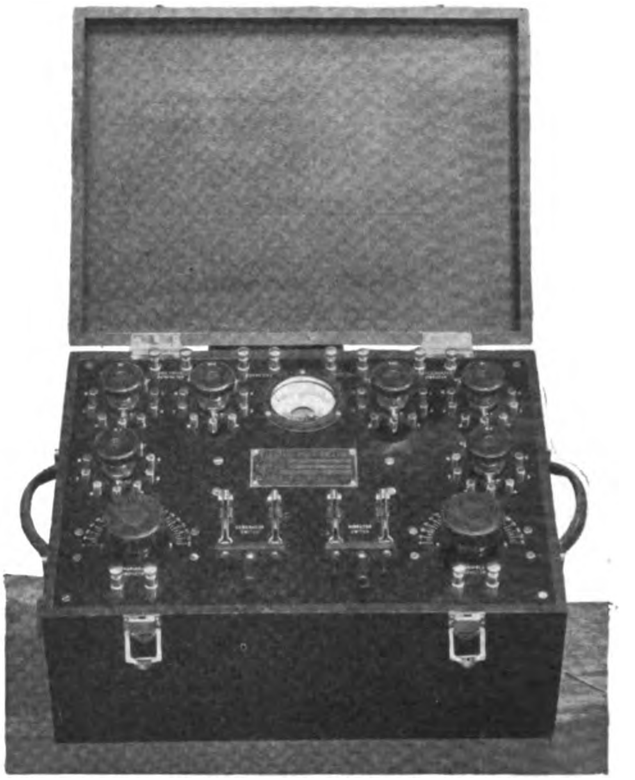

Fig. 1. The Electric Micrometer, which May be Used for Measuring Small Mechanical Vibrations and Transient Pressures

Fig. 2 shows a dummy unit and a pressure test unit used with the electric micrometer. In making the tests the dummy unit was located in the gate house near the oscillograph. The pressure test unit was mounted in a one-inch steel boiler plate located under the waterwheel in the draft tube. Some of the test units were located in portions of the draft tube where the metal wore away rapidly. Others were located where there was less wear. Diaphragms of different thickness were used in the test units, but a diaphragm of high speed steel 0.025 in. thick was found suitable. The coil shown to the right in Fig. 2 has a special sheathed lead 175 ft. long which is capable of preventing any leakage of the 500-cycle current between the outside sheath and the inner conductor.

⑴ A more detailed description of this apparatus is given in the GENERAL ELECTRIC REVIEW, Nov., 1926, p. 815; the *A.I.E.E. Journal*, Sept., 1926, p. 820.

The pressure test units were calibrated after installation in the steel boiler plate in the draft tube. In order to obtain reliable results, the test units were calibrated before and after a test run. Fig. 3 shows a calibration curve made under air pressure before and after a test run; and the points agree within a small percentage. Air pressure was used to force the water out of the draft tube to a sufficient level to allow a man to enter the scroll case and to apply known calibrating pressures to the test units in position. Corrections are shown for air pressure as well as for the head of oil in the calibrating pressure gauge.

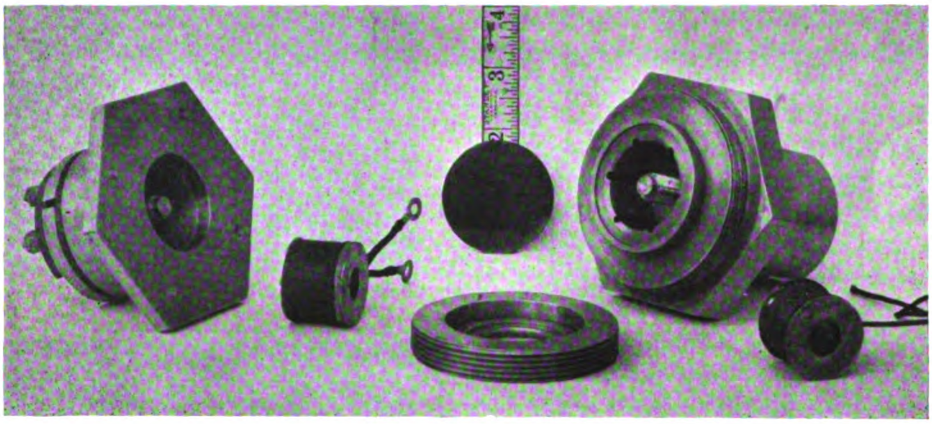

Fig. 2. The Pressure Units Used with the Electric Micrometer in Making Tests on a Large Hydraulic Turbine. The dummy unit is shown at the left; the test unit on the right

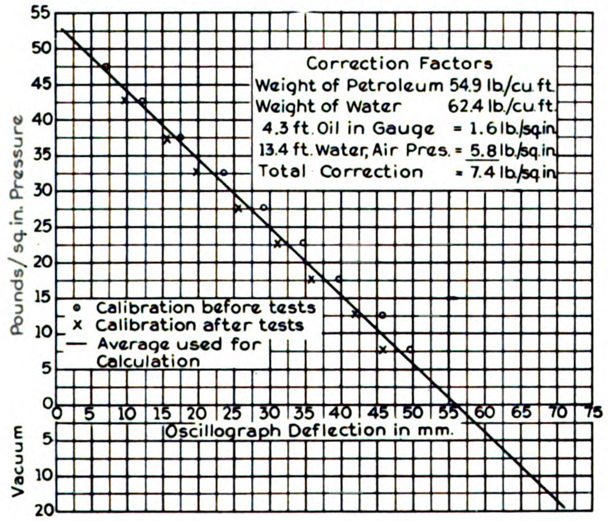

Fig. 3. Calibration Curves for the Electric Micrometer Used in Making Water Pressure Tests on a Large Turbine

The start of the calibration curve is at $7\frac{1}{2}$ lb. per sq. in., which is the total correction factor for air pressure and oil pressure. The circles on this curve sheet represent calibration points before test. The crosses are calibration points taken after the water wheel tests were made. Averages of these points, represented by the heavy line, were used to calculate the pressures from the oscillograms. Tests were not made to determine the lower or vacuum part of the calibration curve, but in previous experiences it has been found that the curve extends below the zero or atmospheric pressure according to a linear function, as shown.

Fig. 4 shows oscillograms taken at 15,000-kw. load and 24,000-kw. load respectively. Six indentations of the lower, or 30 electrical cycles of the upper curve, represent one complete revolution of the turbine runner. The waterwheel runner is of the propeller type having six blades. Atmospheric pressure corresponds to 56 millimeters deflection on the original width of film. Deflections outward on these oscillograms represent vacuum and deflections inward, therefore, represent pressure. By applying the calibration curve to these oscillogram deflections the

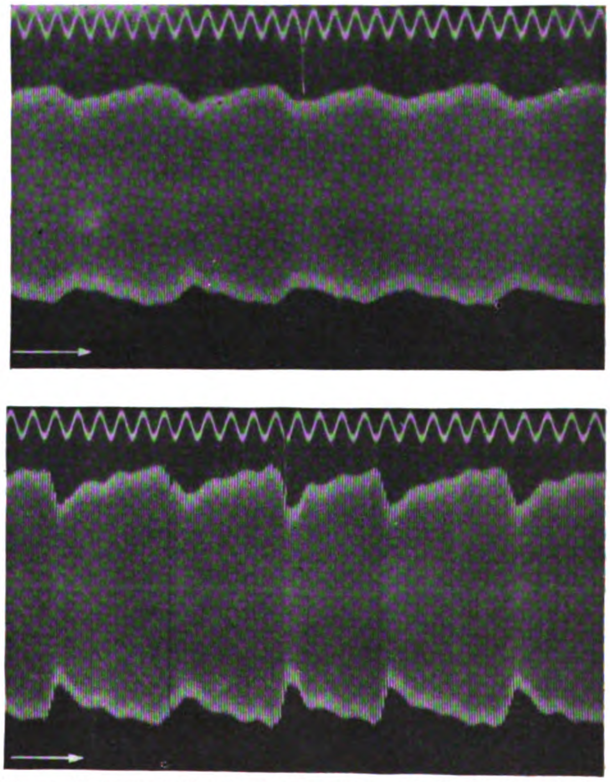

Fig. 4. Oscillograms Showing Water Pressures Under the Runner of a Water Turbine. The upper curve is for a load of 15,000 kw.; the lower for 24,000 kw.

pressure changes under the water wheel can be determined.

The curves plotted in Fig. 5 were calculated from the oscillograms (Fig. 4) and the calibration curve (Fig. 3). Pressure variations are the larger for 15,000-kw. load, and vacuum variations predominate for 24,000 kw. By integrating over each of these individual pressure and vacuum areas with a planimeter, the relative areas have been determined. The figures written in each pressure and vacuum area represent the number of planimeter units. Table I, based upon these planimeter readings, was computed as follows: The curves in Fig. 5 were originally plotted on rectangular centimeter cross-section paper. By integrating over one square centimeter in a similar manner gave 147 planimeter units. Subtraction of the totals of planimeter area units for pressure and vacuum gave the total difference in area representing a load condition. By dividing the number of planimeter units thus obtained by 147, the number of square centimeters difference in area was found. Since one revolution of the turbine runner is plotted over a distance of 30 centimeters, this area difference divided by 30 gives the average height in centimeters and hence the average pressure within the draft tube. In Fig. 3 one centimeter on the vertical axis represents two pounds per square inch.

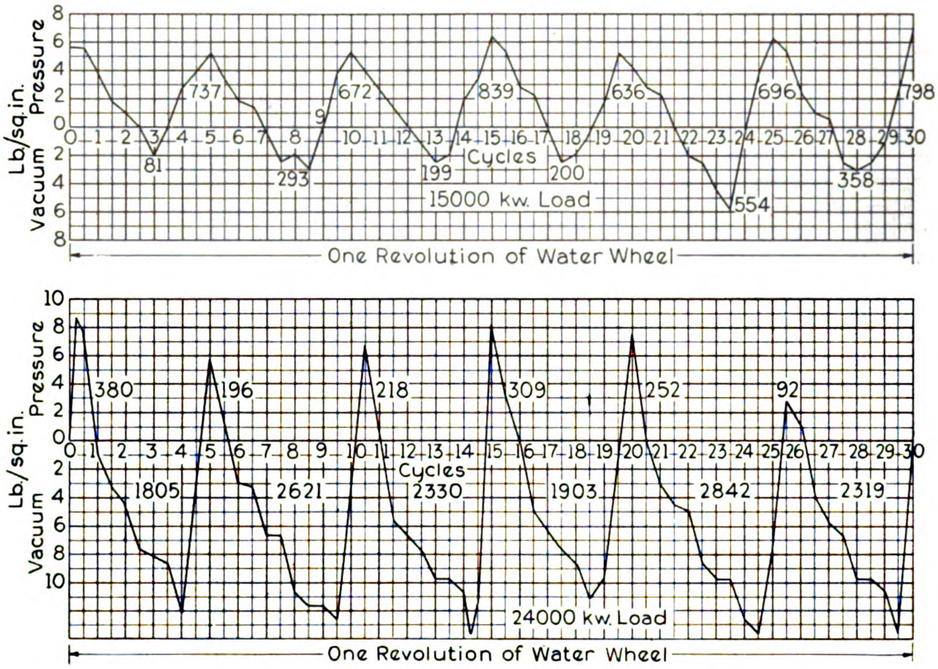

Fig. 5. Calculated Pressure Curves for Water Turbine Tests, Derived from Calibration Curves and Oscillograms of Figs. 3 and 4

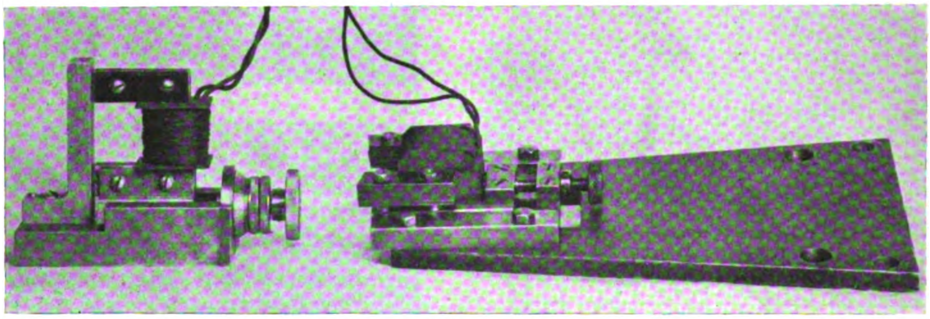

Fig. 6. Vibration Units for the Electric Micrometer Used in Measuring Vibration Amplitudes in Steam Turbine Rotors. Dummy unit at left; test unit at right

TABLE I

CALCULATIONS FROM CURVE SHEETS AND FILMS

Kilowatt Load	Planimeter Units	Area Sq. Cm.	Ave. Height Cm.	Ave. Vac. or Pres. Lb. Per Sq. In.	Equivalent in Feet of Water
15,000	+ 2,693	18.2	0.607	1.22 Pres.	2.8
18,000	+ 1,322	9.0	0.30	0.60 Pres.	1.36
20,000	− 4,262	29.0	0.968	1.93 Vac.	4.44
22,000	− 4,797	32.62	1.09	2.18 Vac.	5.02
24,000	−13,373	84.1	2.8	5.8 Vac.	12.9
25,000	−17,128	116.4	3.88	7.76 Vac.	17.85

Data taken at position "A" in the draft tube using an 0.025-in. diaphragm in the pressure test unit. Other points in the draft tube show independent data.

The table shows calculations from a group of test films for different kilowatt loads. A 15,000-kw. load produces an average pressure of 1.22 lb. per sq. in.

which corresponds to a head of 2.8 ft. For a 24,000-kw. load, a vacuum of 5.6 lb. per sq. in. exists under the wheel, which corresponds to a head of 12.9 ft.

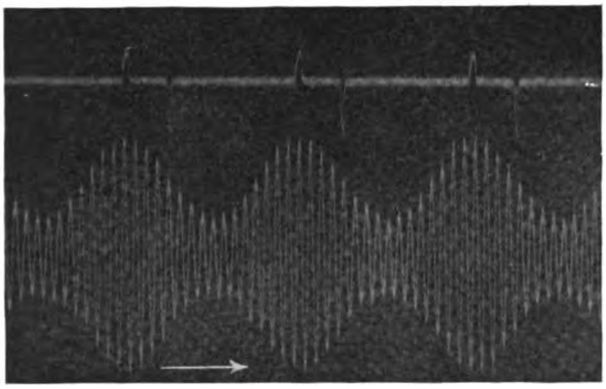

Fig. 7 (*A*)

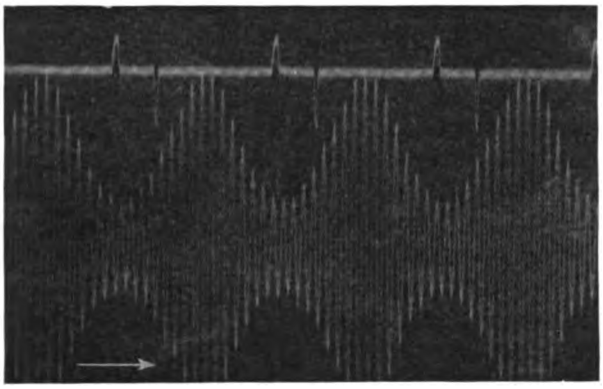

Fig. 7 (*B*)

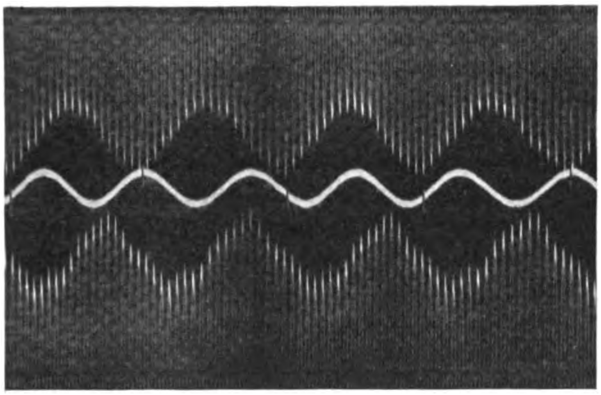

Fig. (*C*)

Fig. 7. Oscillograms Made in an Application of the Electric Micrometer to Balance Tests on a 32,000-kw. Steam Turbine

Both *A* and *B* show vibration amplitude of rotor and the rotational shift of the vibration; *C* shows vertical and horizontal rotor amplitudes.

II. BALANCE AND VIBRATION TESTS ON ROTORS OF LARGE STEAM TURBINES AND WATER-WHEEL GENERATORS

In testing the rotors of large steam turbines for mechanical vibrations, the electric micrometer has been used to measure the vibration amplitude. Weights were placed in the last stage wheel at various locations to obtain the best balance. After the rotor was balanced, starting conditions were studied which showed periodic changes in the amplitude of the shaft vibration until conditions become constant.

Fig. 6 shows a dummy vibration unit on the left and a test vibration unit on the right. Both have micrometer screw adjustments for calibrating their air gaps in thousandths of an inch. In measuring shaft vibrations the test unit was placed near the shaft with an air gap of 0.025 in. and was so adjusted that movements of the shaft towards the test coil increased the amplitude of the 500-cycle deflections on the oscillograph.

Fig. 7 shows two oscillograms (*A* and *B*) taken during a balance test on a 32,000-kw. 1800-r.p.m. steam turbine rotor. The top curve on each of these oscillograms shows two interruptions in the zero line produced by a stud on the shaft making contact at two stator positions 90 deg. apart. These interruptions are always in a fixed position with respect to the turbine shell. In comparing oscillogram *A* with oscillogram *B* the large peaks on the 500-cycle deflections are at different rotor positions with regard to the fixed stud on the rotor, as shown by the interruptions in the top wave. Changing the balance weights causes the high spot on the shaft to change with respect to the fixed stud. Oscillogram *A* shows the 500-cycle high spot lagging the stud 87 mechanical degrees; and oscillogram *B* shows the high spot on the shaft lagging the stud 240 mechanical degrees. Heat changes over a period of several hours can be observed in the shaft, which cause the high spot to shift its angular position. The difference in height of the 500-cycle waves at the high spot as compared with those at the low spot indicates the shaft movements in thousandths of an inch when a calibration is applied. Oscillogram *A* represents 3.6 mils vibration and oscillogram *B* 6.2 mils vibration.

Oscillogram *C* in Fig. 7 shows two 500-cycle waves. The upper record was taken on the top of a rotating turbine shaft for the purpose of showing vertical shaft movements; the lower one was taken on the side of the same shaft to obtain horizontal shaft movements. The middle curve is a 40-cycle timing wave with one interruption superimposed per revolution. Ten millimeters change in height of the 500-cycle peaks represents one thousandth of an inch shaft movement.

Fig. 8 shows the vibration amplitude of the rotor in inches during a period of two and one-half hours immediately after applying 33,000 kw. load. This curve was taken on the turbine end bearing after a shut down period of 12 hours. The turbine mid-bearing showed similar variations. The shaft amplitudes were oscillatory for one and one-half hours after which period they became steady. The occurrence of these variations is due to unequal heating when the turbine is started up.

Fig. 9 is a curve plotted from data taken from oscillogram *C* in Fig. 7. Simultaneous reading on the top and at the side of the shaft, 90 deg. apart, determines the exact radial movements of a point on the shaft. In this curve vertical motion of the shaft is plotted as ordinate and the horizontal motion as abscissa. Three consecutive revolutions are plotted showing the clockwise path described by a point on the shaft. This figure shows the form of the path described by a point at the center of the shaft in making one complete revolution at normal running speed, 1800 r.p.m. The small dip to the left in the figure is only 0.0001 in. and it is the same depth for all three revolutions. At different turbine speeds the shape of this figure changed. The greatest vertical motion of the shaft shown in the figure is 0.0015 in. and the greatest horizontal motion is 0.0015 in. This figure was taken to show the movement of the shaft outside the turbine end bearing. The turbine mid-bearing showed figures of a different shape.(2)

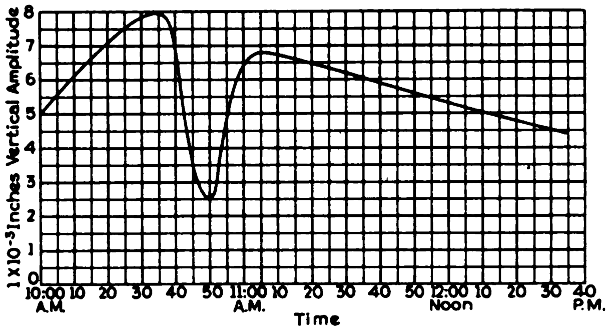

Fig. 8. Vibration Amplitude of a Turbine Shaft Immediately Following the Application of a Load of 33,000 kw.

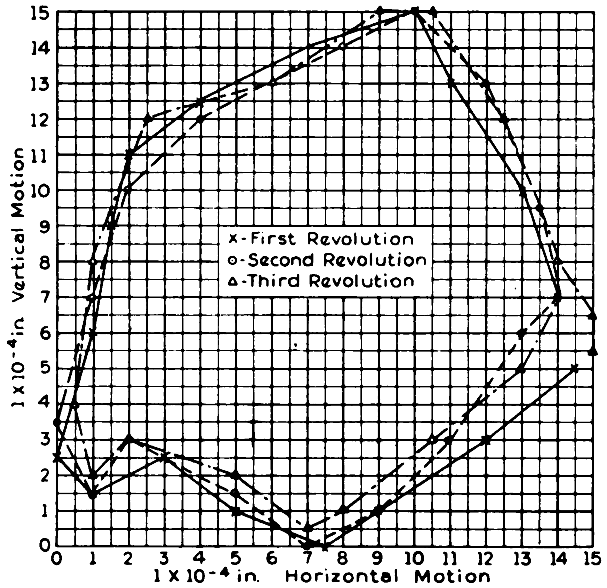

Fig. 9. Three Consecutive Figures of Revolution Made by the Point at the Center of the Turbine Shaft When Rotating at 1800 r.p.m.

When used in connection with over-speed tests on large waterwheel-generator rotors in the new high-speed test pit at the Schenectady Works of the General Electric Company, the electric micrometer provided the means of making visual observations of the test from a sheltered observation building, one-eighth of a mile away. Moreover, these observations showed vibration amplitudes of the order of thousandths of an inch.

III. COMPRESSION INDICATOR DIAGRAM OF AN ELECTRIC HOUSEHOLD REFRIGERATOR UNIT

The two foregoing applications of the electric micrometer circuit were on relatively large rotating apparatus. The use of this instrument to obtain compression indicator diagrams of an electric household refrigerator unit is somewhat on the other extreme. The refrigerator unit has a one-inch bore and a 0.625-in. stroke.

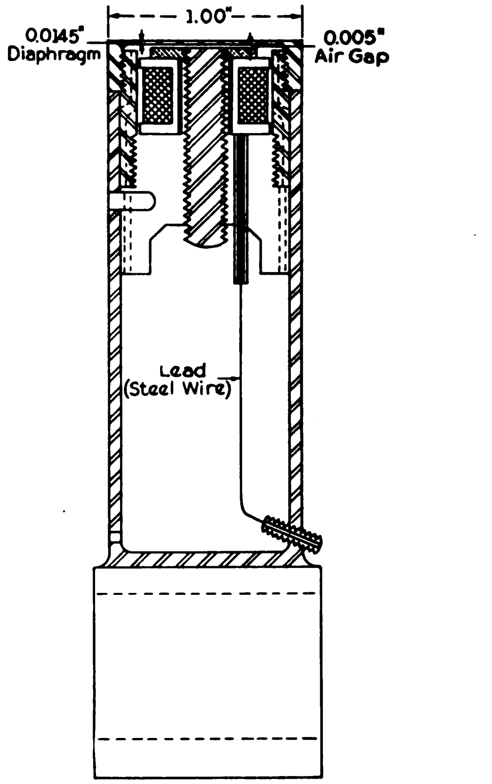

Fig. 10. Sectional Drawing of the Piston Used in Securing Compression Indicator Diagrams by Means of the Electric Micrometer

The use of mechanical devices for obtaining indicator diagrams requires that the compressor cylinder be accessible, and on account of the inertia of the moving parts, that the speed of the compressor should not be over fifteen compression cycles per second. The compressor cylinder in the refrigerator unit, however, is inaccessible as it is mounted entirely within a sealed vessel; and the piston travels at 1800 r.p.m., producing 30 compression cycles per second.

In applying the electric micrometer in this case, the test unit or hollow piston, Fig. 10, is provided with a high-speed-steel diaphragm 0.0145 in. thick which deflects about 1 mil for 20 lb. per sq. in.

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(2) The author expresses his appreciation to H. L. Davis of the Turbine Engineering Department, for his assistance on the tests and calculations described in this section of the article.

pressure acting on the piston head. This piston pressure test unit, is used instead of the pressure test unit shown in Fig. 2. There is an air gap of 0.005 in. between the diaphragm or the piston head and the core of the test coil. The surface of the core next to the diaphragm is cross-slotted to prevent any cushioning effect between the diaphragm and the core. The test coil is mounted around the core inside of the hollow piston; therefore any magnetic variations produced in the air gap give deflections on the oscillograph proportional to the pressures acting on the piston head. The diaphragm deflects 0.00005 in. at the center for one pound per square inch effective pressure. The diaphragm does not move an appreciable amount, and thus does not create any large change in volume; and the inertia effect is negligible. The magnification ratio of the motion at the center of the diaphragm as compared with the deflections on the oscillograph vibrator is 217 or the ratio of 0.01085 in. to 0.00005 in.

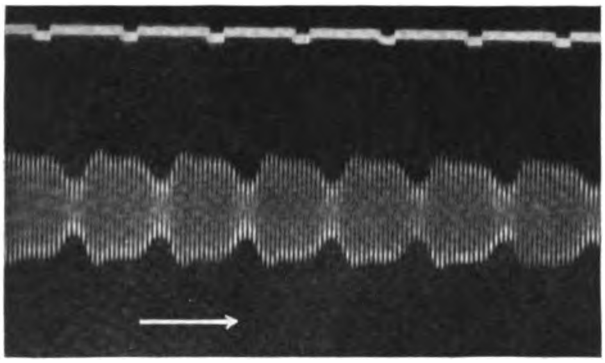

Fig. 11 (*A*)

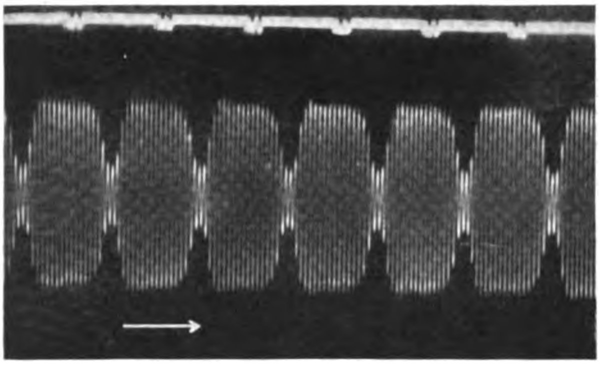

Fig. 11 (*B*)

Fig. 11. Oscillograms Showing Cylinder Pressures for Indicator Cards

A—Piston operating against 17 lb. per sq. in. pressure.
B—Piston operating against 100 lb. per sq. in. pressure.

A suitable calibration curve was obtained by plotting the pounds per square inch acting on the piston head against the millimeters deflection on the oscillograph. A calibration curve having been obtained, transient pressures can be determined from the oscillograms, Fig. 11, for any position of the piston.

It is necessary to know the piston position at a point on the time axis in order to plot an indicator diagram. A contactor mounted on the crank end of the piston closes a battery circuit through an oscillograph vibrator at the instant the piston is at the end of its intake stroke. Since the piston displacement can be calculated for any given crank angle, the displacement at any instant is determined.

Fig. 12 shows a family of compressor indicator diagrams for several refrigerator case pressures. On the lower or horizontal scale is plotted piston displacement in inches. Zero on this scale represents the time at which the piston is at the end of its compression stroke. The crank angle, or piston displacement, varies directly with time. The vertical axis represents pounds per square inch cylinder pressure acting on the piston head.

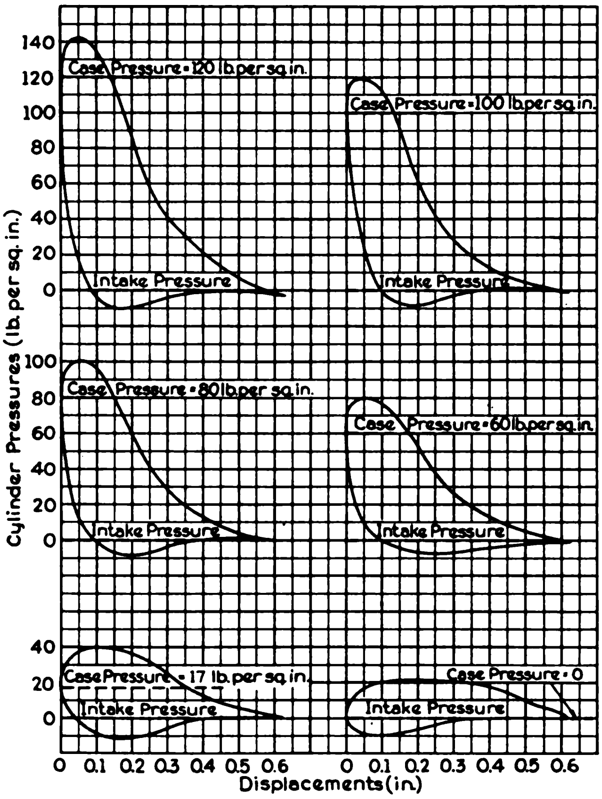

Fig. 12. Typical Indicator Cards for Various Case Pressures in a Refrigerator Compressor

The intake is opened to atmospheric pressure but the lower part of the curve represents a vacuum. The vacuum reaches approximately 10 lb. per sq. in. for all case pressures. The pressure in the cylinder rises to approximately 20 lb. per sq. in. above the case pressure for all case pressures. This pressure of 20 lb. per sq. in. is necessary to open the exhaust valve and overcome the drop in pressure in the muffler.

In a similar manner the electric micrometer may be applied in obtaining indicator diagrams showing the time-pressure characteristics of internal combustion engines as in making tests for rapidity of burning different grades of gasoline.

Temperature and Brightness of the New Standard Type of Incandescent Lamp

Design Features—Coiled Filament—Few Filament Supports—Inside-frosted Bulb—Tables of Comparative Temperature and Brightness

By W. E. FORSYTHE and E. M. WATSON
Research Laboratory, Incandescent Lamp Department, General Electric Company

THE new standard type of Mazda lamp, which has as its most distinctive feature an inside-frosted bulb, is the result of a successful endeavor to improve the construction of incandescent lamps over the range of ratings that is suitable for domestic lighting and services of like caliber. When developing the new lamps to cover this range, the number of different wattage ratings was restricted to that which would adequately fulfill the purpose.

In working out the design and determining the number of lamps which would constitute the series, close coöperation was secured between the development, manufacturing, and illuminating engineers. To satisfy all the demands of good lighting, it was found necessary to include six different sizes of lamps; *viz.*, 15 watt, 25 watt, 40 watt, 50 watt, 60 watt and 100 watt. The lamp engineers were of course most concerned in the various mechanical parts and processes of manufacture; and the illuminating engineers gave most attention to the wattage ratings and general design features. The bulb was designed by an artist without restriction as to its manufacturing practicability and afterward was found to fit in well with production processes.

A number of improvements were made in the design of the lamps during the process of simplification, so that these lamps are decidedly better than those they are intended to replace. The improvement that gave the new lamps their distinctive appearance was the method of frosting the bulb. It was very fortunate that at about the time the new lamps were being developed a method was worked out for frosting (1) the bulbs on the inside in such a manner as to produce a good diffusing surface and still maintain the strength of the bulbs. Among the other improvements may be mentioned a reduction of bulb size, and the coiling of the filament. This last improvement made it possible to reduce the number of supports and still keep the filament in place. Instead of a very long, straight filament draped over 9 to 11 supports, the filament is coiled so that three supports are sufficient. When it is considered that each support causes a loss in efficiency of about one-half of one per cent, due to conduction of heat away from the filament, it will be seen that this reduction in the number of supports results in an appreciable gain in efficiency.

A lamp bulb frosted on the outside by either the sand blasting or acid etching process absorbs from five to ten per cent of the light. A five per cent loss of light may not seem high, but nevertheless it is too high in view of the effort being made to produce the best possible light source consistent with practical operation. Frosting the bulbs on the inside by the new process was found to increase their absorption by less than one per cent of the absorption of clear bulbs of the same size and shape. It had been known for some time that the amount of light transmitted by frosted glass depends upon whether the light first strikes the frosted surface or the clear surface. A very satisfactory explanation (2) has been offered. A clear glass surface reflects about four per cent of the light which falls upon it. A frosted glass has some small parts of its surface so orientated that almost total reflection will take place for light passing from the glass to air, but this is not the case for light moving from air to glass. Thus, with frosting on the outside of the bulb a greater amount of light will be returned to the inside of the bulb where it is scattered and partially absorbed by the filament, base, or other parts of the lamp. With lamps frosted on the inside, there is less light returned to the inside of the bulb to be wasted.

The amount of light absorbed (3) by the new bulbs, which are frosted on the inside, has been measured by two methods. First, clear bulbs were selected and their absorption measured by placing them over small lamps, the light from these small lamps being measured with and without the bulbs over them, thus giving the absorption due to the addition of the clear bulb. These bulbs were then frosted on the inside and the process repeated. The difference between the two readings gave the absorption due to the inside frosting. This method is not entirely satisfactory because the absorption is not measured in exactly the same manner as it occurs when the lamp is used.

The second method is an attempt to overcome this difficulty. In this method a number of filaments were mounted on their supports in as nearly an identical manner as possible. One-half of these mounts were sealed in bulbs frosted on the inside and the other half in clear bulbs. Since this method carries with it the unavoidable variations in the manufacture of the individual lamps, it required a considerable number of lamps to obtain a proper average. Several separate determinations were made with 25 to 50 lamps having clear bulbs and 25 to 50 lamps having inside-frosted bulbs. The lumens per watt were measured and comparisons made. In Table I are given the results of these two tests, from which it can be seen that the average amount of light absorbed by the inside-frosted bulb is but about one-half of one per cent more than that absorbed by the clear bulb.

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(1) "The Inside Frosting of Incandescent Lamps," by M. Pipkin, *Industrial and Engineering Chemistry*, Vol. 18, Aug., 1926, p. 774.

(2) "The Measurement of Transmission-Factor," by M. Luckiesh and L. L. Mellor, *Jour. Frank. Institute*, Vol. 185, 1918, p. 527.

(3) Data furnished by M. Pipkin, Lamp Development Laboratory, Nela Park.

TABLE I

DATA OF ABSORPTION OF INSIDE-FROSTED BULBS

First Method: **Transmission determined by measuring the candle power of small lamps, first, with clear bulbs over these small lamps, and, second, after the same bulbs were frosted on the inside.**

Bulb	Number Tested	Clear	Inside Frosted	Per Cent Change in Absorption
PS-20	5	1.31	1.60	0.29 increase
PS-20	5	2.04	2.21	0.17 increase
PS-20	26	1.11	1.37	0.26 increase
A-21	10	1.49	1.40	0.09 decrease
A-21	30	1.56		
A-19	10		1.45	
A-21	10		1.64	
A-23	10		1.78	
Mean		1.41	1.53	0.12 increase

Second Method: **Comparison of a number of lamps with clear and inside-frosted bulbs, with the lamp filaments as nearly identical as possible.**

Lamp	Number Tested	Clear	Inside Frosted	Per Cent Change in Absorption
50	25	10.58	10.60	0.19 decrease
50	25	10.58	10.48	1.09 increase
50	9	11.19	11.14	0.34 increase
40	15	9.20	9.13	0.77 increase
50	17	11.35	11.26	0.80 increase
Mean				0.56 increase

The main reason, of course, for frosting the bulb of the lamp is to reduce the brightness of the source, because the tungsten filament in the lamp is operated at such a very high temperature that it makes a very bright source if viewed directly. The brightest point on a number of these inside-frosted lamps has been measured, and the results are given in Table II, where the brightness of some other standard sources are included for purposes of comparison. In this table is also given the brightness of the filament of the lamp as it would be observed if it were mounted in a clear bulb. It will be noted that the density of frosting adopted by the engineers is such as to reduce the maximum brightness observed by about a factor of fifty. A number of filaments of different wattages, such as are used in these new lamps, were mounted in clear bulbs and their temperatures measured. These values, together with the efficiencies of the lamps, are also given in Table II.

A coiled tungsten filament is inherently a poorer source of light than a straight tungsten filament when both are operated at the same temperature. This results from the fact that tungsten is quite selective in favor of the short wavelengths and also from the fact that when a filament is coiled part of the energy radiated from the spaces between the coils approximates that from a black body, which is a less efficient producer of light than tungsten. The loss due to the partial black radiation from the inside of coils and the small amount of light absorbed by the inside-frosted bulb combine to lower the efficiency of the new line of lamps. However, there are other factors that tend to increase the efficiency. Two of the latter are outstanding. The rate of evaporation from a coiled filament is considerably less than that from a straight filament at the same temperature. This difference

TABLE II

BRIGHTNESS OF FILAMENTS AND BULBS OF NEW STANDARD LAMPS AND SOME OTHER SOURCES FOR COMPARISON

Lamp	Volts	Watts	Type	Lumens per Watt	Maximum Temp. Deg. K	Color Temp. Deg. K.	Maximum Bulb Brightness cand./cm.^{2}	Filament Brightness cand./cm.^{2}
Kerosene flame, flat wick...						1920		1.2
4-watt per candle carbon lamp..................						2080		55
40-watt tungsten (straight filament................			Vacuum			2500		206
40-watt tungsten (frosted bulb..................			Vacuum				2.5	
Sun as observed at earth's surface................								165,000
Clear sky, average.........								0.4
New standard lamp........	115	15	Vacuum	8.6	2475		4.0	214
New standard lamp........	115	25	Vacuum	9.5	2510		4.5	245
New standard lamp........	115	40	Vacuum	10.2	2545		5.5	278
New standard lamp........	115	50	Gas-filled	10.5	2660		9.0	430
New standard lamp........	115	60	Gas-filled	11.3	2695		9.5	490
New standard lamp........	115	100	Gas-filled	13.4	2755		12.5	600

permits the operation of the coiled filament at a higher temperature with the same life. As previously mentioned, the coiling of the filament permits the number of supports to be reduced from 9 to 3, and this change in construction results in a three per cent increase in efficiency.

Another factor gives an additional though slight gain in efficiency. Because of the mutual heating effect between coils, the same current will heat a coiled filament to a higher temperature than the corresponding straight wire. This means that with coiled filaments a slightly larger filament can be used for the same temperature and the same wattage. It has long been known that the life of a lamp at a certain temperature depends upon the cross-section of the filament. Thus, the use of the larger filament permits the filament to be operated at a slightly higher temperature for the same life and this results in an increase in efficiency.

The combined effect of these five properties of the coiled filament and inside frosting produces a marked gain in efficiency for the new standard type of lamps over the old straight-filament type of tungsten lamps.

New Lighting Units for Airports

An intermediate field boundary light and a ceiling light are two new units that have recently been made available for airport illumination.

The intermediate field boundary light is intended for use on a multiple circuit at small airports where the number of lamps around the entire field will not exceed 30. The energy supply for such installations may be distributed through two-conductor, No. 10 parkway cable laid in two separate circuits to a point half-way around the field. The units are equipped with clear, ruby, or green globes. Clear globes are used to indicate the boundaries, and ruby globes to mark obstructions, while the most favorable approaches to the field are indicated by green globes. The units may be equipped with 25-, 40-, 50- or 60-watt lamps. Straight and tee coupling boxes facilitate the making of connections to the cable.

The ceiling light is employed at airports to indicate to approaching pilots the approximate height of fog or clouds above the ground. It is equipped with a non-corrodible aluminum-alloy drum, attached to the side of which is a pad for setting the beam of the searchlight at 45 deg. instead of horizontally. As this light need not be changed after the initial setting, a permanent method of mounting is provided.

Multiple Boundary Light

Airport Ceiling Light

The front door glass is heat-resisting and is made in a dome shape to provide additional strength. This glass is mounted in a frame which is hinged to the drum, and a rubber gasket is placed between the door frame and the drum to exclude moisture. A universal focusing mechanism enables the lamp to be focused in the mirror from outside the drum. The 500-watt 115-volt incandescent lamp used may be of either the medium-screw or the mogul-screw base. It gives a beam of not more than 8 deg. with a candle power of 400,000.

An Improvement in Single-phase Motor Design

Change in Stator Core Design Makes Possible Better Magnetic Utilization—New Motor More Economical in Material, Lighter, and Smaller

By NEIL CURRIE, JR.
Motor Department, Pittsfield Works, General Electric Company

THE efficient use of materials and the elimination of waste are of prime importance in the manufacture of any device. Engineers and manufacturers are continually striving to reduce the amount of inactive or unnecessary material in their designs, and consequently to reduce the scrap or wastage of active material in the product itself. The outcome of this research work, together with the development and use of materials of better quality and capable of carrying greater loads, has resulted generally in the production of smaller and more economical devices that are of improved quality and give better performance.

In the design and manufacture of single-phase motors, the economical use of material has received much attention. For example, the single-phase repulsion-induction motor (1) manufactured by the General Electric Company has recently undergone various improvements which have already been introduced in the smaller sizes.

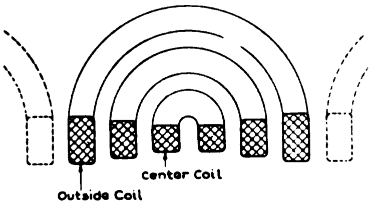

Fig. 1. Graded Distribution of Copper in One Pole Section of a Concentric-type Winding

In the construction of the stators for these motors there is a notably better utilization of punching steel, an appreciable amount of which has heretofore been unutilized.

These improvements are interestingly reflected as an outcome of the following consideration:

The stator winding of a single-phase repulsion or repulsion-induction motor is usually of the concentric type, as this gives the most desirable distribution of the field flux when the coils per pole are assigned numbers of turns graded in such a manner that the outside coil has the most and the center coil the least amount of copper. This general arrangement is illustrated in Fig. 1 which shows one pole of such a winding.

In the usual type of stator punching for the concentric winding all the slots have been of uniform depth, so that with the graded winding the outside slots only of each pole are full of copper, the center slots being only partially filled as shown in Fig. 2. The depth of steel shown at *A* is therefore lost and the motor is unnecessarily large.

If the depth of slots is graded to conform with the distribution of the copper in the coils, the material represented by *A* can be either utilized as a part of the active core *B* or the outside dimension of the punching *C* can be reduced by double this amount (diametrical measure equal to twice radial measure) without sacrifice of performance.

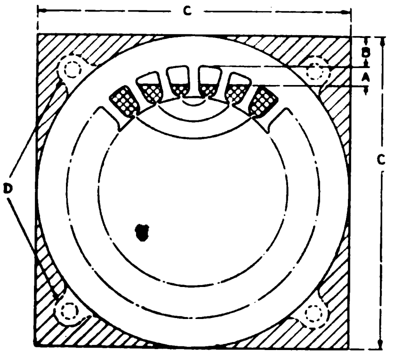

Fig. 2. Diagram of Circular Type of Stator Punching with Slots of Uniform Depth, Showing Wasted Corner Material and Incomplete Utilization of Material at the Pole Centers

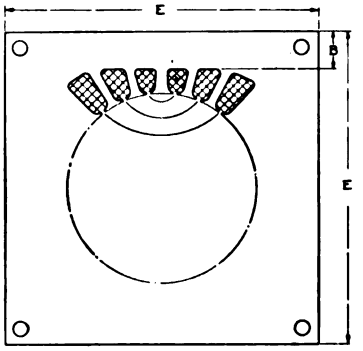

Fig. 3. Square Stator Punching with Graded Slots, Which Form Results in Better Magnetic Utilization of the Steel

In the ordinary circular punching, the material indicated by the shaded portions of Fig. 1 at the corners *D* is routed to the scrap pile. A more efficient employment of this material is therefore desirable.

By combining the use of the graded slot and of the material in the corners, full advantage is taken of the laminated steel employed. With the graded slots arranged around the circumference of the motor bore, the most economical stator punching for a four-pole motor will be a square in which the small slots lie along the sides of the square and the large slots occupy the corners formerly scrapped, thus utilizing the steel at full operating density. The development of the square punching for the four-pole motor is illustrated in Fig. 3, in which the outside dimension E is smaller than the outside dimension C (in Fig. 2) by twice the amount A.

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(1) Identified by the trade designation "Type "SCR."

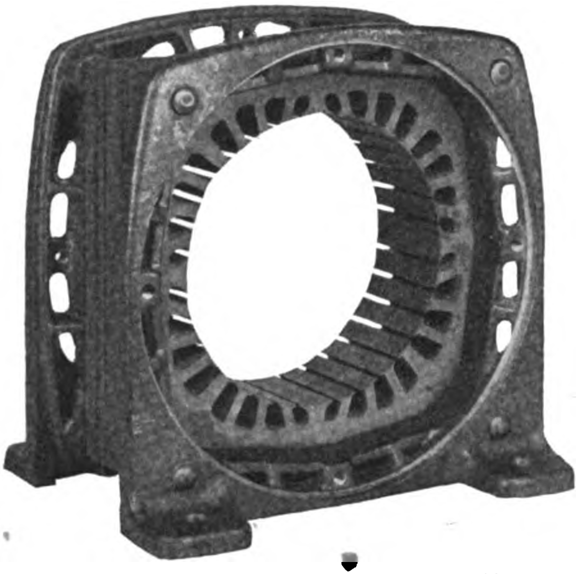

Fig. 4. Stator Frame, with Improved Form of Punchings, Ready for Windings

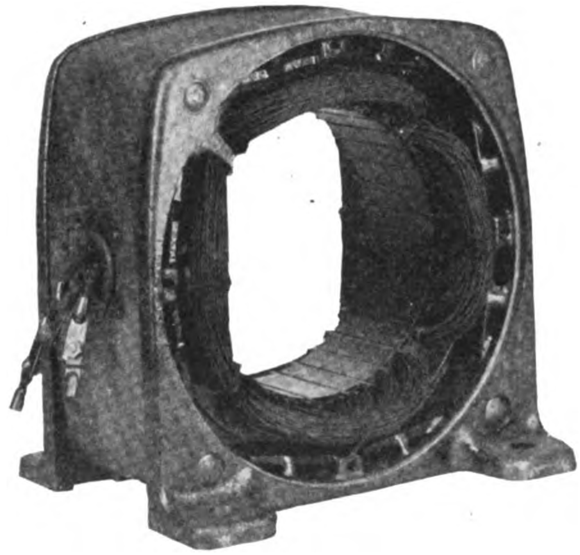

Fig. 5. Completely Wound Stator Frame of the Type Shown in Fig. 4

An additional advantage in this development is that the outside dimensions of the core, instead of being decreased as has been mentioned, can be held fixed with corresponding decrease in length of core stacking. This makes it possible to secure the same length of core for the single-phase motor as is used in the polyphase motor of equal rating and therefore interchangeability in mounting dimensions can be secured. The assembly of these punchings into a stator core is shown in Fig. 4; and the wound stator is shown in Fig. 5.

The motor for which this stator core construction has been adopted is a repulsion-induction motor of high starting torque, low starting current, close speed regulation, and high operating characteristics, including power-factor approaching unity. The repulsion-induction effect, *i.e.*, the combination of the high-starting-torque repulsion motor and the constant-speed induction motor, is secured through two windings, one a squirrel cage and the other of the repulsion type, so related by slot proportioning in the rotor that the squirrel cage is relatively inactive at the start but assumes its proportion of the load when operating under normal conditions. This gradual changeover is effected by the changing magnetic reluctance and consequent shift of torque production between the squirrel cage and repulsion windings as the rotor approaches synchronous speed, resulting in smooth and positive acceleration which contrasts with the sharp break in the accelerating torque of a repulsion-induction motor not provided with a squirrel-cage winding but employing a centrifugal switch.

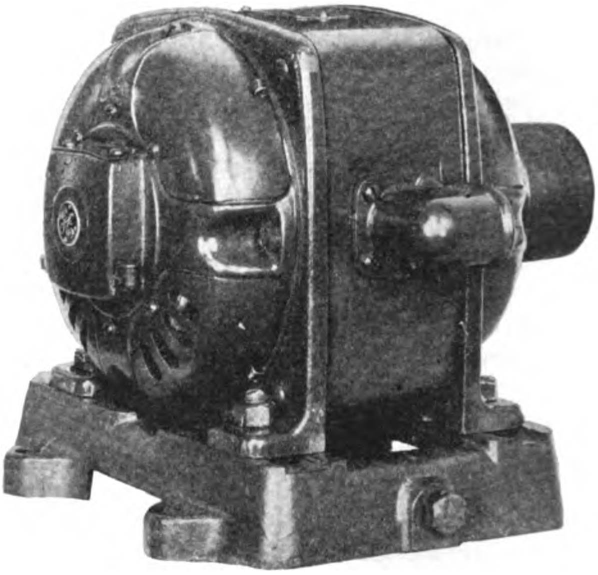

Fig. 6. The Complete Motor of the Improved Type

The complete assembly of the "square punching stator" motor just described is shown in Fig. 6. This construction has made possible a motor of unusual economy in material, weight, and space without sacrifice of operating performance.

Automatic Control for Mercury-arc Rectifiers

Fundamental Divisions of the Control—Temperature—Pressure—Starting and Stopping—Switching and Protection, A-c. and D-c. Systems—Equipment and Operation

By H. T. SEELEY

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Philadelphia Works, General Electric Company

A MERCURY-ARC rectifier functions as a group of electrical valves controlling the flow of current from a star-connected polyphase network to a direct-current load. The valve elements—anodes and cathode—are assembled in a tank, and their valve action depends on the maintenance of the internal temperature and pressure below certain limiting values, and on the initial ignition. There is also a practical lower limit to the temperature. It has not been found feasible to construct glass tanks for units of the sizes required for traction systems, where the unusual efficiency appears most attractive; but improvements in the art of welding and in the technique with exhaust pumps have made possible the commercial manufacture of metal tanks having such small leakage that very low pressures can be maintained continuously under operating conditions. Fig. 1 shows two 500-kw. 600-volt mercury-arc rectifiers of this type with their necessary auxiliary equipment.

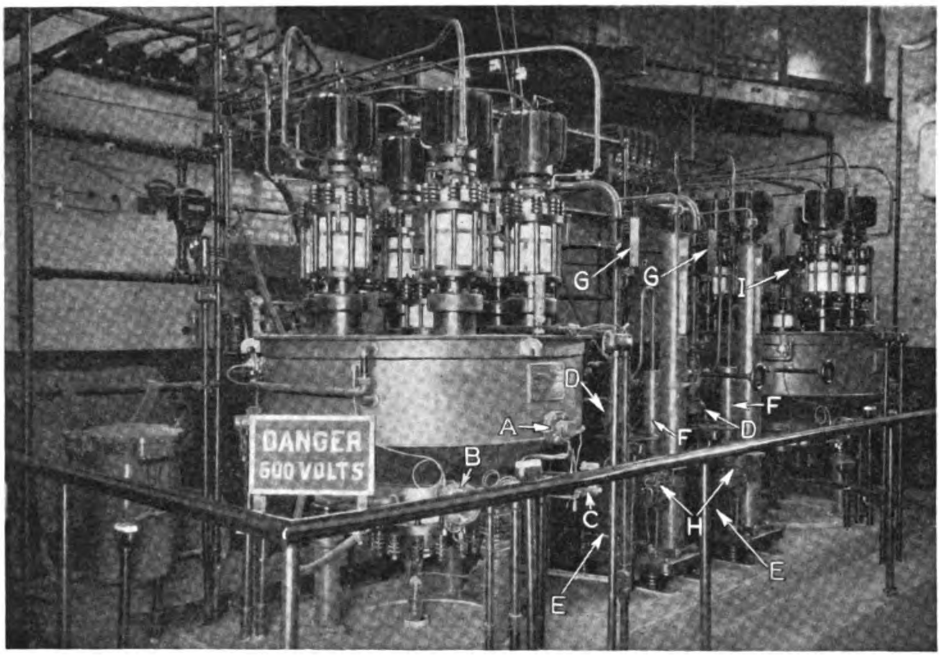

Fig. 1. An Installation of Two 500-kw. 600-volt 60-cycle Mercury-arc Rectifiers

A: Jacket heater.
B: Temperature regulating relay, *No. 23.*
C: Apparatus thermostat, *No. 26.*
D: Mercury condensation pump.
E: Rotary exhaust pump.
F: Gas receiving tank and oil trap.
G: Pressure relay bulb housing.
H: Solenoid for operating oil valve.
I: Exciting anode solenoid.

The control equipment for a rectifier may consist of: (1) temperature control equipment, (2) pressure control equipment, (3) ignition equipment, (4) switching and protective equipment for the polyphase system, (5) switching equipment for the direct-current system.

Alternating-current control is generally used in order to eliminate the initial investment and maintenance cost of a battery. Alternating-current service is usually employed for the auxiliary motors because their drain on a battery would require excessively large charging equipment. When alternating-current control is used, the necessary direct-current operated devices are fed from the unit itself.

Temperature Control Equipment

The function of this equipment is the regulation of the cathode and tank temperature to the range where best operation is obtained. With the exception of tropical installations, jacket heaters (*A* in Fig. 1) are required in order to maintain the desired tank temperature and to avoid the possibility of the cooling water freezing. These heaters are controlled by a thermal relay (*No. 23* in Fig. 2 and *B* in Fig. 1) which energizes them by means of an auxiliary contactor *No. 23 R* when the jacket water surrounding the sensitive bulb is below a normal operating temperature, and deënergizes them when normal temperature is regained. If the temperature falls too far below normal (possibly due to an interruption of alternating-current power), protective relay *No. 26* prevents the unit from going into operation until the jacket heaters have restored normal operating temperature. This and the other sections of the complete control are included in the schematic connection diagram shown in Fig. 2.

Usually, water is used to cool the rectifier by flowing through the cathode and exhaust-chamber jackets. Tap water is used where it is free enough from impurities. If suitable well or tap water is not available, a closed cooling system may be used in which the circulating water is cooled by air, using a blower fan and radiator. The air tunnel which houses the radiator

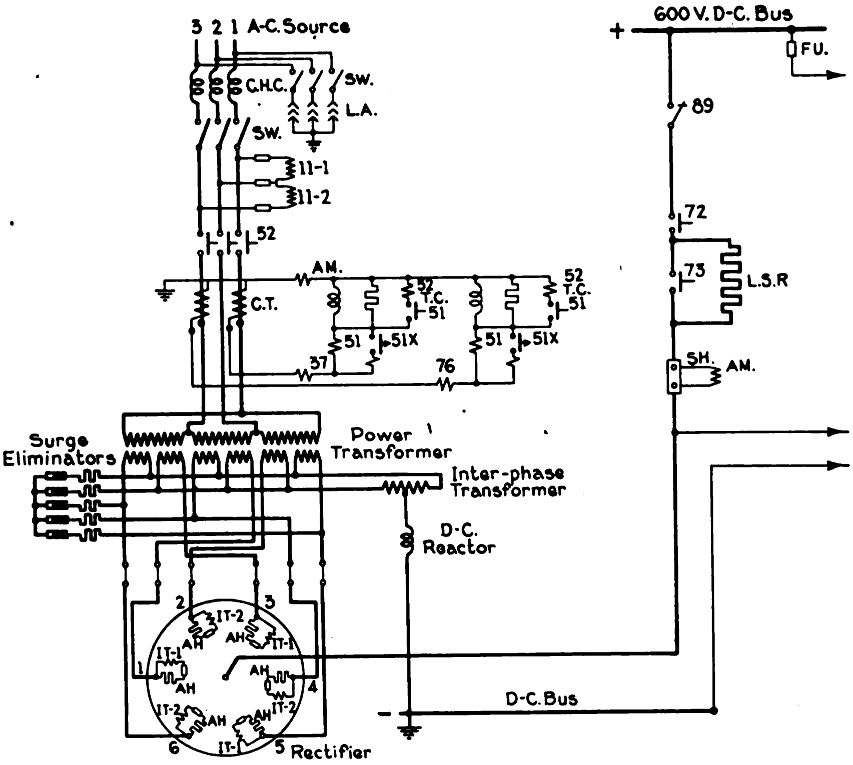

Fig. 2. Schematic Diagram of Connections for the Automatic Switching Control of a Mercury-arc Rectifier

T. C.:	Trip Coil
SW.:	Switch
S. R.:	Stabilizing Resistor
SH.:	Shunt
L. S. R.:	Load Shifting Resistor
L. A.:	Lightning Arrester
I. T.:	Insulating Transformer
H. C.:	Holding Coil
FU.:	Fuse
C. T.:	Current Transformer
C. R.:	Control Relay
CH. C.:	Choke Coil
C. CR.:	Control Contactor
C. C.:	Closing Coil
AM.:	Ammeter
A. H.:	Anode Heater
63WX:	Auxiliary Relay for 63W
63W:	Condensation Pump Cooling Water Flow Relay
89	D-c. Line Switch
88W:	Cooling Water Circulating Pump Motor
88VX:	Auxiliary Contactor for 88V
88V:	Rotary Vacuum Pump Motor
88F:	Cooling Fan Motor
81E:	Auxiliary Generator (arc striking set)
88E:	Auxiliary Motor (arc striking set)
86	Locking-out Relay (hand reset)
81EX:	Arc Striking Anode Solenoid
79	A-c. Reclosing Relay
76	D-c. Overcurrent Relay
73	Load Shifting Resistor Shunting Contactor
72X:	Auxiliary Relay for 72
72	D-C. Line Contactor
63VY:	Auxiliary Relay for 63V
63VX:	Auxiliary Relay for 63V
63V:	Air Pressure Relay
62	Time Delay Stopping Relay
52	Oil Circuit Breaker and Mechanism
51X:	Auxiliary Contactor for 51
51	A-c. Overcurrent Relay (protective)
37X:	Auxiliary Contactor for 37
37	A-c. Undercurrent Relay
28	Resistor Thermal Relay
27X:	Auxiliary Relay for 27
27	A-c. Undervoltage Relay
26	Apparatus Thermostat
23R:	Auxiliary Contactor for 23 (raise)
23L:	Auxiliary Contactor for 23 (lower)
23	Tank Temperature Regulating Relay
20V:	Solenoid-operated Vacuum Valve
14	Underspeed Device
11-2	Control Power Transformer
11-1	Control Power Transformer
8	Control Power Switch
5	Stopping Switch
4	Master Contactor
2	Time Delay Starting Relay
1	Master Element

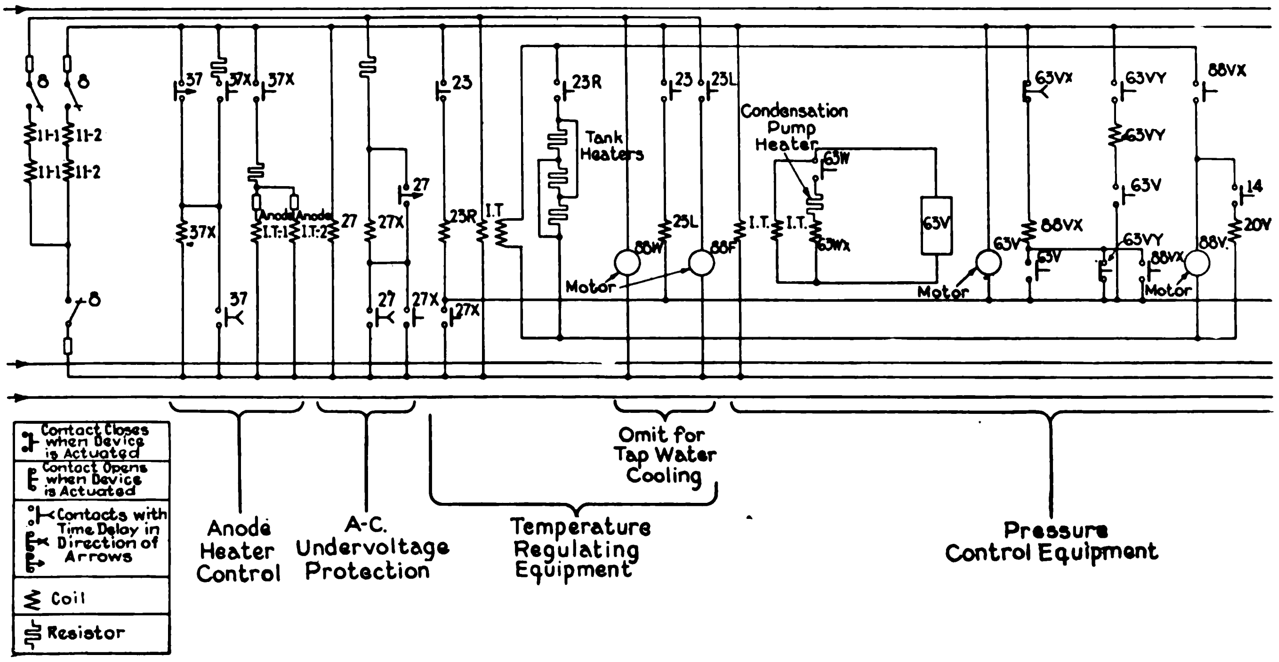
8
8
11-1
11-2
37
37X
37X
23
23R
23
23L
Tank Heaters
Condensation Pump Heater
63W
63WX
63V
63VX
63VY
88VX
14
20V
27
27X
1.T.
1.T.-1
1.T.-2
Anode
88W
88F
88V
Motor
Contact Closes when Device is Actuated
Contact Opens when Device is Actuated
Contacts with Time Delay in Direction of Arrows
Coil
Resistor
Anode Heater Control
A-C. Undervoltage Protection
Omit for Tap Water Cooling
Temperature Regulating Equipment
Pressure Control Equipment

Fig. 2 (*Cont'd*). Schematic Diagram of Connections for the Automatic Switching Control of a Mercury-arc Rectifier and fan for a cooling system is shown in the upper right-hand corner of Fig. 1. The piping schemes for these two methods of cooling are shown in Figs. 3 and 4. In either system, the temperature regulating valve is the main cooling control, and is all that is required for tap-water cooling. In the recirculating system, the pump runs continuously in order that the temperature regulation be as gradual as possible; and at loads where the natural cooling effect of the radiator is insufficient, a second contact on thermal relay *No. 23* starts the fan by means of auxiliary contactor *No. 23 L* when the water temperature reaches a predetermined value. The relay stops the fan after it has cooled the water sufficiently. In either system, if the jacket-water temperature rises to a critical value, relay *No. 26* (*C* in Fig. 1) shuts down the equipment until the temperature falls to an operating value, when it can restart.

In order that relay *No. 26* may not be open when the unit is called upon to start, relay *No. 23* is made to regulate the temperature continuously with the unit either in or out of service.

Pressure Control Equipment

The exhaust system is shown schematically in Fig. 5. A cooling jacket is placed on the exhaust line of the rectifier to prevent mercury vapor from passing into the pumping equipment. A hand-operated valve in this line allows the tank to be isolated in testing the line for leaks, and in making repairs or adjustments to the gauges or pumping equipment. The vacuum is measured by both a McLeod gauge and a vacuum-indicating meter *No. 63V*.

In the exhaust system are connected a mercury condensation pump, gas receiver, and rotary pump.

The two-stage mercury condensation pump (*D* in Fig. 1) gives very rapid evacuation at back-pressures well below atmospheric pressures, and for this reason a rotary exhaust pump (*E* in Fig. 1) is used to reduce the back-pressure to an amount at which advantage can be taken of this valuable characteristic of the two-stage mercury condensation pump. A gas receiver (*F* in Fig. 1) is interposed between the two pumps so that the rotary pump need operate only intermittently, even though the mercury condensation pump (which has no moving parts) operates continuously in order to maintain low pressure. With this method of operation the rotary pump starts possibly once a day, and runs only a few minutes at each time. The useful life of the pumping equipment is therefore greatly increased.

In the event of failure of the supply of cooling water to the mercury condensation pump, it is necessary to shut down the equipment, which operation is accomplished by a water-flow relay *No. 63 W* and an auxiliary relay *No. 63 WX*.

The control of the rotary pump and the protection of the rectifier against operation with too high an internal pressure are both effected by a hot-wire manometer having the necessary auxiliary relays. A fixed current is passed through a filament mounted in a removable bulb (*G* in Fig. 1) tapped into the exhaust line. Since the rate of cooling of a heated object depends on the density of the surrounding gases, the temperature of this filament is an indication of the gas pressure in the pipe line. This temperature is measured by means of a relay *No. 63 V* which closes a contact, when the pressure becomes slightly higher than that necessary for proper operation, to start the pump motor *No. 88 V* by means of an auxiliary contactor *No. 88 VX*. As this motor approaches normal speed, a speed switch *No. 14* energizes a solenoid-operated valve *No. 20 V* (*H* in Fig. 1) which opens

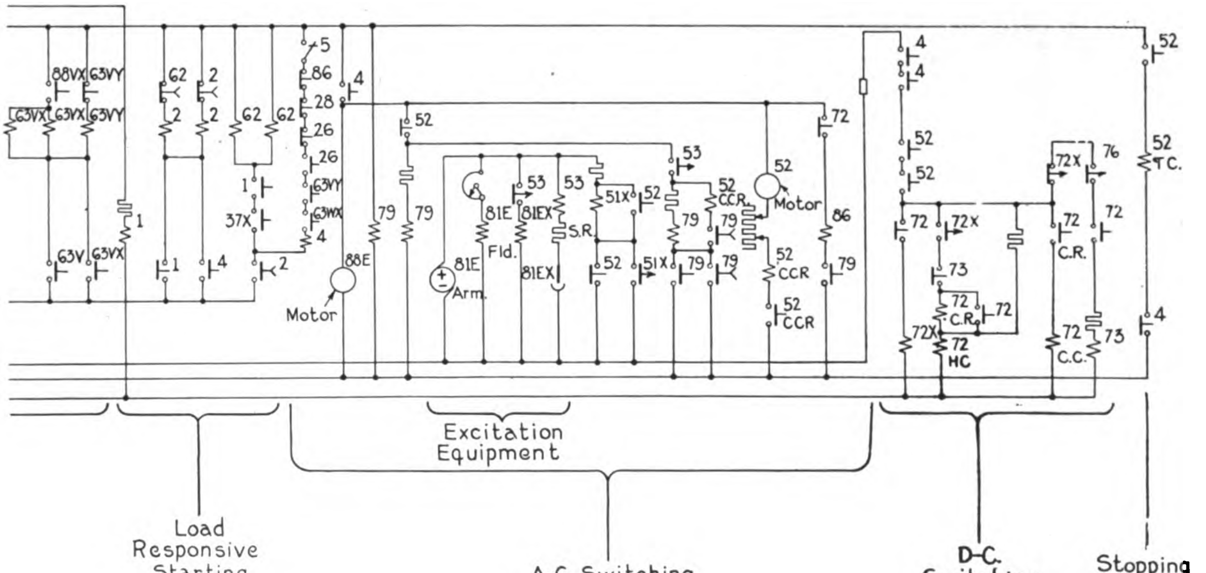

Fig. 2 (*Con'td*). Schematic Diagram of Connections for the Automatic Switching Control of a Mercury-arc Rectifier

the passage from the gas receiver to the rotary exhaust pump. A second contact of *No. 63 V* closes at a critically increased pressure within the rectifier to energize auxiliary relay *No. 63 VY* which opens the master contactor *No. 4* to prevent the operation of the rectifier and also insure that auxiliary contactor *No. 88 VX* is energized to exhaust the tank. A third contact closes, when the pressure becomes proper for operation, to energize auxiliary relay *No. 63 VX* and reset relay *No. 63 VY*, which allows contactor *No. 4* to place the rectifier in operation once more. After a time delay to insure that a proper operating pressure is attained within the rectifier, relay *No. 63 VX* opens contactor *No. 88 VX* to stop the rotary pump.

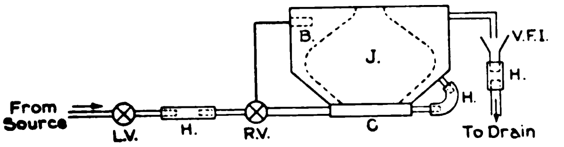

Fig. 3. Elementary Diagram of Tap-water Cooling System for Rectifier

B: Thermosensitive bulb.
C: Cathode.
H: Insulating hose.
J: Rectifier water jacket.
LV: Flow limiting valve.
RV: Temperature regulating valve.
VFI: Visual flow indicator.

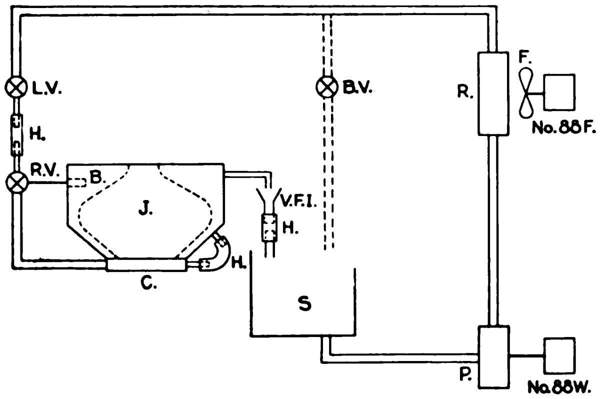

Fig. 4. Elementary Diagram of Recirculating Cooling System for Rectifier

B: Thermosensitive bulb.
BV: By-pass valve (if used).
C: Cathode.
F: Fan.
H: Insulating hose.
J: Rectifier water jacket.
LV: Flow limiting valve.
P: Centrifugal water pump.
R: Radiator.
RV: Temperature regulating valve.
S: Sump tank.
VFI: Visual flow indicator.

To maintain the unit in constant readiness for service, the pressure control is made to function irrespective of whether the unit is in operation.

Starting and Stopping Indication

The unit may be started and stopped like a synchronous converter by remote control, local lever switch, time switch, or load-responsive starting equipment. The latter method is favored as being the most flexible and completely automatic. With this form of control, low trolley voltage causes relay *No. 1* to energize time-delay relay *No. 2*, so that if the low voltage continues for a definite time the starting indication is given. When the unit starts, the trolley voltage rises to normal; and, after the load demand has passed, relay *No. 37* operates auxiliary contactor *No. 37 X* to energize time-delay relay *No. 62* so that if the load demand remains below a set value for a predetermined time, the stopping indication is given. In the case of succeeding units in a multiple-unit station, overcurrent and undercurrent in the bus are used as starting and stopping indications instead of low voltage and underload; and provision is made so that if any required unit fails to start, the next succeeding available unit will start in its place.

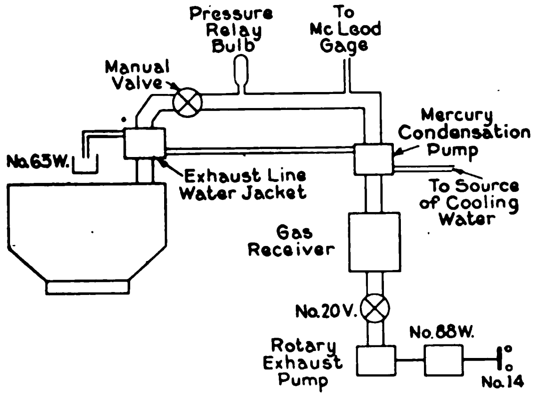

Fig. 5. Elementary Diagram of Exhaust System for Rectifier

The small number of devices involved in the starting sequence of a rectifier makes it possible to start and stop it oftener than a synchronous converter without excessive wear on the apparatus.

Excitation and A-c. Switching Equipment

With alternating-current control, low-voltage protection is required by the control devices. Protection against single-phase voltage is required because single-phase operation reduces the number of active main anodes in service, and thus causes them to overheat at direct-current loads lower than that for which protection can be provided. Single-phase operation would also increase the magnitude of ripple voltage on the direct-current side. Since a rectifier has no rotating magnetic field it cannot function as a phase converter, and the separate current phase-balance protection required for rotating machines is not necessary.

When the starting indication is given in any manner, the unit is checked for:

(1). Safe tank temperature. (See "Temperature Control Equipment")

(2). Safe internal pressure. (See "Pressure Control Equipment")

(3). Cooling water for mercury condensation pump. (See "Pressure Control Equipment")

(4). Low or single-phase alternating-current voltage by relays *No. 27* and *27 X*.
(5). Safe temperature of load-shifting resistors. (See "Direct-current Switching Equipment")
(6). Freedom from previous repeated alternating-current overcurrent.

If all of these conditions are correct, master contactor *No. 4* closes and the auxiliary generator *No. 81 E* energizes the exciting anode solenoid *No. 81 EX* (*I* in Fig. 1). The exciting anode then touches the mercury cathode and current flows through the stabilizing resistor to energize exciter relay *No. 53* which deënergizes exciting anode solenoid *No. 81 EX*. A spring then withdraws the exciting anode from the cathode and an arc is drawn and maintained. Auxiliary contactor *No. 51 X* is also energized from generator *No. 81 E* and closes around the alternating-current overcurrent relay *No. 51*. Reclosing relay *No. 79* closes oil circuit breaker *No. 52* after a few seconds if relay *No. 53* indicates that the exciting arc is being maintained. Contactor *No. 51 X* opens after sufficient time has passed for the power transformer magnetizing transient to disappear.

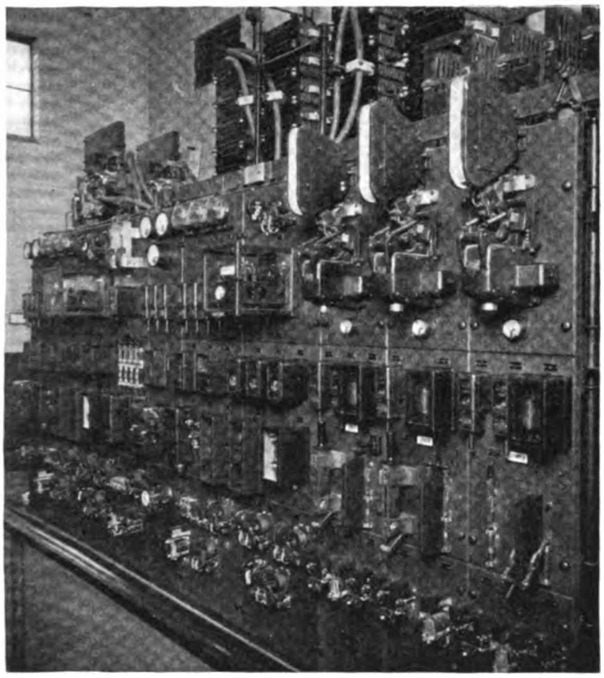

Fig. 6. Switchboard for the Automatic Control of Two Mercury-arc Rectifiers and Three Feeders

D-c. Switching Equipment

When the exciting arc is maintained and the alternating-current breaker is closed, direct-current voltage appears between the cathode and the neutral of the star-connected alternating-current network. This energizes auxiliary relay *No. 72 X* which closes the direct-current line circuit breaker *No. 72*. In the absence of direct-current overcurrent, contactor *No. 73* shunts out the load-shifting resistor. The whole starting sequence may be completed in less than ten seconds, if the first closure of the reclosing relay is adjusted for its shortest interval.

If breaker *No. 72* fails to close, or if it opens and remains open for three minutes while master contactor *No. 4* is closed, relays *Nos. 79* and *86* lock out the unit, thus giving the starting indication for the succeeding unit in a multiple-unit station.

If the direct-current load exceeds the capacity of the rectifier, relay *No. 76* opens shunting contactor *No. 73* to insert the load-shifting resistor. If the overload continues long enough to overheat the resistor and rectifier, the heated air from the resistor causes thermal relay *No. 28* to open contactor *No. 4*, shutting down the rectifier. It will restart when the equipment has become cool.

The valve characteristic of the rectifier makes it unnecessary to check the relative voltages of the unit and the bus (which operation is necessary with other conversion apparatus) and likewise the system voltage may rise above the rectifier voltage without requiring a reverse-power relay to disconnect the rectifier from the system.

The rectifier is relatively insensitive to direct-current short-circuits, so that the direct-current feeder breaker and the load-shifting resistor-shunting contactor need not be especially fast in operation.

Parallel operation with compound conversion equipment of the same or different type is obtained by a saturating winding on the interphase transformer, and requires no more switching or regulating equipment than for the parallel operation of synchronous converters.

Comparison with Synchronous Converter Equipment

Compared with a synchronous converter, the rectifier eliminates the following operations:

Starting and running voltage application.
Brush raising and lowering.
Over-speed protection.
Polarization.
Polarity checking.
Bearing temperature protection.
Current phase balance protection.
Voltage directional protection.

On the other hand, new functions are assumed by the rectifier equipment as follows:

Tank temperature regulation.
Tank temperature protection.
Tank pressure regulation.
Tank pressure protection.
Mercury condensation pump protection.

The result of these differences is that the number of devices required for the automatic control of a rectifier is less than four-fifths as many as for the similar control of a synchronous converter. This fact indicates that even greater reliability may be expected of the switching equipment for rectifiers, with the same maintenance.

Balancing of Rotors in Factory and at Installation

Measurement and Location of Bearing Forces—Simple Beam and Lever Relations Applied to Unbalance—Balancing of an Assembled Machine—Mathematical Analysis of Forces—Quartering Curves

By S. H. WEAVER

Turbine Engineering Department, General Electric Company

Where a pound of unbalanced weight may produce a ton of effective force in a rotating machine it is necessary to seek out and eliminate any such potential malefactor. This is a basic article on the art of balancing rotors. The principles used to obtain perfect balance are given in simple terms, and a systematic procedure is outlined for their application.
—EDITOR

THE effect of an unbalance in the weight of any part of a running rotor is a centrifugal force which places a reaction on each bearing, as in the case of a simple beam. No matter how many of these unbalances there may be or how they are distributed along the shaft or around the rotor, all the unbalances combine as their sum into one reacting force on each bearing. In Fig. 1 these bearing forces are represented by C_1 and C_2. They are radial, rotating with the rotor; and one force is usually in advance of the other. It is these forces that produce the vibration of the bearings, stator, foundation, etc. If these bearing forces can be neutralized, *i. e.*, reduced to zero by counterbalance weights, the machine cannot vibrate and the rotor is in perfect balance.

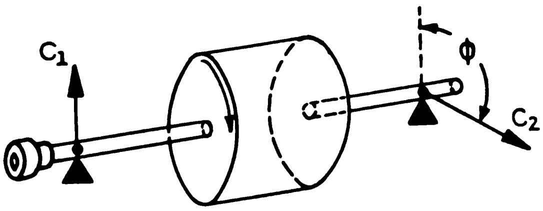

Fig. 1. An Elementary Diagram of a Rotor Showing Condition of Unbalance as Forces at Bearings

The problem is (1) to measure the value and locate the positions of the bearing forces C_1 and C_2; and (2) with these forces known, to place counterbalancing weights on the rotor to bring C_1 and C_2 to zero.

Factory Balance

Rotating parts are balanced in the factory before final assembly of the rotor in the machine. For steam turbine work, the rotor is run in a balancing machine to obtain readings from which the bearing forces C_1 and C_2 are estimated. In principle this embodies a spring-supported bearing at one end and a rigidly supported bearing at the other. The springs compressed by the rotor weight have a natural frequency somewhere between 150 and 400 r.p.m., at which speed the rotor is run because the resonance vibration will give the largest amplitude for a given unbalance. The underlying principle of the machine is the imparting of sensitiveness to the unbalance by running at this very low critical speed given by the springs.

The rotor is first run at this exact critical speed and a pointer held lightly to the chalked surface of the whirling shaft to leave a mark on part of the circumference. The center of the mark gives the direction of the shaft flexure, or high spot of rotation. At critical speed, however, the centrifugal force of the unbalance is 90 deg. ahead of the high spot so that a counterbalancing weight must be added 90 deg. back of the center of the mark. The counterbalance weight M_1 in Fig. 2 is thus located in direction and may then be run out on the balance arm until vibration is zero at the spring-supported bearing. This balance weight (M_1) gives a centrifugal force which acts as a load or force on the shaft. The weight of the rotor always acts downward while the centrifugal force of the balance weight rotates with the shaft. Hence the shaft itself is a simple beam supported at the bearings and loaded with this rotating centrifugal force from M_1. The centrifugal force has exactly neutralized the unknown bearing force C_1 which caused the bearing to vibrate. Thus the reaction C_1 due to the centrifugal force of M_1 can be found by taking moments about the

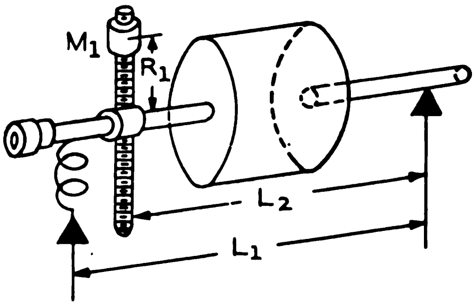

Fig. 2. Diagram of a Rotor in the Balancing Machine in Position for First Reading

rigidly supported bearing. The reaction, expressed as a product of the unbalanced weight and its radius is

$$C_1 = M_1 R_1 \frac{L_2}{L_1} \qquad (1)$$

As speed for the centrifugal force enters all equations, this factor cancels out; and weight multiplied by radius can be used as a measure of centrifugal force.

Repeating the tests for the other end of the rotor, as in Fig. 3, with the coupling bearing rigid and the other bearing elastically supported gives

$$C_2 = M_2 R_2 \frac{L_3}{L_1} \qquad (2)$$

located at the angle ϕ from C_1.

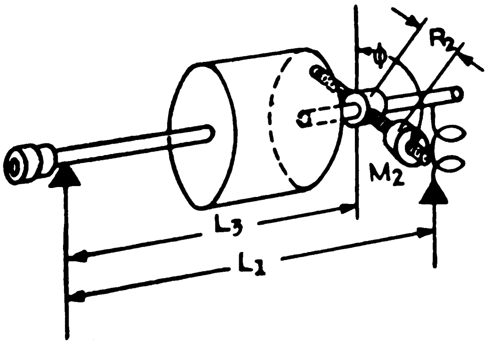

Fig. 3. Diagram of a Rotor in the Balancing Machine in Position for Second Reading

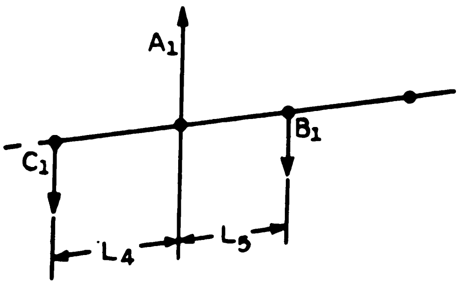

Fig. 4. Diagram of Forces Involved in the Leverage Principle of Balance Weights

From these readings C_1 and C_2 with their location on the rotor circumference are known.

Determining Balance Weights

The next step is to place weights on the rotor that will neutralize the bearing forces C_1 and C_2. Each bearing force can be neutralized independently of the other by means of balance weights placed on each end of the rotor. For example, in Fig. 4 the force C_1 on the bearing can be overcome by a balance weight for force A_1 on one end of the rotor and a balance weight for force B_1 on the opposite end of the rotor; and these two balance-weight forces can be given values that will neutralize C_1 and yet produce no force or disturbance at the other bearing where C_2 acts.

Since no force is wanted at the opposite bearing (C_2) that bearing should not be considered as a support for the beam. Instead of the simple supported-beam equation previously used the equally simple lever formula will give values for A_1 and B_1 when the distances and C_1 are known. This simple lever relation is

$$A_1 = C_1 \frac{L_4 + L_5}{L_5} \qquad (3)$$

$$B_1 = C_1 \frac{L_4}{L_5} \qquad (4)$$

If balance weights producing the forces A_1 and B_1 are placed on the ends of the rotor in the directions given in Fig. 4 and in the same plane as C_1, they will neutralize the bearing force C_1 and will have no influence at the other bearing.

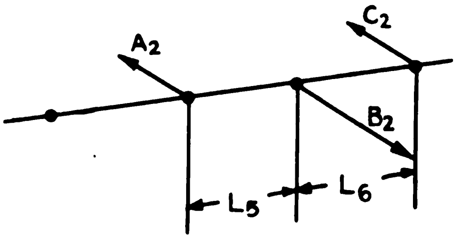

Fig. 5. The Leverage Principle Applied to the Other End of Rotor (Second Position)

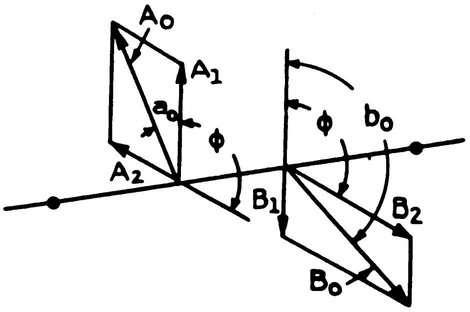

Fig. 6. Force Diagrams for Rotor Unbalance Showing Graphical Method of Combining Weights

In the same manner, in Fig. 5, the balance weight forces A_2 and B_2 acting in the place of C_2 and having the simple lever values of

$$A_2 = C_2 \frac{L_6}{L_5} \qquad (5)$$

$$B_2 = C_2 \frac{L_5 + L_6}{L_5} \qquad (6)$$

will neutralize the bearing force C_2 and have no influence on the other bearing where force C_1 acts.

If the four weights represented by the forces A_1, A_2, B_1, and B_2 are placed on the rotor, both C_1 and C_2 are neutralized and the rotor is in perfect balance both statically and dynamically.

While it would be satisfactory to place two weights (as A_1 and A_2) on one end of the rotor, it is customary to place only one weight on each end. The two weights on either end can be replaced by a single weight which is determined by means of a graphical force diagram. Fig. 6 shows a perspective force diagram where the weights A_1 and A_2 are drawn to scale in value and direction, then combined into the single weight A_0 at the angle a_0. Likewise, B_1 and B_2 may be replaced by a single weight B_0 at the angle b_0. Note that A_1 is in the direction of test weight M_1, A_2 opposite to the direction of M_2, B_2 in the direction of M_2 and B_1 opposite in direction to M_1. A_0 and B_0 are in terms of balance weight multiplied by radius.

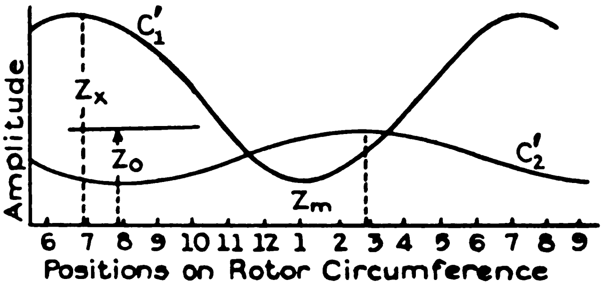

Fig. 7. Quartering Curves of Unbalance for a Trial Weight at Different Positions on C_1 End of Rotor

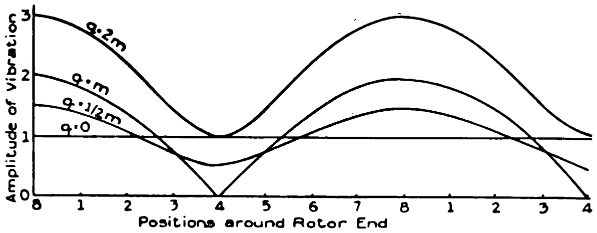

Fig. 8. Theoretical Quartering Curves

q = quartering or trial balance weight
m = unknown unbalance at position 8

If formulas are preferred to the graphical method of combining two weights they can be written from the usual triangle formulas applied to Fig. 6.

$$A_0^2=A_1^2+A_2^2-2\,A_1A_2\cos\phi \tag{7}$$

$$\sin a_0=\frac{A_2}{A_0}\sin\phi;\ a_0=\,? \tag{8}$$

$$B_0^2=B_1^2+B_2^2-2\,B_1B_2\cos\phi \tag{9}$$

$$\sin(b_0-\phi)=\frac{B_1}{B_0}\sin\phi;\ b_0=\,? \tag{10}$$

$$\text{Weights: } w_a=\frac{A_0}{r_a} \tag{11}$$

$$w_b=\frac{B_0}{r_b} \tag{12}$$

where r_a and r_h are the radii of the weights.

Balancing an Assembled Machine

If the rotor is in a machine already installed one can proceed on the same general plan as in factory balancing. Consider first a rotor with two bearings, or a four-bearing set with a flexible coupling between units where the condition of balance is similar to that represented in Fig. 1.

The first part of the problem is to find the value and location of the bearing forces C_1 and C_2. The "quartering" method may be used as follows: Divide the circumference of one end of the rotor into 12 equal spaces and number the spaces in the direction of rotation. Choose a trial balance weight about $\frac{1}{150{,}000}$ of the weight of the rotor. Insert the trial weight, run the rotor and measure the vibration amplitude at both bearings. The same trial weight is then moved to another position but at the same radius and vibrations are again measured. Repeat for other positions and plot the vibration amplitude against position of the trial weight as in Fig. 7. Read also amplitude Z_0 without the trial weight inserted. All readings must be taken at the same speed. Theoretical shapes of quartering curves are given in Fig. 8 wherein the trial weight ranges from zero to twice the value required for balance.

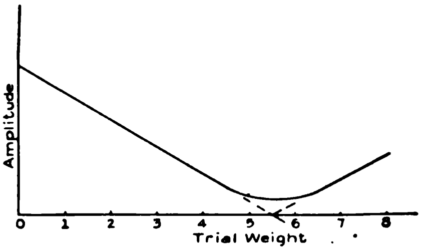

Fig. 9. Curve Showing the Amount of Balance Weight at Position 1 for C'_1 in Fig. 7

The shape and value of the test curves at maximum and minimum vibration are of importance. It is evident that at maximum amplitude of the bearing vibration the bearing forces from the known trial weight and unknown unbalance both point in the same direction, and that these are additive and give the location of the unknown bearing force C_1. In Fig. 7, with the quartering weight placed on the C_1 end of rotor, C_1 points through Position 7 and C_2 through Position 8. The bearing forces are proportional to amplitude of vibration or

$$C_1+qr\,\frac{L_5+L_6}{L_1}\propto Z_x$$

where q is the weight of the trial balance weight and r the radius at which it rotates.

The minimum amplitude of the bearing vibration is exactly 180 deg. away from the maximum value on the curve and the known and unknown balance forces point in opposite directions.

$$C_1-qr\,\frac{L_5+L_6}{L_1}\propto Z_m$$

These two proportionalities give

$$C_1 = qr\,\frac{L_5+L_6}{L_1}\,\frac{Z_x+Z_m}{Z_x-Z_m} \qquad (13a)$$

or

$$C_1 = qr\,\frac{L_5+L_6}{L_1}\,\frac{Z_x-Z_m}{Z_x+Z_m} \qquad (13b)$$

for the following conditions:

Z_x = maximum amplitude with quartering weight
Z_m = minimum amplitude with quartering weight
Z_o = amplitude without quartering weight
Use equation (13a) when Z_x+Z_m equals 2 Z_o.
Use equation (13b) when Z_x+Z_m is greater than 2 Z_o.

If the bearing at the other end (C_2) of the rotor gave sufficiently large vibration when quartering on the C_1 end, then from those readings

$$C_2 = qr\,\frac{L_4}{L_1}\,\frac{Z_x+Z_m}{Z_x-Z_m} \qquad (14a)$$

or

$$C_2 = qr\,\frac{L_4}{L_1}\,\frac{Z_x-Z_m}{Z_x+Z_m} \qquad (14b)$$

with the Z values read from the C'_2 curve of Fig. 7 and with conditions similar to Equation (13).

With C_1 and C_2 known in direction and value the balance weights can be determined as in the section under the heading of "Determining Balance Weights."

Another method of balancing a rotor in an installed machine is to complete a quartering curve as in Fig. 7 and from that curve locate the angular position of the balance weight at the minimum amplitude of vibration as at Position *1* from curve C'_1. At that position use different balance weights and plot readings of the amplitude of bearing vibration against the amount of balance weight used as in Fig. 9. This should be a straight line; but owing to the foundation vibrating from the vibration of the other bearing or of nearby machines, the amplitude may not go to zero value. The weight for the zero or minimum amplitude given by the curve in Fig. 9 multiplied by the radius of the weight can be used in place of M_1R_1 in Fig. 2 and the bearing force C_1 can be found from the simple beam formula of Equation (1).

The simple weight from Fig. 9 that neutralizes the bearing force C_1 and makes the amplitude at that bearing zero is not the balance-weight force A_1 but is less than A_1. This force, while it has neutralized C_1 still has the reaction of a beam at the other bearing and has changed the balance there. That is the reason why improving the balance on one end of a rotor very often increases the vibration at the other end of the machine. This difficulty can be overcome only by the use of two weights in the final balance, one being placed on each end as the A_1 and B_1 of Fig. 4 so that only one bearing is influenced by the balance weights. Since A_1 and B_1 point in opposite directions, the A_1 is greater than the single-weight value just found in Fig. 9.

If the balance is still imperfect, a quartering curve with the trial weights placed on the other (C_2) end of the rotor should be made. This curve can be taken with the weights A_1 and B_1 in place or with A_1 and B_1 omitted, as these two weights have no influence on the C_2 end. With the new value found for the force C_2 the four weights A_1, B_1, A_2, and B_2, two weights on each end, can be combined into a single weight for each end of the rotor as given under section on balance weights.

Resumé

While balancing is a dynamic problem, it is peculiar in that a simple solution can be given by the elementary principles of statics.

One should picture the radial rotating bearing forces of Fig. 1 for it is these bearing forces that produce the observed vibration of the machine.

The test value of the bearing forces as found in Fig. 2 involves only the principle of reaction at the support of a simple beam with the load M_1R_1.

The counterbalancing of Fig. 4 involves only the principle of the simple lever by which the balance weights are estimated so as to influence the vibration and force at one bearing only.

The difficult part of the problem is to actually measure the bearing forces for value and direction. This is accomplished by the balancing machine in the factory or quartering curves in the field.

Remodeling of Cahokia Station Includes New 75,000-kw. Turbine

The Union Electric Light and Power Company of St. Louis has made plans to remodel and expand the Cahokia station, the largest of its generating plants. The plans include the installation of a new 75,000-kw. turbine-generator set. Instead of making an addition to the present station building, space for the 75,000-kw. unit will be provided by removing a 30,000-kw. machine. The foundations of this machine will be strengthened to suit the larger unit, and the condenser which was used with the 30,000-kw. turbine will be altered and connected to the new 75,000.

The new turbine will be a single-cylinder machine receiving steam at 315 lb. gauge, 300 deg. superheat, and will operate against a back pressure of 1 in. absolute. It will be rated 75,000 kw. at 1800 r.p.m. The generator will be rated 60 cycles, 13,800 volts, three phase, and 90 per cent power-factor.

Vacuum Tubes as Oscillation Generators

PART VI

SECONDARY GRID-COUPLED CIRCUITS—POWER-AMPLIFIER AND MASTER-CONTROLLED CIRCUITS

By **D. C. PRINCE** and **F. B. VOGDES**
Research Laboratory, General Electric Company

IF the secondary circuit in a set driven by a vacuum tube can be used to supply the tube with grid excitation, it is possible to make an arrangement such that oscillations can occur at only one of the resonant frequencies corresponding to the coupled circuits used. At the frequency for which the secondary is in resonance, there is a reversal of the reactive component of the voltage which it induces in the primary. This means a reversal of the secondary current with respect to that in the primary. Now the grid voltage will be obtained by taking the drop produced by the secondary current in some reactance in that circuit and will, therefore, reverse when the current reverses. The plate voltage, on the other hand, must always maintain the same relation with respect to the current in the transformer primary. If it did not, it would mean that the transformer primary would be required to show an apparent reactance of the nature of a capacity—a phenomenon entirely incompatible with resonance in that circuit. The two frequencies giving stable operation with zero apparent primary circuit reactance lie on opposite sides of the secondary resonant frequency; hence, if the grid excitation is correct for one, it is incorrect for the other, being 180 deg. out of phase. Therefore, a set with two coupled circuits in which the grid is connected to the secondary will have only one stable operating frequency.

The reversal of the secondary current does not represent a discontinuity as might be inferred. The secondary impedance at resonance changes from reactive in one sense through pure resistance and then becomes reactive in the opposite sense, the change in the power-factor being quite smooth although compressed almost entirely into a very small range of frequency. The reversal of the current follows the same law as regards its phase. It is, of course, greatest in magnitude when there is only resistance to be overcome; *i. e.*, at the point of resonance. As the grid voltage should be opposite in phase to the plate voltage, it will be found necessary to have the two circuits run at a frequency different from their resonant frequencies in order that the overall voltages will have somewhat the same phase, and displacements of 90 deg. will be avoided.

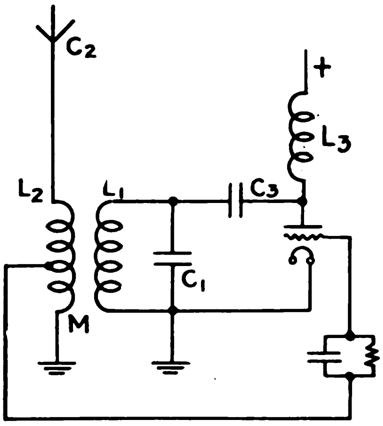

Fig. 44. Secondary Grid-coupled Circuit

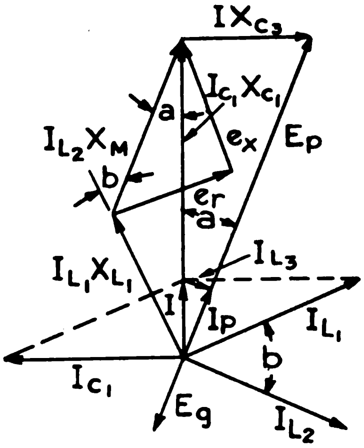

Fig. 45. Vector Relations in Secondary Grid-coupled Circuit

Fig. 44 shows a secondary grid-coupled circuit and Fig. 45 presents the corresponding vector diagram. To examine this diagram, assume the antenna current I_{L_2} flowing in the antenna loading coil L_2. This current induces in the primary inductance L_1 the voltage $I_{L_2}X_M$, due to the coupling. The voltage $I_{L_2}X_M$ has two components, e_r at right angles

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References made to material hitherto published in this series of articles may be readily located from the following table:

	Issue	*Figs.*	*Tables*	*Footnotes*
Part I	June, 1927, p. 320	1 to 4	——	——
Part II	Oct., 1927, p. 501	5 to 21	I to III	(1) to (4)
Part III	Feb., 1928, p. 97	22 to 23	IV	——
Part IV	Mar., 1928, p. 147	24 to 34	——	——
Part V	July, 1928, p. 388	35 to 43	——	(5) to (7)

to the self-induced voltage in the primary ($I_{L_1}X_{L_1}$) and e_x parallel with it. The former is the power component due to the antenna resistance. The latter represents a wattless interchange between the primary and the secondary circuits. Both $I_{L_1}X_{L_1}$ and $I_{L_2}X_M$ are induced in L_1, so that their sum is impressed upon C_1 and the current in C_1 produces the voltage I_{C_1} X_{C_1}. The capacity current I_{C_1} leads its voltage 90 deg. while the current in L_1 lags 90 deg. behind its self-induced voltage.

Since the grid is connected to the antenna coil L_2, the grid potential will be at right angles to I_{L_2} and the plate potential E_p may be made 180 deg. out of phase by proper choice of C_3, which gives the voltage IX_{C_3}.

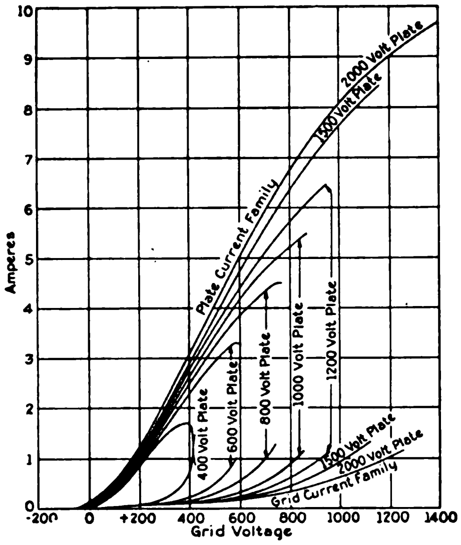

Fig. 46. Characteristic Curves of a Water-cooled Pliotron

It is apparent that, as drawn, $I_{L_1}X_{L_1}$ and $I_{C_1}X_{C_1}$ are not equal. In other words the primary circuit is not in tune. However, it is also apparent that an increase in $I_{L_1}X_{L_1}$ must be at the expense of the e_x component of $I_{L_2}X_M$, since e_r represents the energy transfer which must be maintained. The angle b is therefore determined by the ratio between $I_{L_1}X_{L_1}$ and $I_{C_1}X_{C_1}$ and the more nearly tuned the primary circuit the greater will be the angle b. If the primary were entirely tuned, the angle b would be 90 deg. and it would become entirely impossible to bring E_g and E_p into line by any drop across C_3.

Coupled oscillating systems result in the radiation of a purer wave than can be obtained from the usual simple circuit. Some irregularities are certain to be present in the circuit next to the tube due to the fact that the input and the output are not continuously equal. If this circuit is also the radiating circuit, these irregularities will succeed in producing more or less harmonic radiation. If, however, the radiating is done by a second circuit coupled to the tube circuit, the harmonics usually will be transferred through the coupling with much more difficulty than the primary frequency power, and very little harmonic radiation will result.

It has been seen that with the grid connected to the secondary, the oscillating circuits must be operated at a frequency appreciably different from their natural frequencies when separated. With the circuit shown in Fig. 44, it is advisable to have the primary tune by itself at a frequency above the operating frequency in order that the voltage induced in the primary by its own current will be in approximately the same direction as the voltage induced in it by the secondary current.

On the other hand, if $X_{C_1}/X_{L_1}=4$, any second harmonic component in the plate current will be in resonance and a large second-harmonic wave may be radiated. If this ratio be greater than 4, the phase relations will be such that a second-harmonic current in the antenna will be amplified; and continuous second-harmonic oscillations might even be built up, provided the fundamental wavelength were twice the natural wavelength of the antenna. It seems desirable, therefore, to keep the tuning point of the primary circuit between the fundamental and second harmonic.

Having avoided accentuating harmonics by not making the ratio X_{C_1}/X_{L_1} too large and dephasing the grid by not making this ratio too near unity, the amount of harmonic radiation, for any value of C_1, can be determined by plotting the plate current and making an harmonic analysis of it. This current is then assumed to pass through C_3 to the primary circuit and the portion of it which finds its way into the antenna may be calculated directly.

Once C_1 has been made large enough to keep harmonic radiation within the required limits, the whole circuit may be calculated exactly by making a quantitative determination from the vector diagram, bearing in mind the values for E_p and E_g from the tube characteristics and that these voltages must be displaced 180 deg. in phase. A sample calculation in which this procedure has been followed will now be given.

Calculation of a Secondary Grid-coupled Circuit

This circuit is to feed 25 kw. at 2500 meters into a multiple-tuned antenna having two downleads. The full power requires 46.5 amp. per downlead. The characteristics, efficiency, and optimum operating conditions of the tube to be used are shown in Figs. 46, 47, and 48.

Fig. 48 shows that for 25-kw. output the alternating grid voltage should be 2300 and the minimum instantaneous plate voltage should be 1700. The direct plate voltage is 15,000, and deducting 1700 from this and multiplying by 0.707 gives an alternating plate potential of 9400 volts.

No requirements have to be met regarding harmonics, so the primary volt-amperes will be made equal to approximately 4π times the watts. The wavelength of 2500 meters is equivalent to a frequency of 120,000 cycles, which corresponds to an angular velocity of 753,000 radians per second for the vectors representing the voltages and currents. If the primary condenser C_1 has a capacity of 0.005 mfd., it will give the required volt-amperes. Its impedance will be 266 ohms, and the voltage across it will be practically equal in magnitude to the alternating plate voltage. At 9400 volts it will carry 35.3 amp., giving slightly more than $4\pi \times 25{,}000$. The current through the primary inductance may be taken as equal to that through the condenser, making it also 35.3 amp.

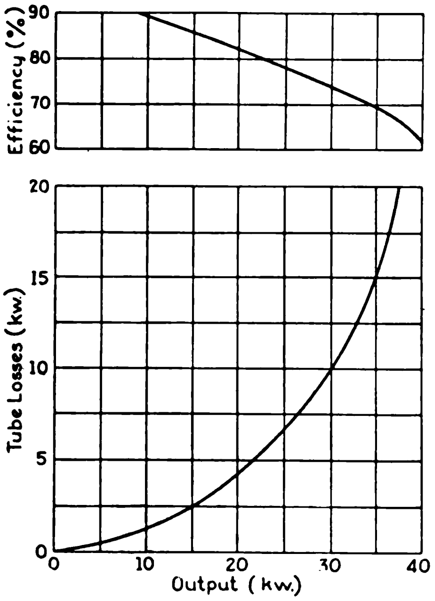

Fig. 47. Loss and Efficiency Curves at 15,000 Volts Direct Potential for a Tube Having the Characteristics Shown in Fig. 46

Knowing the primary current and the power transferred through the coupling, the unity power-factor component e_r of the voltage induced in the primary can be found by dividing the power by the current, and this gives 25,000 watts divided by 35.3 amp. or 707 volts.

Now $I_{C_1}X_{C_1}$, the drop across the primary condenser, is to be equal to the three vectors comprising the voltage in the primary inductance (see Fig. 45). $I_{L_1}X_{L_1}$ represents the voltage induced in the primary due to its own current and reactance, and e_r and e_x are respectively the unity power-factor and zero power-factor components of the voltage induced in the primary by the secondary current. As e_r is only 707 volts and is perpendicular to the other two which are much larger, they will be very nearly 9400 volts together, or each may be taken as 4700 volts. This will result in a resonant frequency of the primary (by itself) 41 per cent higher than the operating frequency, which is within the range of desirable values.

Under these conditions, the reactance of the primary coil X_{L_1} is half that of the condenser, or 133 ohms which corresponds to 177 micro-henries at the frequency used. Similarly, the mutual reactance of the primary and secondary coils is obtained by dividing the voltage which the secondary current induces in the primary winding by the value of the current. The quotient of 4700 volts and 46.5 amp. is 101 ohms, and this is, therefore, the value of X_M. At 2500 meters this is equivalent to 134 micro-henries.

The angle a will have a sine given by $\frac{707}{2}$ volts divided by 4700 volts; hence a is 4.33 deg. This is the angle which the drop in the plate blocking condenser must produce between the plate alternating voltage and the primary voltage. Therefore, IX_{C_2} is given by 9400 volts times the sine of 4.33 deg. and is 707 volts. The input current I is equal to the power divided by the voltage, or 25,000 watts divided by 9400 volts and is 2.66 amp. At 707 volts this means an impedance of 266 ohms for X_{C_2} or 0.005 mfd.

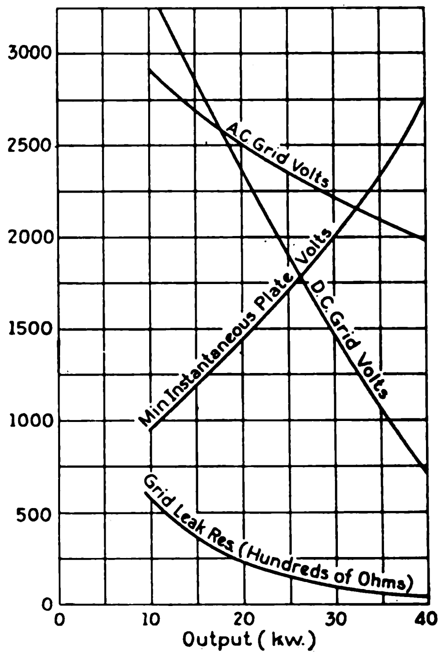

Fig. 48. Optimum Operating Conditions, at 15,000 Volts Direct Potential for a Tube Having the Characteristics Shown in Fig. 46

In order that unity power-factor current may be taken by the tube, the plate choke must draw a reactive current I_{L_2} equal and opposite to that taken through the plate-blocking condenser. The latter is given by $I \sin a$ and is 0.20 amp. With 9400 volts impressed on the choke, this requires a reactance of 47,000 ohms or an inductance of 62.4 milli-henries.

The grid excitation is supplied from the antenna circuit. When the antenna coil is at hand, it should not be a difficult matter to pick a point on it giving 2300 volts, which is the potential required, remembering that part of the voltage in this coil is induced by the primary current.

At 25 kw., the antenna voltage is 30,900 volts, and the potential induced in the antenna coil by the primary is $I_{L_1}X_M$ or the product of 35.3 amp. and 101 ohms which is 3565 volts. This is practically in phase with the antenna voltage, leaving the difference of 27,335 volts to be given by the reactive drop in the secondary circuit due to its own current and reactance. Hence, $I_{L_2}X_{L_2}$ is 27,335 volts, and dividing by 46.5 amp. for I_{L_2} gives 589 ohms reactance for the secondary coil system or 782 micro-henries.

POWER AMPLIFIER AND MASTER-CONTROLLED CIRCUITS

Many present-day radio stations use circuits containing special control means. These schemes vary widely in nature, and a single transmitter may contain more than one of them. Two principal objects are attained by their use. One is the production of modulated waves for radio telephony, and the other is the maintenance of a fixed frequency of oscillation.

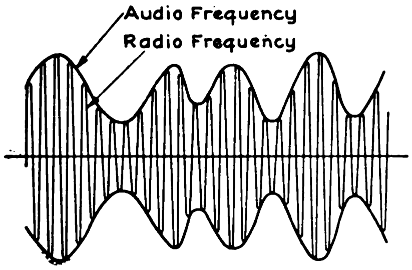

Fig. 49. A Modulated Wave

Radio-telephone stations (of which broadcasting stations are a familiar example) usually radiate a modulated wave as shown in Fig. 49. The "carrier" consists of waves of radio frequency which vary in magnitude at the frequency of the sound being transmitted. The signal itself is, therefore, represented by the envelope drawn through the peaks of the radio-frequency oscillations. The antenna system of the receiving station carries a current similar to the radio-frequency oscillations being transmitted. The receiving apparatus amplifies and rectifies this current in such a way that the telephones carry a current which varies in the same manner as the envelope of the radio-frequency waves, although the radio-frequency variations themselves are no longer present.

Due to the great number of stations which desire to operate, it is necessary for them to transmit on wavelengths which are very close together. This means that each must maintain its frequency constant in order that it may not interfere with the others. For this purpose, master oscillator sets are often used, in which the grid of the transmitting tube has impressed upon it a radio-frequency voltage which is not derived from the circuit into which the output of the tube is delivered but from a separate source. By this means, the changes of the antenna constants due to wind and temperature variations are prevented from having any effect on the frequency of the oscillations.

Plate-voltage Modulation

Fig. 50 shows a common form of circuit used for radio telephony. The oscillating circuit itself is of one of the types which have already been described as producers of unmodulated radio-frequency currents and includes the usual radio-frequency choke. Modulation of the oscillations is obtained by varying the plate voltage. This is accomplished by forcing the supply current for the oscillating circuit through an audio-frequency choke and then with a second pliotron draining away the current not required for the oscillating circuit. When the modulator tube draws a pulse of current, both the input voltage and current to the oscillating circuit decrease, and, if the modulator tube ceases to draw current, the input voltage and current to the oscillating circuit increase. The change in current drawn by the modulator tube is obtained by applying to its grid the signal which is to be transmitted. The tube driving the oscillating circuit operates as though it were supplied with a direct voltage having a variation impressed on it corresponding to the audio-frequency signal. The changes at audio frequency are so slow compared with the radio frequency that effects due to the rate of change of the input voltage are negligible. Hence, the tube will drive the oscillating circuit at the various magnitudes of voltage and current with the same instantaneous efficiencies as though it were supplied with steady inputs. It is not difficult to arrange the circuit so that these efficiencies are always good.

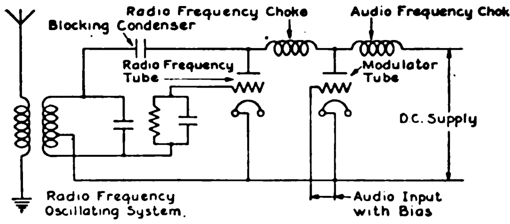

Fig. 50. Circuit with Plate-voltage Modulation

The oscillating circuit, therefore, offers a design problem but slightly different from a set running under steady conditions. The range of the applied voltage is from low values to a value almost double that of the direct-current supply. The oscillating circuit should have sufficient insulation to stand the highest voltages involved, and the tube should have enough emission to carry the highest current. The circuit adjustments for operation at the voltage of the supply source will usually give good operation over the range involved, and, if any changes are required, they will probably consist of no more than slight adjustments in the value of the grid-leak resistance and capacity. The latter should be kept small in order that it may charge and discharge rapidly and follow the changing conditions without any appreciable lag.

The tube losses with high modulation are greater than for no modulation, but this load is carried for only a short time, and, if the heating with no modulation is kept somewhat below the maximum permissible, no heating troubles should result. It will be seen that the problem in this type of transmitter is not in the design of the oscillating circuits but in the arrangement of the tubes which modulate the voltage of the supply source.

With complete modulation, the voltage and current supplied to the tube driving the oscillating circuit vary from zero to twice the average value which prevails when no sound is being transmitted (see Fig. 51). If E and I are the average voltage and current, the total input to the oscillator tube with complete modulation is $EI+\frac{EI}{2}$, the first term representing the direct-current and the second the alternating-current component. The alternating current through the modulator tube is in the opposite phase to that drawn by the oscillator tube; hence, the loss in it is $EI-\frac{EI}{2}$. The total power supplied to both modulator and oscillating circuit combined is 2 EI; and, when no signal is being transmitted, this is divided equally between the two. Under ordinary transmission conditions, there will be periods running into seconds when little or no sound is being transmitted. The modulator tube must, therefore, absorb in itself an amount of power approximately equal to the amount being radiated. In case the modulator tube is not arranged so as to be able to give complete modulation of the radiated wave, the service required of it may be less severe. On the other hand, it may carry a higher current than the oscillator tube at no load in order to utilize a section of its characteristic giving less distortion.

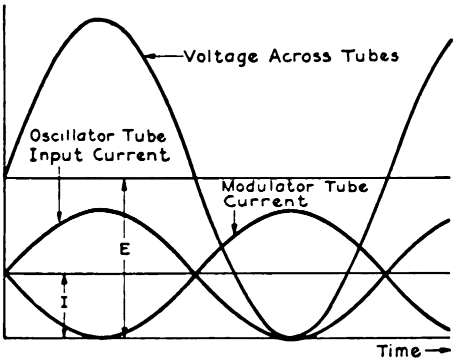

Fig. 51. Input to Tube Driving an Oscillating Circuit, Assuming That Modulation is Complete

Distortion of the signal is a problem which must in the final case be solved by test means, for some distortion will always exist and with any given equipment will increase with the strength of the signal. The problem is to obtain the greatest amount of modulation with a minimum of undesirable effects. Workers with amplifiers have, however, certain rules by which they can obtain uniformly good results.[8] Almost needless to say, these rules partake of the nature of personal opinion, for distortion which is disagreeable to one investigator may be allowed by another because of some other benefits he obtains thereby.

Plate-voltage Modulation with Master-controlled Grid

Plate-voltage modulation with the grid of the tube driving the oscillating system fed from the system itself does not give the desirable constant-frequency effect, for the oscillating circuit can change its frequency if the antenna constants vary, and a slight variation with output may also be expected. By exciting the grid of the oscillator tube at radio frequency from a separate source, as in Fig. 52, all variations in frequency may be avoided. The grid excitation will be constant in magnitude and of the frequency to be radiated. The magnitude will be fixed by conditions existing when maximum power is being radiated. This will result in unusual although not necessarily excessive grid losses at all times. Modulation will be very much the same as before, for low plate currents can be drawn even though the grid current be higher than necessary. It would be well, however, to limit the amount of modulation so that the lowest voltage applied to the oscillator tube (due to the audio-frequency variations) will not approach too near to the maximum positive grid swing.

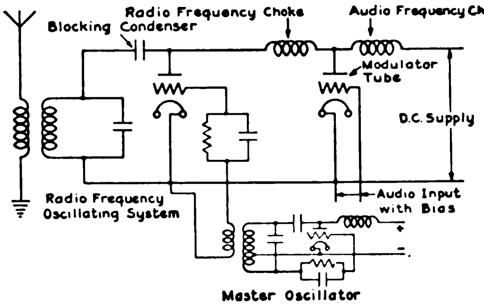

Fig. 52. Circuit with Plate-voltage Modulation and Master-controlled Grid

While constancy of frequency is attained with such a circuit, the variations of the constants of the output circuit may affect its operation in other ways, as will be shown later.

Power-amplifier Circuits

Instead of having an oscillating circuit which is driven by one tube fed with power at a voltage modulated by a second tube, a type of circuit may be used in which small modulated radio-frequency oscillations are amplified by a single tube to the volume required to be radiated. A circuit of this general type is shown in Fig. 53. It may take other forms including the two-tube "push-pull" arrangement, but discussion of the circuit shown will indicate the nature of such circuits in a general way.

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[8] For example: "Design of Non-Distorting Power Amplifiers," by E. W. Kellogg. *A.I.E.E. Journal*, May, 1925, pp. 490-498.

By impressing a modulated radio-frequency wave on the grid, the plate current and voltage may be made to vary as shown in Fig. 54(a) and (b) which will cause oscillations following the same law to be set up in the antenna system. The loss here is EI when the magnitude of the radio-frequency oscillations is zero, and the greatest output is $\frac{1}{2}\,EI$ when the oscillations are a maximum. The input is steady and equal to EI.

Since the oscillating system stores considerable energy, it is not necessary to feed it sine-wave current as in Fig. 54(b), but irregular impulses of plate current as in Fig. 54(c) may be made to answer the same end. This greatly reduces the tube losses as the bias current I is then disposed of, thus eliminating the steady loss. Such an arrangement, however, requires more than usual care in order to avoid distortion.

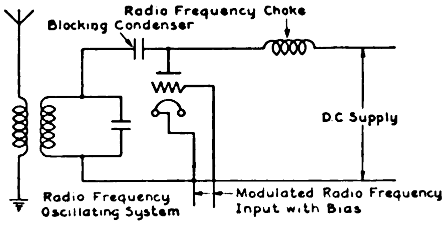

Radio Frequency Choke
Blocking Condenser
D.C Supply
Radio Frequency Oscillating System
Modulated Radio Frequency Input with Bias

Fig. 53. Power-amplifier Circuit

Power-amplifier circuits also are affected by detuning of the antenna due to unavoidable changes in its constants.

Effect of Detuning an Antenna Driven by Master-oscillator Set

The effect of detuning an antenna driven by a circuit with a fixed frequency can be illustrated by considering a continuous-wave telegraph set. Such an arrangement is shown in Fig. 55. In this diagram, a multiple-tuned antenna receives power from an oscillating circuit driven by the main pliotron through coupling with one tuning coil. Grid excitation for the main tube is supplied from an oscillating circuit driven by a second tube which can be much smaller than the first. The grid excitation of the main oscillator tube will be so arranged that this tube will operate under the conditions of high efficiency described in Part II of this serial, when the antenna is in its normal condition.

In order to insure constant frequency, the circuit supplying the grid excitation must be so arranged that nothing can affect its operation. This means that reliable apparatus must be used and arranged so that it will not be disturbed. Also, no coupling must exist between the master circuit and the main oscillating circuit, for the latter may then be able to affect its own grid circuit to some extent. In this connection, the grid circuit of the main tube should be so arranged that it is not likely to change its value as a load on the master circuit, for changes in load can produce changes in frequency as was shown in Part IV. Electric and magnetic coupling of the circuits may be avoided by shielding the master-oscillator circuit with sheet metal.

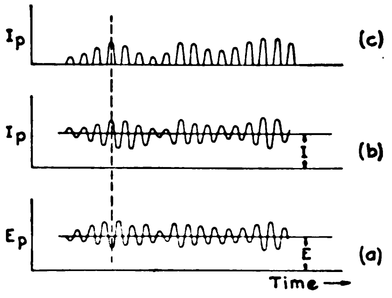


Fig. 54. Wave Forms Obtained with a Power-amplifier Circuit

The calculation of master-controlled circuits will follow the same general methods which have been used with the other types of circuits already discussed. The real problem is not the arranging of the voltages and currents, etc., but in determining the disturbances which may be caused by variations in the values of the reactances employed and in determining the variations in output which may result.

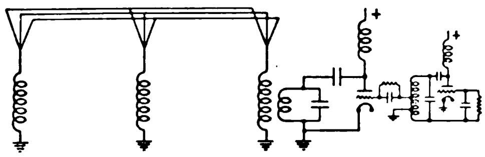


Fig. 55. Continuous-wave Telegraph Set with Master-oscillator Control and an Intermediate Oscillating Circuit

Other circuits can adjust themselves to slight irregularities by changing their frequency, but a power amplifier circuit must do the best it can without this property.

When the antenna becomes detuned due to wind or other cause, it changes both the apparent resistance and reactance of the primary circuit which drives it. This circuit has been arranged so that it requires no reactive component of input current when the antenna is in its normal condition, and such a component can be supplied only by seriously disturbing the arrangement of the currents and voltages which comprise the load on the vacuum tube.

In order to simplify the calculation of the disturbance, it will be assumed that both primary and secondary circuits tune by themselves to the operating frequency. Then, let the reactance of the primary coil be X, and the apparent reactance produced in the primary coil by disturbing the antenna will be taken as ΔX. The apparent resistance will be R. The susceptance of the primary coil is $\frac{X}{Z^2}$, but the resistance is so small in comparison with the reactance that $\frac{1}{X}$ may be used and the change in susceptance taken as $\frac{\Delta X}{X^2}$. Before the unbalance occurs, the primary circuit has zero susceptance between the points where it is connected to the tube. After the unbalance, it will have a susceptance given by $\frac{\Delta X}{X^2}$. The conductance will be given by $\frac{R}{Z^2}$ or $\frac{R}{X^2}$ (approximately). Hence, the power-factor of the load driven by the tube will be the power-factor of the apparent impedance of the primary due to the secondary. The reactive and real power supplied will be equal to that resulting from the current in the primary flowing through the apparent resistance R and apparent reactance ΔX. To arrive at this result, the current is assumed to be unaffected by the slight changes in the primary reactance due to the secondary.

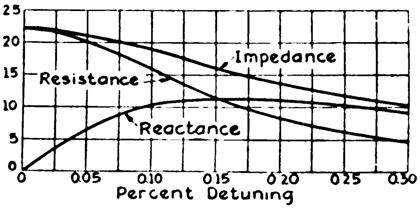


Fig. 56. Effect of Detuning of Antenna Upon the Apparent Resistance and Reactance of the Intermediate Circuit

Antennas which have a low resistance compared with their reactance are the ones most sensitive to frequency change. The characteristics of such an antenna have been shown in Fig. 37. Changes in wind and temperature may change the antenna capacity sufficiently to vary its resonant frequency by 0.3 per cent. The effect is, for all practical purposes, the same as though the antenna constants had remained fixed and the applied frequency had been changed by this amount. Taking, for convenience, a coupling of 20 ohms and calculating the effect on the primary circuit, there result the curves shown in Fig. 56 which show the apparent resistance and reactance of the primary due to the secondary for various amounts of detuning.

Fig. 57 illustrates the manner in which the oscillating circuit receives its input. The direct voltage of the power source is D, and the oscillating circuit has a voltage wave of amplitude A. The plate-current wave is dependent primarily on the grid excitation and is not much affected by changes in the plate voltage unless it swings below the instantaneous value of the grid voltage. The plate-current wave may be analyzed into its components. Of these only the fundamental-frequency component, which is shown by the dotted line, is of interest at present. The other components do not represent input to the oscillating circuit, hence, the fundamental-frequency component will be considered as though it actually were the entire plate current. It reaches a maximum at the center of the actual wave and both may be considered to have a phase displacement ϕ with respect to the voltage wave. This angle is fixed by the power-factor of the oscillating circuit as determined by the effect of the antenna upon it.

If the oscillations are constant in magnitude, the volt-amperes input to the circuit will be constant and the power will depend on $\cos\phi$. A constant current flowing through the impedance produced by the antenna detuning as shown in Fig. 56 will not absorb this input, however, for the impedance decreases as detuning advances and the resistance decreases more rapidly than $\cos\phi$. Therefore, to keep a balance between input and output, the oscillations in the primary circuit will increase in magnitude as detuning advances.

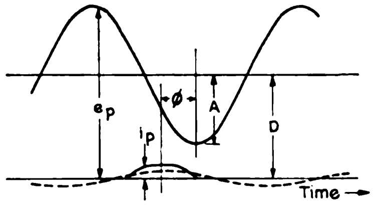


Fig. 57. Manner in Which the Intermediate Circuit Receives Its Input When the Antenna is Detuned

This increase in magnitude of the primary voltage is readily calculated if it be assumed that the plate-current wave remains unaffected. It can, moreover, be expressed in terms of the power-factor in such a way as to have general application. Returning to the starting point for all the calculations involving coupled circuits, it is remembered that the total apparent complex impedance of the primary circuit is given by

$$Z'_1=R_1+\frac{M^2\omega^2}{Z^2_2}R_2+j\left(X_1-\frac{M^2\omega^2}{Z^2_2}X_2\right)$$

The primary resistance R_1 will be assumed so small that it can be neglected. The reactance X_1 of the primary by itself is zero and $M^2\omega^2$ will be a constant. The secondary resistance R_2 will be constant, and the secondary impedance can be expressed as $\frac{R_2}{\cos\phi}$. Combining the two terms comprising the effective impedance of the primary due to the secondary, it will be found equal to $\frac{M^2\omega^2}{Z_2}$ or $\frac{M^2\omega^2}{R_2}\cos\phi$. The apparent resistance of the primary in terms of the secondary resistance and power-factor will be $\frac{M^2\omega^2}{R_2}\cos^2\phi$.

Under steady conditions, the volt-ampere input to the primary circuit must be equal to the volt-ampere output. The former will be proportional to the voltage since the input current is constant and the output will be proportional to the product of the square of the circulating current and the apparent impedance due to the secondary. As the circulating current and voltage are proportional, it may also be said that output is proportional to the product of the impedance and the square of the voltage. If the voltage varies inversely as the impedance, the volt-ampere output will vary as the first power of the voltage which would make it obey the same law as the input. Hence, the voltage of the primary circuit will vary inversely as the impedance or will be proportional to $\frac{R_2}{M^2\omega^2 \cos \phi}$. The watts output will be proportional

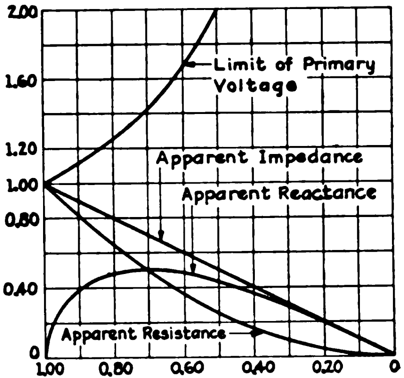


Fig. 58. Conditions in Intermediate Circuit When the Antenna is Detuned

to the current squared, or voltage squared, multiplied by the apparent primary resistance. This results in a value proportional to $\frac{R_2}{M^2\omega^2}$ which is a constant. In other words, the detuning does not decrease the output under the assumed conditions because the primary oscillations increase in magnitude. As both input direct-current power and output radio-frequency power are constant, the losses also must be constant.

In practice, the state of affairs just discussed is not likely to be completely realized. The plate current is almost certain to suffer somewhat from the increase in the alternating plate voltage, for the current falls off very rapidly if the instantaneous plate voltage is not greater than the instantaneous grid voltage. A decrease in the plate current will cause a decrease in the input to the primary circuit. This will cause a decrease in the output and will also cause the remaining plate current to encounter a higher tube drop which is, of course, the instantaneous difference between the direct and alternating plate voltages. Hence, in a master-controlled set with an intermediate circuit, detuning of the antenna system will usually result in somewhat decreased radiation with the possibility of increased tube losses, although this type of circuit has a natural tendency to hold both output and losses constant. The increase of voltage in the primary circuit is an undesirable property inasmuch as it increases the voltage strain on the tube and other insulation, but it is necessary if the output is not to suffer serious fluctuations due to the detuning of the antenna system. Fig. 58 shows the apparent primary resistance, reactance, and impedance due to the secondary as functions of the power-factor. The theoretical limit of the primary alternating voltage is also shown.

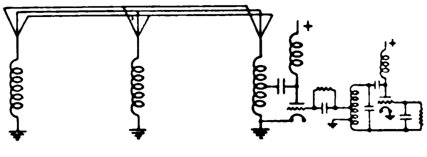


Fig. 59. Master-oscillator Controlled Set without Intermediate Circuit

Master Oscillator Controlled Set Without an Intermediate Circuit

It may be inquired what the effect of eliminating the intermediate circuit from a master oscillator set might be. Such an arrangement is shown in Fig. 59. It is apparent as a first requirement that the section of antenna tuning coil across which the tube is connected must have a voltage equal to the proper alternating plate voltage for the tube and must carry a current sufficient to give a volt-ampere product for this section at least 4 π times the load in watts. The question of the effect of the detuning of the antenna then arises. This can be answered by considering the antenna to be a secondary circuit fed by the tube through an auto-transformer arrangement. The primary and secondary and mutual inductance of this transformer will then be identical. The impedance of the primary circuit will consist of the blocking condenser and the section of antenna coil plus an apparent impedance due to the antenna system.

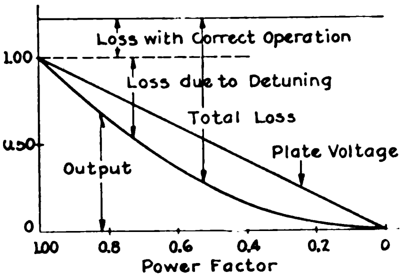


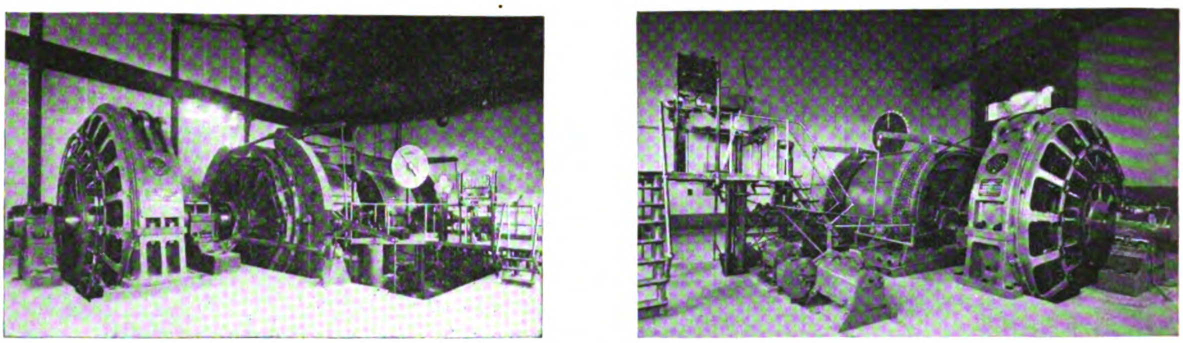
Fig. 60. Output and Losses in a Set without an Intermediate Circuit When the Antenna is Detuned

The actual impedance of the antenna coil is ωM where M is the common inductance, and the apparent impedance of the primary due to the antenna system will be $\frac{\omega^2 M^2}{Z_2}$. The voltage induced in the antenna system by the plate current is $I_p\omega M$ where only the fundamental-frequency component is considered. This will be equal to $I_A Z_2$, the impedance drop in the secondary. Now I_A is much larger than I_p, hence, ωM will be much larger than Z_2 and can be neglected in comparison with $\frac{\omega^2 M^2}{Z_2}$. In other words, the inductance of the path taken by the primary or plate current may be neglected in comparison with the apparent reactance due to the antenna system. The reactance of the plate-blocking condenser will also be negligible. In fact, these reactances which are dropped from the calculations are opposite in sign and could be balanced out by being made equal in magnitude.

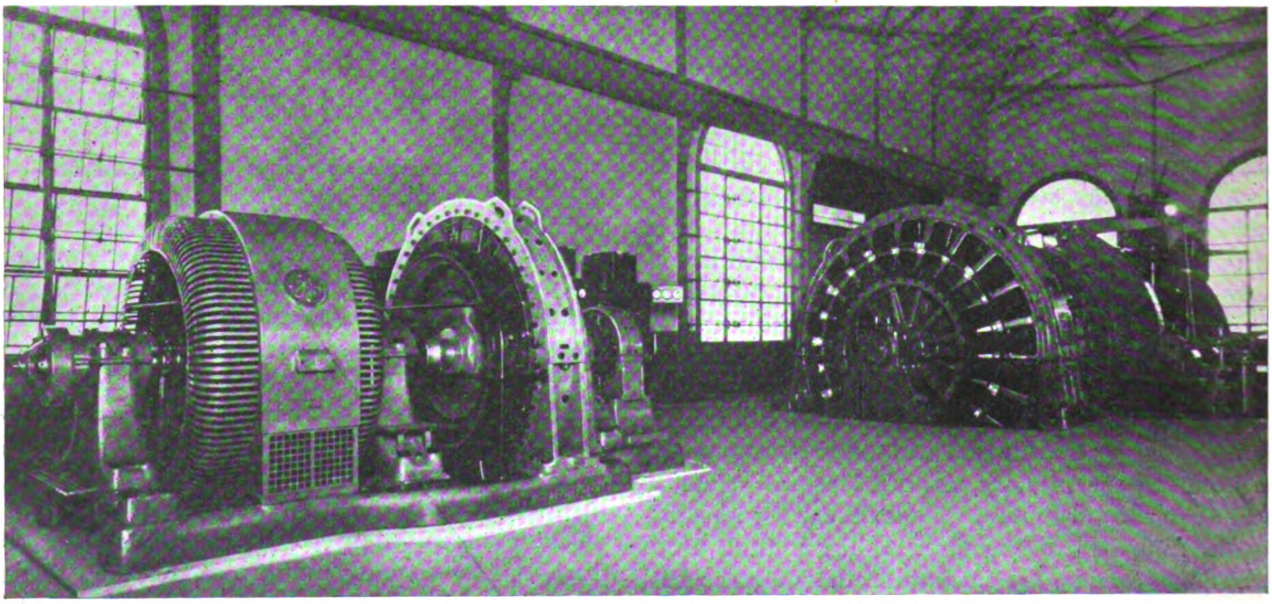
When the volt-ampere input and output to the primary or plate-current circuit are balanced, it is found that the input is proportional to the voltage and the output varies as the square of the voltage divided by the apparent impedance. This is satisfied by making the voltage proportional to the apparent impedance due to the antenna. The apparent resistance, reactance, and impedance of the circuit traversed by the plate current will be the same as already calculated for the primary coil of the arrangement with the intermediate circuit, as shown in Fig. 58. The plate alternating voltage in the arrangement without the intermediate circuit is shown in Fig. 60. It is proportional to the antenna voltage and is plotted as a function of the power-factor. The output and tube loss are also shown. Obviously, it is very undesirable to omit the intermediate circuit from a master-oscillator set for the slightest detuning cuts heavily into the radiation and causes excessive losses in the tube due to the decrease in the plate alternating voltage.

(To be continued)

Electric Hoists at the Belmont and Badger Mines



The Main and "Chippy" Hoists at the Belmont Mine of the Anaconda Copper Mining Company, wherein a Motor Capacity of Over 3000 hp. is Provided by Two Motors



The Main Hoist Motor and Synchronous Motor-generator Set at the Badger Mine. By means of such equipment seven tons of ore may be lifted at the rate of 2250 ft. per min. from depths approaching one mile

Features of 20-amp. A-c. Series Street Lighting Circuits

By HENRY E. BUTLER
Illuminating Engineering Laboratory, General Electric Company

FOR many years 6.6-amp. a-c. straight series circuits have been used for street lighting. In place of the 6.6-amp. lamps, 20-amp. series Mazda lamps were later used in many such circuits, since the latter give more lumens per watt.

The present practice is to burn these 20-amp. lamps from individual series transformers located in the base of each pole. These transformers consist of a 6.6-amp. primary winding and a 20-amp. secondary winding, the latter being highly insulated from the primary. This means that low voltage is used in the pole, which is highly insulated from the high-tension constant-current secondary line, and at the same time permits the use of 20-amp. lamps burning at a 3 per cent greater lumen output than a straight 6.6-amp. lamp. This system is possibly the most popular in use for White Way installations as it fills the requirements for this class of service, where low voltage up the pole is particularly required.

For some time the 20-amp. straight series constant-current circuits have been given considerable study by many central stations for White Way installations where the linear distance is short and the I^2R losses and losses due to inductance will not be objectionable. Local conditions will determine largely whether it is economical or good illuminating engineering practice to install a straight 20-amp. constant-current series lighting circuit, or a 6.6-amp. series circuit with each standard equipped with a pole-base series transformer; *i.e.*, an individual lamp transformer wherein the secondary is highly insulated from the primary or high-tension circuit.

In order that the engineer can conveniently determine the most efficient and most economical system that will meet his particular local requirements, the writer has included in this article a number of charts which may be referred to for assistance in studying the advantages of either system.

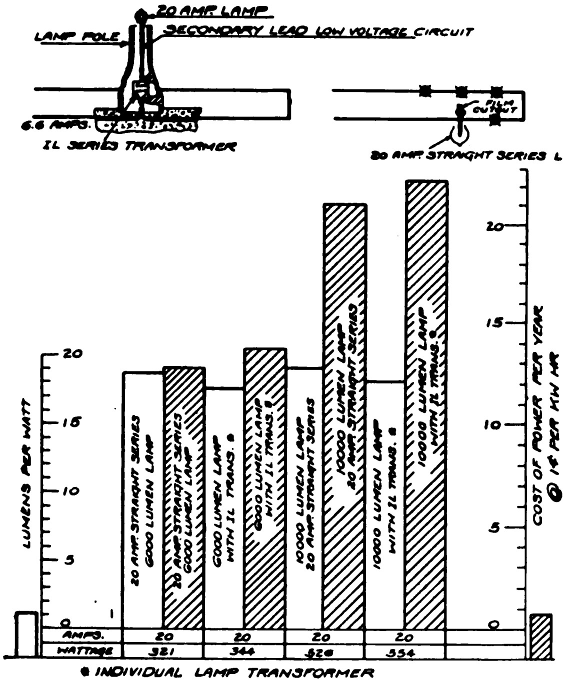


Fig. 1. Elementary Sketches of a Section of a 6.6/20-amp. Circuit with Individual-lamp Series Transformer and a Straight 20-amp. Circuit with Film Cutout; and (below) a Comparative Chart of the Lumens Per Watt and Power Cost

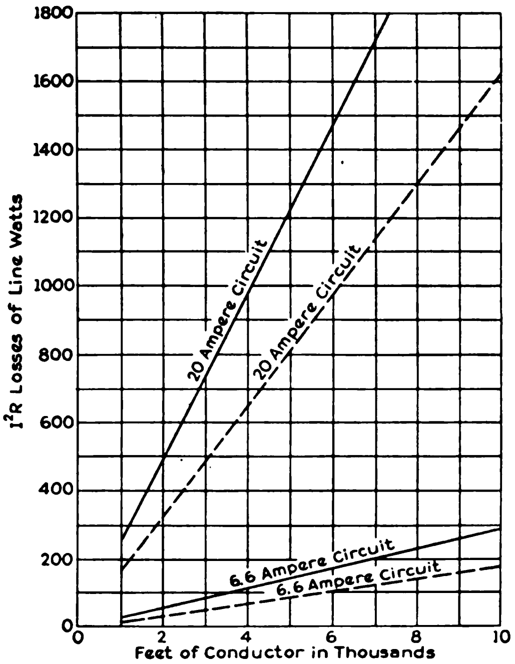


Fig. 2. Line Losses in 6.6-amp. and 20-amp. Series Circuits. The dotted lines indicate losses in No. 6 copper conductor; the full lines are for No. 8 copper conductor

Fig. 1 shows a summary of the advantages of the use of a series circuit with individual 6.6/20-amp. transformers instead of a straight series circuit. This should be helpful in determining the advantages from this angle.

Fig. 2 indicates the line losses of the 6.6- and 20-amp. a-c. series circuit using both No. 6 and No. 8 copper conductor. These data will also be helpful in determining whether the line losses from a 20-amp. series circuit will be excessive and thus prohibit its use.

This, of course, will depend largely on the linear distance which is to be illuminated. It will be found, however, that for short linear distance or White Way installations where small circuits are used a 20-amp. series circuit may be used to advantage both from an economic and the illuminating engineering standpoint.

Figs. 3, 4, 5 and 6 show the losses⁽¹⁾ under various conditions in armored cable with lead or steel sheath either open-circuited or short-circuited, which condition will depend upon local requirements. It should be noted in these charts that there is a rapid increase of losses due to inductance in going from a 6.6- to a 20-amp. circuit; and also that these losses are slightly lessened when cables are touching. These line losses will be of particular concern to those contemplating the use of a 20-amp. series circuit and will perhaps indicate the need of fibre, clay, or other non-metallic

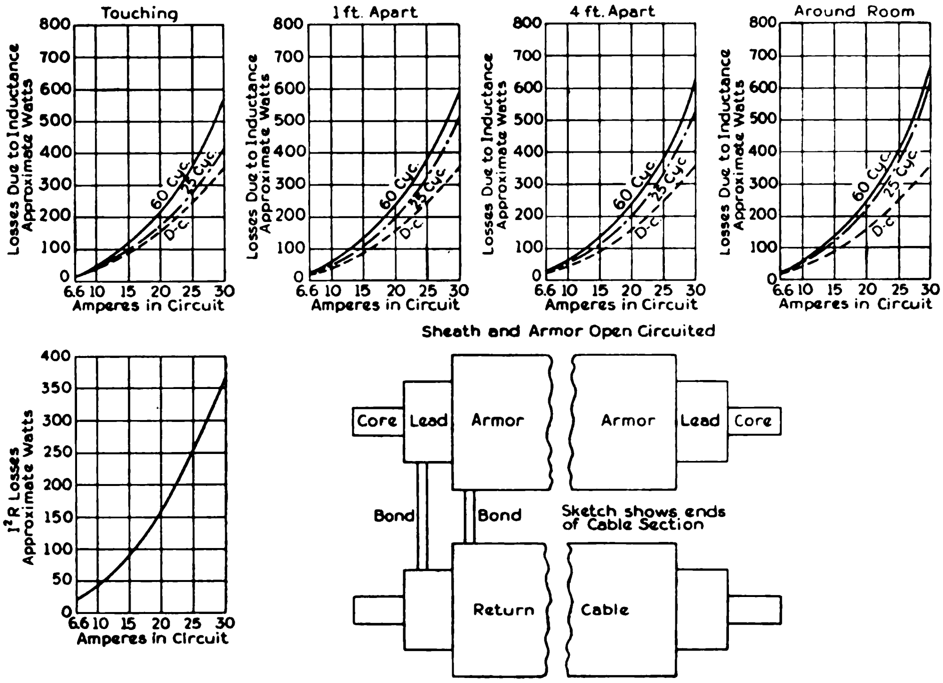


Fig. 3. Inductance and Resistance Losses in 1000-ft. Lengths of No. 6 Steel-armored Lead-covered Cable, with Both Armor and Sheath Open-circuited

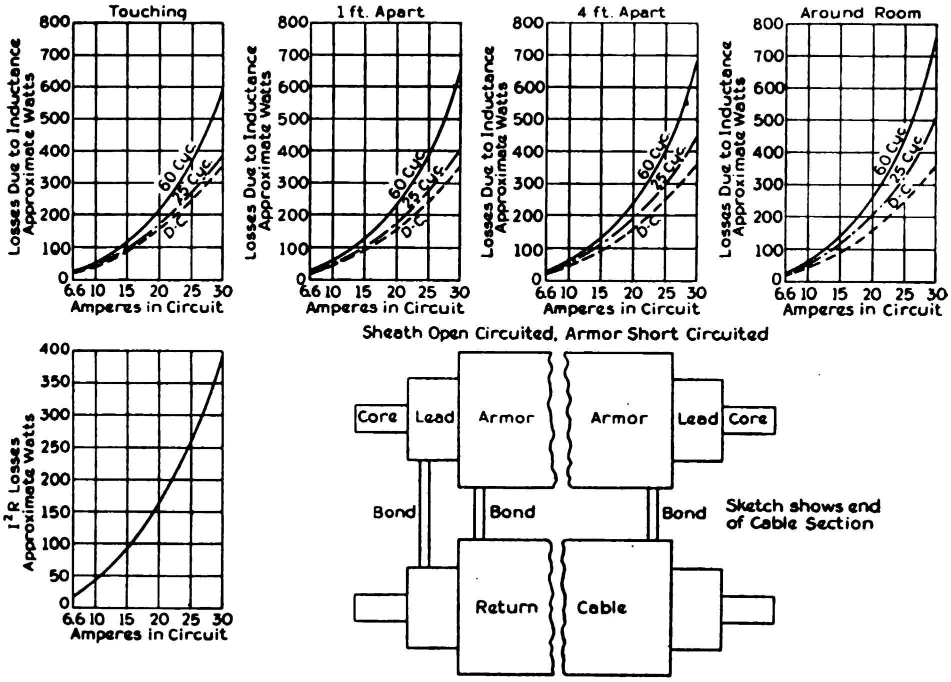


Fig. 4. Losses in Armored Cable in which the Lead Sheath is Open-circuited and the Steel-tape Armor is Short-circuited

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(1) The data for these curves are obtained from the *Electrical World*, Nov. 18, 1916.

conduit with plain lead-covered cable for such circuits.

Fig. 7 shows the full-load and open-circuited secondary voltages for pole-type and station-type constant-current transformers of various sizes when feeding 6.6-amp. and 20-amp. circuits, also standard voltage ratings of cable for underground and aerial installations, together with relative costs of the different cables. In each case the insulation for the cable used must be of sufficient strength to withstand the open-circuited secondary voltages. It will be observed that the insulation requirements of a 20-amp. series circuit are approximately one-third of those of straight 6.6-amp. series circuit and that the underground cable cost of a 20-amp. series circuit is approximately one-half the cost of a straight 6.6-amp. series circuit. The question of the relative transformer cost for the various series systems in use is a very interesting factor and should be given consideration, especially when the appropriation for street lighting is limited.

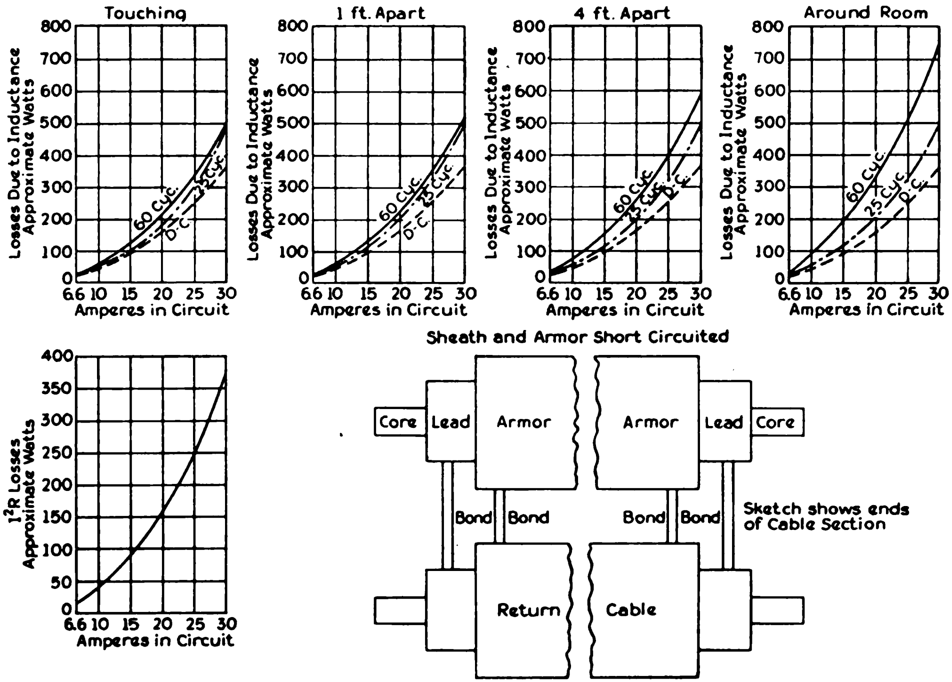


Fig. 5. Losses in Armored Cable as Obtained by Tests in which Both Lead Sheath and Steel-tape Armor Were Short-circuited

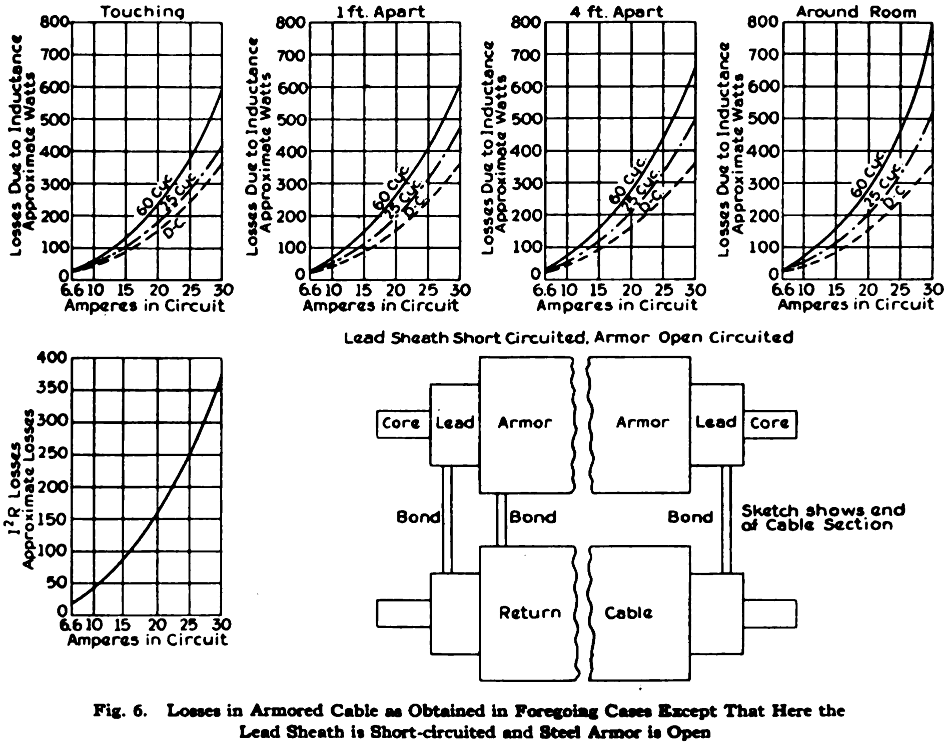


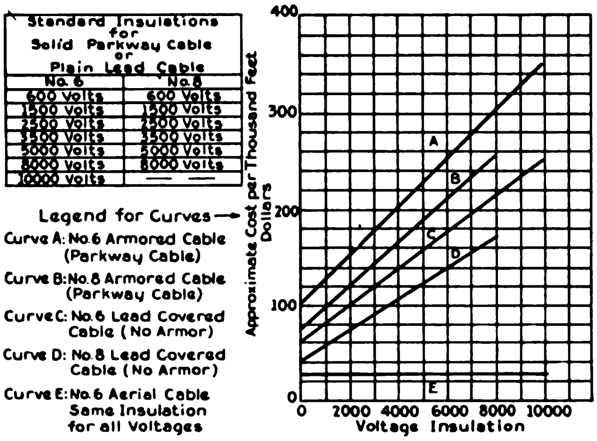
Fig. 6. Losses in Armored Cable as Obtained in Foregoing Cases Except That Here the Lead Sheath is Short-circuited and Steel Armor is Open

Fig. 8 shows the relative transformer cost and operation cost per unit of the following series systems which are in standard use today:

6.6-amp. straight series system
20-amp. straight series system
6.6-amp. series system with auto-transformer
6.6-amp. series system with insulating transformer.

These comparisons were based on five 3-kw. pole-type constant-current transformers in one case, and in another on the basis of one 15-kw. pole-type constant-current transformer. The latter fulfills the average local conditions. It will be observed upon analysis of Fig. 8 that the 20-amp. series circuit is the more economical, especially for short distances; but it must be remembered, however, that there is an advantage in the use of the transformer, although the cost is higher than the straight 20-amp. series installation.

Pole Type Constant Current Transformer, 6.6 and 20 Amperes Secondary 60 Cycles					Station Type Constant Current Transformer, 6.6 and 20 Amperes Secondary, 60 Cycles				
Capacity in Kilowatts	Secondary Voltages				Capacity in Kilowatts	Secondary Voltages			
	Full Load		Open Sec.			Full Load		Open Sec.	
	6.6 Amp.	20 Amp.	6.6 Amp.	20 Amp.		6.6 Amp.	20 Amp.	6.6 Amp.	20 Amp.
1	152	50	221	73	5	758	250	910	300
2	303	100	435	144	10	1515	500	1820	600
3	455	150	650	215	15	2275	750	2725	900
5	758	250	1075	355	20	3030	1000	3640	1200
7½	1136	375	1600	530	25	3790	1250	4550	1500
10	1515	500	2100	690	30	4550	1500	5455	1800
15	2270	750	3180	1050	35	5300	1750	6365	2100
20	3030	1000	4250	1400	40	6060	2000	7275	2400
25	3790	1250	5260	1700	50	7580	2500	9100	3000
30	4550	1500	6375	2090	60	9100	3000	10910	3600
—	—	—	—	—	70	10600	3500	12750	4200



Standard Insulations for Solid Parkway Cable or Plain Lead Cable	
No. 6	No. 8
600 Volts	600 Volts
1500 Volts	1500 Volts
2500 Volts	2500 Volts
3500 Volts	3500 Volts
5000 Volts	5000 Volts
8000 Volts	8000 Volts
10000 Volts	— —

Fig. 7. Chart Showing Voltages for Constant-current Transformers, Together with Voltage Rating and Comparative Costs for Various Types of Underground and Aerial Cable

There has been considerable question as to whether the standard series socket and receptacle are suitable for 20-amp. series circuits since the standard socket was originally designed for 6.6-amp. service. It has been found from experience, especially in actual installations, that the standard series socket and receptacle operate satisfactorily on 20-amp. circuits. The following are a few of the systems which have been installed using the 20-amp. series circuit with the standard series socket and receptacle, and are today operating satisfactorily:

New York & Queens (New York City)
Racine (Wis.)
Ft. Worth (Texas)
St. Clair (Pa.)
Birmingham (Ala.)
Sudbury (Ont.)
Crystal Beach (Ont.)
Santiago (Chile)
Richmond (Va.)
Seattle (Wash.)
Chicago District
St. Louis District
Bridgeburg (Ont.)
Welland (Ont.)

As has been indicated, neither the 20-amp. a-c. series circuit nor the 6.6-amp. circuit with individual lamp transformers can be stated as being superior to the other for all purposes and for all conditions of installation. Each of these systems of street-lighting distribution has certain merits that are not possessed by the other; and, in consequence, the choice of system should be based upon a comparison of their respective merits with regard to each installation under consideration. It is hoped that the foregoing description and the following analysis will materially assist in making an advantageous choice for any given set of conditions.

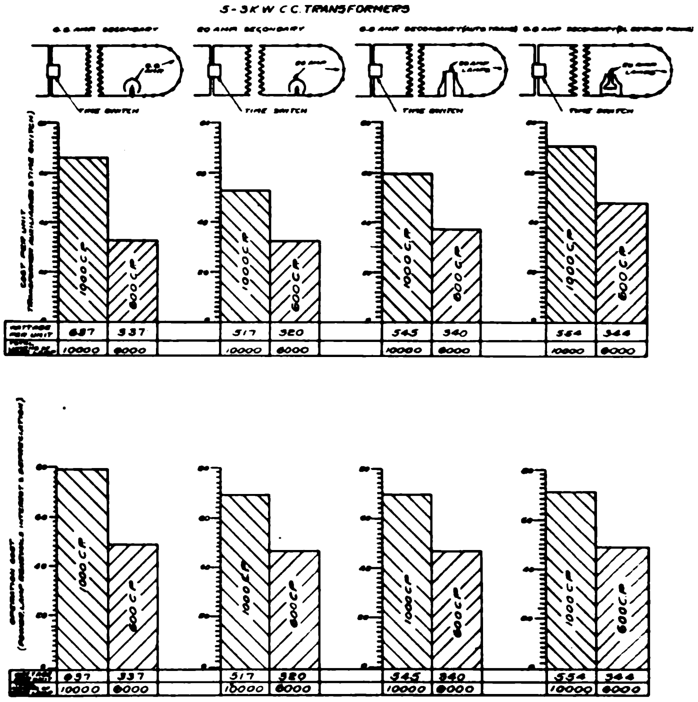


Fig. 8. Relative Costs of Transformers and Auxiliaries, 15-kw. Capacity, of 6.6-amp. Secondary with Auxiliaries *vs.* 20-amp. Secondary, Operating 20-amp., 6000-lumen and 10,000-lumen Mazda Series Lamps

Advantages of 6.6-amp. Series Lighting Circuit with Individual Lamp Transformers

Low voltage up the pole.

Line losses with 6.6-amp. approximately 1/10 as great as the corresponding losses in a 20-amp. circuit.

From the standpoint of standard lamp sizes, there are more sizes available for 6.6-amp. circuits than for 20-amp. circuits.

Lamp outages do not interrupt the high-tension series circuit and the burned-out lamps are therefore readily located.

Elimination of film cutouts on the main series circuit.

Protection of lamps from surges.

Advantages of a 20-amp. Series Lighting Circuit

Voltage of the constant-current secondary series circuit is less than ⅓ that of a 6.6-amp. circuit.

The prevailing circuits permit the straight 20-amp. circuit and lamp to be used economically without the necessity for an investment in a special transformer.

A larger number of lamps can be installed on the circuit because of the elimination of the inductive transformer load.

Simplicity of installation due to the elimination of individual-lamp transformers.

Disadvantages of a 6.6-amp. Series Circuit with Individual-lamp Transformers

Transformer involves an added investment cost per standard.

Cable connections to transformers are a possible source of trouble arising from poorly wiped joints.

Transformers represent an inductive load, reducing lamp capacity of the constant-current transformer; *i.e.*, it lowers the power-factor of the system.

Underground cable insulation cost is approximately twice that of a 20-amp. straight series circuit.

Disadvantages of a 20-amp. Straight Series Circuit

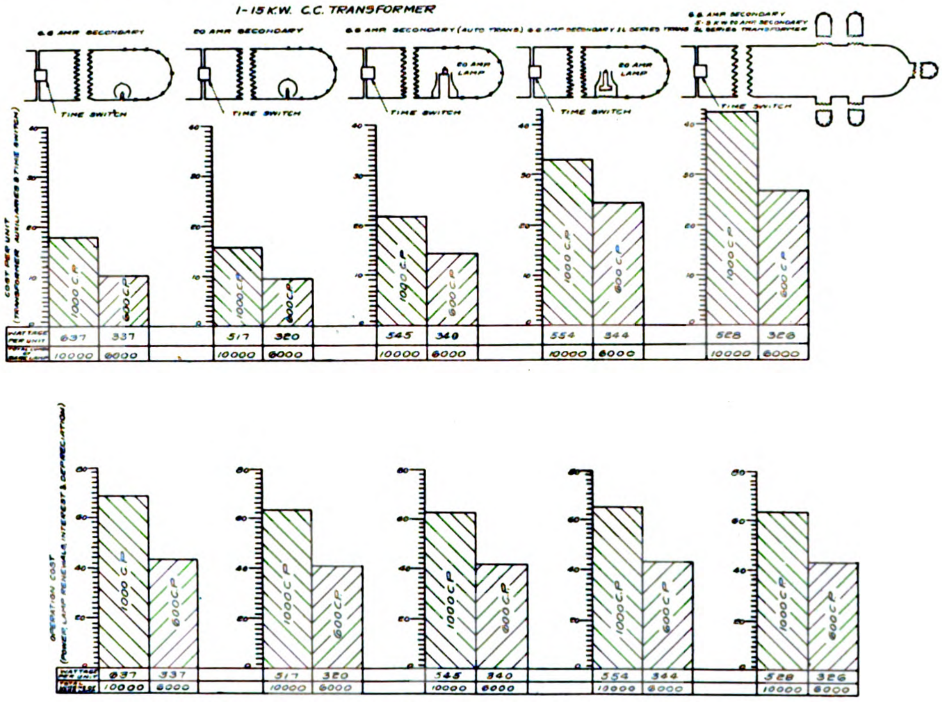
High I^2R losses in line.

High inductance losses where standard Parkway cable is used.

More lamp outages caused by failure of improper film cutouts.

High voltage up the pole.

More lamp failures due to surges.



Charts computed for a 4000-hr. year.
Power at $0.02 per kw-hr. (includes line loss, transformer loss, and lamp watts).
Line loss for 6.6-amp circuit, 1 per cent; for 20-amp. circuit, 9.2 per cent.
6 per cent interest and 10 per cent depreciation on investment.

Fig. 8 (cont'd). **Relative Costs of Transformers and Auxiliaries, 15-kw. Capacity, of 6.6-amp. Secondary with Auxiliaries *vs.* 20-amp. Secondary Operating 20-amp., 6000-lumen and 10,000-lumen Mazda Series Lamps**

Industrial Electric Heating

PART VII: THE RESISTOR FURNACE

By **N. R. STANSEL**

Industrial Engineering Department, General Electric Company

THE statistics of the resistor furnace are not complete. The chart given in Fig. 100 indicates the growth in the output of these furnaces since 1920. The figures include what is estimated to be about 85 to 90 per cent of the total. Some allowance must be made for furnaces discontinued so that, on the whole, it is believed that the chart gives a fair picture of the growth in the use of this type of furnace during recent years. The significant feature of these data is the continuously increasing kilowatt capacity installed year by year, showing that the resistor furnace has obtained a permanent place as a tool in industrial operations.

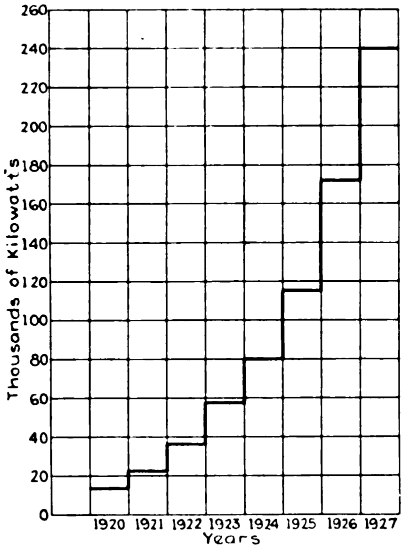


Fig. 100. Accumulation in Kilowatts of Installed Capacity of Resistor Furnaces Since 1920

The only limit in the size of the resistor furnace is the practicability of utilization. A parallel case is that of the electric motor which, as regards size, finds its widest use in small units. By far the majority of resistor furnaces are within the size range of 5 to 100 kw. Sizes from 100 kw. to 500 kw. are common, and a number between 500 kw. and 1000 kw. are in use. The upper limit in size at present is a furnace rated 3000 kw. Like the electric motor, the resistor furnace is efficient in all sizes, large and small, and its easy divisibility adds much to its value as a manufacturing tool.

The temperature range within which the resistor furnace operates is 750 to 2400 deg. F., with some latitude as to these limits. The general use of this furnace is for heat treatment processes;(36) although the entire list of its applications includes many heating operations which do not come altogether, if at all, within the definition of this term.

Resistor furnaces are divided into two types, *viz.:*

(*a*). Stationary-charge furnaces, known generally as batch furnaces.

(*b*). Moving-charge furnaces, a type which includes the various conveyor furnaces, rotary-hearth furnaces, pusher furnaces, etc., according to the mechanical arrangement used for moving the material to be heated through the heating chamber.

The batch-type furnace—as the name implies—is used for heating individual charges. The charge may or may not be cooled within the furnace enclosure. Several standard mechanical methods of conveying the charge to and from the heating chamber of the larger furnaces are in use and such furnaces are generally known by a descriptive title according to the method used, *e.g.*, car-bottom furnaces, in which the hearth is mounted on a truck; elevator furnaces, in which the hearth forms the platform of an elevator, etc. The selection of the method of handling the charge is a matter of consideration of the size, weight and character of the charge, the duration of the heating period, and the rate of production required. The smaller furnaces are usually served by hand; and as the size increases an overhead crane facilitates loading and unloading.

The moving-charge furnace has a continuous feed (the motion may be either continuous or intermittent) the material to be heated passing through the heating chamber, past the stationary heating units, at a speed suited to obtaining the desired temperature of the charge. In some cases a cooling chamber is included in the structure of this type of furnace. A selection of the form of conveying mechanism involves consideration of the character of the material to be heated, the process, the size and shape of the pieces, the volume, and the rate of production desired. These factors do not introduce any new problem in

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(36) A heat treatment process is an application of heat to produce a product in which quality and uniformity are essential. Part II, p. 564.

The following table facilitates the location of material hitherto published in this serial:

Part	*Issue*	*Figs.*	*Equations*	*Examples*	*Tables*	*Footnotes*
Part I	Oct. 1927, p. 488	1 to 23	(1) to (8b)	1 to 3	I to III	(1) and (2)
Part II	Nov. 1927, p. 551	24 to 43	(9) and (10)	4 to 6	IV to VII	(3) to (13)
Part III	Dec. 1927, p. 600	44 to 57	(11) and (12)	8 to 11	VIII to IX	(14) and (15)
Part IV	Mar. 1928, p. 125	58 to 73	(13) and (14)	——	X to XX	(16) to (26)
Part V	Apr. 1928, p. 204	74 to 83	(15) to (21)	12 and 13	XXI to XXIII	(27) to (29)
Part VI	Sept. 1928, p. 485	84 to 99	——	14	XXIV to XXVI	(30) to (35)

the movement of material in a definite sequence. However, with these factors must be combined the problems involved in the handling of material at comparatively high temperatures; and, following this, the method of cooling required which is often incorporated in the general scheme of the furnace. In some cases the problem of the flow of heat into the charge has a deciding influence. Descriptions of various arrangements of resistor furnaces will be given in later paragraphs.

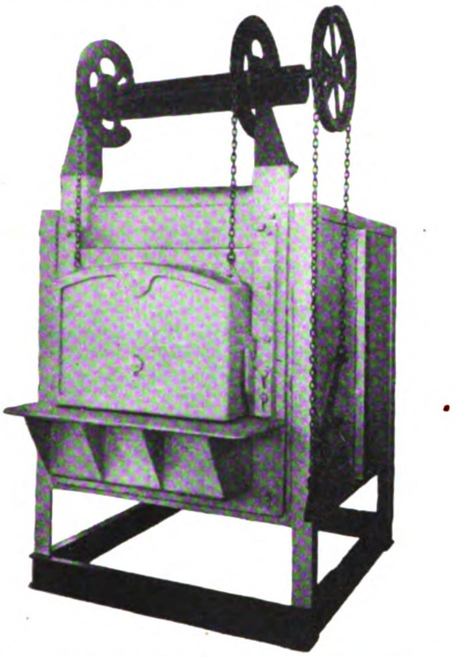

Fig. 101. A 30-kw. 220-volt Single-phase Resistor Furnace
Heating chamber dimensions: Length, 36 in.; width, 18 in.; height, 18 in.

The larger part of a treatment of the resistor furnace as a machine for the utilization of electric energy can be based upon the simple batch-type furnace, examples of which are shown in Figs. 101 and 102. The general construction of a moderate-sized resistor furnace of this type is shown in Fig. 103. The main elements of this furnace are:

(*a*). The walls of the heating chamber. (This includes the hearth, the roof and the door.)

(*b*). The heating units or resistors.[37]

(*c*). The atmosphere within the heating chamber.

The functions of these elemental parts are, of course, the same for both types of furnaces.

If there were no limitations as to the shape of the heating chamber for a given volume, the first thought would be the economics of a cube or sphere. This would hold were it not for the necessary door and the space factor of the charge. Also in case of large furnaces the space occupied by the furnace and its weight distribution may be deciding factors in the proportions of the heating chamber. With reference to these two last named factors a corresponding example is the hot-water tank. This has no door and the space factor of the content is a maximum, *i.e.*, unity; yet an elongated tank is the shape more often adopted.

On account of its movable construction, the door of a furnace cannot be made as effective as the wall structure proper in its resistance to the flow of heat to the air. Hence, the area of the door opening should be as small as other considerations will permit. The space factor of a charge is always less than unity; and the hearth area is increased accordingly. The length (or depth) of the heating chamber of a batch furnace is limited by the facilities for loading and unloading the charge. The width is limited by the design of the roof. The height is fixed by the height required for necessary clearance in moving the charge into and out of the heating chamber. The result of all of these factors is the general adoption of the shape of heating chamber for horizontal furnaces in which the length is the greatest dimension. Similar considerations apply to vertical furnaces.

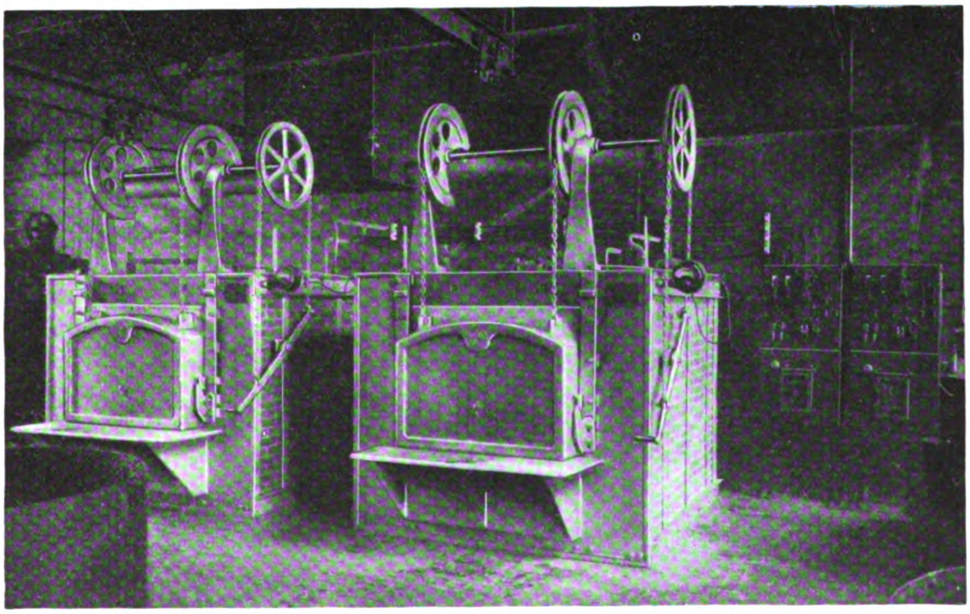

Fig. 102. An Installation of Two 50-kw. 220-volt Single-phase Resistor Furnaces
Heating chamber dimensions: Length, 54 in.; width, 36 in.; height, 23 in.

Resistor furnaces are classified according to the service conditions as follows:

(1). Daily use, one shift per day with partial cooling overnight and more complete cooling over the week-end.

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[37] Resistor: A device used primarily because it possesses the property of electric resistance.

(2). Heating—cooling service. In this service the charge is cooled to some predetermined temperature (frequently the handling temperature) before withdrawal from the furnace structure.

(3). Continuous use.

(4). Intermittent use.

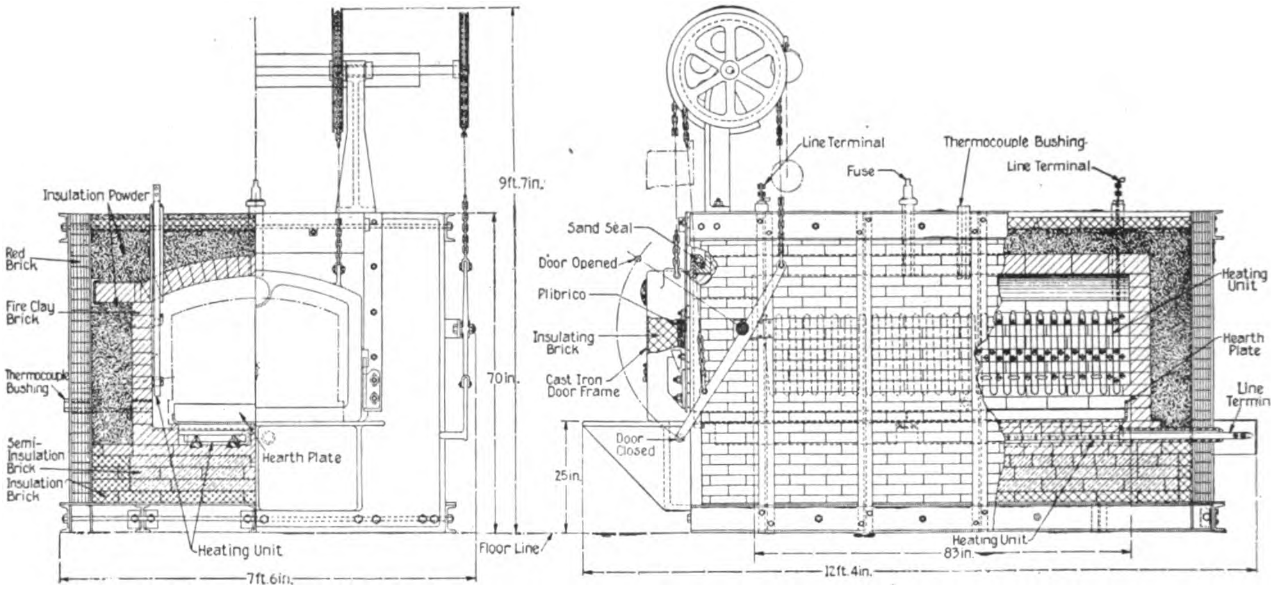

Fig. 103. The Typical Construction of a Batch-type Resistor Furnace

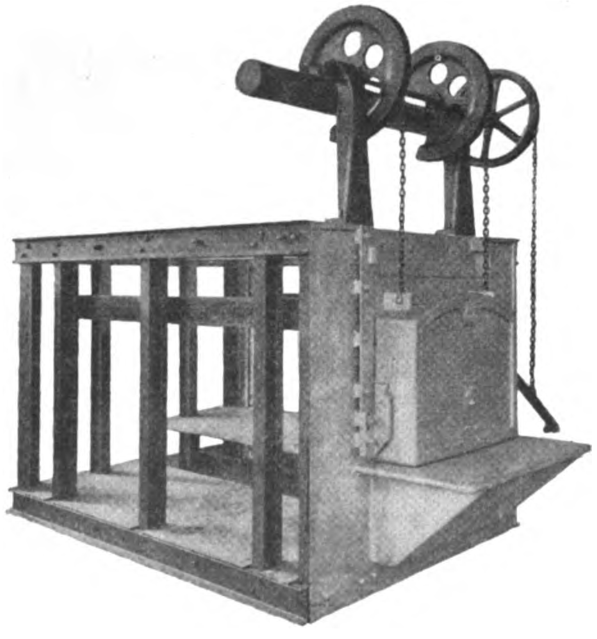

Fig. 104. The Steel Framework for a 60-kw. Batch-type Resistor Furnace

The enclosure of the heating chamber of a resistor furnace has a threefold function, providing for:

(*a*). Mechanical strength.

(*b*). Heat storage.

(*c*). The conservation of the energy supplied to the furnace.

In the case of arc furnaces—in which the temperature of the source of heat exceeds the fusion temperature of the materials available for the furnace walls—heat is sacrificed to prevent the building up of a temperature within the furnace which would destroy the structure. As it is, the wall problem of the arc furnace is to obtain an economical life of the furnace, particularly of the roof, where the temperature conditions are most severe. The temperature of the source of heat in the resistor furnace is below that at which materials available for the lining are affected to any great extent; and the lining of the heating chamber can be surrounded with heat-insulating material with a thermal resistance between the heating chamber and the surrounding atmosphere as high as may be economical.

Adequate mechanical strength in the furnace structure is obtained in the case of small and medium size furnaces by adding to the wall structure an outer steel casing, as shown in Fig. 101. For the larger furnaces a supporting steel framework (as shown in Fig. 104) is provided. In this case the outer casing is made of common brick as shown in Fig. 103.

Heat stored in the walls of a furnace is analogous to energy stored in a rotating flywheel. In both cases we have mass. Temperature corresponds to velocity. A flywheel is used to absorb and deliver energy corresponding to a varying load so that the power demand upon the source of energy supply will not exceed a certain value. Heat stored in the walls of a resistor furnace is a similar reservoir of energy and is utilized in three ways: *First*, to limit the demand on the electric circuit to the furnace; *second*, to maintain the flow of heat to the charge when the power is off through the functioning of the "On and

Off" control equipment(38) (the type of control equipment in general use with resistor furnaces); and *third*, to assist in equalizing the distribution of the heat flow to the charge.

The maximum flow of heat to a charge placed in a heating chamber occurs when there is the maximum difference of temperature. This, in the case of a heating chamber held at a constant temperature, is at the beginning of the heating period. The flow decreases as the temperature difference becomes less and less and becomes zero when temperature equalization is attained. This can be pictured as shown in Fig. 105. Such a curve represents an undesirable load characteristic; and the reservoir of heat formed by the walls is drawn upon to supply the peak load of the flow of heat to the charge.

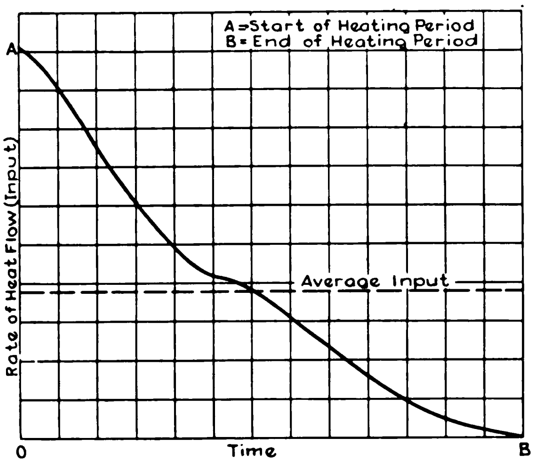

Fig. 105. Power Input Curve for a Charge of Steel in a Heating Chamber Held at Constant Temperature

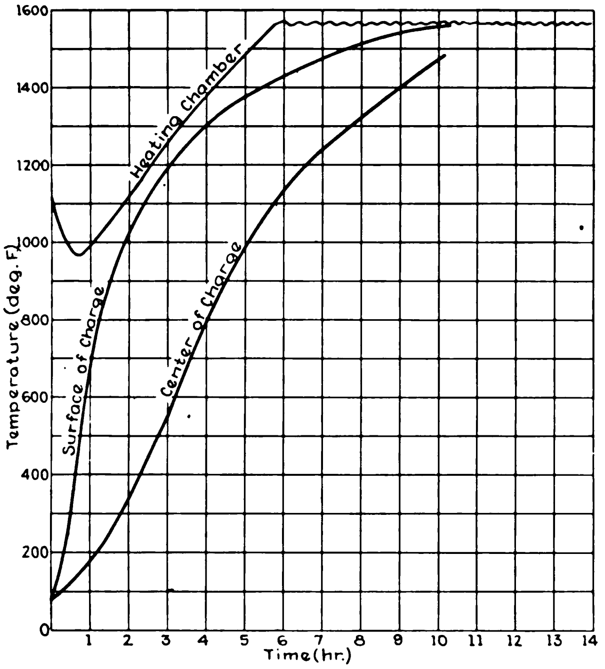

Fig. 106. Temperature—Time Curves of a Furnace and Its Charge of Steel

When the rate of flow of heat to the charge, in diminishing, reaches a value which plus the rate of flow of heat to and through the walls is equal to the kilowatt capacity of the heating units in the furnace, the control equipment begins to function; and the power in the electric circuit is alternately switched on and off. (When the power is on the walls absorb heat and when the power is off heat flows from the walls to the charge.) The frequency of this operation decreases as the flow of heat to the charge becomes less and less and becomes a minimum when temperature equalization of all parts, including the charge, within the heating chamber is attained. Through this graduation of the supply of power corresponding to the demand a practically constant heating-chamber temperature is maintained.

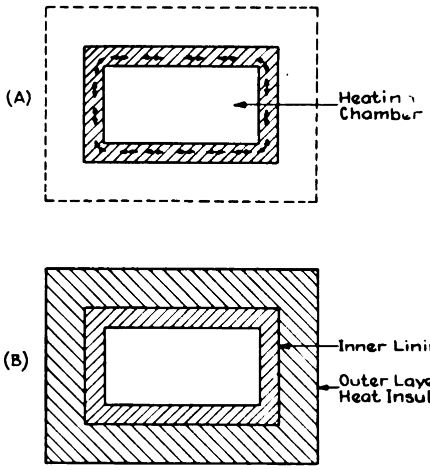

Fig. 107. Elementary Diagrams of a Furnace Showing (*A*) the Paths of Heat Flow in Lining, and (*B*) the Heat Insulation

The utilization of stored heat requires, of course, a drop in temperature of the heat reservoir. Hence, at the beginning of the heating period there will be a temperature drop in the heating chamber due to the flow of heat from the walls to the charge. Also there is the loss of heat through the door area while the door is open to admit the charge. The result is that the usual temperature-time curve of a furnace shows a drop in temperature at the start of the heating period, an example of which is shown in Fig. 106. This drop in temperature and the time for recovery in any particular case will depend upon the design of the furnace, the space factor of the charge, the weight of the charge, the normal temperature of the heating chamber, the initial temperature of the charge and upon the duration of the period that the door is open. Thus in case of a large furnace and a small charge quickly inserted, the temperature drop would be small and the recovery rapid. The effect of this drop in the temperature of the heating chamber is only to reduce the rate of heating of the charge during the initial part of the heating period.

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(38)"Industrial Heating Control," by P. H. Clark, GENERAL ELECTRIC REVIEW, September, 1927, p. 446.

A constant temperature of the heating chamber usually refers to the temperature of the point in the chamber at which the controlling thermocouple is located. It is desirable to maintain the temperature of the heating chamber constant both to secure a continuous rising temperature of the charge and to obtain full output from the furnace. In heat treatment processes, however, the uniform heating of the charge is a primary consideration. This requires—as stated in Part II, p. 551—that there be a uniform flux of heat supplied to each unit area of the surfaces of the charge. This uniformity is obtained in the resistor furnace by a wide distribution of the heating units, as shown in Fig. 103, which will be referred to later; by radiation (see Part II, p. 553); and with the aid of the thermal storage capacity of the inner lining of the heating chamber and of the thermal conductivity of this lining. The conductivity referred to is that along the paths of heat flow parallel to the sides of the heating chamber, as indicated by the double-headed arrows (indicating flow in either direction according to the direction of the temperature difference) in Fig. 107(A). The higher this conductivity the more effective will be the heat distribution within the inner lining. The less the quantity of heat stored in this inner envelope, and the higher the thermal resistance of the lining, the less effective are the thermal properties of the lining in aiding radiation toward providing uniformity in the heat distribution.

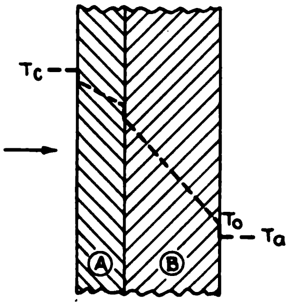


Fig. 108. Temperature Gradient of a Composite Type Furnace Wall

(*A*) is the fire brick lining.
(*B*) is the outer layer of heat insulation.
T_c = Temperature of heating chamber.
T_o = Temperature of outside surface.
T_a = Temperature of air.

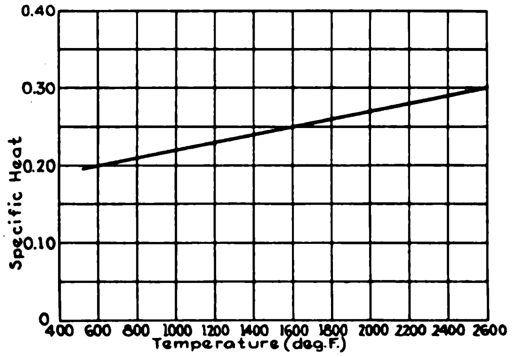
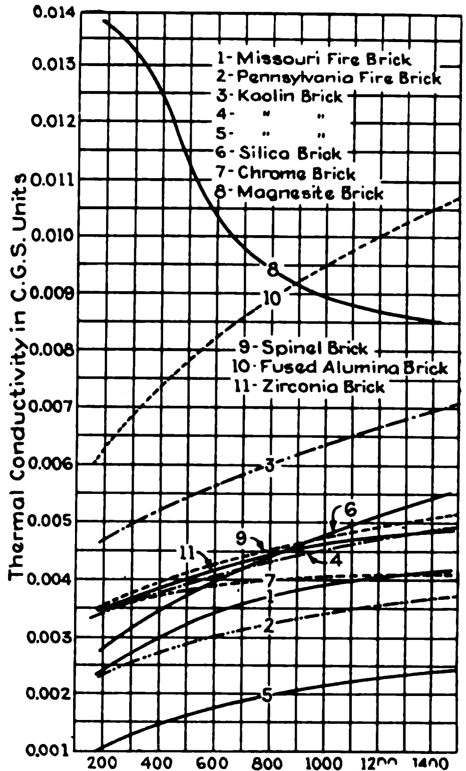


Fig. 109. Mean Specific Heat Curve for Fire Brick (*Bradshaw and Emery*)

k = B.t.u. per sq. ft. per hr. per in. thickness per deg. F. temperature difference.

As regards this thermal storage capacity and its effect upon temperature control and heat distribution there is no rule to follow, as both the thermal storage capacity of the lining and of the charge must be considered jointly. In general, the quantity of heat stored in this lining—when the steady state of heat flow through the walls is attained—should exceed by a considerable margin the thermal storage capacity of the charge at the given temperature to which the charge is to be heated. It will be found that there is not a wide range of choice as to the thermal conductivity of the inner lining.



(*Courtesy of A.S.M.E.*)

Fig. 110. Relative Thermal Conductivities of Refractory Materials (*Phelps*)

As previously noted (Part II, p. 563), the total loss of energy from a resistor furnace—with respect to the wall structure—during a given period of operation is the sum of the loss of stored heat and of the loss of heat by conduction through the walls. Hence the thermal storage capacity and the thermal conductivity as obtained in the complete wall structure determine the conversion efficiency of the furnace.

The material most generally available for the inner lining of the furnace is fire brick. This material has sufficient strength at high temperatures and sufficient resistance in other ways against the effects of heat. In addition, fire brick has a comparatively high thermal storage capacity; and, from the viewpoint of heat insulation, a comparatively high thermal conductivity. The last named property is, of course, opposed to the requirement for the conservation of heat. Hence, in order to keep the structure within economic limits it is necessary to surround a fire brick lining with materials which have a low coefficient of thermal conductivity, *i.e.*, heat insulation, as indicated in Fig. 107(B). This in practice is the composite wall construction shown in Figs. 29 and 30 (Part II). The temperature gradient through a wall of this construction is shown in Fig. 108.

The mean specific heat curve of fire brick given by Bradshaw and Emery(39) is shown in Fig. 109. The average weight of this material is 130 lb. per cu. ft. The *k-T* (coefficient of thermal conductivity and temperature) curves of fire brick vary with the analyses and the temperature at which the brick were burned; the higher the burning temperature, the higher the thermal conductivity. The chart given in Fig. 110 by Phelps(40) gives *k-T* curves for fire brick and of other refractory products.

The earth product diatomite—a form of a silica described in Part II (p. 562)—in general use as the basis of heat insulation for resistor furnaces is available in a number of manufacturing shapes and in the form of powder. In some cases, the raw material is simply cut to size and dried. In others binders of different kinds are mixed with the raw material, which may or may not have been purified to a certain degree; and after being formed into shapes the material is given a certain degree of burning or heating.

The mean specific heat—temperature (*k-T*) curve of diatomite is practically the same as that of fire brick (Fig. 109). This earth in its natural state weighs from 25 to 30 lb. per cu. ft. The thermal storage capacity per unit volume and the coefficient of thermal conductivity of the manufactured forms vary with the method of manufacture. A *k-T* curve for diatomite in its natural state is shown in Fig. 111. The corresponding values of *k* at one temperature for some of the manufactured forms of this material as published by the manufacturers are given in Table XXVII.

TABLE XXVII

DIATOMITE HEAT INSULATION

Form	*k* at 1000 deg. F.	Lb. per cu. ft.
Brick A	0.85	28
Brick B	0.68	30
Brick C	1.22	48
Block	0.65	30
Powder	0.52	(x)

(x) Tamped to approximately 14 lb. per cu. ft. The loose powder weighs about 8 lb. per cu. ft.

Depending upon the method of manufacture, the diatomite heat insulating products each have an upper limiting temperature above which the insulating quality is impaired. As a rule, the higher this temperature limit the higher is the value of *k* for the product at a given temperature. With the composite wall construction the inner face of the inner layer of heat insulation is protected by the inner lining of fire brick. The drop in temperature through the inner lining is proportional to the rate of flow of heat through the lining. Hence the barrier to the flow of heat through the wall to the atmosphere imposed by the heat insulation must permit a sufficient flow of heat through the wall to keep the inner face of the layer of heat insulation cool, *i.e.*, below the maximum temperature specified as the limit in any particular case. (If the temperature of the heating chamber is below the maximum value specified as a limit for the preservation of the heat insulation, this provision of course does not apply.) From this viewpoint, low thermal conductivity of the inner lining is desirable. However, by the nature of materials, high thermal storage capacity and high thermal conductivity go together and, conversely, low thermal storage capacity and low thermal conductivity are twins also. Hence, in the general class of materials used for furnace walls, weight alone is a fair guide to the thermal specification of a material.

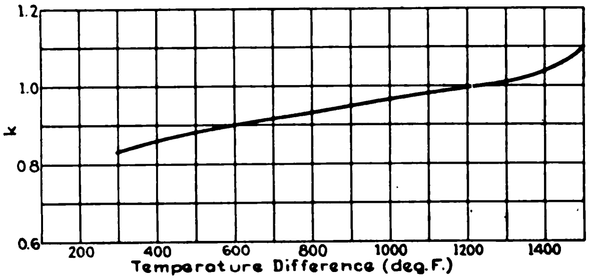


Fig. 111. ***k-T* Curve for Natural Diatomite**
(Armstrong Cork and Insulation Co.)

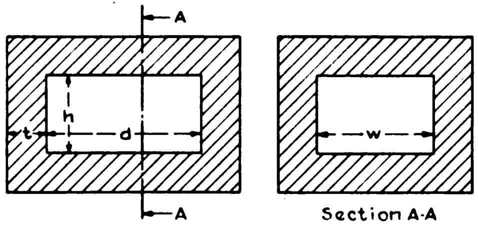


Fig. 112. Shape Factor Diagram for Rectangular Furnaces

As stated in previous paragraphs, the primary requirement for the outer layer of the composite wall is high thermal resistance. The bottom of the furnace, however, requires in addition sufficient mechanical strength to support the weight of the charge; and the roof design is individual. Hence the construction of the enclosure of the heating chamber cannot be uniform, but must be of the general arrangement shown in Figs. 29, 30, and 103. This variation in construction results in variations in the thermal resistances of the paths of heat flow through different sections of the heating-chamber enclosure. Furthermore, the different positions of the outside surfaces of the furnace, top, bottom, and sides, also introduce variations in the outside surface thermal resistance (see Fig. 11 in Part I, p. 494). The result is that the temperature at the inner face of the layer of heat insulation (which is the heating chamber temperature minus the temperature drop in the inner lining) for a given total kilowatt loss by conduction through the walls will vary at different points on the inner face of the layer of heat insulation. With a constant heating-chamber temperature and a steady state of heat flow, the temperatures at different points on the outside surfaces of a furnace are an index to the variations in the temperatures around the inner face of the layer of heat insulation. The measured values of outside temperatures in one case are given in Table XXVIII.

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(39) *Trans. Ceramic. Soc.* (England), Vol. 19, p. 84.
(40) *Trans. Amer. Soc. Mech. Engrs.*, January-April, 1928.

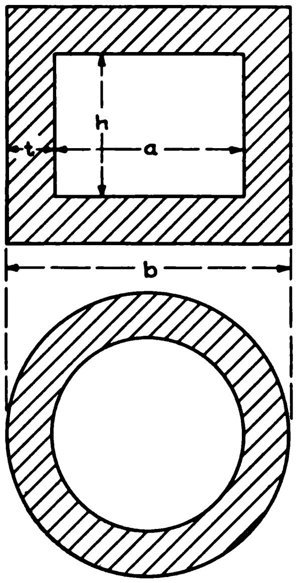

Fig. 113. Shape Factor Diagram for Cylindrical Furnaces

TABLE XXVIII

AVERAGE OUTSIDE SURFACE TEMPERATURE OF A FURNACE

(Heating chamber temperature constant at 1800 deg. F.)

Location	Temperature, deg. F.
Front wall (door end)	300
Right side wall	141
Left side wall	165
Rear wall	124
Top	145
Bottom	167

Hence, as we deal only with average temperature values, some margin should be allowed between the maximum permissible temperature in any given case and the average value for the inner face of the layer of heat insulation so that the maximum sustained temperature at any point will not be above the safe value.

By reference to the temperature gradient curve in Fig. 108 it is noted that the average temperature of the inner lining of fire brick is much higher than the average temperature of the outer layer of heat insulating material. Thus, referring to the specific heat curve in Fig. 109, the mean specific heat of the fire brick will be higher than that of the heat insulation. The unit weight of fire brick is also much greater than the unit weight of the material used for the heat insulation. Therefore, for a given temperature gradient the thermal storage capacity of a unit volume of the fire brick lining of a composite wall is several times that of a unit volume of the heat insulation. The greater total volume of the outer layer of the wall does not go far in compensating for this difference. Hence in the composite wall construction we have the larger part of the thermal storage capacity of the structure in one part—the inner fire brick lining—and the larger part of the thermal resistance in the paths of heat flow from the heating chamber to the surrounding atmosphere in the other part—the outer layer of heat insulation.

The general equation for the flow of heat through a wall by conduction (Equation (5) Part II, p. 493), is based upon a wall with parallel sides and of great area. The walls of a heating chamber being joined, the area of heat flow is enlarged at each junction. The correction of Equation (5) for this condition, termed the "shape factor," determined by Langmuir(41) is as follows:

For heating chambers as shown in Fig. 112

$$S=\frac{A}{t}+0.54\ \Sigma l+1.2\ t \qquad (22)$$

in which

S = shape factor for rectangular chambers
A = inside area of walls in square inches
t = thickness of walls in inches
Σl = sum of the lengths of all inside edges.

For heating chambers as shown in Fig. 113, the shape factor is

$$S=\frac{A}{t}+0.54\ \Sigma l+\frac{2\ \pi\ h}{\log_e\left(\frac{b}{a}\right)} \qquad (23)$$

in which

S = shape factor for cylindrical chambers
A = inside area of the top and bottom surfaces in square inches
Σl = sum of the lengths of the inside top and bottom edges in inches
h = inside height in inches
b = outside diameter in inches
a = inside diameter in inches.

Equation (5) as applied to the walls of heating chambers thus becomes

$$P_c=\frac{S\times k_m\times(T_c-T_o)}{3412\times 144} \qquad (5a)$$

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(41) *Trans. Am. Electrochem. Soc.*, Vol. 24. pp. 53-84.

in which

P_c = flow of heat through the wall in kilowatts.
S = shape factor
k_m = mean value of k for the material of the wall corresponding to the temperature gradient through the wall
T_c = temperature of the hot side of the wall
T_o = temperature of the cool side of the wall.

The door, the roof, and the bottom of a furnace must be constructed to suit individual requirements and hence all differ from the side-wall construction. The outline of a section of the furnace is not a simple rectangle. Terminal connections through the walls for the heating units and for the control equipment must be provided. All of these contribute toward lowering the average thermal resistance of the enclosure of the heating chamber. Hence the actual flow of heat from the heating chamber to the atmosphere in any given case is greater than the value given by Equation (5a). It is therefore necessary to supply a suitable constant greater than unity to take into account the effect of these factors in an actual furnace. This constant varies according to the size and proportions of the heating chamber; and various individual conditions relating to furnace construction have an influence such that, on the whole, the constant represents the summation of experience which cannot be set down in a table of values. However, the departure of an actual furnace from the simple outline of Fig. 107(A) does not introduce any change in the principles involved; and an example not complicated by practical details can be used for illustration.

Example 15. For this example is taken a batch-type furnace similar to that shown in Fig. 101 for the daily use noted as Class 1 service. The inside dimensions of the heating chamber are:

Length	54 in.
Width	24 in.
Height	20½ in.

The wall construction will be of the composite type. A thickness of 4½ in. for the fire brick lining is a tentative selection—as a matter of obtaining ample mechanical strength and of the use of standard material. The temperature conditions are:

Heating chamber	1800 deg. F.
Permissible average temperature of the inner face of the layer of heat insulation	1500 deg. F.
Temperature of the outside surface of the furnace limited to approximately	150 deg. F.
Temperature gradient of the wall	Fig. 108

The shape factor of the fire brick lining as given by Equation (22) is $S_b = 1505$. The temperature drop through the fire brick is $T'_b = (1800-1500) = 300$ deg. F. From Fig. 110 the coefficient of thermal conductivity of fire brick is $k_m = 9.5$. By Equation (5a) the flow of heat through the wall is

$$P_c = 8.75 \text{ kw.}$$

Tests of a furnace having this wall construction show a total heat loss at 1800 deg. F. of 10 kw. Hence the multiplying constant in this case is $C = \frac{10}{8.75} = 1.15$, and we have

$$P'_c = 8.75 \times 1.15 = 10 \text{ kw.}$$

The temperature drop through the layer of heat insulation is $T'_i = (1500-150) = 1350$ deg. F. The coefficient of thermal conductivity of the wall is the average value resulting from the variations in the wall, hearth, and roof constructions. A tentative assumption is $k_i = 2.0$. The heat flow through the layer of insulation is the same as that through the fire brick lining. Hence we can place Equation (5a) applied to the layer of heat insulation in equality with that equation applied to the fire brick lining, *i.e.*,

$$S_i \times k_i \times T'_i = S_b \times k_b \times T'_b \tag{24}$$

and by substituting values we have as the shape factor of the layer of heat insulation,

$$S_i = 1412$$

This value substituted in Equation (22) gives a value for the thickness of the layer of heat insulation, $t_i = 9$ in.

In the foregoing the temperature drops at the inner surface of the fire brick lining and at the joint between the two layers of the wall has not been included. These temperature drops are both comparatively small and variable and their effect may be considered as giving a certain margin to the assumptions it is necessary to make.

The total wall thickness is (4½+9) = 13½ in.; and this together with the thickness of the outer steel plate casing gives an outside surface area of approximately 165 sq. ft. For a total flow of 10 kw. of heat through the walls, the average rate of heat dissipation from the outer surface is 66 watts per sq. ft. From Fig. 14 (Part I, p. 495) this corresponds to an average outside surface temperature of approximately (100+70=) 170 deg. F., which in the majority of cases would be permissible.

The volume of the inner fire brick lining is 20 cu. ft. The average temperature of this lining is 1650 deg. F. The specific heat from Fig. 109 is $s = 0.25$. By Equation (2b) the heat stored in the fire brick is $h_a = 290$ kw-hr.

The volume of the layer of heat insulation is 78 cu. ft. The average temperature is 825 deg. F. and the corresponding value of the specific heat is $s = 0.21$. This gives 80 kw-hr. stored in the heat insulating materials.

The total calculated stored heat is (290+80) = 370 kw-hr. Some allowance must be made for heat stored in other parts of the structure and for excess material over that required for the elementary furnace outline assumed. In an actual furnace of this size and type the total quantity of heat stored in the structure with a heating chamber temperature of 1800 deg. F. is approximately 420 kw-hr.

From the records of a resistor furnace of this size and construction it is found that, after cooling overnight from a temperature of 1800 deg. F., the energy required to bring the heating chamber temperature back to 1800 deg. F. averaged 85 kw-hr. and that the total loss of stored heat for a week period averaged 600 kw-hr. The corresponding loss of heat by flow through the walls during a week period averaged 480 kw-hr. This is a fair balance between these two losses for this method of operation as regards the economic utilization of materials in furnace construction.

(To be continued)

Floodlighting of the New City Hall, Los Angeles (Cal.), on the Occasion of the Dedication of the Building and the Lindbergh Beacon which Surmounts It

LIBRARY SECTION

Condensed references to some of the more important recent articles in the technical press, and to new books of interest to the industry, as selected by the General Electric Main Library.

Antennas

Characteristics of Certain Broadcasting Antennas at the South Schenectady Development Station. H. M. O'Neill.
Inst. Radio Engrs. Proc., July, 1928; v. 16, pp. 872-889.

Bus Structures, Power Plant

Concrete Bus-Structure Tests. Willard A. Kates.
Elec. Wld., June 30, 1928; v. 91, pp. 1379-1382.

Capacitors

Direct-Current Transmission Approximated With Alternating Current.
Power, July 10, 1928; v. 68, p. 80.
(Short account of an installation of three series capacitors on a New York Power & Light Corp. 33,000-volt, 7,500-kv-a. tie-line at Ballston Spa, N. Y.)

Cars, Gas-Electric

Gas-Electric Motor Cars as Applied to Steam Railroads. P. M. Gillilan.
Rwy. Age, June 2, 1928; v. 84, pp. 1273-1275.
("A presentation of the desirable features of differential-field generator control.")

Electric Arc

Probe Measurements in the Normal Electric Arc. W. B. Nottingham.
Franklin Inst. Jour., July, 1928; v. 206, pp. 43-55.

Electric Drive—Power Plant Auxiliaries

Factors That Influence the Selection of Motors for Power-House Auxiliaries. H. L. Smith.
Power, June 26, 1928; v. 67, pp. 1145-1148.

Floodlighting

Motion-Color Floodlighting. Arthur A. Brainerd.
Elec. Wld., June 23, 1928; v. 91, pp. 1346-1348.
(Illustrated description of the installation used on the new 26-story building of the Philadelphia Electric Co.)

Glass

Dielectric Data on "Pyrex." C. L. Dawes and P. H. Humphries.
Elec. Wld., June 23, 1928; v. 91, pp. 1331-1332.
("Tests of material over wide range of frequency and temperature establish a standard of comparison for dielectrics exhibiting ionization and hysteresis lag.")

Inductive Interference

Effect of Street Railway Mercury Arc Rectifiers on Communication Circuits. Charles J. Daly.
A.I.E.E. Jour., July, 1928; v. 47, pp. 503-506.
(Compares the telephone interference effect of mercury arc rectifiers and rotary converters as measured in the Bridgeport, Conn. district.)

Lightning

Lightning Investigation on New England Power Company System. E. W. Dillard.
A.I.E.E. Jour., July, 1928; v. 47, pp. 489-491.
(An analysis of the surges recorded during 1927 on a 75-mile, 110-kv. double-circuit transmission line.)

Oscillographs

High-speed Recorder. C. I. Hall.
A.I.E.E. Jour., July, 1928; v. 47, pp. 516-519.
(Illustrated description of the design and construction of the high-speed recorder. Abridgment.)

Radio Communication

Development of Radio Aids to Air Navigation. J. H. Dellinger and Haraden Pratt.
Inst. Radio Engrs. Proc., July, 1928; v. 16, pp. 890-920.
(On the construction and theory of radio beacon devices for the guidance of airplanes in flight.)

Ship Propulsion, Electric

Electrical Propulsion of Ships. Alfred Regnauld.
Wld. Power, July, 1928; v. 10, pp. 11-16.
(Special attention to American practice.)

Tidal Power

Tidal Power, and Turbines Suitable for Its Utilization. A. H. Gibson.
Engng., June 22, 1928; v. 125, pp. 785-786.
(From a paper read before the Institution of Civil Engineers.)

Voltmeters

High-speed Graphic Voltmeter for Recording Magnitude and Duration of System Disturbances. A. F. Hamdi and H. D. Braley.
A.I.E.E. Jour., July, 1928; v. 47, pp. 512-515.

X-Rays

X-Rays and Their Application to Industry. G. L. Clark.
Tech. Assoc. Pulp & Paper Ind., June, 1927; Ser. 10, pp. 63-76.

NEW BOOKS

(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW)

Aufbau und Entwicklungsmöglichkeiten der Europaischen Elektrizitatswirtschaft. 511 pp., 1928, Berlin, Schwarz, Goldschmidt & Co.

(This volume presents a statistical study of the economics of the electricity supply industry in Europe. It was prepared, with the collaboration of two specialists in the subject, by the Economics Division of Schwarz, Goldschmidt & Co., a banking firm in Berlin. It consists of three parts. Part I considers the electricity supply industry of Germany. Part II reviews briefly the electrical industry of the leading countries of continental Europe other than Germany. Part III gives data on the financial structures of the leading electric utilities and manufacturers of electrical machinery in Europe and South America. The descriptive accounts are arranged by countries, and under each country alphabetically by the name of the utility or firm. An alphabetic index of the firm names mentioned is provided.)

Die Wanderwellenvorgange auf Experimenteller Grundlage
Ludwig Binder. 200 pp., 1928, Berlin, Julius Springer.
(This book gives us a systematic presentation of the experimental work on transient waves in electric circuits which has been conducted since 1914 by Professor Binder and his graduate students. It is issued on the occasion of the one hundredth anniversary of the Dresden Technical Institute, Germany, of which Prof. Binder is Director. The work is divided into four main parts. Part I deals with the electric spark as a generator of surges and traveling waves, and with methods of measuring or observing transient phenomena; Part II is devoted to transient phenomena in high-tension installations; Part III considers our present-day protective devices, such as reactors, condensers, resistors and lightning arresters, including a twenty-page chapter on American types of oxide-film and autovalve arresters; and Part IV deals with the behavior of insulating materials when subjected to the action of transients. A short bibliography concludes the volume.)

Meeting the Demand

Present-day tendencies toward the use of powdered fuel and the installation of electric equipment in the boiler-room require equipment enclosed against conductive dust. These cubicles and floor stands were designed by General Electric to meet this demand and to provide a uniform mounting for field switches, instruments, meters, and control equipment.

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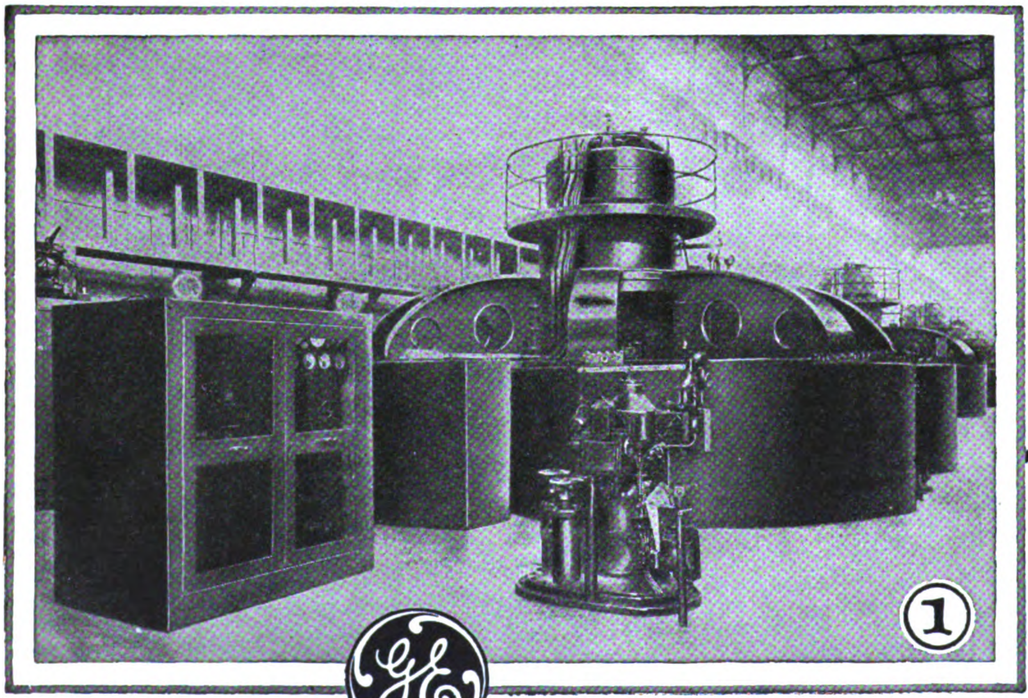
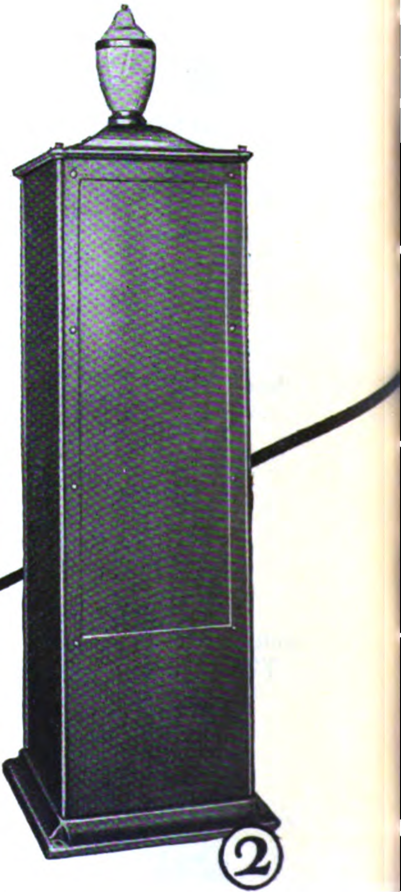
At the Louisville Hydro Electric station, the automatic control and indicating equipment for each of eight hydroelectric generators is housed in a cubicle. These units can be disconnected immediately, as the connecting leads are plugged into the proper terminals by means of multi-plug car couplers. Normal operation is possible without opening the cubicles, as pull-button switches are built into the front doors.

Important features which mark these stands are: flexibility as to mounting and arrangement of electric equipment; accessibility of parts; neatness and compactness of design.





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OWEN D. YOUNG

GENERAL ELECTRIC REVIEW

Vol. 31 November, 1928 No. 11

STREET RAILWAYS

We publish in this issue Mr. Owen D. Young's address before the American Electric Railway Association. His remarks are so authoritative that we give them in full.

Whether much can be done to help the street railway situation is another matter.

Both our city and suburban railways are only *items* in the broad field of transportation. The question that faces us today is: "Can they provide satisfactory transportation on an economic basis under our modern system of life?"

All systems and modes of transportation have to meet the same problems as other manufacturing concerns, because they are manufacturers, although it is unusual to look at them in this light—their product is transportation. They are engaged in manufacturing ton-miles and passenger-seat-miles, and their economic success depends upon selling their product at a profit.

If they produce more passenger-seat-miles and more ton-miles than they can sell, they, like any other manufacturer, are in a bad way. If it costs more to produce a passenger-seat-mile and a ton-mile than they can get, they are again in a bad way. If their product is unattractive to their potential customers, they are in a bad way.

Unlike many manufacturers, street railway companies are a Public Service and, as such, are quite properly regulated in the interests of the public they serve. Perhaps they have been over-regulated.

A brief glance at some phases of their history may help us to sympathize with them in their problems.

In their beginning, aside from the stage coach days, street railways were the only means of local public transportation. They did not displace anything older—there was no competition. The first in the field were the horse car and the mule car. Then came the cable car, and then the trolley.

The trolley car gave such improved service that it was supreme in the field of transportation.

But then the privately owned automobile came along and the street railways lost part of their traffic, but they did not regard their loss very seriously. Next the jitney entered the field of transportation, and then the automobile looked like much more of a menace, and the trolley companies began to realize that there was a new vehicle in the field; but still they did not view the situation as really serious—they were more annoyed than alarmed. Because after all the jitney was usually only a second-hand car and was driven by a man who was more concerned with the number of nickels at the end of the day than by his profit-and-loss account at the end of the year. The jitney was an unorganized individual effort.

Then, again, the number of automobiles materially increased, and there was soon an appreciation of the fact that the privately owned cars were beginning to make real inroads upon the number of riders in the street cars.

Then came the commercial truck which was the logical development of automobile equipment, and the bus now made its appearance, which was substantially only a different type of body on a truck chassis.

Now the automobile manufacturer woke up to the fact that there was a new field of enterprise which offered them an attractive market, and so it came about that the bus became an active competitor of the trolley car. Automobile salesmanship was more of the go-get-it type than prevailed in the sale of trolley cars.

The manufacturers of trolley cars were accustomed to build standard equipment and to submit bids on it to the trolley companies. On the other hand, the automobile salesman sought an interview with the railway management and pointed out the superior qualities of his vehicle; and, furthermore, the automobile industry carried out an active campaign in improving the buses as it learned the requirements by experience. It retained those features which were desirable and further improved its buses by eliminating features that had proved unsatisfactory.

It was not until then that the manufacturers of the trolley car woke up to the fact that a very rapid advance had taken place in the development of buses, and, being faced by a serious loss of business, they earnestly undertook to improve the trolley car by reducing weight and enhancing the appearance.

The production of these new means of transportation—the buses with mechanical drive—new buses with electric drive—the trackless trolleys, or electric coaches as they are now called—the numerous new developments and improvements in the former conventional trolley car—presented a problem of "choice" to the operating companies quite different from that of the day when they had only to decide how large a trolley car they wanted to buy.

Whatever types of equipment survive must give an attractive service to its patrons and must win sufficient patronage to guarantee the general expenses, depreciation, and maintenance charges involved in operation.

The trolley car is becoming something of a bus, and the bus is becoming something of a trolley; which will stand the test of the survival of the fittest remains to be seen. The time has now arrived to build a new structure on what values are still left in our old trolley systems. The form which this new structure shall assume must be left to our transportation experts.

That some change in the general condition of our street transportation is necessary is apparent to all who observe the conditions of our city streets. The ever increasing numbers of automobiles that have made such inroads upon the receipts of our transportation companies have also resulted in a serious interruption of the automobile traffic itself.

The streets of our cities are intended for, and should be used for, the movement of traffic. The parking of cars in our streets seriously curtails their use for this purpose. It interrupts the movement of all vehicular traffic, including automobiles and trolley cars.

Will restrictions as to parking prove the logical consequence?

When the solution has been found, the days may return when the transportation companies will again enjoy an increasing demand upon their facilities, whether it be trolley car, bus, or electric coach. J. R. H.

The Street Railway Industry

By OWEN D. YOUNG
Chairman of the Board of the General Electric Company

> *We publish with much pleasure Mr. Young's address before the American Electric Railway Association delivered at Cleveland, O., on September 25th. This address is so forcible and useful that we trust that it will help in moulding the happier future for our street railway industry.*—Editor

Mr. President, and Gentlemen of the American Electric Railway Association:

Up in the country where I live there is an old farmer, a friend of mine, who has a pair of horses. They are good horses, for he knows good horses. He likes them so much that he has fallen into the habit of talking to them. So now every morning when he starts out he bets them five dollars that they will get a licking before the day is done.

The public keeps the street railway industry just as the farmer keeps his horses, because it needs the service and knows it to be indispensable. The public is less wise than the farmer, because it does not feed the railways well enough to enable them to do their best. It has fallen into the same kind of habit, after years of practice, of talking to the industry. It is indeed rather a dull morning when the public, in one form or another, does not threaten to lick the industry before the day is done.

Now that talk, like the farmer's, is partly a matter of habit. It is partly a matter of psychology too. Neither expects to execute the threat but both get a certain satisfaction from this indirect declaration that they are masters and that the horses or the railways, as the case may be, are servants—and only servants.

My old friend would not keep a balky horse, or a kicking horse, or one that would not respond to the rein, one determined to go where he wanted rather than where his master wanted.

There was a time when the industry was none too well broken to harness. It rather wished to dictate than to serve. Those days have long since gone. Perhaps the public wished to tame the spirit of its carrier by underfeeding. If so, it is clear that it has done it. In fact, it is equally clear that it has overdone it. It is time for the public to know that it has not only weakened its servant but it has impaired its service.

Mr. President and gentlemen, I salute you as men of courage and determination. For fifteen years you have had a most difficult job to do. One day I hope that the American people may understand that in the face of adverse economic tendencies, in spite of the rapid technical advance in motor cars, in defiance of unjustifiable political assault, you have patiently rendered the best transportation service which you could, under the circumstances, in the congested areas of the United States. You have held the fort while the battle was going on between these great conflicting forces, the battle which was to settle, perhaps once and for all, the important question of what agencies are best adapted to local transport. Has not that battle reached its equilibrium? If so, why not survey the lines and face the facts? After all, what the public wants and should have, and what it will ultimately get, is the best means of public transportation. You are the men who have been trained in that undertaking. You are the men who are skilled in that business. To you the public has a right to look to provide means to meet its pressing needs.

We shall have to learn in this country that as our life becomes more complex, as our tools become less simple, as large numbers of our people become segregated into specialized groups, each group doing its particular part in the operation of the vast machine of modern life, that the way of progress is through the utilization in the highest degree of the knowledge and experience of those specialized groups. And conversely, such groups will have to learn that they are the trustees for the public as a whole of their specialized art and business, and to the extent which they fail in the performance of that trust they injure themselves individually and the group to which they belong. Therefore, I say it is your duty, as specialists in the field of local transportation, to advise the public how their needs can be most economically, efficiently, and adequately met. It is also true that the public will do well to take your advice.

It is interesting to see how thoroughly we have accepted this principle in some things and how strangely we resist it in others. If we wake up some morning and find our eyesight impaired, and face the dark tragedy of what might further happen, we do not call up our ward leader nor even our alderman for advice as to what we should do. Even our trained family physician will not do. Even the general practising oculist will not do. No one but he who has spent long years in study and experience of highly specialized cases similar to our own will do. Yet when we ride down to his office on the local transportation system, we may be impatient of the delays; we may be crowded and not comfortable; and the thought comes through our mind in angry protest that we will complain to our politicians and our office holders and office seekers, in order that they may cure our transportation ills. How often in such a situation do we ask the men responsible for the transportation why it is they cannot do better? How often do we pledge ourselves to aid them in getting the conditions which are necessary for our convenience and our comfort. We take the doctor's prescription for our eyes, but we refuse it for our railways.

Has not the time come for the public to take account of stock? Conditions of local transportation are growing worse. If we were to find a new program now there would be a long interval before it could be made effective, and during that time conditions will continue to grow worse. Plainly, it is time to act. Private automobiles must have reached the maximum, or at least near the maximum, in per capita distribution. Motor buses and motor cabs have been operated on a sufficiently large scale and over an adequate length of time to enable us to determine the conditions under which they can render the best service. The electric cars on rails, on, above or below the ground, have demonstrated that under certain conditions they are an absolutely essential means of transportation. Why not, therefore, meet the problem now? Why should electric cars be run where a motor bus can do the job better? Why should private motor cars be parked on our streets as a slight convenience for their owner when they clog the way for motor buses or railway cars and thereby waste the time of hundreds? I have often wished that some competent statistician would compute and state the cost to the community of parking a car in a congested street on land of tremendous value. Let him figure the value of the space, measured by surrounding private values. Let him figure interest and carrying charges. Let him figure the cost of the time lost by the hundreds or the thousands of people who are delayed in the parking of that car. I venture the guess that it would not take long to show that from the standpoint of the community we could better afford to buy private lots on Fifth Avenue and raze the buildings for parking spaces rather than have our streets cluttered as they now are in the congested communities of the country. Why permit motor buses to occupy our streets for long haul business when subways or elevated roads can do it quicker and better? Why not reduce our public transportation in congested areas to an orderly, controlled, and regulated system best adapted to our needs? And why not do it now?

So much for the determination and allocation of the agencies of transport. This, in a way, is the simplest and the easiest part of the problem. Much more must be done. If new and adequate facilities of transportation are to be provided, money must be found to pay for them. By and large, I believe that our means of local transportation are more inadequate and unsatisfactory than any other part of our modern machine. That means that the facilities should be in substantial part rebuilt, and that they should be vastly improved and extended. That requires large sums of money. Now there are only two ways of securing capital for such an enterprise. One is to create a situation where private capital will voluntarily invest. The other is to take the capital away from the individual involuntarily through taxation and provide facilities by public ownership. I say in this case, as I have said elsewhere, that it is vastly more important to the public to get the job done than to debate forever how, as between these two systems, it ought to be done. Let no one misunderstand that statement whether it be applied to railways or to other public utilities. I believe it to be in the interest of the community that such facilities should all be provided by private capital. It is not because the government could not build, manage and operate these systems. The government can. It is because there can be no economic check on the enterprise unless it has to go into the market for its capital. And unless there is an economic check on the enterprise there is no real way of measuring the value of its service. There are those who believe that in this country we can substitute the political check for the economic one. I do not. Whenever public facilities are to be provided, it is the safest for the community as a whole that the cost should be commensurate with the need, and the best way to determine that is to see whether people will pay enough for the service to attract the capital to that enterprise. Then you have a constantly working referendum. If, however, you delegate that question to a few politicians, they will be acting upon the political expediency of the moment, and they have the power to take the capital out of the individual pocket involuntarily. Therefore, I say that in so far as the community can secure the public service needed through private ownership and private capital, it is wise and more economical in the long run for the community to do it so. Even if the private capital gets one or two per cent more than it should, it will still be more economical for the community as a whole. It will be the part of wisdom to resolve the doubt on the side of good credit as an insurance of ample capital supply even in the worst of times, and therefore of service at all times.

Now I must hasten to say too, lest I be misunderstood, that there are some public services which have such large social significance that they cannot be translated into economic terms, and therefore private capital cannot be called upon to meet such needs. In such cases I shall always be in favor of taking the capital involuntarily from the taxpayer, because in a country as rich as this we must keep our social machinery in line with our economic resources.

So, Mr. President, I think our facilities for local transportation should be paid for by private capital, at least until it is demonstrated that our social needs for distribution of the population are so great that we cannot get the results through our usual financial machinery. I can well understand that situations may arise, and perhaps even exist, in this country today, where it is of vital importance to a congested area to distribute its population throughout suburban districts. It may be important as a matter of health to get the sunshine and open spaces and air of the country. It may be important as a matter of morals not to pack our people too closely. It may be important as a matter of mob psychology not to encourage congestion. It may be that the price which individuals will pay to avoid those conditions is not such as would enable private capital to carry large numbers long distances. In such cases there is no way for the community to solve its social problems except to provide transport facilities through taxation or to make up the normal economic deficit between what individuals are willing to pay to escape, and what the community is willing to pay to induce them to escape, the evils of congestion. By and large, I think in this country we are still able to get private capital to do the job, but that means coöperation on the part of the public authorities with you gentlemen who understand the business of transportation, to create a situation in which the credit of local transportation companies can be established and maintained, in order that private capital may be acquired.

What are the elements necessary to establish such credit? First, the rate of fare, must be such as to provide a gross income adequate, after deducting operating expenses, including maintenance and reserve, to yield a fair return on the capital required for the facilities. Second, that situation must be insured by satisfactory franchises for such a period as will warrant permanent investment. Third, in my judgment, a monopoly should be established of local transportation facilities of all kinds, and the rates and service should be regulated by the government so as to give to the public the best service and lowest rates consistent only with such return as will always enable them to get the capital desired for their needs. Unless we have a monopoly and public regulation, we must do the job by competition and I believe that in this field of transportation competition is not an effective regulator; it is merely a destructive one which can come to nothing else but government ownership in the end, and if that is so we might as well take it in the beginning.

So I say let us provide ample credit for our transportation companies. Let us give them sound fares. Let us demand from them adequate facilities. Let us build up a situation where we get sufficient capital to meet all of our transportation needs, and let us do it quickly in order that the horrible congestion which now exists in some of our great cities, which is not only uneconomic but immoral, shall be abolished from this country, which at least is rich enough to provide, one way or another, decent transportation facilities for its people.

All of the financial water has been drained off the railways as a result of the experience of the last few years. Some part of the substance has gone too, but the values which now remain are, in my judgment, reduced to the lowest terms and should furnish a sound basis for new capitalization and new capital, provided satisfactory conditions for the future are granted by the public. In a word, I am not pessimistic about the street railway values which are left. It is now time to build on them the new structure which the public needs and which the initiative of you, men skilled in the business, must provide.

So, Mr. President, if we can determine what agencies are the best for the job and we can determine how we will pay for them, whether by public or private capital, and if by private capital at the lowest rate, then we shall be on our way toward a real solution of this vexed question which at present is a disgrace to the intelligence of the American people. May I mention some of the slogans which bar the way. First and most persuasive of all, the 5-cent fare for all distances and under all conditions. Well, the 5-cent fare is the disease of our coinage and our fluctuating price level. It is the attractive germ by which politicians inoculate the public with an apparently incurable and progressive ailment.

Let us look for a moment at the justice of this fare question. The average increase in the general price level of this country between 1913 and 1928 is approximately 60 per cent. In other words, it takes at least $1.60 to buy goods today which could have been bought in 1913 for $1.00. If this same percentage of increase were applied to street railway fares, it would mean that an 8-cent fare is now, measured by commodity prices, only the equivalent of a 5-cent fare before the war. This is, as a matter of fact, the general average now attained, but unfortunately this does not answer the whole question. Everyone knows that the law lags behind economic change. Increase in costs and competition from the motor industry did not wait upon the deliberations of public service commissions; increase in the fare necessarily did. The result speaks for itself. I need only say that, as a consequence, there is an enormous slack to be taken up, and the inevitable losses of the past few years, not so much to stockholders through the suspension of dividends as to the service through depletion of funds for maintenance and reserve, make adequate rates today all the more imperative if the service is really to be restored.

Even this is not the whole story. For example, the level of wages in this country has increased during the same period on an average, not of 60 per cent, but of 120 per cent, and it is a fact that the price of labor is a much more important item in transportation costs than the price of commodities. If we were to apply that percentage of increase to the 5-cent fare, we would find that 11 cents now is no more than the equivalent of 5 cents in the earlier days. So it may not be far out of the way to say that somewhere from 8 cents to 11 cents is the equivalent of the old 5-cent fare. This only assumes, of course, that the service of today shall be no better than the old service, but we all know that improvement is demanded, and rightly demanded, everywhere. High-speed service, overhead service, underground service, more reliable service, is the rightful expectation and demand of the public. Improvements can be made and will be made, but it is neither wise nor just for the public to make demands and then set up obstacles to performance.

Certainly the farmer in 1928 does not expect to feed his horses half the oats he did in 1914 and yet get the same service from them. That is exactly, however, what the public is demanding where it still insists on the 5-cent fare. If a 5-cent fare is insisted upon where on the average it costs 8 cents to transport, why talk about private capital doing that job? It simply cannot be done. Nor can it be done any cheaper by public capital, but we can tax the community instead of the individual rider to make up the difference. To all advocates of a 5-cent fare when it costs 8 cents to transport I say, not in the interests of the street railway industry but in the interest of the public, be fair with your slogan. Do not expect private capital to do the job under those circumstances. Frankly say that you are for government ownership and that you intend to tax all people to give riders carriage for 3 cents less than cost. Let us cut out the nonsense and get down to business. The fare, however, cannot be such as to defeat the very distribution which is necessary to the social needs of the community, and when it does we shall have to take the social penalty or go to government subsidy or ownership.

There are many minor charges, Mr. President, against local transportation companies, such as contributions to paving and premiums for this, that or the other which I will not mention. Once they were a vital item in the street railway business. Now it is well understood that they are only special taxation upon the riders themselves. All of such special charges will have to disappear.

Mr. President, I shall be bold enough to speak of one more condition affecting the electric railways of the country which has tended to create some confusion in the public mind, and has perhaps to some extent delayed both the public and the industry in meeting the basic issues involved in adequate street railway service. I refer to the practice of unifying, either in fact or in treatment, the investment in the local railway service with that of the light and power service. In such a case, if the light and power rates provide a satisfactory return on the total investment, there is no great pressure on the part of the owners to force the issue with the public of a sound economic rate of fare on the railways. By and large, it is conducive to the comfort of the local utility management and to the comfort of public authorities to permit the continuance of a low rate of fare on the railways which will be locally popular, and continue the light and power rates on a higher level in order that they may supplement the return on the railway investment.

Now, I do not mean that this situation results from bad faith on the part of anyone. In many situations in this country the railways and the electric power and light properties have been combined for many years. The naturalness of the combination is self-evident. The railway is a large consumer of power. It turned out to be economically unsound, for the most part, for the railway to maintain its own separate power station. It therefore became a large customer of the central power system. Besides this, the overhead and management and engineering costs of the railway could frequently be diminished by unification with the power and light organization. And latterly, as the railway credit became impaired, it was often possible to get new money for railway purposes when the railway properties were unified with power and light properties. In a word, while railway property might not be good credit, power and light property was. The consolidation of these properties very largely took place prior to the time of large unification of power properties. Consequently, during the last fifteen years, it has frequently been impossible for the power companies to acquire important units of local distribution in their systems without buying at the same time the local street railway property. Having acquired it they naturally and appropriately wanted a fair return on the invested capital. But the point which I wish to make is that they were not greatly concerned whether each property stood on its own economic feet. Being human, they were willing to acquiesce in an uneconomic railway situation if the valley of the railway net could be made up by an adequate peak in the light and power earnings. For many communities I am not saying that this was a bad method, certainly as a matter of expediency. The harm came to the industry when the fares on such unified properties were compared with the railway fares in some other community where the transportation property was not unified with the power and light service. There is no more persuasive argument against an increase in fare than to point out that another community, perhaps of about the same general size and having similar operating conditions, is paying less for its transportation than either the present or a proposed new rate. So there are today throughout the United States certain railway situations where uneconomic fares are being charged without protest by the owners, and of course with the full approval of the public authorities. These situations tend to delay the complete rehabilitation of the railway industry as a whole, and are therefore regrettable.

By and large, I believe it will be better for the industry and for the public to face frankly the question of providing an adequate return on its transport capital either from fares or through a frank subsidy from the public treasury. To levy a special tax on power and light users to supplement an inadequate railway fare is, in my judgment, unsound in prinicple, and will in the long run delay the best development of both industries. If that be true, then each community should provide for each service such rates as will yield a fair return on the investment in that particular service. It would be highly unfair in cases where they have acquiesced in unification to reduce the power and light rates without providing at the same time an adequate return on the railway investment. Perhaps that is too much to expect of politics, but I, for one, shall never lose faith in the soundness of judgment nor the fairness of spirit of the American people if problems are put to them frankly and clearly. Secrecy and confusion are the enemies of confidence. Confidence is the basis of business. It is the sound insurance of fair treatment for men and money that render a public service.

The public must know that while it can exhaust the capital now sunk in the industry by unfair treatment it cannot by such means induce new capital to go in. That, unfortunately, is our situation in many places today. This must be corrected, and I have confidence that the masses of the people, even over the heads of their political leaders, will insist on such correction if the matter be properly presented. So too, the public must remember that while it may keep the old men in the business, it will not be able to impress young men into its service unless the conditions are sound and attractive. The young men of these days will not embark on the voyage of life in an unseaworthy ship. There are too many fine vessels in other lines putting out of the harbor every day. I say this not to discouarge young men, but to warn the public. If the conditions are corrected, I know of no field of service in which ability and leadership should be more generally appreciated or more generously rewarded than in this great field of local transportation. For, as I have said before, this is a business which meets not only an economic requirement but a highly pressing social need in all great congested communities.

And so, if I may summarize the things which I believe in, they are:

First: That the business of local transportation should be a monopoly, publicly regulated as to rates and service, with a fare adequate to attract private capital if such fare can be obtained, and if not then the distribution needs of the community should be satisfied through the tax levy, either under public ownership or as a contribution to the deficit in private ownership.

Second: That private ownership be adopted in all cases where it can meet the needs of the community, and when adopted that the conditions be made such that the safety of the capital be adequately insured and the rate of return be as amply provided for as in any general business in order that the cost of the capital may be not greater for these enterprises than for others.

Third: That the transportation company, whether it be publicly or privately owned, use the means best adapted for the most efficient and economical transportation of the public which it serves. I hold no brief for electricity, except that if it cannot be used in local transportation it will have met its first great defeat. If it can be used advantageously, then our research and engineering, under the impetus of a new spirit and in the light of a pressing need, should be drafted into service to redeem an industry and an art which has been able to survive during these last fifteen years only by reason of the courage and determination of the men who have administered it.

Design and Construction of the 110/33-kv. Outdoor Substation of the Elmira Water, Light, and Railroad Company

By L. G. SAWYER
Electrical Engineer for E. L. Phillips and Company

TO supply the rapidly increasing demands for electric power in the section of New York State which it serves, the Elmira Water, Light, and Railroad Company has recently put into operation a 110-kv. to 33-kv. switching and transformer substation near the city of Elmira. General views of the substation are shown in Figs. 1, 2, and 3.

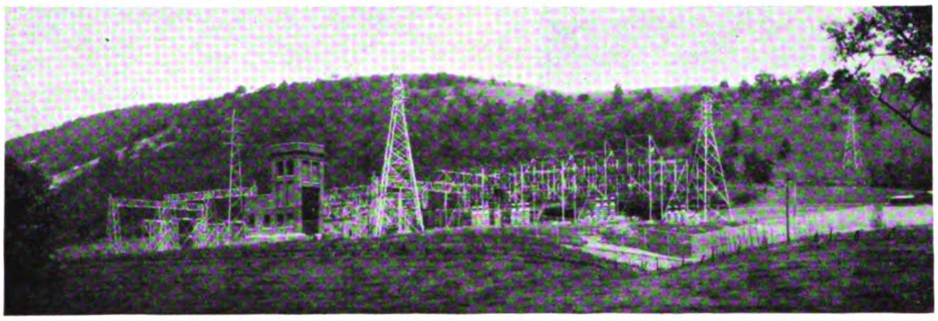

Fig. 1. Outdoor Substation of the Elmira Water, Light, and Railroad Company, Elmira (N. Y.)

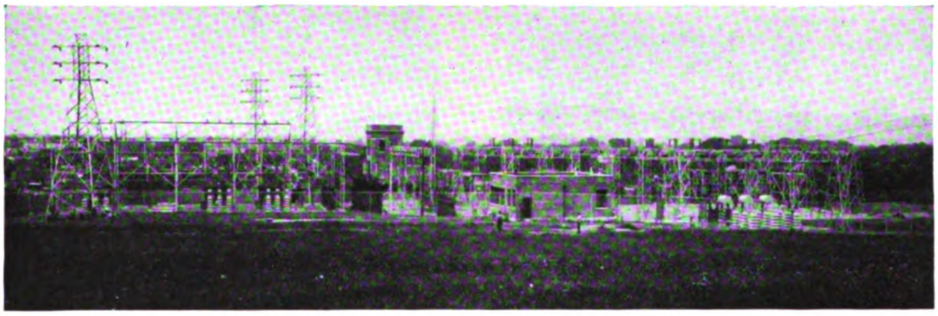

Fig. 2. The Substation as Viewed from the Side Opposite to That in Fig. 1

This substation will be the connecting link between three independent generating and transmission systems. Two of these systems are of 110 kv. and the third is of 33 kv. The generating stations for the 110-kv. systems are over 100 miles apart, and the generating station for the 33-kv. system is but a few miles from the substation. Energy will be transmitted to the substation, over distances of approximately 50 and 70 miles, from each of the 110-kv. systems at 60 cycles by two aluminum-cable steel-reinforced 3/0 lines, supported on steel towers. A portion of the electrical energy will be transmitted between the 110-kv. systems through this substation, and the remainder will be tapped off and transformed to 33 kv. for distribution to various other substations and a secondary transmission system having a total capacity of approximately 27,000 kv-a. The 33-kv. system will receive energy also from a generating system which has an approximate capacity of 21,000 kv-a.

The present capacity of the substation as equipped is 10,000 kv-a. in transformers, with four present and two future 110-kv. lines and with eight 33-kv. lines.

The ultimate capacity of the station has not been determined, but provision has been made for an additional bank of transformers and the 33-kv. section has been so designed that several bays of two feeders each may be readily added. The size of the transmission lines is the only limit to the ultimate capacity of the station. No provision has been made for the correction of power-factor by the use of synchronous condensers at the station.

110-kv. Structure

Owing to the wide spacing required for the 110-kv. conductors, it was decided that the skeleton and box-girder or sectional type of construction would be the most economical. This construction consists of several detached towers built of light, galvanized steel shapes

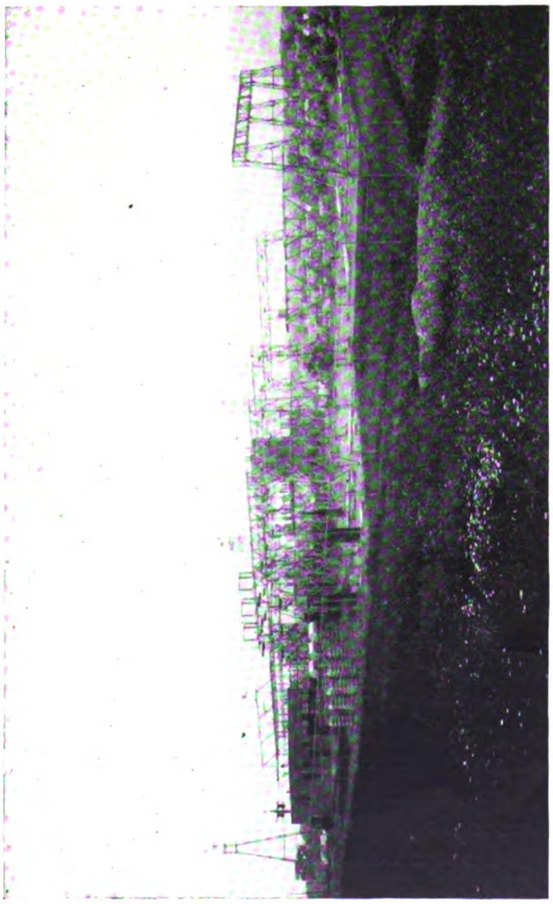

Fig. 3. A Quartering View of the Substation

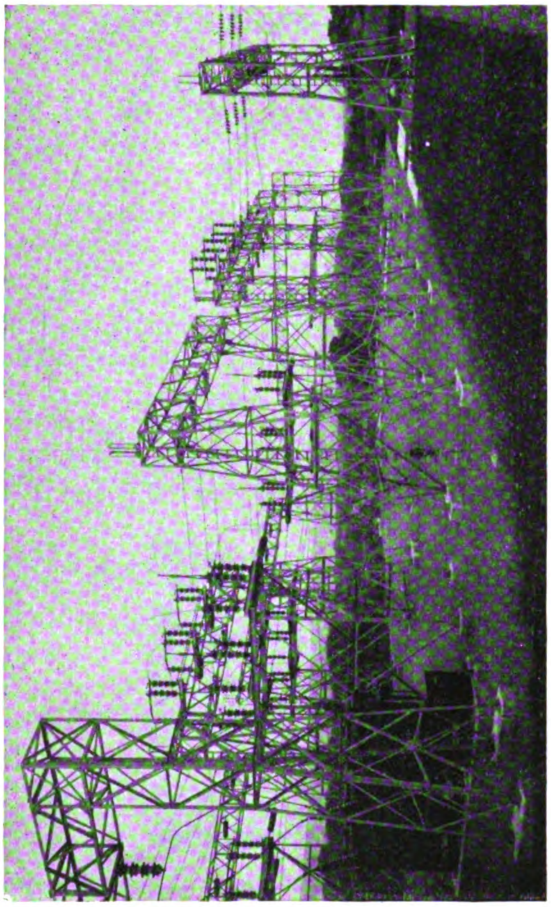

Fig. 4. A View of the 110-kv. Switch Yard with Incoming Line

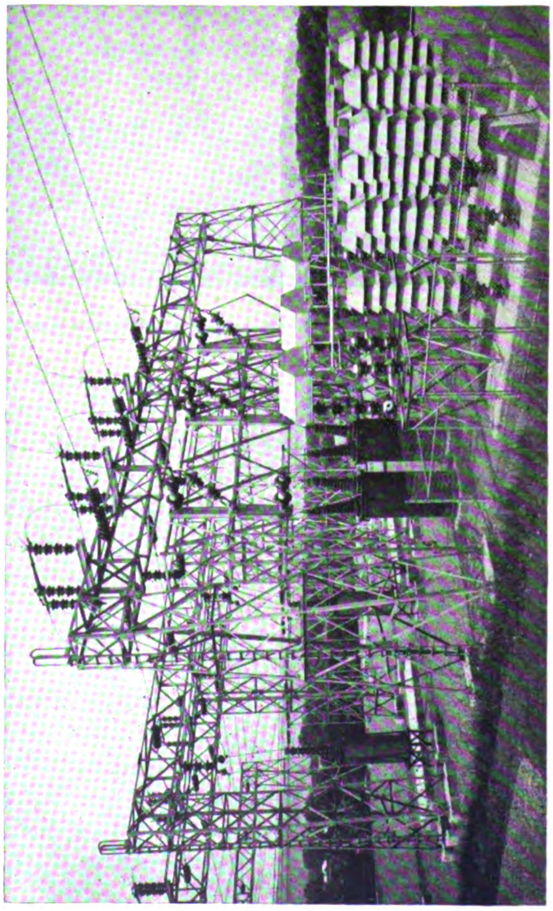

Fig. 5. Another View of the 110-kv. Switch Yard, Showing the Incoming Line, Lightning Arresters and Potential Transformers

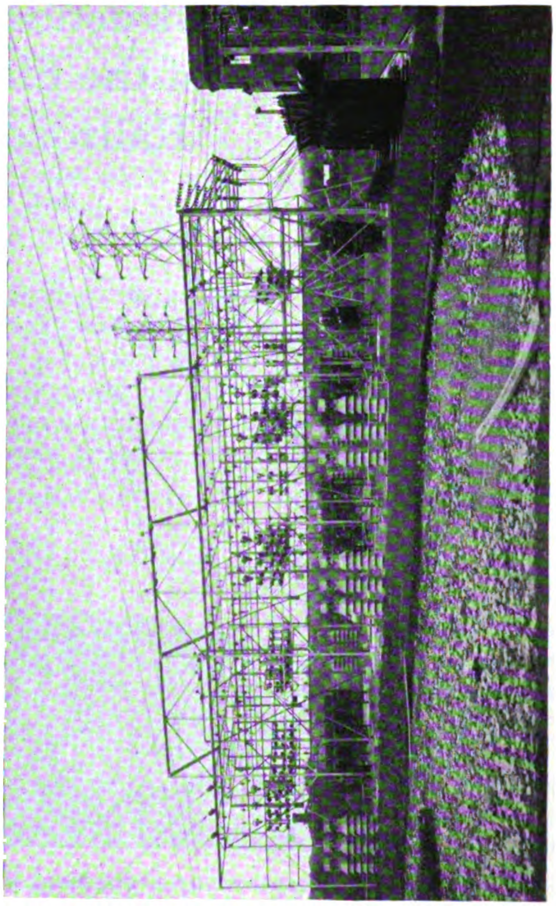

Fig. 6. Side View of the 33-kv. Switch Yard

which are set up with buses of stranded wire suspended between them by the aid of strain insulators, as illustrated in Fig. 4. This type of construction lends itself readily to the topography of the country where it is to be located as well as to changes in length of bus. For instance, it is possible to sectionalize a bus by the aid of a strain insulator and insert an oil circuit breaker by moving one or more of the steel structures a few feet without the purchasing or erecting of more steel, which is not the case with the framed-in or composite type of structure. Cross buses were installed by using different lengths or heights of structures. The connections between the buses were made by vertical drops at points where the buses crossed.

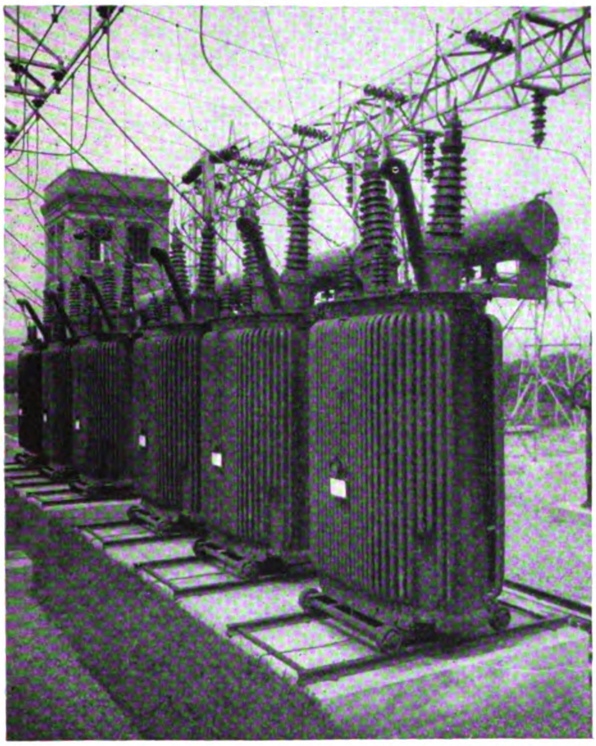

Fig. 7. The Two Banks of Three 1667-kv-a. 110/33-kv. Transformers

The substation throughout is of the double-bus type and has a tie bus which serves the two-fold purpose of tying the two lines together (enabling energy to be fed in on one line and out on another, or both in the case of an emergency) as well as a suitable place for tapping off the transformer feeds, so that the transformer banks may be fed from either or both lines as desired, by the use of air-break horn-gap switches.

For the protection of the substation against surges and lightning discharges, oxide-film lightning arresters were installed as shown in Fig. 5. Ultimately, each 110-kv. line will be additionally protected by oil circuit breakers similar to those referred to later in this article. At this time, however, only one 110-kv. line is in service; the three additional lines are expected to be placed in service in the near future.

Energy as received over the 110-kv. lines will be measured through two oil-insulated 110-kv. current and potential transformers, shown in Fig. 5, which will supply energy to the meters.

33-kv. Structure

The 33-kv. structure, which is shown in Fig. 6 and which is adjacent to the 110-kv. structure, is of the solid beam and girder or framed-in type of construction with rigid double bus of copper tubing and post-type insulators. The circuit arrangement of the structure is such that either or both banks of transformers may be switched to either or both buses at the same time. The same condition holds true for each of the eight 33-kv. feeders. Each transformer bank and each feeder are protected by 33-kv. oil circuit breakers which will be described in another paragraph of this article. All of the oil circuit breakers can be isolated by the use of 37-kv. remote-operated disconnecting switches.

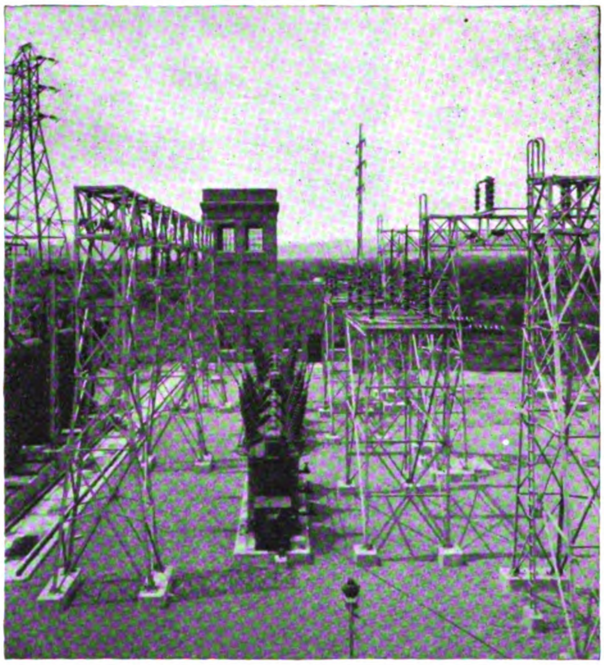

Fig. 8. The 110-kv. Circuit Breakers and Transformer Servicing House

110/33-kv. Transformers

The transformation of energy from 110 kv. to 33 kv. in this substation is accomplished by two transformer banks each consisting of three 60-cycle 1667-kv-a. 55-deg. C. single-phase oil-insulated self-cooled outdoor-type transformers having one approximately 5 per cent tap above and two approximately 5 per cent taps below normal voltage rating. These units are shown in Fig. 7 and are protected on both the high- and low-tension sides by oil circuit breakers. At the present time only overload protection is used, but when the complete equipment for the substation is installed the transformers will be provided with differential protection together with overload protection on the high-voltage side.

Oil Circuit Breakers

As will be seen from Fig. 9, only two 110-kv. circuit breakers are now installed, and these are on the high-tension side of the transformers. Provision has been made for the future installation of breakers on the four 110-kv. lines. The present breakers have a published rupturing capacity of 2600 amp. at 110 kv. sustained voltage, and are illustrated in Fig. 8.

In the 33-kv. side of the transformers and in the eight 33-kv. feeders are 800-amp. 37-kv. triple-pole single-throw oil circuit breakers, Fig. 11, having a published rupturing capacity of 4480 amp. at a sustained voltage of 33 kv., based on an OCO plus an OCO duty cycle. The breakers are so designed that their rupturing capacity can be increased when necessary by the substitution of new tanks and contacts without disturbing the main switch structure.

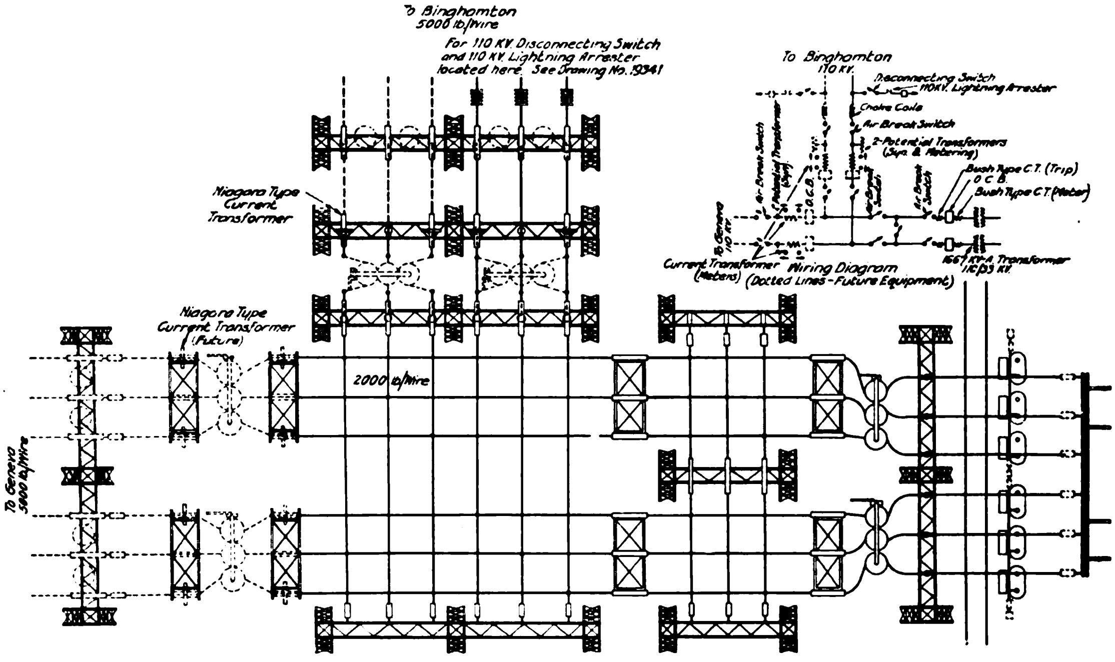
To Binghamton
5000 lb/Wire
For 110 KV. Disconnecting Switch and 110 KV. Lightning Arrester located here. See Drawing No. 19341
Niagara Type Current Transformer
Niagara Type Current Transformer (Future)
2000 lb/Wire
To Geneva 5000 lb/Wire
To Binghamton 110 KV.
Disconnecting Switch
110 KV. Lightning Arrester
Choke Coils
Air Break Switch
2-Potential Transformers (Syn. & Metering)
Bush Type C.T. (Trip)
O.C.B.
Bush Type C.T. (Meter)
Air Break Switch
Potential Transformer (Syn)
O.C.B.
Air Break Switch
Air Break Switch
To Geneva 110 KV.
Current Transformer (Meters)
Wiring Diagram
(Dotted Lines - Future Equipment)
1667 KV-A. Transformer 110/33 KV.

Fig. 9. Diagrams of the 110-kv. Section of the Substation

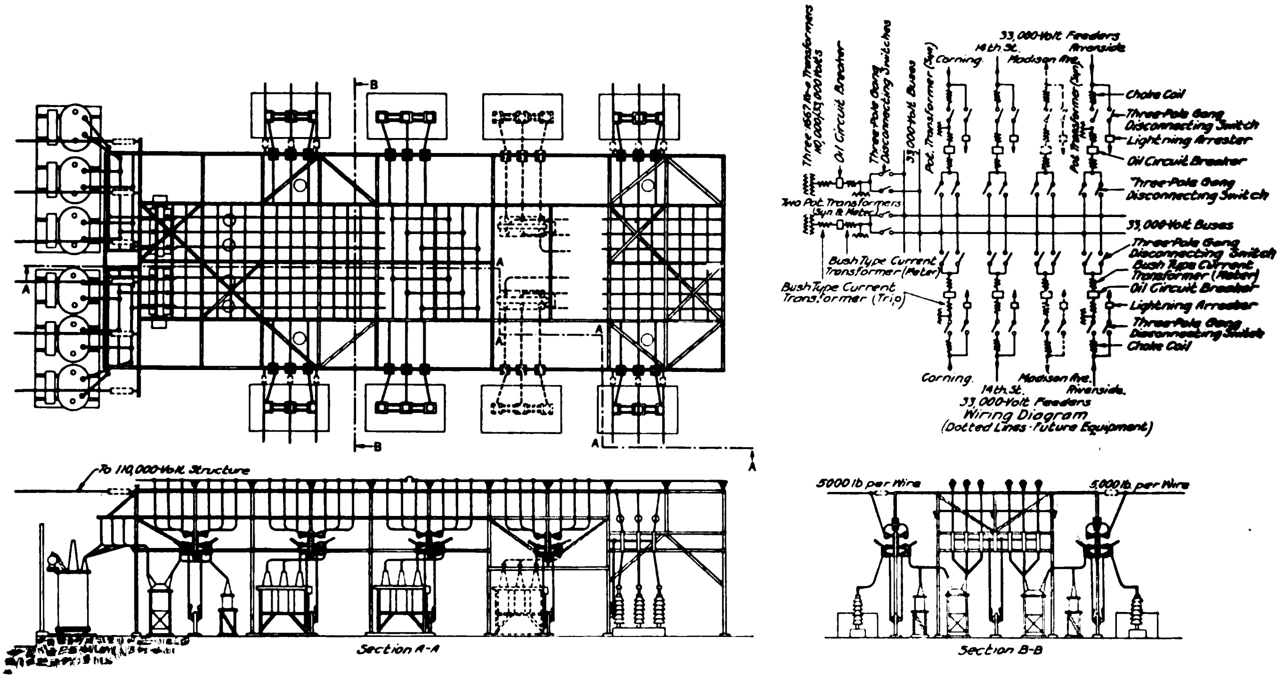
B
B
A
A
A
A
To 110,000-Volt Structure
Section A-A
Three 1667 Kv-a Transformers 110,000/33,000 Volts
Oil Circuit Breaker
Three Pole Gang Disconnecting Switches
33,000-Volt Buses
Pot. Transformer (Syn)
33,000-Volt Feeders
Corning
14th St.
Madison Ave.
Riverside
Pot. Transformer (Syn)
Choke Coil
Three-Pole Gang Disconnecting Switch
Lightning Arrester
Oil Circuit Breaker
Three-Pole Gang Disconnecting Switch
Two Pot. Transformers (Syn & Meter)
33,000-Volt Buses
Three-Pole Gang Disconnecting Switch
Bush Type Current Transformer (Meter)
Bush Type Current Transformer (Meter)
Oil Circuit Breaker
Bush Type Current Transformer (Trip)
Lightning Arrester
Three-Pole Gang Disconnecting Switch
Choke Coil
Corning.
14th St.
Madison Ave.
Riverside.
33,000-Volt Feeders
Wiring Diagram
(Dotted Lines - Future Equipment)
5000 lb per Wire
5000 lb per Wire
Section B-B

Fig. 10. Diagrams of the 33-kv. Section of the Substation

Both the 110- and 33-kv. oil circuit breakers are operated by universal motors on either 220 volts a. c. or 125 volts d. c. Normally, the breakers are closed by alternating current and tripped by direct current. The direct current is used for closing only in emergency. All the breakers are equipped with bushing-type current transformers, for relaying.

Switchboard

The control and relay apparatus as well as the metering instruments are mounted on a sixteen-panel switchboard consisting of two rows of panels, back to back, separated by a distance of six feet. The front row of panels (Fig. 12) carry the control swiches and indicating instruments. The rear row of panels (Fig. 13) support the relays and watthour meters. The space at the end of the panels are closed by a grating, so that unauthorized persons cannot go behind the boards. In the floor between the switchboard panels is a trench which is approximately three feet in depth and which is covered by removable checker plates. All control and metering cables are brought into this trench through a conduit from its respective transformer or switch, and then along the trench to a point just beneath its panel where it is brought up through the stationary checker plate

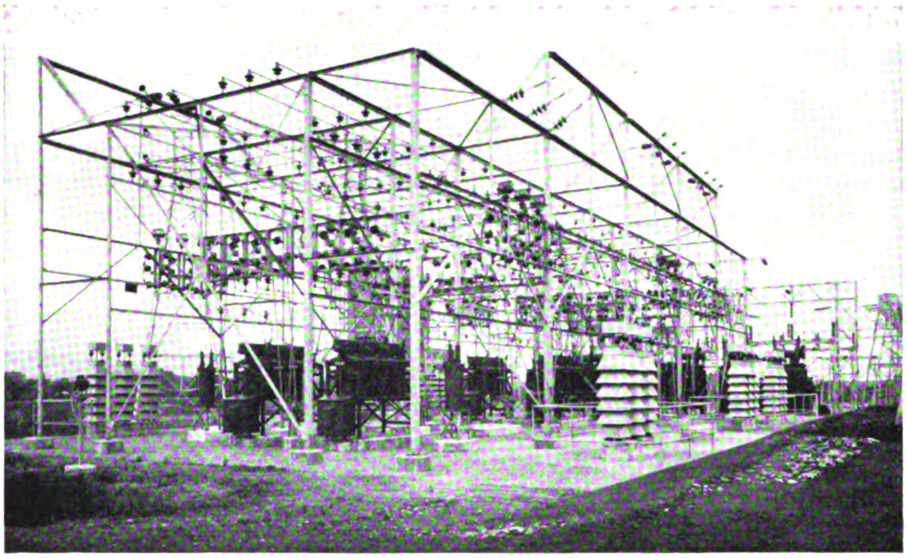

Fig. 11. The 33-kv. Section of the Substation, Showing the Oil Circuit Breakers and Lightning Arresters

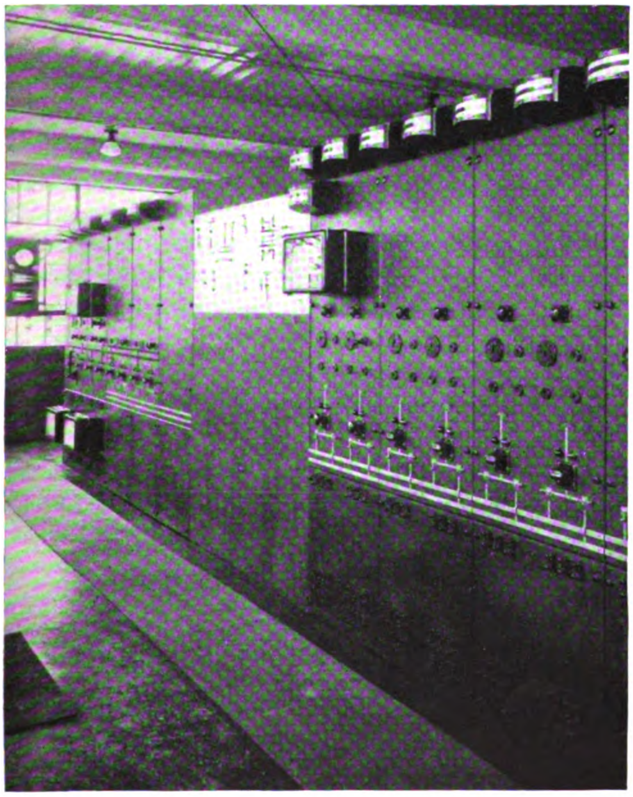

Fig. 12. The Control and Indicating Instrument Panels

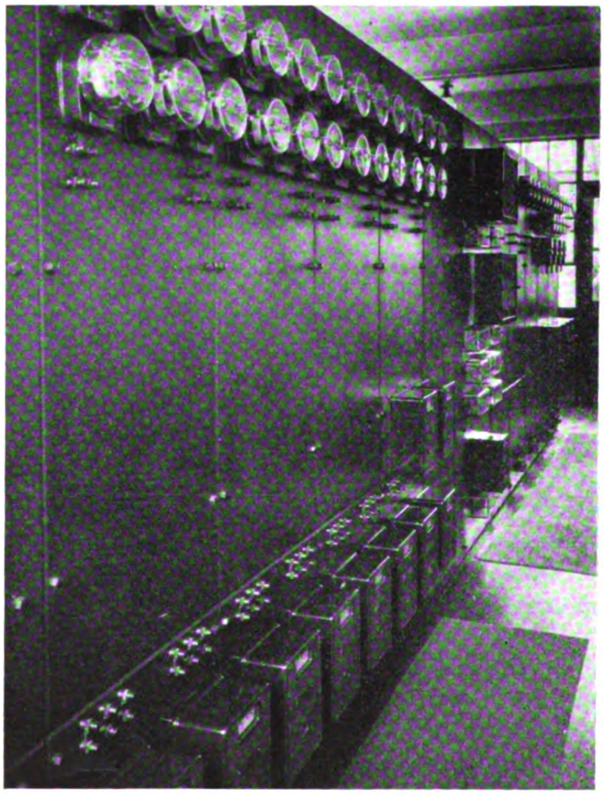

Fig. 13. The Relay and Watthour Meter Panels

at the rear of the panels in a chase nipple. All control cables in this trench are lead covered and run in flexible steel conduit.

The cables between the control and relay boards also pass through this trench. The trench serves as a very flexible means for connecting the cables to the boards; and it enabled the construction crew to lay the concrete floor long before the locations of the terminal blocks or arrangement of panels were decided upon. The control of the substation required the laying of approximately 18,000 ft. of underground duct and approximately 25,000 ft. of wire and cable.

Control Battery

For the tripping of the oil circuit breakers and the emergency closing of them, it was necessary to provide a storage battery, which would also supply lighting for the station during an emergency. The current required for closing the largest breaker by direct current is 180 amp. at 125 volts. In predetermining the size of battery it was estimated that the most severe service would be the closing of the 16 breakers one at a time in rapid succession and the carrying of an emergency load of one kilowatt in lighting for a period of three hours before the battery voltage would drop to 1.75 volts per cell, or 105 volts across the battery.

For this service there was chosen an Exide battery which consisted of 60 cells in self-sealing glass jars, and which carried a guarantee of 25 amp. discharge for 8 hr., 100 amp. for 1 hr. or 250 amp. for 1 min., to a final voltage of 1.75 volts per cell.

Battery Charging Unit

The battery manufacturer recommended that the battery be floated across a charging device, or be given a continuous trickle charge, thus keeping it in a fully charged condition at all times. For a continuous trickle charge, it was recommended that the battery be charged at a rate equal to one-eighth of its 8-hr. normal charging rate. In order to supply current at the recommended charging rate, and at the same time carry the pilot lights on the switches, it was necessary to have a charging device capable of delivering 750 watts.

A 1-kw. generator having a continuous rating of 8 amp. was chosen as it could furnish an increased charging rate after there had been a severe drain on the battery. This machine is driven by a direct-connected three-phase 220-volt induction motor; and the combined unit is controlled and protected by an automatic charging panel.

Building

In order to house the switchboard and control equipment, the one-story building shown in Fig. 14 was erected. This building is divided into three rooms, *viz.*, switchboard or operating room, battery room, and motor-generator room. The original plan was to provide space in this building for a transformer repair room, but due to the contour of the plot it was decided to construct a second building for this purpose, shown in Fig. 8. The transformer repair room has a tower over one section. From the roof of the tower is suspended a ten-ton hoist which may be used to lift the core from a transformer tank, the tank being then moved to one side and the core lowered to the floor where it can be inspected or repairs made. The transformers are moved from their position in the substation to the untanking house by a specially designed carriage which travels on steel rails. The transformers themselves are provided with wheels that allow them to be moved in one direction. By means of these wheels a unit can be rolled upon the carriage, which carries a set of rails, and then can be moved in a direction at right angles to its individual movement. Therefore, by arranging tracks in two or more places at right angles to the carriage track, and at the required elevation (20 in.) above it, it is possible to move the transformers from place to place with very little effort.

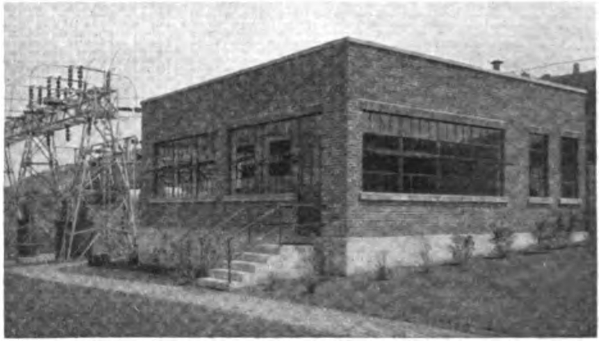

Fig. 14. The Control Building and a Portion of the Incoming 110-kv. Line

Each transformer and oil circuit breaker are equipped with oil-drain and filter-press connections; but no elaborate scheme of piping to a central filtration plant was considered for this substation.

Conclusion

In designing this substation, every effort was made to minimize the effect of service interruption, and at the same time avoid complexities that might prove bewildering to the operators during an emergency.

E. L. Phillips and Company designed and erected the station, and the General Electric Company furnished the material and equipment.

Carboloy—A New Tool Material

By DR. SAMUEL L. HOYT
Research Laboratory, General Electric Company

> *The discovery, development and perfecting of a new material can revolutionize our workshop practice. The author describes a wonderful new material called "Carboloy," gives a description of its characteristics, and tells of some of the tests that have been made with it. The subject matter of this article is from a paper read by the author before the American Society for Steel Treating at the Annual Convention held at Philadelphia, October 8 to 13, 1928.*
> —EDITOR

THE great hardness of tungsten carbide has been known for a long time and many attempts have been made to take advantage of it commercially. Tungsten carbide, however, as commonly manufactured, is too weak and porous for those industrial applications in which its high cost would permit it to compete with other hard materials. This circumstance accounts for the efforts which have been made, both here and abroad, to eliminate or control the porosity and to increase the strength of tungsten carbide. Obviously, this strengthening must not be obtained at too great a sacrifice in hardness.

German investigators have been particularly active in this field, as shown by the various patents covering tungsten carbide. One of their important developments, a contribution from the incandescent lamp industry, is the tungsten carbide and cobalt combination represented by the "Hartmetall" of the Osram Company. By means of cobalt, tungsten carbide is made about half as strong as high-speed steel, while still retaining sapphire scratching hardness. In the writer's opinion, great credit is due the inventors of this carbide tool material. A super-hard carbide tool material of this type, which has somewhat unusual properties, has now been developed in the Research Laboratory.

These carbide tool materials seem destined to play a dominant part in the industrial life of the future, and, on that account, the Research Laboratory has actively pursued the study of them for the past few years. This work was initiated, by the writer, in the Research Laboratory but it has since been greatly extended in the Company's shops as a large scale investigation. Various forms of carbide tool materials have been studied and it is the experience thus gained which will be described here. The name Carboloy has been adopted as the trade name for materials of this type.

Characteristics of Carboloy

The density of Carboloy is about 14 grams per cubic centimeter and upward depending on the amount of cobalt used. This characterizes it as a heavy metal, almost of the tungsten class. It does not tarnish and, when ground, resembles steel in appearance. It resists chemical attack remarkably well and is a pleasing and satisfactory material for use in the arts.

The hardness of such materials can be specified in a number of ways. Sapphire is the mineral which comes next below the diamond in hardness. Carboloy is capable of producing a well defined scratch in the natural sapphire. When it is borne in mind that the sapphire has an "absolute hardness" of 1150 kilograms per square millimeter, and that a mineral which is as hard as feldspar has an "absolute hardness" of only 300, the hardness of Carboloy receives new significance.

A sample of Carboloy ground to an edge of about 90 deg. is capable of cutting a rather deep, narrow groove in an Alundum wheel without suffering much loss itself; whereas our ordinary tool materials are worn away by an Alundum wheel. This behavior on the Alundum wheel gives a very good idea of the hardness of the material; even the manner in which a sample scratches a plate of glass serves as a crude test. However a quantitative, or at least a numerical, expression of the hardness is much more illuminating, as is given by the more formal methods of testing hardness.

Engineers will be more interested in the Brinell or Rockwell tests; and, in our work, we have used the Rockwell test almost exclusively to get the correct evaluation of the hardness. Obviously it is not possible to secure the Brinell number in the usual way. For the sake of obtaining quantitative values of the hardness of our samples, and for comparing different materials and treatments, we find the actual unit load supported by the sample during the Rockwell test. This gives a load in kilograms per square millimeter, and it is so closely like the Brinell number in the higher ranges (within 100 to 200 points) that its designation as such seems justified.

In the earlier tests we used the 150-kg. load with the diamond penetrator and recorded hardness numbers of around 70 on the "C" scale. This load caused the point of the diamond to wear and crumble in too short a time and we have since changed to the "no load" scale, which simply means that no weight is put on the lever arm. The hardness numbers are read on the "C" scale and have come to be known as the Rockwell "A" numbers. To get the unit load on the sample, it was necessary to measure the actual load for the "no load" loading. By making impressions in a sample of hard steel with 150 and 100 kg. and with "no load" and then measuring the three impression diameters with an accurate comparator, this load was found to be 64 kg. for our machine by utilizing the principles of Meyer's analysis. The load for our modified test, having been determined, it is then possible to calculate the unit loads for different Rockwell "C" scale readings, or Rockwell "A" numbers.

This can be done by the relationship given in Fig. 1. It is not likely that the designation of hardness numbers secured in this way as Brinell numbers will be misleading; and their use brings out differences in hardness much better than the Rockwell numbers do. Certainly nobody accustomed to measuring penetration hardness will be misled thereby. In practice we measure the "no load" Rockwell hardness on the "C" scale and use the numbers so obtained without converting them into Brinell numbers. The "A" readings obtained on one sample do not differ by much more than ± 0.5, while the readings of different diamonds may differ from each other by a little more than this as an average maximum.

We may now compare the penetration hardness of different materials on a rational basis. O'Neil has shown that the Brinell number of the hardest steel may be taken as about 1000, using a diamond ball point as the penetrator. Common hardened tool steel probably runs closer to 850 as a maximum. The Brinell number of the original German material runs from 1250 to 1400, so that it represents a considerable increase over the hardest steel, while the newer grade runs from 2000 to 2500. The Rockwell "A" numbers for these two materials run 85 to 87 for the former and 90.0 to 91.5 for the latter. An interesting check on the relative hardness is secured by comparing the size of the two impressions made, even with the unaided eye. To those who might wish to know the ordinary Rockwell "C" number, the only information that can be given at present is that the original German material runs around 70, while the super-hard material has been found in a few trials to run 75 and above. The wear and tear on diamond points is too great for testing with the 150-kg. load, but, of course, the sensitivity is reduced by going to the "no load" reading.

Inasmuch as the strengthening of tungsten carbide has been the feature which has given these materials promise of industrial use, this property has more than common interest. D'Arcambal has shown that quenched and tempered high-speed steel has a modulus of rupture in a transverse or cross-bending test of around 425,000 lb. per sq. in. Similar tests have been run on the new tungsten carbide tool materials at the Research Laboratory, using a section one-quarter of an inch square with supports one inch and a half apart. The German material is found to run around 225,000, and the super-hard material around 250,000 to 275,000 lb. per sq. in. We have not found it feasible to get a figure for the pure carbide, (*i.e.*, without the cobalt), to compare with these figures. It is not likely that a good figure could be obtained, but we may safely say that the pure carbide would come under 50,000. Our experience indicates that such a material would be very weak.

These figures show that tungsten carbide has had its strength increased to somewhat over half that of high-speed steel. Viewed in one light, this is to be considered as a very substantial achievement, but this strength must also be regarded as a factor which places a definite limitation on the use of tungsten carbide in commercial practice.

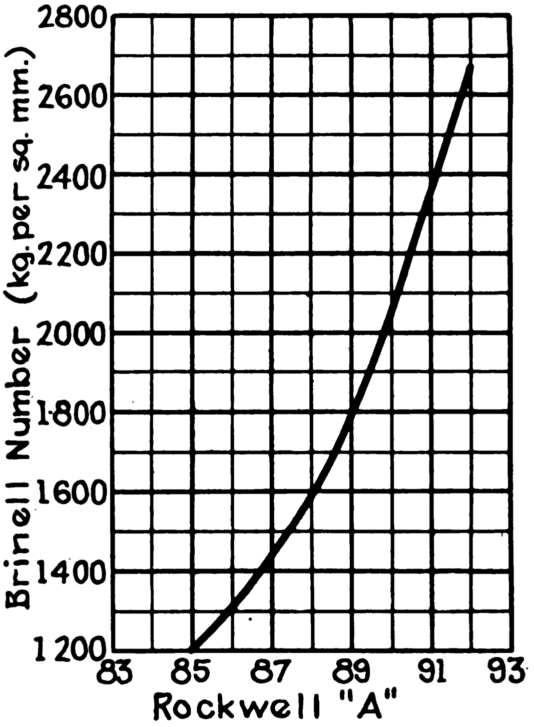

Fig. 1. Relationship of Rockwell "A" Numbers to Brinell Hardness Numbers

Another important property of a material of this type may be called "toughness," the ability of a material to stand up under stresses or impacts imposed on relatively small areas. A weak, or fragile, material would chip and flake off, but a tough one would be able to stand stresses of this type without injury. The new carbide tool materials possess their share of toughness, since they withstand quite severe blows, or sufficient to indent a supporting block of copper. Inasmuch as the break comes abruptly when the bar does break, the material is brittle, in that sense. The fracture is of the curved type, while a weaker, or less "tough," material would break straight across. A further demonstration of strength and toughness is afforded by the use of Carboloy on intermittent cuts on metals.

The thermal expansion and thermal conductivity of Carboloy are low, but quantitative data will not be given at this time.

The chemical stability of a new material is naturally of considerable interest. Carboloy is probably permanently stable at room temperature and under normal conditions, for we have observed no deterioration in several years. It does not pit or tarnish, and is dissolved in acids only with great difficulty. In many respects it may be expected to show somewhat similar properties and behavior to tungsten. At elevated temperatures the carbide oxidizes, as other tungsten products do when heated in an oxidizing atmosphere. On this account the various operations which are performed on Carboloy at high temperatures are carried out under non-oxidizing conditions. The moderate temperatures involved in cutting metals at the speeds which we generally use have not been observed to be harmful. If the speed is raised too high and the abrasion is severe, small particles of carbide may oxidize and shoot off the edge as sparks. Ordinarily, the temperatures involved simply produce temper colors, much the same as they do on high-speed steel.

Finally, we have to consider the way in which Carboloy retains its strength and hardness at elevated temperatures. No quantitative test data can be given at this time, but we know from the performance that Carboloy retains its hardness at a bright red heat to a most remarkable degree. Tools have been observed cutting nickel steel when the point is at this temperature and quite without ill effects.

That such strength, hardness, toughness, and chemical stability just cited, plus the ability to take and keep a cutting edge, must characterize a material of most extraordinary possibilities, as compared to our present tool materials, is self evident to all who are conversant with this field. But we have found that the performance of Carboloy on the lathe and in the shop does actually exceed the predictions which one might make from a knowledge of these properties. This, in turn, is doubtless due to two circumstances which are highly favorable to Carboloy. Carboloy has no "temper" to be "drawn " by the heat generated, and it is actually much harder than the materials machined, even at elevated temperatures, while steel tools are frequently softer. To get a better understanding of the performance of Carboloy tools, let us turn to a review of our experience in several exemplary fields.

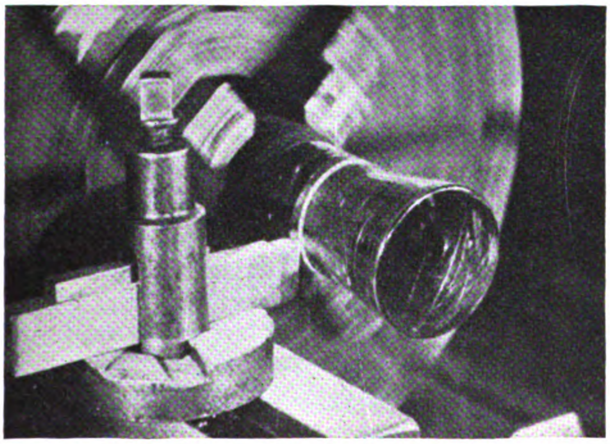
Fig. 2. Cutting Glass on a Lathe with Carboloy—Start of Cut

One's first experience with a material having such remarkable properties is apt to mislead him into believing that it is capable of almost anything; but Carboloy, like other materials, has its breakdown point, first through deficiency in one property, and then in another. Early in the work it became of interest to determine this breakdown point as one means of estimating the economic value of the tools and, particularly, to compare it in this respect to the tool materials with which it would have to compete. In this way we have been led to try Carboloy tools on jobs which are not at all "commercial" and have secured some very valuable information. In this light it may not be amiss to review some of these experiences.

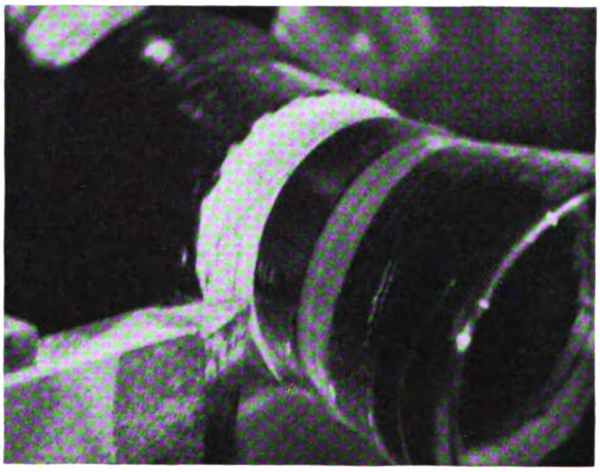
Fig. 3. Wide and Deep Groove in Glass, Cut with Carboloy

Tests

Glass proved to be quite easy to machine, or to cut with screw threads; and we soon found that hard porcelain insulators could be machined on a shaper. It is not often that one wishes to machine anything like hard porcelain; but since discovering that we could do so, we have found that these tools have been used for such work to a limited extent. Hadfield's manganese steel has always belonged to the non-machinable class; but it was found to yield easily to Carboloy, so easily, in fact, that it would seem that the operation could be developed commercially. For a check on the Carboloy test, a similar test was run with a standard steel tool on the same material, and it was found that the steel tool lost its edge almost at once, as was to be expected. Since then we have found Carboloy tools to be of great assistance in handling many of these hard, tough, and non-machinable steels. At the Research Laboratory we have occasion to prepare and test many different alloy compositions, and frequently find that it is only with the new tools that such alloys can be shaped or surfaced. This experience suggests that steels with higher alloy contents than are now commercially machinable will some day be brought into the machinable class. So far this work has consisted simply in doing jobs which could not ordinarily be done.

At one time we attempted to machine a block of quenched high-speed steel on the shaper. This attempt appeared to be the height of folly, but at least we satisfied our curiosity as to our ability to take a cut on such a hard, tough material; in that we were successful, but the tool performance was not at all satisfactory, for the edge crumbled. The same was found to be the case on several attempts to machine tool materials of the cobalt-chromium alloy type. If one were obliged to machine these alloys, he would be fortunate if he had tools of the tungsten carbide type at hand; but the grief involved, in tool breakage, would be uncommonly great for ordinary machine shop practice.

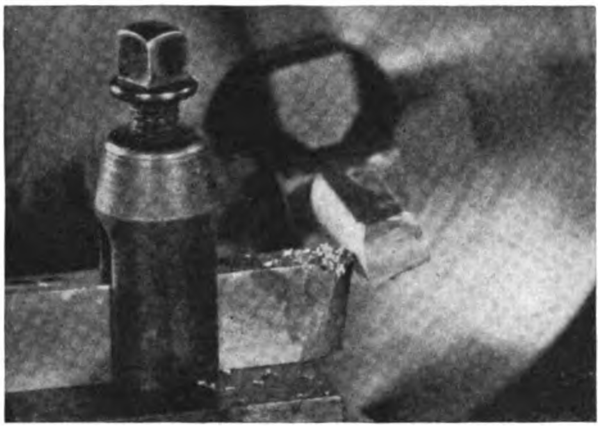

Fig. 4. Cutting a High-speed Steel Cutter in a Lathe with Carboloy

A complete knowledge of the essential properties of Carboloy and a correct understanding of the requirements of a particular service would enable us to predict the behavior of Carboloy in that service. Of course our knowledge is not so complete, but we probably do know enough to enable us to select the most promising fields. Thus we should expect the peculiar virtues of Carboloy to be most prominent in the field of weak or low-tensile materials, which are also uncommonly abrasive to our present tools. Fortunately, for our experimental purposes, the electrical industry furnishes a goodly number of such materials which must be machined in daily production. Without attempting to observe any particular order, our experience with a number of these will be given.

In the production of fused quartz, it has been found advantageous to use molds of carbon—the hard abrasive variety. A considerable amount of experimenting had shown that even the best tool steel lost its cutting edge and began cutting a taper soon after starting a cut on carbon. The grief involved in machining a hollow cylinder of carbon may be easily imagined. On putting Carboloy tools on this job we found at once that we could take the entire cut without tool wear, and therefore without tapering. This statement means, of course, that the cut was parallel within the tolerance allowed.

Genelite is composed of copper, tin, and carbon. It is so soft, in one sense, that it can be whittled with a knife blade very easily, but it is also so abrasive that a lathe operation dulls a steel tool almost at once. A cut on a small cylinder of only a few inches in length is always on a taper. This again demonstrates the extreme abrasion which may be involved in the machining of carbon. Carboloy tools were found to machine Genelite with no difficulty and accurate tests of the cuts made showed that they were not tapered. A careful inspection of the cutting edge confirmed this observation by showing that no wear had occurred.

Commutators of electric motors are composed of alternate layers of copper and mica. The bearing surface of these has to be machined after assembly on the shaft. The mica is quite abrasive and has always presented a problem to the machine shop, for the operation is one which must be carried out very accurately. With high-speed steel a sacrifice must be made on this job, either in speed, or in the finish of the surface; whereas the Carboloy tools cut through the mica without undue wear and give a smooth finish to the commutator.

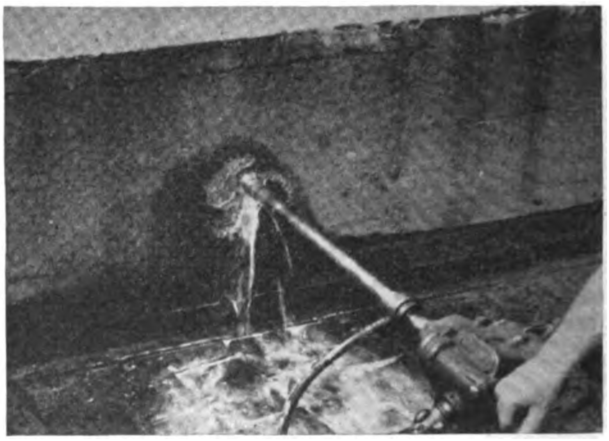

Fig. 5. Using the Carboloy Concrete Drill

Mycalex is an insulating material of such excellent properties that there is considerable demand for it in the electrical industry, in spite of the extreme difficulty with which it can be shaped or machined. It is composed of fine mica particles and lead borate, the latter acting as a bond for the mica. Ordinary tool materials are worn away at once on Mycalex, so quickly that it would be difficult to give quantitative data on their performance. The effect and noise produced brings to one's mind a picture of an attempt to machine a grinding wheel. Carboloy tools, on the other hand, are capable of machining Mycalex commercially, and we have observed tools cutting over 1000 ft. linearly, before losing their edge. Other molded compounds come under the same category and need not be considered separately.

Materials like Bakelite and hard rubber are very abrasive when machined at high speeds. One of our operators showed me that even though he had made his steel tool as hard as "fire and water could make it," the cutting edge was rounded off very soon on hard rubber. Although the ease with which this machining was done made this result surprising, the evidence was very clear. Carboloy tools keep their cutting edge very well on hard rubber, and have been used with very satisfactory results.

Bakelite offers somewhat similar problems and, on account of the necessity of machining Bakelite at high speed, diamond tools have been commonly used. This was the only tool material which had been found to stand up at the speeds used, but the diamonds possessed one disadvantage which was overcome by substituting Carboloy. If the part machined contained a metal insert, a special operation was required to recess this part below the cut of the diamond, for if the diamond hits the metal at high speed, it breaks off at once. Here the Carboloy tool necessitates no such special operation for it cuts both Bakelite and metal insert.

On a somewhat similar operation we compared Carboloy tools with Stellite tools which had given the best performance on this operation up till then. Three tools were used on this job on two different heads, finishing three faces on the part. The Stellite tools required redressing every 150 parts machined, on the average, before the tools became dull enough to require sharpening. Carboloy tools were substituted under the same operating conditions, and finished 11,000 parts before they became dull enough to be redressed.

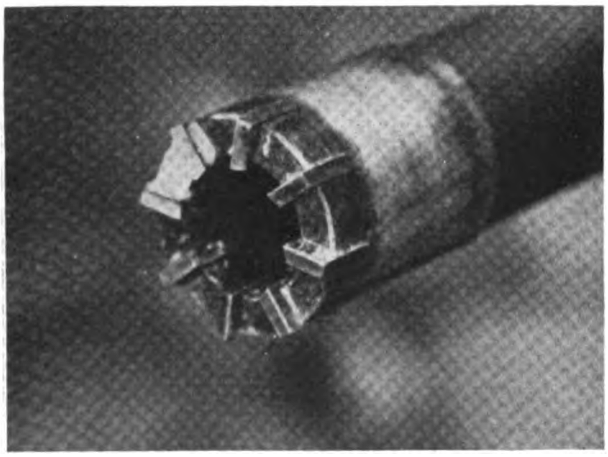

Fig. 6. Tip of Carboloy Drill for Drilling Concrete

In the "low tensile" field the durability of Carboloy tools has been found to be of the order of 25 to 75 times that of high-speed tools, or so great that the simplest kind of a test demonstrates the vast superiority of the carbide tool. Here the tool pressure is always light, the cuts are small, and the cutting speeds are high. Resistance to either wear or "burning" is the chief requirement of the tool.

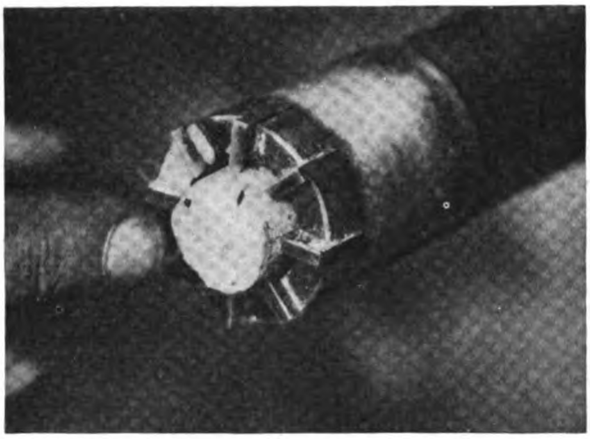

Fig. 7. Tip of Carboloy Drill, Showing Core of Concrete

Coming to cast iron we find that the cutting conditions are apt to be radically different. The parts are frequently large, the cutting speeds are much lower, the cuts are heavier, and the tool pressures are greater. The heavier pressures are due in part to the higher unit pressures exerted by the iron, but more particularly to the larger cuts. In this way the tool pressure becomes a more prominent factor in tool performance, but our experience shows that cast iron comes well within the field which can be handled by Carboloy.

The machining of cast iron by Carboloy offers a variety of problems so that it will be well to consider the subject from several different points of view.

One of the most difficult jobs to do on cast iron with the usual tools is that of removing the surface layer, particularly if the surface contains sand. This is due to the fact that the chilled iron and sand are harder than the tool used to cut them. As has been pointed out, Carboloy is harder than sand and chilled iron, so that castings which would take the edge off high-speed steel almost at once can be handled with but little difficulty by Carboloy. In fact, we commonly use about the same speeds and feeds on these cuts that are used on the sub-surface cuts. Even cuts which travel in and out of the surface layer cause no particular trouble, and such cuts are recognized as being the most abrasive on the tool.

A somewhat similar problem is presented by "hard" castings. If a machine shop is accustomed to machining castings of some particular grade, a run of castings which is appreciably harder is certain to increase the machining costs whether the increase be due to annealing, to a lower production rate, or to scrapped castings and lost time. These castings may be, and quite likely are, of a better grade than the softer castings with which the shop is accustomed to deal. From that point of view it is unfortunate that they cannot be satisfactorily and economically handled. Carboloy tools machine these "hard" castings with ease, and it has been our experience that a run of such castings will go through the machine shop without disturbing the practice in any way. Naturally, the life of the tool is diminished according to the increased abrasion of the harder iron, but the tool life of Carboloy on cast iron is normally so great that the decrease involved is not a serious matter. Furthermore, it is not an uncommon occurrence that the use of Carboloy tools obviates the necessity of scrapping a large casting on which a considerable amount of work has already been done. The cost of the casting may be many times as much as that of the tools used. The improvement in shop practice and morale can be easily pictured by those who have this type of work under their supervision.

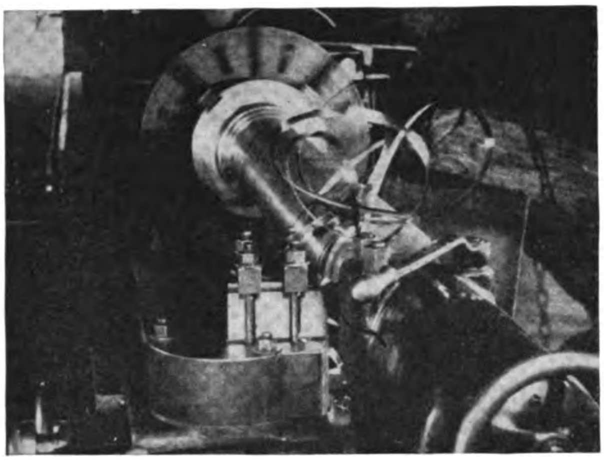

Fig. 8. Testing the Carboloy Cutter on a Nickel-steel Test Log, Showing Long Turning Coming from Log. The usual high-speed cutter is quickly dulled on this test

In the manufacture of armature spiders for electric motors it is necessary to take a cut over the ribs to true up the spider. This cut is an intermittent one, and has given us an excellent opportunity to test the behavior of Carboloy tools under impact. The impact on the Carboloy tool is generally greater than on the high-speed tool on account of the higher speeds used with the former; but, in spite of this factor, we find that Carboloy tools stand up very satisfactorily under this service.

Not much can be said at this time on the use of Carboloy as a finishing tool on cast iron for, obviously, this will require quite a special study. In tests, I have observed that a speed of 300 ft. per min. is sufficiently high to take the edge off high-speed steel and other tools almost at once. In fact, in one such test the cast iron acted more like a grinding wheel and wore off the tool as rapidly as it was fed into the work. The Carboloy tool used in a parallel test was ground to a rather keen edge and was able to take a light cut for over a foot in length along a block about six inches in diameter with no appreciable wear on the edge. This test suggested that cast iron is more abrasive at high speeds than steel is, but I am not sure this is correct.

As the hardness of the cast iron decreases, or as the grain becomes more and more open, the abrasion on the tool becomes less and heavier cuts and higher speeds can be used. Abrasion resistance becomes less of a factor in tool performance, and the strength of the tool, particularly in the case of Carboloy, enters more prominently. In practice this is readily taken care of by being sure that a tool of sufficient size is used for the cut taken, for the unit pressure in machining cast iron is not large. Quite naturally this type of iron is most commonly met with in large castings. Here the speeds which can be used are correspondingly small, because large masses of metal are not moved as rapidly as small masses, and the necessity of reducing the time of machining requires the use of heavy cuts. Even this cursory discussion, which cannot possibly depict the situation fully, is probably sufficient to show that the advantage Carboloy possesses over high-speed steel is greatest for small castings, and hard iron, and that it decreases as the size of the castings increases and the hardness of the iron decreases.

Our work on cast iron has shown clearly that Carboloy tools can machine harder and denser grades than can be economically handled by high-speed steel. It is logical to assume that the iron is made soft enough to avoid running up machining costs inordinately, and that the most important factor in the situation is the cutting efficiency of the tool material. It is also logical to assume that practice aims at producing iron which, on the average, is well below the grade which begins to give trouble. Such being the case, an economical advantage is bound to follow the introduction of a cutting tool whose cutting efficiency is well above that of our present tool materials.

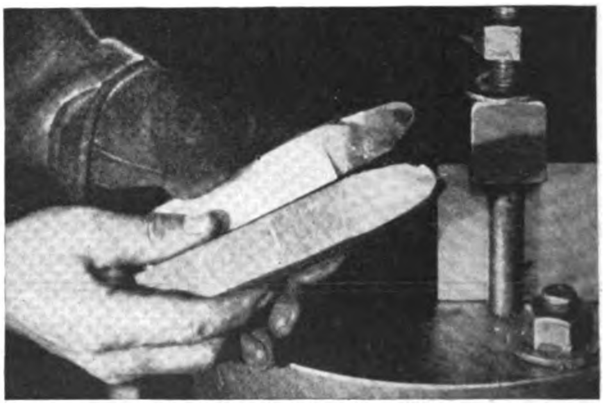

Fig. 9. Comparison of High-speed Steel and Carboloy Cutters After Use on Nickel-steel Log. The steel cutter (lower) is burned; the Carboloy cutter (upper) has not been affected

Coming to steel, we enter what is probably at once the most important and the most difficult field for Carboloy. The difficulties arise mainly from the high tool pressure involved. Even so, our early tests on annealed tool steel, mild and medium carbon steel, and wrought iron led us to expect that Carboloy would show equally as spectacular a performance here as in the work already mentioned. These expectations have been adequately realized in tests such as the following.

At one time we ran a series of tests on a nickel-steel test log, the Taylor speed of which was about 54 ft. per min. under our standard conditions for testing high-speed steel. We soon found that if we were to get any adequate estimate of the life of Carboloy tools on this log, we should have to increase the speed to around 200 ft. per min. This threw us quite out of the range of speeds used with high-speed steel, and we decided to run the high-speed steel under the conditions set for the Carboloy tools. The calculated tool life came under 30 seconds, while the tool on test failed in 16 seconds, having had its edge burnt off almost at once. The Carboloy tool was then set up in the same way, and after running for one hour the test was arbitrarily stopped for the tool was still cutting and capable of cutting for a much longer period. The difficulties involved in determining the tool life of these Carboloy tools may well be imagined from the results of this test. In fact, they are so great that we do not commonly run the tools to failure, but stop the test at some predetermined point and inspect the edge of the tool to see how it has worn. One of our test logs is a chromium-nickel steel with a Brinell hardness number of just over 250. With a cut of $3/8$ in. and feed of $\frac{1}{16}$ in., we found that good high-speed tools fail at 35 ft. per min. in about 6 to 8 minutes. We ran the Carboloy tools with the same cut and feed, but at 70 ft. per min., and stopped the test after 10 minutes. The tools showed but slight wear at that point.

In this work on steel we have carried the thought in mind that the principal factor limiting the performance of Carboloy tools is the strength of the material. This is predicated on the use of the proper tool shape, cutting angles, size of tool, etc. To test out this viewpoint we ran a series of tests to determine the proper ratio between the chip cross-section and tool section. These tests showed that normal service may be expected if the chip area is in the order of 1 per cent of the tool area. Expressed otherwise, we may expect good service from Carboloy on steel if a sufficiently large tool be used for the job at hand, provided that the feed be not too great. This, in turn, will vary with the steel machined, so that definite recommendations could be given only for specific cases.

Conclusion

This account of Carboloy has shown that the use of tungsten carbide as a tool material has increased the cutting ability of the tool by a whole order of magnitude. This is the first time this has been done since the introduction of high-speed steel by Taylor and White. Granting this improvement in cutting ability, the extent of the use of Carboloy will depend largely on the cost of operating with Carboloy tools and their output, as compared to that of high-speed steel. The ingredients of Carboloy and the process of making it are expensive so that we may safely say that Carboloy will cost more than high-speed steel; but we have found by an extended study of many fields that the improved performance of Carboloy tools more than offsets the greater cost of the material. In these cases our shops find that they cannot afford, now, to operate without the carbide tools. If the present limited experience with Carboloy does not mislead, because of its very sensational properties and behavior, further possibilities of development are both manifold and attractive.

The research and development work on Carboloy has been carried out as a large scale investigation, in which members of the staff of the Research Laboratory and of other departments of the General Electric Company have participated. It is due to this large scale, coöperative effort, that we know so much about the production and utilization of Carboloy.

High-voltage Transmission in Canada

The utilization of the highest transmission voltage in Canada has been started with the opening of the new Quebec section between Paugan Falls and Toronto, with operation at 220,000 volts. Twelve transformers for operation at 220,000 volts were supplied to the Gatineau Power Company by the Canadian General Electric Co., Ltd. Nine of these transformers are rated at 19,000 kv-a., 25 cycles, and step up the generator voltage from 6600 to 220,000 volts. The remaining three are rated at 20,000 kv-a., 25 cycles, and will be used to supply power to a 110,000-volt transmission system from the 220,000-volt line. The generating stations of the Gatineau Power Company are approximately 250 miles from Toronto.

New Portable Four-tube Kenotron Testing Set of the Edison Electric Illuminating Company of Boston

The Consummation of Nine Years' Experience in D-c. Testing of Cables—Features of Outfit—Portability—Testing Flexibility—Measuring Equipment—Protective Devices

By C. L. KASSON and R. W. CHADBOURN
Superintendent of Standardizing and Testing Dept. Head of Standardizing Division
The Edison Electric Illuminating Company of Boston

FOR many years, when only alternating-voltage sources were available, the problem of cable testing on a large system was a difficult one, owing to the bulky test apparatus necessary to take care of the high capacity of cables. For fifteen years a large portable a-c. testing set has been used on the system of the Edison Electric Illuminating Company of Boston for cable tests involving potentials below 31,000 volts, and cable lengths of not more than seven miles. For higher voltage and longer cable lengths, various temporary testing sets have been installed in the substations.

As the system expanded and operating voltages increased, the difficulties multiplied—particularly where the test apparatus had to be portable. Obviously on an extensive network, involving many substations, permanently installed test apparatus at many testing centers is not commercially feasible. With the advent of rectifying tubes, or kenotrons, the possibilities of portable testing equipment were tremendously expanded. Whereas, with the a-c. outfit, the size limitations were determined by current; with d-c. equipment, the limitations are largely set by voltage.

Nine years ago (November 1919) the Edison Company made its first kenotron tests on installed cables, and for this work a General Electric one-tube 50-kv. set was employed. This equipment was mounted on rollers and could be loaded onto a truck for transportation from station to station. Later, the outfit was mounted on a small trailer which could be easily attached to a truck and hauled from place to place as desired.

At the time this set was put into operation, comparatively little was understood about the d-c. testing of cables. Just what d-c. test voltage should replace a certain a-c. test voltage was not known—in fact, is not accurately known today. Obviously, comparative d-c. and a-c. tests involved more than the mere ratio of peak to r.m.s. voltage, as the oscillatory stress set up by the alternating voltage is not duplicated by the d-c. voltage; the latter, therefore, must be greater than the peak of the r.m.s. voltage in order to compensate, so far as is possible, for the lack of frequency stress. This additional increment of voltage, in view of our exceedingly limited knowledge of dielectric action, has had to be determined largely by experience. The first kenotron tests were therefore made at rather moderate voltages, and these voltages were later raised as experience seemed to justify. Some of the relations between the a-c. and d-c. testing of cables were described by one of the writers at the A.I.E.E. Regional Meeting at Pittsfield (Mass.) in May, 1927. (1)

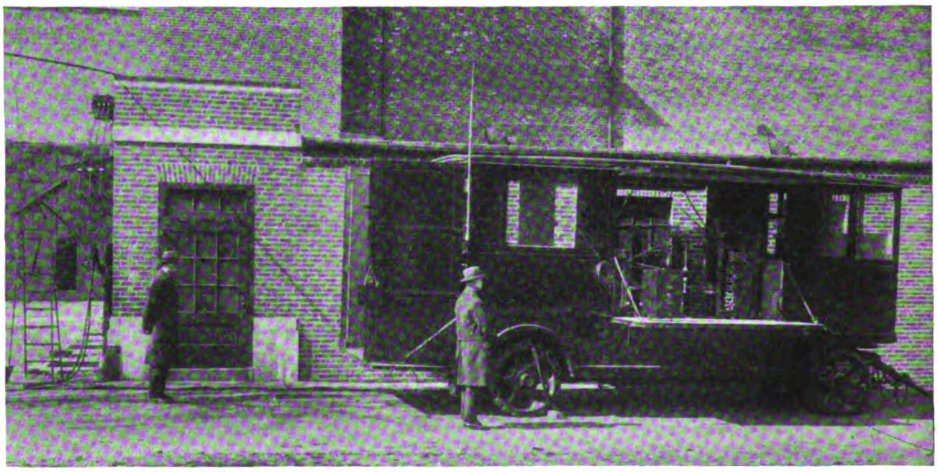

Fig. 1. Side View of the New Four-tube Cable-testing Set, Mounted on a Special Four-ton Trailer and Opened up As for Test

About three years ago (May 1925) the Edison Company, desiring a larger set, put into operation a

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(1) "High-voltage Measurements on Cables and Insulators," by C. L. Kasson, *Transactions of the A.I.E.E.*, May, 1927, p. 634. Abridgment *A.I.E.E. Journal*, Oct., 1927, Vol. 46, p. 1065.

General Electric two-tube outfit, mounted in a special enclosed body on a one-ton Ford truck. Until recently, all the high-voltage d-c. tests on the Company's transmission circuits were made with this outfit. During this period, 1336 tests were made—an average of one and one-third lines per day. The experience of these three years alone has proved beyond question the value of kenotron testing as a partial insurance against service interruption.

Most of these tests were made on circuits of 13,800 volts or less. As circuits of higher voltage rating were installed, and test voltages gradually increased, as experience indicated advisable, a point was finally reached where the two-tube set proved inadequate. Accordingly, the Company has just put into operation a portable four-tube outfit. This will be used for whatever tests cannot be handled by the smaller outfit, which, for purposes of convenience, has been reduced to a single-tube set.

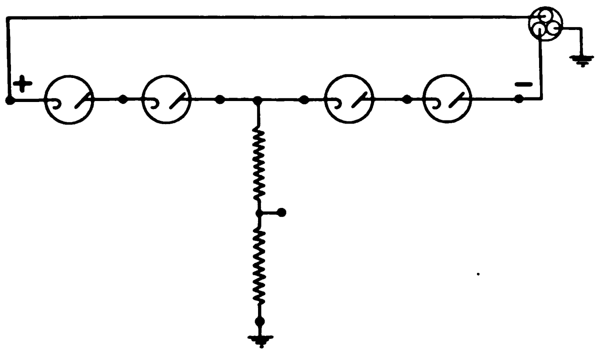


Fig. 2. The T-connection of the Four Tubes and Main Transformer. Output rating: 200-kv., 0.25 amp.

The new four-tube kenotron testing apparatus was designed and built by the General Electric Company. The various units of the set were installed upon a trailer in accordance with a plan and design of the Standardizing and Testing Department of the Edison Company. All additional auxiliary apparatus was designed and installed by the latter company in accord with its previous cable-testing experience.

For this new outfit, a special trailer of four tons capacity was constructed. This trailer was provided with a totally enclosed body approximately 20 ft. long, 7 ft. wide, and 8 ft. high, having a cubical content of about 1050 cu. ft.—about half the size of an ordinary house room. The floor of the forward part of the truck was raised about 18 in. to accommodate the front wheels. A small door was placed at one side of this raised section, large double doors at the rear, and sections of the sides of the truck were arranged to swing outward. Fig. 1 shows the truck opened up for test. This trailer, despite its size, is easily drawn by a five-ton truck. It is equipped with a special brake mechanism which operates automatically when the brakes of the driving truck are applied.

Most of the electrical equipment, including step-up transformer, filament transformers, protective gaps and resistors, control panel, and voltage regulator, was purchased to order from the General Electric Company and assembled, mounted, and wired at the laboratory of the Edison Company. The apparatus, as purchased, was intended to be operated from a 230-volt 60-cycle source. However, as 230 volts might not be always available, a 25-kv-a. 2300/230-volt transformer was also installed, so that tests can also be made direct from a 4000/2300-volt distributing circuit.

The set is arranged to be operated from the rear of the truck, the control board being placed just inside the large double doors. A wide tail-board drops down and serves as a platform for the operator during the tests. Through flexible leads, on reels with handles for winding and unwinding, power is brought from either a 230-volt or a 2300-volt source to one of two pairs of studs on a small wooden panel underneath the switchboard. A swinging wooden box-like cover which normally hangs in a closed position, but which may be lifted up readily for the attachment of the leads, protects the operator from live parts. The 2300-volt studs supply the 25-kv-a. step-down transformer directly through an automatic circuit breaker. Through a double-throw switch, power may be taken directly from the 230-volt studs or from the secondary of this 25-kv-a. transformer to energize the main step-up transformer, the voltage being controlled by an induction regulator. The main transformer is rated at 25 kv-a., 88,000-volts, with both terminals insulated, and special provision is made for grounding the mid-point through a ground rod. The circuit to this transformer is provided with an automatic breaker the function of which is to protect the tubes and transformer on short-circuit or heavy overload without at the same time opening the auxiliary circuits, such as that to the tube filaments, etc. The auxiliary circuits are tapped off the 230-volt supply, through a fused switch, just outside the automatic breaker in the transformer circuit. Each filament transformer circuit has its own switch, its rheostat for voltage regulation, and voltmeter for indicating filament voltage. The filament voltmeters are of the small-size switchboard type, and have been provided with cushions of sponge rubber to reduce the effect of road shocks.

The layout provides for three schemes of tube connections as follows:

T-Connection. In this arrangement, Fig. 2, one side of the transformer is grounded; at the other terminal is a divided circuit, one part containing two tubes in series and connected so as to allow current of one polarity to flow, the other part containing the other two tubes connected in the reverse direction so that only current of the opposite polarity can flow through them. By connecting these two branch circuits to two conductors of a cable and grounding the sheath, twice the transformer peak voltage is impressed upon the insulation between the conductors, and half as much between either conductor and ground.

Bridge Connection. This connection, Fig. 3, receives its name from the arrangement of the tubes, which are connected like the four arms of a Wheatstone bridge, the transformer being connected between opposite points, the other two opposing points serving as terminals to the apparatus under test. The arrangement allows a test voltage equal to twice the voltage of a single tube and a current flow equal to twice that permissible for a single tube. In effect, it is a series-multiple arrangement.

Parallel Connection. In this scheme, Fig, 4, the transformer mid-point is grounded and forms one side of the test circuit. Each live terminal is connected through the tubes in multiple to a single terminal post, the pairs on opposite terminals being of course oppositely connected. This is in effect a parallel arrangement, permitting four times the current of one tube, but only the voltage permissible for one tube.

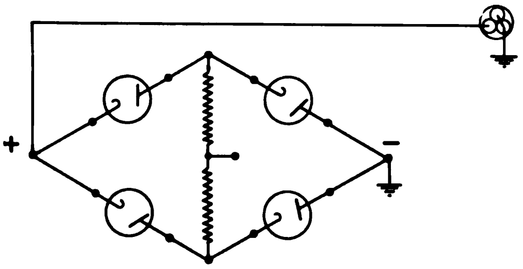


Fig. 3. Bridge Connection. Rating: 100 kv., 0.5 amp.

The change from one scheme of connections to another is made very simple and quickly. Connections among tubes, on the high-tension end, are made by a system of sliding concentric tubes, with hooks at each end. The necessary changes in the metering circuits are made by a simple three-position drum switch on the switchboard.

To measure the test voltage and current, portable d'Arsonval type instruments are used. For use, these are placed on a bench in front of the switchboard, and connected to suitable studs on the boards. When not in use, they are stored in individual compartments in a small cabinet. The compartments are lined with heavy felt to absorb the road shocks. Instruments of the portable rather than the switchboard type were chosen because of the better protection which could be given them from vibration during transit.

The voltage is measured by means of the rectified current from a voltmeter coil in the main step-up transformer. Two small rectifying tubes, oppositely connected, each with its own voltmeter, are used for measuring the opposite sides of the wave. This is necessary for the T-connection. For the other connections, one instrument sufficies.

For measurements of current, two three-scale milliammeters are provided. One of these is arranged for 1, 10, and 300 milliamperes full scale, the other for 300, 600 and 1200. For the bridge and parallel combinations, one of these meters is connected directly, by means of the drum switch, into the ground end of the test circuit. By means of a four-pole double-throw switch, the milliammeters are made interchangeable without the necessity of changing leads. Thus currents can readily be measured from a few microamperes up to 1200 milliamperes. In the T-combination, where current connections must be made in the main transformer circuit between one end of the transformer and ground, a current flow in both directions is encountered, since one-half of the wave forces current through one of the kenetron circuits and the other half through the other kenotron circuits, the two currents being of such direction as to produce additive potentials in the kenotron circuits, but being of opposite directions in the transformer. The currents in the two kenotron circuits are easily measured by the simple expedient of introducing into the transformer circuit two tungar rectifiers connected in multiple but with plate circuits reversed. One tungar supplies one of the milliammetres and measures the current on the positive side of the test circuit; the other supplies the other instrument and measures only the current in the reverse direction—which is that flowing in the negative side of the circuit.

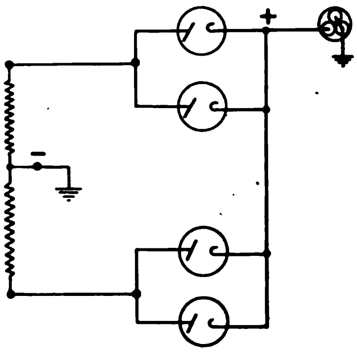


Fig. 4. Parallel Connection. Rating: 50 kv., 1.0 amp.

Fig. 5 shows the control board with the portable instruments connected in the circuit, together with the oil circuit breakers, connection box, etc.

The kenotron assemblies are shown in Fig. 6. Each filament transformer has, mounted on top of the insulating bushing, a special assembly consisting of a two-arm bracket, with the rectifying tube mounted in a socket on one arm of the bracket, and a protective gap for the tube mounted vertically on the other arm. At the center is a long insulating tube, with two sets of resistors mounted on top. One set of resistors is in series with the gap, the other with the tube, the latter set serving to limit the current taken from that particular tube. Two of the four tube assemblies have condensers mounted within the central insulating tube; these are used to balance the voltage between tubes where tubes are used in series.

The main transformer is also protected by a gap, mounted on the roof of the truck, just above the transformer. In series with each live test connection is a resistor to limit the initial rush of current into the cables. These resistors are mounted on top of insulating posts at one side of the truck. One of these is shown in the right foreground in Fig. 6. On the side of each of these posts is a discharge resistance, connecting with an automatic discharge switch at the bottom, the other end of which is grounded. The discharge switches are actuated by solenoids, fed from the 230-volt auxiliary supply, through a pallet switch on the circuit breaker on the main transformer supply. When the circuit breaker is closed to energize the transformer, the solenoids come into operation and hold the discharge switches open. As soon as the breaker is opened, the solenoid circuit is broken, and the discharge switches spring back into a closed position, thereby automatically providing a discharge path for the cable under test.

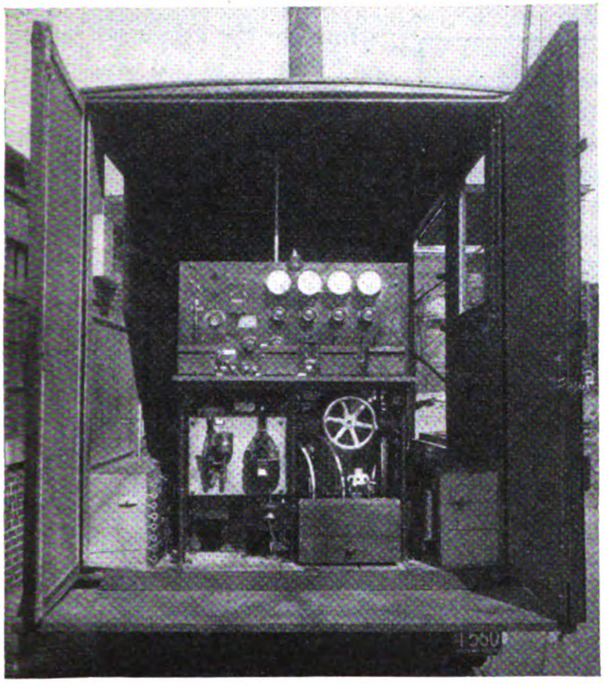


Fig. 5. Control Board at the Rear of the Trailer

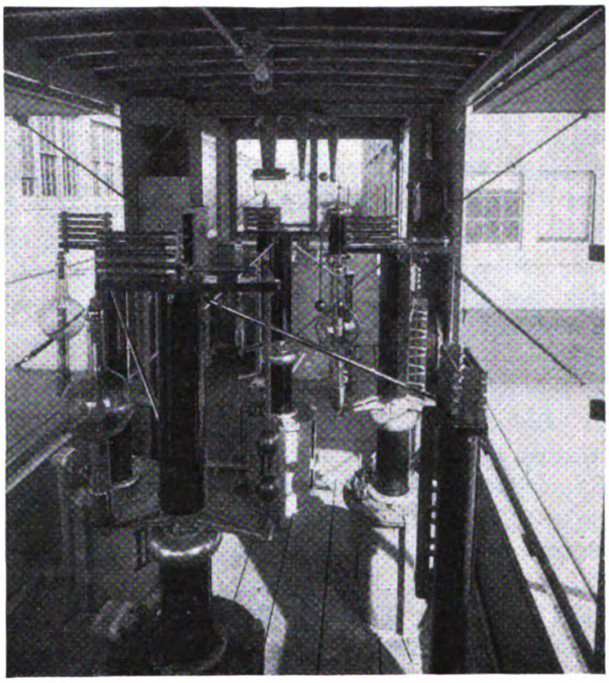
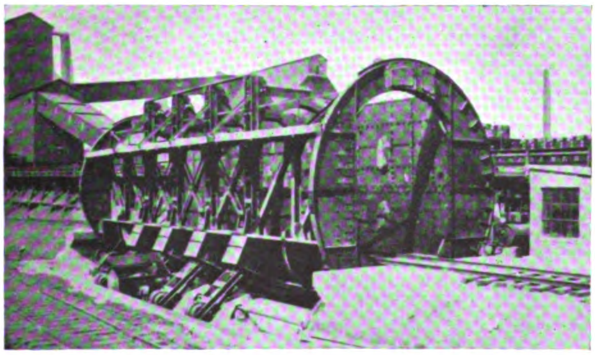


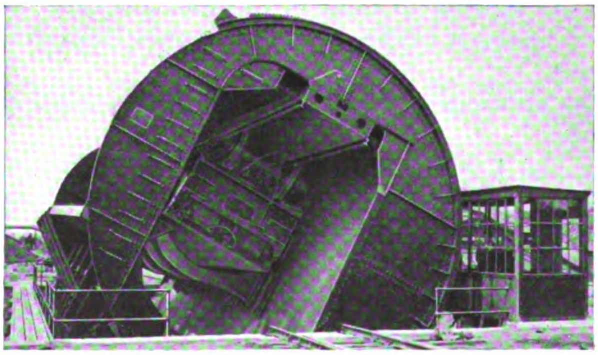
Fig. 6. Interior of the Trailer Showing the High-tension Assembly

For limiting the rush of current at the time of cable failure, a reactor is connected in series with the main transformer primary. This can be cut out of circuit, if desired, by means of a short-circuiting switch on the switchboard. The large kenotron tubes, when not in use, are supported between canvas straps in a special box, which protects them from damage by road shocks. Small tubes, such as voltmeter kenotrons and ammeter tungars, are carried in small felt-lined compartments in a separate box.

In conclusion, the results obtained have fully warranted the great effort that was expended to make the entire outfit as flexible, as safe, and as easily handled as possible.

Revolving Car Dumpers in Operation





Direct-current Generators of Very High Voltage

Ratings from 5 to 150 Kw. and 7500 to 15,000 Volts—Employ Fully-distributed Compensating and Exciting Windings—Efficiency High—Remarkable Freedom from Voltage Ripple—Design Features

By S. R. BERGMAN

Consulting Engineer, General Electric Company

> *The machines described in this article were developed for service in radio transmission, and the author frankly acknowledges that at present he does not know of any commercial applications for them outside of this field. The machines represent a noteworthy extension in electrical development, however, and who can say with confidence that they will be of service only in radio? For instance, unexpected uses have been found for the electric arc, the x-ray tube, etc.*—EDITOR

FOR radio telegraphy and telephony there is a demand for direct-current energy of relatively high voltage to furnish the plate voltage for vacuum tubes. In this article will be described a novel type of high-voltage direct-current generator which has been developed for this purpose during the last few years.

A number of such direct-current generators have been built in ratings varying from 5 to 150 kw., and ranging in voltage from 7500 to 15,000 volts. Examples of such ratings are: 5 kw. 10,000 volts 1450 r.p.m.; 10 kw. 7500 volts 1150 r.p.m.; 30 kw. 12,000 volts 1150 r.p.m.; 150 kw. 15,000 volts 850 r.p.m. Two of these generators are installed at the Radio Research Laboratory at South Schenectady (N. Y.), the ratings being 30 kw. 12,000 volts and 150 kw. 15,000 volts. These machines are used regularly to supply energy for broadcasting WGY programs.

The principle of this type of generator is novel in character. In appearance, the machine resembles more an induction motor than a direct-current generator. From an inspection of some of the photographs in this article, the similarity may be readily recognized, the field consisting of a slotted sheet-steel structure and the winding of a plurality of coils assembled into a lap winding. These features are radical departures from the conventional type of direct-current machines with salient poles carrying concentrated field coils. This new type of machine has certain characteristics that are superior to those of the standard type, for which reason it has been adopted to meet this severe service.

The principle of the machine may be explained as follows:

In Fig. 1 is shown schematically a direct-current armature *A* which may be wound with any of the usual types of windings. Surrounding this armature is a field structure carrying a compensating winding which is in series with the armature winding and distributed evenly around the armature in such a manner as to oppose the reaction of the armature at each point. This compensating effect of winding *C* is made equally as strong as the armature reaction. and thus these two windings constitute a nearly non-inductive pair; *i.e.*, no flux except a small amount of leakage flux will be produced. In order to provide for excitation, there is a second winding, the exciting winding *E*. The compensating and the exciting windings are located in quadrature in space, and these two windings are comparable with a two-phase winding as usually applied in polyphase machines. It is obvious that the shorter the coil pitch, the better utilized is the material. As shown in Fig. 2, the exciting winding and the compensating winding form a lap winding of 50 per cent pitch. It will be noted that in each slot there are two coils, one exciting coil and one compensating coil.

The strength of the compensating winding is made. as nearly as possible, equal to the armature reaction and the over-compensation is provided by a separate winding C_1 as shown in Figs. 2 and 3. The commutating teeth are bolted to the frame and can thus be easily removed, together with the commutating winding, without in any way disturbing the distributed windings. The commutating flux in this machine may be changed, either by the use of shims behind the commutating teeth, or by a change of turns in the commutating winding. After the over-compensation has once been determined for a certain rating, it never need be changed.

As is shown in Fig. 2, the field structure consists of punchings with open slots. These slots are closed by magnetic wedges which are made up of soft-iron wire that has been cotton-covered and then woven into a cloth. This cloth is dipped in a phenolic-resin solution and then shaped and cured in a steel mould. These wedges are found to be very effective; for example, by removing them the core-loss is doubled and the excitation increased by approximately one-third. The wedges also reduce the noise considerably

The structure shown in Fig. 2 seems to be an entirely practical solution of the high-voltage problem. The coils are easily assembled; and the over-compensation can be readily changed, thus providing a flexible combination.

The characteristics of this type of machine depend mainly upon the fact that the armature reaction is completely neutralized by the compensating winding; there is no distortion of the field flux as is the case with non-compensated machines. This important consideration leads to the following advantages:

First: There are no extra load losses due to field distortion which, in non-compensated machines, may reach considerable amounts. Therefore, the efficiency of the compensated machine is greater than that of the conventional type. Actual tests show that the full-load efficiency of the generator rated 30 kw. 12,000 volts is 92 per cent, and the efficiency of the 150-kw. 15,000-volt generator is 94 per cent, thus demonstrating the high efficiency claimed for this type of machine.

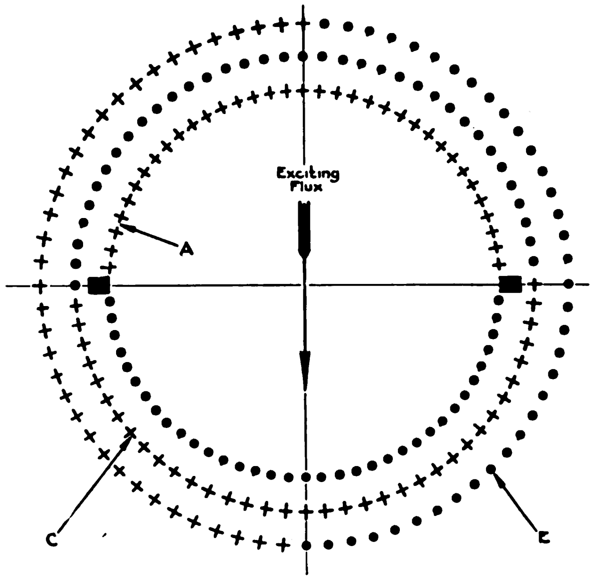


Fig. 1. Schematic Diagram of the Conductor Arrangement in the High-voltage Machine

A: Armature winding *C:* Compensating winding
E: Exciting winding
The crosses indicate current going down into the plane of the paper.
The dots indicate current coming up from the plane of the paper.

Second: Due to the fact that the field flux is not distorted, the field in a compensated machine may be weakened without any penalty. Thus, any compensated generator may be adjusted over a wide range of voltages without instability.

Third: All of these machines run without sparking, even at very heavy overloads. Many of them have carried three times normal load with black commutation. When applied to vacuum tubes a generator may be momentarily short-circuited, because of failures in the tube circuits. One important requirement for generators in this service is that they withstand short-circuits without injury. Tests have shown that this type of generator will withstand short-circuit, not only momentarily but for a sustained time without flashing. Repeated short-circuits on the generator have proved that no injury is caused by these severe conditions, which is rather remarkable considering the extremely high voltage of operation.

Fourth: The fact that there is no field distortion leads to a stable commutating zone. There is no undue distortion in the commutating zone, which means that there is at all times the proper field for commutation. This is not the case in non-compensated machines since the field distortion piles up the voltage on the commutator in the neighborhood of the brushes, which condition leads to sparking and flashing. Thus, with the compensated design, machines have been built with higher voltage between bars than heretofore. In salient-pole machines, it has been found that 25 to 30 volts per bar is the limit. On the other hand, compensated machines with fully distributed windings have been run successfully with the difference of potential between bars as high as 100 volts. In the largest machines, *viz.*, the 150-kw. 15,000-volt generator, the average voltage between bars is 60, or about twice as much as would be considered good practice in salient-pole machines.

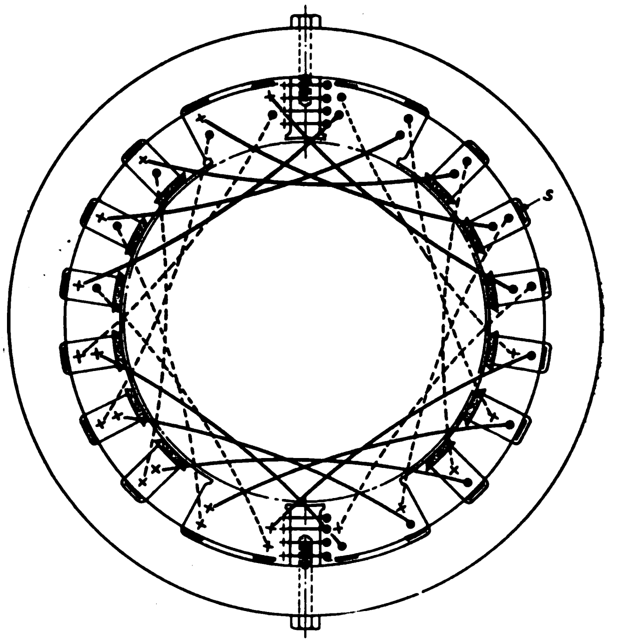


Fig. 2. Diagram of Two-pole Field Winding

The full line represents the compensating and commutating winding.
The dotted line represents the exciting winding.

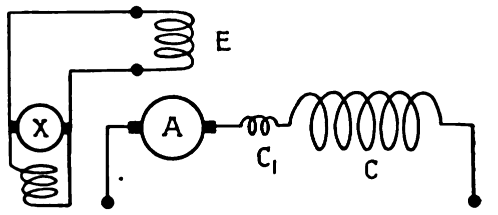


Fig. 3. Connection Diagram of the Separately-excited High-voltage Generator

A: Armature *C:* Compensating winding
C_1: Commutating winding *E:* Exciting winding
X: Exciter

So far, the advantages of only the fully-distributed compensating winding have been outlined. The fully-distributed exciting winding offers additional advantages. The distribution of the exciting winding improves the flux distribution, as shown in Fig. 4. Usually, due to saturation, this curve is somewhat flatter than a sine wave. The sides of the curve slope gradually and the neutral between the poles is absolutely devoid of flux. As may be seen from the illustration, the main flux is zero over the whole neutral zone, which is an important condition that does not exist in salient-pole machines. In the latter type of machine, the massed field-coils set up stray fluxes and fringings which cannot be governed, and the neutrals of such machines are usually narrow.

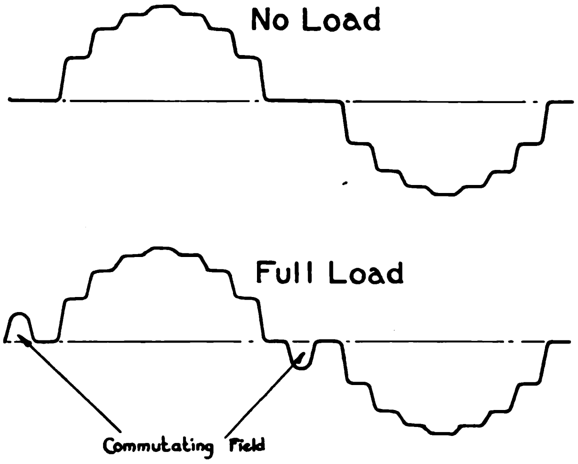


Fig. 4. Flux Distribution Across a Pair of Poles as Produced by the Fully-distributed Exciting Winding

A second advantage of distributed windings lies in the uniformity of the heating. In machines of the salient-pole type, the field coils are often several inches in thickness, and the heating inside the coil is far in excess of that observed by the thermometer on the service. In compensated machines, with fully distributed windings, the temperatures are more uniform, similar to those in alternating-current machines.

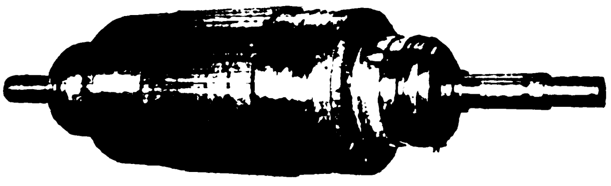


Fig. 5. Double-winding Two-commutator Armature of the 150-kw. 15,000-volt Direct-current Generator

For generators applied to radio service, the voltage drop from no load to full load, known as the voltage regulation, must be good in order to secure clear signals. The voltage regulation of this new type of generator is very low, not exceeding 2.5 per cent when run at constant speed. Since these generators are usually driven by a motor, the drop in speed of the motor, when loaded, must be taken into account and this drop will naturally lower the voltage of the generator somewhat more. Actual tests have shown that the regulation of a motor-driven generator will not exceed 5 per cent.

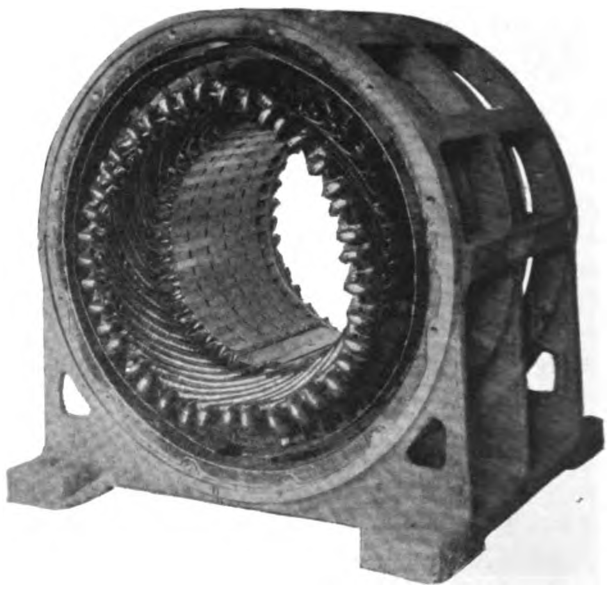


Fig. 6. Field Structure of the Machine of which the Armature is Shown in Fig. 5

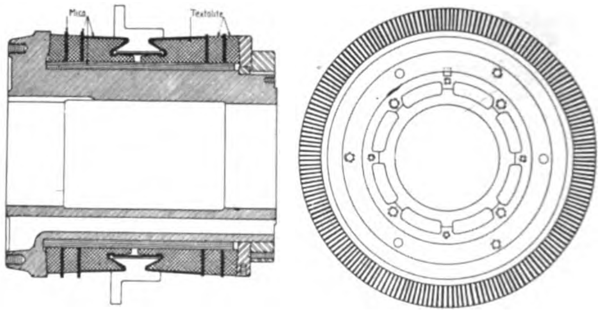


Fig. 7. Section and End Elevation of High-voltage Commutator

Another requirement for radio service is that the generator supply a current of steady value. It is well known that any direct-current generator does not produce a current of absolutely steady value. In its output there exists a ripple which, in machines of standard construction, may amount to 5 per cent. The cause of this ripple is often laid to the fact that a direct-current machine has a commutator, and may be considered as a rectifier consisting of a large number of coils. For each pair of commutator segments there is therefore a small wave. Thus the commutator produces a ripple in the current wave, but this ripple is extremely small and calculations for the machine that are being described show that it amounts to less than $\frac{1}{300}$ of one per cent. This contribution to the pulsation in the current wave is therefore so small that it may be overlooked as compared with another influence which is caused by the slotted construction of the armature. Owing to the fact that the armature is slotted, there is a pulsation of reluctance when the armature revolves, thereby causing pulsation in the main flux which, in turn, causes pulsation in the induced voltage. This is the principal reason for the ripple in the current wave. For radio applications it is desirable that this ripple be eliminated. In order to minimize it, use is made of a squirrel-cage consisting of copper strips placed in the bottom of the field-slots, as shown in Fig. 2 and marked *S*. This squirrel-cage embraces each pole without short-circuiting the commutating flux. When the main flux pulsates, due to the revolution of the armature slots as described, currents are induced in the squirrel-cage which tend to stop the pulsations of this flux; and oscillograms taken of the current wave show that the pulsations may be kept as low as one-quarter of one per cent. This small ripple is further reduced by capacitors shunted across the terminals of the generator; the result being that the pulsations do not exceed 1/100 of one per cent, which has proved to be entirely satisfactory for radio applications.

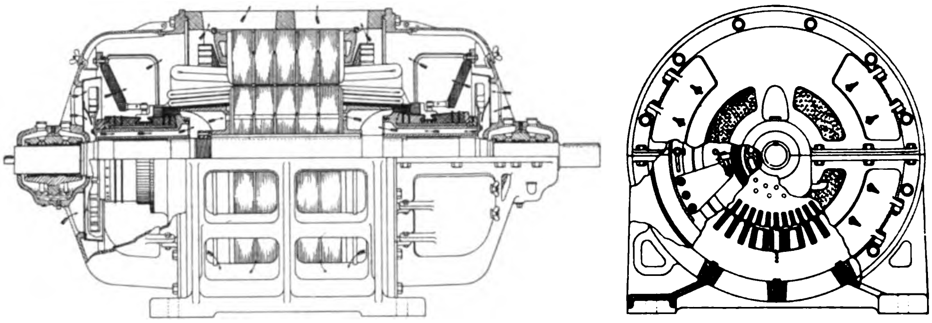


Fig. 8. Side and End Elevations of the 150-kw. 15,000-volt 850-r.p.m. Direct-current Generator

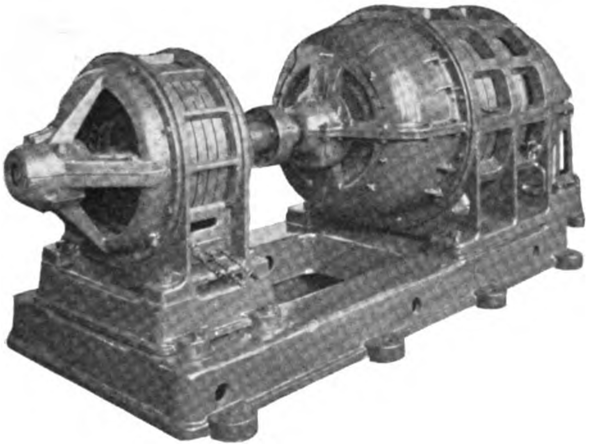


Fig. 9. Motor-generator Set, Consisting of a 150-kw. 15,000-volt Direct-current Generator (right) Coupled to a Three-phase Induction Motor (left)

In order to handle potentials above 10,000 volts, it was found advantageous to use two armature windings, each connected to a separate commutator. Since these two windings are connected in series, each commutator carries half the rated voltage. The armature for a machine rated 150 kw. 15,000 volts is shown in Fig. 5.

The field structure of the same generator is illustrated in Fig. 6, where the arrangement of the completely distributed windings is clearly shown.

A problem that proved difficult was the design of the commutator for these high voltages. In Fig. 7 is shown a section of one of the commutators. The principle of its design is similar to the standards used for lower voltages, except that the cones which hold the segments together are made from textolite instead of cast iron. It will be noticed that there is a mica cone between the textolite cones and the segments.

A sectionalized view of the generator is shown in Fig. 8. Owing to the high voltage involved, separate excitation is employed. For service where no direct current is available, a small exciter is assembled on an extension of the generator shaft. In Fig. 9 is shown the complete motor-generator set, the driving motor in this case being a three-phase induction motor.

In industrial applications, particularly traction work, high-voltage direct current has been more or less commonly employed up to 1500 volts per single motor. High-voltage direct current has also been tried abroad by Thury for power transmission. The Thury system consists of a large number of direct-current generators connected in series, each generator however not being rated higher than 4000 volts. The author is not aware of any instances where direct-current power transmission has been attempted in this country, although it has been under consideration in cases where a big block of power is to be transmitted over a very long distance. Due to recent improvements in alternating-current power transmission, it does not seem likely that high-voltage direct current will be competitive with the highly-developed alternating-current system. These high-voltage direct-current machines therefore seem to be limited at present to radio applications.

Low-voltage A-c. Networks

PART VII

NETWORK RELAY CHARACTERISTICS

By D. K. BLAKE
Central Station Engineering Department, General Electric Company

THE network protector was mentioned in Part VI of this serial as having a relay equipment that performed the two functions of tripping on reversals of energy and reclosing when the incoming circuit is energized. The required characteristics for such functions are influenced by the transformer connections, transformer impedance, primary voltages and induction feeder regulator equipment. The prevailing practice is to utilize Y-connected transformer secondaries; therefore, this treatment will be confined to that connection. The principles involved, however, may be readily applied to other connections.

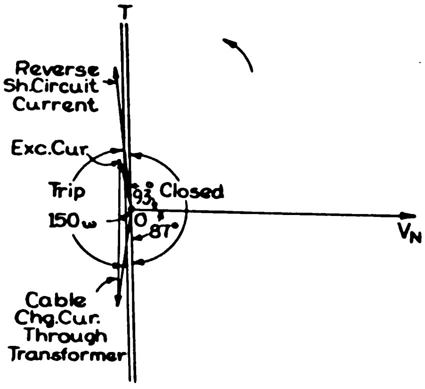

T
Reverse Sh.Circuit Current
Exc.Cur.
Trip
Closed
93°
150ω
O
87°
V_N
Cable Chg.Cur. Through Transformer

Fig. 34. Vector Diagram of Tripping Characteristics for Network Relays

Tripping Function

The relay must trip the breaker on short-circuit currents in the reverse direction. It must also trip on the transformer exciting current in the reverse direction in order to permit the disconnecting of a feeder and its transformers from the network by opening the feeder breaker at the substation. The characteristic of the relay must be such that the addition of cable-charging current to the exciting current will not prevent tripping in the reverse direction.

The vector diagram in Fig. 34 indicates a suitable characteristic for the tripping function. The short-circuit current will probably lag at a maximum angle when a short circuit occurs at the high-voltage terminals of the transformer. This angle is 84.3 deg. with a transformer having a one per cent resistance and ten per cent reactance drop. If the parallel lines *TT* are perpendicular, or slant in the leading direction as shown, it is obvious that if the relay will operate on the transformer exciting current it will also operate on any amount of charging current which may be added. The watthour meter has a similar characteristic. The saturation of the current-transformer iron causes the lines *TT* to bend to the right at the top or to the left at the bottom.

On polyphase systems, it is desirable to use polyphase relays to prevent tripping on single-phase line-to-line faults which may occur on the network. The vector diagram in Fig. 35 shows this condition for a three-phase four-wire system. The short-circuit current in phase *c* will go on the tripping side when the current lags more than 60 deg. The same effect is obtained on feeder No. 1 when a high-voltage phase-to-phase fault occurs on feeder No. 2 of Fig. 36. The polyphase relay will not trip because the *a* and *b* elements exert a greater and opposite torque.

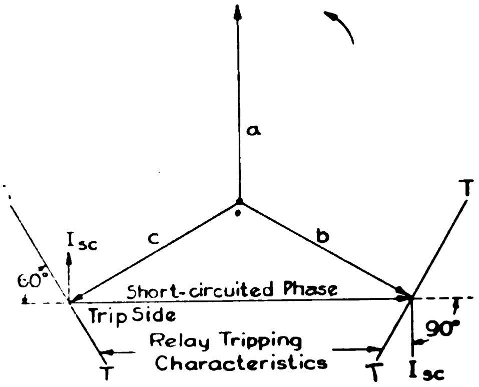


Fig. 35. Diagram of Tripping Characteristics for a Three-phase Four-wire System

The tripping function then requires no new, untried device, since it is possible to utilize watthour-meter parts and construction to form a simple induction-type reverse-power relay. The connection shown in Fig. 37 is suitable, being similar to the connection for measuring power. The current transformer and current coil of the relay are designed together to operate over the entire range of reverse current. This range may be from 1.5 to 15,000 amp. The current transformer has a relatively small number of turns to give sufficient secondary current for relay operation on exciting current and a small amount of iron to limit, by saturation, the secondary current to a safe value for the relay-current coil. The time-current characteristics should be such as to require, under short-circuit conditions, not longer than ½ sec. for operation. A much shorter time, such as ¼ sec. is preferable

Reclosing Function

The network breaker should reclose only when the incoming transformer voltage is of the proper value and phase relation to cause the transformer to take its share of the network load. This requirement does not permit connecting the closing circuit directly to the transformer secondary, because after opening the circuit breaker at the substation it would be possible for some network breakers of the particular feeder to open and reclose again before all the breakers opened. The result would probably be a "seesaw" operation of opening and closing. Induction regulators may also cause low voltage on a feeder by being in the buck position when the feeder is energized. Other factors that may cause improper voltage relations will be considered later. It is obvious, therefore, in view of these possibilities that the relay should measure the voltage across the open breaker contacts and complete the closing circuit only when the voltage difference is of a certain value in the proper direction.

The voltage difference may be measured by a separate relay, by an extra winding on the reverse-power relay, or by the current coil of the reverse-power relay. This article will treat of the latter arrangement (Fig. 38) and the use of a separate phasing relay where the connection shown in Fig. 38 would not be suitable.

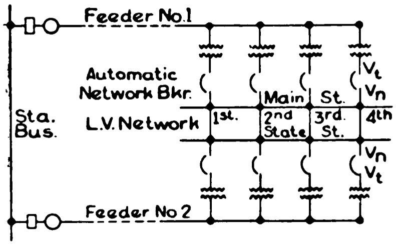


Fig. 36. Typical Low-voltage A-c. Network Utilizing Automatic A-c. Network Circuit Breakers. *(Fig. 33 repeated)*

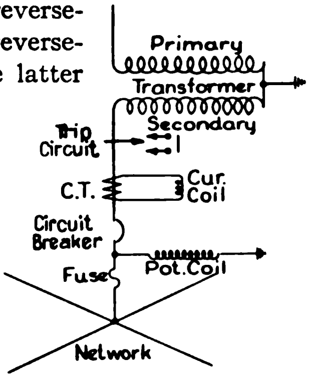


Fig. 37. Relay Connection Diagram for the Tripping Function

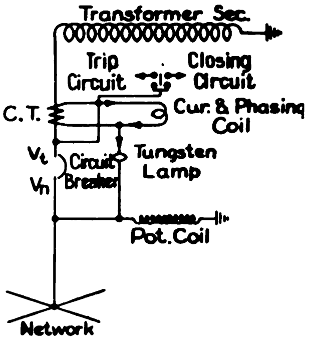


Fig. 38 Relay Connection Diagram for Both the Tripping and Reclosing Function

The circuit of Fig. 38 is similar to that of Fig. 37, with the addition of an extra relay contact for the closing circuit and the connection of the current coil across the circuit-breaker contacts. When the circuit breaker is closed, the relay acts as an ordinary reverse-power relay; and when the circuit breaker is open, the relay acts as a voltage-direction relay. The tungsten lamp has an approximate resistance of 70 ohms when cold and 700 ohms when hot. It therefore prevents short-circuiting the relay coil when the breaker is closed and protects the relay coil when the voltage across the open breaker is high, *e.g.*, 115 to 230 volts.

The current through the relay coil does not increase directly with the voltage, because the resistance of the lamp increases with the voltage also. The high value of the lamp resistance limits the total possible current through the relay coil to a value appreciably below the value of the secondary current corresponding to the transformer exciting current. It is obvious, therefore, that the time required for operation is appreciably longer. The impedance of the current transformer at current values passed through the lamp is much greater than the impedance of the relay coil. Thus its connection in parallel does not interfere with the relay operation.

The potential coil is connected on the network side to keep the relay energized when the transformer is deënergized. This arrangement permits a strong torque in the tripping direction with the breaker open, insuring against the breaker's reclosing as soon as the transformer is energized. The relay must close the closing circuit by a spring when the network, and consequently the potential coil of the relay, are deënergized. It is also desirable, but not necessary, to have the relay protect against reclosing when phases are crossed or reversed by workmen during installation or repair work. Obviously, such protection would not be obtained if the potential coil is on the transformer side, because it is reversed with the transformer voltage.

The current coil of the relay has an impedance of less than three ohms. The resistance of the tungsten lamp cold or hot determines that the current through the relay coil be practically in phase with the voltage across the breaker, enabling the relay to operate according to the direction of this voltage as it does according to the direction of the line current. The lines *RR* are similar to the lines *TT* in Fig. 34, and therefore may be drawn through the end of vector V_n as indicated in Fig. 39. The direction and magnitude of the voltage across the switch is represented by a vector drawn from V_n as its origin, to V_t as its terminus. Therefore, V_t must go beyond *RR* in order to reclose the breaker. This characteristic obviously protects against reclosing on cross or reverse phases since the voltages would lead or lag the network voltage 120 deg. or 180 deg.

Reclosing Where No Feeder Regulators Are Used

Where no feeder regulators are used, the difference in voltage that usually exists across the network breaker is that caused by the impedance drop from the substation to the network. The direction of this voltage depends on the impedance angle and the power-factor. At unity power-factor it leads the network voltage at an angle equal to the impedance angle. For power-factor less than unity, this impedance voltage lags its unity power-factor position by the same angle that the current lags the network voltage. Therefore, for lagging power-factors, it is always on the right-hand side of the lines *RR* of Fig. 39. This impedance voltage will always cause the incoming transformer to carry its share of the load.

It is necessary to design the relay with a reclosing characteristic so that it will be certain to close in on the impedance voltage which does not exceed the impedance voltage corresponding to full-load current. It is desirable to have the breaker reclose at an impedance voltage corresponding to a value considerably below full-load current. Otherwise it would not be possible to switch in a new transformer or circuit at times other than full load.

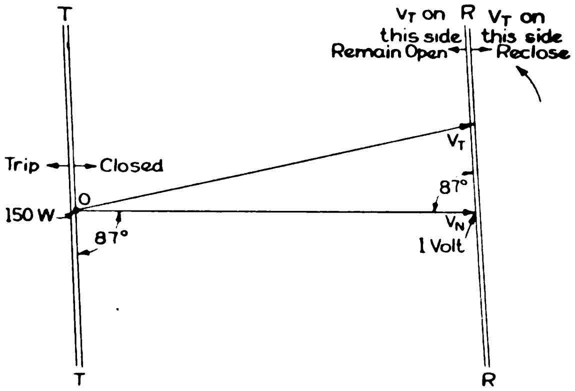


Fig. 39. Vector Diagram for Both Tripping and Reclosing Function

The total impedance voltage across the breaker is the resultant of the primary feeder, transformer, and secondary main impedance voltages. It is advisable to use only the transformer impedance voltage in the relay design because it is possible for one breaker to reclose ahead of another, leaving only the transformer impedance voltage for the remaining breaker in cases where two or more transformers are located at the same point or close together. In Fig. 40 is shown the impedance voltage of transformers radiating from the end of V_n. The long vector corresponds to full-load impedance voltage of a high-reactance (10 per cent) transformer, and the short vector a low-reactance transformer. The resistance component of each vector is the same. Two positions are indicted for each vector, corresponding to unity power-factor and 95 per cent power-factor.

If the relay is designed for conditions making the line *RR* perpendicular and adjusted to close on a component of one volt in phase with the network voltage, it is obvious that approximately full-load current at unity power-factor is required on the adjacent transformer, of either high or low reactance, to give this one volt. If the power-factor lags to 0.95, the conditions are considerably improved, the high-reactance transformer closing in on 20 per cent load and the low reactance on 37 per cent load. If the relay is adjusted for ½ volt instead of one volt, these values are reduced to ½ for either the unity or 0.95 power-factor position. Now suppose the line *RR* is advanced about three degrees to the position *R′ R′*. Then the unity power-factor condition is considerably improved, being reduced from 100 per cent load to 60 per cent for the high-reactance transformer, and to 80 per cent for the low-reactance transformer. The low power-factor condition is not changed very much. Notice that the high-reactance transformer is much better for this operation than the low-reactance. The line *RR* should be much further advanced if closing in on leading currents is required.

It is probable that an adjustment of about one volt will be satisfactory, because most network transformers have high reactance, and the power-factor is not likely to be higher than 0.95. It seems, therefore, that the relay should have a range of adjustment from ½ to two volts in phase with the network voltage. This may be accomplished by using a device similar to the usual light-load adjustment supplied with standard watthour meters. This adjusting device gives a torque that opposes the closing torque exerted by a spring.

It has been previously explained that the relay will operate on cable-charging current in the reverse direction. This charging current does not usually become appreciable until high voltage (such as 13,200 volts) is used for the primary feeder. Suppose a 13,200-volt feeder has only two transformers feeding the network and that the substation feeder breaker is opened. Then the two relays go to the tripping position. Breaker No. 2 of Fig. 41 trips satisfactorily but, for some reason, breaker No. 1 does not trip. The cable may have a charging current of one ampere per phase per mile and may be five miles long, giving a total charging current (I_c) of five amperes or $5 \times 13.2 \times 1.73 = 96.8$ kv-a., which is approximately ⅓ load on a bank of 300 kv-a. This charging current flowing through the transformer causes a voltage to appear across the open breaker No. 2. This voltage (V_n V_t) leads the charging current by an angle equal to the impedance angle of the transformer which is about

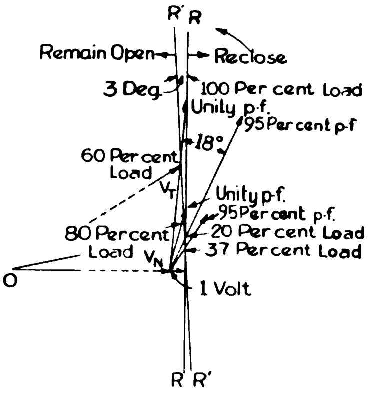


Fig. 40. Vector Diagram Showing Transformer Impedance Voltage Used for the Reclosing Function

84.3 deg. for a 10 per cent reactance transformer. Obviously, breaker No. 2 will reclose but will trip again because of the reverse current. It will periodically reclose and trip (called "pumping") as long as breaker No. 1 remains closed, or until something happens to breaker No. 2. If the closing circuit of the breaker does not require very much current, voltage fluctuations are not likely to be noticed by the consumers. If the relays are designed so that it takes a much longer time for reclosing than for tripping, it is not possible for a "seesaw" action to take place because of some breakers opening before the others.

Trouble from this pumping may be avoided in several ways. The substation operator could stop the pumping by simply reclosing the feeder circuit breaker, for a voltmeter would indicate back feed which means that one or more switches failed to open. Two or three operations of the substation breaker might clear the breaker which had stuck. If not, an inspection would be necessary to locate the faulty breaker. The exciting current of the other transformers on the feeder will greatly reduce the charging current flowing through transformer No. 1. Suppose the total transformer capacity on the feeder is 5000 kv-a.; then the exciting current may be approximately two per cent, or 100 kv-a., more than enough to overcome the charging current. If the transformer exciting current is lower than the charging current, it could be increased by connecting a reactor to the feeder at the substation.

The relay characteristics may be modified by changing the design of the relay itself or by adding a separate relay to prevent reclosing on voltages that lag the network voltage more than three to five degrees.

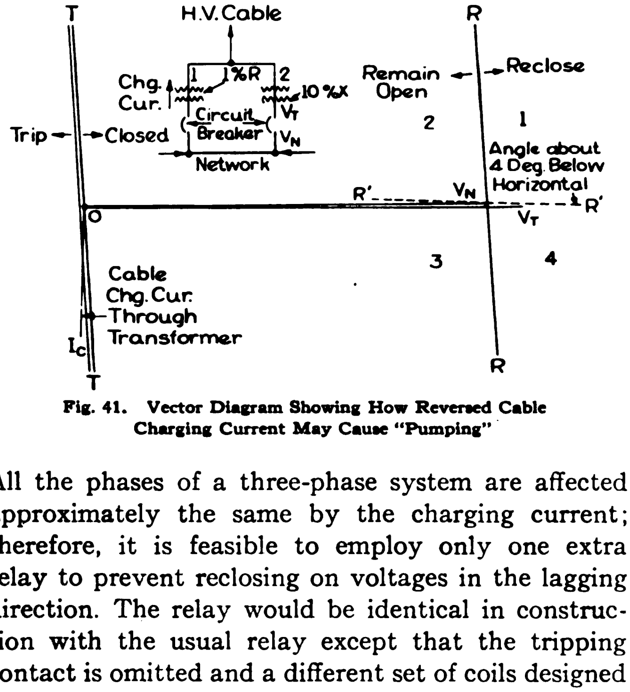


Fig. 41. Vector Diagram Showing How Reversed Cable Charging Current May Cause "Pumping"

All the phases of a three-phase system are affected approximately the same by the charging current; therefore, it is feasible to employ only one extra relay to prevent reclosing on voltages in the lagging direction. The relay would be identical in construction with the usual relay except that the tripping contact is omitted and a different set of coils designed so as to give the characteristic shown at *R'R'* in Fig. 41. The phasing coil is connected across the contacts of one pole of the breaker and closes its contacts on any voltage vector in sectors 1 and 2. The contact of this relay is connected in series with the closing circuit of the standard relay, which closes its closing contacts when voltages occur in the sectors 1 and 4. The closing circuit, therefore, is energized only when voltage vectors occur in sector 1. The connections of the two relays across one pole of the breaker and the resulting characteristics are shown in Fig. 42.

Case of Three Single-phase Regulators

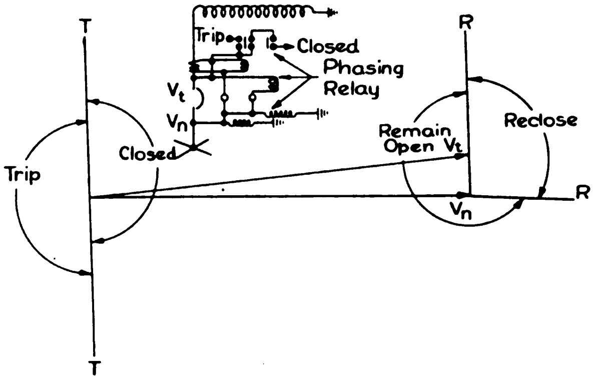


Fig. 42. Relay Characteristic That Will Prevent "Pumping"

When the feeder voltage is regulated by individual induction regulators, the voltage existing across the contacts of the breakers of the incoming feeder may not be that due to the impedance voltage but may be modified in direction and magnitude by the position of the regulators. The simplest case involving regulators is the three-phase four-wire feeder regulated by three Y-connected single-phase regulators and supplying Y-Y connected transformers. In this case the regulators always boost or buck the voltage in phase with the substation voltage. Therefore, with this combination, the incoming transformer voltage is always in phase with the substation voltage. The cable charging current has a negligible effect on the receiving voltage because of the low resistance and reactance of the cable. The substation voltage can lag the network voltage only when the load power-factor angle at the network is greater than the resulting impedance angle of the primary feeder, transformers, and network mains. Suppose this resulting impedance angle is 60 deg.; then the power-factor must be lower than 50 per cent in order that the substation voltage may lag the network voltage. If the impedance angle is 30 deg. the power-factor must be lower than 86.6 per cent. As long, therefore, as the combination of impedance angle and load power-factor is such as to keep the substation bus voltage in the leading position, as shown in Fig. 39, the regulators cannot set up a condition that will cause pumping. If the regulator bucks the voltage V_t (Fig. 39), the breaker cannot reclose. If they boost the voltage V_t, the voltage difference across the breaker cannot cause pumping, because the lag of the current set up cannot be over 90 deg. and will be on the closing side of the lines *TT*.

Use of Polyphase Regulators

If the polyphase regulators are used, it is possible for the regulators to cause a voltage difference across the open breaker of the incoming feeder that will lag the network voltage and cause pumping with the relay characteristic shown in Fig. 39. An inspection of Fig. 43 will show how this is possible. It is more convenient in this case to draw the substation voltage V_s as the reference vector. The polyphase regulator voltage is a constant value and changes the feeder voltage by moving from x to y along a circular path. The diagram indicates that one regulator is at the neutral position V_n and the other is appreciably advanced to the boost position V_t. The voltage across the open breaker is $V_n - V_t$ and sets up the circulating current I_c. This would cause pumping at no load or at light loads.

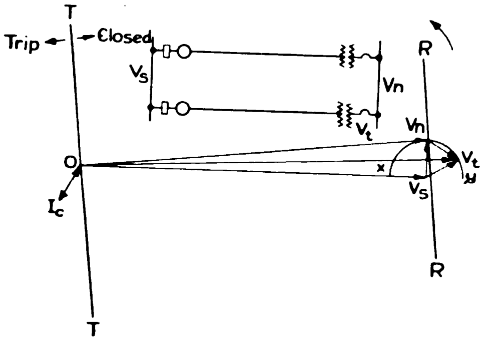


Fig. 43. Vector Diagram Showing How Polyphase Regulators May Cause "Pumping"

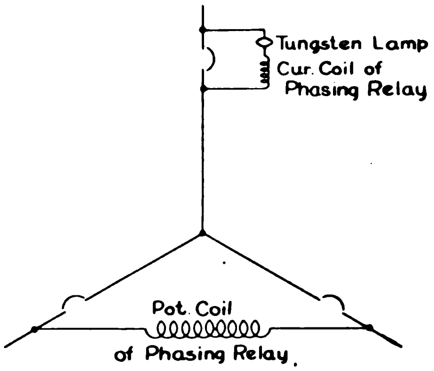


Fig. 44. Actual Connection of Potential Coil of Phasing Relay

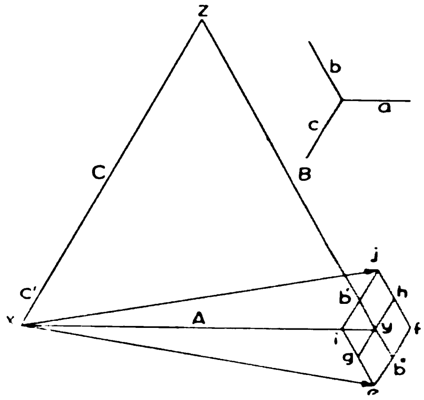


Fig. 45. Vector Diagram Showing How Two Single-phase Regulators, Open-delta Connected, May Cause "Pumping"

Hand control of the regulators would be likely to give trouble from pumping. The mechanical interconnection of the regulators would eliminate pumping caused by regulator displacement. The relay characteristic may be so modified that the switch cannot reclose on voltage differences that will set up a circulating current on the tripping side of the Line *TT* in Fig. 39. The relay characteristic shown in Fig. 42 is suitable. Only one phasing relay is necessary, because the regulator affects all phases alike. This relay potential coil is actually connected as shown in Fig. 44 to permit the closing characteristic to be designed as close as three degrees to the horizontal. The breaker cannot close in on a voltage difference that is lagging the network voltage more than three degrees. The current lagging this voltage will not exceed 84 deg. on modern systems. Therefore, no phase angle adjustment is necessary.

Use of Two Single-phase Regulators

When two single-phase regulators are used in open-delta connection, it is possible for one of the phase voltages to be displaced by the regulators to either the lag or lead position. A reference to Fig. 45 will explain how this takes place. The equilateral triangle *ABC* represents the bus voltage and also the line voltage when the regulators are in the neutral position. The normal operation of the regulators increases or decreases the voltage triangle symmetrically. No phase displacement occurs. The voltage range of the regulator in *C* phase is shown as c'—x—c'' and in *B* phase as b'—y—b''. Therefore, phases *B* and *C* cannot be displaced by the regulators. The only phase affected is *A* phase.

Suppose the line voltage is impressed on a transformer bank with the high voltage connected delta and the secondary connected *Y* with the neutral grounded; then the secondary voltage *a*, *b*, *c* correspond in phase with the primary voltages *A*, *B*, *C*. Phases *B* and *C* are not displaced by the regulators. Suppose regulator *C* is in the neutral position and regulator *B* is run to the full buck position at b''; then voltage *A* or *a* is shifted to the lagging position. Now when regulator *C* is operated from buck to boost (c' to c'') the end of vector *a* travels from *e* to *f*. When regulator *B* is in the neutral position and regulator *C* is varied from c' to c'', the voltage vector moves from *g* to *h*. The same thing is true with regulator *B* in the buck position where *a* varies from *i* to *j*. Obviously, the regulators can be made to cause the secondary voltage *a* to occur at any point in the diamond-shaped area *e*, *f*, *j*, *i*. The normal operation of the regulators bucks or boosts the secondary voltage in phase with it along the path *i*—*y*—*f*.

The characteristics indicated in Fig. 42 may also be used to prevent reclosing on transformer voltages that are in the lagging direction. This characteristic is necessary only on the one phase affected by the regulators. The other two phases can use the characteristic indicated in Fig. 39.

(*To be continued*)

Impulse Characteristics of Driven Grounds

Tests Made with Cathode-ray Oscillograph—Comparison of Impulse Resistance with 60-cycle Resistance—Single and Multiple Pipe Grounds—Effect When in Series with Arrester

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CONSIDERABLE information has been published in recent years concerning the characteristics of grounds. Studies by numerous investigators in widely separated geographical sections have disclosed the behavior of various forms of ground electrodes under greatly differing soil conditions. The industry has benefited greatly from this work through a general cognizance of the factors influencing the grounding problem.

The purpose of grounds in general is to hold various apparatus or parts of an electric circuit at or near earth potential. In fulfilling this purpose the ground electrode or multiple system of electrodes must present a low electrical resistance to earth at large, so that the possible current flow from the electrodes to earth will not produce any appreciable voltage drop. The great majority of electrical grounds are subject to current flow from only the electrical system itself, involving either direct currents or alternating currents of commercial frequencies, so that the efficacy of the ground needs to be determined only by a study of its electrical resistance with respect to those commercial direct-current or alternating-current sources. Grounds for modern lightning arresters however are subject to little or no system current flow, but pass impulse currents which are of a transient nature and which may amount to hundreds or perhaps thousands of amperes. It is therefore desirable to know how the resistance to relatively large impulse currents compares with the resistance as shown by the smaller and steady-state current flow involved in the ordinary measuring practice.

It is a well-known fact that many materials of non-homogeneous character do not obey Ohm's law but decrease in resistivity as the current density increases. With possibly rare exceptions, soil is non-homogeneous with conducting particles separated by the more non-conducting particles, and the physical and chemical structure of the particles varying widely. The current density in the soil around a lightning-arrester ground is indeterminate, and depends upon the magnitude of the lightning discharge current and the physical features of the electrodes. Nevertheless, it has been suspected that the enormously greater currents of lightning discharges together with the non-homogeneity of the conducting soil would account for some difference in the effective resistance from that indicated by the ordinary ground-measuring practice.

Some studies of the impulse characteristics of several driven-pipe grounds have been made by the Research Laboratory of the Lightning Arrester Engineering Department at Pittsfield, using the cathode-ray oscillograph⁽¹⁾ for recording the transient behavior. While the work is by no means complete or conclusive, the results secured are interesting and hint, at least, of the probable tendencies of the majority of arrester grounds under impulse conditions.

The electrodes employed were one-half inch standard (0.84 in. outside diameter) galvanized-iron pipes which were driven into loose, gravelly soil outside of the laboratory. The soil is partly of artificial fill of some twenty years ago and offered little resistance to driving. Frequent rains during the investigation kept the soil always damp just a few inches below the surface. Electrodes of various depths and multiples of them were studied. Connection was made from a 50-kv. impulse generator and the oscillograph to the ground electrodes by an 18-ft., No. 6, stranded, cambric-covered cable having an ohmic resistance of 0.01 ohm. As the basis of reference for both the steady-state alternating-current and impulse measurements, use was made of a ground consisting of a pipe and conduit system which combined had but a fraction of an ohm resistance to true ground.

The impulse generator was adjusted to deliver a current of several hundred amperes (depending somewhat on the ground resistance being studied) which rose in 20 to 30 microseconds and decreased rather slowly, being about two-thirds of the maximum value at the end of 100 microseconds. The voltage followed approximately the same wave shape and is shown by the oscillogram in Fig. 1. Voltage and current were recorded simultaneously, and the ground resistance at any point of the impulse wave can be calculated from the volt-ampere characteristic so produced.

Before studying the grounds, a few preliminary tests on water-tube resistances were made to check the adjustments. Figs. 2 and 3 (made by the cathode-ray oscillograph respectively without and with sweep motion) show the volt-ampere characteristic of a water tube as an absolutely straight line, denoting a constant resistance irrespective of current density. This tube measured 51 ohms by 60-cycle test; and the impulse current of 810 amp. produced a voltage

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(1) A description of a portable type of cathode-ray oscillograph was given in the article, "Cathode-ray Oscillographs and Their Uses," by E. S. Lee, GENERAL ELECTRIC REVIEW, August, 1928, p. 404.

drop of 42 kv., thus indicating 51.7 ohms, which is a good check.

The impulse was then passed through the 18-ft. lead with no water tube or ground resistance, to determine any inductive drop which the lead might introduce. Fig. 4 shows the volt-ampere test, and indicates negligible voltage drop across the lead. Fig. 5 shows the volt-ampere characteristic of a 38-ohm water tube in series with the 18-ft. lead. It will be noted that the lead negligibly modifies the straight-line volt-ampere characteristic of the water tube. This test proves that in the subsequent tests on the driven grounds no corrections need to be made for the effect of the lead.

Fig. 6 is the volt-ampere test on 11 driven-pipe grounds in parallel. The 60-cycle measurement showed the resistance for this group of pipes to be 24 ohms. On the impulse test, the start of the volt-ampere curve indicated 27 ohms; but at the maximum current of 880 amp. the voltage drop across the ground resistance is 15 kv. indicating a resistance of 17 ohms, which is 71 per cent of the 60-cycle measured value.

Fig. 7 is a similar test on a single 10-ft. driven-pipe ground. The a-c. 60-cycle measurement is 82 ohms; the initial volt-ampere slope indicates 78 ohms, and at the maximum current of 660 amp. the resistance is 50 ohms, or 61 per cent of the 60-cycle measured value.

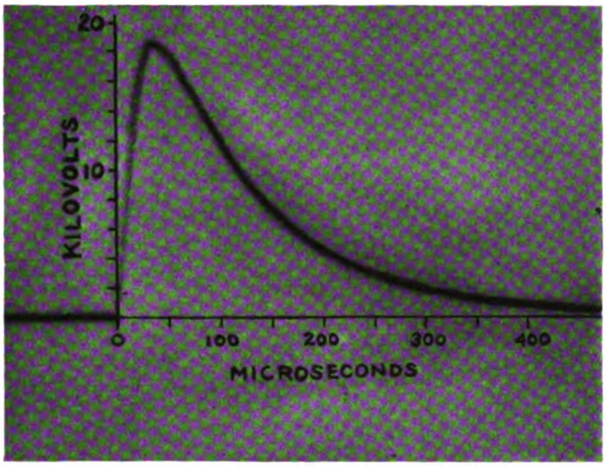

Fig. 1. Cathode-ray Oscillogram of the Typical Impulse Test Voltage Applied

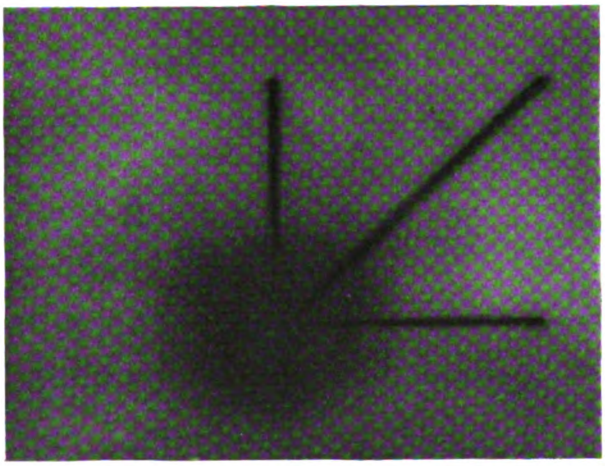

Fig. 2. Cathode-ray Oscillogram (Without Sweep) Indicating the Volt-ampere Characteristic (and Zero Lines) of a Water-tube Resistance

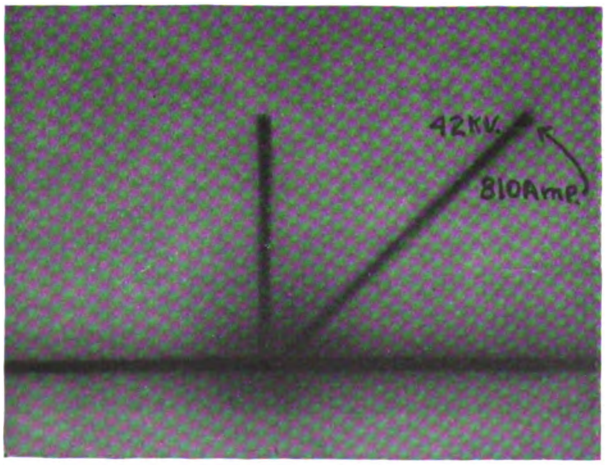

Fig. 3. Oscillogram (Employing Sweep) Made Under the Same Conditions as Those for Fig. 2

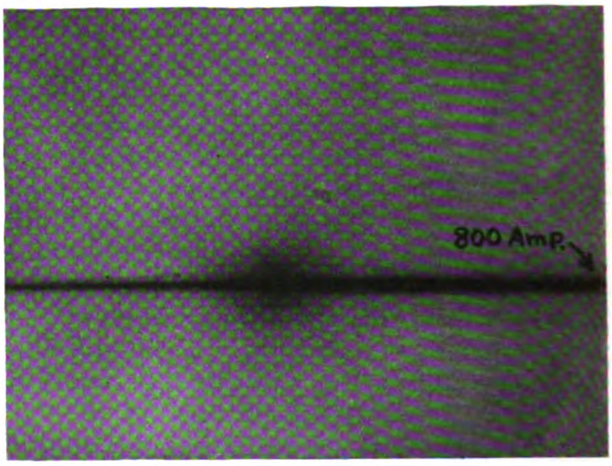

Fig. 4. Volt-ampere Characteristic of an 18-ft. Lead Grounded to Conduit (no Ground Resistance)

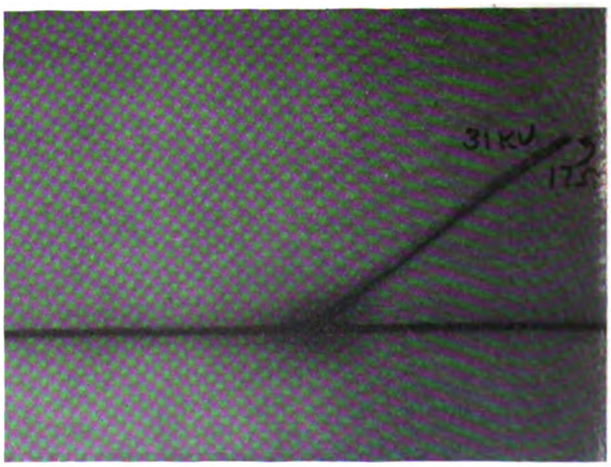

Fig. 5. Volt-ampere Characteristic of the **18-ft. Lead (Fig. 4) and a 38-ohm Water-tube in Series**

Fig. 8 was found for a pipe driven 3 ft. and the 60-cycle measured resistance was somewhat variable, averaging 110 ohms. At the maximum impulse current of 583 amp. the resistance is 55 ohms, which is 50 per cent of the 60-cycle resistance value.

Fig. 9 is for a 6-ft. pipe with a 60-cycle resistance of 165 ohms; and the impulse resistance is only 77 ohms at the impulse current of 494 amp. It will be noted that this 6-ft. electrode presented higher 60-cycle and impulse resistances than the 3-ft. electrode referred to in the preceding paragraph. This was due to appreciably poorer soil conditions at the location of the 6-ft. sample. Had the soil been uniform for both locations the 6-ft. electrode would naturally have presented appreciably lower resistances than the 3-ft. electrode.

Several studies were made of the behavior of still higher resistance grounds. Fig. 10 is representative, being for a 6-ft. pipe ground having a 60-cycle resistance of 330 ohms. The resistance values under impulse currents are designated for the initial upward slope, the maximum current, and the final downward slope of the volt-ampere characteristic. It will be seen that these values are 204 ohms, 113 ohms, and 79 ohms respectively. At the maximum current of 240 amp. the resistance was 113 ohms, or about 34 per cent of the measured 60-cycle resistance.

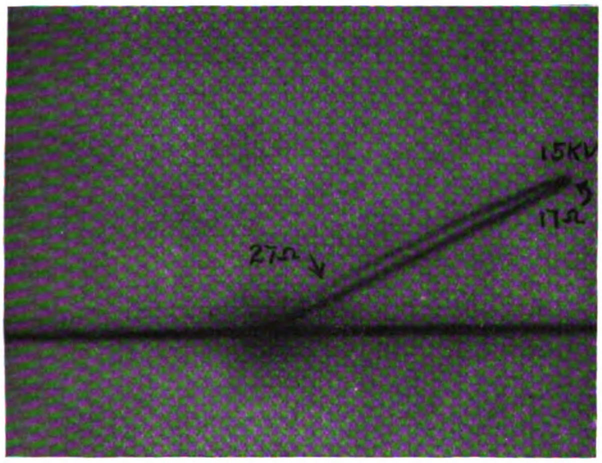

Fig. 6. Volt-ampere Characteristic of 11 Driven-pipe Grounds in Parallel. The resistances given are those at the beginning and at the peak of the impulse application

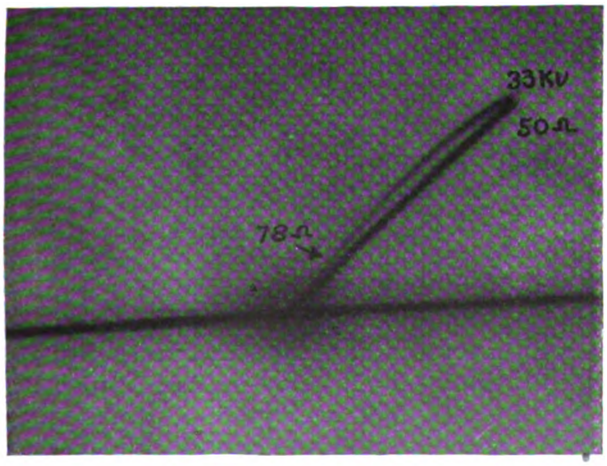

Fig. 7. Record Similar to That in Fig. 6 Except for Being Applied to One 10-ft. Pipe Ground

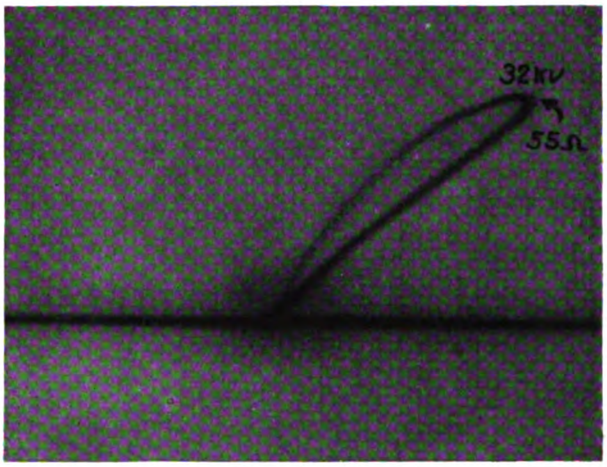

Fig. 8. Volt-ampere Characterstic of One 3-ft. Pipe Ground

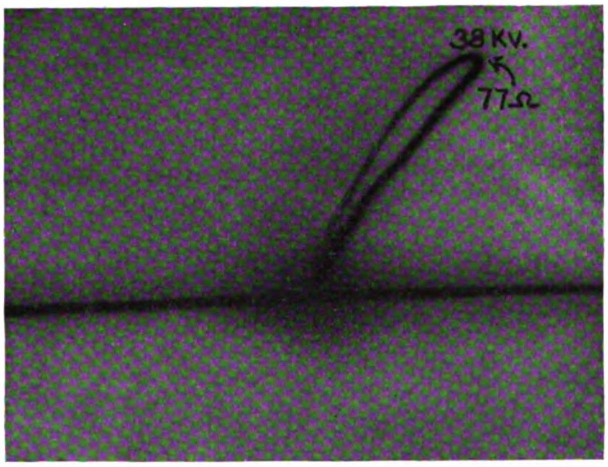

Fig. 9. Volt-ampere Characteristic of One 6-ft. Pipe Ground

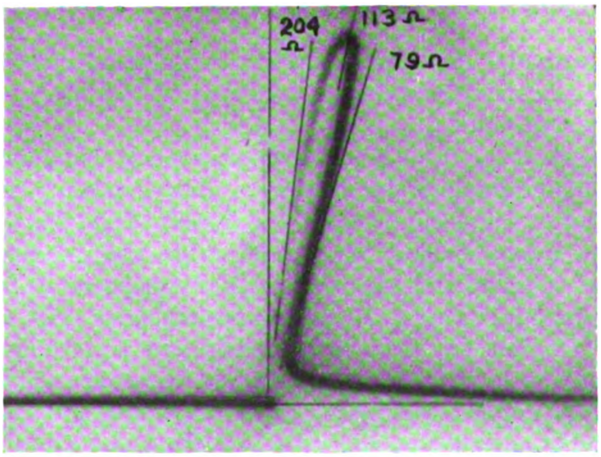

Fig. 10. Volt-ampere Characteristic of a 6-ft. Pipe That Formed a Ground of Higher Resistance Than That in Fig. 9

Some of the grounds were salted liberally to determine if this changed the impulse characteristic. Fig. 11 is for a 3-ft. pipe which before salting measured 160 ohms on 60-cycle current; and at maximum impulse current was 69 ohms, or 43 per cent of the 60-cycle value. After salting the 60-cycle measurement was 19 ohms; and the impulse resistances for the initial upward slope, maximum current, and final downward slope are 23 ohms, 15 ohms, and 10 ohms respectively. At a maximum impulse current of 660 amp. the resistance was 79 per cent of the 60-cycle measured value.

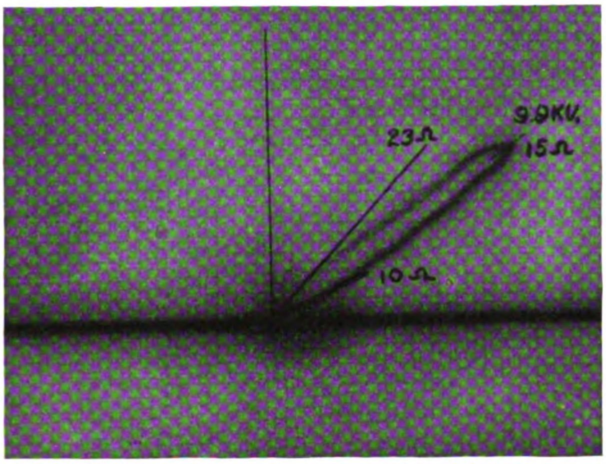

Fig. 11. Volt-ampere Characteristic of a 3-ft. Pipe Ground After Salting

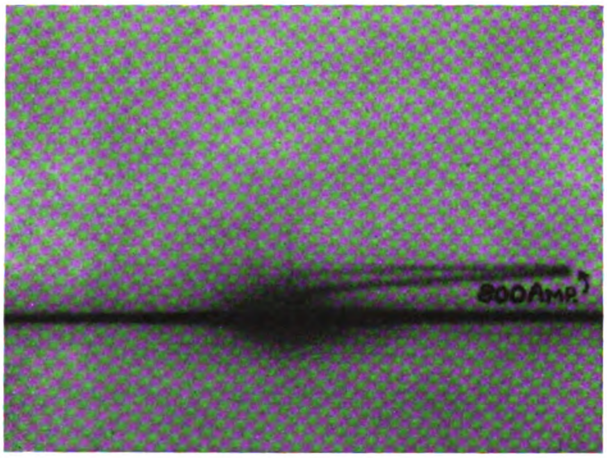

Fig. 12. Volt-ampere Characteristic of a 3-kv. Pellet-type Arrester

In all of the tests, 60-cycle measurements using the voltmeter-ammeter method were always taken just before or immediately after the impulse test. Sixty-cycle measurements before and after the impulse were substantially the same. These measurements were made using a 1-kv-a. hand regulator and a 220/440-volt transformer, enabling any voltage up to 440 to be applied to the ground resistance. The voltage applied depended on the resistance, and the range of current was generally between 1 and 4 amp.

The initial upward slope of the impulse volt-ampere curves in some cases indicates resistance values slightly above the 60-cycle values. This is probably due to error in the determination of the initial upward slope, which will be understood from a study of the oscillogram. The slope is indicated on the oscillograms largely to show more conspicuously how the volt-ampere curve falls off, indicating a continuously decreasing resistance.

It is interesting to note how the presence of a series ground resistance affects the protective ability of a lightning arrester. Fig. 12 shows the volt-ampere characteristic of a 3-kv. pellet-type lightning arrester. The characteristic is ideal in that the voltage across the arrester is practically constant regardless of discharge current through it. The instantaneous or crest voltage drop at 800 amp. is much less than twice the crest voltage of the arrester's 60-cycle rating. If the impulse discharge current is increased to 1000 or 1500 amp. the voltage drop through the arrester remains substantially the same. Fig. 13 is the impulse volt-ampere characteristic for a 20-ft. driven-pipe ground which had a resistance of 55 ohms by 60-cycle measurement; and at 758 amp. impulse current had a resistance of 41 ohms. If the arrester is connected in series with the ground resistance, as in the practical case, the protective ability of the installation is determined by the impulse volt-ampere characteristic of the series combination, which is shown in Fig. 14. In this the voltage drop of the arrester is added to that of the ground resistance, and the higher the ground resistance the greater will be the voltage across the combination for any value of discharge current. In Fig. 15 the record of Fig. 14 is superimposed on that of Fig. 12 to show more directly the influence of the ground resistance to reduce the protective performance of an arrester in practice. It becomes evident that the maximum value of ground resistance which can be tolerated in any installation, and still give protection to the adjacent apparatus insulation, depends on the value of the lightning discharge current and the voltage which the adjacent apparatus insulation will withstand for the

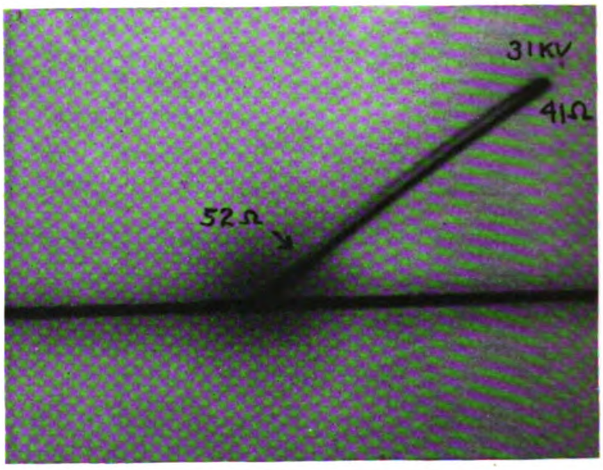

Fig. 13. Volt-ampere Characteristic of a 20-ft. Pipe Ground

duration of the particular discharge. Unfortunately, the magnitude and duration of the discharge current, and the impulse strength of apparatus in service, can rarely be predetermined. It is therefore obvious that protection of weaker insulation against the severest discharge demands low ground resistance, which keeps the overall volt-ampere characteristic little or no different from that of the arrester. It is also apparent that the lower the volt-ampere characteristic of the arrester itself, the greater the value of series ground resistance which can be safely employed in any case.

It is recognized that in confining these studies to one class of soil and one form of ground electrode, the data are not complete in their application to the widely varying conditions in practice. However, it seems probable that soil, except perhaps the most homogeneous variety, can be expected to present a fairly pronounced loop type of impulse volt-ampere characteristic, in which the resistance decreases with increase in applied voltage and resultant current. The initial slope of the impulse characteristics indicates a resistance approximately the same as the 60-cycle measured value. The resistance at maximum voltage and current of the oscillograms can be very accurately measured and was found from many tests to be from 34 to 80 per cent of the measured 60-cycle values.

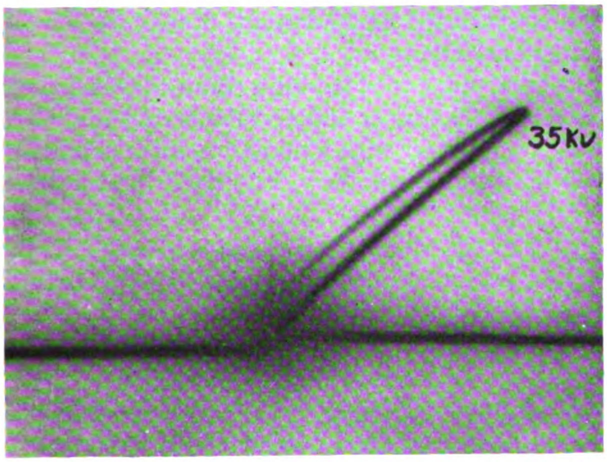

Fig. 14. Volt-ampere Characteristic of the 3-kv. Arrester (Fig. 12) and 20-ft. Pipe Ground (Fig. 13) in Series

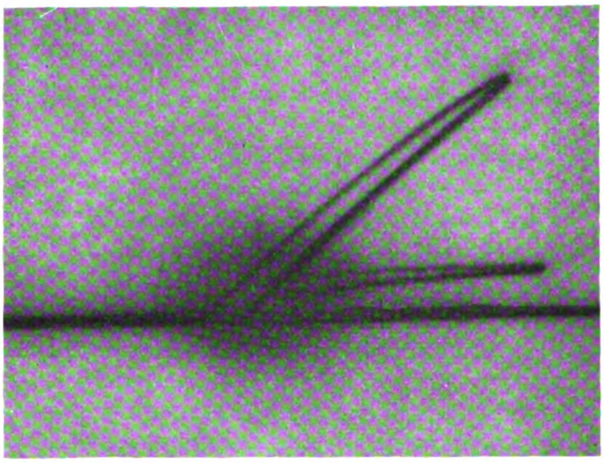

Fig. 15. Comparison of the Arrester and Ground (Fig. 14) with the Arrester Alone (Fig. 12)

In general, the greatest difference between 60-cycle and impulse resistances is found for the higher resistance grounds with which, of course, the voltage drop is highest. However, the difference is not always proportional to voltage, but depends also on the character of the ground and soil. For instance, in one test the resistance under impulse was 79 per cent of the 60-cycle resistance in which case the impulse voltage across the ground resistance was 9.9 kv. In another case, the resistance under impulse was 74 per cent of the 60-cycle resistance when the impulse voltage across the ground resistance was 31 kv. While in the latter case the voltage was three times that in the former, the ratio of impulse resistance to 60-cycle resistance is substantially the same. The size of the loop or the difference in resistance with increasing and decreasing current probably depends on the physical character of the soil, as well as on the magnitude and time element of the impulse. In some cases the resistance becomes fairly constant after the crest of the impulse current is reached, while in others the resistance decreases throughout the entire loop. The decrease in resistance shown by the impulse volt-ampere characteristics is probably due to high-resistance contacts between the more conducting particles of soil, at or near the surface of the electrode, which are bridged over by arcs or sparks when sufficient voltage is applied. These arcs continually decrease in resistance throughout the duration of the impulse current. In some of the tests the arcs were observed both visually and audibly.

Some tests were made using a quicker impulse in which the voltage and current rose to maximum in 5 microseconds. This faster wave caused a large inductive drop across the 18-ft. lead and oscillations on the wave front, both of which somewhat obscured the true volt-ampere characteristic of the ground resistance, especially at rising currents. However, in general, similar loop characteristics were found for the ground resistance and the reduction of resistance at maximum current and voltage was of the same order as shown for the slower wave.

In conclusion, it is probable that for practically all classes of soil and all forms of electrodes the loop volt-ampere characteristic will exist. However, the decrease is not sufficient to justify counting on this action to make high-resistance grounds effective for lightning protection. For lightning arresters of the pole type, 60-cycle ground-resistance values of 15 ohms or less are desirable. Where such low values cannot be obtained, a higher density of arresters per unit length of the line, to diminish the net resistance from line to earth, will improve protection. For station-type arresters, where the larger discharge currents may be expected, 5 ohms by 60-cycle measurement is probably more desirable.

Decrease of Luminosity of Self-luminous Materials Over a Five-year Period

By **W. S. ANDREWS**
General Electric Company

THE decreasing luminosity of materials containing radioactive matter was made the subject of a series of observations extending over a period of five years from January 23, 1923, to January 23, 1928, inclusive. Two objects were sought: (*a*) to compare the performance of radium content with that of mesothorium content, and (*b*) to show the relative performance of mixtures containing different amounts of these two elements.

Five samples were used, each containing one gram of zinc sulphide. Three contained respectively 100 micrograms, 50 micrograms and 15 micrograms of radium element; and two contained respectively 50 micrograms and 15 micrograms of mesothorium element.

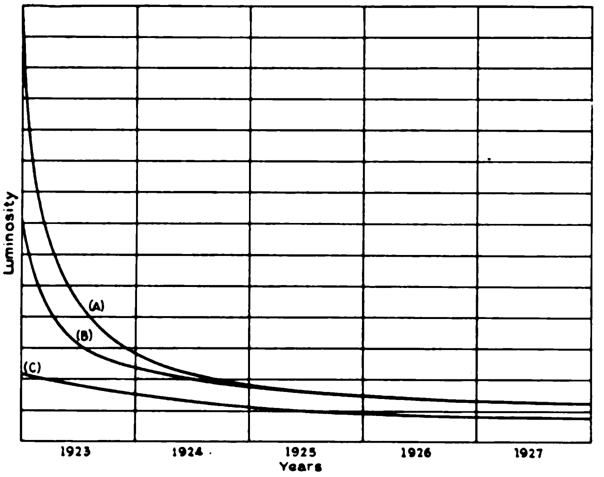

Fig. 1. Curves Showing Comparison of Decreasing Luminosity, with Time, of Samples Containing Respectively (*A*) 100 Micrograms, (*B*) 50 Micrograms, and (*C*) 15 Micrograms of Radium Element Mixed with Zinc Sulphide

These self-luminous materials were in the form of dry powder in flat glass tubes, measuring approximately 2¼ in. long, ¼ in. wide and 3/32 in. thick, the open ends being sealed with wax.

The luminous value of these materials is so low that no effort was made to measure them in terms of candle power and a photometer of the ordinary kind was not employed.

The method used was to produce alongside the sample an equal degree of luminosity from a frosted miniature lamp, measuring at each observation the distance of the lamp from the plane of the sample, this distance increasing of course as the sample grew dim. Following the law of inverse squares, the reciprocal of the square of the distance was used as the measurement of relative luminosity. These values, plotted from successive observations, give drooping curves that bring out very clearly the comparisons which were the object of the test.

The instrument used is what has been called a comparator and which was designed for this purpose and used in a previous similar five-year test, described in an article in the GENERAL ELECTRIC REVIEW of October, 1916, entitled, "A Luminescense Comparator." With certain features added to increase the convenience of operation, it may be again described as follows:

A telescoping tube having a plano-convex lens at the fixed end. The inner sliding tube carries an "all-nite-lite" transformer with a miniature incandescent lamp in a horizontal position which may be moved towards or away from the lens. Covering the inner surface of the lens is a color screen made of several plates of carefully selected colored glass, which give to the lamp-light emitted from the lens a color that matches the color of the luminous samples.

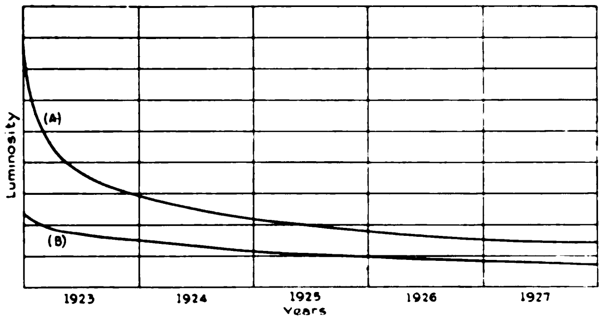

Fig. 2. Comparison of Decreasing Luminosity, with Time, of Samples Containing Respectively (*A*) 50 Micrograms, and (*B*) 15 Micrograms of Mesothorium Element Mixed with Zinc Sulphide

The outer surface of the lens is masked except for a narrow slot of the same width and length as the sample to be measured. The sample, held by clips, is placed beside this slot so that the light from the lamp and the light from the sample present two parallel and contiguous bands. By moving the lamp back and forth, a position is found where the two bands of light appear to be of identical value.

The sliding tube holding the lamp is supported by a carriage that is moved by a thin metal band passing over a sprocket wheel at the front of the instrument and an idler pulley at the rear; holes in the band fit the sprocket teeth. A handwheel on the spindle of the sprocket wheel affords convenient control of the moving system.

The metal tape is marked with millimeter graduations and a fixed pointer allows readings of the distance of the lamp from the outer face of the lens. A shelf at the front of the instrument and beneath the lens accommodates the paper on which one records the readings; and the graduated tape is seen through an opening in the shelf. Light is thrown upon the tape at this point by a carefully shaded miniature lamp controlled by a push button.

The process of testing is perhaps obvious from the foregoing description. The sample is slipped under the clips that hold it beside the band of lamplight and motion of the handwheel serves to adjust the position of the lamp until the two light bands match. The reading lamp is then flashed on and observation is made of the scale marking, after which the light from the lens is dimmed or brightened by turning the handwheel and the process is repeated until a number of readings have been made from which to secure an average.

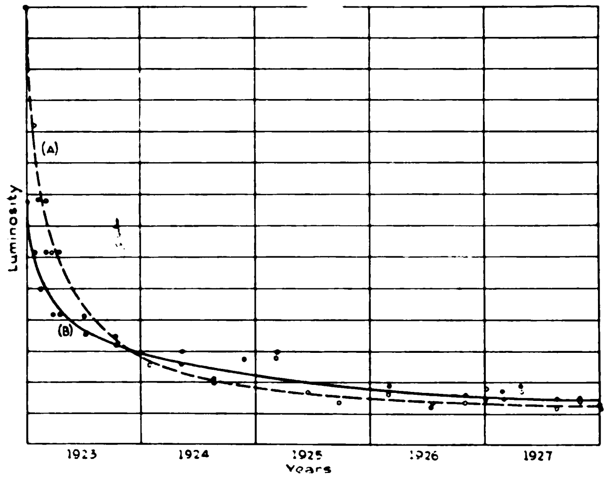

Fig. 3. Comparison of the 100-microgram Radium Element Sample (*A*) with the Sample Containing 50 Micrograms of Mesothorium Element (*B*). Note that after the first year the mesothorium sample makes the better showing

The light values are very low and a few millimeters one way or another make but little difference to the eye. It requires very deliberate observation to convince one's self that a balance has been struck. One may obtain a succession of identical readings and find the next is surprisingly higher or lower. Two observers making independent tests would often check accurately with each other in some readings and disagree in others, and in such cases the average of their respective averages was the figure used.

All observations were made in the same room and in the dark. The samples were kept from the light between tests, being individually wrapped in colored paper and kept in a box by themselves.

For the first six months, observations were made every two or three weeks, and after that at intervals of about 100 days with occasionally shorter intervals. The tests were conducted at the writer's residence using the city lighting supply which during the winter months was sometimes below normal voltage. While this and the great difficulty of accurately comparing lights of such very low intensity make some of the readings appear erratic, it is believed that the general trend of luminosity is shown by the curves with sufficient accuracy to permit a comparison of the different samples as regards the rate and extent of their decline from initial brightness.

From the curves showing the recorded data the following conclusions may be drawn:

(1). The mixtures which have the highest energizing element suffer rapid decline during the first year.
(2). After five years there is but little practical difference among the samples in the matter of light emitted.
(3). After the second year the curves of the 50-microgram and 100-microgram radium samples are practically identical.
(4). Samples containing mesothorium decline more slowly than those containing corresponding weights of radium element.
(5). The 50-microgram mesothorium sample, after the first year, is brighter than any of the others, including the 100-microgram radium sample.

It is believed that these tests may serve as a guide in the selection of materials for self-luminous paint which has acquired such an important place in connection with illuminated dials and markers for switches, socket pendents and other applications.

The writer is glad to acknowledge the assistance of Mr. J. H. Jenkins in the conducting of these tests, for conveniences added to the comparator and for aid in the preparation of this report.

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375 Miles of Michigan Highways Lighted

Michigan's trunk line highways, which in a decade or so have advanced from gravel to pavement, are taking another step forward. They are becoming lighted paths, as safe for travel at midnight as at noon. So rapid has been progress in this direction that within the next five or ten years, it may be expected that the state will be criss-crossed with illuminated highways. With recent additions, the lighted rural highways now stretch a total of more than 375 miles in that state, and the mileage is growing with increasing speed.

A Study of Crystal Structure and Its Applications

PART XII

CRYSTAL ORIENTATION AND THE STATIONARY-OSCILLATING FILM METHOD FOR ITS DETERMINATION

By DR. THOMAS A. WILSON
Research Laboratory, General Electric Company

Lattice Orientation and Its Significance to Industry

X-RAY analysis of crystalline materials is employed in industry at the present time to furnish information regarding their lattice structure and the lattice orientation. Many measurements of the former quantity, for a wide range of substances, have been made by a large number of investigators. Any pure substance will show only one regular arrangement of its constituent atoms or molecules within any particular crystal. No substance shows more than a very few fundamental lattices. Hence the lattice structure of a substance is characteristic of that substance.

The lattice orientation of any particular specimen on the other hand is only to a very slight extent characteristic of the substance of which the specimen is composed. For instance, a cube of iron appears cubic no matter from what position it is regarded; *i.e.*, whether a cube face, a cube edge, or a cube corner is directed toward the observer. But these three positions of the iron cube represent three different orientations of the cube with respect to the observer. The lattice arrangement of the atoms forming this cube of iron is the same in this cube irrespective of its orientation, since it is the one which is characteristic of iron at room temperature. But the orientation of the cube relative to its surroundings can be changed at will. An infinite number of such positions are possible.

Let us imagine a successive subdivision of the massive iron cube into smaller and smaller cubes, even into cubes far below the range of microscopic visibility. Eventually a cube will be obtained containing the equivalent of only two atoms of iron. The length of the cube edge will be 2.86 Ångström units. (140) This is the body-centered unit cube of which α iron is composed (141) but as shown in Fig. 78, several strikingly different orientations of this unit cube within a sample of iron are possible. It is not so easy to control the orientation of this unit cube within the specimen as it was to control the orientation of the massive iron cube. Many processes extensively employed in metallurgical work, however, such as rolling, wire-drawing, and bending (142) do produce changes in its relative position in the specimen. Even a strip of iron cut parallel to the edge of a sheet of iron, as shown in Fig. 79A, will show a different orientation of the elementary cubes than a strip cut diagonally across the sheet as shown in Fig. 79B, if the sheet originally showed a preferred orientation of these cubes.

The industrial world is interested in the physical properties of the materials which it employs, such as their tensile strength, ductility, and rigidity. These properties are particularly influenced by differences in orientation. (143) It follows then that in any specimen which shows preferred orientation, these properties will not be the same in all directions throughout the specimen. Neither will they be the same in the same direction throughout different specimens of the same material, unless these different specimens show the same preferred orientation. They will also be governed largely by the previous history of the material. Further investigation may show that many other properties are also definitely controlled by the lattice orientation. Hence the investigation of the influence of various manufacturing processes on the lattice orientation of materials and the relationship existing between the orientation and the properties of these materials is a task which industry can not afford to overlook.

The Importance of Single Crystal Specimens

Most of the metal specimens, at least, encountered in industry are composed of fairly small crystals which probably will show, because of their previous

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(140) Hull, *Phys. Rev. 10*, 661 (1917), *14*, 540 (1919). McKeehan, *ibid. 21*, 402 (1923). Davey, *ibid. 23*, 292 (1924).

(141) Westgren, *Jour. Iron & Steel Inst. 103*, 303 (1921). Westgren & Phragmen, ibid. *105*, 241 (1922).

(142) Taylor & Elam, *Proc. Roy. Soc. 112-A*, 337 (1926). Nishikawa and Asahara, *Phys. Rev. 15*, 38 (1920). Mark, Polanyi and Schmid, *Z. f. Physik. 12*, 58 (1922). Hoyt, Tech. Paper No 1624 E. *Am. Inst. Min. Met. Eng.* (1926). Tanaka, *Memoirs of the College of Science, Kyoto Imperial University, 8A*, 319 (1925), *9A*, 197 (1925). Tammann, *Z. f. tech., Physik. 7*, 531 (1926).

(143) Mathewson & Phillips, Tech. Paper No. 1623 E. *Am. Inst. Min Eng.* (1926). Mathewson, Trewin, and Finkeldey, *Trans. Am. Inst. Min. Met. Eng. 64*, 305 (1920).

References made to material hitherto published in this series of articles may be readily located from the following table:

Part	*Issue*	*Figs.*	*Equations*	*Tables*	*Footnotes*
Part I	Nov., 1924 p. 742	1 and 2	1 to 16	I and II	1 to 21
Part II	Dec., 1924 p. 795	3 to 10	17 to 32	III	22 to 27
Part III	Feb., 1925 p. 129	11 to 22	33 to 37	IV	28 to 42
Part IV	Apr., 1925 p. 258	23 to 28	38 to 41	None	43 to 48
Part V	May, 1925 p. 342	29 to 35	None	None	49
Part VI	Aug., 1925 p. 286	36 to 51	42	V to VII	50 to 58
Part VII	Oct., 1925 p. 721	52 to 59d	43 to 47	VIII and IX	59 to 65
Part VIII	Feb., 1926 p. 118	60 to 67	48	X to XIII	66 to 82
Part IX	Apr., 1926 p. 274	68 and 69	49 to 56	XIV to XXVI	83 to 117
Part X	Aug., 1926 p. 580	70 and 71	57 to 66	XXVII to XXXI	118 to 131
Part XI	Sept., 1928 p. 496	72 to 77	67 to 84	XXXII to XXXIII	132 to 139

treatment, a preferred orientation. Such specimens are not so desirable for experimental study as those in which only one orientation is present. Furthermore, the relationship between properties and preferred orientation in polycrystalline materials may be complicated because of the different sizes of crystals present and the conditions obtaining at the crystal boundaries.(144) Single-crystal specimens eliminate these two variables and also assure the presence of only one lattice orientation. Many methods have been developed for producing metal specimens composed of only one crystal,(145) or of producing specimens composed of such large crystals that the individuals may be isolated and investigated separately. Therefore, the determination of the lattice orientation of single-crystal specimens is of particular value at the present time.

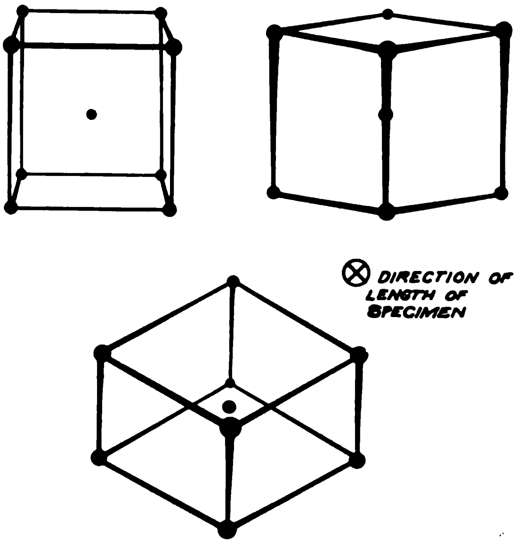

Fig. 78. Some Different Orientations of the Unit Cube of Iron

Methods of Determining Lattice Orientation in Single Crystal Specimens

Except for a few instances in which optical methods can be utilized for the determination of lattice orientation, x-ray analysis must be employed. Two general methods of x-ray analysis have been found suitable for the purpose: the Laue method(146) and the oscillating crystal method.

The Laue method must be employed if the specimen under investigation can not be rotated or oscillated. With monochromatic x-radiation penetrating a stationary single-crystal specimen, or one possessing a strongly preferred orientation, the chances will be small that any considerable number of planes will be properly placed to produce reflection. With white radiation, reflections from more planes are produced. Uspenski and Konobejewski(147) have employed a modified form of the Laue method in which a series of photographs are taken of the specimen set at a number of slightly different angles with respect to the incident x-ray beam. From the change in the shapes of the different zone ellipses produced in the photographs of these settings, it is possible to calculate the orientation of the elementary prisms in the metal strip. This method has also been chosen by Fujiwara(148) for his investigation of rolled tungsten strip. In order to increase the number of plane reflections produced by the specimen, Seeman(149) has employed either a divergent incident x-ray beam or an oscillation of the specimen. In Seeman's method, the photographic plate is caused to oscillate in unison with the specimen. Very complex patterns, called by Seeman "complete diagrams," are thus obtained. These diagrams were first obtained by de Broglie.(150) The origin of these diagrams and the manner of their use in determining crystal orientation have been described by Polanyi and Weissenberg.(151)

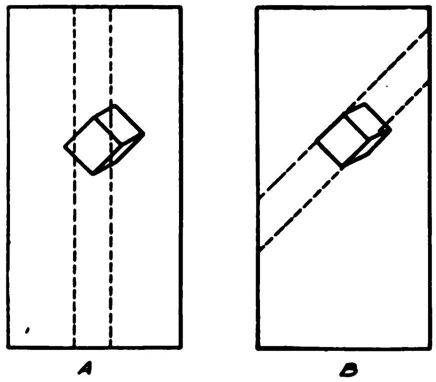

Fig. 79. Producton of Different Orientations of the Unit Cube by Varying the Direction of Cutting a Strip Specimen

In the oscillating crystal method, a directed beam of monochromatic radiation is caused to graze the surface of the crystal at various angles determined by the oscillation of the crystal. In the arrangement of Schiebold(152) the film remains stationary, while the specimen is caused to oscillate. The angle of oscillation may be varied in order to obtain the necessary number of plane reflections for a determination of the lattice orientation. The photographs are best solved by graphical means, provided the crystal can be set by some optical method so that its axis of oscillation coincides with some simple zone axis. To avoid the necessity of making this previous setting, a complicated x-ray goniometer for monochromatic radiation work has been devised by Weissenberg.(153)

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(144) Sykes, *Trans. Am. Inst. Min. Met. Eng. 64*, 780 (1921). Jeffries' *Chem. Met. Eng. 25*, 775 (1921). Carpenter and Tamura, *Proc. Roy. Soc. 113A*, 28 (1926).

(145) Czochralski, *Z. phys. Chem. 92*, 219 (1917). von Gomperz, *Z.f. Physik, 8*, 184 (1922). Carpenter & Elam, *Proc. Roy. Soc. 100A*, 329 (1921). Edwards and Pfeil, *Jour. Iron & Steel Inst. 109*, 129 (1924). Bridgman, *Proc. Am. Acad. Sci. 60*, No. 6, 7, 8. Davey, *Phys. Rev. 25*, 248 (1925). Gerlach, *Z.f. Physik, 38*, 828 (1926). van Liempt, Tech. Pub. 15, *Am. Inst. Min. & Met. Eng.* (1927).

(146) Davey, General Electric Review, *28*, 129 (1925).

(147) Uspenski and Konobejewski, *Z.f. Physik, 16*, 215 (1923).

(148) Fujiwara, *Publication of Tokyo Elec. Co. 1*, 83, (1925).

(149) Seeman, *Ann. d. Physik, 49*, 470 (1916), *Physik Z. 20*, 51 (1919).

(150) M. de Broglie, *Compt. rend. 156*, 1011 (1913).

(151) Polanyi, *Naturwissenschaften, 9*, 337 (1921). Polanyi and Weissenberg, *Z.f. Physik, 9*, 123 (1922).

(152) Schiebold, *Z.f. Physik, 9*, 180 (1922), *28*, 355 (1924).

(153) Weissenberg, *Z.f. Phsyik, 23*, 229 (1924).

The Laue method permits the determination of the internal orientation of the specimen—which may be different from the orientation prevailing at the surface. No mechanism need be provided for oscillating the specimen. On the other hand, the complicated patterns obtained from the Laue method are the more difficult of solution if no clue as to the possible orientations of the lattice is available. Particularly is this true of the complete diagrams. Bernal (154) has recently collected the methods to be employed for solving these complete diagrams for the indices of the planes producing the observed reflections. Another disadvantage of the Laue method is common to any method which depends on penetration of the specimen by a beam of x-rays; the allowable thickness of the specimen is limited by the absorption of the material for the x-rays employed. (155) A specimen of any appreciable thickness demands an inconveniently long time of exposure.

This objection does not apply to the oscillating crystal method. Owing to the fact that the x-ray beam is caused to graze the surface of the specimen, the effect of absorption is diminished and the necessary time of exposure is shortened. The thickness of the specimen under investigation is of no moment. The orientation of crystals in thick plates or bars may be thus determined. But by this method, only the orientation of the surface layers of atoms is determined. If the specimen is etched so that purely surface effects are eliminated, this objection is invalid. Sometimes indeed it is an actual advantage to be able to analyze layers of atoms at different distances beneath the surface. (156) The oscillating crystal method permits such an analysis to be made, since successive etchings and exposures can be made on the same specimen.

Practically all of the materials in common commercial use have had their atomic arrangement determined. The use of such data should render possible a simpler method of ascertaining the lattice orientation of a particular single-crystal specimen. With this end in view, Schiebold and Sachs (157) have devised a simple stereographic method of determining the crystal orientation of a face-centered cubic single-crystal specimen from a Laue photograph of the stationary crystal. Muller (158) making use of the intensity of x-ray reflections from different planes of a material has devised an easy way of ascertaining the crystal orientation of a wire or bar-shaped specimen composed of that material by means of an adaptation of the oscillating crystal method. But in this method, the constant attention of the operator is required during the time in which the series of exposures is being made.

The Stationary-oscillating Film Method for Determining Crystal Orientation

In this method which has been previously announced by Davey (159) the monochromatic x-radiation and the oscillating crystal are employed. Two photographic impressions of the resulting glancing beam spectrogram are obtained simultaneously, on two films curved to form arcs of concentric cylinders about the axis of rotation of the specimen as the axis of the cylinders. The radii of the two films are as nearly identical as the construction of the apparatus will permit. One of the films remains stationary during the exposure; the other is caused to oscillate together with the crystal. The record of the stationary film is utilized for determining the planes producing x-ray reflection, and for determining the horizontal angle formed by these planes; the oscillating film supplies information as to the amount of rotation experienced by the specimen before a particular reflection is produced. No goniometric setting of the specimen is necessary. The exposure time is short, the patterns obtained are simple and easily solved by graphical means. Since the x-ray beam grazes the surface of the specimen, the crystal may be of any thickness.

Fig. 80 represents a sectional diagram of the experimental apparatus; *a* and *b* represent two films curved concentrically about a center of curvature *d*, perpendicular to the plane of the paper, and in the surface of the crystal under examination; while *c* represents the usual ZrO_2 filter employed with the K radiation from the Mo target of the tube to prevent radiation of other wavelengths from reaching the films *a* and *b*. The film *a* and the crystal are caused to oscillate simultaneously and slowly through the same angle ϕ. Before the specimen has experienced any rotation, the situation is that shown in Fig. 80A. The beam grazes the surface of the specimen, and makes an impression on the two films at the points *e* and *f*. If there are no planes inclined at the proper angle to produce reflection, as required by the Bragg reflection equation, no further impression will be produced on the films. Now let the specimen and the film *a* be rotated through a certain angle ϕ (Fig. 80B), in which location some atomic plane in the specimen is brought into the proper position for x-ray diffraction. Again impressions will be made simultaneously on the two films at points *g* and *h*. The zero beam impression on the rotating film is now found at *f*. Let the angle subtended by the arc *g e* at the axis of rotation *d* be ϕ_1, and the angle subtended by the arc *hf* be ϕ_2. The amount of rotation experienced by the specimen before the reflection from this atomic plane was produced is accordingly:

$$\phi = \phi_1 - \phi_2 \qquad (85)$$

Fig. 81 represents the situation when reflection is recorded on the stationary film. The line *ge* representing the intersection of the plane of Fig. 80 with the stationary film is perpendicular to the axis of rotation of the specimen at the midpoint of the zero beam impression. Let k be the spot produced by the diffracted beam and kg its perpendicular distance from ge. Let a plane be erected tangent to the cylindrical-shaped film at the point e. The plane cutting the cylinder at ge will intersect this tangent plane along $g'e$. The spot k will appear at k', at a distance $k'g'$ from this intersection, and the angle $k'eg'$ will be the azimuthal angle α'. The angle $k'de$, or kde, is the angle between the primary and the reflected beams, which is usually designated as $2\,\theta$: twice the angle of incidence of the x-ray beam. The angle $k'dg'$, or kdg, will be designated as χ. This angle must have the same value on both the rotating and the stationary films, although the length of the line $k'g'$ will be less than kg owing to the shorter radius of curvature of the film a as compared with b (Fig. 80). It can be easily shown that the angles $2\,\theta$, ϕ_1, and χ are related by the equation

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(154) Bernal, *Proc. Roy. Soc. 113A*, 117 (1926).

(155) A. H. Compton, "X-rays and Electrons" (D. Van Nostrand Co. 1926), Chap. 6.

(156) Bozorth, *Jour. Amer. Chem. Soc. 49*, 969 (1927).

(157) Schiebold and Sachs, *Z.f. Krist. 63*, 34 (1926).

(158) Muller, *Proc. Roy. Soc. 106A*, 500 (1924).

(159) Davey, *Phys. Rev. 23*, 746 (1924).

$$\cos 2\,\theta = \cos \chi \cos \phi_1 \tag{86}$$

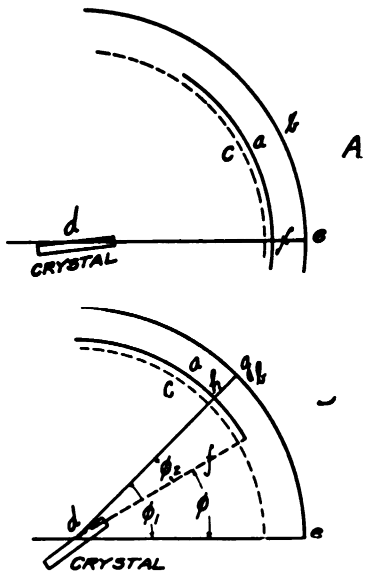

Fig. 80. Principle of the Stationary-oscillating Film Method

For purposes of solving the films obtained by this method by means of the stereographic projection the angle α must be determined. From consideration of Fig. 81, it is seen that

$$\tan \alpha' = \frac{k'g'}{g'e} \tag{87}$$

$$\tan \phi_1 = \frac{g'e}{de} \tag{88}$$

$$\text{and } \tan \chi = \frac{k'g'}{dg'} \tag{89}$$

But

$$dg' = \frac{de}{\cos \phi_1} \tag{90}$$

Substituting this value in Equation (89)

$$\tan \chi = \frac{k'g' \cos \phi_1}{de} \tag{91}$$

Rearranging terms and making the substitution of $g'e$ as given by Equation (88)

$$\tan \alpha' = \frac{\tan \chi}{\cos \phi_1 \tan \phi_1} = \frac{\tan \chi}{\sin \phi_1} \tag{92}$$

It is seen that this is also the value of α.

The determination of the orientation of a single crystal by means of the stationary-oscillating film method requires a knowledge of the angles $2\,\theta$, ϕ_1, ϕ_2, and α. From inspection of Fig. 81, it is seen that

$$\frac{ge}{de} = \phi_1 \qquad \frac{kg}{dg} = \frac{kg}{de} = \tan \chi \tag{93}$$

while for the rotating film,

$$\frac{g_2e_2}{d_2e_2} = \phi_2, \ \frac{k_{-}g_2}{d_2e_2} = \tan \chi \tag{94}$$

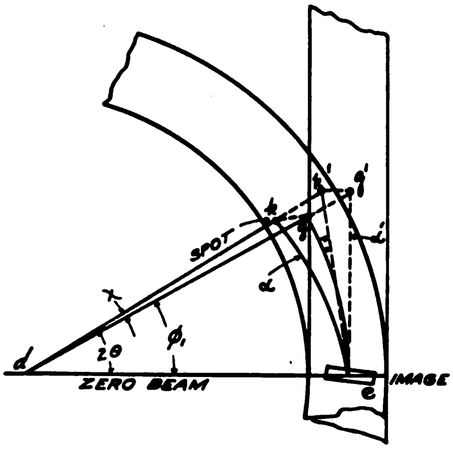

Fig. 81. Determining the Position of a Spot on the Stationary Film

But the other angles involved are difficult of direct measurement and calculation is tedious. It is better to read these angles from the films directly by means of charts. On the apparatus actually used, the stationary film was curved about a radius of 81 mm., the radius of the rotating film being 68 mm. Of the necessary angles, $2\,\theta$, ϕ_1, and α are obtained from the stationary film, ϕ_2 from the rotating film. Using the proper value of the radius, it is possible to calculate the value of ϕ_1, ϕ_2, and χ from Equations (93) and (94), and of α and $2\,\theta$ from Equations (92) and (86) respectively. Figs. 82, 83, and 84 show plots of the various combinations of angles needed in reading the films for every 5 deg.; Fig. 82 showing ϕ_1, α and χ; Fig. 83 showing ϕ_2, and χ, and Fig. 84 showing $2\,\theta$ and χ. It is very convenient when reading films to use tracings of these charts on transparent paper, over which the film can be laid, and under which a source of illumination can be placed.

While the values of χ are not used in solving stationary-oscillating photographs, it is nevertheless desirable to list these values for the different spots as read off from the films. Some of the spots may be of such characteristic appearance on the two films that there can be no doubt concerning the spots produced by the same plane reflection. Yet the shapes of the spots produced by the same reflection may be quite different on the two films. In cases like this, in which doubt may exist as to the manner in which the spots should be paired, the χ values of the spots may be conveniently used. If the spots on the two films have the same value of χ, they may have been produced by reflection from the same plane, otherwise they certainly were not. In the latter case, the recorded pairing of the spots must be changed until a match is secured.

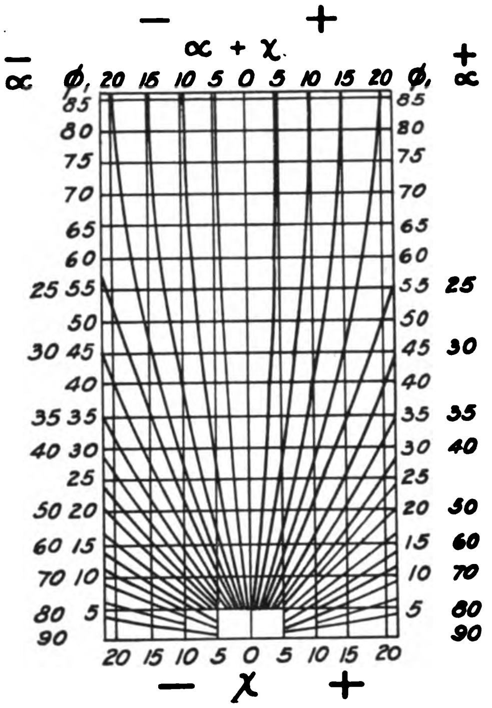

Fig. 82. Plot of the Angles ϕ_1, α, χ

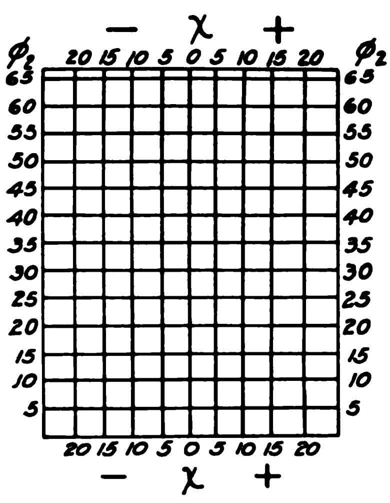

Fig. 83. Plot of the Angles ϕ_2, χ

Method of Reading the Films

The films obtained from an exposure of a single-crystal sample of silicon steel are reproduced in Fig. 85. Fig. 85A shows the stationary film pattern, while the pattern of the oscillating film is reproduced in Fig. 85B. Molybdenum Kα radiation was used. The lattice of iron is of the body-centered cubic type. The solution of the films in Fig. 85 for the orientation of the elementary cubes in the strip specimen involves the determination of five quantities:

(1). The indices of the planes corresponding to the spots on the films.

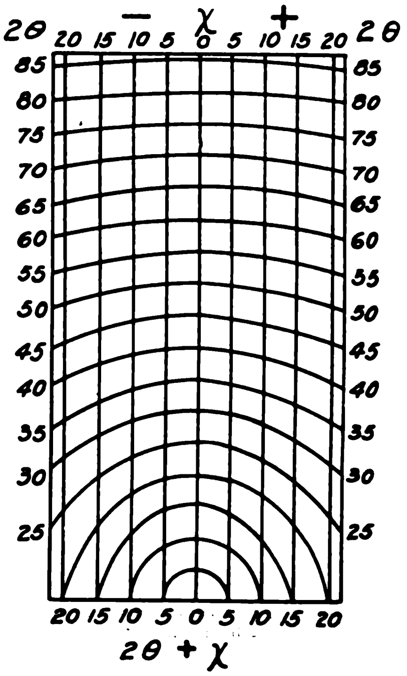

Fig. 84. Plot of the Angles 2θ, χ

(2). The location of these planes at the moment their spots were produced.

(3). The amount of rotation of the specimen at the time each spot was produced.

(4). The location of these planes before any rotation had been experienced by the specimen.

(5). Finally, the orientation of the unit cube corresponding to these locations.

The indices of the planes corresponding to the spots on the films are determined from the values of 2θ for each spot on the stationary film. In Fig. 85A, the image made by the rotating specimen in the zero beam impression on the stationary film is seen at the bottom of the print. To obtain the values of 2θ, the chart in Fig. 84 is laid over the film so that the lower *O* coincides in position with the center of this image and the bottom line of the chart coincides with the direction of the zero beam impression on the film. The *O–O* line now corresponds to the line *ge* of Fig. 81. The values of 2θ for the separate spots are recorded in column 2 of a table (Table XXXIV). From the known lattice structure of the specimen the interplanar distances and the angles of reflection from the different planes can be calculated.(160) From the values of θ the values of 2θ can be obtained. For iron, the results of such a calculation are shown in Table XXXV. By reference to this table, the planes corresponding to the different spots of the stationary film can be tabulated in column 3 (Table XXXIV).

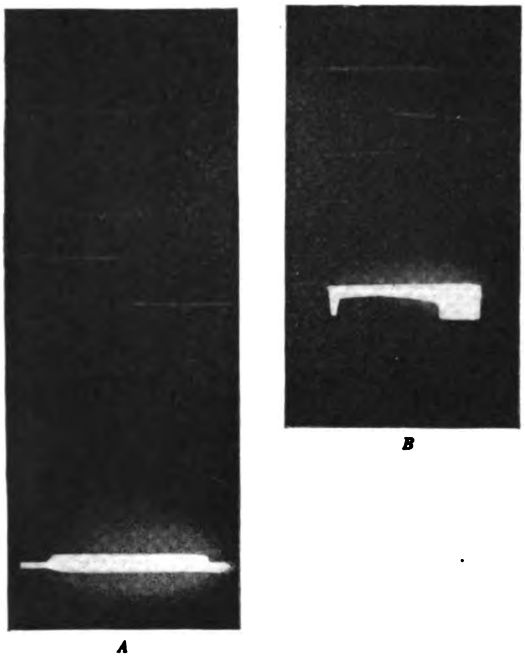

Fig. 85. Pattern of Silicon Steel Sample Obtained by the Stationary-oscillating Film Method

To ascertain the location of the atomic planes producing reflection at the instant of reflection, it is necessary to determine accurately the position of the spots on the films. This involves the determination of the angles ϕ_1 and α corresponding to each spot. For this purpose, the chart in Fig. 82 is laid over the stationary film in exactly the same position as that previously occupied by the chart in Fig. 84. The vertical distances now yield the values of ϕ_1 for the different spots, and the values of α may also be obtained from the appropriate curves shown on the chart. These angles are recorded in columns 4 and 5 of Table XXXIV respectively. Finally, to assist in properly pairing these spots with their reflections on the oscillating film, the values of χ for each spot are also recorded as in column 6. It will be noticed that, from the shape of the curves, the value of ϕ_1 for any particular spot will be identical with its 2θ value if the spot is not far removed from the *O–O* line of the chart, otherwise the ϕ_1 value will be somewhat less. The value of α for any spot will always be somewhat numerically larger than the value of χ for the same spot. Angles measured to the left of the *O–O* line are given the negative sign. Comparison of the readings of the angles for the spots in this manner will often prove of value in preventing errors.

The third step in the complete solution, the determination of the amount of rotation of the specimen at the time each plane reflection was produced, depends upon the pattern recorded on the oscillating film. The chart in Fig. 83 is laid over this film so that the *O–O* line and the bottom line of the chart bear the same relation to the zero beam impression as existed with the charts previously used on the stationary film. The angles ϕ_2 and χ corresponding to each spot are now read and perhaps best written down on some scrap paper. The spots may now be paired off with the spots previously listed from observation of the stationary film by comparing the different values of χ as tabulated, and also by examining the films themselves for similarity of appearance of certain spots. In the films in Fig. 85, the χ values were particularly useful, since they show that the order of spots 3 and 4 is reversed on the rotating film, and spot 6 can not be seen. Close inspection of the oscillating film with a bright light showed that this spot was only very slightly removed from the zero beam, so that its position could not be accurately determined. After the spots have been satisfactorily paired, the ϕ_2 and χ values obtained from the oscillating film may be entered in the table as shown in columns 7 and 8 of Table XXXIV. From the tabulated values of ϕ_1 and ϕ_2 for each spot, the angle ϕ

TABLE XXXIV

Readings of the Spots in the X-ray Diffraction Pattern of Silicon Steel of Fig. 85

COLUMN									
	2	3	4	5	6	7	8	9	10
No. of Spot	2θ Deg.	Plane	Stationary Film			Rotating Film			
			ϕ_1 Deg.	α Deg.	χ Deg.	ϕ_2 Deg.	χ Deg.	ϕ Deg.	θ Deg.
1	39	110	38	12	8	31	9	8	20
2	61	100	61	13	12	42	13	19	30
3	66	210	65	−5	−4	38	−4	27	33
4	50	111	51	−5	−4	46	−5	5	25
5	45	310	44	−12	−8	24	−7	20	22
6	28	100	25	−27	−12	Not visible			

TABLE XXXV

Interplanar Distances and Angles of X-ray Diffraction for Iron

Plane	Interplanar Distance Angstrom Units	θ Deg.	2θ Deg.
110	2.03	10.0	20.0
100	1.43	14.4	28.8
211	1.17	17.7	35.4
110 (2)	1.01	20.6	41.2
310	0.905	23.1	46.2
111	0.825	25.7	51.4
321	0.764	28.0	56.0
100 (2)	0.715	29.8	59.7
411 110 (3)	0.675	31.8	63.7
210	0.638	34.0	68.0
332	0.610	35.6	71.3
211 (2)	0.584	37.7	75.3

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(160) See for instance Davey, GENERAL ELECTRIC REVIEW 28, 592 (1925).

through which the specimen had been rotated at the time the spot was produced may be easily obtained by Equation (85). These values are recorded in column 9. The table is completed by tabulating the values of θ in column 10 for later use.

The determination of the position of the planes before any rotation had been experienced by the specimen is best made graphically. Of the devices which have been tried, the most satisfactory has proved to be the stereographic rotation net.

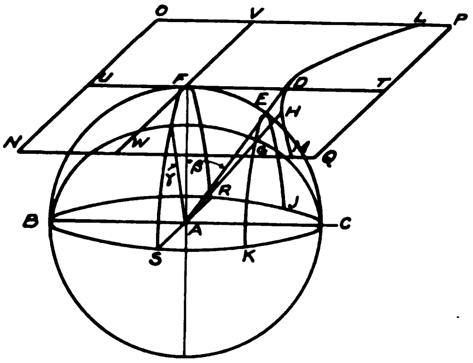

Fig. 86. Derivation of the Rotation Net

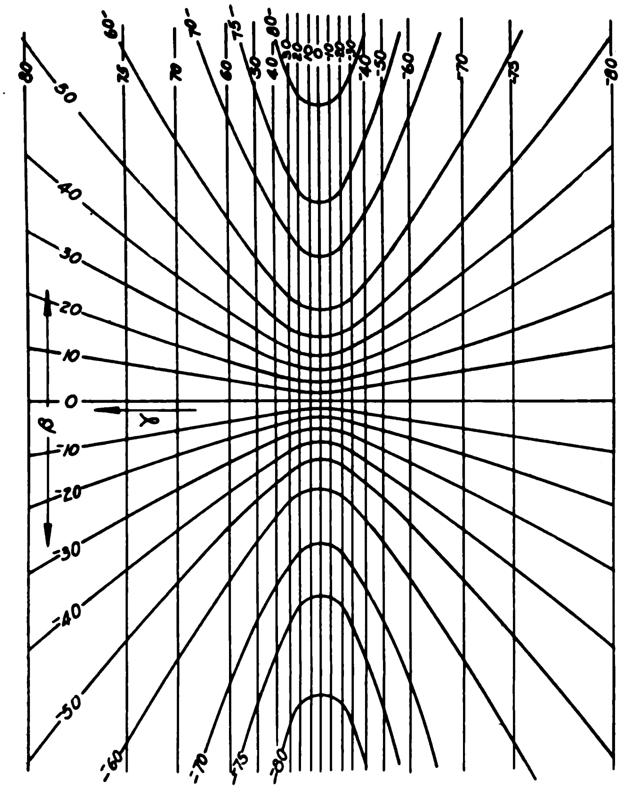

Fig. 87. Rotation Net

The Stereographic Rotation Net and Its Use

In Fig. 86, let *NOPQ* be a projection plane, tangent at the point *F* to a sphere of radius *AF* drawn about the point *A* as a center. Let *AD* be the perpendicular to some atomic plane in a crystal at *A*, which intersects the plane of projection at *D*, and the sphere at *E*. Let *BC* be an axis of rotation passing through *A* parallel to the plane of projection and let *RFS* be a plane perpendicular to *BS* passing through *A*. Then the position of the plane perpendicular *AD* at any instant may be defined by specifying the angles β and γ measured respectively perpendicular to the plane *RFS* and from the radius *AF* in this plane. Let the initial value of the latter angle be 0.

If the crystal is now caused to rotate about the axis *BC*, the angle β remains constant. Consequently, the path of the point *E* on the surface of the sphere is small circle *JEK*, and after a rotation through the angle γ, the point *E* will occupy the position *G*. But the path of the point *D* on the plane of projection will be the hyperbola *LDM*, and after a rotation of γ, the point *D* will lie at *H*. All points having equal values of γ, however, will lie along the great circle *CGB*, the corresponding points on the plane of projection will lie along straight lines parallel to *TU*. Let distances on the plane of projection parallel to the line *TU* be designated as ζ, and those perpendicular to the line *TU* be designated as ξ. Then for the purpose of calculating the position of the point *D* after the crystal has been rotated through an angle γ, use may be made of the following equations due to Smith:(161)

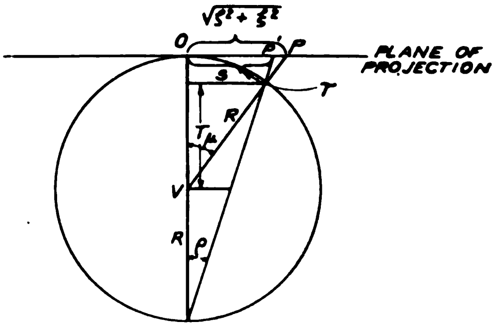

Fig. 88. Relation Between a Point *P* and Its Stereographic Projection *P'*

$$\xi = AF \tan \gamma \qquad (95)$$

$$\zeta = AF \tan \beta \sec \gamma \qquad (96)$$

By calculating the values of ζ and ξ for 5 deg. intervals of both β and γ, the net shown in Fig. 87 may be plotted. Because of the fact that this net shows the paths followed by the points of equal β as the specimen is turned through an angle γ, it will be called a rotation net. This net is similar in form to the gnomonic rotation net of Wyckoff.(162)

The relation existing between the distance *OP* of the point *P* from the origin of coördinates *O* and the stereographic projection *OP'* of that distance is shown in Fig. 88. By geometry,

$$OP = \sqrt{\zeta^2 + \xi^2}$$

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(161) G. F. H. Smith, *Z. f. Krist.* 32, 142 (1904).
(162) R. W. G. Wyckoff, "The Structure of Crystals" (Chem. Cat. C., 1924), 135.

Let the distance OP' be designed as τ. Draw the line S perpendicular to OV as shown. Let the distance of the intersection of S and OV from the center of the circle be denoted by T. Let R be the radius of the circle. Then by inspection it is seen that

$$\frac{\sqrt{\zeta^2+\xi^2}}{R} = \tan \mu \qquad (97)$$

and $\tau = 2\, R \tan \rho$

where μ and ρ have the significance shown in Fig. 88. But

$$\tan \rho = \frac{S}{R+TR} = \frac{R \sin \mu}{R+R \cos \mu} = \frac{\sin \mu}{1+\cos \mu}$$

or

$$\tau = 2\, R\, \frac{\sin \mu}{1+\cos \mu} \qquad (98)$$

By the use of the values of ζ and ξ calculated for the points of the rotation net, and from Equations (97) and (98) the corresponding points of the stereographic rotation net may be easily plotted. The value of R may be conveniently taken as 5 cm. The stereographic rotation net calculated from the rotation net in Fig. 87 is shown in Fig. 89. In this net, if AB coincides in direction with the axis of rotation, the arcs represent the paths followed by the stereographic projections of the intersections of the plane perpendiculars with the plane of projection.

By the use of this net, the positions of the plane perpendiculars corresponding to the different spots on the stationary film may be plotted. These perpendiculars may then be rotated into the positions which they occupied before the specimen had experienced any rotation. On a sheet of paper a circle is drawn with the same diameter as that of the stereographic rotation net. A diameter is drawn through the circle representing the line *ge* in Fig. 81. This is represented by CD, Fig. 90. Radii are drawn forming the angles α (Table XXXIV) with this diameter. Since the positions of the plane perpendiculars are to be plotted, the complements of the θ angles (column 10, Table XXXIV) are to be laid off along the appropriate radii from the center of the projection net. This is most easily done by plotting the values of θ along these radii starting from the circumference of the net. These values may be easily obtained by dividers along either of the lines AB or CD in Fig. 89. These plots are shown by the black circles in Fig. 90.

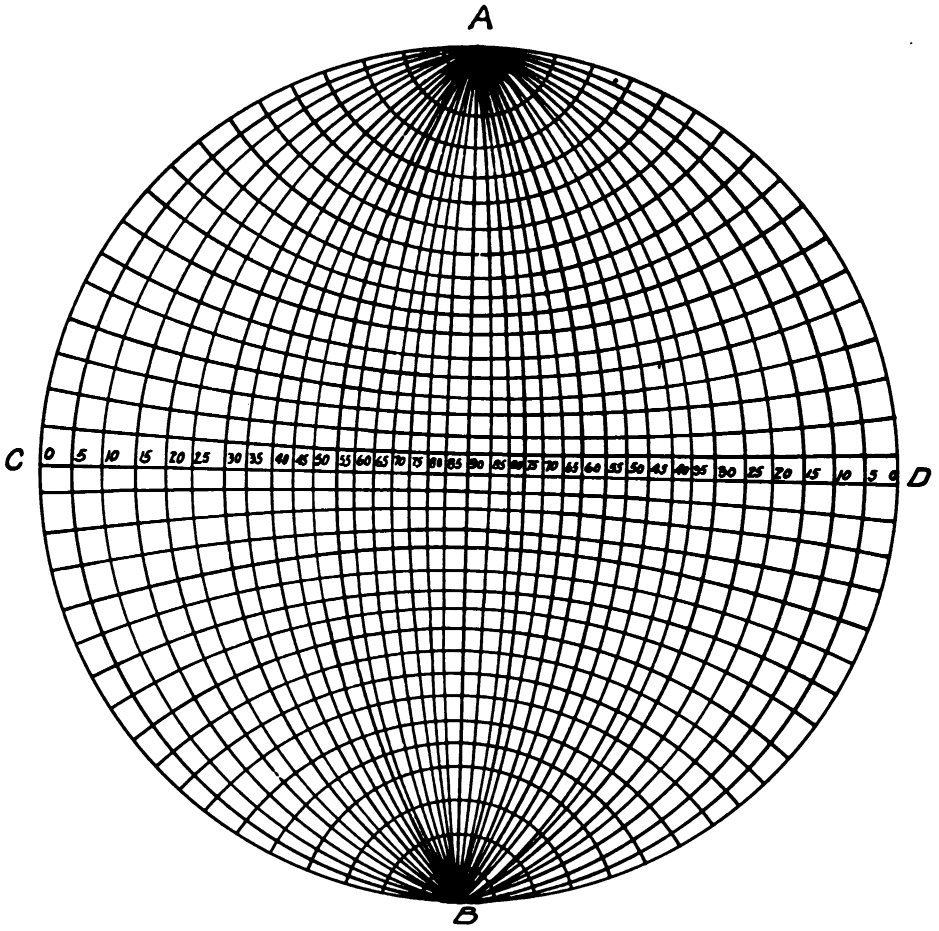

Fig. 89. Stereographic Rotation Net

These spots represent the stereographic projections of the plane perpendiculars at the time when their reflections were produced. Ordinarily, no two reflections are produced with the crystal in the same position. The plot of points is now placed over the stereographic rotation net so that the diameter *AB* coincides in direction with the axis of rotation (*AB* in this case), and the spots are moved along their appropriate arcs through the angles ϕ (Table XXXIV). The new locations thus obtained are shown by the triangles in Fig 90. They represent the stereographic projections of the intersections of the plane diameters with the plane of projection before the specimen under investigation had experienced any rotation.

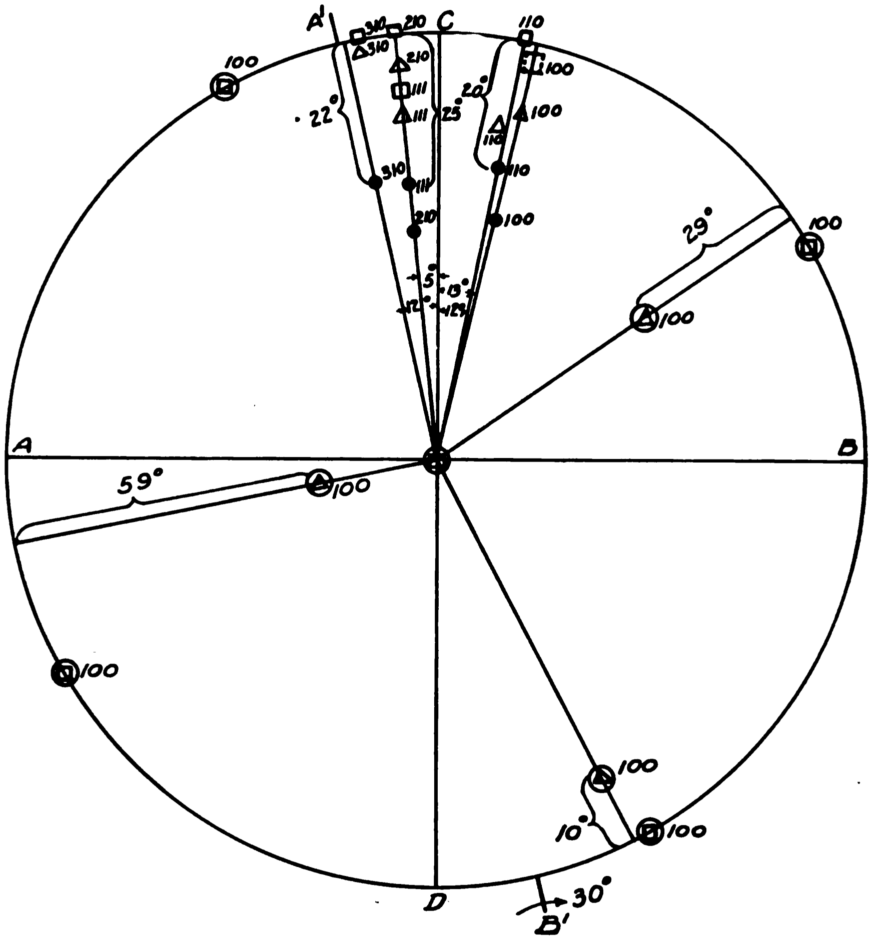

Fig. 90 Obtaining the Orientation of the Unit Cube in the Silicon Steel Specimen from the Pattern in Fig. 85

The Use of the Standard Stereographic Projection

To complete the solution by determining the orientation of the unit prism which will account for the initial positions of the plane perpendiculars now obtained, the positions of the spots must be made to agree with the position of the same spots on some standard stereographic projection for the particular material of the sample. Such a projection is shown in Fig. 91 for a cubic material such as steel. On this standard projection the perpendiculars of the planes 110, 210, 310 appear on the 90 deg. circle. Accordingly, the stereographic rotation net is placed over the plot in Fig. 90, so that the centers coincide. The rotation net is turned until the triangle plots of these plane perpendiculars lie on the same great circle. The axis of rotation is now situated at *A'B'*. By a 30-deg. rotation about this axis, the plots of the 110, 210, and 310 plane perpendiculars may be brought into the 90 deg. circle corresponding to their position on the standard stereographic projection, as shown by the squares in Fig. 90. From the standard projection, the position of the 100 perpendiculars may be plotted as shown by the circumscribed squares in Fig. 90. The positions occupied by these perpendiculars before the specimen had experienced any rotation may now be found by reversing the previous 30 deg. rotation about *A'B'* by means of the stereographic rotation net, bringing these perpendiculars into the positions shown by the circumscribed triangles in Fig. 90. By measuring the distance from the circumference of the projection to these last plots along their radii, the original angle of incidence of the x-ray beam with the cube faces may be obtained as described above. The values of these angles are shown on the plot beside the appropriate perpendiculars.

A construction showing the orientation of the unit cube of iron in the specimen follows at once, if it is remembered that in the cubic system, the plane perpendiculars are the cube edges. The orientation of the cube so obtained is shown in Fig. 92. The angles of incidence of the x-ray beam with these planes while the specimen was in its initial position are shown in the appropriate faces. The direction of the x-ray beam is perpendicular to the plane of the paper.

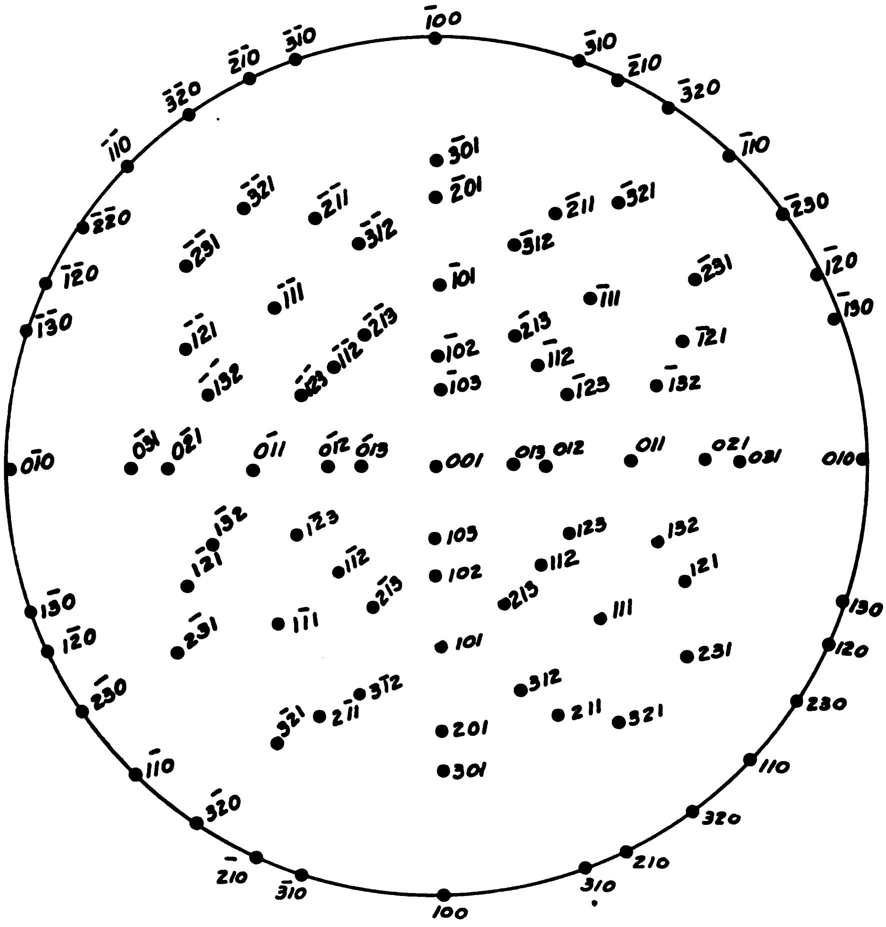

Fig. 91. Standard Stereographic Projection of Cubic Lattice

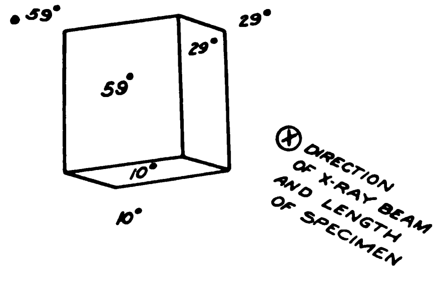

Fig. 92. Orientation of the Unit Cube of Silicon Steel from the Pattern in Fig. 85

In the rotation of the plot in Fig. 90, it was found impossible to reconcile the positions of the plots of the 100 and the 111 perpendiculars with the standard projection. These reflections on the films appear weaker than the others and are not as sharply defined. They are therefore caused by "included crystals" imbedded in the single crystal of silicon steel used to illustrate the method. Such a crystal was found on subsequent examination of the specimen in the area grazed by the x-ray beam. Single crystals of a metal grown by any method which involves straining and subsequent annealing are apt to contain these included crystals which will produce anamolous reflections. A great advantage of the method of solution of the photographs which has been described is that it makes possible the definite selection of the reflections caused by the major crystal, and the exclusion of all others. Should the orientation of these subordinate crystals be desired, however, it may be obtained by the same process as was employed in determining the orientation of the major crystal.

(To be continued)

Virginia Electric and Power Co. Winner of the Charles A. Coffin Award for 1927

How an electric street railway company in Virginia transformed its patrons from a state of hostility to one of friendship, increased earnings and raised wages, was revealed when the Charles A. Coffin gold medal was awarded to the Virginia Electric and Power Company during the annual convention of the American Electric Railway Association at Cleveland, on September 25th. The award was made by R. P. Stevens, president of the association and chairman of the Coffin prize committee. It was received in behalf of the railway company by President William E. Wood. In addition to the medal, a check for $1000 was given to the treasury of the employees' benefit association.

This is the sixth year in which the Coffin medal and cash award have been presented under the terms of the Charles A. Coffin Foundation, established in 1922 by the General Electric Company as a perpetual memorial to its founder and first president. Awards are also made each year to an electric light and power company, to graduates of technical colleges, and to employees of the General Electric Company. The award to electric railway companies has been competed for, in the six years since the Foundation was created, by 48 different companies.

Three years ago, the public of Richmond, Norfolk, Portsmouth and Petersburg, the principal communities served by the Virginia Electric and Power Company, was extremely bitter toward that company. The company's physical property was deteriorating; renewals of franchises could not be obtained; and on every hand there were criticism and condemnation.

Then came a change of attitude on the part of the company, which started in to build a well-founded public confidence and respect. It began by modernizing its rolling stock and providing new track facilities. The response from the public was so generous as to be a revelation.

An intensive plan of employee training was then undertaken. Many methods of giving better service were adopted, with special attention to the safety of both employees and public. In 1927 the number of riders was 102,000,000, compared with 98,000,000 in 1926.

Operating costs were reduced 1.77 per cent, although mileage was increased 2.06 per cent. Gross earnings were augmented 6.30 per cent. The speed of cars was increased, and economies of various sorts were introduced.

As public friendliness increased, millions of dollars of stock was sold to customers in various offerings at low sale cost. The time finally came, during the last year, when there was no controversy or litigation with any community or state body. Newspapers commented editorially in complimentary words; city officials and civic leaders wrote commendatory letters; and coöperation is now the rule all along the line.

Previous railway awards were made to the following companies:

1922—Chicago, North Shore & Milwaukee Railroad Company
1923—Northern Texas Traction Company
1924—Pittsburgh Railways Company
1925—Pennsylvania-Ohio Electric Company
1926—Grand Rapids Railroad Company

Seventh National Exhibition of Power and Mechanical Engineering

The National Exhibition of Power and Mechanical Engineering will be held this year from December 3rd to December 8th at the Grand Central Palace, New York City.

All four floors will again this year be occupied by booths displaying the many and various types of equipment apparatus and supplies that have made our age known as "The Mechanical Age."

Keen competition for the markets of the world dictate the absolute necessity for providing our industries throughout the country with the most economical and efficient methods of power production.

Not only the production of power, but, also, the control and distribution of power are all-important factors in our industrial life, and all those who are responsible for our commercial preëminences find it exceedingly hard to keep abreast of our fast-moving times. They must be ever alert to know the latest developments in both the electrical and mechanical world. To such—Engineers and Executives alike—these exhibitions form a wonderful medium for the acquisition of knowledge and the exchange of ideas.

That the value of these exhibitions is realized is evident by the fact that the registered attendance has grown from a few hundred to over twenty-five thousand; the general attendance at previous exhibitions has been as high as one hundred thousand in one week.

We wish the Seventh Annual Exhibition every success.

LIBRARY SECTION

Condensed references to some of the more important recent articles in the technical press, and to new books of interest to the industry, as selected by the General Electric Main Library.

Antennae
Effect of the Antenna in Tuning Radio Receivers and Methods of Compensation for It. Sylvan Harris.
Inst. Radio Engrs. Proc., Aug., 1928; v. 16, pp. 1079-1088.

Arc Welding
Electronic Tornado Principle of Welding. J. F. Lincoln.
Am. Weld. Soc. Jour., July, 1928; v. 7, pp. 48-51.

Bearings
Investigation of the Performance of Waste-Packed Armature Bearings. G. B. Karelitz.
A.S.M.E. Trans. (*APM*), Jan.-Apr., 1928; v. 50, Paper No. 1, pp. 1-10.

Circuit Breakers
High-Speed Circuit Breaker in Service on the Illinois Central Railroad. W. P. Monroe and R. M. Allen.
A.I.E.E. Jour., Aug., 1928; v. 47, pp. 571-575.

Commutators
How Motor Service Dictates Commutator Assembly. Jesse M. Zimmerman.
Ind. Engng., Aug., 1928; v. 86, pp. 410-414.

Condensers, Steam
Condenser Leakage; Its Detection and Effect on Boiler Water Concentrations. W. C. Carmichael.
Power, Aug. 14, 1928; v. 68, pp. 267-270.

Electric Cables
Residual Air and Moisture in Impregnated Paper Insulation—II. J. B. Whitehead and others.
A.I.E.E. Jour., Aug., 1928; v. 47, pp. 565-566.
(Abridgment.)

Electric Control Systems
Control Starts Machines in Proper Sequence. J. J. Steinke.
Power, July 31, 1928; v. 68, pp. 200-201.

Invisible Relay in Vari-Time Control. L. A. Watson and others.
Iron & St. Engr., July, 1928; v. 5, pp. 323-326.
(Illustrated description of a steel mill control system.)

Electric Distribution
Common-Neutral Network. M. T. Crawford.
Elec. Wld., July 28, 1928; v. 92, pp. 157-158.
(Explains a distribution system of the Puget Sound Power & Light Co. which embodies a common neutral.)

Electric Drive—Mining
Use of Electric Power in Iron Mining. A. C. Butterworth.
Mech. Engng., Sept., 1928; v. 50, pp. 670-675.
(Its development and possibilities in the Lake Superior district.)

Electric Drive—Pumps
Submerged Motor Pumps. H. Sauveur.
Engng. Prog., July, 1928; v. 9, pp. 191-194.

Electric Drive—Steel Mills
Electric Control Enables Continuous Sheet Rolling. T. J. Flaherty and A. F. Kenyon.
Elec. Wld., July 21, 1928; v. 92, pp. 103-107.
(Illustrated account of the electrification of the Ashland, Ky., plant of the American Rolling Mill Co.)

Steel Mill Drives Made Flexible by Speed Control Systems. L. A. Norris.
Iron Tr. Rev., Aug. 9, 1928; v. 83, pp. 321-323, 326.

Electric Motors, Synchronous
Design and Application of Two-pole Synchronous Motors. D. W. McLenegan and Ivan H. Summers.
A.I.E.E. Jour., Aug., 1928; v. 47, pp. 585-590.
(Abridged.)

Electric Welding
Study in Welded Plate Girders. Frank P. McKibben.
Am. Weld. Soc. Jour., July, 1928; v. 7, pp. 13-21.

Electro-Culture
Electric Heat in the Garden. R. Borlase Matthews.
Elec. Rev., Aug. 10, 1928; v. 103, pp. 229-231.
(Electrical heating of the soil is described.)

Hydroelectric Development
Rocky River Hydroelectric Development of the Connecticut Light and Power Company. E. J. Amberg.
A.I.E.E. Jour., Aug., 1928; v. 47, pp. 567-571.
(Abridgment.)

Insulating Oils—Testing
Insulating Oils for High-Voltage Cables. T. N. Riley and T. R. Scott.
I.E.E. Jour., Aug., 1928; v. 66, pp. 805-840.
(Includes a bibliography of 71 references.)

Insulation—Testing
Make Insulation Resistance Tests. H. E. Stafford.
Power, Aug. 7, 1928; v. 68, pp. 238-240.
(On practical methods of testing insulation of machines.)

Lightning Arresters
Lightning Arrester Problems; New Light from Klydonograph Studies. A. L. Atherton.
A.I.E.E. Jour., Aug., 1928; v. 47, pp. 591-594.

Load Dispatching
Telltale Load Dispatcher Board. W. S. Cronin and A. J. Davis.
Elec. Wld., Aug. 4, 1928; v. 92, p. 213.
(Short description of a board used by the Indianapolis Power & Light Co.)

Magnetic Properties—Testing
Effect of Methods of Demagnetization on the Energy Required to Magnetize Steel. John D. Ball.
Franklin Inst. Jour., Aug., 1928; v. 206, pp. 181-200.

Piezo-electricity
Some Practical Applications of Quartz Resonators. G. W. N. Cobbold and A. E. Underdown.
I.E.E. Jour., Aug., 1928; v. 66, pp. 855-871.

Prime Movers
Development of the Steam Prime Mover from Hero to Emmet. David O. Woodbury.
Elec. Lt. & Pr., Aug., 1928; v. 6, pp. 22-25, 107-109

Pulverized Coal Engines
Coal-Dust Engine Upsets Traditions. R. Pavolikowski.
Power, July 24, 1928; v. 68, pp. 136-138.
(Illustrated description of a German internal combustion engine burning pulverized coal. Includes some performance figures.)

Radio Direction-Finding
Rotating Radio Beacon.
Engng., Aug. 3, 1928; v. 126, pp. 130-131.
(Discusses Radio Research Special Report No. 6, Dept. of Scientific and Industrial Research, England.)

Radio Engineering—Crystals

Quartz Plates, Air Gap Effect, and Audio-Frequency Generation. August Hund.
Inst. Radio Engrs. Proc., Aug., 1928; v. 16, pp. 1072-1076.

Radio Engineering—Power Supply

Development of a System of Line Power for Radio. George B. Crouse.
Inst. Radio Engrs. Proc., Aug., 1928; v. 16, pp. 1133-1148.

Radio Engineering—Reception—Interference

Cause and Prevention of Hum in Receiving Tubes Employing Alternating Current Direct on the Filament. W. J. Kimmell.
Inst. Radio Engrs. Proc., Aug., 1928; v. 16, pp. 1089-1106.

Correcting Radio Interference. G. W. Shaver.
Elec. Rwy. Jour., Aug. 4, 1928; v. 72, pp. 193-194.
(Methods used by the San Diego Electric Railway for locating and correcting interference.)

Study of Signal Fading. E. V. Appleton.
I.E.E. Jour., Aug., 1928; v. 66, pp. 872-885.
("An account of the work of Peterborough Radio Research Station of the Department of Scientific and Industrial Research.")

Railroads, Electric

What Are You Doing to Meet Competition? E. P. Waller.
Elec. Rwy. Jour., Aug. 11, 1928; v. 72, pp. 223-224.
(Discusses questions of obsolescence of electric railway equipment.)

Railroads—Electrification

Water Power a Factor in the Electrification of Swiss Railways. Kent T. Healy.
Elec. Rwy. Jour., Aug. 4, 1928; v. 72, pp. 176-181.
(Illustrated description of equipment and methods used.)

Ship Propulsion, Electric

Propulsion of Ships by Electricity. S. B. Jackson.
I.E.E. Jour., Aug., 1928; v. 66, pp. 898-899.
(Abstract of a paper read before the North-Eastern Students' Section.)

Substations

New Features Embodied in Century Receiving Substation. C. P. Garman.
Elec. Wld., Aug. 4, 1928; v. 92, pp. 209-213.
(Illustrated description of a substation of the Bureau of Power & Light, Los Angeles.)

Substations, Automatic

Favors Supervisory Control for Substations. W. F. Sutherland.
Elec. Wld., Aug. 11, 1928; v. 92, pp. 251-255.
(All recent stations of Toronto Hydroelectric System are equipped with supervisory control.)

Switches and Switchgear

Contact Resistance of Electric Switching Apparatus. W. Schaelchlin.
Elec. Jour., Aug., 1928; v. 25, pp. 386-391.

Transient Phenomena

Effect of Transient Phenomena on the Design and Selection of Alternating Current Equipment. E. G. Merrick.
Iron & St. Engr., July, 1928; v. 5, pp. 316-323.

Voltmeters

Thermionic Voltmeter for Measuring the Peak Value and the Mean Value of an Alternating Voltage of Any Wave-Form. E. B. Moullin.
I.E.E. Jour., Aug., 1928; v. 66, pp. 886-895.

NEW BOOKS

(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW)

Analytic Geometry. R. L. Borger. 334 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Business Cycles, the Problem and Its Setting. Wesley C. Mitchell. 489 pp., 1928, N. Y., Nat. Bur. Economic Research.

Code for Electricity Meters. Ed. 3. 122 pp., 1928, N. Y., Nat. Elec. Lt. Assoc.

Elements of Industrial Engineering. George H. Shepard. 541 pp., 1928, Bost., Ginn & Co.

Elements of Physics. A. Wilmer Duff and Henry T. Weed. 565 pp., 1928, N. Y., Longmans, Green & Co.

Industrial Management. Ed. 2. Richard H. Lansburgh. 509 pp., 1928, N.Y., John Wiley & Sons.

Lineman's Handbook. Edwin Kurtz. 547 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Mechanical Engineering. A Text Book for a Short Course. W. A. Mitchell. 402 pp., 1928, N. Y., John Wiley & Sons.

Mechanics for Engineers; Statistics and Kinetics. Julian C. Smallwood and Frank W. Kouwenhoven. 185 pp., 1928, N. Y., D. Van Nostrand Co., Inc.
("The authors claim nothing new except an arrangement and a selection of material, a combination of problems with text for the exercise of the best pedagogy, and an emphasis upon subject important to engineering students with corresponding omission or reduction of the less important ones." The above quotation from the Preface aptly characterizes this short text for engineering students. Numerous problems are interspersed throughout the text so that the student may apply the principles set forth and thereby obtain a working knowledge of the subject.)

Modern Picture Theater Electrical Equipment and Projection. Ed. 2. R. V. Johnson. 189 pp., 1927, Lond., Crosby Lockwood & Son.

Outlines of Public Utility Economics. Martin G. Glaeser 847 pp., 1927, N.Y., Macmillan Co.

Physics of Crystals. Abram F. Joffe. 198 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Practical Television. E. T. Larner. 175 pp., 1928, N. Y., D. Van Nostrand Co., Inc.
(In a Foreword by John L. Baird, himself an experimenter and inventor in the field of television, we are told that this book "deals very fully with the fundamental principles from which Television was developed, and deals with them in such a way as to interest the general reader without departing from strict scientific accuracy." An enumeration of the ten chapter headings gives a fair conception of the scope of the volume: "Introductory," "Historical," "Selenium and the Selenium Cell," "Photo-electricity and the Photoelectric Cell," "Continental and American Researches," "Researches with the Cathode-rays," "Images and Their Formation," "The Baird Televisor," "Television Technique," and "Recent Developments.")

Principles of Mechanism. F. Dyson. 296 pp., 1928, N.Y., Oxford Univ. Press.

Radio; A Study of First Principles. Elmer E. Burns. 255 pp., 1928, N. Y., D. Van Nostrand Co., Inc.
(An elementary text in which there is made "an attempt to present, simply and clearly, the fundamental principles of electricity as applied in radio." The author hopes "that the book may be of use to radio experimenters who are looking for a simple exposition of radio theory and to those who are well along in experimental work and wish to review the fundamentals." The principles of direct currents and of high and low frequency alternating currents are presented. The explanations of the electrical phenomena involved are based on the electron theory as being in accordance with latest conceptions. The mathematical formulae employed are of the simplest nature.)

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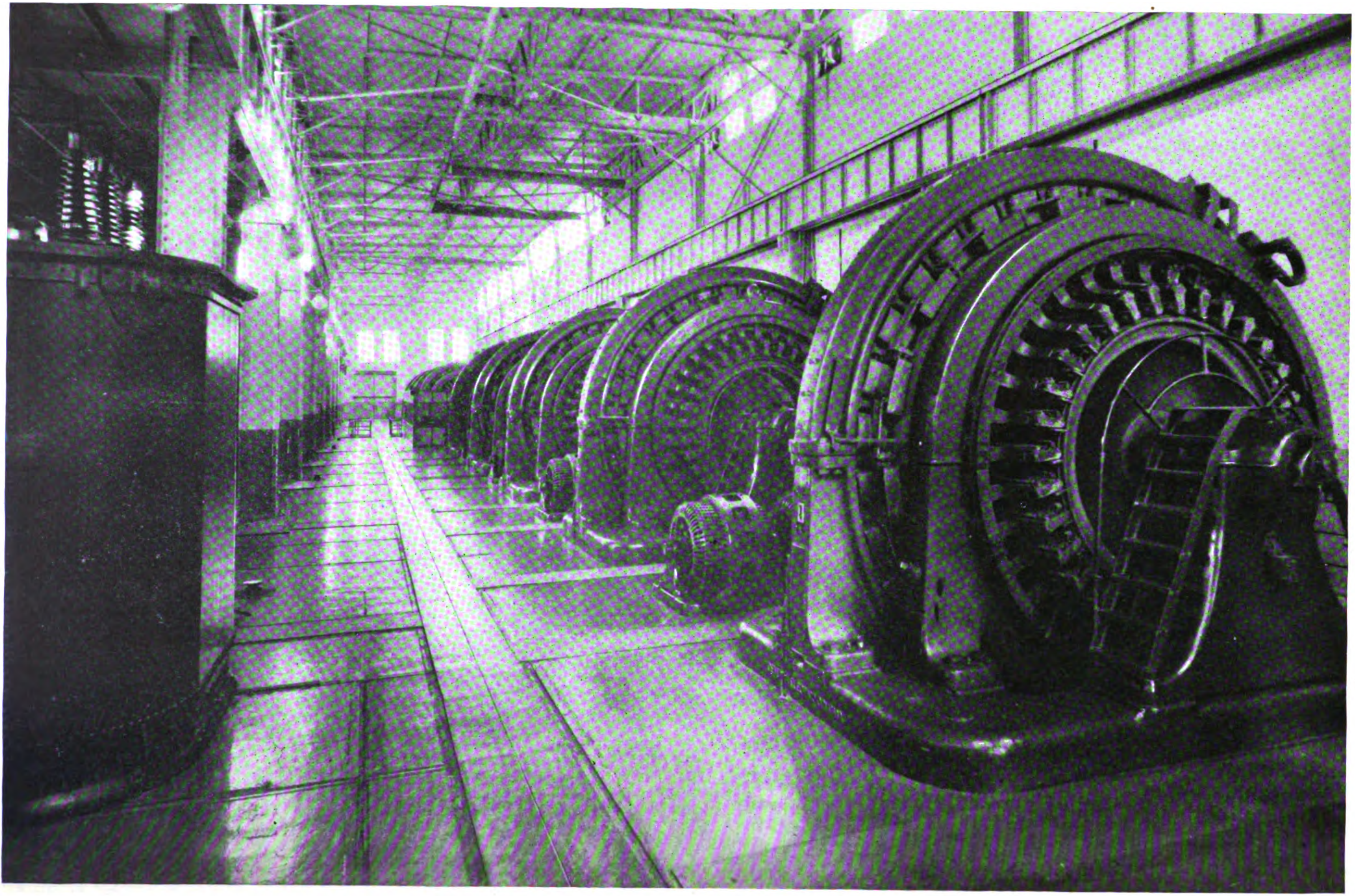

LARGE CONVERTERS STEADFASTLY CONTRIBUTE TO INDUSTRY

Five of the nine converters in this group have been operating in electrolytic zinc reduction for twelve years; and in the earlier part of this period—during years demanding peak load—a service-factor of 99 per cent was maintained. A presentation of factors largely affecting such continuity of service is found on p. 635

GENERAL ELECTRIC REVIEW

Vol. 31 **December, 1928** **No. 12**

CREATION

In this issue, we publish an article by Dr. W. R. Whitney, the opening sentence of which reads: "Scientific research, in its various fields, develops in one who watches it an ever increasing appreciation of Creation." We take this sentence as the text for our present editorial.

If we were asked to write a story of Creation we should be tempted to commit an act of plagiarism, and just copy the first chapter of Genesis and possibly add the first three verses of the second chapter. We should feel quite sure that no better account ever has been, ever will be, or ever could be written.

Unless this story is true we know absolutely nothing about the act of Creation; we know no more about the beginning of "things" than we do about the ending of "things." But how can we know anything? In the infinite universe, our own little universe, although of such stupendous proportions to us, is but a little dot, our own solar system a tiny speck, our habitation, the earth, a microscopic grain, and we, as Bertrand Russell puts it, "tiny lumps of impure carbon and water of complicated structure, with somewhat unusual physical and chemical properties."

We are aware that this definition neglects the spiritual and intellectual side of man, but nevertheless with these facts in mind, we are only apostles of the obvious when we say that we know nothing of the act, or story, of Creation. But the very knowledge of our ignorance makes us both admire and respect our remote ancestors who penned so beautiful an allegory from the inspiration they attained in their own efforts to reach the truth.

There is always an early morning twilight, or half light, before the dawn, and in the first rays of knowledge many laughed at this first story of Creation, but as the light of knowledge grows we find a new respect and a new love for it.

It is easier to believe blindly than it is to think accurately, but believing, and controversy about beliefs, must have stimulated men to think. But now in the light of modern knowledge which, however, is still very meager compared with what there is to know, man must both believe and think differently from his ancestors.

Since the days of those five great "Torch Bearers," Nicolaus Copernicus (1473-1543), Tycho Brahe (1546-1601), Johann Kepler (1573-1630), Galileo Galilei (1564-1642), and Isaac Newton (1642-1727), we are living in a different world, so far as human knowledge goes. The first fruits of this new knowledge should have been to humble arrogant spirits, as the earth lost its proud position, assigned to it by man, as the center of all things, and was seen to be but a tiny microscopic grain in an indefinite universe.

Before the days of the work and discoveries of these "Torch Bearers," the earth was the center of the universe, and the sun, the moon, the planets, the stars, and the heavens themselves were all thought to have been created to dance attendance on our globe, and solely for the service of mankind. But since their work our knowledge has so increased that wise men have lost their arrogance, and with reverence, bow their heads and acknowledge frankly that finite man can, perhaps, never know the infinite Creator or Creative Power, who must be infinitely greater, infinitely wiser, infinitely more powerful, infinitely more wonderful, infinitely better than any man-like God, as we know man. In all probability we are absolutely wrong in applying human, limiting attributes to the Creative Power, but it must have been useful in the schoolroom days of man's history when the foundations on which we have built our civilization were laid.

We always like to remember with respect and affection those forerunners of the "Torch Bearers," the old Babylonian Shepherds, "who watched their flocks by night" and at the same time studied the stars, and were the first to find and name our planets, and called them "the wanderers." They were the first observers, and observation is the father of inspiration and inspiration the father of knowledge.

Knowledge advanced but slowly in those long ages between the days of our first observers, the shepherds, and the era of the "Torch Bearers," but in the interval between Copernicus and Newton there came a change in the spirit of man's dreams, which paved the way for the still greater change which came about between the days of Newton and the present.

Man now began to realize that it was better to think than to believe, and that his only chance of rising higher in the scale of existence, and of knowledge, was to study the Works of the Creator, and by seeking, to learn as much as he could about the wonderful world he was living in, and, if possible, something about the universe of which his world is such a tiny speck.

And it thus came about that the age of science was born.

With the birth of science came that great conflict between those who wanted to learn the truth of Nature's laws through research, and those who wanted to control the beliefs of man by dogma. And there followed prosecutions, persecutions, and crucifixions. But that is another story. "Where passion rules, how weak does reason prove!"

But finally out of the confusion grew the age of inquiry when many men gave up the struggle of certifying the unknowable, and devoted their lives to putting questions to Nature and trying to get answers that would enable them to turn the resources of Nature to the service of man, so that they could live securer, better and more useful lives.

Since then we have learned much, and the seemingly unknowable has often become the known, and more miracles are recorded now almost every day because of inquiring minds than were anticipated anywhere before in all the ages.

We were not put into the world merely to believe. We were apparently put here to improve, to develop, to think, to work and to learn; one of the greatest blessings the Creator has given us in our schoolroom is the eternal question mark. The How? The Where?

The When? The Why? And little children seem to be almost the only ones that understand this.

But Longfellow knew it when he wrote:

> "Not enjoyment, and not sorrow,
> Is our destined end or way;
> But to act, that each tomorrow
> Find us farther than today."

The Creator gave us the most wonderful textbooks if we will only learn how to use them. The Creator's manuscripts are a million times more durable, instructive, and interesting than any old papyri, parchments, or printed books. The rocks are some of the manuscripts of the Creator. Longfellow, who presided at Agassiz's 50th birthday party, read on that occasion a poem he had written to commemorate the event, from which we quote two verses.

> "And Nature, the old Nurse, took
> The child upon her knee,
> Saying: Here is a story book
> Thy father has written for thee."
>
> "Come wander with me, she said,
> Into regions yet untrod;
> And read what is still unread
> In the manuscripts of God."

Indeed, the rocks are the manuscripts of the Creator. In them lie the history of the world, the history of the microbes, the history of the plants, the history of the animals, the history of man, the history of so many things, but not the history of Creation—the beginning of things. That has always had an eternal question mark against it.

Lord Kelvin, then Sir William Thomson, in his presidential address before the British Association, in the year 1871, suggested that the germ of life might have reached our planet on a meteorite, and we believe appointed a committee to examine every meterorite that fell to the earth to see if any signs of life could be found in them. But that would not have settled the question, as Tom Taylor so wittily said in *Punch* for August 12th of that same year; from his poem, which was entitled "The Truth After Thomson," we take the following lines:

> "But say, whence on these meteors life began,
> By whose collision came the germs of man?
> Still hangs the veil across the searcher's track,
> We have but thrust the mystery one stage back.
> Below the earth the elephant we've found
> Below him of the tortoise touched the ground;
> But what the tortoise bears? Dig as we will,
> Beneath us lies a deep unsounded still."

Science is only organized knowledge, and, as such, is the great antidote for the poison of superstitition and fear. Those that think that Science has dethroned God are mistaken. There shall ever be the unknowable, but science has given us respect, veneration, love, and worship for the Creative Power. It has put the mainspring into modern life, where every one of intelligence has an aim and object in living. It has given us the privilege and pleasure alike, of wanting to find out the Creator's laws—which are the laws of Nature—the laws that govern the movement of the heavenly bodies, and every act, and every motion of human beings, and everything that has life, and has not life in all the universe.

We have only been on the right track for a very short time, but just consider what we have learned from the Rocks alone since the days of "Strata" Smith, Charles Darwin, and Alfred Russell:

The story of the evolution of the species, as unfolded by the study of the fossil-bearing rocks, has given us a reasonably perfect picture of the tree of life from the single-celled Protoza to the most complicated, highly organized, but still developing, "Supreme Mammal." If we could once learn the story of how Nature first organized unorganized inert matter in the first single cell of life we possibly should have solved the problem of the origin of life. At present we can conceive of life only as coming from life. The origin of life is still a mystery. We are, however, beginning to learn something of how the green leaf —that small, but most important factory in Nature's extensive realm—through the catalytic action of its chlorophyl, is able to use the sun's rays in the process of photosynthesis, and thus build up the carbon compounds, such as the starches and sugars, from air, water and salts.

We believe that everything that has life, with the possible single exception of the bacteria, in the last analysis, depends upon the green leaf for food—and so, for life.

If we give the reins to our imagination and let it run wild for a moment, we can picture in a period a few millions, or few hundred million years ago, before the first Protist (those single-celled entities that had not settled whether to become vegetable or animals) had appeared, a single cell—not quite a living cell—but almost a living cell—a partly organized cell—that in some unknown way manufactured the first green pigment on our earth. It (the green pigment) might have had chlorophyl, or something like chlorophyl, which made it possible for it to utilize the energy of the sun's rays in breaking up the carbon dioxide, and thus building up in itself, the first spark of life on our earth—and the first speck of food for life on our earth—because life must feed on life. What a story from the first organized single cell to man!!!

We are constantly walking into new worlds; that is the joy of life. The Microbe Hunters opened up a new and astonishing world for us. William Dalton with his atomic theory in the last years of the 18th century opened up another world in the realm of chemistry. The telescope has opened up new worlds to us. Sir Joseph Thomson in the last year of the last century propounded his electron theory and thus opened up one of the most wonderful worlds of all. Each sucessive new world we discover reveals more of the wonders of the Creator's work, and makes our world a better and pleasanter place to live in.

No form of society can be considered successful where the greater number are poor and miserable. Science is the greatest agent of democracy in the world. Scientists are not working to make the rich man richer and the poor man poorer. They are working so that the blessings that the Creator has given us can be more equally shared by all. And they are doing it.

We feel that Dr. Whitney is right when he says "Scientific Research, in its various fields, develops in one who watches it an ever increasing appreciation of Creation." So—

> "Let Knowledge grow from more to more
> But more of reverence in us dwell."
> —*Tennyson*

J. R. H.

Electrical Research and Progress

By DR. W. R. WHITNEY
Director, Research Laboratory, General Electric Company

*An address before the Fifty-first Convention of the National Electric Light Association,
Atlantic City, N. J., June 4-8, 1928*

SCIENTIFIC research, in its various fields, develops in one who watches it an ever-increasing appreciation of Creation. All that men have done to make life seem happier or more interesting is merely to test new arrangements of old parts of the eternally-created whole. On top of this recognition grows the conviction that the process of improvement has been barely started. There is no sign of exhaustion of new, raw materials, nor of scientific and constructive principles. If there is anywhere failure to continue the advance in general welfare, whether in a material or in a purely spiritual sense, it will certainly be due to inappreciation or lassitude on our part rather than any deficiency in the limitless possibilities of Nature.

If we control our fears and taboos, as now seems possible through more educated inquisitiveness, we may look forward to interesting occupations provided for everyone, mechanical work everywhere, but only for machines, and perhaps permanent world peace.

Babies Start Right

As babies, we start right. The normal child begins life intensely inquisitive and appreciative. Whoever knew a healthy baby that did not taste everything it could get into its mouth? They seem to know that the world is particularly theirs to test and develop new tastes. Everyone has wondered how man learned that raw oysters were edible. Certainly an inquisitive baby showed him. Man never needed food seriously enough to investigate such things; they had to be discovered first and then found worthwhile.

As children grow up we put counter pressure and control of all kinds upon their inquisitiveness. Elders find it easier to drown than direct the natural researches of youth and "safety first" gets to be a common aim.

I think there may be some truth in this picture to account for the limited ambitions and activities of maturity in a world which is replete with novelty. You may say, "Never in history has the rate of increase of knowledge and its utilization been so high as at present." That is true, but even so the limitations are still in us rather than in our environment.

Let us view for a moment what we shall arbitrarily call "Progress," with reference to the scientific research which precedes it. I say arbitrarily because in this way we can shorten that discussion which inevitably arises when one asks whether, with all our new needs and novel pleasures, we are any better off than the cave-dweller who had just learned to warm and light his home. He must have felt greatly satisfied at that time, and could have felt no need for motorizing his industry and pleasures nor for refrigerating his foods and drinks. It is difficult for us to go further back than the heated and lighted home, but I might as well have referred to the long-haired monkeys in tropical cocoanut trees. Admitting with some philosophers that these seem as happy as anyone, we still arbitrarily call progress the motion away from the less helpless though satisfied state and toward the greater appreciation of infinite new possibilities which we may easily bring within our reach.

Man might possibly exist indefinitely in a stationary state of cultivating the soil and killing and eating just enough of the wild animals, and he might have got along without much mental growth, but he has already proved that the creative saliency which Emerson called "the importation and domestication of divine effort in man," suits him better.

I think I can show that while we have progressed with unprecedented strides in the recent past, there is no danger that an upper limit has been approached. As this should apply to all mental activities, I wish to refer to several different sciences before considering electricity.

Material Investigation Productive Beyond Measure

Encouraging men in every field to devote effort to material investigation is productive beyond measure or contemporary comprehension, because what is learned in any one field always and forever forms part of the foundation on which all the following work in that field has to be built. Beyond that is the immeasurably great effect which one branch of science exerts upon all the others, because each builds with reference to the rest. I feel quite powerless to explain adequately this point, but I will digress just enough for an illustration. At this moment, science knows very little about the two most fundamental things in life, the chlorophyl of plants and the protoplasm of animals. The chemist is getting excited over a few facts which we might call his present ignorance of these chemical compounds. He is almost frightened at the consequences of what a clear understanding of this subject might involve. What is it to understand the life process? The physicist and the astronomer, looking at the subject as a radiation problem, contribute new knowledge of specific radiation, wavelengths, whether they be ultra-violet light, the ever-present radium emanation, radio itself, or the newest cosmic rays. The biologist makes his corresponding experiments, and the expert on heredity, working at his fruit flies, takes the contributions of these diverse fields and applies them to his researches. Finally his conclusions are tentatively extended to heredity in man. In this way our progress becomes exponential. It rises as the "*n*th" power of the number of even widely remote investigators.

This may be expressed in another way. With our shortsightedness, we cannot see the paths along which we shall finally proceed in any particular progression. Take, for example, artificial illumination. If we knew nothing more about it than we see in the present systems of lighting, we would probably confine our attempts to improve artificial illumination to what I call the contiguous areas—those that call for no great external scientific contributions nor surprising disclosure—such as hotter gases, or more refractory metals. But when we draw from comparisons of radiations the fact that light should be capable of much more perfect and economical production, we begin to look at our present lamps as devices which only feebly punctuate the darkness—small spots which no one could possibly mistake for real daylight. Radio broadcasting, a radiation phenomenon, has been developed in a different field, but now we wonder whether light, being but a very short-wave radio, could not be benefited by contributions from the radio art. We turn to astronomy and there find a wonderful analysis of the production of light which might also direct the next step in artificial illumination. Through the correlated investigations of modern astronomy and atomic physics, then, we are rapidly acquiring a new conception of light and its generation. This could not have come to us through any extension of ordinary incandescent lighting studies.

The Nature of Light

All light in its infinite wavelengths or colors is apparently generated in accord with partially understood electro-mechanical laws. In some way, most wonderfully pictured as the readjustment of electrical charges in orbits of atoms, light waves are sent through space, just as are radio waves when suitable motion of more massive electric charges is produced by some sending station. We supply energy to our broadcasting stations whether they produce radio or light waves, but modern astronomy is showing that the light energy for the sun and stars is taken directly from matter itself; that is, probably from the potential energy of the electrical charges of the atoms themselves. Thus our daylight is due to the partial neutralization of positive and negative charges which in their proper orbits constitute atoms, but which, by their occasional neutralization, emit their electrical energy as the radiant energy we call light. Here, then, atoms become light, and matter is changed to radiant energy. It has been estimated by Jeans that the light from a 50 hp. searchlight, if it burned like the sun (*i.e.*, was not supplied from a power station), would turn matter into radiant energy at a rate which would cause it to lose about 1¼ grams in weight per century. As the sun radiates about that much energy per square inch, it is decaying from the electrical state which we call matter, into the immaterial thing we call radiant energy at the rate of 360 million tons daily! If the sun were not built on rather a generous scale, this rate of loss might be quite serious to mankind. Fortunately its rate of loss is still quite low compared to its stock in hand.

Magnitudes

If astronomy had no other wonders to expand the mind, it would still be our greatest scientific field, because the one thing which seems to delay or impede progress is our failure to appreciate magnitudes. We have long been used to million dollar congressional appropriations, but in very recent years we had our first billion dollar congress. Slowly, but surely, our understanding of numbers grows, but astronomical numbers (values) rise always beyond our grasp. Thousands and tens of thousands were once the peak, myriads were later an upper limit; a light year was once almost inconceivable, but now universes a hundred million light years away are things of nightly observation. What is more interesting than that the laws of celestial affairs are directly applicable to, and adequate in explaining, the general properties of matter, the structure, so-called, of the elementary atoms? It will be surprising, indeed, since matter is electrical and light is attributable to certain defined motions or changes of electrons obeying electrical laws, if the final efficient artificial light process is not traceable to astronomical studies.

We have become hardened to such expressions as, "We are living in a new and different world." Someone says, "The new things transcend our imagination." But this has always been true and will be as long as experiments are made, for limitations are indeed in the mind of man, not in Nature.

We are beginning to realize that new products and processes do not come exclusively or even usually from a mere physical change, like dryness, thickness, etc. With knowledge, it becomes not more of the same that is important, but some of a different sort; not merely the same old frontal attack—a new flank movement will often do better. The gases added to lamp bulbs after forty years of work to improve evacuation is an illustration of this point.

Ramifications of Radio

Every week I sit in a conference of fifty or more young radio engineers. I often realize that hardly a word of their hour's discussion was known to the scientists of twenty years ago. With this new vocabulary came countless corresponding pieces of new apparatus and new uses. New principles are involved, and the electrical industry will be influenced much more by this novelty than by the mere addition of the field of broadcasting. It is already becoming obvious that any kind of starting, stopping or control of remote electrical operations can be brought about by the merest trace of energy. A wave of the hand, the casting of a shadow, even the sound of a voice that is still may now be amplified so as to control any desired energy anywhere. This is part of the unexpected contribution of radio. But this is only a crude illustration of the manner in which experimental work well done and generally-known contributes to other fields. It seems as if no simple known fact anywhere need long stand unemployed in a world which is animated with the will to appreciate and utilize.

Because I think this talk ought to serve to strengthen the hands of men in all true scientific research, I wish I could point out some of the countless interlocking ramifications of the work of a single general physical research laboratory, the Cavendish Laboratory of Cambridge, England. But this would be going entirely too widely afield, and time forbids. For nearly 30 years, however, there has been a steady flow of new science from that laboratory and everybody interested in light has been continually benefited by it.

The Public Always Benefits

It may be assumed that separate industries are working out by experiment the value to them of various grades of pure and engineering research. They have ways of determining the value to them of new things. Research is more or less rampant. The public wins always, but the company supporting research, much like the unfortunate inventor of the past, may still suffer in some cases.

For example, the atomic or inherent energy of a drop of oil, properly utilized, would be sufficient to drive the engines of a liner for a complete trans-Atlantic trip. Suppose some research laboratory made the feat possible, it is still a question whether the discovery would yield anything to the company which supported the research. Many reasons for such failure might be found, both in the action of the owners and in those of their competitors. But there is one sure winner, and his name is Public. This is another way of expressing the thought that appreciation of the general services of science is an opportunity for the public.

But a talk of this kind may well be criticized as too general, and I might properly be expected to refer more specifically to researches of interest to the electrical industry. So now I wish to paint a picture, and I will select one particular thing on which to focus attention, just as the impressionist in painting emphasizes one peculiar color.

The Beginning of Industrial Use of Hydrogen

The lightest thing in the world is hydrogen gas. Its historical trail through twenty-five years in the laboratory will serve as the dominant color for my picture. We began using hydrogen in 1900, at the rate of a few cubic feet monthly. In 1903 it became a few hundred cubic feet. In 1907 we were using 17,000 cubic feet per month. In five years it was 175,000 cubic feet. In five more it reached 1,000,000 and it is now about 2,000,000 per month.

Everybody recalls the little hydrogen generator which is made in every elementary chemical laboratory and is exploded there with more noise than damage. These use zinc and sulphuric acid, and such generators, on a large scale, are still used by lead burners, *i.e.*, men who construct lead apparatus by melding the joints together with oxy-hydrogen flame. These chemical generators furnished a few cubic feet of hydrogen per day and served for many researches in the laboratory twenty years ago. Among the first of these experiments was the use of hydrogen in high temperature anneal and purification of iron for special purposes. Here it proved valuable in preventing scaling or burning, and also in slowly removing carbon and similar foreign elements. These experiments on heating iron alloys in hydrogen were done many years ago, and applications of the process have continued to increase not only in the Laboratory, but for various purposes in the factory.

Over a hundred hydrogen furnaces are in use daily in the Laboratory for production of special alloys of tungsten, molybdenum and other metals, and over a hundred more are in use for various experimental purposes. Most of these use hydrogen not only for the atmosphere within the furnaces proper, but also as a protecting gas to preserve the molybdenum or tungsten heating units of the furnace. In this way we are able to make use of temperatures higher than we otherwise could, even if the heating units were platinum.

The Electrolytic Generator Increases Production

In our work on pure metals, many of which are made from their oxides by reduction, we soon found the chemical methods of producing hydrogen too expensive, so we constructed a home-made electrolytic generator. This gave us a few hundred cubic feet of hydrogen per day. Then we began to pipe the gas to work rooms, and 95 rooms in the Laboratory are now so supplied.

Our first electrolytic generator probably cost a thousand dollars, and this gave gas enough for the early work on annealing experiments and on reducing tungstic oxide to metal for lamp filaments. In what we call the early days, this metal was pressed into lamp filaments by use of an organic binder, and the next considerable use of hydrogen was to remove the carbon of this organic matter from the filaments so as to give a pure metallic product. This became a large-scale factory process. In this work we became better and better acquainted with the gas. Its dangers shrunk and its cost was reduced as improved electrolytic gas generators were obtained. Soon we had installed, for combined factory and laboratory uses, several large electrolytic gas generators, and we then began to pipe the factory yard, so that the gas could be brought into scattered buildings for special work. The furnaces for the reduction of tungstic oxide are still in use, for there seems no better way to prepare pure tungsten than this, but from time to time many other metallic oxides are reduced by the same process. For example, all the molybdenum used in our radio tubes is made this way. Many new alloys have also been made by the reduction of corresponding oxides in hydrogen atmosphere. Copper, cobalt, nickel and iron are most easily prepared pure by use of these furnaces.

Such details are not particularly interesting to you, but I want to show how extensive the application of a single element may become when it is allowed to expand.

After tungstic oxide is reduced in a hydrogen atmosphere the powder is pressed into bars which are sintered in hydrogen and these are then suspended in another hydrogen atmosphere and several thousand amperes sent through to produce compact metal. The bars are then worked down mechanically while hot, and frequently reheated in other hydrogen furnaces on the way to the finished tungsten wire, sheet, or other special shape. Thus at least four different hydrogen furnaces are required in the manufacture of finished tungsten metal.

Hydrogen in Cutting and Welding Metals

About as soon as we had apparatus for supplying hydrogen in quantity by electrolysis of water, engineers brought us problems such as cutting and welding metals by its use. The electrolysers supplied both hydrogen and oxygen. A gas flame using these gases cuts iron and steel cheaper and better than power saws can do it, and so the risers of steel castings, which are difficult to saw, were cut by gas. These "risers" are large masses of steel which are a part of steel castings as these come from the molds, but are not a part of the finished product. At about this time the hydrogen and oxygen began their work for us in cutting iron plate and structural steel. This line of work has continued to expand, has quite revolutionized motor frame construction and has economically displaced a great deal of foundry equipment. A gas torch became a rapid jig saw to cut out the patterns, another torch or an electric arc became the sewing machine for properly stitching them together. The cast-iron bodies or frames of large generators which constitute about all one sees of such apparatus, were relatively expensive. Replacing cast iron was only a question of cutting rolled plates into suitable form, arranging them in place and welding them together. The cutting was not practicable by old methods, but the gas torch solved the problem. Oxy-acetylene, oxy-hydrogen and oxy-illuminating gas are now used for this cutting. The welding of the assembly into a solid frame is now done by one of the various electric arc processes. By this improvement, large foundries have been made quite obsolete. Only about one per cent of the frames of large electric generators are cast nowadays, the other 99 per cent being gas-cut and arc-assembled. This entire change has come about within the past five years.

Hydrogen as a Flux

While we were first working with ductile tungsten we produced tungsten contacts for automobile coils and vibrators. Millions of these were used. They are practically little steel tacks with heads of pure tungsten. Dr. Coolidge found that the proper way to stick the tungsten to the steel was to melt a very little copper between the two when they were heated in a pure hydrogen atmosphere. No flux was necessary, for the hydrogen performs this office. The fluxes used in metal brazing dissolve metallic oxides, thus giving clean metallic surfaces for welding. The hydrogen removes the oxides by reducing them to the corresponding metals, so hydrogen has also become a flux.

This contact-process led to general studies of brazing in hot hydrogen, and the advantage of using copper for brazing became more and more apparent. Melted copper in the presence of hydrogen has a very remarkable tendency to flow even into the narrowest cracks and crevices in steel. And it becomes an easy matter to fill whatever pores there may be in iron by impregnating it with copper. This is a very convenient way of insuring vacuum tight iron apparatus. It would now be hard to get along without it. In this connection we found that highly-compressed and sintered powders like tungsten and iron could drink up and absorb large quantities of molten copper in hydrogen, and in this way a number of practical applications have been found where the electrical or thermal conductivity of the copper combined with hardness or infusibility of the tungsten was desired.

Copper brazing in hydrogen has now been used on a large scale for years. This is done in electric furnaces 12 to 24 ft. in diameter. Turbine parts, such as steam labyrinths, and many devices such as steam plates for hydraulic presses are thus produced. At present, the evaporators of all of our electric refrigerators are copper brazed after first having been welded by the new atomic hydrogen process. In other words, our latest factory product uses hydrogen twice.

The Atomic Hydrogen Arc

In Langmuir's study of reactions within partially-evacuated lamp bulbs, a group of interesting chemical phenomena had been observed. They led to the conclusion that hydrogen molecules were decomposed into atoms under the operating lamp conditions and that, while these could exist in the atomic state, they would readily recombine again. After a study of the energy relationships it appeared that the mere reunion of hydrogen atoms should give a very localized and exceedingly high temperature. As hydrogen can be dissociated into atoms in an electric arc and combine again a short distance away, we have a method of transferring electric energy from an electric arc to the spot where excessive temperature is desired, for the gas atoms will give up this extra energy there. The heat of combustion of the hydrogen with oxygen may be looked at as an added source of heat to this atomic heat. The advantages of the atomic hydrogen arc are two-fold. The metals to be arc-welded are protected by an envelope of hydrogen, and in the hottest part, where welding is being done, there is no danger from oxygen, but, furthermore, atomic hydrogen is an excessively powerful chemical reducing agent, more powerful than molecular hydrogen. For this reason arc welding of otherwise refractory substances has become possible, and the surety of the weld increased.

Another intermediate application of hydrogen arc welding is also important. In this case hydrogen is fed into the ordinary low-voltage welding arc, thus surrounding it with the atmosphere of hydrogen which prevents the oxidation of the metal during welding. In other words, we now have three hydrogen arc types of welding: the direct gas flame without electrical energy except the original production of the gases from water, then the ordinary electric arc surrounded by hydrogen in place of air, and last the electrically dissociated hydrogen arc which really also includes in effect both the others.

Hydrogen and Refrigeration

In the new refrigerator evaporators, where every precaution must be taken against leaks of SO_2 gas, the main joints are first arc-welded with atomic hydrogen and then made doubly sure by an added copper braze in hydrogen gas. This double use of hydrogen is mentioned not so much to advertise refrigeration as academically to emphasize the wide application of the single element, hydrogen. And yet hydrogen differs from the other known elements in being the simplest of them all.

I take a sort of wicked delight in mixing the sublime and the ridiculous, passing directly from electric refrigerators to the structure of the atom, for example. A whole hour could easily be devoted to describing the use Professor Bohr and others made of the known spectral lines of hydrogen as first explaining the inner structure of our simplest atom. Other structures followed, and also the interpretation of spectral lines themselves, and consequently of light production in general. In other words, the hydrogen spectrum with its orderly arrangement of separate lines was made the key for unlocking atomic structure, the source of light waves, and for locking into the two the entire celestial mechanics.

We look at light nowadays as very short wave radio sent out by radio stations within the atoms. Electrons in atoms, like satellites in a solar system, return to their proper atomic orbits when electrically activated—*i.e.*, disturbed, and this return sets up characteristic electro-magnetic waves in space which we call light of various colors or wavelengths. In this way hydrogen has helped us form some ideas of the future experiments to be directed toward more effective artificial light.

Hydrogen and Ultra-violet Light

In speaking about spectral lines, it is also a temptation just to refer to the extensive field opened up by the studies of the ultra-violet light and vitamin production, because similar studies must eventually be made of other spectral regions. One narrow spectral band from a mercury arc in a quartz container is very much more effective than sunlight in bringing about certain reactions of living processes, and as a result we may now purchase valuable medicinal foods which owe their importance to an exposure to this spectral band. Moreover, much of the difficult glass and quartz-blowing for special lamp bulbs for such uses is done in the oxy-hydrogen flame, and so our laboratory glass-blowers use large quantities of the gas in this quite prosaic way, for purposes of assisting in very complex organic reactions.

To return to hydrogen phenomena again:

If a piece of ordinary copper is heated red hot, but below its melting point in hydrogen or other reducing atmosphere, it loses its strength and becomes entirely useless. There have been many cases where this has developed as a real trouble in the factories. It is due to the reduction of the traces of oxide which are present in all ordinary copper. It can be obviated by using a special metal entirely free of oxide, but this is not readily procurable. This effect was studied and obviated by the use of hydrogen.

An equally interesting action between steel and hydrogen is also worth describing, because I am trying to illustrate the spread of contiguous hydrogen knowledge. If a strained steel spring is exposed to processes which produce atomic hydrogen at room temperatures, for example, in acid pickling or in electrolysis, even where corrosion is obviated, as in alkaline solutions, the fatigue value of the steel is greatly reduced, though its tensile strength is not much affected. This is a very important matter and has wide applications. One would not pickle or electroplate an important steel spring nowadays without in some way taking care of this effect. Here is the peculiarity, that cold atomic hydrogen penetrates the steel easily, while molecular or gaseous hydrogen cannot do so. At red heat, molecular or ordinary hydrogen passes readily through solid iron, and is apparently the only gas that does so. This is probably why it is relatively easy to make copper penetrate nearly dense iron, because oxide and carbide are removed and a way opened up.

We have long been using mixtures of nitrogen and hydrogen where the cost of pure hydrogen was too great. This mixture is made by burning the oxygen out of air by an excess of hydrogen, then removing the water thus formed. The hydrogen actually burned by the process is replaced by the reaction in the resulting gas-mixture by twice its volume of nitrogen. This mixture has long been in use in tungsten lamp factories. Nowadays it is cheaper to produce nitrogen directly by the liquid air process and mix it with electrolytic hydrogen. This mixture constitutes both an economy and an additional safety service. We have ahead of us still cheaper hydrogen when the work demands it, because it may be made directly from water and carbon.

Hydrogen as a Cooling Agent

As hydrogen naturally diffuses faster than other gases, it can be used to advantage where it is important to remove heat rapidly. For this reason high-speed motors and generators are being operated in this gas in developmental tests. On reflection, we see that it is only a sort of natural accident that we cool most electrical apparatus by the air which is about it. Hydrogen is better than air for the purpose. Much of the difficulty in connection with such applications of hydrogen consists in keeping the gas from leaking out of the system, but as it is cheaply produced from water by electric current, it will probably be used for this purpose also.

In these superficial notes on hydrogen I have referred only to the major part of our Schenectady applications. But its use elsewhere is not confined to balloons! It was once used in the Drummond light, a stick of lime incandescent in oxy-hydrogen gas flame. It is worth while pointing out, however, that its study elsewhere had led to the production of artificial ammonia, and to the so-called hydrogenation of oils, which opened new fields for use of cottonseed oil. It is an important constituent of water gas and is being expanded into the field of artificial motor fuel, because it may be artificially combined with carbon as benzine and countless other combinations.

This growing demand for hydrogen has also brought along with it substitutes for many of the original uses, such as nitrogen-hydrogen mixtures and illuminating gas-oxygen mixtures for metal cutting.

My idea in this discussion has been that any one subject or element might be chosen to illustrate the effect of continued research and that, while I might be expected to choose light, radio or tungsten, which we are also studying, it is more novel and instructive to consider our volatile old friend, hydrogen.

Turbine Exhaust Steam to Save Equivalent of 222 Tons of Coal a Month in Heating the Buildings of the University of Kansas

In the combined heating and power plant of the University of Kansas, at Lawrence (Kan.), two condensing turbine-generators were installed in 1922. These are of 200-kw. and 500-kw. capacity, and operate at 175-lb. gauge pressure. For heating the buildings, steam at 10-lb. gauge pressure has been obtained from a reducing valve between the 175-lb. boiler header and the heating mains.

Because of increasing load, the 200-kw. turbine became inadequate and was not being used. A study of the heating and load conditions which was later made showed that the removal of the load of the electrically-driven auxiliaries of the smaller turbine would enable this unit to carry the University load for the greater part of the day, and its exhaust steam then furnished to the heating system would save the equivalent of 222 tons of coal per month during the winter season—provided such non-condensing operation were possible. Further investigation revealed that the addition of an oil-operated governor and the substitution of new buckets and nozzles would permit the unit to deliver 200 kw. while operating non-condensing at 10 to 15 lb. back pressure, thus making it suitable to function also as a reducing valve for supplying low-pressure steam to the heating system. The cost would amount to no more than the fuel saving during the first month.

Flashing of Synchronous Converters

Cause and Development of a Flash-over—Effect of Armature Reaction—Results of Sudden Application and Release of Load—Influence of Frequency and Voltage—Prevention and Protection

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As the authors mention, in the concluding paragraph, the performance of modern synchronous converters is not handicapped in commercial service by influences tending to cause flash-over. However, the phenomenon of flash-over and the developments that have combined to eliminate it and its consequences are subjects that are rich in engineering interest, and are explained with unusual clarity in this article.—EDITOR

IT is a generally recognized fact that the behavior of the synchronous converter with respect to flashing at the commutator is different from that of the direct-current motor or generator. But it appears, however, that the phenomena accompanying flashing are not clearly understood. Very frequently the question arises as to the direct cause of this disturbance, with queries concerning some of the theory involved. Various articles have been published on the subject, but it is felt that they are either lacking in sufficient detail to give a good physical picture of flashing or are so technical as to be suitable only for specialists on such matters. It is the purpose of this article, therefore, to present in a simple manner a clear conception of the flashing of converters, to discuss the physical relations of the quantities concerned, and to describe the order of occurrence of the events accompanying a flash-over.

Definition and Cause of Flashing

By flashing is meant the establishment of a complete arc path across the commutator surface from one brush group to an adjacent group, or from a brush group to grounded metal parts. This phenomenon is distinct from a simple "spit," or case of severe sparking at the brushes, which is sometimes called a flash. Arcing often occurs between adjacent bars, either as they leave the brushes, or because of lint, dust, or foreign material falling across them; but such bad sparking cannot be classed as a complete arc-over between brushes which is the phenomenon of primary importance.

Flashing is not instantaneous but is a progressive event initiated by the generation of ionized conducting vapor at the brush contact surfaces, either by excessive overload current or by the short-circuit currents in the commutated armature coils. It is necessary that there be generated between adjacent commutator segments, as they emerge from under the brushes, a voltage of sufficient magnitude to maintain an arc through the conducting vapor which has been formed. If this action continues during the time required for the first arc to be carried to the next brush group, by the rotation of the commutator, a complete train of these arcs is formed and flashing results. This entire operation may be called a "primary flash-over."

Because of the primary flash-over or severe sparking, sufficient conducting vapor may be produced to fill the space between a brush group and any grounded metal parts, thus permitting the formation of an arc by way of this external path. An arc of such a character is then termed a "secondary flash-over." Although this flash-over may sometimes occur alone, it usually follows a primary flash-over, which is believed must first be present so that the required ionized vapor is produced to permit a conducting path to ground.

Very few flashes are caused directly from excessive load current. The greater part of the conducting vapors are created by the short-circuit currents flowing in the armature coils and the brush faces. These short-circuit currents are in turn produced by distortion in the magnetic fields resulting from synchronizing currents which follow large changes in direct-current load or abrupt alternating-current disturbances, as are discussed later. This article considers mainly the effect of the direct-current load swings as they are of fundamental importance.

The principal cause of flashing is the phase-angle displacement of the slip-ring voltage from the main-bus voltage. This angular displacement is composed of two elements. The chief component results from the reactance drop in the transformer and supply lines due to energy currents, and the second component is caused by the shift of the magnetic center of the total field flux. The latter displacement is small, however, and amounts to only a few degrees. Furthermore, the presence of damping windings in the converter pole faces prevents any rapid motion of the flux wave and its effect is therefore greatly reduced.

With no direct-current load on a converter, any angular displacement between slip-ring and bus voltages results in the flow of corresponding synchronizing currents which distort the main field because there is no compensation for the armature reaction of these currents. The distortion shifts the edge of the flux wave over the commutating zone and the armature coils short-circuited by the brushes then have corresponding voltages induced in them by rotation through the unbalanced field. Sparking occurs as the segments emerge from under the brushes, and the voltage between bars may be sufficient to maintain the little arcs.

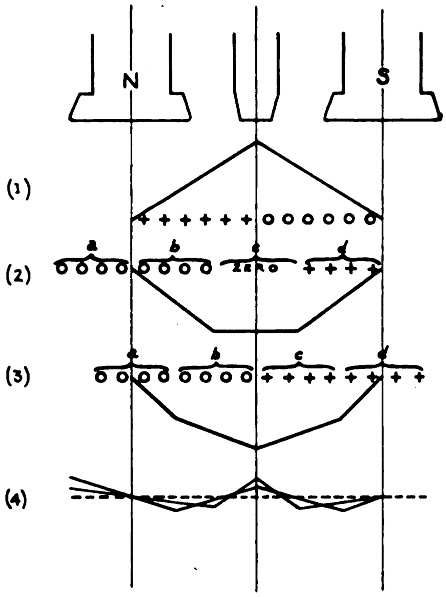

Fig. 1. Graphical Analysis of the Armature Reaction of a Synchronous Converter

Curve (*1*): Direct-current distribution
Curve (*2*): Alternating-current distribution
Curve (*3*): Alternating-current distribution, 30 deg. rotation of armature
Curves (*4*): Resultant distribution of armature reaction

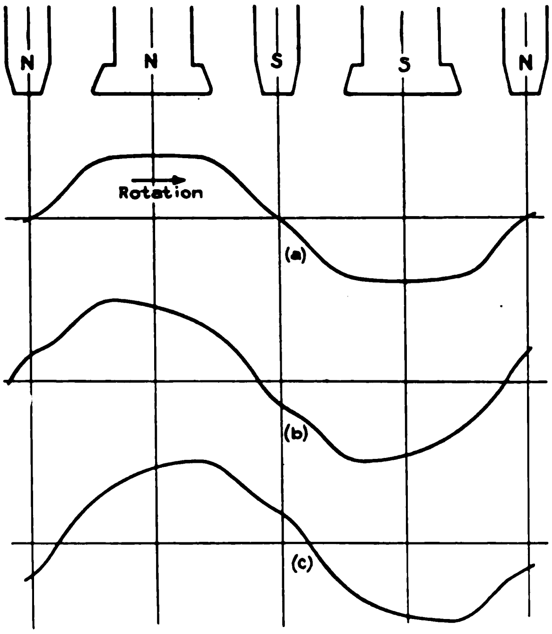

Fig. 2. Flux Wave Shape of Converter

Curve (*a*): Main field acting alone
Curve (*b*): Total field showing effect of armature currents when converter operates as a synchronous motor
Curve (*c*): Total field when converter operates as an alternator

Armature Reaction

The subject of armature reaction of a synchronous converter is directly associated with a consideration of flashing, and the topic is therefore treated at this point. The consideration precedes that of flashing proper since specific reference will be made to the points with which armature reaction is concerned.

The synchronous converter is usually characterized as the combination of the synchronous motor and the direct-current generator. According to this analysis, the total armature reaction of the machine can be treated by separately considering the armature reactions of the assumed motor and generator. Fig. 1 shows graphically the physical relations and magnitudes of the quantities involved.

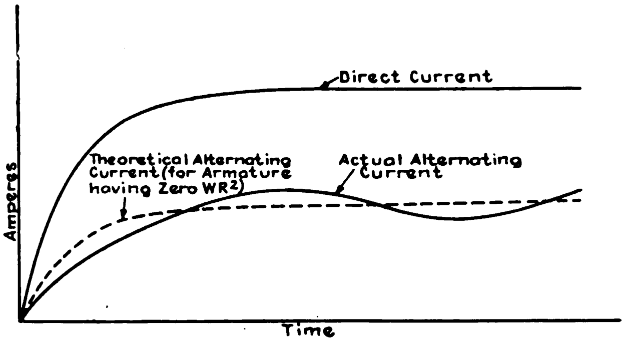

Fig. 3. Curves Showing the Rise of the Direct and Alternating Currents in a Converter as Load is Suddenly Thrown on the Machine

The direct-current armature reaction given as Curve (*1*) in this illustration is of the well-known triangular shape; while that of the alternating-current varies between the two forms shown as Curves (*2*) and (*3*). A six-phase converter has been chosen, with full-pitch infinitely-distributed windings, each phase belt covering 60 electrical degrees. Over a pole pitch the integrated value of the direct-current armature reaction is practically equal to the alternating-current armature reaction, save for the slight difference due to the current supplying the machine losses. But at any point along the periphery, the net armature reaction is not necessarily zero. It has a value given by the resultant space waves shown as Curves (*4*).

The net value of armature reaction which is shown in the quadrature axis is due to the direct current and is that resultant for which the commutating poles of a converter must compensate. Its value varies between 11 and 23 per cent of the direct-current armature reaction, and for this reason the commutating field of a converter is approximately only one-third as strong as the commutating field of a direct-current generator. The influence of this weak field on sparking will be considered later in this article.

Under the main poles, the armature reaction varies in a linear manner from a definite value at one pole tip to the equal and opposite value at the other pole tip. The effect of such a variation is to produce a so-called cross-magnetizing field which increases the flux density at one pole tip and decreases it at the other. The "center of gravity" of the total field is thus shifted toward the side of greater flux density. Calculations and test show this displacement to be rather slight, ranging from two to six electrical degrees at full load.

The normal flux wave-shape due to the main-field m.m.f. acting alone is flat-topped, and symmetrical about the main-pole center as shown by Curve (*a*) in Fig. 2. The presence of the alternating-current armature reaction acting alone alters this flux wave. If the machine is operating purely as a synchronous motor, the resultant wave is shifted backward against the direction of rotation and is peaked under the leading pole tip. This is shown by Curve (*b*). Similarly, if the converter operates as an alternator, the flux wave is moved forward and the wave becomes peaked under the trailing pole tip Curve (*c*). The influence of this wave shift has a direct bearing upon flashing as will be explained later.

It is perhaps well to point out the distinct effect of the commutating pole upon the flux in the quadrature axis. The reluctance of the magnetic path at this position is considerably reduced by the presence of the commutating pole iron, and a greatly increased flux variation is allowed with changes of armature m.m.f. over the neutral zone. Commutating pole converters are therefore more sensitive to flashing than are non-commutating pole machines, but this undesirable characteristic of the former is reduced by the use of high-reluctance commutating poles.

Operation Under Sudden Application of Load

When a fairly heavy direct-current load is suddenly placed on a converter, the effect of the rising alternating-current flowing through the transformer reactance causes the secondary voltage to lag that of the primary. This is equivalent to a momentary change in frequency at the slip-rings; and the converter, like the synchronous motor, will tend to slow down and drop back a few degrees with respect to the main bus. While changing its angular relation the armature will give up kinetic energy, and since the total output of the converter is composed of the energy input from the alternating-current lines and the rotational energy from the rotor, the effective alternating current will be less than that corresponding to the output.

As with all rotating apparatus when suddenly subjected to an increased load, the converter overshoots the mark and the actual alternating-current wave thus oscillates about the theoretical value as the armature of the machine oscillates about its steady-state position. Under normal operation, the alternating and direct currents are in a constant ratio to each other.

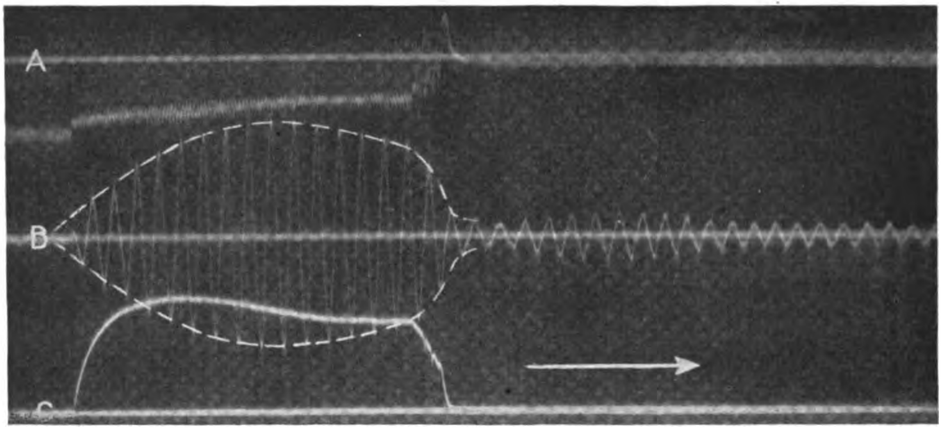

Fig. 4. Oscillogram of a Short-circuit on a 1000-kw., 600-volt Synchronous Converter Showing No Record of Flashing

A: Zero line of direct voltage
B: Zero line of alternating current
C: Zero line of direct current (580 per cent normal)

Fig. 3 shows the rise of the direct and alternating currents with respect to time. The direct-current and the theoretical alternating-current curves are exponential and rise quickly to their final values. Assuming a six-phase machine, the effective value of the alternating current under steady conditions is approximately one-half of that of the direct current. Because of the variation in armature speed just pointed out, the actual alternating-current wave lies below the theoretical curve until the time at which the direct current has reached its maximum value. The difference between the ordinates of these alternating-current curves represents the stored energy given up by the converter armature. Similarly, when the actual wave lies above the theoretical curve, the difference in ordinates indicates the electrical energy from the alternating-current lines which is converted into mechanical energy of rotation.

While the alternating and direct currents are rising the converter acts as a pure direct-current generator to the extent of the kinetic energy of rotation which is converted into electrical output. Under such conditions, the ratio of the direct-current armature reaction to the alternating-current armature reaction is greater than normal, and the resultant value of these reactions under the commutating poles is also greater. As previously pointed out, the commutating poles of a converter are weak during this particular part of the cycle and poor commutation with sparking may result. If the short-circuit or overload on the machine is extremely severe, commutation may be so bad as to cause flashing. However, such an occurrence is exceptional.

The oscillogram in Fig. 4 is of a short-circuit test on a converter in which the load current rose to 580 per cent of normal value without flashing. The alternating current is correspondingly high. This oscillogram is typical of such short-circuits and clearly shows the alternating-current wave shape as it varies with time. It will be noted that during the rise of current there are no breaks in the wave, which breaks would be present (Fig. 5) had any unusual occurrence, such as flashing, taken place. In the many tests conducted by the General Electric Company, a converter has only once or twice flashed over upon the sudden application of a very heavy load, and it can be definitely stated that converters very rarely flash over when large loads are suddenly impressed on them.

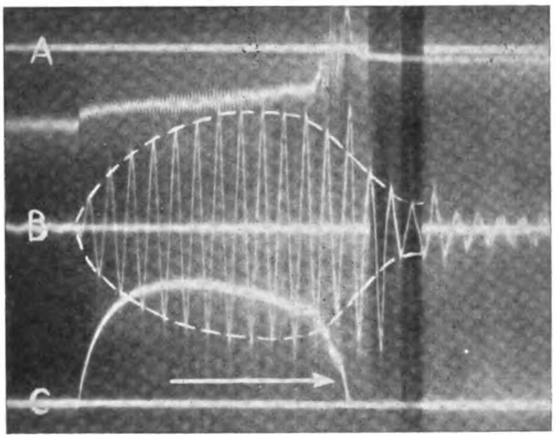

Fig. 5. Short-circuit on a 1000-kw., 600-volt Synchronous Converter Showing a Record of Flashing

A: Zero line of direct voltage
B: Zero line of alternating current
C: Zero line of direct current (600 per cent normal)

As an explanation of the foregoing statements, let Fig. 6 represent an alternator supplying power through a transformer and a converter to a direct-current load. The sketch is made single-phase for the sake of simplicity, but the generator is assumed to be of infinite-bus character so that the primary voltage of the transformer is not affected in magnitude or in phase position, with respect to the system which the generator represents, by any load on the secondary. When the switch connecting the load is closed the direct and alternating currents immediately rise, the converter armature supplying some of the output energy mechanically as just described.

The vector diagram in Fig. 7 conveniently explains the operation. Let OA be the voltage impressed on the primary of the transformer. The vector OB, equal and opposite to OA, then represents the no-load secondary voltage; and correspondingly, OC is the load secondary voltage of the transformer impressed on the converter slip-rings. This makes BC the impedance drop due to the reactance and resistance in the transformer and converter supply lines. Assuming that the diagram is drawn for unity power-factor at the slip-rings, the locus of the reactance drop is the circle built upon OB as a diameter.

The displacement angle α is thus the angle by which the slip-ring voltage of the converter drops behind the system voltage; or expressed in a slightly different manner, α is the shift in voltage between the converter rings and the source of constant potential, wherever it may be. The reference voltage is usually taken to be that of the transformer primary.

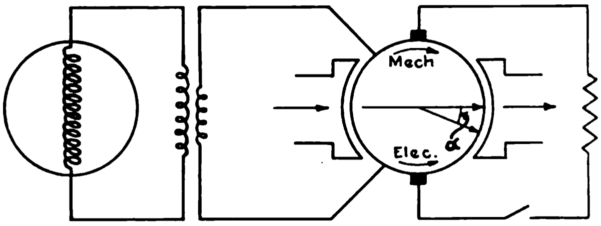

Fig. 6. Alternator Supplying Power Through a Transformer and Converter to a Resistance Load

During the application of load the frequency at the converter rings is momentarily altered and it decreases to the extent that the angle α is lost. Obviously, however, it will be gained when the load is released and the converter moves back into its no-load position. Unlike the synchronous motor, the converter does not drop behind its field, but, so to speak, continues to run in phase with its field. This is due to the fact that a converter carries no mechanical load save windage, friction, and copper and iron losses. A stroboscope, if connected across the converter rings, would therefore detect only a rotor displacement resulting from the losses plus the shift due to the field distortion. If the stroboscope were connected to the primary lines, however, it would then measure directly the total displacement angle which is the sum of the transformer shift α and the field shift.

Fig. 6, to which reference has already been made, represents a simple two-pole converter rotating in a clockwise direction. At no-load the fields are in line as shown. As the load comes on and the frequency is momentarily altered, the rotor field tends to shift clockwise with respect to the armature, producing a torque which drops the rotor backward through the angle α. The fields will thus remain with their original relationship.

The question might be asked: Why, when a load is suddenly thrown on a converter, does not the machine behave as a synchronous motor and immediately draw synchronizing current from the alternating-current lines? This action does not take place because the voltage at the slip-rings and the converter armature drop back together so that there is no relative motion between them. The alternating current is completely rectified and does not circulate in the converter windings as synchronizing current.

The following analogy might be used to illustrate the point in question. Consider Fig. 8, which shows an alternator driving a synchronous motor. Suppose that the prime mover momentarily lags and then continues to run at some small angle behind its former position. The generator will of course be retarded similarly. Due to its own inertia the synchronous motor will not follow the generator exactly but will oscillate slightly before falling into its final position. The oscillation will be accompanied by the flow of synchronizing currents.

If, however, the same retarding force be applied to the motor as it is to the alternator, and at the same time, both machines will drop back together and will always be in step. While the motor is lagging there is no relative change between its terminal and generated e.m.f.'s and no reactive current will flow other than that which might be already present.

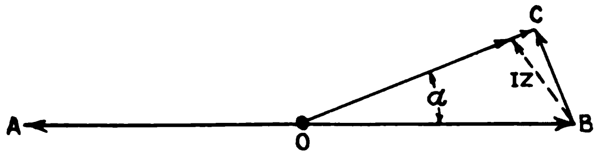

Fig. 7. Vector Diagram for Converter with Unity Power-factor at the Slip-rings, Showing the Angular Displacement Between the Main-bus and Slip-ring Voltages

Such is the condition really met in a converter, and the generator and motor action of the converter may be represented by the generator and motor in Fig. 8. Figuratively speaking, the converter is not aware of any change in phase angle. Both the alternator and synchronous motor, on the other hand, show displacement angles between their rotors and rotating fields proportional to their loads.

Operation Upon Sudden Release of Load

Contrary to the opinion which is frequently held concerning flashing of converters, it is upon the sudden release of a large load that the machines are most susceptible to arcing, and not upon the application of the load. When the direct-current circuit is opened, both the direct-current and alternating-current energy components become zero in a very few thousandths of a second. But because of the angular displacement between the main-bus and slip-ring voltages of the machine, large synchronozing currents flow and pull the machine back into its no-load position.

Owing to its inertia, the armature cannot immediately swing forward, and the voltages just mentioned will not coincide until the rotor has reached its initial position. What formerly was the impedance drop due to the alternating current which was rectified, *i.e.*, the closing vector shown in Fig. 7, is now the impedance drop due to synchronizing currents. These currents are energy currents as can be seen from the diagram. Since the current vector would be practically normal to the impedance drop, it would then be almost wholly in phase with the slip-ring voltage.

Formerly, the machine was delivering direct current and the armature reactions of the direct and alternating currents approximately balanced each other. Now, however, only the alternating-current armature reaction exists and there is no compensation for it. The total field flux is thus distorted. The synchronizing currents act so as to accelerate the machine in the forward direction, and the converter operates strictly as a synchronous motor. The flux wave will therefore shift backward as shown in Fig. 2 (b).

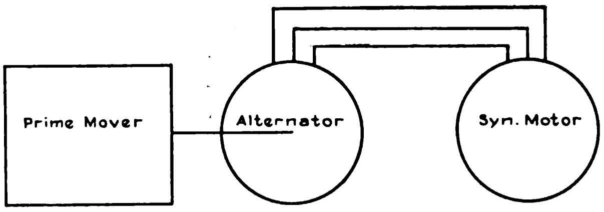

Fig. 8. Alternator Driving Synchronous Motor, Showing Analogy of Converter Upon Sudden Application of Load

It will thus be noticed that the sides of the coils which are short-circuited by the brushes are not in a field of negligible intensity, but cut a considerable amount of flux as they pass through the neutral zone. The voltages induced in the coils by this flux cutting produce short-circuit currents which complete their paths by way of the brush faces. Sparking, which may be of a severe character, then takes place as the commutator segments move from under the brushes.

It will also be noticed in Fig. 2 that the sides of these coils are in a field which is rapidly increasing in value as they leave the commutating zone. The voltage between bars is therefore distinctly higher than usual and arcs between commutator segments are more easily maintained. If the little arcs are sustained until the next brush group is reached, the converter will flash over.

The converter, in being pulled forward into the no-load position, will overshoot its mark, because of rotor inertia. The machine then undergoes a period of generator operation during which kinetic energy of the armature is converted into electrical energy and returned to the alternating-current lines. Owing to the generator action, the standing flux wave is moved forward in the direction of rotation, as previously explained in the treatment of armature reaction. The sides of the short-circuited coils are thus in a field which is decreasing in value as they leave the commutating zone and they actually pass a point at which the voltage between bars is zero, Fig. 2 (c).

This decreasing field, which passes through zero, is not of sufficient strength to induce voltages that will maintain arcs between commutator segments. So on the retarding swing, or during generator action, flashing will not take place. This statement is quite sufficiently borne out in the many tests made by the General Electric Company. All but one or two of the oscillograms ever obtained in such tests show that flashing occurred on the motor swing.

It might be remarked that during the accelerating swing a point of zero voltage between commutator bars is met just before each brush group is reached, and that this voltage in itself is insufficient to maintain the small arcs. But, by the time the first arcs reach the weak field, which is fairly close to the brush group, the expanding ionized vapors form a path between that brush group and the train of arcs along the commutator.

The complete record of flashing, with its accompanying details, is registered by the oscillogram. During the occurrence the ionized vapor, which has been created by sparking as the short-circuited coils emerge from under the brushes, forms an external path between brush groups. This path is in parallel with the armature winding, so that superposed upon the current which flows through the armature coils is the additional current of this external circuit. If flashing has taken place it is therefore distinctly indicated by pronounced peaks which extend outside the otherwise smooth envelope to the alternating-current wave. A break in this envelope is thus a criterion of flashing.

The oscillograms in Figs. 4 and 5 are illustrations of short circuits on a converter in which both possibilities of load removal are clearly shown. In Fig. 4, the envelope of the alternating-current wave retains its smooth characteristic during the entire time of short-circuit, and hence denotes that flashing did not occur. The alternating current dropped rapidly to a small value after the short-circuit had been cleared, and then, as synchronizing current, oscillated with a varying amplitude for many cycles while the machine was settling down to its no-load running position.

On the other hand, the envelope to the alternating-current wave in Fig. 5 contains a definite peak which extends above the upper dotted line, and two peaks which extend below the lower line. The machine flashed over in this interval of time. It will be noted that the flash took place after the direct current had become zero, and during the period favorable to flashing, *i.e.*, during motor operation of the converter.

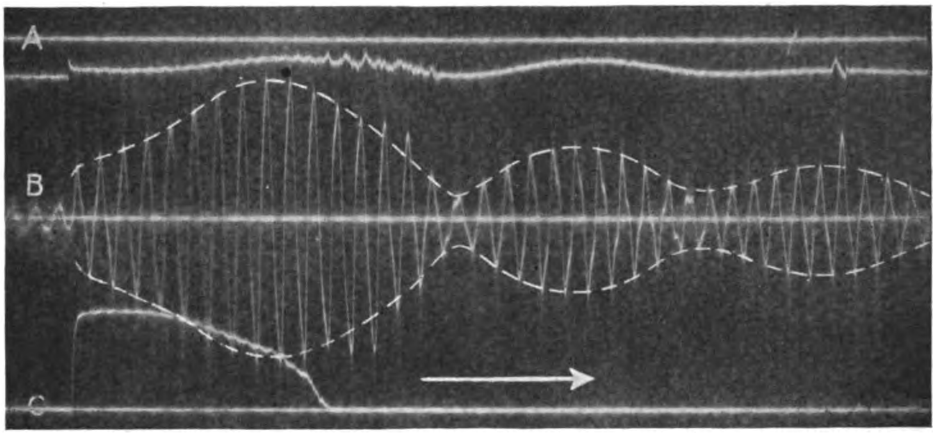

Fig. 9. Short-circuit on a 300-kw., 750-volt Synchronous Converter, Showing Two Flashes
A: Zero line of direct voltage *B:* Zero line of alternating current
C: Zero line of direct current (550 per cent normal)

The oscillogram in Fig. 9 is the record of a short-circuit following which the machine flashed over twice. The abrupt rise of the direct current, which reached 550 per cent of its normal value in about one-quarter of a cycle, is very noticeable and shows the suddenness with which the load was placed on the converter. The only effect of this action to be seen on the film is the dip in the alternating current envelope as the armature dropped back and gave up of its kinetic energy. When the armature responded it undoubtedly swung backward too far because of its inertia, and a large synchronizing current accounts for the high peaks in the alternating wave after the direct current had decreased considerably.

The oscillogram shows two regions of motor operation of the converter and one region of generator action. Just beyond the point at which the direct current became zero, the machine flashed over, as evidenced by the breaks in the alternating-current envelope. This flash occurred during the favorable conditions accompanying motor action. The converter then passed through the period of generator operation and on into motor action again, during which the machine flashed over the second time. The fact that the converter flashed during both intervals of motor action, and not during generator action, bears out the theory that the conditions attending the first operation are conducive to flashing while those accompanying the second operation are not favorable.

Influence of Frequency and Voltage Upon Flashing

The higher the frequency of a synchronous converter the greater is the flashing tendency. Being a synchronous machine, the frequency of the converter must bear a fixed relation to the speed and number of poles, assuming the same distance between poles and brush groups, and it thus follows that the peripheral speed of the armature and commutator increases with frequency. Although in some instances commutator peripheral speeds have risen to 6500 ft. per min., the generally accepted value is approximately 6000 ft. per min.

Since the distance between two points that are a pole pitch apart on the commutator surface represents one-half cycle, it requires only 0.02 sec. on a 25-cycle machine for a segment to pass from one brush group to the next. On a 60-cycle machine the time is lowered to 0.00833 sec.; and if a succession of small arcs between commutator bars persists for this length of time, flashing will occur. The diameter and peripheral

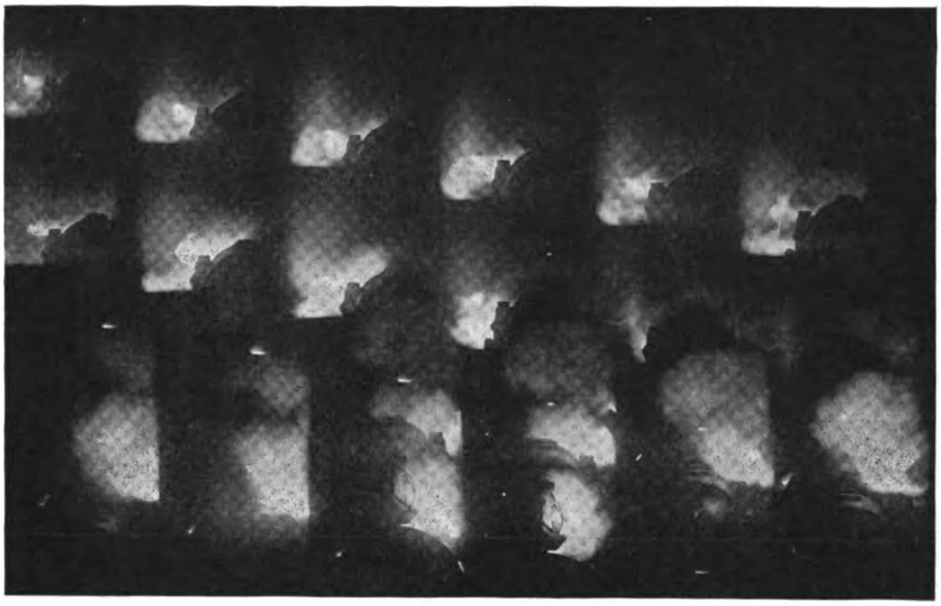

Fig. 10. High-speed Photograph of Short-circuit Flashing on a 500-kw., 600-volt Synchronous Converter Having a Standard Circuit Breaker as Its Only Protection

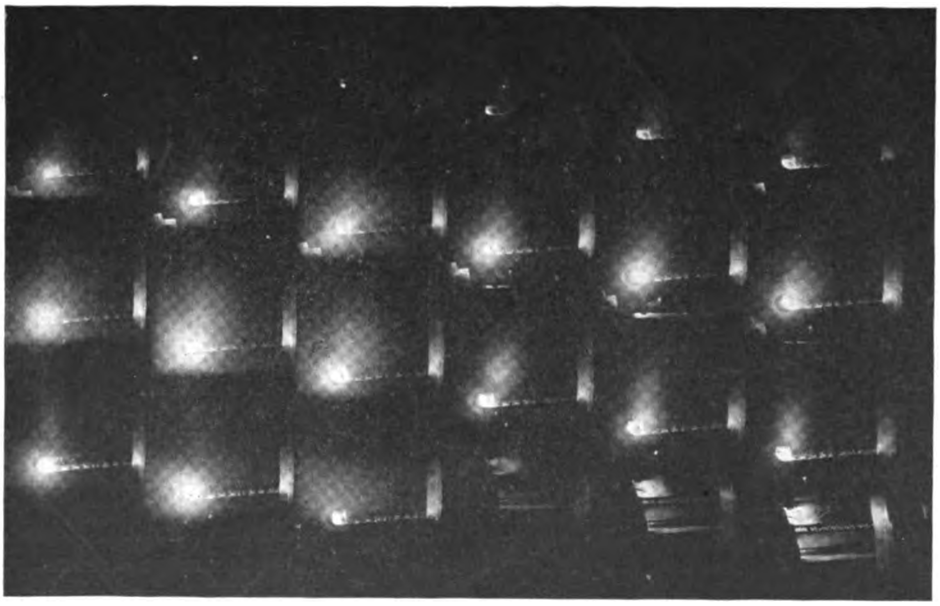

Fig. 11. High-speed Photograph of Short-circuit on Same Machine as Recorded in Fig. 10 After the Addition of Flash Barriers. Had a high-speed circuit breaker been used in place of the standard breaker, even less arcing would have occurred

This photograph and that in Fig. 10 were taken with a special multi-lens camera. In each photograph the same brush group is shown in each exposure. The first exposure appears at the lower right and those which follow it appear in succession toward the left across the bottom row, then toward the left across the row above, etc.

velocity of the commutator do not influence these values of time; they are dependent only on the frequency. It is therefore observed that a 60-cycle converter is more prone to flashing than is a 25-cycle machine.

With the same diameter and peripheral speed of the commutator, the brush groups on a 25-cycle converter would be spaced 2.4 times as far apart as the brush groups on a 60-cycle converter. Hence, for the same potential between brush groups, the voltage gradient of the higher-frequency machine is much greater than that of the lower-frequency machine. A high gradient promotes flashing, since it tends toward increased arcing at the brushes and will better sustain the arcs once they are started. For this reason also, 60-cycle converters are more sensitive to flashing than are 25-cycle converters. Without exceeding normal speeds and gradients, it is possible to go to higher voltages with lower frequencies; and for a fixed potential gradient a 60-cycle machine, for example, could be designed for 750 volts and a 25-cycle machine for 1800 volts.

The foregoing figures refer to converters operating under normal conditions. As has already been described, abnormal conditions—such as sudden large fluctuations in load—produce a voltage distribution along the commutator surface that may result in a gradient twice the average value.

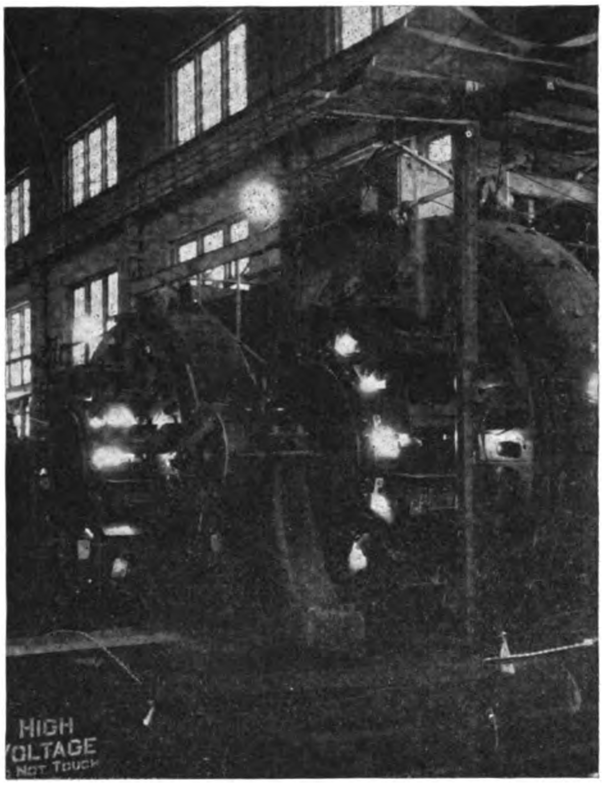

Fig. 12. Two 1500-kw., 750-volt Converters in Series, Equipped with Flash Barriers and Protected by a High-speed Circuit Breaker, Photographed While Subjected to a Dead Short-circuit. No flash-over followed the disturbance

It will be of interest to consider briefly the prevention of, or protection against, flashing. This subject may be divided into these two parts: prevention and protection.

Prevention

The synchronizing currents, which are one of the primary links in the chain of events causing a flash-over, may be reduced in magnitude by preventing or limiting the magnitude of direct-current short-circuit currents. This may be accomplished in either of two ways: by inserting limiting resistance in the direct-current feeders, or by using high-speed circuit breakers. The resistance method was early recognized and has been in use practically as long as traction systems have existed. In application, it is sufficient to carry the feeders out from the substation about 1500 or 2000 ft. before connecting them to the contact system (trolley or third rail). This permanent resistance in the feeders limits the maximum value of short-circuit current and therefore limits the maximum displacement of the converter armature and the resultant synchronizing currents and reactions. Such protection is far from complete because economy will not permit use of sufficient resistance to afford full protection.

The type of protection offered by the high-speed circuit breaker is of more recent development.

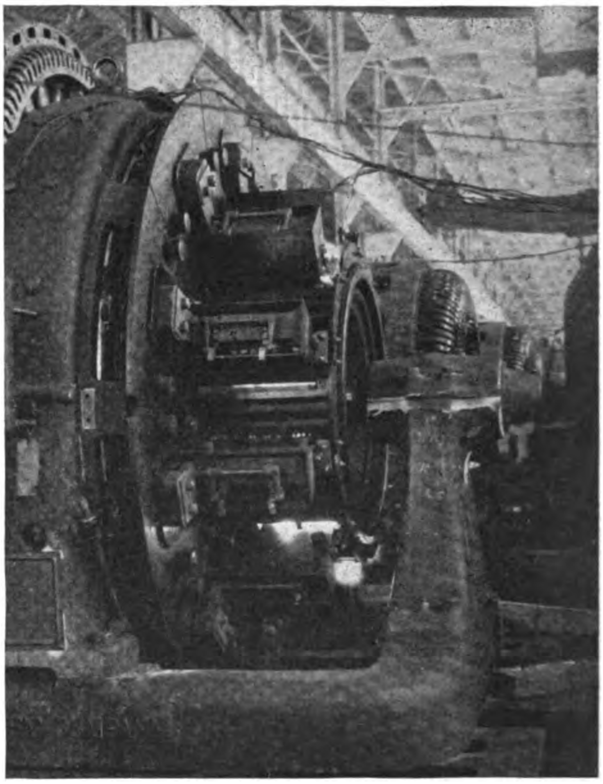

Fig. 13. A 1000-kw., 750-volt Converter, with Flash Barriers and High-speed Circuit Breaker, Photographed While Being Dead Short Circuited. No flash-over took place

The standard type of circuit breaker requires approximately 0.25 sec. to completely interrupt the circuit. This is more than enough time for the armature of the average converter to fall back to the maximum displacement, which usually is reached in 0.1 to 0.15 sec. The high-speed circuit breaker will completely interrupt the circuit in 0.015 sec. It is evident that the armature can decelerate but little during this short period. Consequently, the synchronizing currents and reactions are reduced in proportion.

To decrease the amount of flux through the commutating zone under conditions of unbalanced armature reaction, and hence to prevent severe sparking as the short-circuited coils leave the brushes, "high reluctance" commutating poles are used. These poles are built with a thick section of non-magnetic material placed in the magnetic circuit next to the frame. To obtain the required commutating flux under normal operation the field turns are increased, raising the excitation from 40 to 120 per cent of that of the corresponding direct-current generator.

Protection

Various means are available for protecting against flashing. The simplest of these is to surround each brush group and its supporting parts with a non-conducting arc-resistant material. This does not prevent flashing and the formation of an arc across the commutator segments between points of maximum potential difference. It does, however, prevent the transfer of arcing to the brush gear and thereby protects the vulnerable parts from damage.

Still greater protection may be had by using flash barriers. These barriers constitute individual inclosures around each brush group being made up of arc-resistant inert material and arranged so that they may be placed with the minimum clearance to the commutator. Their purpose is to prevent the spread of ionized conducting vapor and to deflect such vapors outward, away from the commutator, where they are mixed with cool air and rendered non-conducting.

The formation of a complete arc between brush groups may also be prevented by directing a current of air axially across the commutator surface from the armature toward the outer end of the commutator, between each group of brushes. This current of air will sweep the arc off the commutator before it can form a complete path between brush groups. The air blast may be provided either by a separate blower or fan furnishing air to an inclosure about the armature from which the air issues through a circular opening at the inner end of the commutator, or a fan that forms an integral part of the armature may be used. Either of these arrangements for their best functioning requires that the diameter of the commutator be approximately equal to that of the armature. With this arrangement it is usual to extend the shaft so as to remove the bearing some distance from the end of the commutator. The bearing and its supporting pedestal are then completely insulated with non-conducting arc-resistant material.

With the application of the means described for reducing the causes of flash-over and with the foregoing methods of protecting against the damaging effects of a flash-over—should it take place—this arcing no longer presents the problem it did years ago. As the result, converters of large capacity operate today under very severe conditions with perfectly satisfactory commercial performance.

C. B. & Q. R. R. to Have Powerful Gas-electric Cars

Eight of the most powerful single-unit gas-electric railway cars ever built will be used by the Chicago, Burlington and Quincy Railroad as a part of the large group of these vehicles being built up by that railroad. The program involves the use of more than 50 gas-electric cars. As fast as the new cars are delivered, they are replacing steam trains in light traffic and branch line service on various parts of the Burlington system. Early in 1927 five gas-electric cars were placed in service. In the middle of the same year eleven more were added; and an additional 36, built during the summer of 1928, are expected to be in service by the end of the year.

The new and unusually large power plant with which eight of the latest cars are equipped represents a marked advance in the development of internal combustion engines for railway use. Each car is equipped with a 400-hp. eight-cylinder Winton engine. The engine drives an electric generator supplying power to two 220-hp. motors on the front trucks. A total of 400 hp. is thus available from the power plant, whereas the maximum available power from a single car heretofore has been from 275 to 300 hp. The new cars with the 400-hp. engines are provided with single-end remote control.

Gas-electric drive was chosen for this service by the Burlington because of the saving in operating and maintenance expense, as well as increased reliability. It has been demonstrated that this saving with the use of gas power is from 40 to 60 cents a train mile. The 65-ft. cars are normally operated in two-car trains, the second car being a light 35-ton coach or combination baggage and mail car. The 75-ft. cars normally operate as a single unit. Loaded and equipped the 275-hp. cars each weigh approximately 50 tons, and the 400-hp. cars 65 tons.

Recording Instruments in Cost Analysis

By **H. K. NOCK** and **H. S. EDGERLY**
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FOR many years recording instruments have been used in electric generating stations to make a continuous record in permanent form of such factors of generating and distributing circuits as may interest the plant management. These records are a measure of the quality of service which the generating utility furnishes to its customers. They also make data available for the analysis of power production costs. In the latter respect, recording instruments find their greatest service in general industrial applications.

The investment in these instruments was not questioned because after all it constituted only a very small percentage of the cost of the entire equipment. As to size, well, in the days gone by, space was available, so why not utilize it?

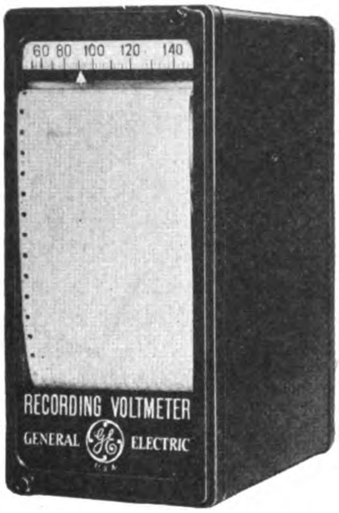

Fig. 1. The New Type of Recording Instrument, Specially Designed for Industrial Applications

Today, however, space is a very important factor as the varieties of instruments, control switches, relays for protection, etc., are increasing; while the panel size, if anything, must decrease. Especially is this true in the larger cities where space is at a premium.

The United States is an industrial country, and the war and the succeeding years have given a tremendous impetus to industrial development. The country has demonstrated its ability to absorb the products of industry in quantity. This market and the development of industry through quantity production have gone hand in hand.

It is almost axiomatic that production in large quantities should be accompanied by profits small in percentage. This combination requires of the industrial organization a heretofore unthought of degree of attention to detail. Wrong methods in one part of an organization, laxity of employees in following prescribed methods in another; these and other similar items quickly absorb the small differential between profit and loss. Analysis, and analysis in detail, is necessary to prevent such happenings.

Not only does this natural tendency toward analysis exist but competition makes industry work for its bread and butter daily. Detailed analysis is most necessary to obtain a proper understanding of all the cost-making factors and their relation to the competitive situation.

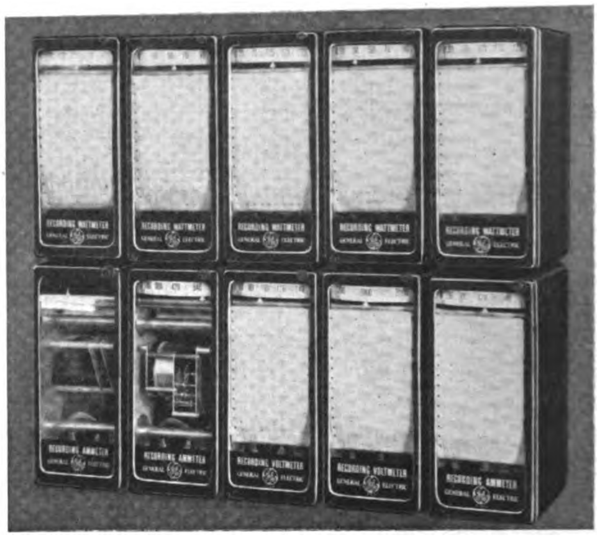

Fig. 2. Ten Recording Instruments of the New Type Mounted in Compact Arrangement on a Panel

Industry has come to depend almost entirely upon electric power. The factors of electric power are most convenient of measurement. It is possible, therefore, to obtain a general analysis of industrial-plant operations and detailed analysis of their power-driven equipment very simply with electric instruments. Particularly is the recording instrument useful in this respect. It may be placed anywhere. Attendance is unnecessary. After a prescribed interval it may be examined and its story found as a permanent record.

Recording instruments have not hitherto found general application in industry at large, mainly because they have not been produced in forms directly suitable to the purpose. The instruments that were designed primarily for generating-station service were designed for maintenance by skilled operators. Such devices could not take a proper part in a study of industrial operations. Recording instruments are now built which no longer require the services of a skilled attendant for routine up-keep.

Designers no longer attempt to modify for industrial use the recording instruments which are primarily intended for central-station service. Instead, they have developed a complete new design from the industrial point of view. The new device is an unusually sturdy recording instrument. Electrically, however, its record has all of the high-accuracy features of its counterpart on the generating-station switchboard.

The question of cost has also received careful consideration, because of the great need for this type of instrument in the industrial field. It is self-evident that it would not be profitable to invest a large sum of money in a device to check operations when several months or perhaps years would be required to liquidate the investment through the savings that may accrue through its use.

The requirements of a compact instrument having been fulfilled, attention was given to improving the details of construction with respect to maintenance. On the front of each cover are two pull buttons. With one pull on each of these, the cover comes completely off the instrument, leaving it entirely free for inspection. The carriage, on which is mounted the scale and which also carries the record roll, may be gripped with the hand and pulled forward to any position. Here its paper may be replaced with a new roll as easily as loading a kodak. After servicing with new paper, the chart carriage may be pushed back into place and the cover shoved into position. The pull buttons may be sharply slapped with the hand or fist, locking the cover in place. In brief, the modern recording instrument may be serviced by unskilled attendants.

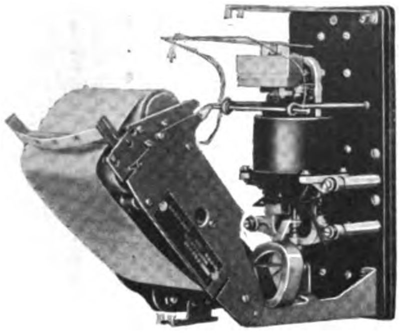

Fig. 3. The Alternating-current Voltmeter with Chart Carriage in Forward Position

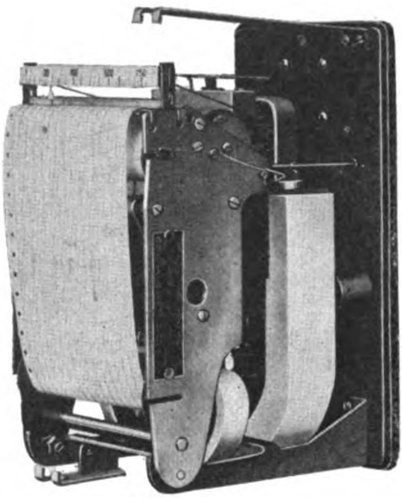

Fig. 4. The Direct-current Recording Voltmeter with Cover Removed and Chart Carriage in Place

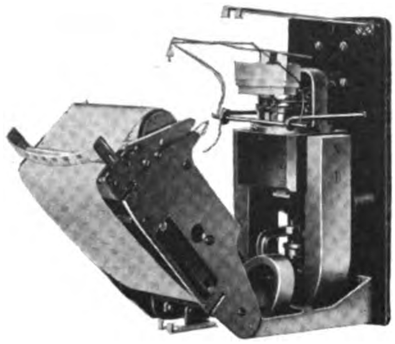

Fig. 5. The Direct-current Voltmeter with Chart Carriage Tipped Forward for Reloading or Inspection

The requirements of today are for a recording instrument of such size that it can be mounted in four vertical rows on a panel 24 in. wide, that will be of reasonable cost, be a high-grade instrument electrically, and have mechanical features that make it of sturdy construction and easy to service by an inexperienced man.

Recording instruments are now available which are only 5½ in. wide, of 12-in. vertical dimension, and project but 9 in. from the mounting surface. The back-connected type may be mounted on 5½-in. centers. They may be mounted side against side and top against bottom. Ten such instruments occupy less than three square feet of panel space. The records are 4 in. wide from zero to full scale. Each instrument presents at a glance the picture of several hours of past operation and a clear indication on a large indicating scale of the pen's present position. These are seen for each instrument as one eye-full. As one looks at such an instrument these factors of interest predominate; its cover appears as a small frame about the picture.

These small-space requirements and the improved facilities of routine servicing have made recording instruments attractive to industrial management and they now play a daily part in industry. Their records form the basis of the necessary analyses of plant operations. Recording instruments in the power circuits that feed a plant inform the management as to how much time is wasted. Graphically is told a story of how long each morning or afternoon a plant takes to reach normal productive activity. Recording instruments on individual units of equipment reveal stories of idle time, or show details of operating cycles. In the case of inter-related equipment, recording instruments determine whether each part is performing properly in relation to each other part.

The peak load of an industrial plant that purchases its power may be largely instrumental in determining the cost of the power. A recording instrument in the circuit feeding such an industrial plant would draw a graphic picture of the power fluctuations. Such a picture would immediately suggest the making of a study to determine if several large power-consuming

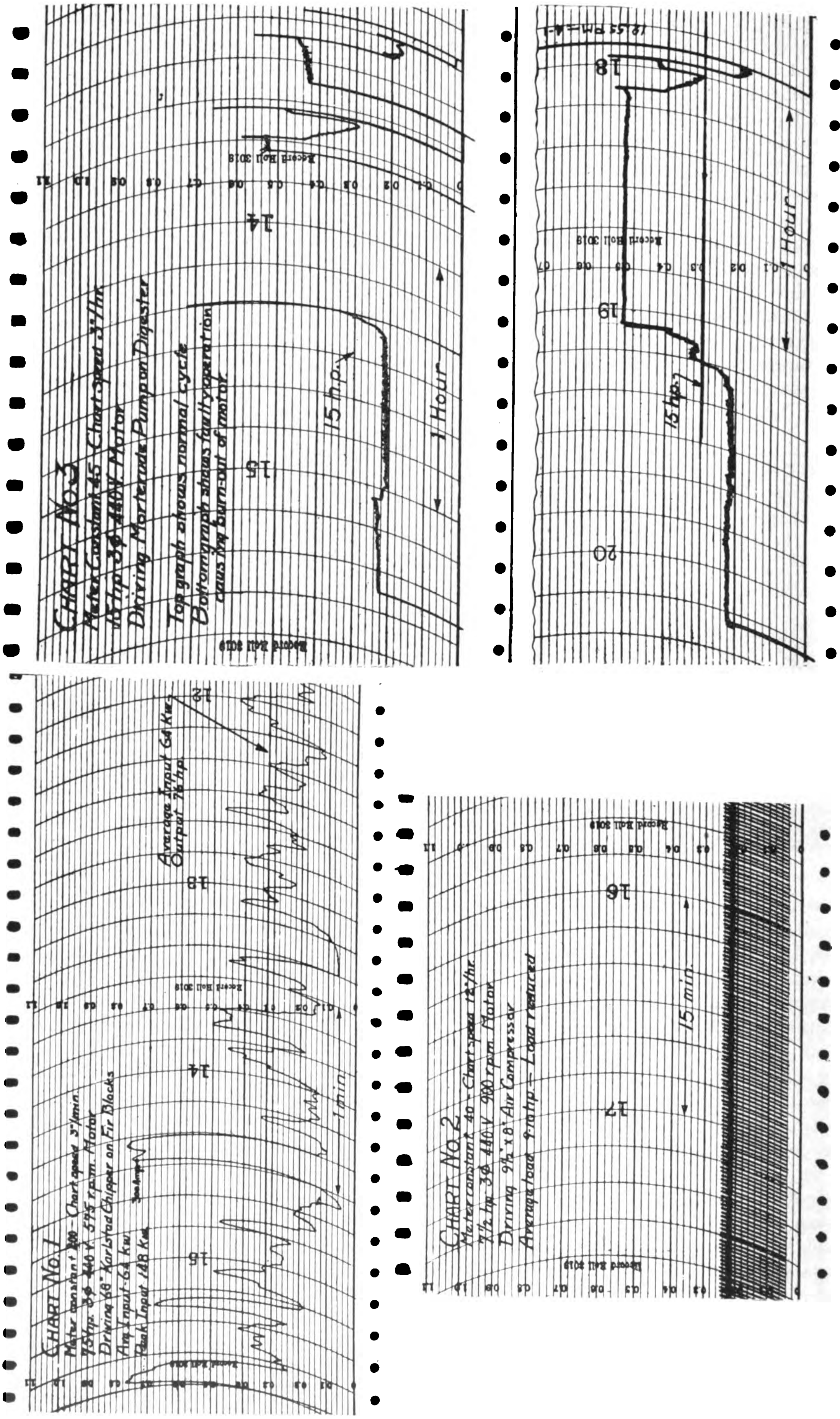

Fig. 6. The Operation of Three Industrial Drives as Charted by the New Type of Recording Instruments. These records revealed faults in the drives and indicated their solutions

devices then working simultaneously could not be made to operate alternately, thereby greatly reducing the peak-power demand. The maintenance department of any industrial manufacturing plant requires recording instruments of the portable type. With these it is possible to make checks for overloads on circuits or machines. Additions to power requirements of a plant, accompanying expansion, and the choice of proper horsepower capacity of motor-driven equipment are thus made intelligently.

Centralized measuring of energy as supplied to departments has become standard practice, and has proved valuable for general accounting purposes. With each department installation of this kind there should be a recording instrument to graphically draw the picture of the department's activities in total. The major machines should also be similarly equipped. A large machine-tool investment when idle or improperly utilized wastes money. Recording instruments tell a story of operation which cannot be refuted.

Competitive industry today cannot live without analyses of its whole and component parts in ever-increasing detail. Recording instruments remove the human element from such analyses and in fact do more than could ever be accomplished with strictly human agencies.

Some possible applications for recording instruments are as follows:

Assume a large building in which several elevators are installed. By using zero-center wattmeters, it is very easy to determine from the records as to whether the elevators are operating to capacity.

In steel mill work it is very easy to learn how many times the mill was idle, and subsequent investigation will disclose the cause and may provide a remedy for the future.

Another application is in the checking of machines that operate intermittently. By the use of a recording wattmeter, the total power consumption can be determined as well as a graphic picture showing how often and when the unit starts and stops.

The advantage of installing recording instruments on individual machines is mutual to the operator and the company. Both are furnished with an actual picture of the work done. As an example, in one factory a bias cutter was piece-rate priced on the basis of a definite number of strokes per minute. The operators complained that the machine was running at less than the agreed speed and they could not therefore earn the specified rate. After the installation of a recording wattmeter the operators became satisfied for, with a chart speed of 12 in. per min., it gave a clear record of operation at the agreed speed.

Almost everyone connected with industry realizes the importance of maintaining a graphic curve to show progress in production, volume of output, etc., because curves can be visualized much quicker than a corresponding list of figures. The same argument holds true of records of detailed operations, and recording instruments furnish these records automatically.

Giant Reactors

The world's largest reactor, from the point of view of both size and electrical capacity, is a neutral grounding reactor rated 80,000 kv-a, on a four-minute basis. It was built at Pittsfield, (Mass.) and is installed at the Westport Station of the Consolidated Gas and Electric Light and Power Company of Baltimore, (Md.). It stands over nine feet high, and weighs eight tons. The reactor is of standard cast-in concrete design and is supported on eight extra-heavy porcelain insulators.

This reactor is the head, so to speak, of a family of giant reactors manufactured in recent months. Several others have been built for the Brooklyn Edison Company for its Hudson Avenue station. The line bus for the 137,500-kv-a. turbine generator is connected to the synchronizing bus through these reactors. Each unit is rated 4583 kv-a., 2880 amp., and introduces 10 per cent reactance between these two buses. These reactors are the largest ever built for a circuit voltage as high as 27,600 volts. In physical dimensions they are over nine feet high and of similar construction to the giant supplied to the Consolidated Company.

Another group belonging to this family are 19 2750-kv-a. oil-immersed units to be used in sectionalizing the 22,000-volt bus of the State Line Generating Company's station at Hammond (Ind.). The ultimate generating capacity to be connected to this bus is expected to be in excess of 1,000,000 kv-a. These reactors are the largest ever built of the type in which all live parts are completely inclosed in metal. Each reactor will be connected to the bus by means of lead-covered cable, the lead covering making a wiped joint with a brass casting bolted to the transformer tank. In this particular station the buses, switches, and reactors are completely inclosed in metal and require no protection from the weather.

Still another large reactor is the iron core type unit recently installed at the Amite (La.) substation of the Louisiana Power Company for neutralizing part of the charging current on their 240-mi. 110-kv. transmission line. This reactor is connected directly across the secondary bus; and when operating at 24,000 volts, draws a current of 109 amperes. It is the largest for this class of service ever built.

Motion Picture Photography with Mazda Lamps

Color Quality with Respect to Panchromatic Emulsion—Intensity—Distribution—General and Modelling Lighting Requirements—Lighting Units Are Convenient to Move About, Are Clean and Quiet in Operation, and Require Minimum Attention

By **C. E. EGELER** and **R. E. FARNHAM**
Engineering Department, National Lamp Works, General Electric Company

> *The general trend toward incandescent lighting is now extending successfully into the taking of "movies," because the color spectrum of such lamps so affects a panchromatic emulsion as to produce an exceptionally good monochromatic record of color variations. In addition to possessing this combined scientific and artistic qualification, these lamps have many operating advantages with respect to production work.*
>
> —EDITOR

DURING the past year the outstanding advance of the motion picture producing industry has been the increasing adoption of sound business policies, involving a careful analysis of every phase of operation with a view to reducing production costs. A second development of importance has been the great popularity of the panchromatic film, due to its ability to register all colors.

These two factors have had a profound effect upon studio lighting practice. Together, they account largely for the rapid trend toward incandescent lighting, partly because of the economies it affords and partly because the spectral characteristics of the light of the high-efficiency gas-filled Mazda lamp meet the requirements of the panchromatic emulsion particularly well. Curve A in Fig. 1 shows that the panchromatic emulsion, while it registers all colors, is not as sensitive in the yellow-red as in the violet-blue regions; therefore relatively more light is needed in the yellow-red region to obtain the desired photographic registration of these colors. As shown by Curve B, the characteristic energy distribution of the incandescent lamp meets this requirement, and it is evident from a comparison of the two curves that light from the incandescent source is particularly suited to provide balanced photographic registration on the panchromatic film. No filters are required.

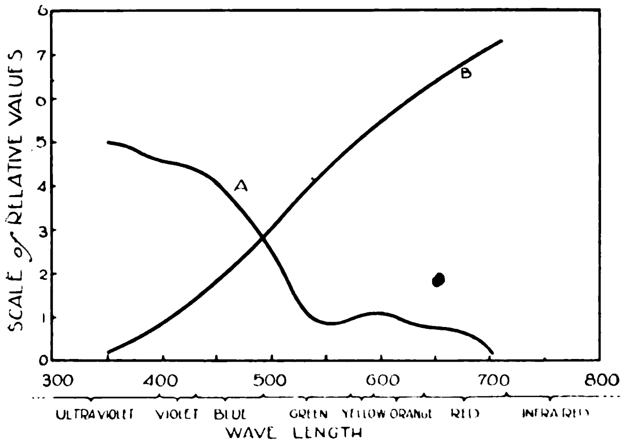

Fig. 1. The Mazda Lamp and the Panchromatic Emulsion Provide a Well-balanced Photographic Effect Throughout the Visible Spectrum, Since the Light Output of the Mazda Lamp is Greatest in Those Regions Where the Panchromatic Film is Least Sensitive

Curve A: Relative sensitivity of the panchromatic emulsion.
Curve B: Relative energy and light output of the clear Mazda lamp.

As a result of their experience in the use of Mazda lamps for motion-picture photography, studio staffs have found that:

(1). The number of men necessary to handle incandescent lighting equipment for a set of medium size is only 8 to 12, where 20 to 30, or even more, have heretofore been employed.

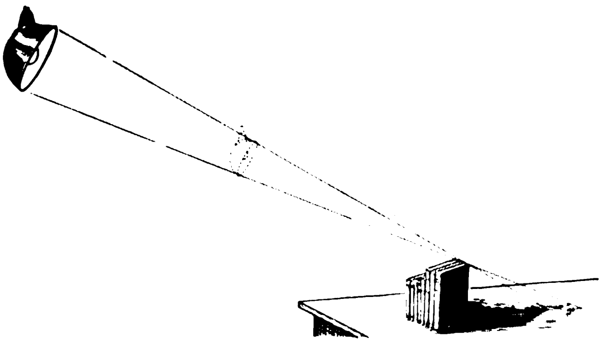

Fig. 2. The Sharpness of a Shadow Depends Upon the Size and Distance of the Light Source

(2). The lighter weight of the incandescent lighting equipment allows it to be handled more easily and quickly, and it is now possible to photograph more sets in a given time. Likewise, lighter and less expensive overhead supporting structures suffice.

(3). Since well-designed incandescent equipment utilizes the light very efficiently, the electrical energy which studios have found necessary to illuminate sets has been reduced to one-half or even one-third of that previously employed.

(4). The light from incandescent lamps is absolutely steady, thus eliminating retakes because of intensity variations.

(5). Since the incandescent lamp operates equally well on alternating and direct current, the heavy investment and operating expense of motor-generator equipment is no longer necessary. Thus the electrical installation can be extended at a fraction of the cost formerly required.

In fact, the incandescent lamp has always been favored over other illuminants where it could be applied, because of its inherent operating advantages, such as constant color quality, quiet operation, cleanliness, and minimum of attention required. In the studios lighted by incandescent lamps, the actors do not photograph differently as they move about the set in the range of the several lighting units; and since less make-up is necessary the effects are more natural. In addition, there is no danger of the actors' old bugbear, "Klieg-eye." Delays incident to lamp operation are practically eliminated and periodic airing of the set to remove smoke is no longer necessary. The noiseless feature of the incandescent lamp is a quality which is essential for pictures in which sound is recorded.

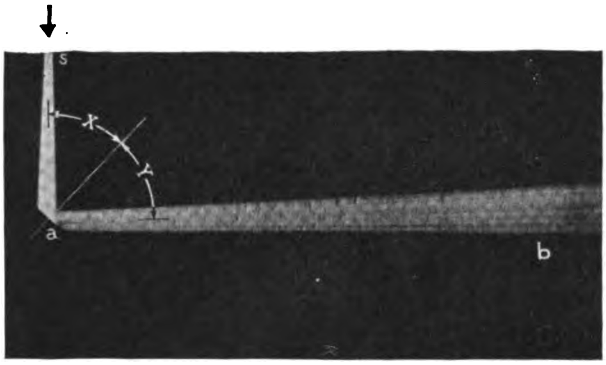

Fig. 3. Reflection of Light from Polished Metal or Mirrored Glass Has Directional Control but no Diffusion

Since the lamps may gradually be brought from total darkness to full brilliancy, or vice versa, by means of regulators of the induction type or resistors similar to those used in theatre service, effects that could not be obtained heretofore are made possible. A point that is sometimes overlooked is that the light of the incandescent lamps used for motion-picture photography does not follow the cyclic variation of the current when used on alternating-current circuits of standard frequencies; thus there is complete freedom from flicker at all standard camera shutter speeds.

As previously touched upon, an outstanding feature of the incandescent lamp is that it lends itself to the design of equipment which will efficiently utilize and direct the light where it is desired. Full advantage may be taken of this characteristic by using equipment particularly designed for the Mazda lamps which are adapted to this field.

Lighting Requirements

Color quality, intensity, distribution, and direction are the important factors in the illumination of the motion-picture studio. Given a satisfactory color quality, there remains a need for such control in directing the light that the various parts of the set may appear in the proper relative brightness, with the highlights and shadows at the immediate command of the director so that he may give each scene the precise feeling and action he desires.

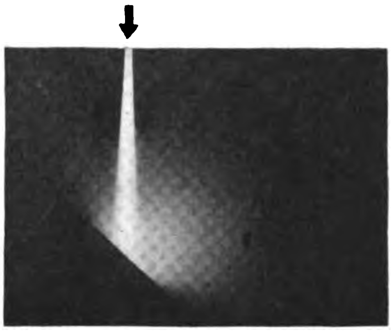

Fig. 4. With Reflection from a Mat Surface the Light Control is Lost

The intensities of illumination should be sufficient to produce the desired densities in the darkest parts of the picture and yet, for the brightest areas, come within the latitude of the film emulsion. Otherwise, lack of detail and contrast can be expected. Data on practical working intensities are presented in a later section of this article.

Photography is a record of contrasts, highlights, and shadows. The location of the highlights and shadows is obviously dependent on the placing of the lighting units. Their size is dependent on the size of the source, its distance from the surface, and the character of the surface. The degree of shadow sharpness depends not only on the size of the source but also on its distance from the surface, as shown in Fig. 2.

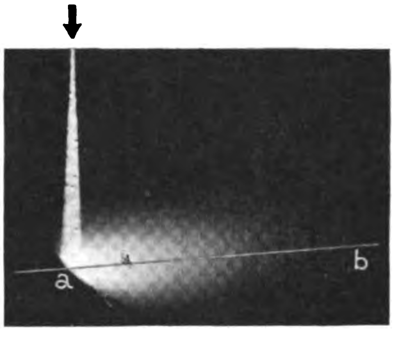

Fig. 5. Reflection from Treated Aluminum Has Directional Control with Slight Diffusion

The contour and nature of the surface of a reflector determine the candle-power distribution which will be obtained with a given light source. Polished metal and mirrored glass, for example, reflect light beams as shown in Fig. 3; the angle Y, which the reflected rays make with the normal to the surface, is always equal to the angle X. A mat surface, on the other hand, diffuses the light in all directions, Fig. 4, regardless of the angle from which it is received, and therefore permits of but little control. Porcelain enamel diffuses most of the light in this manner. Aluminum with a treated or semi-mat finish reflects light as shown in Fig. 5; here most of the light is reflected in the same general direction *ab* as though the surface were polished, but with controlled diffusion, giving what is known as spread reflection. Essentially the same effect can be obtained by rippling or otherwise configurating the surface of mirrored glass. This spread type of reflection is particularly adapted to reflectors for general lighting, since most of the light can be sent into the desired directions by selecting the proper contour and finish, which will also eliminate the beam striations or source images obtained with a smooth polished or mirrored surface.

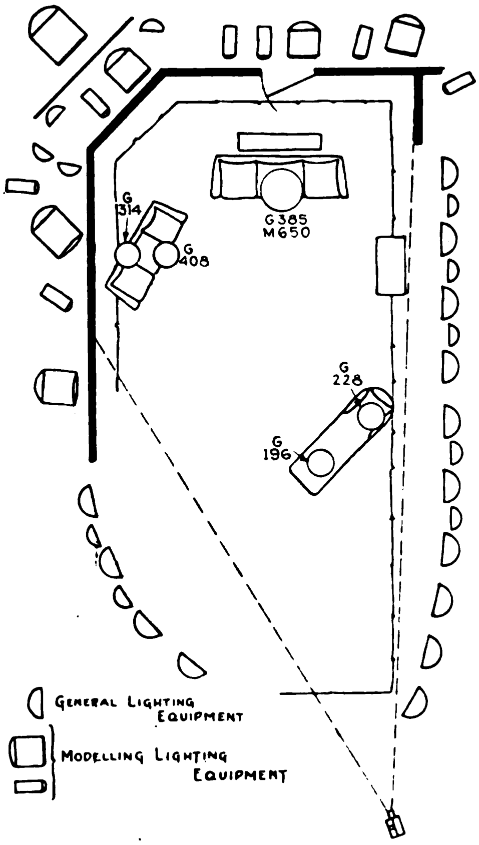

Fig. 6. The Illumination Intensities Employed for This Set of Medium Size Are Shown in Foot-candles by the Figures (*G:* General Lighting; *M:* Modelling Lighting)

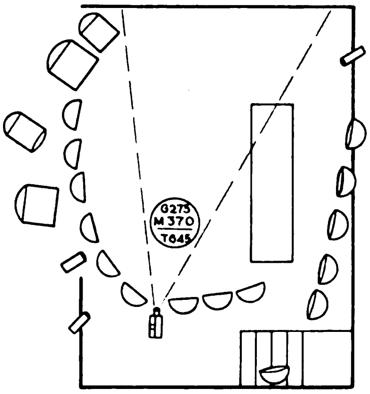

Fig. 7. This Diagram Shows Intensities for a Typical Close-up

For practical consideration, it is convenient to classify motion-picture lighting as general and modelling. The purpose of the general lighting is to provide a ground-work illumination that is fairly uniform in intensity throughout the set. It not only illuminates the areas that the modelling lights do not serve, but allows control of shadow density over the entire set. For the more usual cases, the general lighting should not model the actors or create noticeable highlights and shadows; this is the function of the modelling lights which the lighting director employes to create the effects desired.

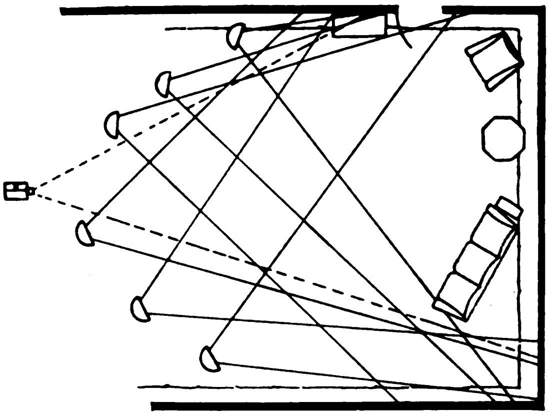

Fig. 8. Broadside Units Directing Most of the Light Through Horizontal Angles of 60 deg. Give Good Coverage with Minimum Waste of Light

General Lighting

The intensities required for general lighting will vary with the nature of the set, the action, and the colors of the properties. An extensive survey was conducted in prominent studios to determine the illumination levels employed in all types of scenes, both with incandescent and arc lighting. Fig. 6 shows the general illumination intensities in foot-candles found in a typical set of medium size. The modelling units were turned off and the illumination measured at the stations indicated. Fig. 7 shows in a similar manner the intensities for a typical close-up. The studies of the general lighting conditions obtained on a variety of sets and of the other factors involved show that the general lighting equipments should be capable of providing from 200 to 500 foot-candles at distances of from 6 to 18 ft.

Broadside Units

Ordinarily the camera is aimed nearly horizontally and hence the illumination on vertical surfaces is most important. For the smaller sets much of the action usually occurs over areas from 6 to 12 ft. deep and the intensities required can be best provided by lighting equipments placed across the front.

The lighting units used for this purpose are called broadsides.

Broadsides should be capable of providing fairly uniform illumination through horizontal and vertical angles of approximately 60 deg., and the horizontal angle of cut-off should not be greater than 90 deg. in order to restrict the illumination intensities on the side walls of the set. Fig. 8 shows the areas covered by units giving 60-deg. distributions horizontally. If the horizontal distribution is much less than 60 deg., the general illumination of the set becomes spotty and difficulty is encountered in securing uniform lighting, or else a larger number of units are required to gain the desired uniformity. For angles greater than 60 deg., much light is wasted and the side walls of the set may become undesirably bright. Reference to Fig. 9 shows that a 60-deg. spread also meets the vertical angular requirements, thus making possible a reflector of symmetrical contour with its attendant higher efficiency and lower cost of manufacture. In reaching this conclusion about the angular requirements of broadsides, sets ranging from a "close-up" to one of almost indefinite width have been considered, although sets 15 to 20 ft. wide by 20 to 35 ft. in depth have been given special consideration owing to the prevalence of these sizes.

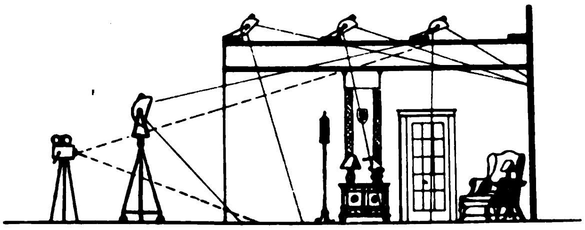

Fig. 9. A Vertical Spread of 60 Deg. Meets the Requirements of Both Overhead and Broadside Lighting Units

Fig. 10 is a candle-power distribution curve for a broadside lighting unit which meets the requirements. The candle-power values at the center of the beam are well maintained between 25 and 30 deg. from the axis, to improve the uniformity of illumination.

Fig. 11 shows the contour of a semi-mat aluminum reflector designed for the PS-52 bulb lamps (1) to give the beam spread shown in Fig. 10. Its diameter is 18 in. and its depth about 13 in. By proper treatment of the surface, the semi-mat reflection characteristics can be held within close limits. It will be noted that the reflector in Fig. 11 is so designed that the lamp is normally operated in the base-up position. The performance of the lamp is materially better if it is operated in this rather than in a horizontal position.

Semi-mat aluminum reflectors for general lighting have the advantages of high efficiency, low first cost, and light weight. In practice, the surfaces require retreating at periodic intervals to maintain their reflection efficiency unless cover glasses are employed on the units. Reflectors in which the surface is configurated mirrored glass also have high efficiency, but their first cost is higher, and their weight is greater; also, they are subject to breakage but are readily cleaned by washing.

When it is necessary for the actors to look directly into a lighting unit, as is usually the case in close-ups, some form of diffusing material should be used to shield the bright lamp filament from the eyes. This shield should be placed near the lamp, and it should be of small size so that it will intercept only a little of the light from the reflector.

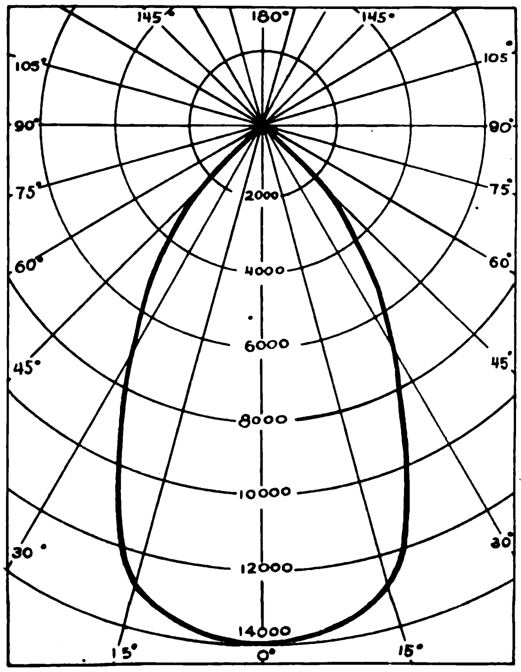

Fig. 10. Desirable Candle-power Distribution for Broadside and Overhead Equipment

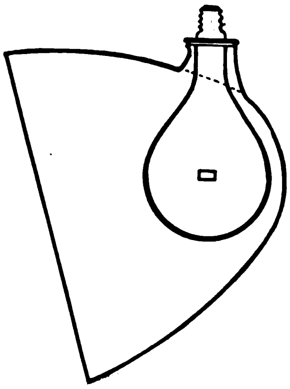

Fig. 11. Contour of Reflector to Give the Candle-power Distribution Shown in Fig. 10

The 1000-watt and 1500-watt PS-52 bulb standard Mazda lamps meet the usual requirements for general lighting. On the larger sets, or where the higher intensities are required, the 2500-watt PS-52 bulb lamps can be employed.

Eight of the 60-deg. broadsides with the 1000-watt lamps arranged in the usual manner across the front of a set 20 ft. wide will give a practically uniform intensity of 250 foot-candles 12 ft. back from the front of the set. At 18 ft., the intensity is over 100 foot-candles, exclusive of the light received from the overhead general lighting equipment. Proportionately higher intensities are, of course, obtained by using 1500-watt or 2500-watt lamps.

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(1) "PS-52" is the trade designation of a particular size of pear-shaped amp bulb. See column 6 of Table III.

Overhead Units

For the deeper sets, overhead units are needed to send light to the rear areas that cannot be illuminated satisfactorily by the broadsides. If the light output of the broadsides were increased sufficiently to provide ample light in the rear, the illumination intensities nearer the front would be so high that the modelling lights would make relatively little impression—hence the need of equipment capable of directing light into the rear parts without increasing the illumination at the front. Such units must be capable of supplying higher illumination on the vertical than on the horizontal surfaces.

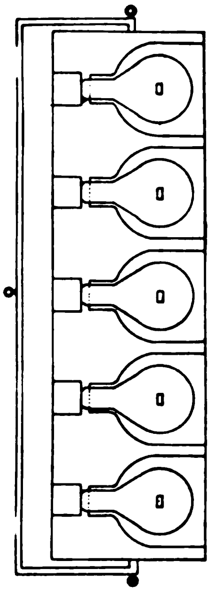

Fig. 12. The Use of Individual Reflectors for Each Lamp in a Strip Unit Increases the Amount of Light in Useful Directions

Since the overhead units are placed across the set similarly to the broadsides, it is found here also that the 60-deg. horizontal and vertical distributions meet the requirements, thus permitting the use of a symmetrical reflector similar to the broadside in contour. As shown in Fig. 9, the overhead units are suspended so that the center of the beam will be directed from about 30 to 45 deg. below the horizontal. Such an arrangement gives a relatively high illumination on vertical surfaces as compared to the illumination on horizontal surfaces, with a minimum of wasted light. As in the case of the broadsides units, the overhead units employ PS-52 bulb lamps of 1000 to 2500 watts.

Strip Units

Frequently, owing to space limitations, it is necessary to illuminate the set from room corners, or through doorways, where it is not possible to place the usual broadsides. For this purpose the so-called "strip light" has application. It is generally mounted vertically, close to the surfaces to be illuminated, and a spread of about 90 deg. is required. It has been the practice to place five lamps in a metal trough but this design has the disadvantage of too wide a beam spread, especially in planes lengthwise through the unit. Placing an individual aluminum reflector behind each lamp, as shown in Fig. 12, greatly increases the amount of light sent in useful directions; 1000-watt PS-52 bulb lamps are used in these reflectors. Switches should be provided to permit turning off any of the lamps of the group and thus control the volume of the light. The unit should be capable of being suspended vertically or horizontally above the set.

Where extreme compactness of lighting equipment is required, as for example for lighting from behind trees and columns, a strip light employing a semicircular enameled-steel reflector, containing five 1000-watt T-20 bulb lamps[2] placed with their axes in the axis of the reflector, has found favor. Here, light control is sacrificed for compactness. A unit of this shape utilizes the light distribution from the modified monoplane filament construction of the 1000-watt tubular lamp to good advantage.

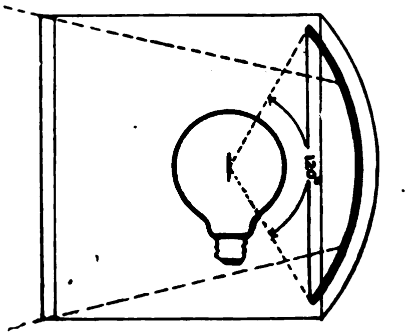

Fig. 13. Optical Elements of Usual Modelling Unit. The light from the outer zones of the reflector is wasted when the beam is spread

It is interesting to note that the two units just described have no counterpart among the older light sources. They illustrate the flexibility of the incandescent lamp for this service and indicate the new lighting effects it makes possible.

Modelling Lighting

Alone, the general illumination of any motion-picture set is not satisfactory, as the faces appear flat and the various actors seem equally conspicuous from the illumination standpoint. To model the features properly, to give emphasis to certain actors, and to create depth, additional lighting is employed to give much higher intensities over limited areas. Intensities ranging from two to four times that of the general illumination are required to produce distinct highlights, although a higher ratio may sometimes be desirable. The modelling units must be capable therefore of producing intensities from 200 to 1500 foot-candles, over areas ranging from 3 to 12 ft. in diameter, at distances from 12 to 30 ft. or more. The modelling lighting intensities for the set shown in Fig. 6 average about twice the general lighting values. Here the modelling effect is taken care of almost entirely by medium and high-power light projectors. A study of the areas in which modelling lighting is employed for

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(2) "T-20" is the trade designation of a particular size of tubular lamp bulb. See column 6 of Table III.

all types of sets shows that the usual requirements for each of these units are as given in Table I.

The desirable characteristics of modelling lighting equipment are:

(1). Continuous range of beam divergence within the limits given in Table I.
(2). No light outside the beam.
(3). Reasonable uniformity of intensity over the area illuminated.
(4). High efficiency of light utilization.
(5). Adjustability from directly downward to directly upward, and 360 deg. in a horizontal plane.

Since it is necessary to confine the light within relatively narrow spreads, mirrored-glass or polished-metal reflectors of accurate contour, or refracting lenses, are possible means of meeting the beam-spread requirements. The single plano-convex lens intercepts only a relatively small percentage of the light and should be used in combination with a spherical mirrored reflector; but even with this combination the highest part of the range (indicated in the last colmun of Table I) is not obtained. These values are possible, however, with the mirrored-glass paraboloids which intercept about four times as much light as the equipment employing a plano-convex lens and a spherical-mirrored reflector. The range of variation in spot diameter with the parabolic reflector is less than with the lens, and therefore cover glasses of 8 to 16 deg. spread should be provided to obtain the full range. There are also possibilities in the way of modifications in the parabolic contour to produce a wide range of beam spreads without too low relative intensities in the center of the beam. However, it is difficult to obtain sufficiently accurate metal paraboloids. Furthermore, in order to produce a surface which is resistant to tarnish and abrasion, such as chromium plate, one must sacrifice as much as 25 to 30 per cent in reflection factor.

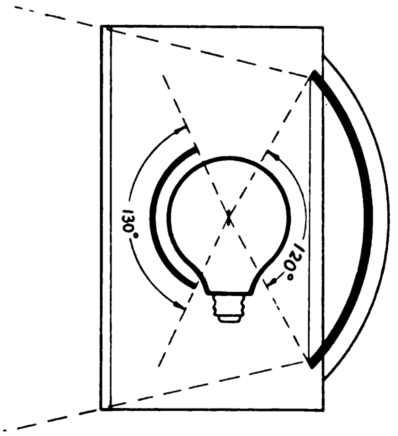

Fig. 14. In This Design, the Front Opening is Large Enough to Pass All of the Spread Beam. A spherical mirrored-glass reflector is used in front of the lamp

The usual design is shown in Fig. 13, where provision is made for moving the lamp along the reflector axis to change the beam spread. Built-up or strip-glass cover doors minimize breakage. Ground surface gelatin has often been used but is less efficient, and it has the further disadvantage that it is difficult to obtain the exact spreads desired. Since the source of the incandescent lamp is somewhat larger than an arc crater, the shadows are not so sharp as those obtained with an arc spotlight. The slightly softer edges of the shadows are of course desired much more often.

The light utilization is increased about 35 to 40 per cent by the use of a spherical mirrored-glass reflector in front of the lamp, as in Fig. 14. This reflector intercepts the light emitted toward the set (little of which falls in the beam) and sends it to the main reflector. The spherical mirror is set so that its center of curvature is in the light source. It should intercept at least the maximum solid angle of light utilized by the paraboloid. The housing need be no deeper than is necessary to intercept the stray light which is not utilized by the optical system. Searchlight cylindrical-type housings have been generally employed with the parabolic reflector units with the result that a considerable amount of light strikes the housing and is lost when the beam is diverged by moving the lamp toward the reflector. The design in Fig. 14 eliminates this loss through the use of a larger housing and cover-glass diameter.

TABLE I
REQUIREMENTS FOR MODELLING LIGHTING UNITS

Type	Spot Diameters (Feet)	Throw (Feet)	Beam Divergence (Degrees)		Light Volume of Beam (Lumens)
Medium Power	3 to 6	12 to 20	12-ft. Throw	14 to 28	1,400 to 42,000
			20-ft. Throw	8 to 17	
High Power	4 to 12	20 to 50 and upward	20-ft. Throw	12 to 33	2,500 to 170,000
			50-ft. Throw	8 to 14	

TABLE II
LAMPS FOR MODELLING UNITS

Parbolic Reflector Unit	Reflector Diameter (Inches)	Lamp 115-volt Class
Medium Power	18	2,000-watt G-48
	24	5,000-watt G-64
High Power	24	5,000-watt G-64 10,000-watt G-80
	36	10,000-watt G-80

The lamps for these units have G-48, G-64, and G-80 bulbs,(3) and have concentrated monoplane filaments to produce beams of minimum spreads with good uniformity. The types that are suitable for the several units are listed in Table II.

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(3) "G-48, G-64, and G-80" are trade designations of particular size of globular lamp bulbs. See column 6 of Table III.

Equipment Maintenance

Dust or dirt on lamps and reflector surfaces greatly reduces their efficiency; the loss may be as high as 50 per cent. The lamps and lighting units should be cleaned periodically, and the reflector surfaces kept in good condition. The cost of this maintenance is really an investment which returns large dividends and its importance cannot be over-emphasized.

Mazda Lamps

Table III lists the principal types of Mazda lamps applicable to studio photography. The pear-shaped (PS) bulb lamps are best adapted to the general lighting units, and the round (G) bulb lamps to the modelling lighting equipments. The tubular (T) bulb lamps are projector types which have special applications in this field because they provide maximum light output for their bulb sizes, subject to certain limitations in operating position.

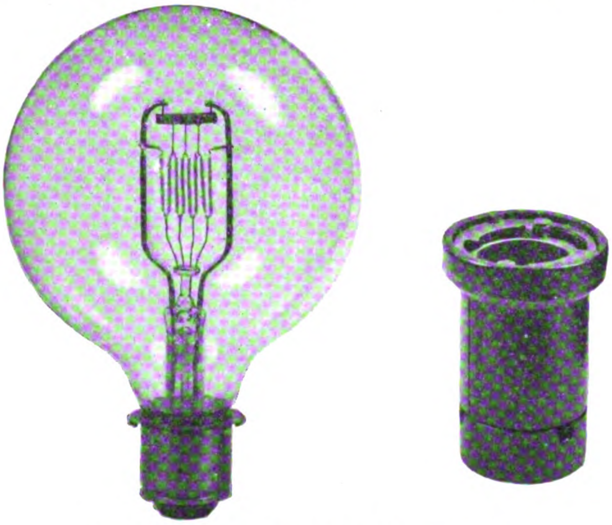

Fig. 15. 2000-watt Lamp with Prefocused Socket

The new prefocused base and socket, shown in Fig. 15, automatically insure accurate positioning of the lamp filament with respect to the reflectors of the modelling units. The bayonet construction permits very rapid insertion and removal of the lamps.

Data obtained on the best filament operating temperatures for panchromatic film indicate that color temperatures of about 3125 deg. should be used. This corresponds to lamp efficiencies of 24 to 27 lumens per watt and 200 to 400 hours' life. Operated at lower temperature and longer life, there is less light in the blue-violet end of the spectrum. The PS bulb lamps, as made for general service, are designed for 1000 hours' life. Since a higher temperature is desirable for photographic use, they should be operated at 10 volts above their labeled value; in other words, 105-volt

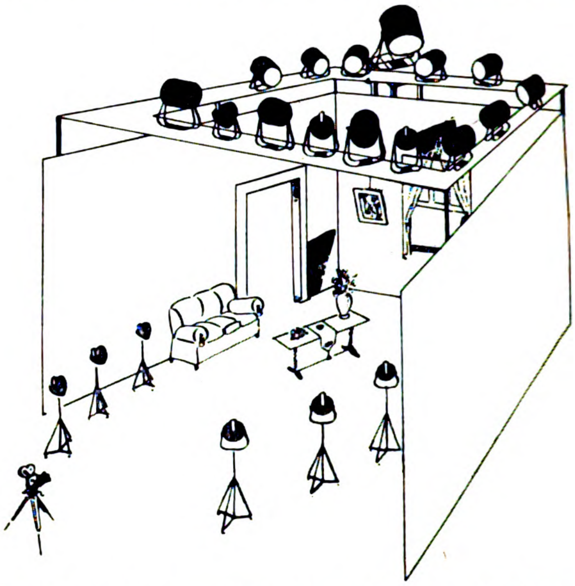

Fig. 16. An Arrangement of Incandescent General and Modelling Lighting Equipment on a Typical Set of Medium Size

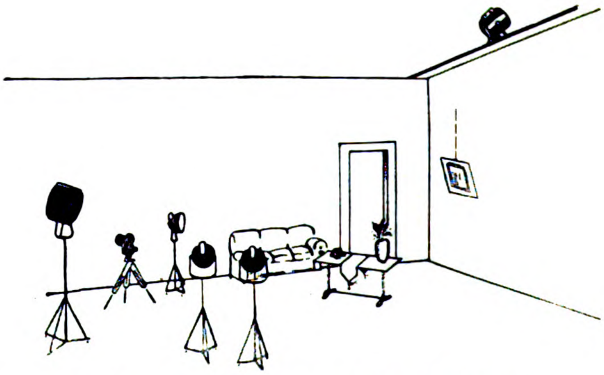

Fig. 17. Artistic Modelling Governs the Location and Choice of Units for Close-ups

TABLE III

MAZDA LAMPS FOR MOTION-PICTURE PHOTOGRAPHY

Service	Watts	Voltage Class	Bulb Shape	Bulb Diameter (Inches)	Bulb Designation	Light Center Length (Inches)	Fil. Const.	Overall Length (Inches)	Base	Operating Position
General	1,000	115	Pear Shape	6½	PS-52	9½	C7A	13⅛	Mogul	Any
General	1,000	115	Tubular	2½	T-20	4¾	C13A	9½	Mogul*	Base down
General	1,500	115	Pear Shape	6½	PS-52	9½	C7A	13⅛	Mogul	Any
General	2,500	115	Pear Shape	6½	PS-52	9½	C7A	13⅛	Mogul	Any
Modelling	2,000	115	Globular	6	G-48	5¼	C13	8⅝	Mogul*	Base down
Modelling	5,000	115	Globular	8	G-64	9	C13	15½	Prong	Base down
Modelling	10,000	115	Globular	10	G-80	12	C13	20	Prong	Base down

*Also made with the prefocused type of mogul base.

PS bulb lamps should be used on 115-volt circuits. Thus the cost advantage of using standard lamps of large production is retained, without the sacrifice of the quality important in photography. The G bulb lamps are designed for photographic service, and therefore should be burned at their rated voltages.

While the PS bulb lamps will operate in any position, they give better life and candle-power maintenance when burned in the base-up position, and it is therefore desirable that the lighting units be designed so that these lamps will normally operate base up, *i.e.*, that the sockets be in the top rather than in the side of the reflector. The G bulb lamps are made to be operated vertically base down, or within 45 deg. of the base-down position. The T bulb lamps are for operation vertically base down, or within 25 deg. of this position. Lamps burned in other than these positions usually give shorter life and poorer candle-power maintenance.

General Considerations

Some twenty units of four different types are shown for the illumination of the set in Fig. 16. For the close-up in Fig. 17, but two different types are employed. These two sets are typical of many of the scenes to be photographed, and the arrangements shown are only two of several which may be employed to advantage.

The lighting of a studio set differs from the usual illumination design problem in that each new set may present aspects which are different from those of preceding scenes. There is no standardization of the lighting, nor is this suggested; however, there can be standardization of the lighting equipment, so that with units of known performance and characteristics there will be a minimum delay for all concerned as the scene and the lights are changed. The wattage required with equipment of best design may be only one-half as much as that needed with some of the units in use today.

The lamps and reflecting equipment are but tools with which the lighting director works to produce attractive and effective photographs. Just as the artist works with paint, brush, and canvas, so the cinematographer uses light, lenses, and film to paint the screen picture. With lighting units available which can produce the fundamental lighting effects with a minimum of attention to their use and operation, the creative ability of the producing artist can be employed with complete freedom and his best talents be exerted in the production of screen pictures.

Captain Remy is Snubbed by Antoinette in "Louisiana" Under Incandescent Lighting

Two-stage Current Transformers

Considerations of Accuracy—The Two-stage Transformer—Electrical Relationships—Watthour Meters for Two-stage Measurements—Testing for Rotation—Summary of Special Features

By W. K. DICKINSON and M. S. WILSON
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General Electric Company

Molehills become mountains in literal truth when sufficiently multiplied. Thus, if an average error of one-tenth of one per cent in power metering had persisted in this country during 1927, seventy-five millions of kilowatt-hours would have been improperly accounted for. This and forthcoming articles will describe improvements by special design, especially in those transformers formerly considered unfit for use in accurate measurements.—EDITOR

SO great has been the increase in the use of electrical energy during recent years that conditions, now involving millions of dollars annually, have made the measurement of power consumption with a minimum possible error a matter of exceeding importance. To meet such exacting requirements, great improvements have been made in the accuracy of measuring devices, especially in watthour meters.

Since the measurement of large blocks of energy involves the use of instrument transformers as well as watthour meters, another source of error is introduced which in these days of high efficiency must be given serious consideration. These errors in the case of current transformers are caused by the failure of the secondary current to bear the desired relation, both in magnitude and phase, to the primary current for all conditions of load, burden, and frequency. This is especially noticeable in the case of low-current and high-voltage lines where it is very desirable to use a single-turn-primary type of transformer, either from the standpoint of cost or overload capacity. Since the accuracy of an ordinary current transformer is roughly proportional to the square of the ampere-turns, it is obvious that the error of a 150-amp. single-primary-turn current transformer for a given set of conditions will be approximately 100 times that of a 150-amp. transformer with 10 primary turns. Consequently, where high-grade metering is desired, it has been impossible to employ satisfactorily the ordinary single-turn-primary transformer in cases involving less than approximately 1000 amp.

Several schemes are in use by means of which the accuracy of this type of transformer is improved; but probably the best accuracy under all conditions of burden, frequency, and load is obtained by the use of the two-stage transformer. Recent developments in this type of transformer have made it possible to measure current accurately under conditions which heretofore have been considered practically impossible.

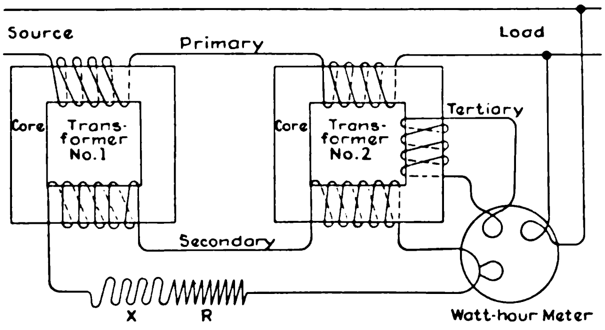


Fig. 1. Simple Diagram of the Two-stage Current Transformer Showing Cores Separated

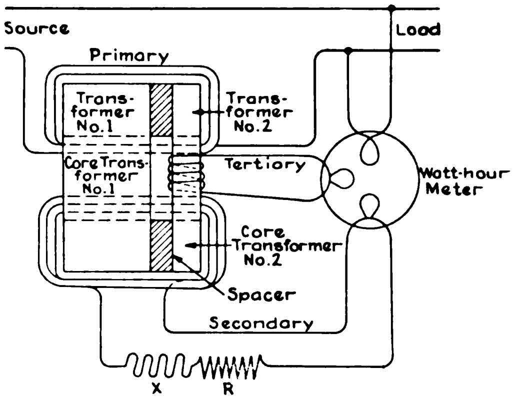


Fig. 2. Diagram of the Two-stage Current Transformer Showing Cores as Combined for Convenience in Manufacture

The two-stage transformer consists of two cores, each having the usual primary and secondary conductors, but with an additional or tertiary winding on the second core which is similar to the secondary. A diagram of this type of transformer with connections to the external burdens and the watthour meter is shown in Fig. 1; Fig. 2 shows the method of combining the cores and windings for convenience of manufacture.

The function of the tertiary winding is to furnish the watthour meter with a current which is very nearly equal in magnitude and phase relation to that which is required for the excitation of the core for the first stage. Hence it functions in a purely corrective capacity and tends to make the resultant ratio correct and to reduce the phase angle.

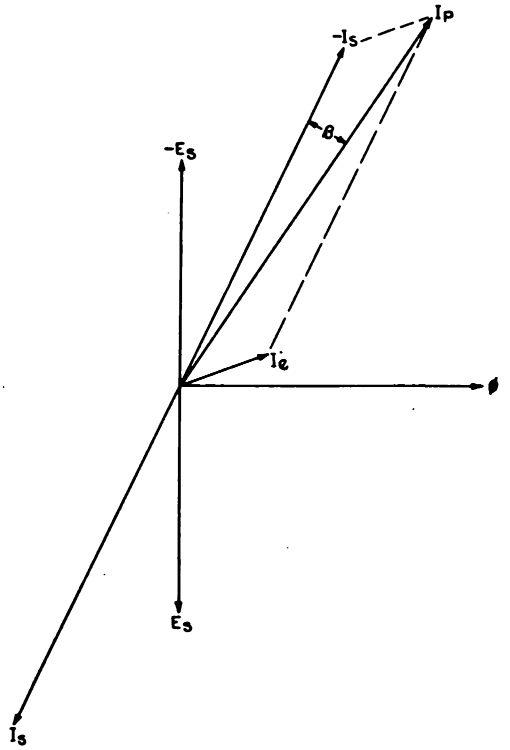


Fig. 3. Vector Relation for First Stage

E_s = Induced secondary voltage
ϕ = Flux
I_e = Exciting current
I_s = Secondary current
I_p = Primary current
β = Phase angle

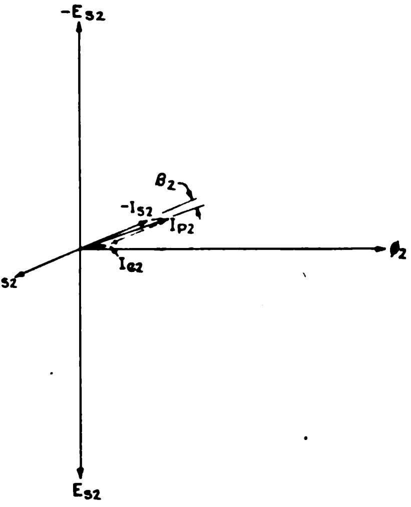


Fig. 4. Vector Relations for Second Stage

E_{s2} = Induced secondary voltage
ϕ_2 = Flux
I_{e2} = Exciting current
I_{s2} = Secondary current (tertiary current)
I_{p2} = Primary current (I_e of first stage)
β_2 = Phase angle

The electrical conditions in core No. 1 may be represented by the usual vector diagram of an ordinary current transformer as in Fig. 3. A similar diagram, Fig. 4, represents conditions existing in the second stage except that the primary current of this stage is equivalent to the exciting current of the first stage. Thus the tertiary or corrective current furnished by the second stage is dependent upon and varies with the exciting current of the main core or first stage. Consequently, the two-stage transformer tends to correct automatically for its own errors under all conditions of burdens, frequency, and load to which it may be subjected.

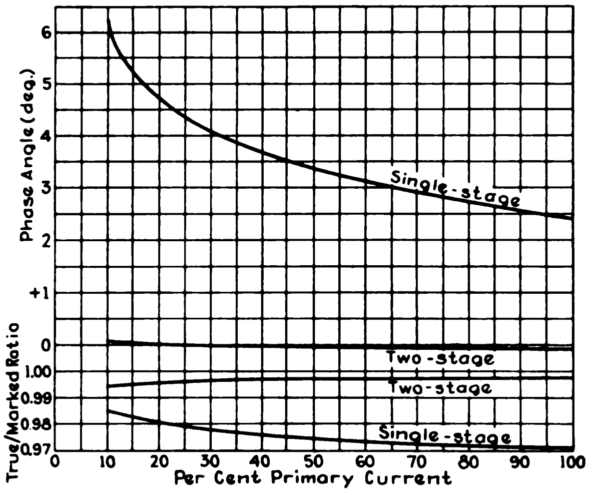


Fig. 5. Characteristic Curves of Ratio and Phase Angle for a Two-stage Transformer-less-primary-type Current Transformer, 100-amp. for 15,000-volt Circuit

Tested at 60 cycles
Secondary burden = 2.25 volt-amp. at unity p-f.
Tertiary burden = 1.9 volt-amp. at unity p-f.

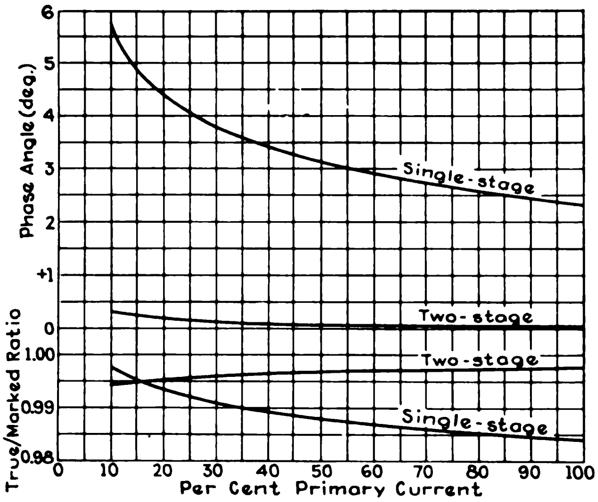


Fig. 6. Characteristic Curves of Ratio and Phase Angle for a Two-stage Bushing-type Current Transformer, 150-amp., for 73,000-volt Bushing

Tested at 60 cycles
Secondary burden = 2.25 volt-amp. at unity p-f.
Tertiary burden = 1.9 volt-amp. at unity p-f.

If it were possible to have no losses in the second stage, the entire amount of the losses of the first stage would be fed back into the line; and a perfect transformer would be obtained. It is of course impossible to entirely eliminate the losses in the second stage; therefore, in order that they be kept as low as possible, it is desirable to keep the burden on this stage low, and to use a nickel-iron alloy for the core. This alloy has the characteristic of low losses, and a very high permeability at low densities, which make it particularly valuable for this application.

With the application of the two-stage principle to the bushing and bar-type transformers, a transformer is made available which is reasonable in cost and high in accuracy even for low values of primary current. High overload capacity is another important feature which is characteristic of transformers of these types.

The curves in Figs. 5, 6, 7 and 8 illustrate the accuracy obtainable with the two-stage bushing and bar-type transformer. The curves which are marked single-stage indicate the results obtained with the transformer in which the total core and main secondary winding are used as in an ordinary single-stage transformer.

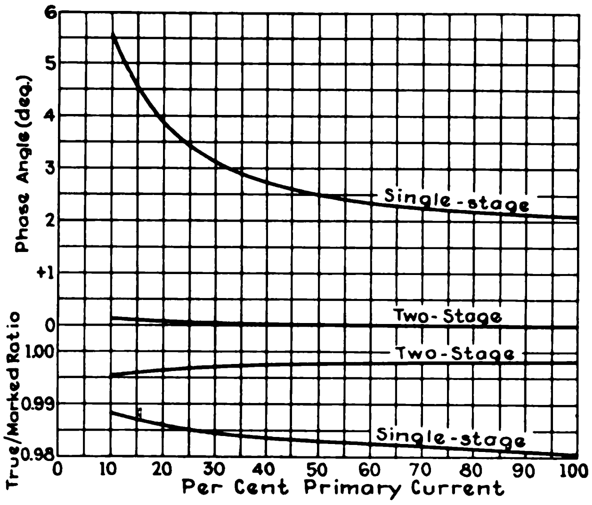


Fig. 7. Characteristic Curves of Ratio and Phase-angle for a Two-stage Bushing-type Current Transformer, 150-amp., for 110,000-volt Bushing

Tested at 60 cycles
Secondary burden = 2.25 volt-amp. at unity p-f.
Tertiary burden = 1.9 volt-amp. at unity p-f.

The bushing-type transformer is so designed that it fits into a pocket in an oil switch cover, the primary consisting only of the conducting core of the standard bushing. Thus the bushing itself furnishes satisfactory insulation between the transformer secondary and the transmission line so that no insulation is necessary on the transformer. For lower voltages, this type of transformer may be slipped over an insulated bus or cable, or the primary, consisting of a bar or rod, may be built into the transformer. In the latter case the transformer is generally known as the bar-type.

The watthour meter designed for use with the two-stage type of transformer is special only in so far as it has a twin current winding and suitable terminals. One winding is provided for the main secondary and the other for the tertiary winding, as illustrated in Fig. 1. In this way the maximum accuracy of the two-stage transformer can be realized.

It may at first appear that the two-stage transformer and special watthour meter are too complicated and specialized to be satisfactory for ordinary meter installation. This impression, however, is felt to be largely due to an unfamiliarity with these devices rather than to the existence of anything inherently complicated about either the devices themselves, or their installation or test. It would seem that the accuracy afforded, particularly in the single-turn, or bushing-type, transformer more than justifies the slight extra care required for installation and test.

Certain rules must of course be followed as to the installation and testing which are slightly different from those pertaining to the standard transformer and meter. They are, nevertheless, easily understood by the ordinary construction or test man.

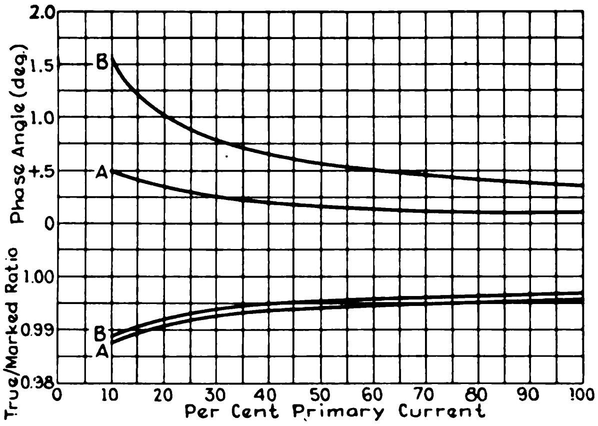


Fig. 8. Characteristic Curves of Ratio and Phase-angle for a Two-stage Bushing-type Current Transformer 150-amp. for 110,000-volt Bushing

Tested at 60 cycles
Curves *A:* Secondary burden = 6.0 volt-amp. at unity p-f.
Tertiary burden = 6.0 volt-amp. at unity p-f.
Curves *B:* Secondary burden = 19.3 volt-amp. at 0.88 p-f.
Tertiary burden = 6.0 volt-amp. at unity p-f.

It is necessary, for instance, to test the meter for rotation due to both the tertiary and secondary currents. This may be done by a sequence of operations as follows:

With the tertiary circuit open, connect the secondary of the transformer to the secondary terminals of the meter, and test for polarity in exactly the same way as in the case of the standard transformer and standard meter. When the proper rotation has been obtained connect the tertiary circuit to the tertiary terminals of the meter. Then connect a resistance in the order of 50 ohms across the secondary winding of the transformer and disconnect the secondary leads from the meter. If the polarity of the tertiary connections is correct, the meter disk will rotate in the same direction as when the secondary only was connected, as described heretofore. The secondary winding should then be reconnected to the meter according to the polarity determined, and the resistance removed.

In some special cases it may be preferable to use a standard watthour meter instead of the special meter as described. By the use of some auxiliary device, one type of which has been developed by Dr. Brooks, a standard watthour meter may be used, but at some sacrifices in the maximum accuracy obtainable with the special watthour meter. Considerable developmental work, however, has recently been done with various devices of this nature, and it seems probable that it will soon be possible, where especially desirable, to use a standard watthour meter with the two-stage transformer with but little sacrifice in accuracy.

The features for consideration in the two-stage transformer may be summarized as follows:

(1). The two-stage current transformer gives better accuracy than any other type known at present. This is particularly true of the bushing type.

(2). To realize the full advantage of the two-stage transformer a special watthour meter is used, although satisfactory results may be obtained with the standard meter.

(3). The installation and test of the two-stage transformers and meters require slightly more care, but no difficulties should be encountered if a few simple rules are followed as explained.

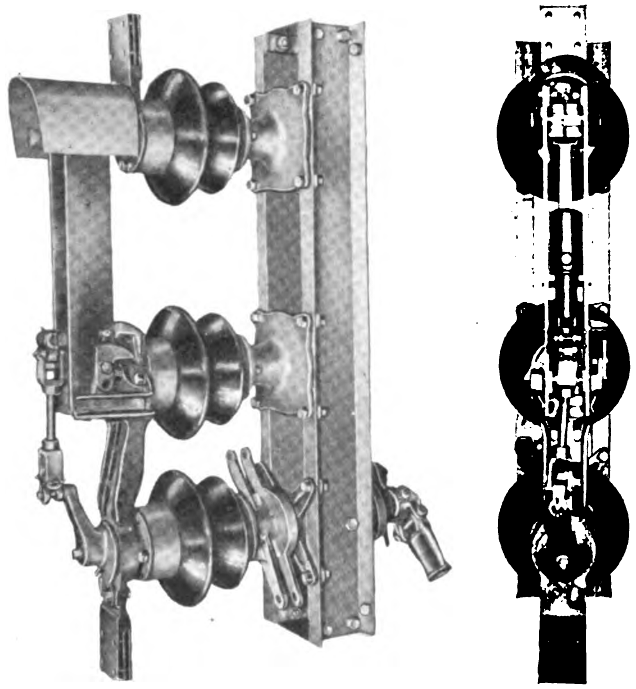
(4). Good metering accuracy may be obtained, as is indicated by the curves on the bushing and bar-type transformer, with primary ratings as low as 100 amp.

(5). In order to obtain the most satisfactory results, especially on bushing and bar-type transformers of low primary rating, each installation should be given special consideration in order that the full advantage of the two-stage design may be utilized.

Further considerations relating to the two-stage principle of metering will be given in future articles which are being contemplated. In these, two-stage transformers and meters will be treated with more specific reference to their applications, and to their construction and calibration.

High-capacity Disconnecting Switch Has New Contact Principle

A new high-current disconnecting switch for group operation has recently been made available, in which contact pressure is obtained independently of the movement of the blade. The switch is designed in ratings of 2000, 3000, and 4000 amp. and for potentials of 7500, 15,000, 25,000, and 37,000 volts.



Switch with Blade in Closed Position, Top View of Switch, Showing Double Blade, Plunger, and Wedges

The new Type RP-2 switch is of the conventional vertical-break type, the blade being swung between the open and closed positions by a linking actuated by a revolving insulator. An entirely new method of securing high-pressure contact is employed, however, which eliminates the heavy contact friction usually encountered in group-operated switches of the higher capacities, dispensing with the spring washers and flexible shunts commonly required at the hinge clip. The blade of the switch consists of two parallel copper bars between which there is inserted a plunger with wedges at each end. When the plunger is moved forward by the action of the operating mechanism, the wedges force the blades outward against the contact blocks. In this way high pressure is developed between the contact surfaces, thereby insuring low contact resistance, and forcing snow or ice formation from the contact area.

Both wedging action and blade movement are accomplished by a single turning of the operating handle. The swing of the rotating insulator is approximately 110 deg. of which 90 deg. is used to move the blade into position, and 20 deg. to operate the plunger and thus wedge the blades into place. The wedging operation cannot take place until the blade is in position between the stationary contacts.

In addition to the reliable low-resistance contact which is obtained in the new switch, the line type of contacts require neither specially machined surfaces nor adjustments for alignment.

The switch is mounted on a channel-iron base, and is supported by heavy-duty pin-type insulators. All pins are of non-corrodible material, and ball bearings are used to support the rotating members. The contacts are of copper-alloy castings.

New Low-voltage Constant-potential Car-lighting Equipment for 600-volt Traction Systems

By F. H. CRATON
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General Electric Company

BECAUSE 600-volt lighting is so widely used on 600-volt car equipments, there is a tendency to regard it as one of those long established institutions whose merit must be without question. Its virtue, however, has been low initial cost rather than any superiority as a lighting system, and it is not improbable that the present trend of railway progress toward higher standards will soon result in 600-volt lighting being replaced in many instances by the low-voltage constant-potential system.

The low-voltage constant-potential system, employing primarily a small motor-generator set, external regulator, and storage battery, possesses two outstanding advantages and several minor ones which justify its higher cost, when used for multiple-unit train and interurban car applications. The low voltage permits the use of a short, thick lamp filament, which is ideal for headlight purposes because it permits a concentration of the light source around the focal point of the reflector, in turn producing a more powerful beam than is possible with a high-voltage filament. The use of constant potential assures a headlight of adequate power at all times, and car lights whose brilliance does not vary with fluctuations in trolley voltage. The results are increased safety and passenger comfort. In addition to these most important features, the low-voltage system also secures the advantages of parallel lamp connections, low-voltage light wiring, and the elimination of wasteful voltage-lowering resistors. When it is considered that the candle power of incandescent lamps varies about as the 3.5 power of the voltage, and that their life is materially reduced by over-voltage, the disadvantages of the 600-volt lighting system become even more apparent.

To make available the advantages of low-voltage power for car lighting, the new motor-generator set illustrated in Fig. 1 has been developed.(1) This set consists of a commutating-pole compound-wound direct-current motor designed for operation on 600 volts line potential and a shunt generator to furnish power at 38 volts for lights and battery charging. The set is rated 1.75 kw. and can deliver this output continuously at 38 volts over a trolley potential range of 450 to 750 volts.

The set has been designed to cope successfully with unusually severe operating conditions such as those encountered on third-rail systems, where requirements are most exacting. The motor will stand power interruptions caused by crossing third-rail gaps where the set may go back on the line at as high as 750 volts. If the master controller is not returned to the "off" position when crossing third-rail gaps, the motor of the set is momentarily short-circuited through the traction motors, and it can withstand this condition without flashover. The set is started by throwing it directly across the line even though the potential may be as high as 750 volts.

The complete set weighs approximately 680 lb. The frame is cylindrical, of the transversely split type with four suspension lugs. Inspection openings are located at each end, on both sides of the frame. The set has two bearings; that at the generator end is a roller bearing and that at the motor end is a ball bearing to take end thrust. The armature is statically and dynamically balanced. The shaft can be removed without disturbing the windings or the commutators. The set is self-ventilated by a multiple fan mounted between the motor and the generator armatures, the air being drawn through protected openings in each end of the frame and exhausted through center openings. The design has been carried out with the intent of securing quiet operation. The brush-holders are radially adjustable and are of the steel clock-spring pressure-finger type.

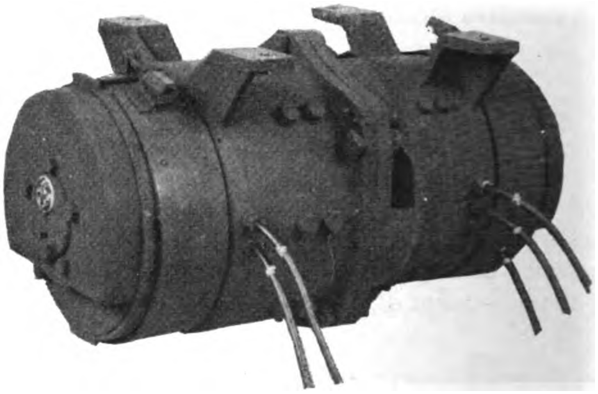

Fig. 1. The New Motor-generator Set for Low-voltage Car-lighting

As applied in service, the set furnishes power for lights and also charges a 16-cell Exide storage battery. With very slight modifications, the set may also be used in connection with a 25-cell Edison battery at 45 volts. It will deliver 2 kw. output continuously at this latter voltage, over a trolley potential range of 500 to 750 volts.

The generator voltage is held constant by a carbon-pile regulator which controls the generator excitation. The battery floats on the generator, while the lamp potential is held at approximately 32 volts by a suitable lamp regulator.

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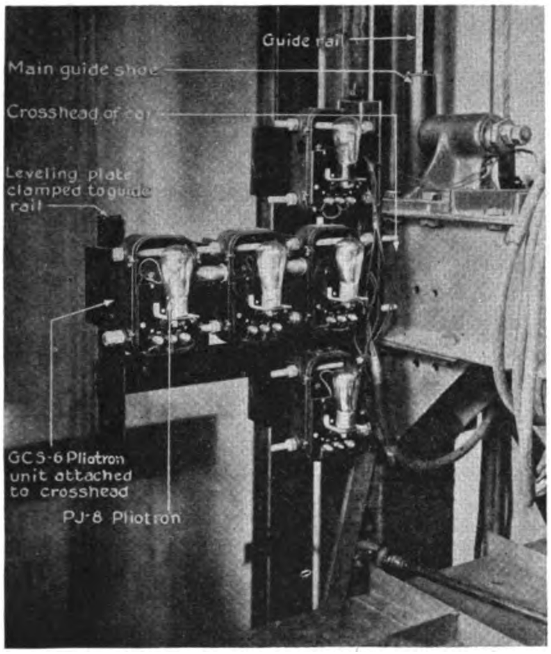
(1) Identified by the trade designation "RMG-115-B."

The motor-generator equipment, because of its exceptionally close voltage regulation, is particularly well fitted to serve also as the power supply for automatic train control in which a source of unvarying potential is an essential feature.

The motor-generator set described has a broad field of usefulness in converting to low voltage the lighting systems on 600-volt equipments. Having been built to meet severe and exacting requirements, it should prove successful under practically any condition of operation encountered in 600-volt railway service.

Vacuum Tube is Heart of New Elevator Control System

A new system of elevator control, by means of which elevator cars are automatically brought to the correct floor level, is among the more recent applications of vaccum tubes to operations in already well-established fields. This method, described as being unusually simple in operation and involving but few extra parts, involves the use of a three-element vacuum tube similar to those used in radio sets.


Guide rail
Main guide shoe
Crosshead of car
Leveling plate clamped to guide rail
GCS-6 Pliotron unit attached to crosshead
PJ-8 Pliotron

The vacuum tube employed in this case is of the type which has been used for a number of years on railroad locomotives for the purpose of transmitting block signals into the engineer's cab in visible form so that the engineer does not have to depend on watching the semaphore and light on the side of the track. This tube, though similar to the common radio vacuum tube, differs from it in the value of operating voltages which are used.

The application of the pliotron in the automatic leveling of elevator cars is based on the characteristic increase of plate current when the tube changes from an oscillating to a non-oscillating condition. A suitable number of tubes, normally in oscillation, are mounted on each elevator car. By an arrangement of coils and vanes the motion of the car approaching a floor level is made to stop the oscillation of the tubes, thus actuating relays. The relays govern control circuits which slow up the car and stop it at the correct position. The new method controls the car when running in either direction.

In operation, the elevator operator throws his car switch to the "off" position as he approaches the floor at which it is desired to stop. On nearing the floor, the relays are actuated by a combination of the coils and vanes, thus reducing the speed of the car and bringing it to a stop at the floor level without any attention or work on the part of the operator. After discharging or receiving passengers, the car is started in the usual manner and operation is continued as before.

Another application of the pliotron tube to elevator operation makes it unnecessary for the operator to watch his position in the hatchway. Devices similar to those used in automatic leveling are employed, in addition to signalling equipment consisting of suitable push buttons, lights, and a bell.

Upon entering the car each passenger indicates the floor at which he wishes to get off, and the operator immediately presses a push button corresponding to this floor. When all passengers are in the car, the operator starts by the usual method. As he approaches the first floor at which a stop is to be made, a signal light flashes and a bell rings, notifying the operator that a stop is to be made. He then throws the car switch to the "off" position and the car continues at full speed to a predetermined point in the hatchway, where it is slowed up automatically and brought level with the next floor by the automatic leveling devices. Operation is continued in a similar manner until the trip is completed.

In addition to this arrangement, push buttons are installed on each floor. A passenger waiting for a car presses a button which lights a signal and rings a bell in the first car approaching in the direction in which the passenger desires to travel. A corridor lantern also lights to show the passenger which car will be the first to reach his floor traveling in the desired direction. An operator, receiving a signal from a floor push button, throws his car switch to the "off" position and the car is stopped in the same manner as described.

With this system, the operator does not need to know where he is in the hatchway; and as long as he pushes the button for each floor called and shuts off his car switch each time the light flashes or the bell rings, every stop will be made as required.

Industrial Electric Heating

PART VIII: THE RESISTOR FURNACE (Cont'd)

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THE electrical element of the furnace, the resistor, is placed directly in the heating chamber as shown in Figs. 114 and 115. There is no intervening medium except a gas between the source of heat and the charge, hence the transfer of heat is direct. The value of this feature lies in the comparatively small temperature gradient required between the resistor and the heating chamber and, as there is practically no thermal storage capacity in the intervening space, the response to the temperature control equipment is immediate. A second feature applying to the resistor is the wide distribution of the heating element over the interior surfaces of the heating chamber enclosure, a factor of importance in obtaining a uniform distribution of the flow of heat to the charge. In addition, there is the merit of simplicity as compared with the type of furnace which requires a muffle between the source of heat and the charge.

The desirable properties of a material for a resistor are summarized as follows:

(*a*). Freedom from oxidation and immunity to the action of other gases which may be present—by intention or otherwise—in the heating chamber. In a large proportion of the resistor furnaces the atmosphere of the heating chamber is largely air; and with these furnaces resistance to oxidation is a primary requisite.

The use of artificial atmospheres in the heating chamber of resistor furnaces is growing and the resistors in these furnaces must be of material suitable for use in such atmospheres. As a general rule an artificial atmosphere selected with reference to the material of the charge serves also as a protection for the resistors.

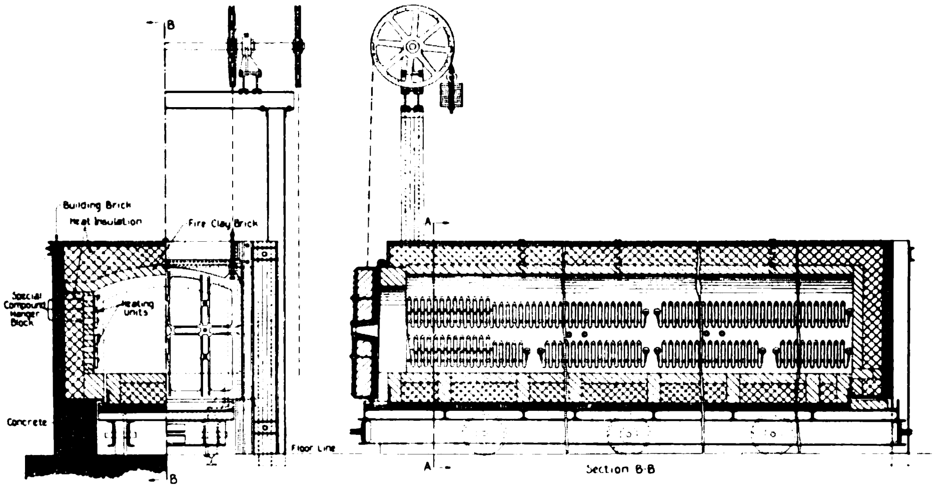

Fig. 114. Sectional View of a Horizontal Resistor Furnace of the Car-bottom Type

(*b*). Comparatively high specific resistance. This is required in order to limit the current in the resistor circuit to a reasonable value and to keep the dimensions of the resistor within the limits of the space available. (Owing to the high specific resistance of the materials used for resistors, the skin effect of alternating current at standard frequencies is negligible.)

(*c*). Low thermal conductivity. The value of low thermal conductivity is in the prevention of an excessive flow of heat from the terminals of the resistor, *i.e.*, a cool terminal is desirable.

(*d*). A low resistance-temperature coefficient, preferably positive and of constant value over the entire temperature range, *i.e.*, from room temperature to the maximum operating temperature. If a low coefficient over this temperature range cannot be obtained, the next best choice is a low

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The following table facilitates the location of material hitherto published in this serial

Part	*Issue*	*Figs.*	*Equations*	*Examples*	*Tables*	*Footnotes*
Part I	Oct. 1927, p. 488	1 to 32	(1) to (8b)	1 to 3	I to III	(1) and (2)
Part II	Nov. 1927, p. 551	24 to 43	(9) and (10)	4 to 6	IV to VII	(3) to (13)
Part III	Dec. 1927, p. 600	44 to 57	(11) and (12)	8 to 11	VIII to IX	(14) and (15)
Part IV	Mar. 1928, p. 125	58 to 73	(13) and (14)	——	X to XX	(16) ro (26)
Part V	Apr. 1928, p. 204	74 to 83	(15) to (21)	12 and 13	XXI to XXIII	(27) to (29)
Part VI	Sept. 1928, p. 485	84 to 99	——	14	XXIV to XXVI	(30) to (35)
Part VII	Oct. 1928, p. 560	100 to 113	(22) to (24)	15	XXVIII and XXVIII	(36) to (41)

value of the coefficient within the working-temperature range of the furnace.

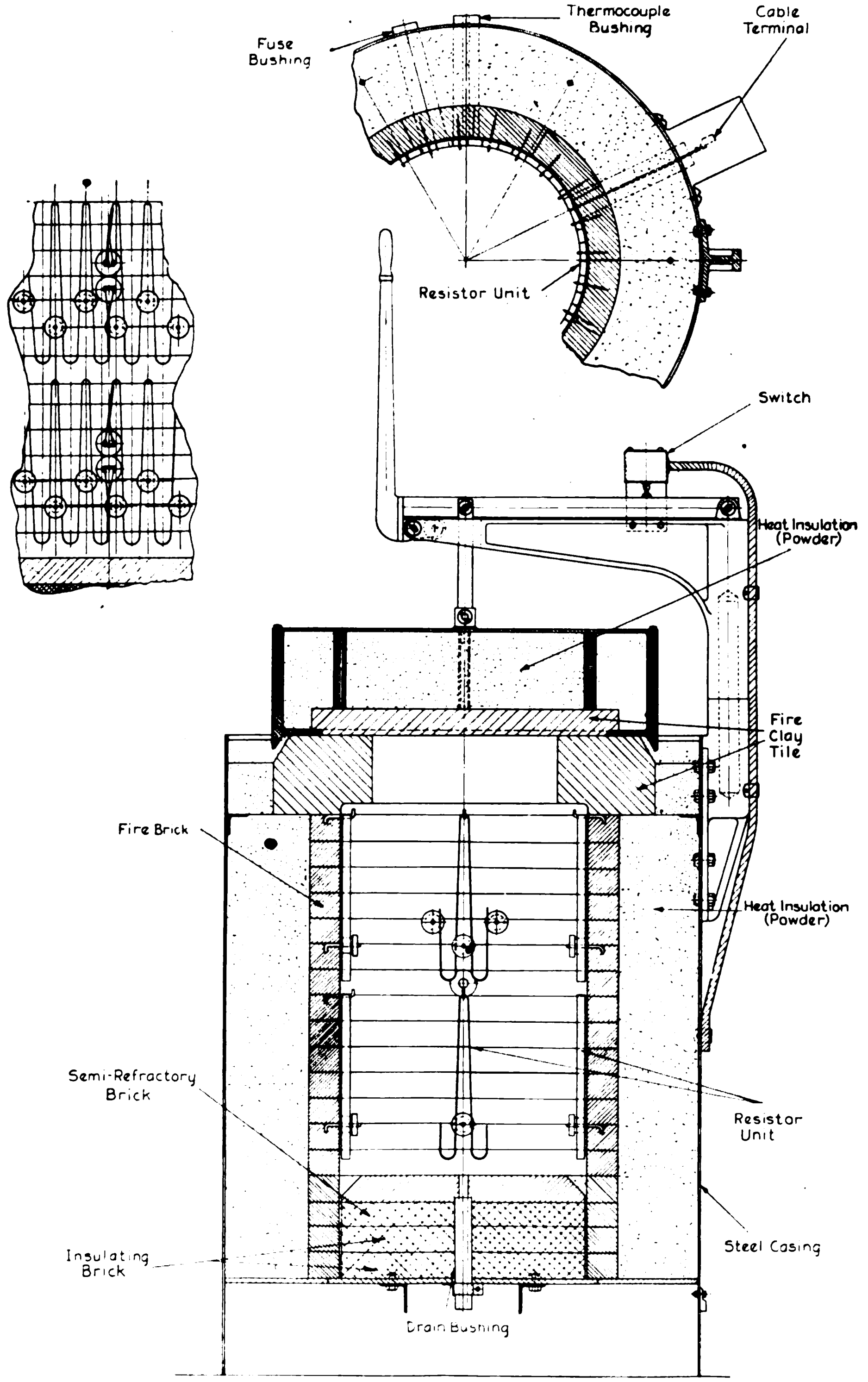


Fig. 115. Sectional View of a Vertical Resistor Furnace

Resistor furnaces are operated on constant-voltage circuits and the method of power control of the furnace depends upon the characteristic of the resistance-temperature coefficient. If there is considerable difference between the cold and hot resistance of the resistor, provision must be made for starting equipment. In general, the less the variation in the resistance-temperature coefficient, the more simple will be the power control equipment of the furnace.

(*e*). The temperature at which the material of the resistor begins to soften should be well above the maximum operating temperature. This has particular reference to the prevention of sagging of the resistor elements.

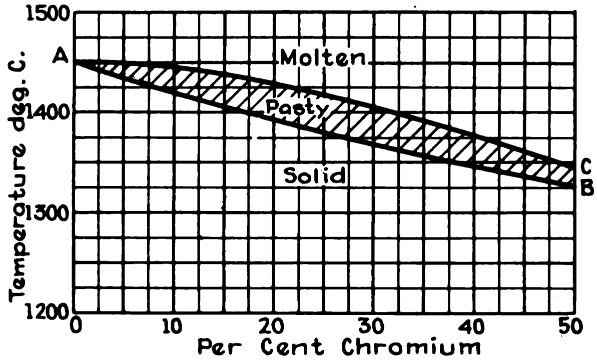
(*f*). The resistor material should be non-magnetic.

(*g*). The expansion coefficient should be low so as to not require a complicated provision for expansion and contraction.

(*h*). There should be no chemical reaction between the resistor material and the supports of the resistor.

(*i*). The structure of the material should be permanent below its softening temperature.

(*j*). Neither specific heat nor unit weight, in this case, is a factor one way or another since the mass of the resistor is not large enough to make the thermal storage capacity of the resistor of consequence.



Note - Point A = 1452°

Fig. 116. The Nickel-chromium Diagram (in Part)

(*k*). It is required that the material have ample mechanical strength at operating temperatures.

(*l*). The resistor material should be capable of being easily shaped in order that any form of the element (wire, ribbon, bars, etc.) can be obtained, and so that the shape selected in any given case can be formed into a resistor to suit the heat distribution desired. It must be practicable to provide terminal connections.

(*m*). The resistor material should have a long life in service.

(*n*). The cost of the material in the shapes needed should be reasonable.

No material, of course, conforms wholly with the ideal specification. As it is, the materials which approach the specification sufficiently close for use are few in number. The higher the operating temperature the more difficult the resistor material problem and the greater must be the compromise in the use of what can be obtained.

The materials in common use for furnace resistors are:

(*a*). Nickel-chromium alloys.
(*b*). Iron and steel.
(*c*). Molybdenum.
(*d*). A silicon-carbide product.

The rapid development of the resistor furnace shown by the chart in Fig. 100 was made possible by the nickel-chromium alloy resistor. The nickel-chromium alloy diagram—in part—is shown in Fig. 116. The maximum operating temperature of the resistor must be below the values of the line *A-B* by a safe margin. The nickel-chromium alloy resistor is protected against oxidation when heated in air by a film of oxide which forms on the surfaces of the element when first heated. This oxide coating has practically the same expansion coefficient as that of the alloy so that the coating is very durable. This coating resists to a high degree the penetration of oxygen; and after the initial coating is formed the rate of oxidation of the alloy is quite slow. The higher the chromium content of the alloy the better is the protection against oxidation; and as the rate of oxidation increases with temperature, the higher the operating temperature of the resistor the higher should be the chromium content of the alloy. Alloys containing more than 20 per cent chromium are difficult to form into shapes and the 80 Ni-20 Cr alloy—practically free from impurities—has been adopted generally for furnace service within its temperature limitation. For this alloy the safe maximum operating temperature is usually specified to be 1150 deg. C. (2102 deg. F.)

The resistance-temperature curve of the 80 Ni-20 Cr alloy is shown in Fig. 117. The values are for the alloy with the initial oxide coating, *i.e.*, black surfaces. It will be noted that the variation in resistance due to changes in temperature are not wide, and resistors of this alloy can be operated on constant voltage circuits without voltage regulating equipment.

As the protection against oxidation is not perfect, the thickness of the oxide coating on the resistor surfaces slowly increases with resulting increase in the resistance of the resistor. In course of time the resistance will have become higher than will permit the needed power input to the furnace. The resistor then must be scrapped. In some cases, however, an increase in the applied voltage can be used to prolong the resistor life. Barring accidents and imperfections in the alloy and in the construction of the resistor, and if the resistor is operated with due regard to its temperature limitation, the life of the nickel-chromium resistor is such that the maintenance cost of this part of the furnace is a small item. As previously stated, the rate of oxidation is measured by the operating

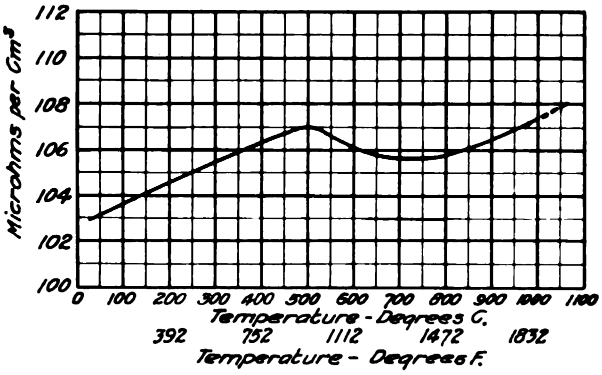


Fig. 117. Resistivity-temperature Curve of 80 Ni-20 Cr Alloy

temperature; hence the lower this temperature the longer is the life which may be expected from the resistor. It is of course possible to use an alloy with a chromium content lower than 20 per cent, particularly in furnaces which operate only in the lower part of the furnace temperature range. In general, the higher the chromium content the higher the cost of the alloy. Thus the selection is a matter of economics in each case.

The main object in using an iron or steel resistor is to obtain a higher heating chamber temperature than can be obtained by using the nickel-chromium alloy resistor. Resistivity-temperature curves of electrolytic iron and carbon steel in the solid state from data by Saldan[42] are given in Fig. 118. A part of the iron-carbon diagram is shown in Fig. 119. The effect of the carbon addition is to increase the electrical resistance and to lower the temperature at which the alloy begins to soften. For example, the addition of 0.5 per cent carbon to iron increases the resistance about 3½ per cent at 2300 deg. F. and lowers the softening points to about 2520 deg. F. Hence if steel is used as resistor material, the lower the carbon content the better. The differences between cold and hot resistances of iron and steel make it necessary to provide means for limiting the current when starting.

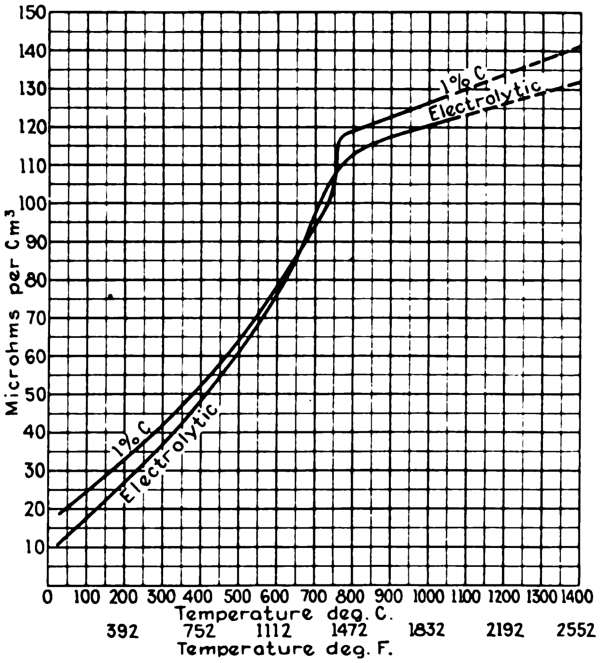


Fig. 118. Resistivity-temperature Curves of Iron and One Per Cent Carbon Steel

The power-factor of an iron or steel resistor, while low when cold, becomes unity when the nonmagnetic state is reached in heating. (See Fig. 120).

The linear expansion coefficients of iron and steel are comparatively high and the arrangement of the resistor in the heating chamber must provide for expansion and contraction. A convenient way of doing this is to corrugate the resistor elements.

Iron and steel resistors must be protected against oxidation; *i.e.*, this class of resistors must be operated in an atmosphere free from oxygen. The life depends both upon the degree of the protection from oxidation (it is difficult to obtain a heating chamber atmosphere entirely free from oxygen) and upon the frequency with which the resistor is heated through the critical range. In passing through the critical range, iron and carbon steel suffer deformation as illustrated by the photograph reproduced in Fig. 121. This block of cold-rolled steel, originally rectangular, was a permanent support in the heating chamber of a batch-type hydrogen-copper brazing furnace operated at 2200 deg. F. After a period of time, the repeated heating wrought the change of shape shown. This phenomenon is often the cause of failure by breakage of iron and steel resistors in batch furnaces.

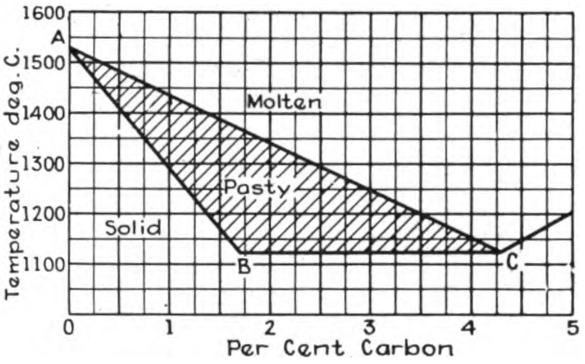


Fig. 119. The Iron-carbon Diagram (in Part)

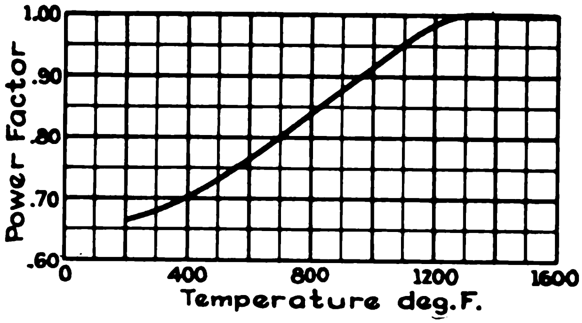


Fig. 120. Power-factor-temperature Curve of a Low-carbon Steel

The life periods of iron and carbon-steel resistors under average conditions in furnaces with hydrogen and hydrogen-nitrogen atmospheres vary. In some cases, with a heating chamber temperature around 2100 deg. F. a year's service has been obtained. In other cases the life has been shorter.

The addition of metallic elements to iron increases the electrical resistance and lowers the temperature at which the alloy begins to soften. Silicon is individual in that it gives a marked increase in resistance with but little lowering of the softening point. Resistivity-temperature curves of silicon steel are given

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(42) "Apparatus for the Measurement of Resistance of Metals in the Solid State at High Temperatures." *The Iron and Steel Institute—Carnegie Scholarship Memoirs.* Vol. 7, p. 195.

in Fig. 122 and the iron-silicon diagram is shown in part in Fig. 123. The upper limit of the silicon addition is the amount above which the metal becomes too brittle for forming into shapes. About 2½ per cent silicon is about as high addition as this condition permits. Another effect of silicon upon iron is the suppression of the transformation points. With about 2½ per cent silicon the transformation points are entirely suppressed as shown in the temperature-time curve of Fig. 106, which is the record of the heating of a 2½ per cent silicon-iron charge. These effects of silicon indicate the value of silicon iron as a resistor material both in its betterment of the resistivity characteristic and in the elimination of the deformation phenomenon noted in case of iron or carbon steel.

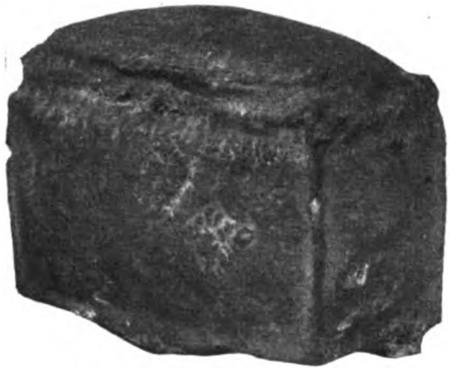
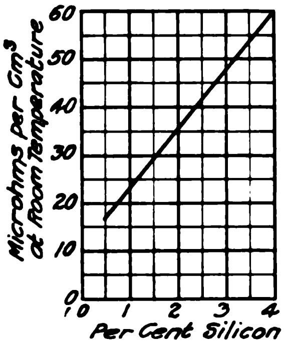


Fig. 121. Deformation Effect Upon Carbon Steel Due to Repeated Heating Through the Critical Range. The original shape of the block was rectangular



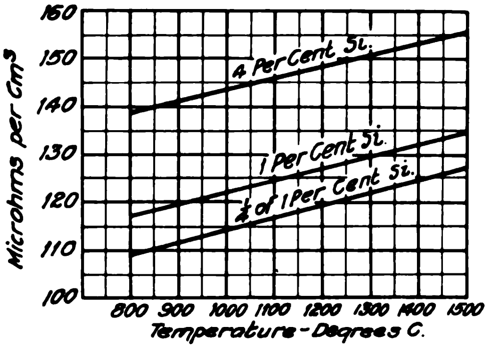


Fig. 122. Resistivity-temperature Curves of Silicon Steel

The use of molybdenum as a material for furnace resistors has so far been confined to high-temperature laboratory furnaces. There is no standard design for these furnaces, which are usually made locally and in design suited to individual conditions. Fig. 124 shows a design frequently used for small furnaces of this type. The molybdeunm resistor is wound upon a refractory muffle, *e.g.*, an alundum tube, which forms the heating chamber. The resistor is protected by an artificial atmosphere, usually hydrogen. The melting point of molybdenum (about 4620 deg. F.) is higher than the softening point of refractory materials. Hence with resistors of that metal the refractory support of the resistor limits the temperature of the heating chamber. The upper temperature limit of this furnace is about 2900 deg. F.

The resistivity-temperature curve of molybdenum is given in Fig. 125. The wide range between the cold and hot resistances makes voltage control equipment necessary for these furnaces. Molybdenum is generally preferable to tungsten as resistor material for the reason that is more easily formed into shape. For high-temperature high-vacuum furnaces, tungsten may be necessary on account of the comparatively high vapor pressure of molybdenum. The resistivity of tungsten is slightly higher than that of molybdenum, *viz.*, 5 microhms per cm^3 compared with 4.5 microhms per cm^3 at 25 deg. C. The resistance-temperature coefficients of the two metals are about the same, so that the curve in Fig. 125 can be used for tungsten without serious error. Both metals must be protected against oxidation when used for resistors.

The non-metallic furnace resistor which within its class is in most general use is a silicon-carbide product. Resistors of this material are available for heating-chamber temperatures up to 2400 deg. F. No protection from oxidation is required. The material is formed into round rods of diameters and lengths to suit the voltage and current requirements. The manufacturer (43) has adopted as the unit of resistance for these elements a rod 1/16 in. in diameter by 1 in. long. For resistor-furnace service the value of this unit is 16 ohms. The elements are made up into multiples of this diameter and length. The unit resistance is expressed as

$$U.R. = \frac{D^2 \times E}{L \times I} \tag{25}$$

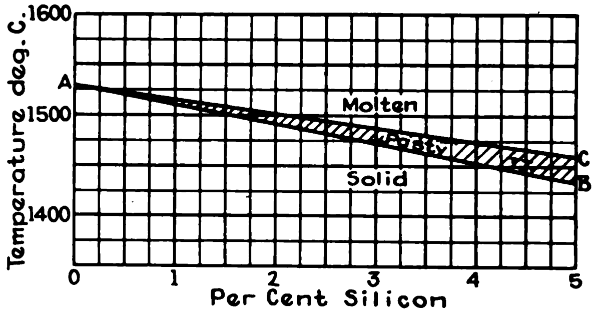


Fig. 123. The Iron-silicon Diagram (in Part)

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(43) The Globar Corporation, Niagara Falls, N. Y.

in which

$U.R.$ = unit resistance
D = number of 1/16 inches in the diameter of the elements
E = voltage
L = length in inches
I = current.

The variation of the resistance with temperature of this type of resistor is shown in Fig. 126. It will be noted that within the operating range of temperature the resistance-temperature coefficient is small and is positive. The range of this coefficient between room temperature and operating temperature is not wide enough to require voltage control equipment for starting.

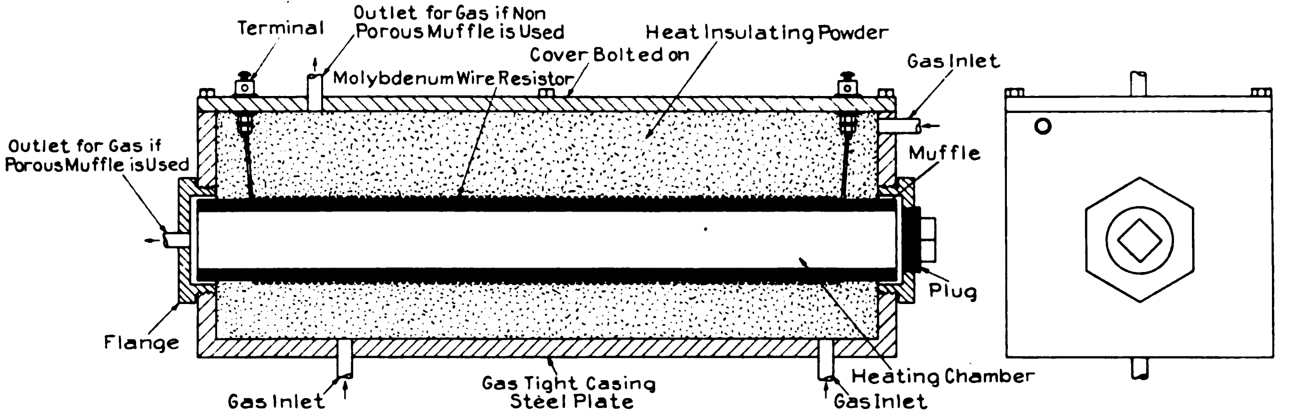

Fig. 124. Laboratory-type Furnace with Molybdenum Resistor

The resistivity of the silicon-carbide resistor increases with hours use and the life of a resistor element is measured by this growth in resistance. After a period the resistance of the elements becomes too high for effective service at the rated voltage, and either renewal or an increase in the applied voltage is necessary.

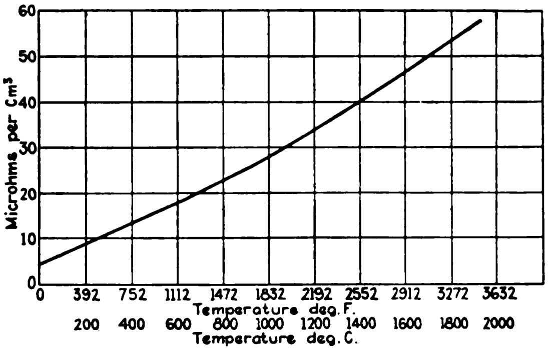

Fig. 125. Resistivity-temperature Curve of Molybdenum

A primary requirement in the arrangement of the resistor in the heating chamber is the spreading of the resistor surface over a large part of the inner surface of the heating chamber enclosure. This is to obtain the wide distribution of the flux of heat noted in the first paragraph of this article. This requires that the resistor be long in comparison with either of its other dimensions.

The selection of the location of the resistors, *i.e.*, the side walls, roof, and hearth, is based upon the nature of the charge and the shape of the heating chamber. The object in all cases is to obtain a path of heat flow to the center of each piece of the charge in which there is the minimum number of surface resistances in series. For example, if the charge consists of a stack of thin steel sheets laid horizontally, resistors placed above and below the charge would not be effective on account of the large number of surface resistances in the path of heat flow from the surface to the center of the charge. For such a charge the best location of the resistors is on the side walls of the heating chamber. Whatever may be the locations of the resistors, much can be done towards speed of heating by so placing the pieces of the charge in the heating chamber as to obtain the minimum resistance to the flow of heat to the center of each piece of the charge. As stated in Part II (p. 554) it is generally necessary—except in cases like that cited above—to place resistors underneath the hearth to supply heat to the bottom surfaces of the charge. There is no rule as to the ratio of the capacity in the hearth resistors to the total resistor capacity. This depends upon the nature of the charge and varies to a considerable extent. An average value of resistor

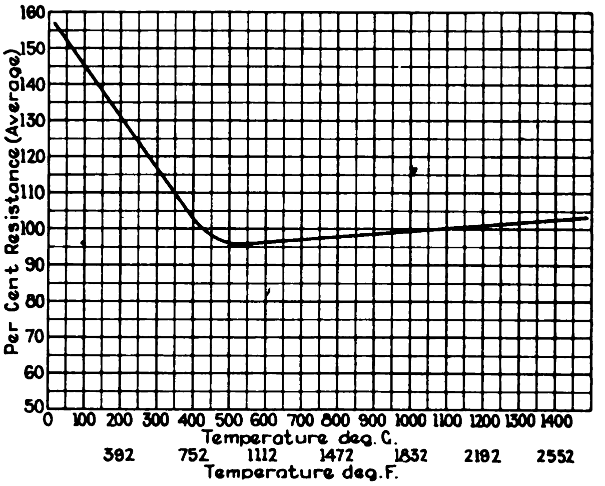

Fig. 126. Variation of Resistance with Temperature of Non-metallic Resistors with Silicon-carbide Base

capacity underneath the hearth is about one-third of the total.

Fig. 127 shows the development of a ribbon-form nickel-chromium resistor; and an end length of a resistor of this type is illustrated in Fig. 128. A typical method of mounting this type of resistor on a side wall is shown in Fig. 129. The photograph of the furnace interior in Fig. 130 shows the mounting of an iron ribbon resistor in a cylindrical furnace. The silicon-carbide resistor is mounted as shown in Fig. 131.

The resistor units are connected in series, multiple, or some combination of the series-multiple arrangement on single-phase circuits and also, for three-phase circuits, in Y or delta as may be required by the current rating of the unit and by the supply voltage. The terminal design provides for easy replacement of a resistor unit while the furnace is hot. A feature of the terminal is the pressure contact which permits but little heat to flow from the unit to the terminal connection yet is of ample cross sectional area for the current flow.

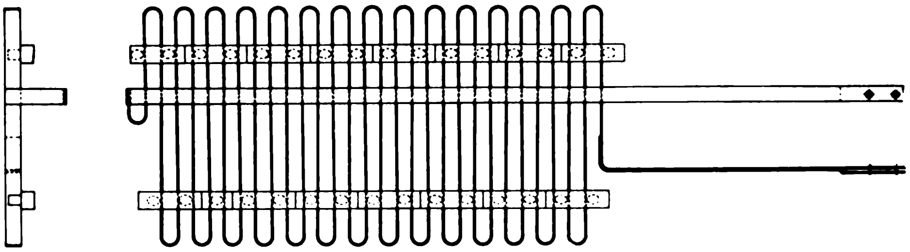

Fig. 127. Development of a Ribbon-type Nickel-chromium Resistor

Frequently, extra capacity is added to the end section of the resistor at the door end of the chamber—as shown in Fig. 114—to compensate for the loss of heat through the door area while the door is open. In case of furnaces which operate with short heating periods and with which the door is open for a comparatively large portion of the time of the cycle the resistor is often divided into two circuits, front and rear, each under separate temperature control.

The metallic resistor, being obtainable in practically any length, has the minimum number of connections through the heating chamber wall. These outlets are a source of heat loss and are thus undesirable. In case of iron resistors, which require voltage control for starting, the usual construction is to divide the resistor into two or more circuits and use the series-parallel method for voltage control. This, of course, increases the number of circuit outlets through the wall.

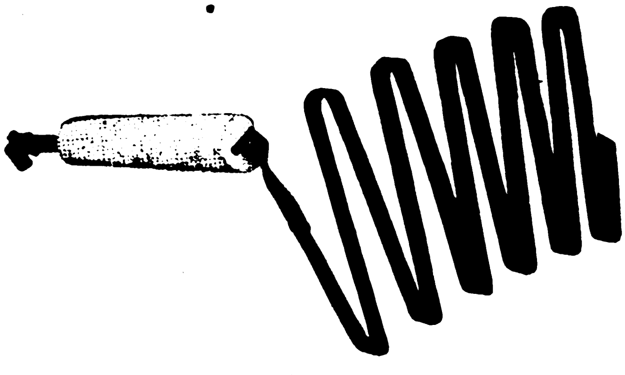

Fig. 128. End Length of Nickel-chromium Ribbon Resistor, Showing Reënforced Terminal and Wall Insulator

A water-cooled terminal for the silicon-carbide resistor unit is shown in Fig. 131. This type of terminal is designed for heating chamber temperatures above 2000 deg. F. Below that temperature a similar terminal without the water cooling feature is used.

The supports of a resistor must be electrical insulators over the entire temperature range of the furnace. Refractory materials are classed as non-conductors. However, these materials have negative resistance-temperature coefficients and at furnace temperatures become partial conductors. Refractory products vary in composition; and besides, for any particular lot of material, the analyses are not apt to be uniform. Hence there is a wide variation in the electrical resistance of any given material. Furthermore, under the influence of sustained high temperature the resistance is subject to change. Hence there is no constant value of resistivity for any given refractory product. Fire brick is among the refractories having the lowest resistivities. The resistivity-temperature curve of Fig. 132 is from

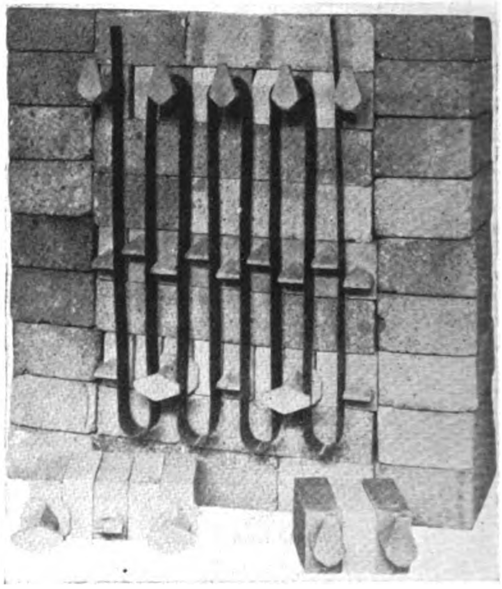

Fig. 129. Typical Method of Mounting Nickel-chromium Ribbon Resistor on Furnace Wall

data by Henry.(44) An insulation breakdown in a refractory is usually due to a local low resistance which provides a selective path for the current.

The supports of the resistor, in addition to being rigid and being insulators at elevated temperatures, should not muffle the resistor surface. This would cause local high temperatures and shorten the life of the resistor. Forms of resistor supports and terminal tubes made of electrically-fused alumina with a clay bond are shown in Fig. 133. The supports in place may be seen in Fig. 129. The electrically-fused alumina has a comparatively high resistivity at furnace temperatures and the method of manufacture makes possible a uniform resistance characteristic. The resistivity-temperature curve of alundum cement —an electrically-fused alumina with a ceramic bond— given in Fig. 132, is from data by Northrup.(45)

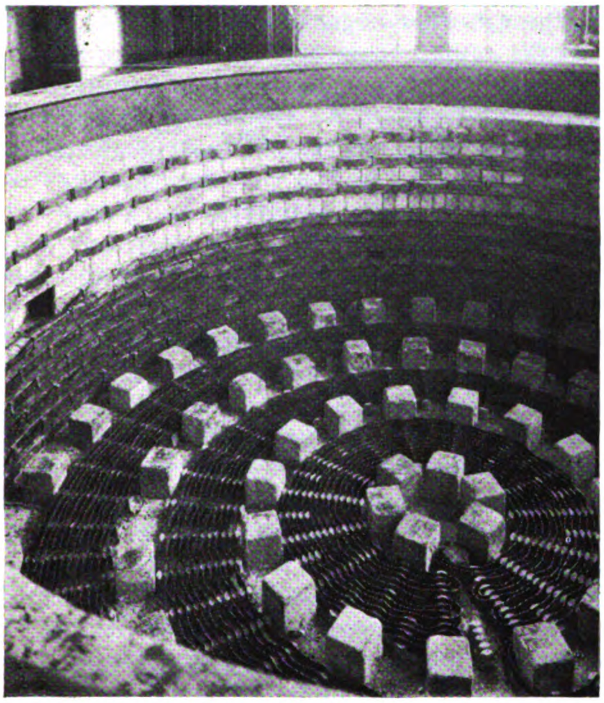

Fig. 130 Method of Installing the Iron Ribbon Resistor in a Cylindrical Furnace

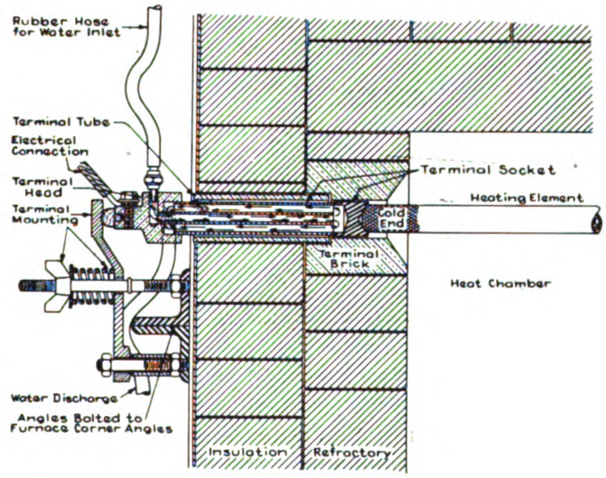

Fig. 131. Method of Installing the Silicon-carbide Resistor and the Water-cooled Terminal

The insulation of the resistor circuit in the furnace is one of the limitations on the voltage of this circuit. About the highest voltage that is practicable at furnace temperatures is 440 volts. The voltage in more general use is 220 volts. For small furnaces, 110 volts or lower is required in order to avoid the necessity for very small resistor elements.

Many large batch furnaces are equipped with removable bottoms. The furnace of Fig. 114 is of the car-bottom type. The medium and small-size batch furnaces usually have fixed hearths. Hearths of this type are frequently made of a plate of heat and corrosion-resisting alloy. Such alloys vary in composition; but all contain nickel and chromium. The alloys are mechanically strong and show but little scaling up to furnace temperatures of 2000 deg. F.

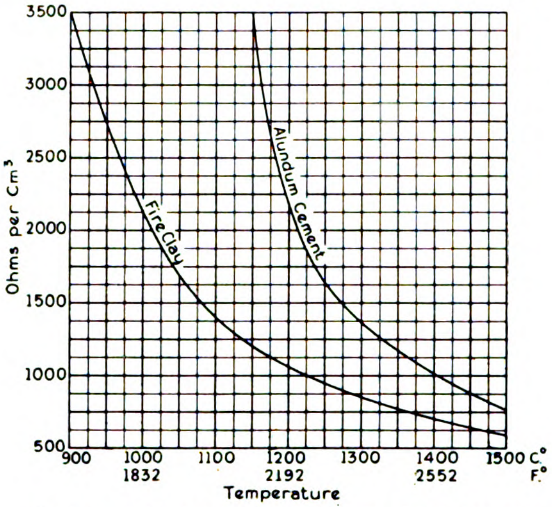

Fig. 132. Resistivity-temperature Curves of Fire Clay and Alundum Cement

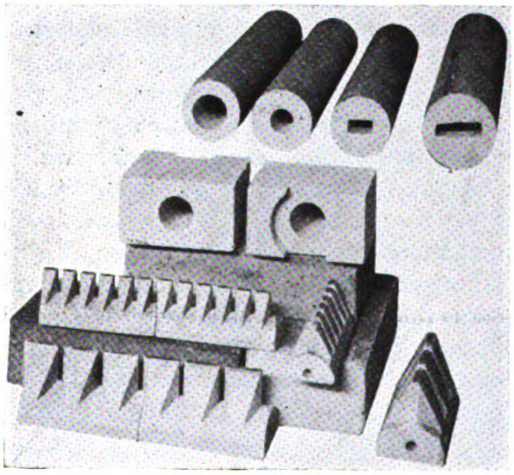

Fig. 133. Insulator Supports and Wall Tubes for Furnace Resistor

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(44) "The Resistivity of Refractories," *Jour. Amer. Ceram. Soc.*, Vol. 7, p. 764.

(45) "High Temperature Resistivity of Refractories." *Met. and Chem. Eng.*, Vol. 12, p. 125.

This type of hearth is indicated in Figs. 29 and 30. The metal hearth may or may not be perforated, and in large sizes is made in sections to minimize trouble from warping. The hearth plates are usually made with upturned edges along the sides to prevent scale from dropping on the resistor below the hearth.

The resistor must deliver its rated output with the heating chamber at the specified temperature and this without reaching a temperature which is higher than permitted by maintaining a safe margin below the softening point of the material of the resistor. It is desirable to keep the temperature gradient between the resistor and the heating chamber—*i.e.*, the inside surface of the heating chamber wall—low; first, so that surfaces and edges of the charge will not be overheated and, secondly, because the lower the operation temperature of a resistor the longer is its life. On the other hand, the higher this temperature gradient the higher is the output of a given resistor. In between these opposing conditions lies conservative design.

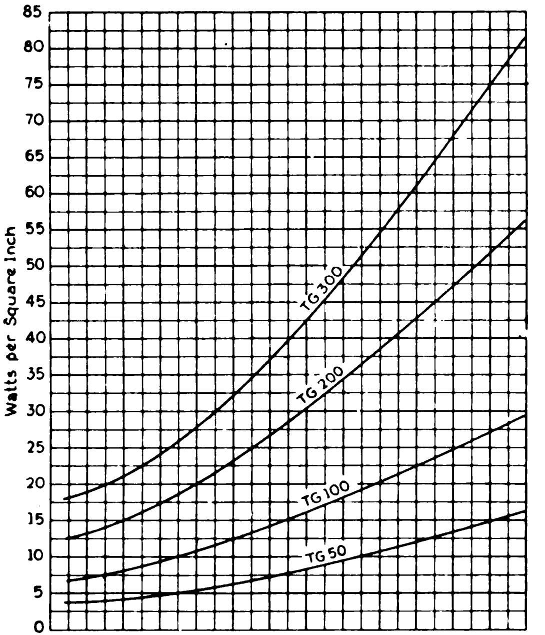

Fig. 134. Chart of Heat Dissipation from Flat Surfaces

Several shapes of resistor elements are in use, the more common being ribbon (rectangular cross section) and wires (round cross section). For any given temperature conditions the shapes of the elements affect the rate of heat flow from the element—this rate being expressed as watts per pound of metal in the element—in accordance with the influence of the three factors:

(*a*). The shape factor
(*b*). The interference factor
(*c*). The surface-area factor.

The chart in Fig. 134 shows the output of resistors having flat surfaces for the range of furnace temperatures and for several temperature gradients. This chart is based upon a constant of 0.29 in Equation (6), and a constant of 0.72 in Equation (8), and upon the condition of an unobstructed flow of heat in an atmosphere of air or its equivalent, as regards thermal conduction, of other gases.

Langmuir(46) has shown that the convection component (natural convection only) of heat transfer from—or to—a surface is accomplished by conduction through a film of gas which envelops the surface. Thus the natural convection component of a flow of heat from a surface under given temperature conditions is determined by the thermal conductivity of this gas film. As in all cases of heat conduction the shape factor of the path of that flow enters into the determination of this convection component. The shape factor in case of resistor surfaces is determined by the degree of curvature of the surface. The curve given in Fig. 135 has as ordinates the ratio of the convection component from a round surface to the convection component from a flat surface. The abscissas are diameters of the round surface. Thus for a round wire 0.3 cm. in diameter the natural convection component would be four times that from a flat surface, all other conditions being the same.

By reference to Fig. 18, it will be noted that at the higher temperatures and with small temperature gradients the part of the total heat transfer by natural convection is very small. For example, referring to Fig. 134, the total heat transfer with a heating chamber temperature of 1700 deg. F. and a temperature gradient of 200 deg. F. is 23 watts per sq. in. from a flat surface. The convection component in this value is 0.45 watt. For a round resistor of No. 1 B.&S. wire (diam. = 0.7348 cm.) the factor c', Fig. 135, is 2.4. Hence the convection component for this wire would be $(0.45 \times 2.4) = 1.08$ watts per sq. in. As a general rule with furnace temperatures and temperature gradients the influence of the shape factor for round resistors of sizes No. 1 B.&S. and larger can be neglected without serious error.

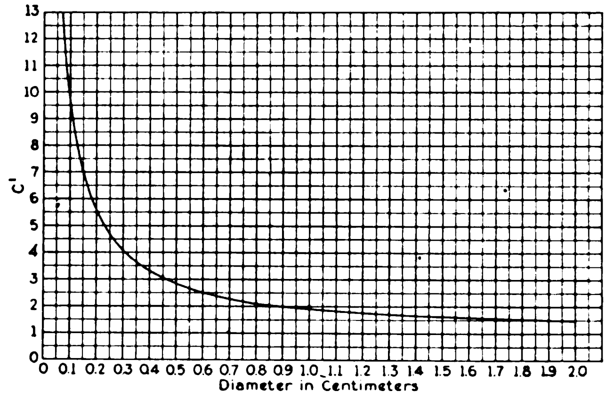

Fig. 135. Effect of Shape Factor on Convection Component.

c' = ratio of convection component from round surface to convection component from flat surface.

Even if by the use of thin, round wires and large temperature gradients the natural convection component is made comparatively large, the resulting increase in the total heat transfer—as compared with that from a flat surface—is not desirable in furnace practice. As previously stated, uniformity of heat distribution is a primary requirement and as heat transfer by natural convection is not subject to control, *i.e.*, it cannot be fixed in distribution like radiation, the convection component works against the attainment of uniformity in the distribution of the flux of heat.

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(46) "Convection and Radiation of Heat," *Trans. Amer. Electrochem. Soc.*, Vol. 23, p. 298.

The chart shown in Fig. 134 is based upon unobstructed radiation. However, the arrangement of the resistor elements in loops (see Fig. 127) causes interference in the radiations from adjacent surfaces and for a given resistor temperature there is a corresponding reduction in the flow of heat by radiation. This decrease is approximately proportional to the angle α as shown for both flat and round surfaces in Fig. 136. The ordinates of these curves are the values less than unity by which the free radiation components must be multiplied. The abscissas are the ratios of the spacing of the resistors—surface to surface—to the diameter or width, as the case may be, of the resistor element. For example, assume as in a preceding paragraph a heating chamber temperature of 1700 deg. F. and a temperature gradient of 200 deg. F. The free radiation component is, as before, $(23-0.45)=22.55$ watts per sq. in. For a ratio of spacing to width of unity in each case the value of c'' for a flat surface is 0.71 and for a round surface 0.78. Hence we have:

Radiation component, flat surface $(22.55\times0.71)=16.0$

Radiation component, round surface $(22.55\times0.78)=17.55$

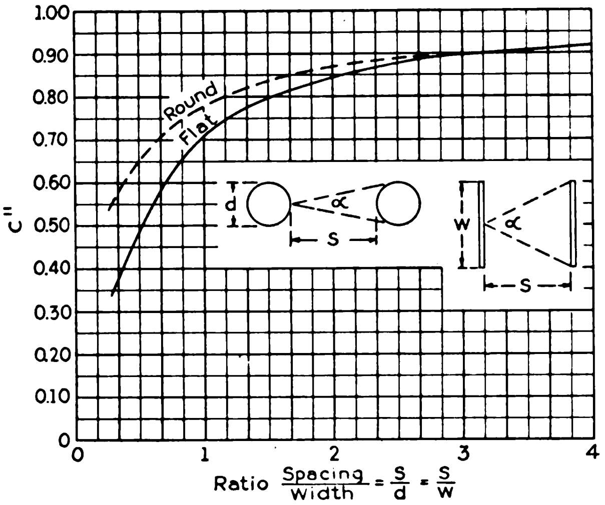

Fig. 136. Effect of Radiation Interference of Adjacent Surfaces.
c'' = ratio of obstructed radiation to free radiation

Combining these figures with those already obtained the total heat dissipation from a No. 1 B.&S. wire is $(17.55+1.08)=18.63$ watts per sq. in. For the flat surface the total head dissipation is $(16+0.45)=16.45$ watts per sq. inch. Thus under the conditions specified the output of the round resistor per square inch of surface is approximately 13 per cent greater than that from the flat surface.

A round wire or rod has the minimum surface area as compared with rectangular shapes of the same mass. If a round rod is rolled to a rectangular cross section the increase in surface area is shown in Fig. 137. The ordinates of this curve are the ratios of the surface areas of rectangular strips to the original surface area of the round rod from which the strip was rolled, the length being unchanged. The abscissas are the widths expressed as multiples of the original diameter of the rod. End areas are not included. For example, if a given length of wire is rolled into strip or ribbon without change in length and to a width five times the diameter, the new surface area is 3.3 times the surface area before rolling.

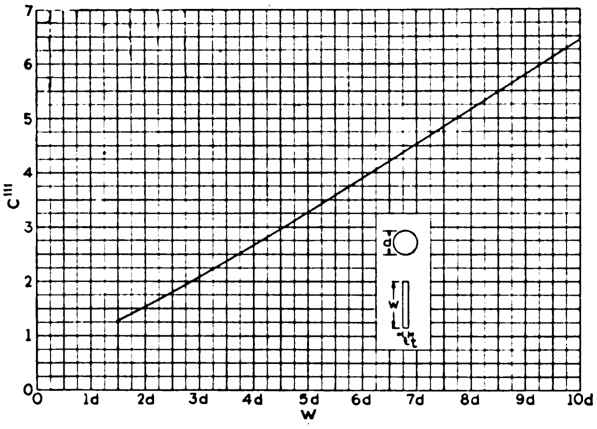

Fig. 137. Surface-area Factor
c''' = ratio of surface area of rectangular resistor to surface area of round resistor

The surface area of No. 1 B.&S. wire is 10.9 sq. in. per ft. of length. Using the value for unit heat dissipation obtained in a preceding paragraph the output per foot of wire is $(18.63\times10.9)=203$ watts. The output per foot of the ribbon which has 3.3 times the surface of the wire is (47) $(16.45\times10.9\times3.3)=591$ watts. As the weight is the same in both cases the output per pound of metal is 2.9 times greater for the strip form.

The fourth factor in the consideration of the shape of the resistor element is its cost expressed as watt-hours output (during its useful life) per dollar. This cost, however, is not the first cost of the metal in the resistor alone, but is the first cost of the complete resistor as installed in the furnace plus the maintenance cost during its useful life. Furthermore, there should be added to this cost the cost of the heat loss through the furnace walls made necessary by the terminal construction of the resistor in each case. The last-mentioned is difficult to evaluate; but in general the fewer the wall entrances required for terminals the lower will be this item.

A cost comparison cannot be expressed in general terms but must be limited to all cases in specific designs of resistors. However, the cost of the resistor is not the end. This cost is not a large item in the furnace expenditure and, alone, is not a deciding factor. The important consideration is the effective use of the resistor in its function as a distributor of heat; and this is the primary basis of resistor design.

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(47) Note: In practice, the cross sections of rectangular shapes are not true rectangles because of rounded corners.

The actual design of a resistor is a matter of a study of the requirements in each case. The problem includes, in addition to the principles discussed in the preceding paragraphs and the observance of the relation expressed by Joule's law, the distribution of the flux of heat, the balancing of the required capacity against the space available for the installation of the resistor, the locations of the elements, the permissible maximum current, the selection of standard size, and the individual problens which must be met in each case. This phase of the subject is reserved for a later article in this series. However, before leaving the subject for a time, it is of interest to make a preliminary estimate of the power required in the resistor of Example 15 of Part VII.

We may assume that the furnace is to be used to heat charges of steel like that noted for the temperature-time curves of Fig. 35. The weight of the charge is 1685 lb., the final temperature 1650 deg. F. and the heating period is approximately 4 hr. This is at the rate of about 420 lb. per hr., which represents an average performance for a batch furnace of this size.

The heat absorbed by the charge (from Fig. 138) is $(1685 \times 85) = 145$ kw-hr. The average input to the steel is $\frac{145}{4} = 36$ kw. The heat loss of the furnace at 1800 deg. F. is 10 kw.; and at 1650 deg. F, will be approximately proportionately less, *i.e.*, 9.2 kw.

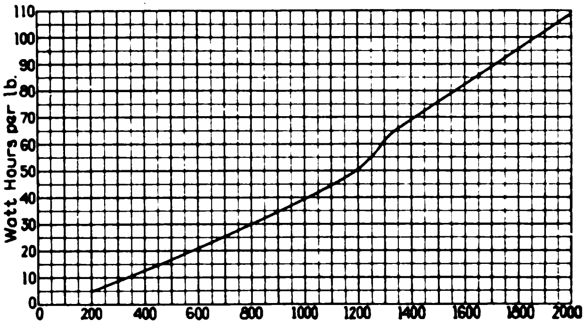


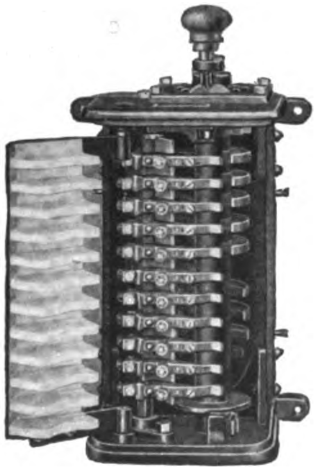
Fig. 138. Heat Content Curve of Iron

Hence the average input to the furnace during the heating period—disregarding the open door loss—is 45 kw. Allowing some margin, the rating of the resistor would be 50 kw. As a preliminaty check on the assumptions so far made, the conversion efficiency of the furnace for continuous operation would be $\frac{1685}{45 \times 4} = 9$ lb. of steel per kw-hr., which is a fair performance for this particular service.

(*To be continued*)

Primary-resistance Drum Switches

A new line of primary-resistance drum switches has been developed for use with squirrel-cage induction motors on small cranes, hoists, machine tools, etc. These switches are made in two forms: one for hoists where the motor is overhauled in the lowering direction, and the other for hoists where the motor is not overhauled in the lowering direction. An example of the former is a hoist with spur gearing and no automatic mechanical load brake. (The switch for this application provides four points hoisting and one point lowering.) An example of the latter is a hoist using low efficiency worm gearing or an automatic mechanical load brake. The switch designed for this service, and also for machine tools and general-purpose applications, provides four points forward and four reversing.



The drum switches are built without operating mechanism so that the user may select the type of operating equipment he desires. The dial plate for the horizontal handle is assembled with the switch but is removed and discarded when a vertical handle or rope-wheel attachment is to be used.

For cement-mill or similar service where the equipment will be subject to excessive non-explosive dust, the covers are made dust-tight by a felt gasket riveted around the edges. They are tightly fitted to the switch frame, and all openings in the frame (including conduit box) are sealed with compound to exclude the dust.

Suitable separate resistance enables approximately 50 per cent of full-load rated torque to be obtained on the first point of the switch. The torque obtainable on the other points can be adjusted over wide limits by shifting the taps in the resistor.

The switches are designed for operation on primary voltages up to 550, and for 15 hp. maximum on two- or three-phase circuits.

Low-voltage A-c. Networks

PART VIII: TRANSFORMER EQUIPMENT

By D. K. BLAKE
Central Station Engineering Dept., General Electric Company

PREVIOUS to the introduction of low-voltage a-c. network systems into the commercial area, it was the usual practice to use 2300-volt single-phase subway-type transformers for the distribution of alternating current. Most of the installations were of 50-kv-a. units and below, having a tank construction similar to that shown in Fig. 46. Larger installations of such as 75- and 100-kv-a. units, used a corrugated tank similar to Fig. 47. Transformer equipment for network systems are shown in Figs. 48 to 57.

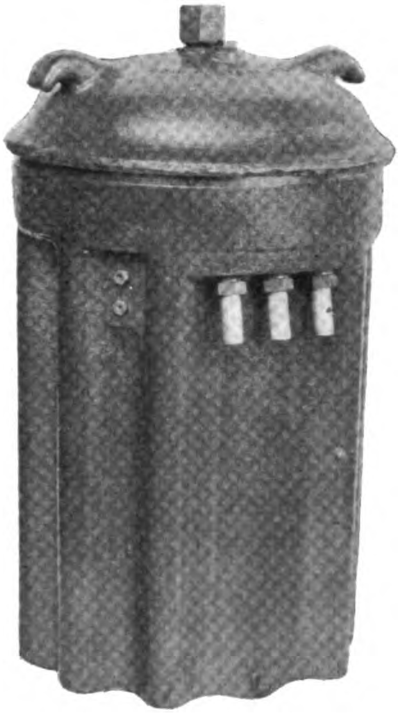


Fig. 46. A Typical Transformer of the Subway Type Commonly Employed for A-c. Distribution Prior to the General Use of Network Systems

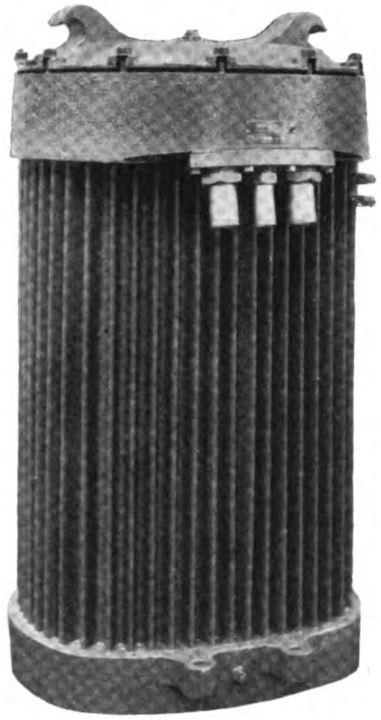


Fig. 47. Subway Transformer of the Type Generally Used for the Large Units in the Older Systems of A-c. Distribution

The network system has influenced the design of distribution transformers with respect to reactance, size, voltage, mechanical features, and accessories. The earlier transformers designed for network operation were designed with 10 per cent inherent reactance for the purpose of better load division. The introduction of the small cable-type reactors constructed of iron punchings has virtually eliminated the necessity of transformers having a reactance above the normal design. A few companies, however, consider the 10 per cent reactance transformer preferable.

The transformers for network systems are of a larger and more uniform kv-a. rating than are used in earlier types of distribution systems. It is seldom that a system requires more than two sizes; and some systems have only one size for the street vaults, and larger ratings in building vaults when very large building loads are involved. In the greater number of systems, transformer installations of 300-kv-a. banks are found more economincal than other ratings. Transformers with ratings of 300 kv-a. are of about as much capacity as can be put in street vaults without encountering very difficult ventilating problems. One company was limited by city authorities in the

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Reference made to material hitherto published in this series of articles may be readily located from the following table:

Part	*Issue*	*Figs.*	*Tables*	*Footnotes*
Part I	Feb., 1928, p. 82	1 to 4	—	(1)
Part II	Mar., 1928, p. 140	5 to 16	—	—
Part III	Apr., 1928, p. 186	17 to 19	I to VII	(2)
Part IV	May, 1928, p. 245	20 to 26	VIII	(3)
Part V	Aug., 1928, p. 440	27	IX to XII	(4) and (5)
Part VI	Sept., 1928, p. 480	28 to 33	—	—
Part VII	Nov., 1928, p. 600	34 to 45	—	—

number and dimensions of street vaults and found it necessary to obtain a special transformer to have as much kv-a. rating as possible without exceeding losses safely dissipated by the vault. The result was a 500-kv-a. three-phase transformer having an efficiency of about 99 per cent.

Where sufficient load and areas are involved, it is more economical to use the generated or transmission

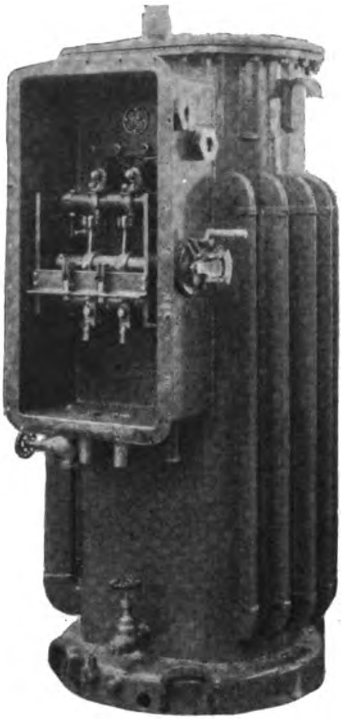
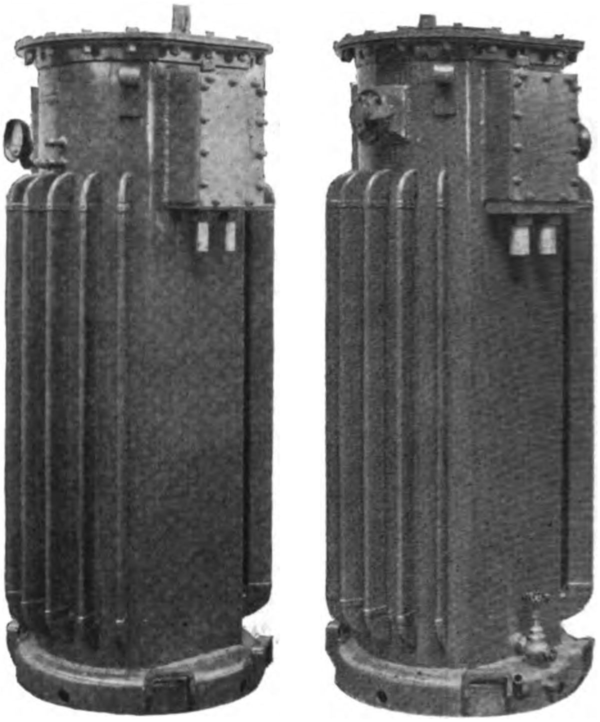


Fig. 48. Single-phase 100-kv-a. Network Transformer for 11- or 13-kv. Circuit. The pothead with grounding and disconnecting switch may be removed to pass the transformer through a 32-in. vault opening



(*A*) (*B*)

Fig. 49. (*A*) High-voltage, and (*B*) Low-voltage Side of 100-kv-a. 11,000-115/199 Y Volt Network Transformer. A two-pole 11,000-volt grounding and disconnecting switch is mounted in the tank under the oil and operated from the handle shown at the top left

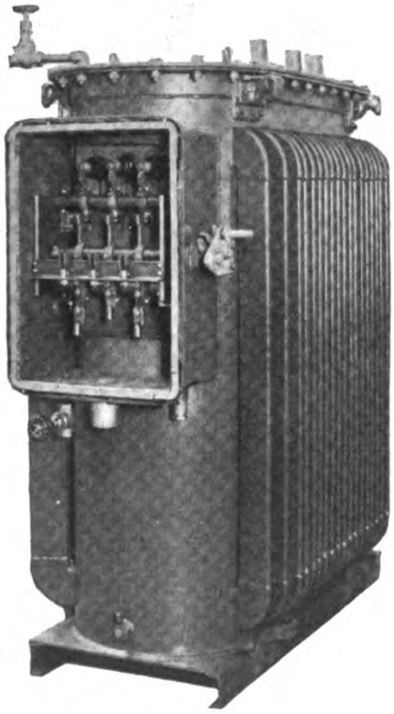


Fig. 50. Three-phase 11- or 13-kv. 300-kv-a. Network Transformer with Pothead Containing Grounding and Disconnecting Switch Mounted on End of the Tank. Removable water-tight caps for giving access to ratio adjuster handles are visible at the top

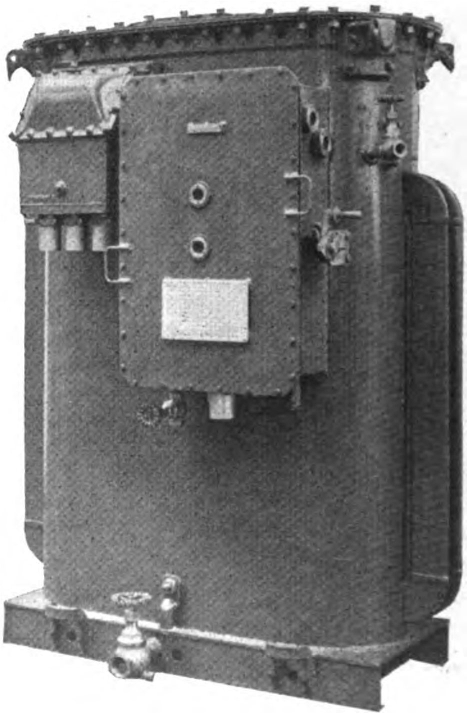


Fig. 51. Three-phase 11- or 13-kv. 300-kv-a. Network Transformer with Both Low-voltage and High-voltage Potheads Mounted on the Same Side. Operating handle for switch and bull's eyes for oil level indication are shown on the side of the high-voltage pothead

voltage for the network feeders. This practice required the development of higher voltage transformers for subway use. The voltages now used are 11,000, 13,200, and 26,400 volts.

The combined light-and-power three-phase network systems commonly used have all transformers installed in three-phase banks, and excess capacity is provided to allow for the loss of any particular bank at peak-load period. Some engineers find the three-phase transformer much preferable to three single-phase units because of the less space required and much simpler vault installation by eliminating much cable splicing and racking; also the losses and costs are slightly lower than single-phase installations, particularly where grounding switches are provided.

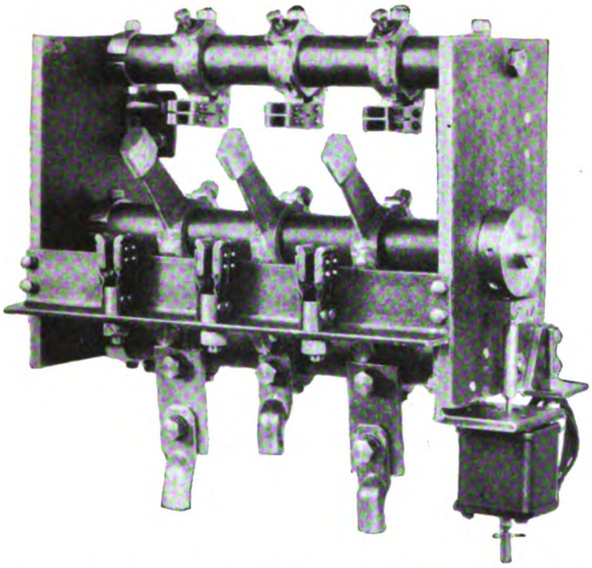


Fig. 52. Unit Assembly of Grounding and Disconnecting Switch for Mounting in Transformer Potheads or in Oil-filled Box for Separate Installation. When energized, the solenoid interlock at lower right prevents movement of shaft

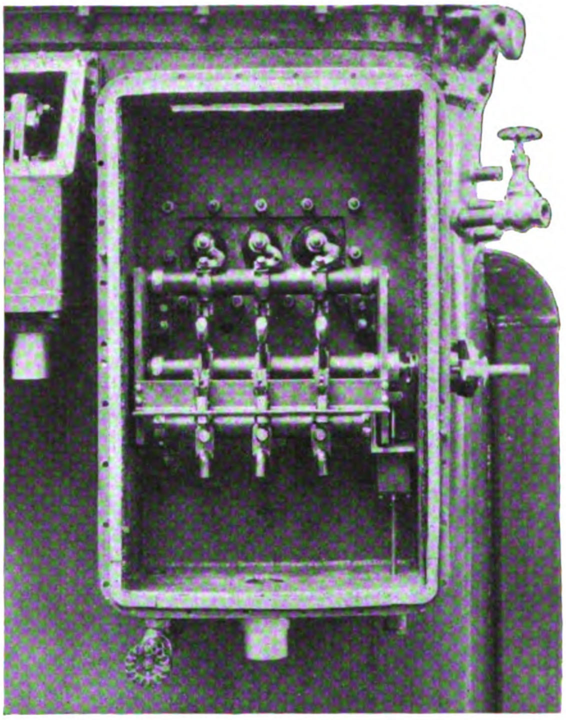
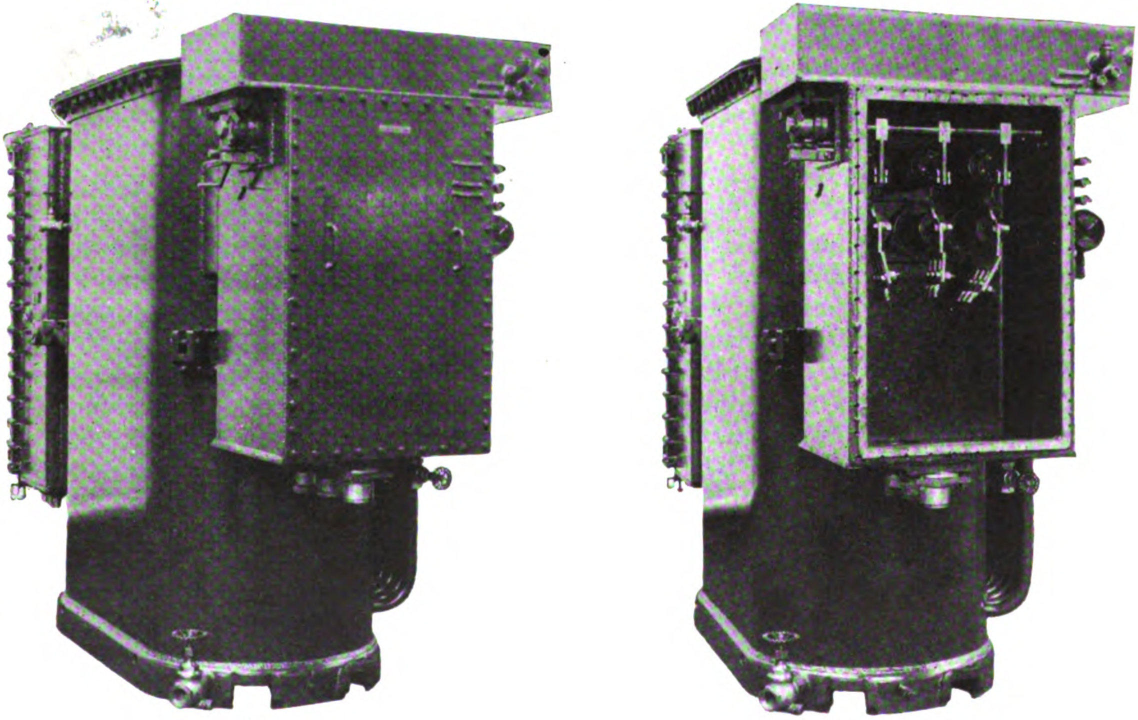


Fig. 53. High-voltage Pothead of a Three-phase Transformer Illustrating the Assembly of Grounding and Disconnecting Switch



(*A*) (*B*)

Fig. 54. (*A*) Three-phase 26-kv. 500-kv-a. Network Transformer with High-voltage Pothead Mounted on One End and Low-voltage Pothead Containing Network Protector Mounted on the Other End. The high-voltage pothead is arranged for three triple-conductor cables, two of which loop through and the third branches off. (*B*) Shows the transformer with high-voltage pothead cover removed and the ground-switch in the closed position

Existing vaults, of course, with the usual 32- or 35-in. openings require single-phase units. Considerable progress has been made in reducing the weight of three-phase units so that they are not difficult to handle.

Changes in mechanical design are caused mainly by the special cable terminations required for high voltages and the operating requirements in the way of safety and convenience for working on the high-voltage cable.

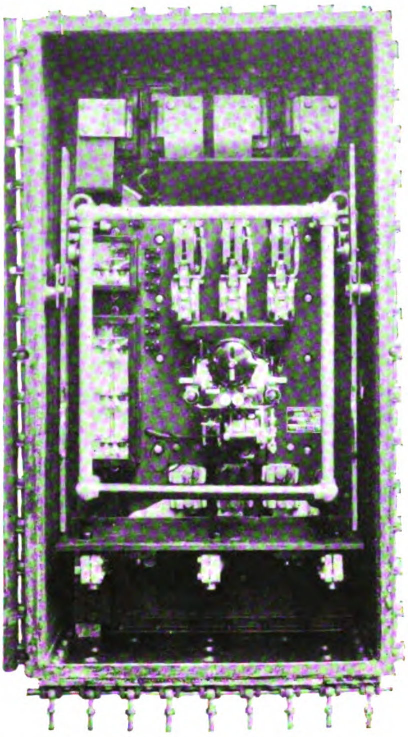


Fig. 55. The Low-voltage Side of the Transformer in Fig. 54 with Box Cover Removed, Showing the Automatic Network Protector Mounted Therein and Arranged for Four Outgoing Cables Per Phase

The termination of the high-voltage cable in the transformer is done in a variety of ways. Early 13,000-volt units were built with standard cable-bells inside of the transformer and the cable was introduced through the ordinary subway bushings. This makes a safe construction, but a rather inconvenient one; and the tendency of late has been to provide junction boxes in which the cable can be terminated and connected to the transformer leads so that the joint can be readily broken in the junction box without taking the cover off of the transformer. This makes the installation much simpler, as all connections can be made through the small openings in the junction boxes.

The ideas of some engineers with regard to the arrangement of junction boxes vary to some extent. The preferred practice is to put in, between the transformer and junction box, bushings which can be sealed tight so that there is no possibility of an interchange of oil or air between the junction box and the transformer. This enables the operator to perform all necessary operations in the installation of the transformer without unsealing the main tank. The construction is, of course, expensive and tends to take more space than a simpler arrangement in which the wall between the junction box and transformer is not hermetically sealed. With the latter construction it is possible to fill the junction box with paraffin or petroleum, but not with oil.

Because of the very limited space available for these junction boxes, the termination of the high-voltage cable by taping the cable from the end of the lead sheath seems to be the most practicable method. The tape is built up at the end of the lead sheath until the diameter of the insulation is considerably increased and is then tapered off for a distance sufficient for creepage to the bare end of the cable. Metal tape is then wrapped over this tape from the lead sheath up to the point of maximum diameter. This tapers the stress through the cable insulation and this type of joint is being used satisfactorily up to 26,000 volts. The cable terminations are made by the cablemen and the glands through which the cable is introduced are large enough to permit the whole end to be pulled out of the junction box. This construction is simple and economical in space.

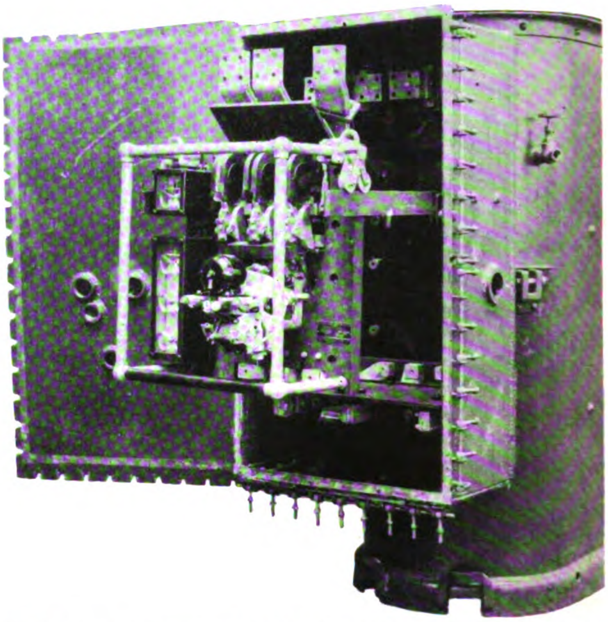


Fig. 56. The Network Protector of Fig. 55 in Extended Position, Illustrating the Roll-out Feature Provided for Convenience in Installing the Protective Equipment in Housing

Flexible low-voltage terminals have been provided to permit the connection of the large secondary cables with ease. Rigid terminals make it very difficult to line up the heavy secondary cables with the terminals.

Operating requirements in the way of safety and convenience for working on the high-voltage cable are in general due to the fact that each feeder and the transformer it supplies are usually handled as a unit, being disconnected at the substation by a circuit breaker and automatically disconnected on the low-voltage side of the transformers by the network protectors. If cablemen are to work on cables or transformers in a vault, safety requirements dictate the grounding and short circuiting of the cable and transformer in that vault and perhaps in one or two adjacent ones. The exact practice differs at present; but in general, since there is no individual protective apparatus on the high side of each transformer, there is no convenient place for the location of grounding devices except in the transformer. The result is the placing in the transformer, or in junction boxes attached to it, a considerable variety of switches for grounding or short-circuiting either the cable alone or cable and transformer. In many cases the switches have an open position in which the cable is not connected to either ground or transformer and hence is in a free condition for "phasing out," *i.e.*, checking the phases to see that they have been properly connected after the correction of trouble or new construction work.

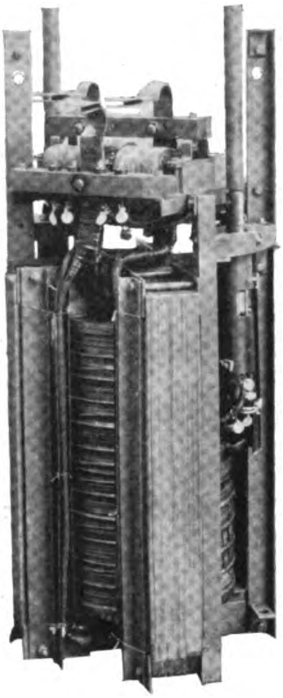


Fig. 57. Core and Coil Assembly of a 13,200-volt Network Transformer, Illustrating the Mounting of Current Transformers Used with Pilot-wire Protection

The majority of these switches are provided with electrical interlocks which prevent their operation as long as there is voltage on the transformer. These interlocks consist of small coils connected to the low-voltage side of the transformer and so arranged as to lock the switch in whatever position it may be when the coil is energized. This prevents any movement of the switch when the transformer is energized from either side. Usually these devices are provided with some form of external indicating mechanism from which the operator can see whether or not the transformer is energized. This may take the form of a target in front of a bull's eye or of a rod in a glass tube extending outside of the pothead.

It is preferable to mount the switches in the high-voltage pothead. This means that the pothead must be filled with oil and must be large enough to provide proper clearances around the switch. For three-phase transformers there is usually sufficient room to make an external pothead large enough for the switch as an integral part of the transformer. Because existing vaults have 32- or 35-in. diameter openings some engineers have required that the switches be mounted inside the transformer tank under the oil of single-phase units, since it is impossible to mount an external pothead large enough for a switch external to a transformer which must pass through a 32-in. or 35-in. opening. It is considered by some that this creates a hazard in the transformer because of the possibility of minute metallic particles, such as copper from the switch tips, getting into the oil and windings. To avoid this, detachable potheads have been developed which can be removed while the transformer is being lowered through the manhole and attached when it is in final position.

Switches for pothead mounting have been developed to constitute a self-contained unit, which is secured in place by a few bolts, and easily removable from the pothead with flexible connections to the transformer and cable terminals. The potheads are provided with an opening in the top through which the blades of the switch can be reached for "phasing out" in the "open" position of the three-position switches. These unit switches may also be mounted in a box for wall mounting. This arrangement has the advantage of being a means of completely deënergizing the transformer and permits the use of fewer switches where they are not considered necessary at every vault.

The switches are manually operated, with an interesting exception wherein one company required that the single-throw grounding switch be closed with a CO_2 jack which can be operated from the street level.

As an additional case of rather exceptional practice, it may be remembered that one company, using loop primary feeders with pilot-wire protection, required that the current transformers connected in the transformer secondary be mounted in the transformer tank, under the oil. These current transformers were of very high ratio and quite a problem was presented in regard to properly mounting them.

Acknowledgment is made to Mr. E. D. Treanor, of the Pittsfield Works, for information contributed.

(To be continued)

Vacuum Tubes as Oscillation Generators

PART VII

METHODS OF IMPROVING THE EFFICIENCY OF VACUUM TUBES DRIVING OSCILLATING CIRCUITS—A HIGH-EFFICIENCY CIRCUIT WITH SQUARE CURRENT WAVES

By D. C. PRINCE and F. B. VOGDES
Research Laboratory, General Electric Company

THE efficiency of vacuum tubes driving oscillating circuits is of importance from two standpoints: The power supply usually consists of a kenotron rectifier set and, while the power may be purchased at a low rate, its cost is fairly high after passing through the rectifier on account of the cost of operation of this piece of apparatus. A second reason for desiring efficiency lies in the fact that the losses which can be disposed of are limited on account of the heating of the anode of the pliotron. Hence, the higher the efficiency, the greater will be the load which can be carried by a given tube.

When driving an oscillating circuit with sine waves of voltage by means of a vacuum tube, certain efficiencies are obtainable as shown in Part II. In this case, the plate potential consists of the steady voltage of the supply source plus an alternating voltage given by the oscillating circuit; and the grid potential consists of a similar alternating voltage plus a steady voltage derived from a grid-leak condenser and resistance. Under good operating conditions, the alternating voltage has a peak value slightly less than the direct voltage of the supply source, and, if the plate current is drawn only at the time of minimum plate potential, power conversion occurs with high efficiency. Such operation is not desirable in practice, however, because it represents a load on the tube much less than it is capable of carrying; accordingly, efficiency of conversion is relinquished in order to get more output from the same apparatus. This is accomplished largely by allowing the plate current to flow through a longer period of time. The additional amount of power is obtained at a much lower efficiency than the first, because of the fact that the extra current is drawn while the alternating voltage is not at its peak and the plate potential is considerably larger than its minimum value.

Effect of Emission on Efficiency and Output

Instead of obtaining the maximum output from a tube entirely by increasing the period during which the plate current flows, the output may be increased partially by increasing the instantaneous value of the current flow while the plate voltage is near its minimum, thus increasing the efficiency and the maximum load which can be carried. This requires that the emission of the tube be increased, for, in the ordinary tube, the most efficient operation already utilizes most of the available emission. Since an increase in emission results in a shortened tube life, there is a limit beyond which it is not economical to increase emission. This limit is different for different operating conditions, and, in some cases, it may be advisable to exceed the rated filament heating current of a tube in order to obtain more output. Fig. 61 shows the calculated output for a given tube with a varying emission and constant tube loss and indicates the order of the increase in output to be expected.

Effect on Efficiency of Harmonic in Plate-voltage Wave

A second way to increase efficiency is in control of the plate-voltage wave. Harmonics might be added to this wave as in Fig. 62, which shows a fifth harmonic impressed on the sinusoidal voltage of the oscillating circuit. The resultant wave is so arranged as to have a flat or slightly cupped peak, and, by this means, two desirable ends could be attained. First, the actual output would be increased. Assuming the plate-current wave to remain constant, it would be possible to use a higher value of the fundamental-frequency alternating plate potential without having the minimum resultant voltage approach too close to the maximum grid potential. This would represent an increase in output proportional to the increase in the voltage. Second, the losses would be decreased. It is obvious that the combined wave would give a lower plate potential during the time current is flowing than would a pure sine wave having the same minimum. The resultant decrease in loss would, for the most part, be equal to the increase in output already described, but the losses would be further decreased if any output were obtained at the harmonic frequency. While no use could probably

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References made to material hitherto published in this series of articles may be readily located from the following table:

Part	*Issue*	*Figs.*	*Tables*	*Footnotes*
Part I	June, 1927, p. 320	1 to 4	—	—
Part II	Oct., 1927, p. 501	5 to 21	1 to III	(1) to (4)
Part III	Feb., 1928, p. 97	22 to 23	IV	—
Part IV	Mar., 1928, p. 147	24 to 34	—	—
Part V	July, 1928, p. 388	35 to 43	—	(5) to (7)
Part VI	Oct., 1928, p. 546	44 to 60	—	(8)

be made of the harmonic power outside of the circuit, it would be better to dissipate it outside the tube than inside.

Production of an Harmonic Voltage

The question of what to do with the harmonic power is closely allied with the question of obtaining the harmonic voltage in the first place. By proper circuit design, the power produced might be used to maintain the voltage. A circuit capable of accomplishing this is shown in Fig. 63. The plate current, which is supplied by the plate-blocking condenser, is drawn through both the main oscillating circuit and an harmonic-trap circuit tuned to the frequency of the desired harmonic. The fundamental frequency component of the current in connection with the voltage of the main oscillating circuit represents power input to that circuit; and the harmonic component in connection with the voltage of the harmonic trap represents power input to the trap circuit. The oscillations in the trap circuit will build up until the input and losses are equal, and by controlling the resistance of the circuit, the voltage may be given the desired magnitude.

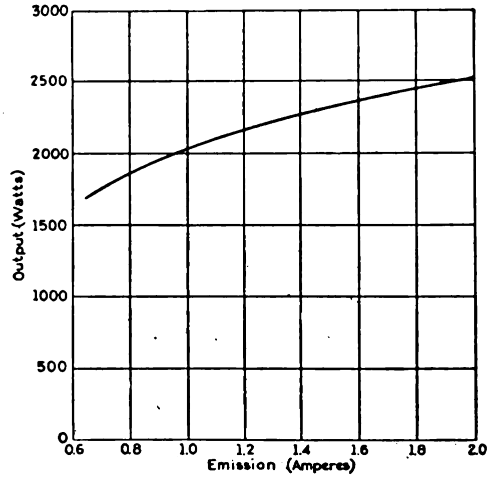

Fig. 61. Calculated Output for a Pliotron with Varying Emissions and Constant Tube Loss

Inspection of Fig. 62 shows that, if current were to flow for a period corresponding to only 180 deg. of the harmonic voltage wave, the input to the trap circuit would be negative. In order to get a positive input to this circuit, the plate-current wave would have to flow for approximately three half cycles of the harmonic voltage wave. The amount of power available for operating a trap circuit at a given voltage and given plate-current wave can be found by a method similar to that described in Part II.

General Problem of High Efficiency

Within the knowledge of the writers, no circuits with harmonic traps have ever been constructed. The subject is of great interest, however, as it shows the nature of the steps to be taken in order to get the highest possible efficiency and output. The problem consists of two parts: The current must be passed through the tube so that the highest average current is passed with the lowest loss, and the direct voltage of the supply source and alternating voltage of the oscillating circuit must be connected together in such a way that the instantaneous difference is not the cause of a loss, as in the case of the excessive plate-voltage drop met with in the simple oscillating circuits with current passing during a considerable time.

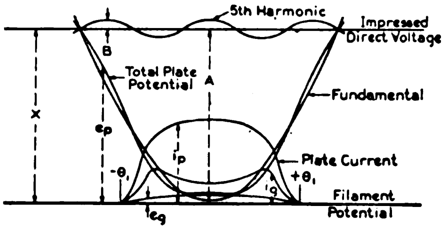

Fig. 62. Effect of Adding a Fifth Harmonic to the Plate-voltage Wave

The Current Problem

Assuming that no loss except the loss due to the space charge occurs in the tube driving the oscillating circuit, let it be seen what the form of the current wave for highest efficiency should be. The instantaneous plate voltage will be assumed only slightly higher than the instantaneous grid voltage, and together these voltages will overcome the space-charge voltage due to the instantaneous current. If the voltages were constant and independent of the current, it would made no difference what the shape of the current wave might be, the loss would always be the same for the same average current. The voltages have to be increased, however, with increased current, and therefore the minimum space-charge loss occurs with a steady current flowing for as large a portion of the cycle as possible.

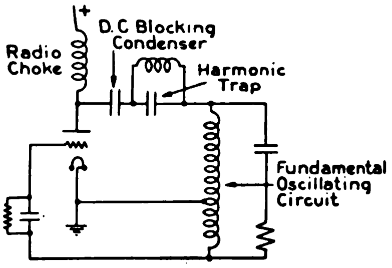

Fig. 63. Oscillating Circuit with Trap to Supply an Harmonic in the Plate-voltage Wave

The Voltage Problem

The passage of current through the tube for any considerable portion of a cycle makes absolutely necessary the balancing of the direct voltage of the supply source and the alternating voltage of the oscillating circuit in such a way that the latter appears as a counter-e.m.f. equal to the former minus the space-charge drop in the tube. To accomplish this, the instantaneous differences in the two voltages must be absorbed by some piece of apparatus which will not act as a sink of power. A choke coil will satisfy the need, for it will alternately absorb and supply energy with no inherent loss.

This choke will have to carry a steady current unless very special means are supplied to stop and start the current through it, and this is undesirable. On the other hand, it is obvious that the tube has no valve action if it carries a steady current.

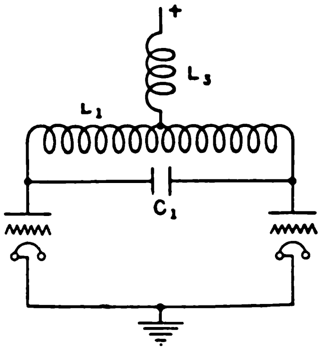

Fig. 64. General Form of a Two-tube Circuit with Square Current Waves

Two-tube Square-wave Circuit

The conditions of voltage and current required for high output and efficiency are satisfied by using two tubes arranged as shown in Fig. 64. The oscillating circuit consists of the elements L_1 and C_1. The steady input current flows in through choke coil (L_3) and passes first through one tube and then the other, resulting in square current waves 180 deg. in length. This gives a high average current carried with a low space-charge loss. The voltages appearing across various parts of the circuit are indicated in Fig. 65.

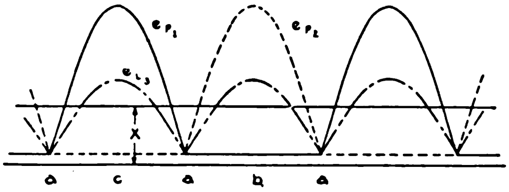

Fig. 65. Voltage Waves of the Circuit Shown in Fig. 64

During the period *aba*, the first tube is conducting current, and the anode potential of the first tube (e_{p_1}) has the small steady value required to overcome the space charge. The oscillating circuit has a sinusoidal voltage, and the potential of the anode of the second tube (e_{p_2}) is equal to the space-charge drop in the first tube plus the voltage of the oscillating circuit. During the periods when the second tube is carrying current as *aca*, its anode voltage is equal to the space-charge drop and e_{p_1} is equal to the space-charge voltage plus that of the oscillating circuit. The voltage of the mid-point of L_1 will be given by the space-charge drop plus half the voltage of the oscillating circuit. Since the conducting path shifts from one tube to the other at the time this voltage is zero, the potential e_{L_3} of the mid point of L_1 will always be positive. The average value of e_{L_3} will be equal to X, the impressed direct voltage of the supply source, and, if e_{L_3} be referred to X as an axis, it will show the voltage across the choke coil L_3.

If it be attempted to analyze and describe the circuit of Fig. 64 in more detail, it will be found that the currents and voltages which can be described in theory are difficult to achieve in practice. For example, the input current cannot be shifted instantly from one tube to the other because this would require an infinite rate of change of their space-charge voltages, and no potential varying at this rate is available with which to cause such a change in the grid voltages. For low- and intermediate-frequency circuits, however, such difficulties can be overcome; and this type of circuit gives promise of great usefulness in many applications where the careful design required for its satisfactory operation is justified.

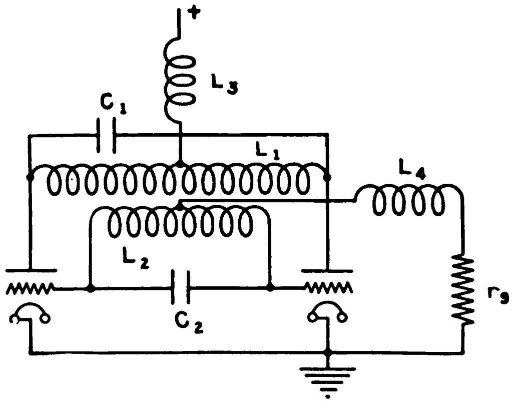

Fig. 66. Grid Excitation for High-efficiency Circuit

At commercial power frequencies, the oscillating circuit may be replaced by rotating machinery, thus storing the necessary energy as kinetic energy of rotation. Circuits of this sort have been described in a previous article;(9) and, in course of time, advances made in the manufacture of the necessary equipment will probably make their use in some instances an economy. With increased frequencies the outlook is slightly different. The difficulty here is not to show the economy of operation, but to find applications where the operating conditions are such that the circuits will not have to be changed after leaving the factory, and sufficient apparatus of any particular rating can be sold to justify the costs of designing. When the square-wave circuit is discussed further, it will be seen how important it is that everything should be exactly as calculated and how the "taps" and "end turns" of the usual radio set would be an almost certain cause of failure.

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(9) "The Inverter," by D. C. Prince, GENERAL ELECTRIC REVIEW, October, 1925, p. 676.

A HIGH EFFICIENCY CIRCUIT WITH SQUARE CURRENT WAVES

It has been shown how square waves of anode current passed through two tubes can be combined with the sinusoidal voltage of an oscillating circuit in such a way as to obtain high efficiency and output, but the actual design of such a circuit presents some difficulty. Fig. 64 shows the elementary circuit schematically, and Fig. 65 shows the theoretical anode voltage (e_{p_1} and e_{p_2}) and the potential (e_{L_3}) of the mid point of L_1 which has an average value equal to the impressed voltage. While either anode is carrying current, the voltage across that tube is equal to the space-charge drop. When no current is flowing to an anode, the potential is allowed to reach a value several times that of the supply source, but this results in no loss since the product of current and voltage is zero.

Current Wave Forms

The plate and grid-current waves should have as nearly a square form as it is possible to give them. The plate-current wave form will have a strong tendency to follow that of the grid, so the latter will be discussed first. The rate of change of the grid current will correspond to the rate of change of the space charge, and this potential is supplied by the secondary or grid circuit which has a sinusoidal voltage. The grids may be arranged as the anodes in an ordinary single-phase rectifier circuit as shown in Fig. 66. The alternating voltage across L_2 will be much in excess of the space-charge drop. The resultant rectified voltage will be absorbed in a resistance r_g, and a choke coil L_4 will hold the current constant. This current is the sum of the two grid currents. When the potential of either grid is considerably more positive than that of the other, it will take all the current. This means that the grid-current waves will have a long, flat, central portion. When the voltages of the two grids are nearly equal, each will draw some current, the total being divided in such a way that the difference in the space-charge drops is equal to the voltage applied between the grid terminals by the secondary circuit. The grid currents will, therefore, have a form of the nature indicated roughly in Fig. 67. The current begins to leave either grid when the voltage of the secondary circuit is equal to the space-charge drop for full current. When the secondary voltage is zero, the grids carry equal currents, and the commutation is completed when the secondary voltage reaches full space-charge value after its reversal.

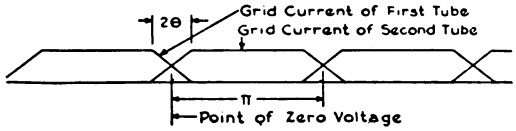

Fig. 67. Approximate Form of Grid-current Waves

The plate-current waves will have a form similar to that of the grid wave. The value of the flat portion will be fixed by the power input. The tapered portions must be similar to those in the grid waves, for the ratio of plate current to grid current can increase only by having an excessive plate voltage; and conditions in the primary circuit will be so arranged as not to favor this. Therefore, when the current to one grid begins to decrease, the corresponding anode current will decrease also. The choke coil through which the input current flows will tend to hold the sum of the anode currents constant. Hence, when one anode begins to lose its current, the other anode will start to carry current. Its grid current begins to increase simultaneously so that no high plate-voltage drop need result.

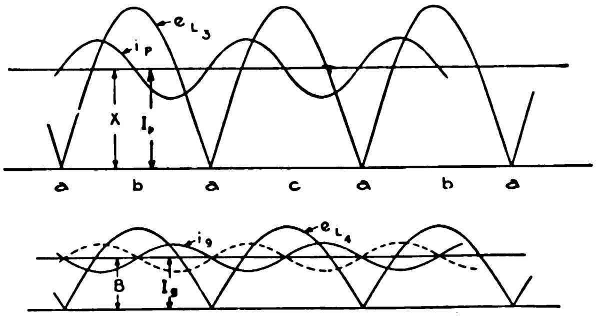

Fig. 68. Ripples Passed by Plate and Grid Choke Coils

Ripples Through Choke Coils

Thus far it has been assumed that the two choke coils are perfect in their action. This is very difficult to achieve in practice, and it is found advisable to allow ripples in the choke current of a magnitude equal to twenty or thirty per cent of the steady current. The particular difficulty which makes this necessary is internal tuning of the choke coils; *i.e.*, storage of so much electric energy between conductors inside the coil that it reacts somewhat as a condenser and inductance in parallel. A moderate amount of such action is not undesirable, but, if present in large amount, it may seriously impair the choke action for the higher frequencies.

The magnitude of the ripples passed by the choke coils is found by dividing the choke voltages by their impedances. To do this exactly, the voltages should be analyzed into components, which are treated separately. In practice, however, only the components at the fundamental choke frequency, which is twice the frequency of the oscillating circuit, need be considered. When this is done, it is found that the plate current tends to increase at the time the grid current tends to decrease. Fig. 68 illustrates this point. The voltage across the plate-current choke coil is indicated by the difference between X and e_{L_3}. This produces a plate current ripple i_p on the steady current I_p, arbitrarily drawn equal to X in order to reduce the number of lines in the diagram. The drop in the grid resistance r_g is shown by B. The voltage impressed on the grid choke is shown by the difference between e_{L_4} and B. This gives a grid ripple as shown by i_g superposed on the grid current I_g.

The difference in phase between the two current ripples results in an excessive plate voltage during the time the plate current is high and the grid current is low. In order to avoid this, the two choke coils may be coupled as shown in Fig. 69, and the ripple in the plate current may be made to cause a ripple in the grid current in the opposite phase to that obtained without the coupling, or as shown by the dotted line in Fig. 68. By this means, plate and grid currents may be kept proportional even though the choke system pass ripples of considerable magnitude. Since the combination of the two choke coils works with the harmonics obtained by rectification of the sine waves of the oscillating system, it has been termed an harmonic transformer.

If the grid-current ripple were forced to flow through r_g, a phase displacement would result which would be difficult to compensate. This is avoided by using a pass condenser c_p in parallel with r_g.

Parasitic Oscillations

On account of the number of parts involved in these circuits, it is possible for oscillations to exist other than those which are desired. These oscillations utilize the unavoidable electrostatic coupling between parts of the circuit. Referring to Fig. 69, it will be seen that L_3 and L_4 may be considered to be connected through a fairly large capacity as C_3. This is due to the supply lines which have considerable electrostatic capacity. These are connected directly to the plate choke on one side and are separated from the grid choke on the other side only by condenser C_p which is used to pass the ripples in the grid current. If there should be no oscillations in C_1L_1 and C_2L_2, it is apparent that L_3 and L_4 may function as the plate and grid inductances of an ordinary Hartley circuit, the condenser of the oscillating circuit consisting of the electrostatic capacity between the coils representing L_1 and L_2 and such other parts of the oscillating circuits as are exposed to one another. The two tubes operate in parallel in supplying power for parasitic oscillations using this circuit, and the frequency of oscillation is the natural frequency of the circuit in which the oscillations occur. The remedy for such parasitic oscillations is to so arrange the circuit in which they occur that high losses will quickly absorb their energy. This may be done by placing some resistance in series with C_p. With good design, the stored energy in the parasitic circuit is small, and this will aid in giving a high decrement.

Another cause which tends to produce parasitic currents in any circuit is insufficient emission, and this must be avoided.

Rating of Circuit

The rating of a circuit of the double-tube square-wave type is very easy to predetermine. A given type of tube will be able to safely withstand the stresses caused by a certain limiting direct voltage and the temperatures due to the passage of a certain current through it for half of each cycle. The product of this voltage and current is the input from the direct-current source. The losses occurring in the tubes are readily determined, and those occurring in the coil systems may be estimated. Hence, the output rating is quickly obtained.

Distribution of Energy Throughout Circuit

The ratio of the energy stored in an oscillating circuit to the energy dissipated per cycle is $\frac{1}{2\pi}$ times the ratio of reactive volt-amperes to watts. Hence, if it is desired to store twice as much energy in a circuit as is dissipated per cycle, the volt-amperes must be 4π times the watts. This has been found to be a very good criterion for the lower limit in the design of simple oscillating circuits, and it is doubtful if it is wise to make any reduction here; for, although energy is now introduced into the circuit in two pulses per cycle instead of one, it is more essential that the voltages throughout the circuit conform to a definite shape, and this is accomplished by utilizing the fairly pure sine wave obtained with moderate energy storage.

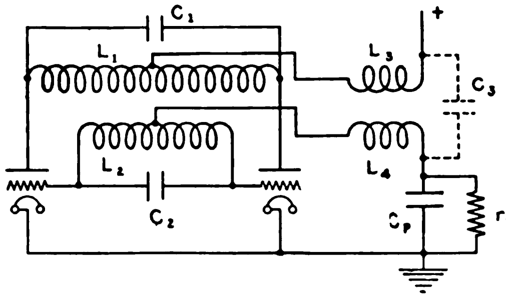

Fig. 69. Coupling of Choke Coils to Form an Harmonic Transformer

The volt-amperes given by the relation just presented is the total for the circuit. Part of this is in the primary circuit, part is in the secondary circuit, and part is held by the two circuits in common. Another way of stating this is to say that the volt-amperes in the two condensers represent the total energy, and their sum is equal to the sum of the volt-amperes in the two coils due to their self-inductances, plus a term involving both currents and the mutual inductance.

As the current required to commutate the grids is drawn from the condenser in the secondary circuit, it is obvious that this is one point in the circuit where it would be well to store an amount of energy quite large with respect to the local energy losses. Several circuits have been designed using

30 volt-amperes in the grid condenser for each watt lost at this point; and this seems to be a very good value. It certainly is generous, and at the same time does not ordinarily tie up too large a portion of the total energy.

Voltages in Plate and Grid Coils

The ratio of average to peak voltage for a double-wave rectifier is $\frac{2}{\pi}$. The grid circuit is obviously an ordinary rectifier, and the plate circuit is the same circuit reversed for the reception of energy at constant potential.

Owing to the action of the grid choke, the voltage applied to the grid resistance is the theoretical average voltage minus the voltage drop in the grids themselves. Similarly, the plate choke takes the rectified voltage of the plate coil and presents it to the direct-voltage source of energy as a steady counter-electromotive force to be overcome. The input voltage must, of course, be equal to this counter-electromotive force, plus the plate drop.

Let E_p = plate drop under full-load current.
E_g = grid drop under full-load current.
E_1 = alternating voltage across primary coil (r.m.s. value).
E_2 = alternating voltage across secondary coil (r.m.s. value).
E_r = voltage applied to grid-leak resistance (r_g).
E_c = voltage of constant-potential source.

Then, remembering that only one-half the voltage of either coil is used at a time,

$$E_c = \frac{1}{2} \times \frac{2}{\pi} \times \sqrt{2}\, E_1 + E_p = \frac{\sqrt{2}}{\pi} E_1 + E_p \qquad (1)$$

and

$$E_r = \frac{\sqrt{2}}{\pi} E_2 - E_g \qquad (2)$$

It will be observed that these values are not entirely exact, as commutation of current from one tube to the other does not occur instantly at the time of zero coil voltage, and other slight variations occur; but the values obtained will be found to be of quite sufficient accuracy for all practical needs.

The direct voltage of the supply source will, of course, be known, and this will determine E_1. The values of E_2 and E_r are not fixed until some assumption is made regarding the time required for the grid current to be transferred from one tube to the other. In general, this period will be short; but, if made too short, E_r will be found to be so high as to cause unwarranted losses in the grid resistance. The whole problem of commutation of the grid currents is quite elaborate and will be discussed later. For the present, it will be sufficient to assume that commutation occupies a time interval represented by an angle θ to either side of the time when the voltage across the grid condenser is zero (see Fig. 67). When commutation starts, there is the drop E_g in one grid and zero drop across the other. The total voltage between grid terminals measured through the grids is E_g. At the same time, this voltage as measured through the grid condenser or coil is $\sqrt{2}\, E_2 \sin \theta$; and, since these two must be equal

$$E_g = \sqrt{2}\, E_2 \sin \theta \qquad (3)$$

As E_g will be known from the tube characteristics, the value of E_2 is determined when θ is assumed. As the volt-amperes in the primary and secondary condensers have already been determined, and the voltages are now known, the currents are easily obtained, thus giving the terminal conditions to be met by the plate and grid coils, which together form an air-core transformer.

(To be continued)

Every Building but Wrecks Electrified, Claims Niagara Falls

With 99⅞ per cent of the homes in the city served by the local electrical utility, Niagara Falls, New York, puts forward its claim to being the most completely electrified city in the world. There are only 22 houses within the city limits which are not connected to the electric lines, and these are old structures destined soon to be torn down to be replaced by modern buildings. When this happens, it is predicted that the community will be 100 per cent electrified.

According to *Electrical Merchandising*, which has compiled a table showing use of electricity in the cities of the country, Niagara Falls heads the list. There are 16,500 residence customers within the city limits. The average household use for this city in 1927 was 1,413 kilowatt-hours. The average residence customer in the United States used 404 kilowatt-hours in 1926, according to the statistics of the National Electric Light Association.

Dolgeville, New York, recently presented its claim to most complete electrification. It has a population of 3,500, and possesses 682 buildings of all kinds, with a maximum possibility of 973 customers. The local electric company serves 971 customers, or all but two families.

The Recording Spectrophotometer for Precise Color Analysis

THE rapid and accurate measurement of the color of reflecting surfaces has been solved by the recording spectrophotometer developed by Professor Arthur C. Hardy of the Department of Physics at Massachusetts Institute of Technology and of the staff of the Research Laboratory of the General Electric Company. The instrument has ample sensitivity at the violet end of the spectrum, where visual spectrophotometers fail because of the low visibility of light in that region.

With the new instrument it becomes possible to duplicate any color exactly, at any place and at any time. For instance, should a fashion dictator in Paris develop a new color, it would be possible to send a photoradiogram of the spectrophotometric analysis to New York immediately. In that city it could be duplicated by dye experts, even though these men would not be able to see an actual sample of the original color until later.

Human judgment, one of the most troublesome elements in the usual methods of color analysis, is entirely eliminated by this device which automatically, rapidly, and precisely measures and records the color and wavelengths of light reflected by any substance.

Construction of the Instrument

Just as ordinary white light, or sunlight, is broken up into a rainbow of colors if passed through a triangular prism, so is colored light broken up by the prism into bands of different wavelengths and colors, with some colors more prominent than others. A pure color would have a minimum number of lines, but most of the colors as we know them are far from pure —they are mixtures of various wavelengths. And that is the reason why some colors which seem to be alike in sunlight differ so much from one another when viewed by artificial light. The prism will show, for instance, that a sample of a certain green dye contains a little of every color, a decided proportion of red, and a maximum of bluish green, green, and yellowish green.

The new color analyzer, a combination of a powerful optical system and electrical devices, automatically analyzes the spectrum of colors and makes a chart of the analysis. A specimen, the color of which is to be analyzed, is placed in a holder and illuminated by a special ribbon-filament incandescent lamp. A surface of magnesium carbonate, the standard of whiteness, is similarly adjusted on the opposite side of the lamp. The light from the lamp falls perpendicularly on both the specimen and the magnesium-carbonate standard, and enters the slit of an ordinary spectograph system after reflection from both.

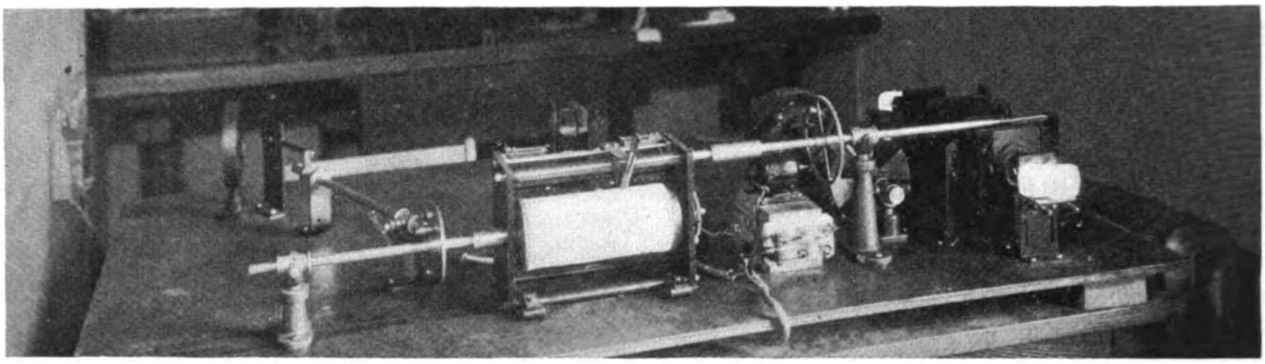

Fig. 1. The Essentials of the New Recording Spectrophotometer Developed by Prof. Hardy

Immediately in front of the slit is a rotating disk with alternate transparent and silvered segments, so located that light from the standard enters the slit when a transparent segment is in the beam, and light from the specimen when a silvered segment is in the beam. The light is dispersed by the spectrograph system, and the proper wavelength band selected by a second slit. Monochromatic, or single-wavelength, light is then received by a photo-electric tube in back of the second slit. This light as received by the tube is of pulsating intensity when different amounts of light in that region of the spectrum are reflected by the sample and the standard. This pulsating light intensity is changed by the photo-electric tube to a pulsating electric current, which is then amplified 10,000,000,000 times and employed to run a small motor. This motor actuates a shutter in the beam between the light source and the standard, automatically adjusting the shutter to that point at which the pulsations in the current cease. This position is independent of the characteristics of the photo-electric tube.

The reflecting power of the specimen is then recorded on a rotating drum by a pen which is attached to the mechanism controlling the shutter just mentioned. A second motor simultaneously rotates the drum and drives the slit across the spectrum, thus giving a complete color analysis in a time which has been reduced recently to less than a minute. In other words, no longer time is now required for a color analysis than for the measurement of some of the simplest physical properties.

Previous Methods of Analysis

Previous methods for determining colors have been of two types, the color analyzer and the colorimeter. Of these, the colorimeter is the simpler to use. The observer looks through an eyepiece at a divided field, one-half of which receives illumination from the specimen under analysis, and the other half of which receives its light from a color that can be controlled at will by the operator, usually by the use of color filters.

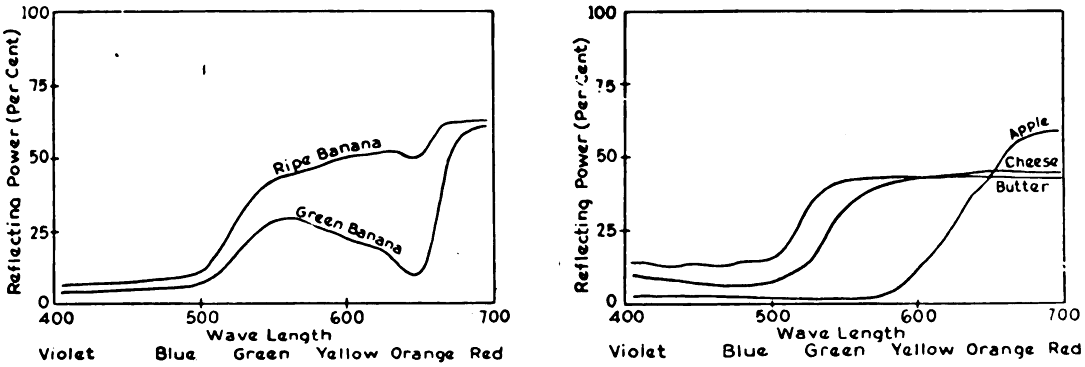

Fig. 2. Representative Color Analyses Made with Recording Spectrophotometer

Physiologically, the eye is affected in three ways by a color—depending on red, yellow, and blue sensations—and three determinations are therefore needed in matching colors. This method depends considerably upon the personal equation of the operator, and much information is still lacking after the analysis has been made. While the instrument will show when the color of the specimen does not match the standard, therefore being applicable to control work, it does not indicate in what respect the specimen varies.

The color analyzer, on the other hand, produces a record similar to that of the instrument devised by Professor Hardy but has handicaps, being difficult to adjust, requiring the making of many readings, depending on the sensitivity of the operator's eye, and requiring considerable time for a complete analysis.

Four settings of the analyzer are required for each wavelength studied, and at least fifty points throughout the visible spectrum must be analyzed before the curve is completed. The four readings for each point are averaged, and the curve of the points then plotted. In a speed test, two operators can complete an analysis by this method in twenty minutes; but fatigue of the operator's eyes makes it impossible to complete more than a few analyses in one day. This type of color analyzer, therefore, is unsuited for routine control work.

Fields of Application

In whatever branch of industry the color of a product is a factor, the new recording spectrophotometer will be of service. In the textile industry, of course, accurate measurement of color is of utmost importance, especially when considering those delicate shades which so easily deceive the eye.

Inks, paints, dyes, paper, lacquer, soap, lard, flour, butter, oleomargarine, cheese, sugar, syrup, chocolate, glass, automobile finishes, tile, brick, roofing materials, carpets, rope, hardware, leather, cement, linoleum, cosmetics—and the list might be continued indefinitely—all have their color problems.

In the sale of cottonseed oil the price is affected decidedly by slight differences of color. An amber hue is demanded by buyers at the New York Produce Exchange. If the oil is slightly red, though the difference in color be hardly noticeable, its price must be less. Lubricating oils are similarly graded on a color basis.

Even when color is not of prime importance, manufacturers usually try to keep it uniform. Fruit canners catering to an exacting clientele endeavor to maintain a standard of colors, *e.g.*, of peaches, since the contents of one can may be placed in the same dish with peaches from another can.

But it is in the cataloging of dyes that the device will probably find one of its most important fields of application. Having a complete set of curves showing the properties of standard dyes and colors, it becomes possible to obtain new colors with predetermined characteristics by properly mixing the dyes already available.

The elapsed time since the recording spectrophotometer was developed has not been sufficient to have permitted the testing of its applicability to all of the industries in which it is expected to be of service; but a sufficient number of tests have been made to indicate that the problem of rapid and accurate color analyses has been solved.

The new spectrophotometer was described by Professor Hardy at the annual convention of the Optical Society of America, held at Washington, November 1st.

LIBRARY SECTION

Condensed references to some of the more important recent articles in the technical press, and to new books of interest to the industry, as selected by the General Electric Main Library.

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Franklin Inst. Jour., Sept., 1928; v. 206, pp. 317-388.

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Stelliting by the Arc Process. C. M. Rusk.
Weld. Engr., Sept., 1928; v. 13, pp. 53-55.
("Nature of deposit secured under different operating conditions—Effect of coating—Hardness and structure of deposits.")

Bearings
Knowledge of Bearing Temperatures Aids Efficient Lubrication. H. L. Kauffman.
Power Pl. Engng., Sept. 15, 1928; v. 32, pp. 977-979.

Busbars
Bus Stresses Under Short Circuits.
Elec. Wld., Sept. 22, 1928; v. 92, pp. 564-565.

Corona
Economics of Corona Loss. J. T. Lusignan, Jr.
Elec. Wld., Sept. 1, 1928; v. 92, pp. 405-407.
("Known facts regarding corona as developed by Ryan and Peek reviewed.")

Cutouts
Critical Study of the Current Rating of Low-Pressure Ordinary-Duty Fusible Cutouts. P. D. Morgan.
I. E. E. Jour., Sept., 1928; v. 66, pp. 926-948.

Electric Cables
Thermal Resistance of Cables. A. M. Taylor.
Elec. Rev., Aug. 17, 1928; v. 103, pp. 270-271.
(Improved thermal dissipative capacity is claimed for an "intersheath" cable as compared with a single-core cable.)

Electric Distribution
Standard Voltage A. C. Network. John Oram.
Elec. Wld., Sept. 1, 1928; v. 92, pp. 415-417.
(Describes the Dallas distribution system of the Dallas Power & Lt. Co.)

Electric Distribution, Underground
Safe, Underground Transmission of Electrical Energy. H. Milliken and L. A. Kenyon.
Elec. News, Sept. 1, 1928; v. 37, pp. 31-37.
(Describes a 66-kv. cable installation of the Montreal Light, Heat & Power Consolidated.)

Electric Lighting—Aviation
Construction and Lighting of the Sacramento Municipal Airport. Charles R. Blood and W. J. Delehanty.
Elec. West, Sept. 1, 1928; v. 61, pp. 134-137.

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Engineering Problems in Theatre and Stage Lighting. W. C. Brown.
W. Soc. Engrs. Jour., Sept., 1928; v. 33, pp. 447-455.

Electric Locomotives
Performance of Motor-Generator Locomotives. P. A. McGee.
Rwy. Age, Sept. 8, 1928; v. 85, pp. 443-447.
(Locomotives on the Great Northern R. R.)

Electric Precipitation
Precipitation in Europe. G. Berg.
Elec. Wld., Sept. 15, 1928; v. 92, pp. 499-503.
(An illustrated description of European electric precipitation equipment, particularly that manufactured by Siemens.)

Electric Traction
High-Frequency Railway Traction. W. Cramp.
Elec. Rev., Sept. 14, 1928; v. 103, pp. 454-455.
(Abstract of a paper read before the British Association for the Advancement of Science. Author proposes a novel scheme for electric traction.)

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Overhead Line Design. J. A. Sumner.
Elec. Rev., Sept. 21, 1928; v. 103, pp. 471-473.
(Serial.)

Semi-Flexible Tower Line to Meet Severe Conditions. Clyde A. Booker.
Elec. Wld., Sept. 15, 1928; v. 92, pp. 507-509.
(Policies of the New England Power Co. in building lines with the conductors in a horizontal plane.)

Transmission System and Conductor Comparisons. William T. Taylor.
Elec'n., Sept. 7, 1928; v. 101, pp. 251-252.
(Comparison of conductor efficiencies of single-phase and three-phase systems; copper vs. aluminum, etc.)

Electrical Machinery—Losses
Calculation of Stray Load Losses. G. H. Rockwood.
A. I. E. E. Jour., Aug., 1928; v. 47, pp. 582-585.
(Abridgment.)

Electricity—Applications—Agriculture
What the Farmer Thinks of Electricity. J. H. Flessner.
N. E. L. A. Bul., Sept., 1928; v. 15, pp. 539-543.
(An address before the Michigan Electric Light Assoc.)

Grounding
Effect of Ground Wire on Traveling Waves. J. H. Cox and J. Slepian.
Elec. Wld., Sept. 22, 1928; v. 92, pp. 551-554.

High Frequency
High-Frequency A . C. Generation. E. D. McArthur.
Elec. Rev., Aug. 24, 1928; v. 103, pp. 303-306.
(An account of experiments in the production of high frequencies by means of vacuum tubes.)

Hydroelectric Development
Conowingo Hydroelectric Development on the Susquehanna River. Alex. Wilson.
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(Abridgment.)

Electrical Features of the Conowingo Generating Station and the Receiving Stations in Philadelphia. R. A. Hentz.
A. I. E. E. Jour., Sept., 1928; v. 47, pp. 649-654.
(Abridgment.)

Insulators—Testing
Time Lag of Insulators. E. J. Wade and G. S. Smith.
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Load Ratio Control, Transformer
Tap-Changing Equipment Controls Transformer Voltage. Arthur Palme.
Power, Sept. 25, 1928; v. 68, pp. 519-522.
(Presents an illustrated description of load ratio control equipment for transformers.)

Three-Winding Transformers With Tap Changing Under Load. K. A. Oplinger.
Elec. Wld., Sept. 1, 1928; v. 92, pp. 410-412.

Oscillographs
An Amplifier to Adapt the Oscillograph to Low-Current Investigations. Sigmund K. Waldorf.
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(List of 11 bibliographic references, p. 674.)

Radio Beacons for Transpacific Flights. Clayton C. Shangraw.
Inst. Radio Engrs. Proc., Sept., 1928; v. 16, pp. 1203-1235.
(An illustrated description of equipment used.)

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Use of Radio Field Intensities as a Means of Rating the Outputs of Radio Transmitters. S. W. Edwards and J. E. Brown.
Inst. Radio Engrs. Proc., Sept., 1928; v. 16, pp. 1173-1193.

Radio Engineering—Transmission

Nature of the Field in the Neighborhood of an Antenna. Raymond M. Wilmotte.
I. E. E. Jour., Sept., 1928; v. 66, pp. 961-967.

Radio Engineering—Transmission, Directional

Chireix-Mesny Directive Antenna for Short Waves. Stuart Ballantine.
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General Considerations of the Directivity of Beam Systems. Raymond M. Wilmotte.
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Relay Protection of Power Systems. W. F. Mainguy.
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Comparison of Turbo-Electric and Diesel Drive in Large Passenger Ships. W. L. R. Emmet.
Am. Soc. Nav. Engrs. Jour., Aug., 1928; v. 40, pp. 504-505.
(Compares operating costs of the *California* and the *Gripsholm*. Extract from Shipbuilding and Shipping Record, May 31, 1928.)

History, Description, and Acceptance Trials of the U. S. S. *Lexington*. C. S. Gillette.
Am. Soc. Nav. Engrs. Jour., Aug., 1928; v. 40, pp. 438-495.

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General Trend of Modern Development in Steam Turbine Practice. Henry Lewis Guy.
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("Reprinted by permission of the Institution of Civil Engineers.")

Substations

Layout of a Hypothetical Substation.
N.E.L.A. Bul., Sept., 1928; v. 15, pp. 544-554.
(Several solutions, by different authors, of a problem submitted by the Electrical Apparatus Committee, Engineering National Section, N.E.L.A. Serial.)

Lighthipe 220-Kv. Substation. R. B. Pollock.
Elec. Wld., Sept. 22, 1928; v. 92, pp. 559-563.
(Illustrated description of a Southern California Edison Co. substation.)

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Hamilton's New Automatic Mercury Arc Rectifier Sub-Station. Charles E. Hutton.
Elec. News, Sept. 15, 1928; v. 37, pp. 29-32.
(Illustrated description of equipment in the Jackson Street substation of the Dominion Power and Transmission Co., Hamilton, Ont.)

Telemeters and Telemetry

Meter Readings and Signals Transmitted by Selsyn Motors. A. E. Bailey.
Power, Sept. 11, 1928; v. 68, pp. 434-435.
(Author is a G-E engineer.)

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Experimental Investigation of Forced Vibrations. L. W. Blau.
Franklin Inst. Jour., Sept., 1928; v. 206, pp. 355-378.

Theory of the Dynamic Vibration Absorber. J. Ormondroyd and J. P. Den Hartog.
A. S. M. E. Trans. (APM), May-Aug., 1928; v. 50, paper No. 7, pp. 9-22.
(A device for reducing vibrations in machinery.)

Vibration of Frames of Electrical Machines. J. P. Den Hartog.
A. S. M. E. Trans. (APM), May-Aug., 1928; v. 50, paper No. 6, pp. 1-6.
(Formulas for the calculation of natural frequencies.)

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(These and other Technical Books may be purchased through the Circulation Dept. of the GENERAL ELECTRIC REVIEW.)

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Engineering Aerodynamics. Walter S. Diehl. 298 pp., 1928, N. Y., Ronald Press Co.

Fundamental Principles of Electric and Magnetic Circuits. Ed. 2. Fred A. Fish. 212 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Petroleum and Its Products. William A. Gruse. 377 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

Physical and Chemical Examinations of Paints, Varnishes, Lacquers and Colors. Ed. 4. Henry A. Gardner. 781 pp., 1927, Wash., Institute of Paint and Varnish Research.

Storage Batteries; Theory, Manufacture, Care and Application. Morton Arendt. 285 pp., 1928, N. Y., D. Van Nostrand Co., Inc.
("This book is a development of lectures to Engineering Students at Columbia University and to Officers at the U. S. Submarine School over a period of years. It is also based upon the author's practical experience in battery manufacture and maintenance." Features of construction and maintenance are emphasized. Sufficient theory to account for reactions and cell characteristics is presented. The influence of plate construction, purity of materials, and electrolyte diffusion upon cell behavior is given special attention. In addition to introductory material on the general theory of the storage battery, the author takes up: lead plates and their manufacture; sulphuric acid as an electrolyte; problems of capacity and efficiency; lead storage cell parts and assembly; methods of installation, operation and maintenance; the nickel-iron-alkaline cell; battery testing and applications. An adequate index is provided.)

Story of Electricity, and a Chronology of Electricity and Electrotherapeutics. Herman Goodman. 62 pp., 1928, N. Y., Medical Life Press.
(A condensed history of the evolution of the electrical science from the earliest times to the present. Special attention is paid to those phases having to do with the development of electrotherapeutics. The many brief biographical sketches of famous scientists in the fields of physics and electricity, together with full-page portraits of several, will be found of interest. A five-page "Chronology of Electricity and Electrotherapeutics," giving some 200 references to note worthy dates, persons and deeds, brings the little volume to a close.)

Water Purification. Ed. 2. Joseph Ellms. 594 pp., 1928, N. Y., McGraw-Hill Book Co., Inc.

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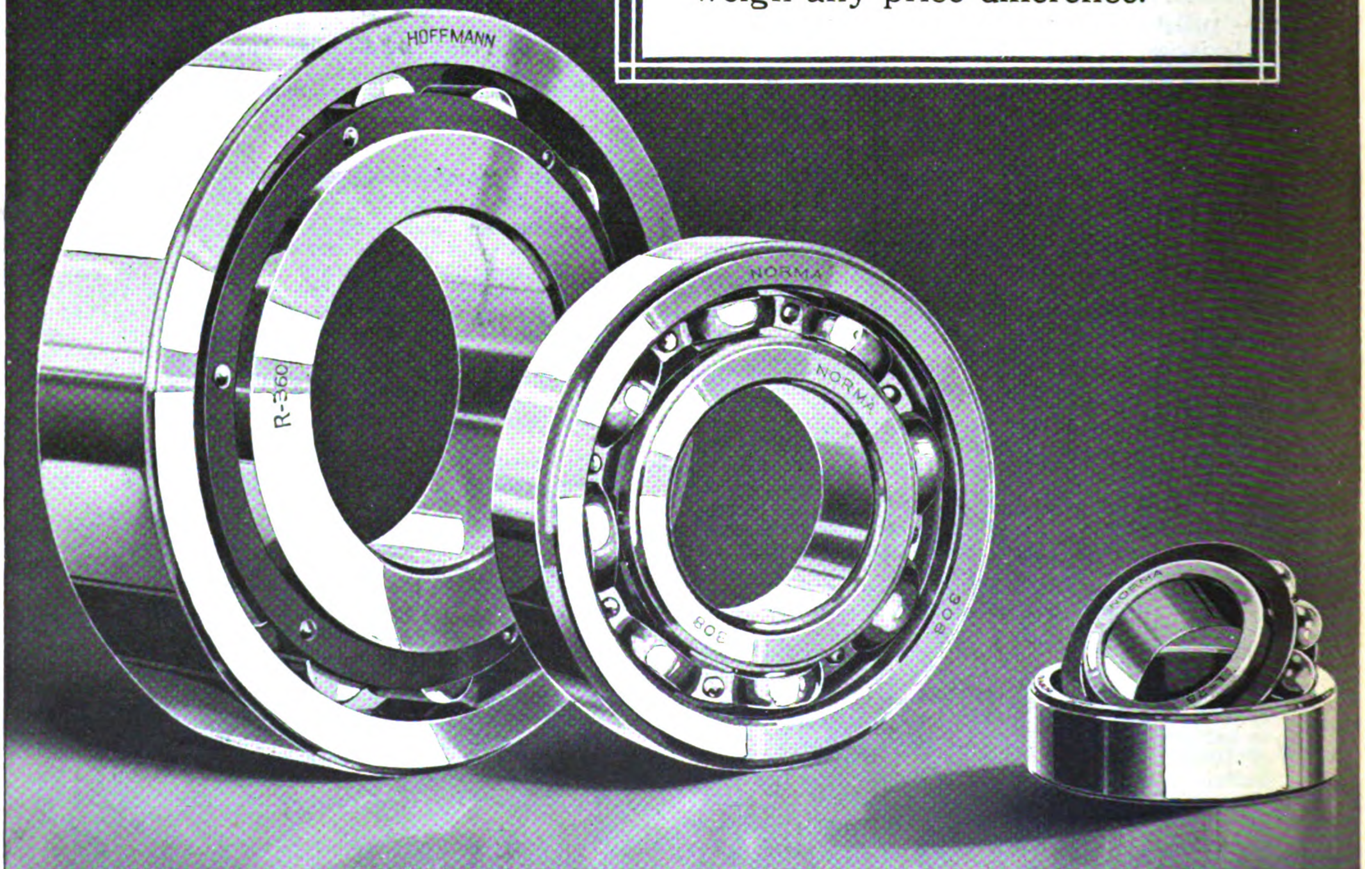
Where capacity for service is the measure of value, prices are to be considered only in so far as they are indicitive of performance possibilities. Motors equipped with "Norma" Precision Ball Bearings, or with "Hoffmann" Precision Roller Bearings, may cost a little more. But their longer life and improved performance return extra earnings which far outweigh any price difference.



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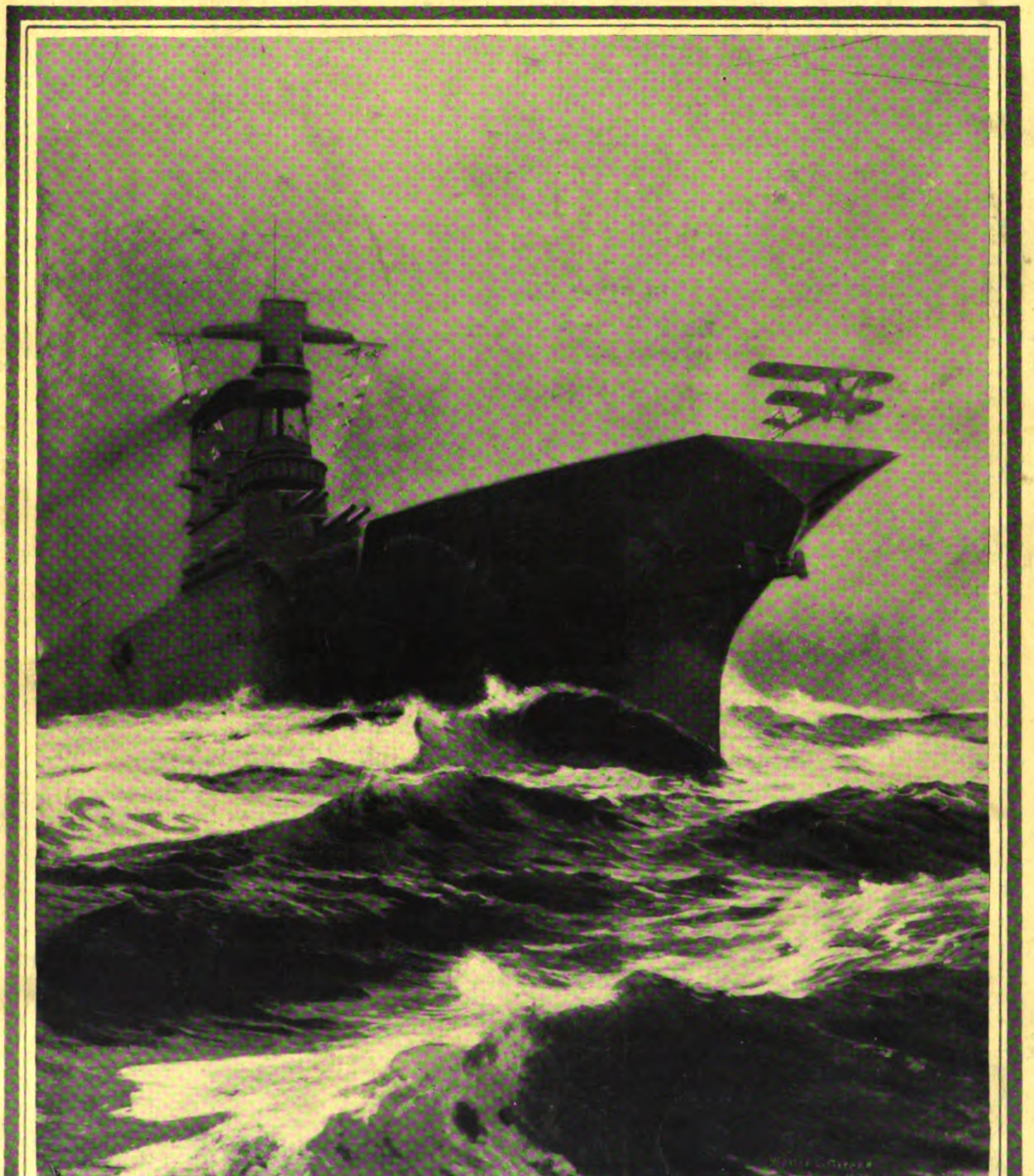
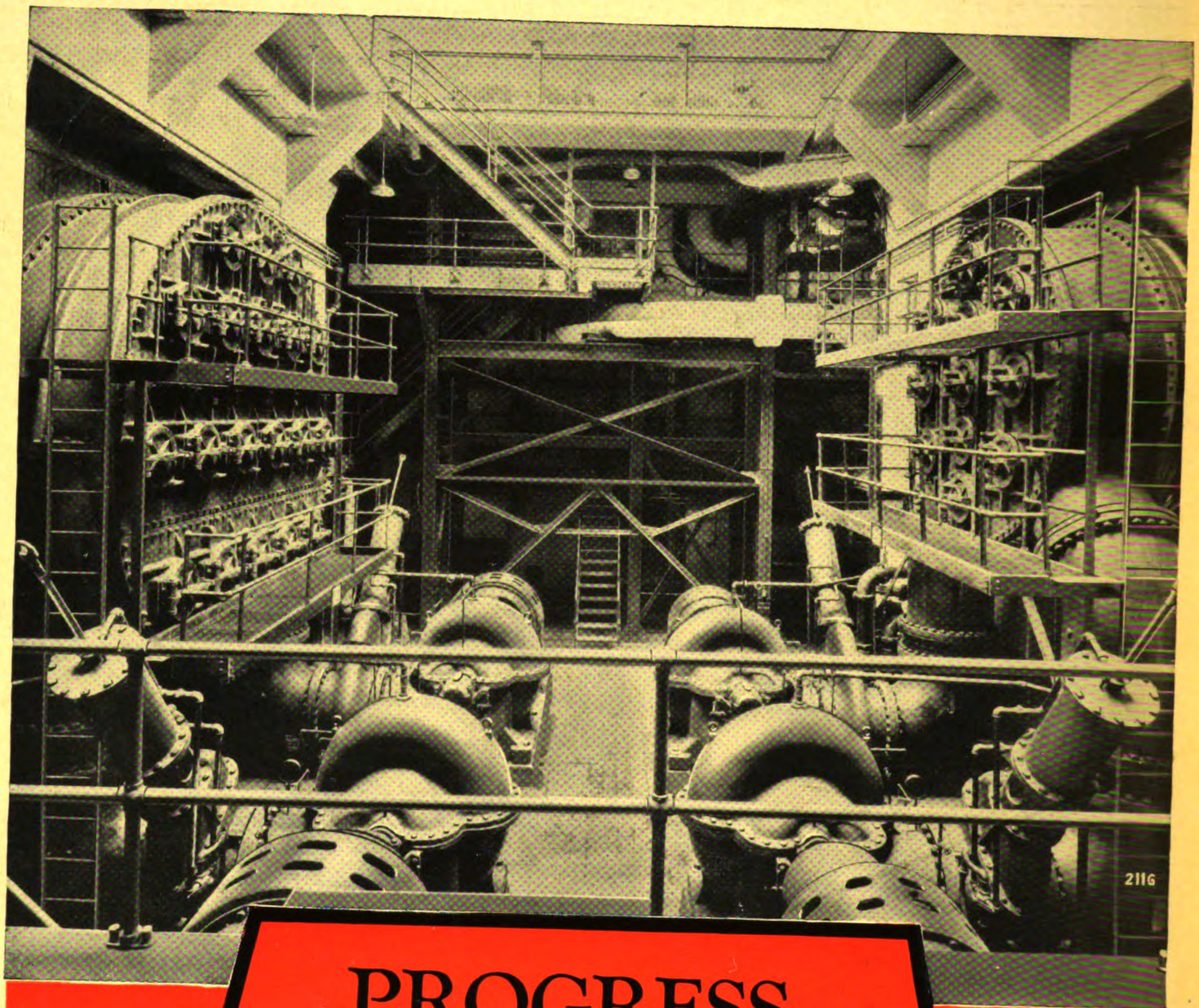


U.S.S. SARATOGA

One of the Developments of Nineteen Twenty-seven

In This Issue: *A Review of the Progress Made in the Electrical Industry, and the Advancement of Radio Reception During 1927*



PROGRESS

THE condenser on the left is a 50,000 sq. ft. Wheeler Condenser installed several years ago, serving a 30,000 Kw. turbine; that on the right, a 25,000 sq. ft. Wheeler Condenser installed last year, serving a 35,000 Kw. turbine in the Lakeside Plant of the Milwaukee Electric Railway & Light Company. Each Condenser requires 50,000 gallons of water per minute and the vacuum results are identical.

Improved design and tube arrangements have raised the heat transfer per square foot of surface so that the same work is accomplished with only one-half the amount of surface previously required.

It is noteworthy that the power required by the circulating pumps of the small condenser is 185 h.p. less than that required for the large unit. There are a total of 6 Wheeler Condensers in Lakeside Station.

Foster Wheeler Corp., 165 Broadway- New York

WHEELER CONDENSERS

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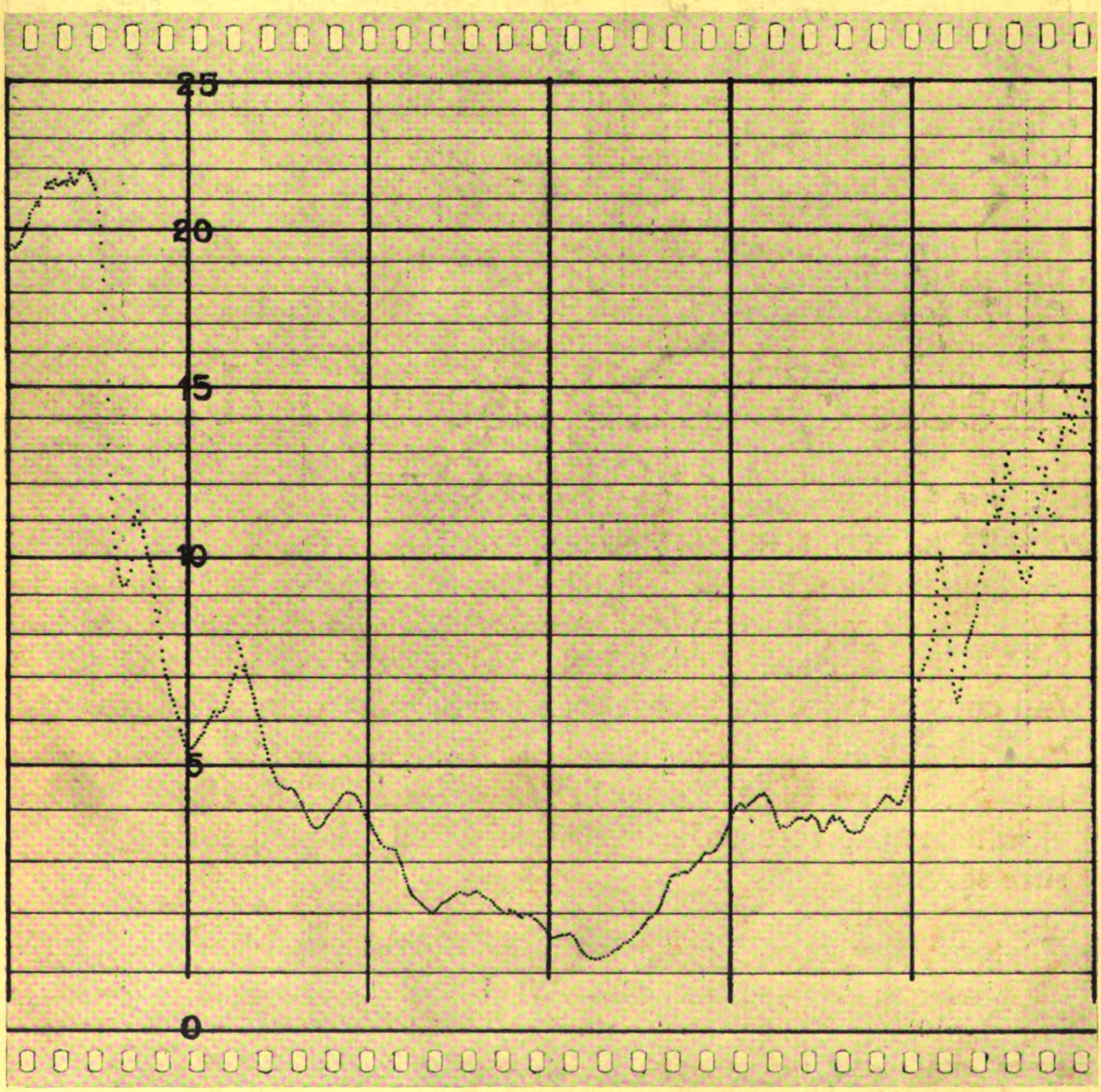
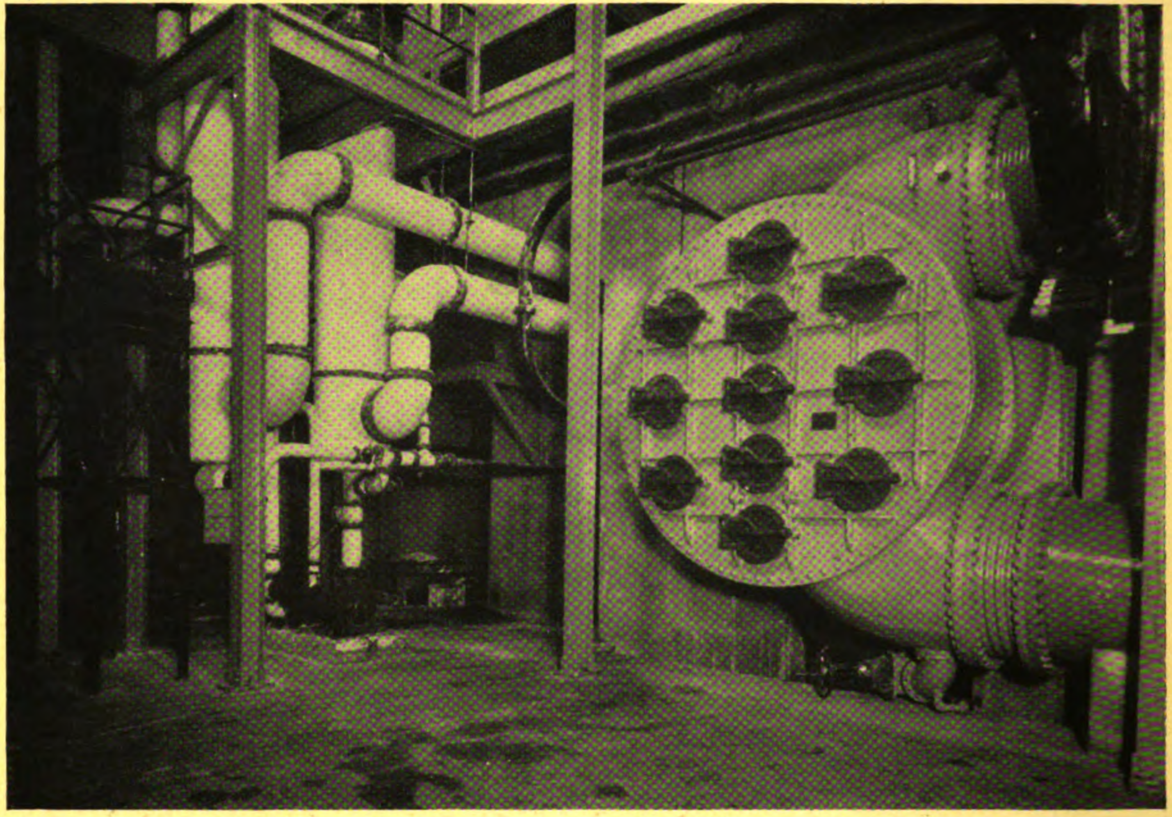


A PASSING CLOUD LEAVES ITS CARD

Floating across the sky a cloud bank progressively shuts off the sunlight and the landscape lies in the valley of shadow until the sun rays in increasing numbers succeed in piercing the trailing edge of the cloud as it moves off

(See p. 85)

In This Issue:
Daylight Recording
Frequency Control
Ground Detection
Low-voltage A-c. Networks
Flux Plotting
Daylight Measurement
Vacuum Tubes as Oscillation Generators
Heaviside's Calculus
Cahokia Substation



Wheeler Condensers and Heaters

at Cabin Creek

THE 35,000 Kw. turbine at the Cabin Creek Plant of the Appalachian Power & Light Co. is served by a 25,000 sq. ft. Wheeler Direct Flow Condenser. Twenty-eight inches vacuum with 80° water and 32,000 Kw. load is shown in recent tests. A full copy of these tests have been published and can be furnished.

To the left of the condenser can be seen the bleed feed water heaters. These are Wheeler U Tube Heaters. The U Tube design eliminates the return header and is well adapted for high pressure service.

Foster Wheeler Corporation, 165 Broadway, New York, N. Y.

A Consolidation of Power Specialty Company and Wheeler Condenser & Engineering Company.

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CONDENSERS COOLING TOWERS
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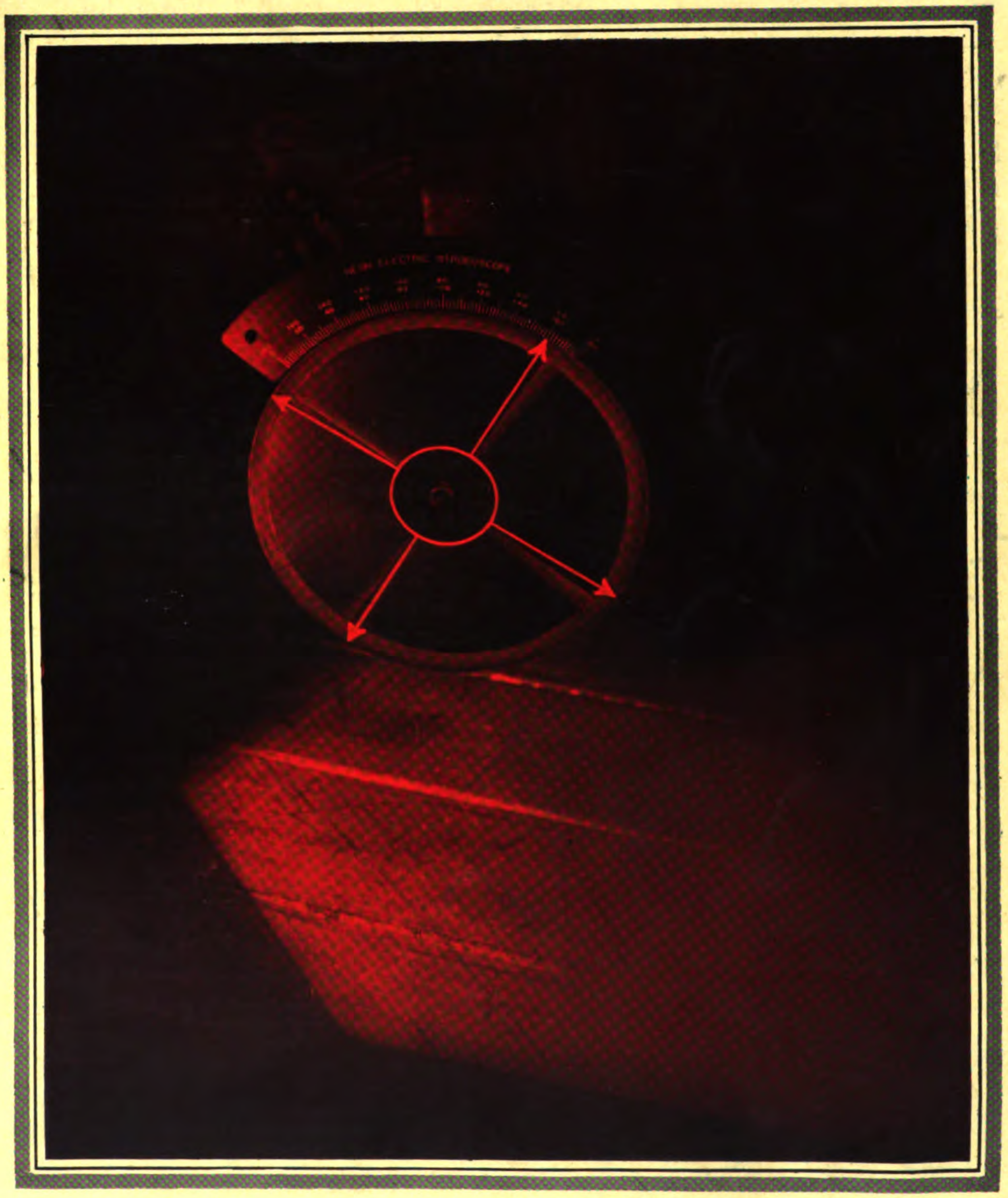


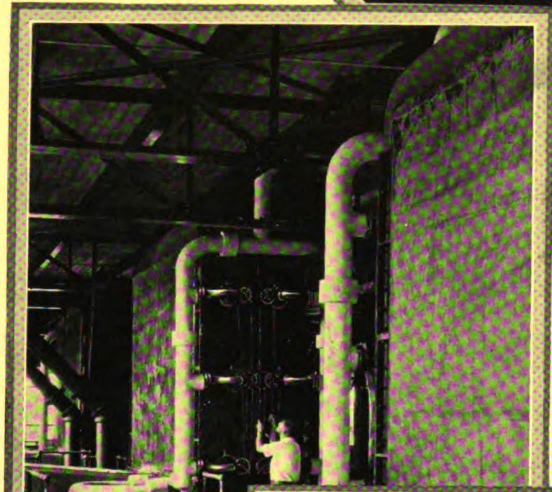
TIME AND PLACE

Continuously rotating arrows as they appear—sharply defined and stationary—in the light of the Neon-electric Stroboscope under the following conditions: disk driven by synchronous motor under constant load and stroboscope operated from same source as the motor. A variation in load causes a displacement in the position of the arrows

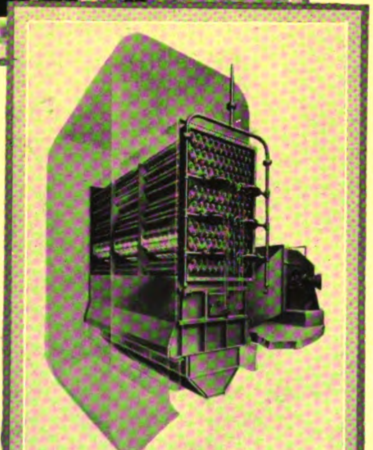
(*See p. 136*)

In This Issue: ***Neon-electric Stroboscope*** ***Load Ratio Control*** ***Heaviside's Calculus***
Vacuum Tubes as Oscillation Generators ***D-c. Motor Development***

The Inglis Station by night. The General Engineering and Management Corporation were the engineers responsible for this attractive installation.



General view of the Foster Economizers installed at the Inglis plant. Characteristic features are — low maintenance, high efficiency, compactness with accessibility.



Phantom view of the Foster Economizer. Catalog upon request.

Inglis, Florida, Uses Foster Equipment

Six 936 hp. Edge Moor longitudinal drum boilers have been installed at the Inglis steam station of the Florida Power Corporation. The boilers are equipped with Foster Superheaters and Foster Economizers, all designed for operating loads up to 300% of rating. The superheaters add 200 deg. of superheat to the steam generated. Economizers of 4368 sq. ft. heating surface raise the temperature of the boiler feed water to 350 deg. F., while handling 85,200 lbs. of water per hour.

Foster Wheeler power plant equipment is being used at leading power stations throughout the world in both large and small units. It is rendering continuous high-grade service and can always be depended upon for unfailing reliability.

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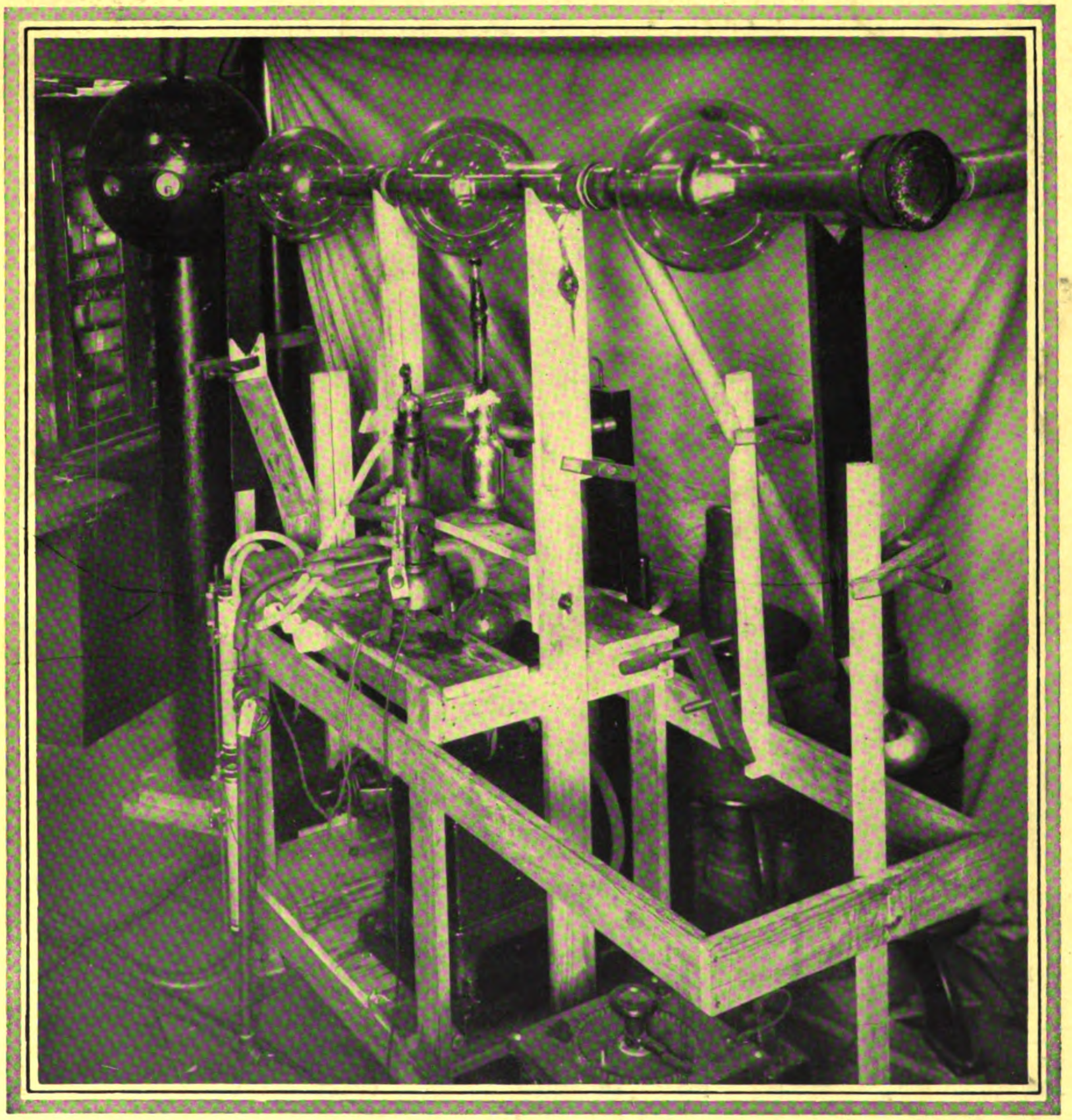
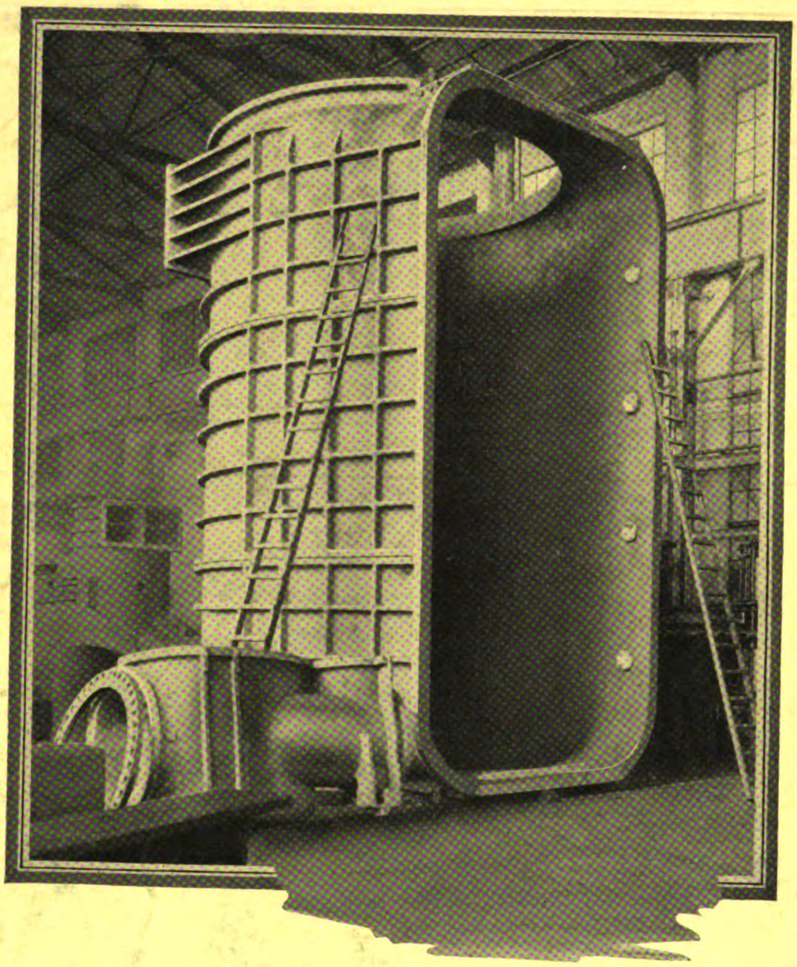


READY TO SHOOT PROJECTILES AT 175,000 MILES PER SECOND

Three-section cathode-ray tube for operation at 900,000 volts. The large metal sphere at the left, on an insulating support, contains a storage battery for heating the cathode filament of the adjacent section of the tube. Beneath the center section, and connected to it, is the exhausting equipment. At the extreme right is the metal window through which the cathode rays are projected from the tube. (*See p. 184*)

In This Issue: ***High-voltage Vacuum Tubes*** ***Load Ratio Control*** ***Oxygen in Industry*** ***Outdoor Switch Houses*** ***Low-voltage A-c. Networks*** ***Liquids as Insulators*** ***Heaviside's Calculus*** ***Industrial Electric Heat*** ***Induction Regulator Switching***



Shell of the 55,000 sq. ft. single-pass Wheeler condenser, under construction. This measures 14 ft. in diameter inside the water boxes and 25 ft. in length. Shop assembly of the condenser shells is always made before shipment to assure perfect alignment. This condenser is to be operated in connection with a 60,000-kw. turbine.

The Largest Single-pass Condenser Yet Built—

is now being installed at the Pekin Plant of the Superpower Company of Illinois.

This equipment is made by the oldest company in this country specializing on condensing equipment. Through years of reliable service, the name Wheeler has become synonymous with the highest type of condensing equipment.

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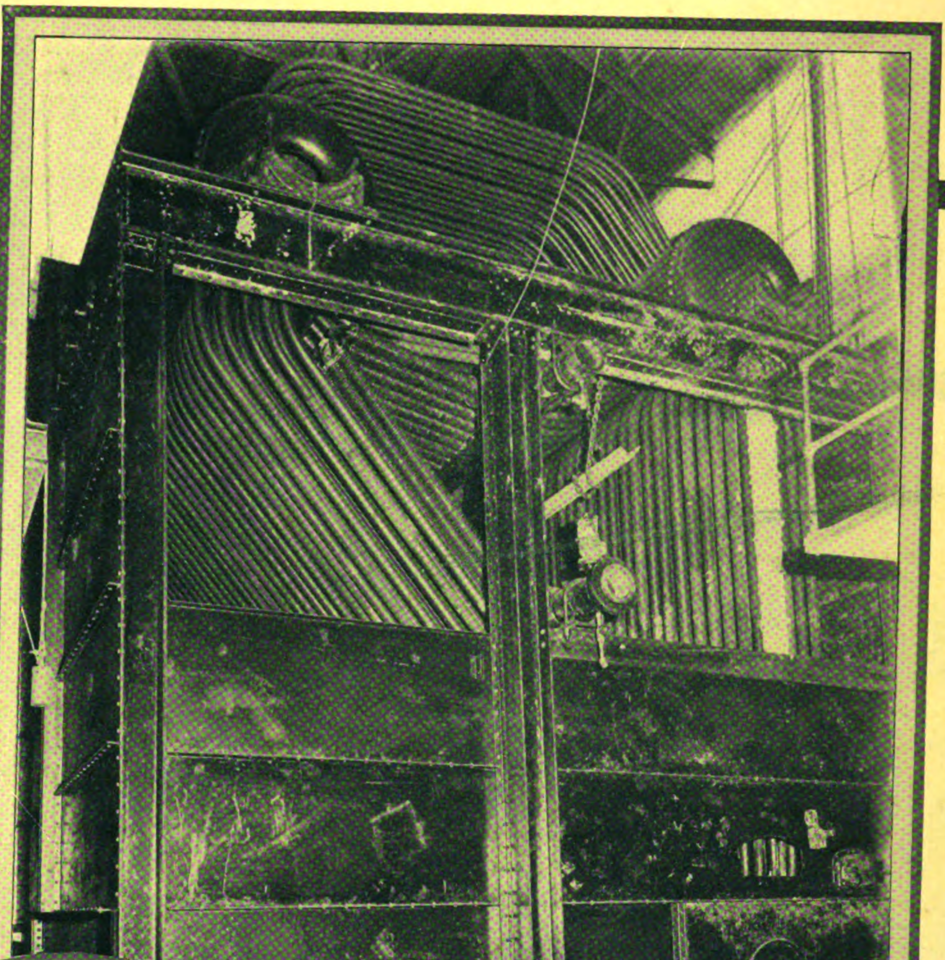
VOL. 31, No. 5 MAY, 1928

BALLOONS—BALLOONS

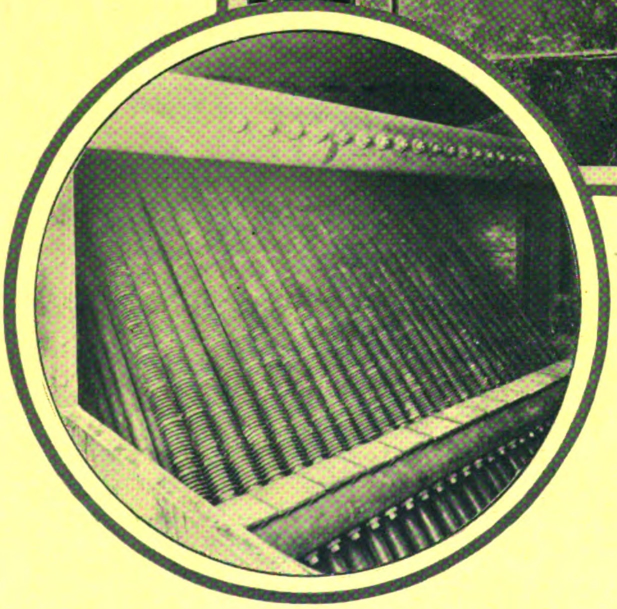
Daylight view of an ornamental lighting installation at the Broadmoor Hotel, Colorado Springs. Thin rubber balloons of a variety of colors enclose Christmas tree lamps connected in series of eight. These groups form units that are readily adaptable to a wide range of decorative lighting effects (*See p. 264*)

In This Issue: *New Arc-welded Building* *Protection of High-voltage Lines* *Low-voltage A-c. Networks* *Ornamental Lighting* *Selection of Gas for Cutting* *Improved Power System for Street Railway* *Heaviside's Calculus* *Wattmeter Connections*



Foster Convection Superheater installed in a semi-vertical boiler with steel casing, typical of high-grade, moderate-sized modern boiler units.

View below shows intake and outlet super heater headers and the inclined legs of the superheater elements. The simplicity and accessibility of the construction appeals to the operating as well as to the consulting engineer.



Foster Superheaters

with the extended surface

are here shown applied to a 601 hp. Wickes semi-vertical boiler and provide 175 degrees of superheat when operating at 200 lbs. pressure and 175% of rating.

Cast-iron extended surface is shrunk upon seamless steel tubes, giving a metal-to-metal contact for maximum transfer of heat. The superheaters are completely drainable in accordance with best practice.

The experience of more than 12,000 installations under widely varied conditions during the past 25 years insures the most satisfactory return on the investment.

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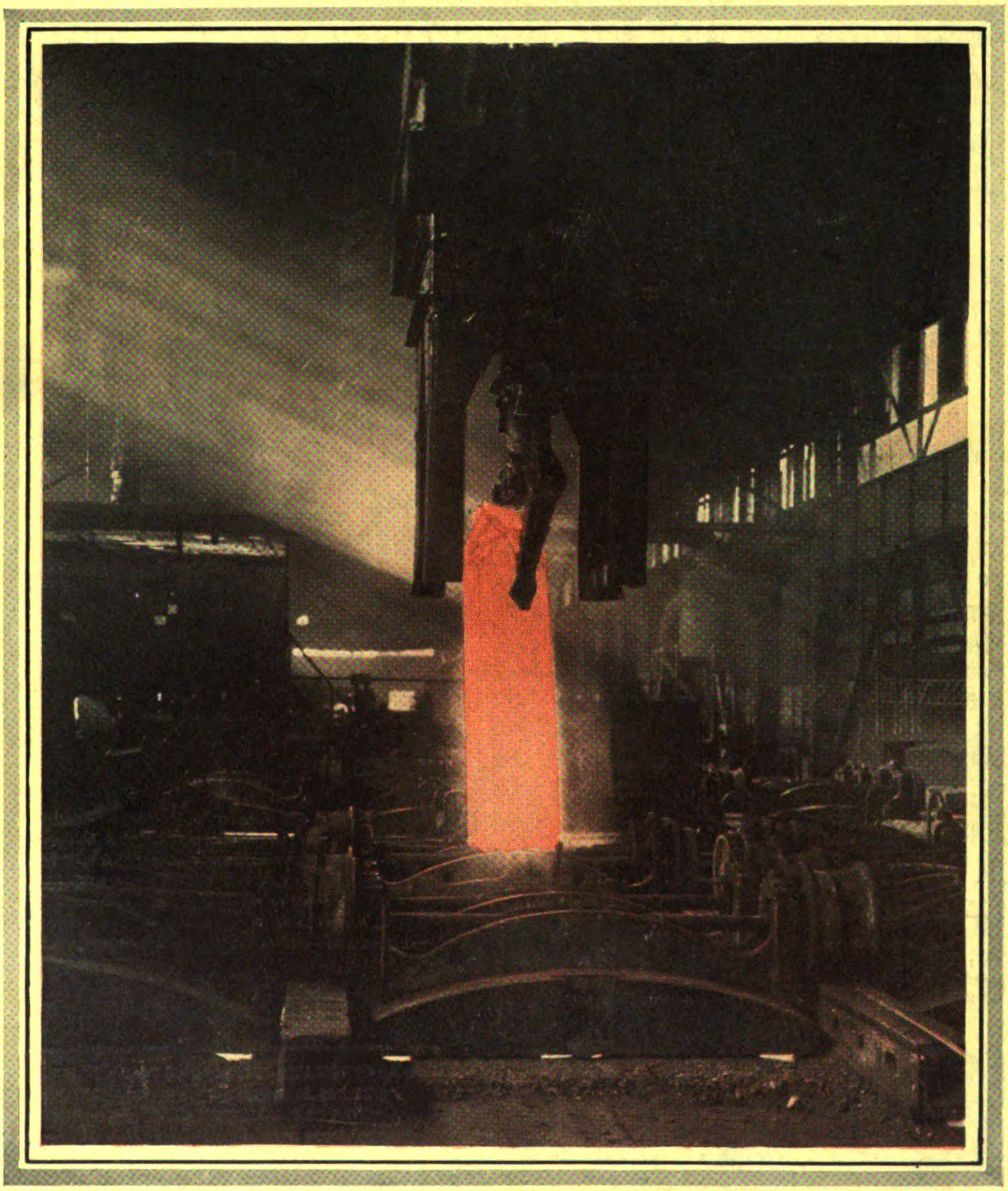
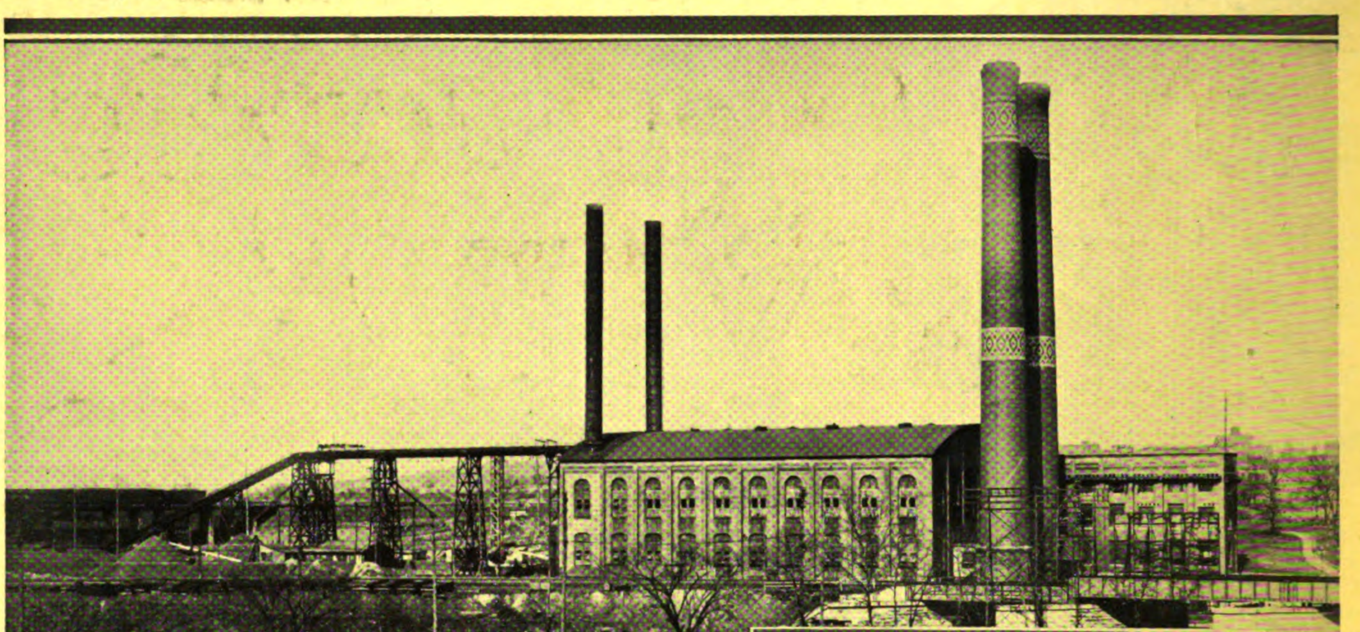
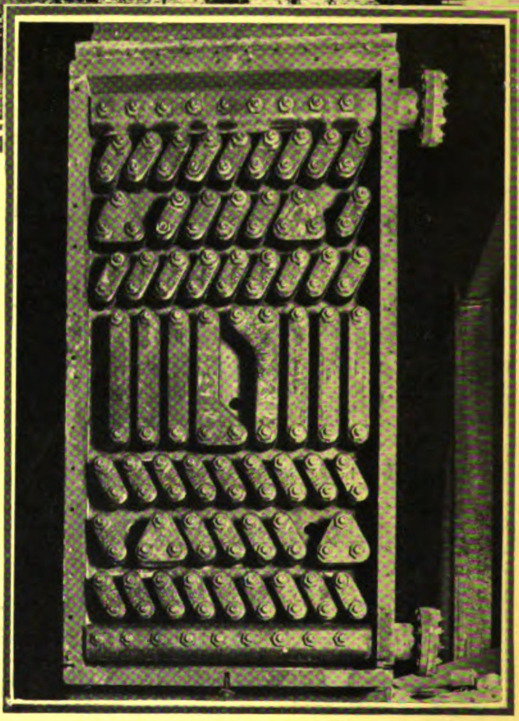


HEAT AND ELECTRICITY COMBINE
TO PREPARE STEEL FOR ITS WORLD'S WORK

In This Issue: *Six articles on Electrified Steel Mill Operation*
together with
Welded Plate Girders *Current Capacity of Wire* *High-speed Recorder*





Foster economizer as installed in the Reading plant of the Metropolitan Edison Company. Hot gases pass upward thru the economizer while water enters the top of the economizer and flows downward thru the tubes, leaving at the bottom.

The Foster application of the counter-current principle, insures maximum efficiency of the unit.

Foster Equipment at
Reading Station
Metropolitan Edison Co.

The Reading plant of the Metropolitan Edison Company has a capacity of 69,000 kw. Foster convection type superheaters are installed in four 1013 hp. Edge Moor boilers designed to give 200 deg. superheat with boilers operating at 175% rating and 265 lb. pressure. Foster radient type superheaters applied to two similar boilers add 200 deg. superheat when the boilers operate at 200% rating.

The plant includes three Foster economizers of 6048 sq. ft. heating surface each. These raise the temperature of the boiler feed water 140 deg. F. at the higher ratings. A [Foster waterback has also been installed in one of the furnaces, adding to the fuel saving effected by Foster apparatus.

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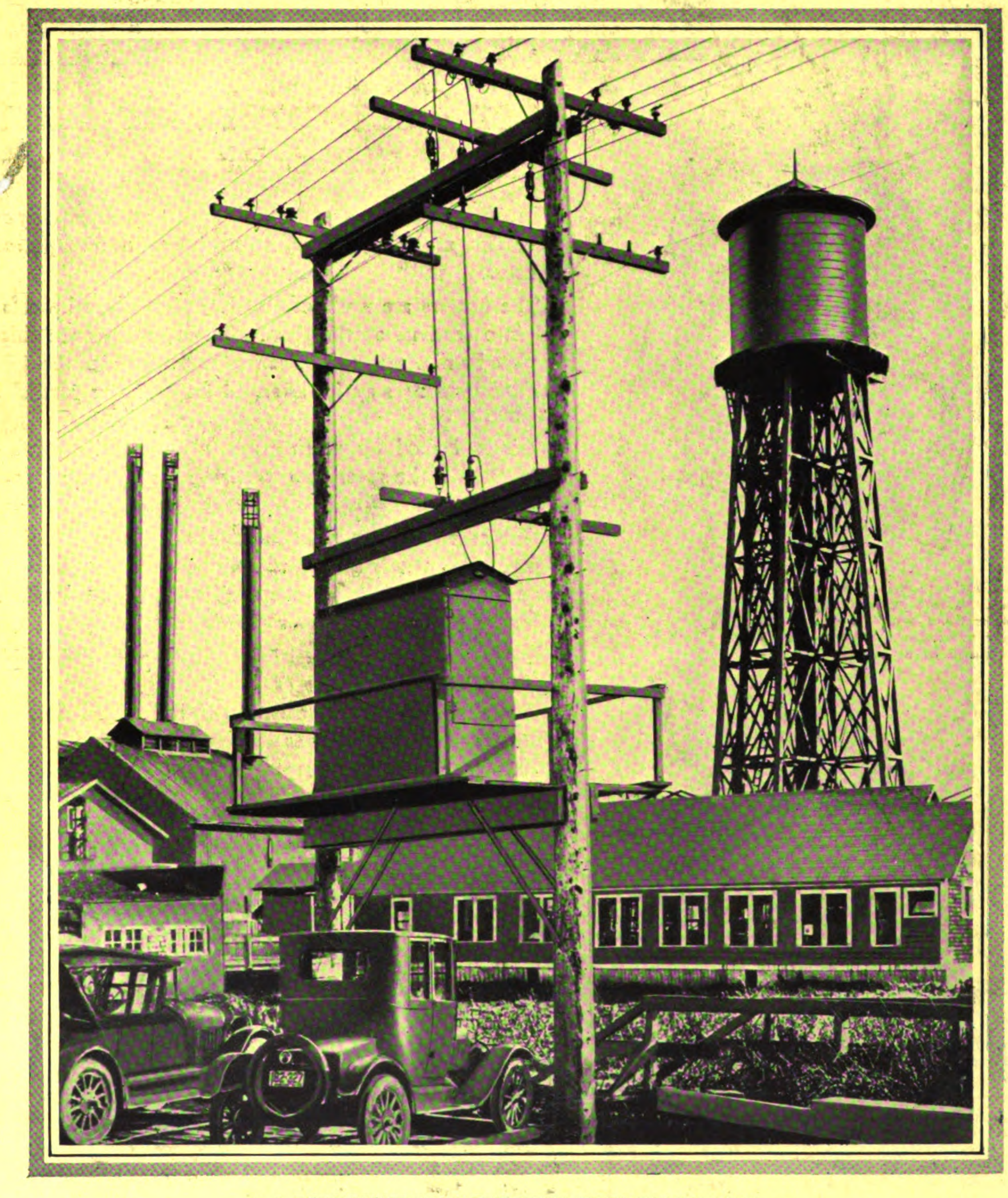
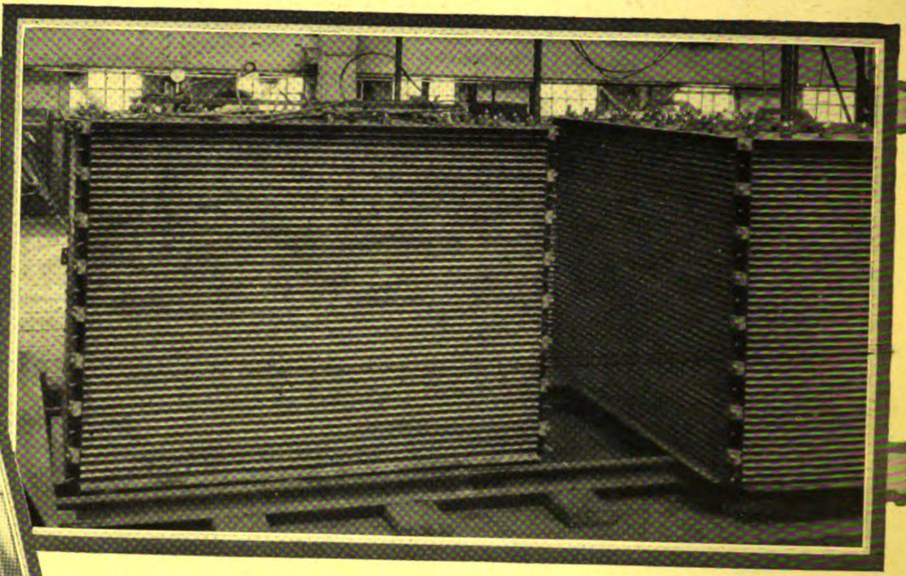


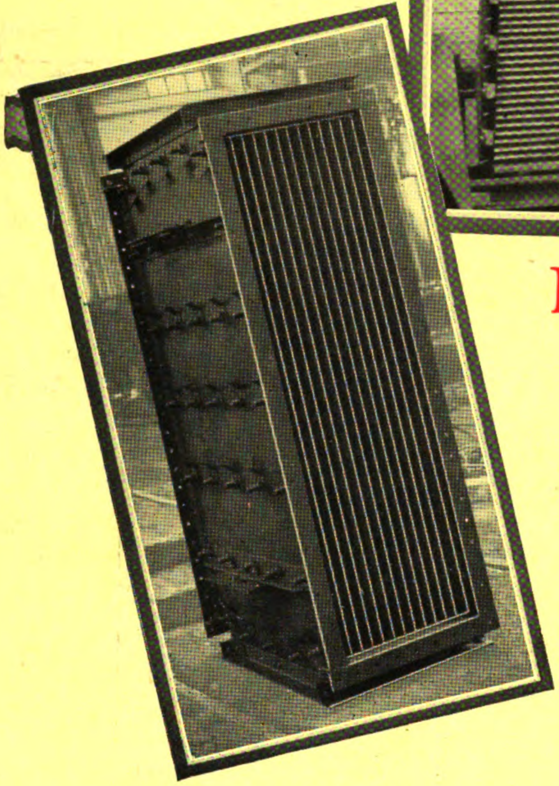
A REDUCTION OF 40 PER CENT IN WATTLESS KV-A.

One of six 300-kv-a. capacitors which raised the power-factor of the Gray's Harbor Railway and Light Company's distribution system from 0.644 to 0.821, or half way to unity. (*See p. 366*)

In This Issue: *Power-factor Improvement* *Wattmeter Connections* *D-c. Transformer* *Oscillation Generators* *Gas Production* *Photo-electric Cells* *Gas-electric Buses* *Synchronous Motor Stability* *Voltage Build-up of Standard Exciters*



Typical Foster air heater sections assembled on jigs preparatory to welding, showing large area of heating surface handled as a unit in each section.



A complete section of a small Foster air heater before shipment. Any number of sections may be connected together to secure series or parallel flow of gases or air, as may be required.

Foster Sectional Air Heaters

(Haber Patents)

The Foster sectional air heater involves new principles in air heater design. They are of great importance to power plant engineers.

The heaters are made of standard sizes of steel sheets assembled into sections at the shop. These sections are shipped as completed units ready for bolting into place on the job. The standard sections may be arranged for series or parallel flow of gases and air.

Air is heated by passing across the heating surface horizontally while the gases flow vertically on opposite sides of the sheets.

The accompanying illustrations show standard units at the factory. Completely assembled heating surface is shown above, before welding and before the application of the framing. The illustration at the left shows a small completed section ready for shipment. The clips on the sides of the casing are for retaining the insulation in place.

Operating records of Stations equipped with Foster sectional air heaters show 5 to 10 per cent savings in fuel and 10 to 20 per cent increase in boiler capacity.

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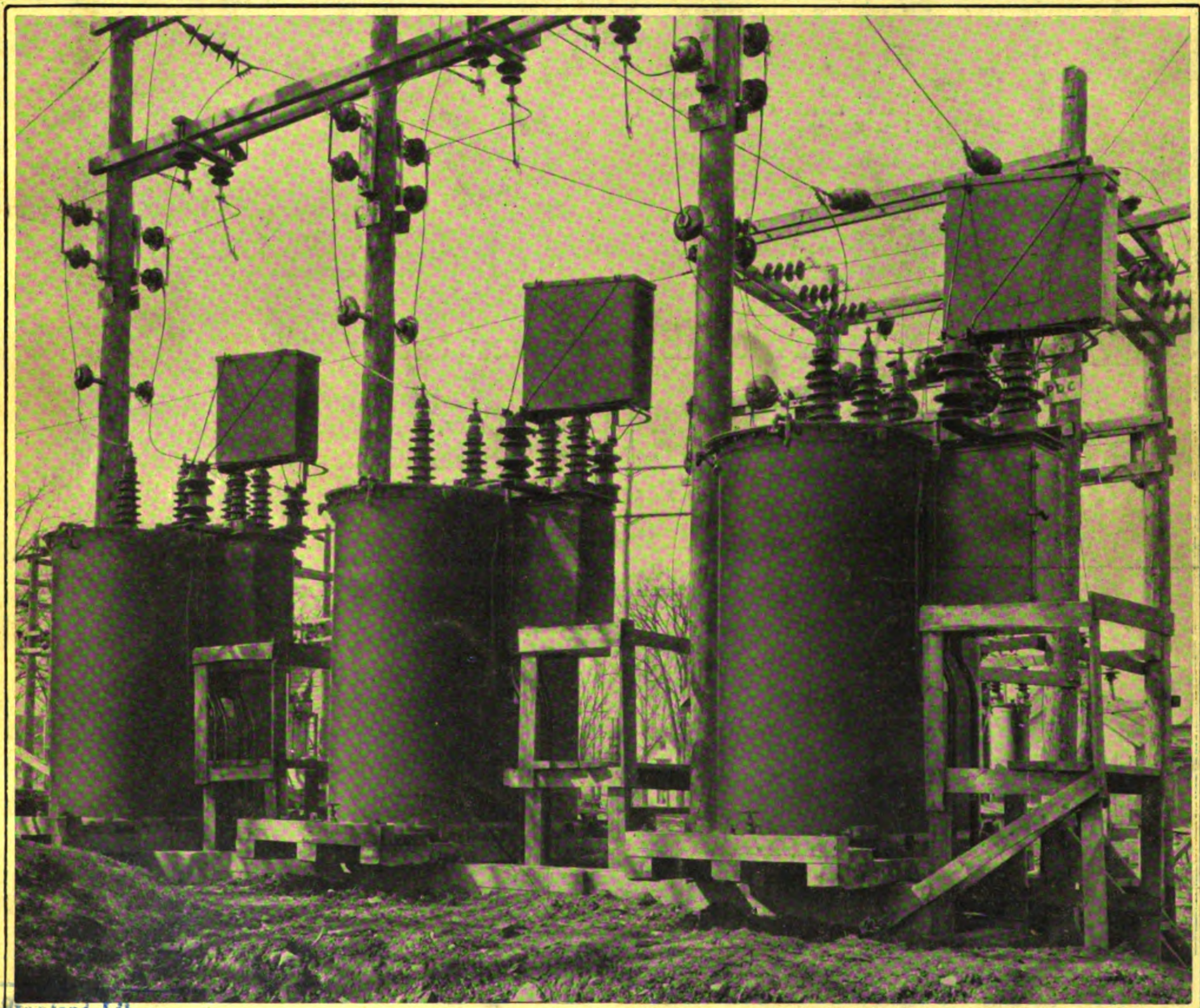
VOL. 31, No. 8 AUGUST, 1928



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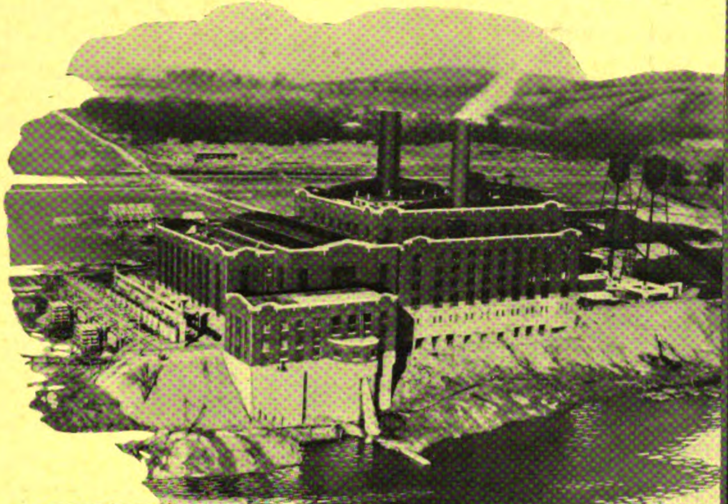
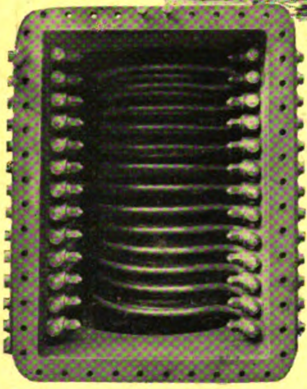
THE FIRST INSTALLATION OF ITS KIND IN THE WORLD

This bank of capacitors connected in series with a three-phase transmission line has demonstrated its ability to improve the characteristics of alternating-current transmission. (*See p. 432*)

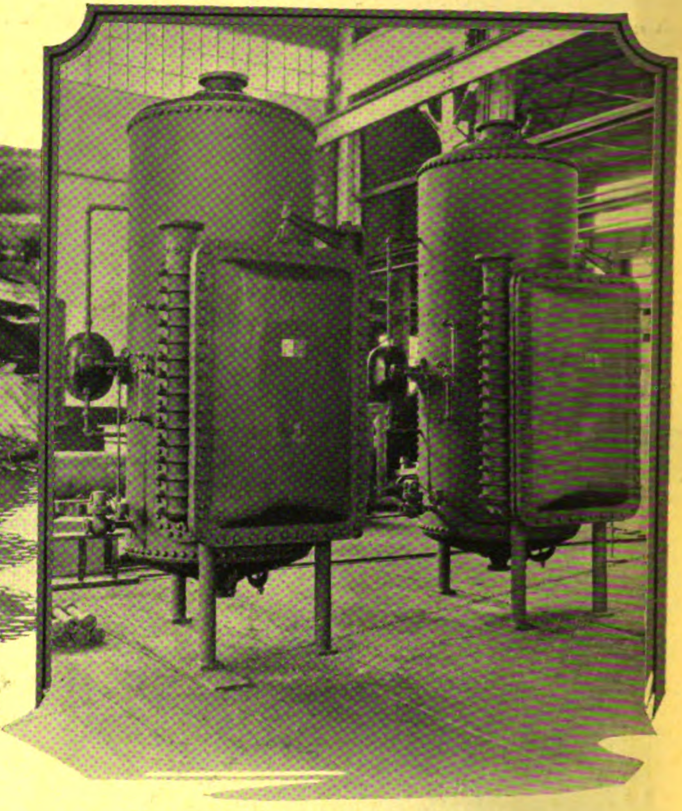
In This Issue: ***Cathode-ray Oscillographs*** ***Series Capacitor Equipment***
Low-voltage A-c. Networks ***Stresses in Bus Structures*** ***Mass Transportation***
Converter Windings ***Compressors for Cupola Blowing*** ***Reflection by Terra Cotta***



Columbia Power Station of the Columbia Gas & Electric Co. Sargent & Lundy, Inc., Engineers.



Door removed from evaporator showing coils and coil couplings. Note simple coil connections. Any coil may be removed without disturbing other coils.



Shop view of Wheeler Contraflo high pressure evaporators, showing davit provided for removing the door; this makes the coils readily accessible.

Columbia Power Station

The make-up water for two 63,000-kw. turbine units at the Columbia Station of the Columbia Gas & Electric Co. is to be supplied by four Wheeler Contraflo Evaporators.

Every power station in this country carrying 600-lb. steam pressure is now equipped with Wheeler Evaporators. These prominent generating stations include Twin Branch, Crawford Ave., Philo and Stanton. Wheeler Evaporators are also being built for installation in the following new power plants: State Line Station (208,000 kw.), Philo Extension (165,000 kw.) and Buenos Aires (100,000 kw.)

Sargent and Lundy, Inc., are the consulting engineers for all of these plants.

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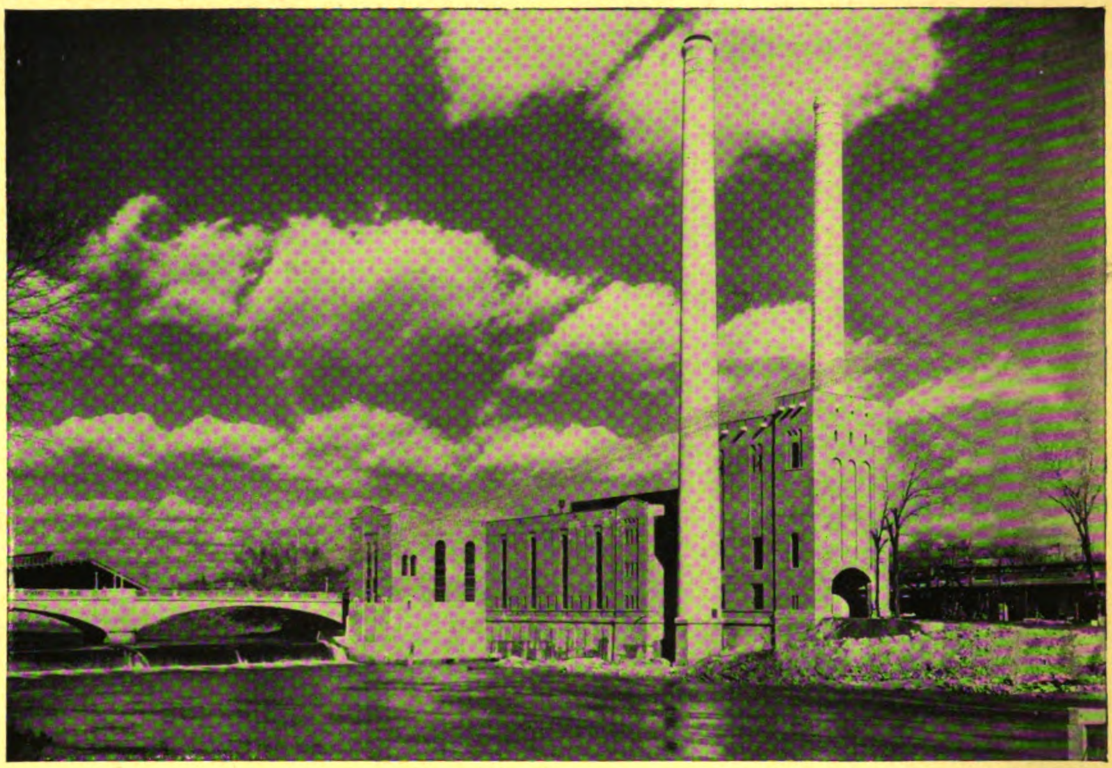
VOL. 31, No. 9 SEPTEMBER, 1928

WITHIN 40 PER CENT OF THE TEMPERATURE OF THE SUN

A 2500-amp. 100-volt 60-cycle single-phase arc between 4-in. graphite electrodes in a brass melting furnace, as viewed through smoked glass. The construction and operation of this type of furnace is described in the article on p. 483

In This Issue: *A-c. High-speed Circuit Breakers* *Rail Bonds* *Single-phase Arc Furnaces* *Carrier-current Street Lighting Control* *Industrial Hydrogen* *Crystal Structure* *Heaviside's Calculus* *Low-voltage A-c. Networks* *Photo-electric Tubes*



View from southwest of heating and power plant
at State University of Iowa, Iowa City, Iowa.

State University of Iowa Adds Two More Foster Superheaters

Additional power at the new plant of the State University of Iowa is to be supplied by two boilers, each equipped with Foster superheaters. The boilers, a 612-hp. Stirling and 631-hp. Murray bent tube type, will operate at a pressure of 175 lb. per sq. in. At 175 per cent normal boiler rating, the Foster superheaters add 100 deg. F. superheat to the steam generated.

Four Foster superheaters were first installed at the State University of Iowa in four 612-hp. Stirling boilers during 1925. They operate at 175 lb. per sq. in. pressure, with 100 deg. F. superheat.

The repeat order for two additional Foster superheaters, bears testimony of the reliability and satisfactory performance of Foster Wheeler equipment.

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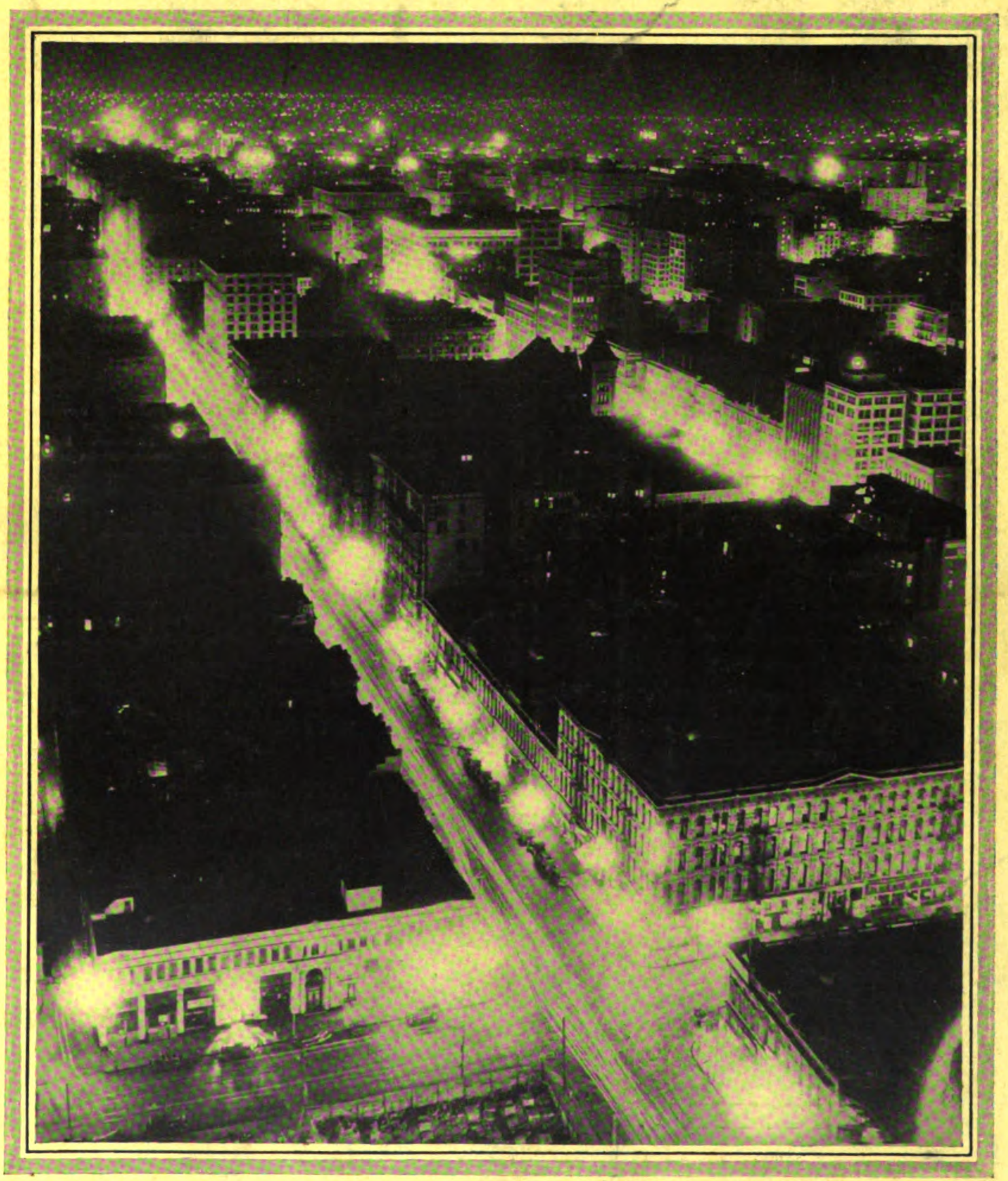
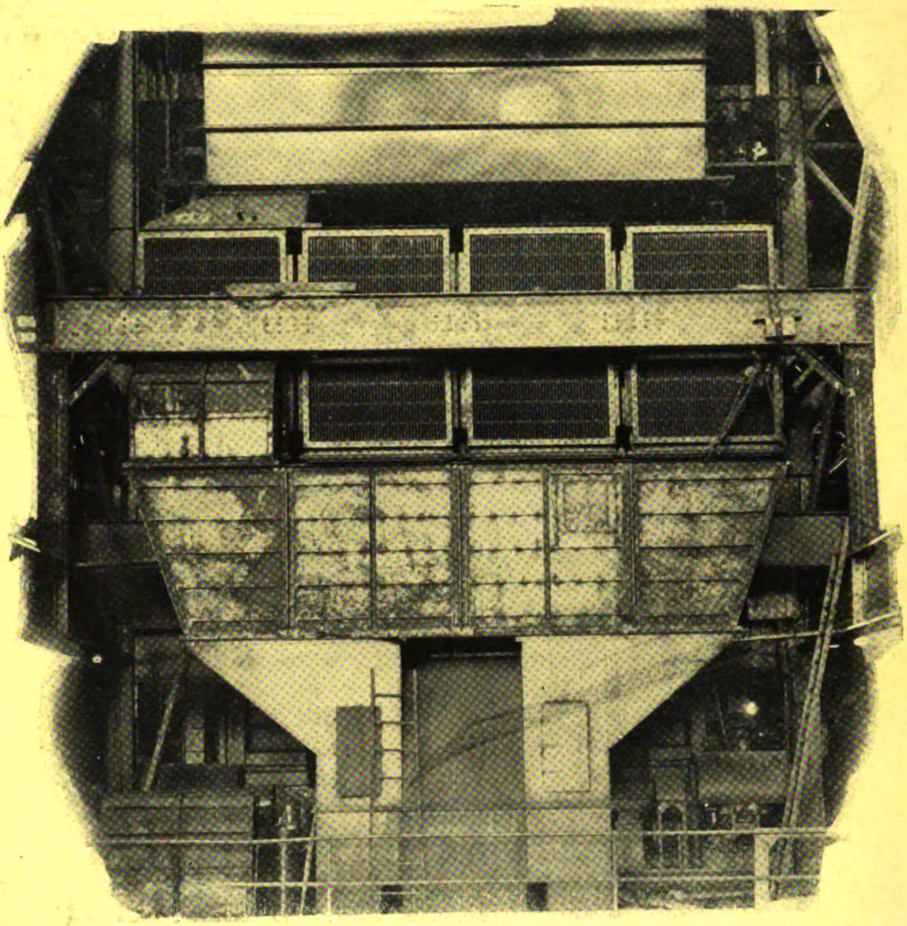


Photograph by International Newsreel

There, side by side, and on into the night
Run avenues of friendly light;
And we behold the busy haunts of men
Revealed as in the day again.
—*Anonymous*

In This Issue: ***Resistor Furnaces*** ***Rotor Balancing*** ***Things We Don't Think Of***
Electric Micrometer ***20-amp. Street Lighting*** ***Improved Single-phase Motors***



Foster sectional air heater in the course of erection. This heater, containing 12,000 sq. ft. of surface consists of 8 interchangeable sections, arranged four passes in parallel. Each pass is made up of an upper and lower section connected in series.

Foster Sectional Air Heater

(Haber Patents)

to Preheat 250,000 lb. of Air per Hour

The Foster sectional air heater, in course of erection as shown above, is now operating in connection with a 1400 hp. B. & W. cross drum boiler.

The eight sections that compose this heater are arranged so that the hot gases go vertically upward through them. Air is admitted at the upper sections, is drawn horizontally through them, then downward through connecting ducts at the back, and forward horizontally, through the lower sections. The heated air then flows down, through the casing shown, to the furnace.

This Foster air heater raises the temperature of 250,000 lb. of air per hour from 80 deg. F. to 204 deg. F. at the same time cooling the flue gases to a temperature of 294 deg. F.

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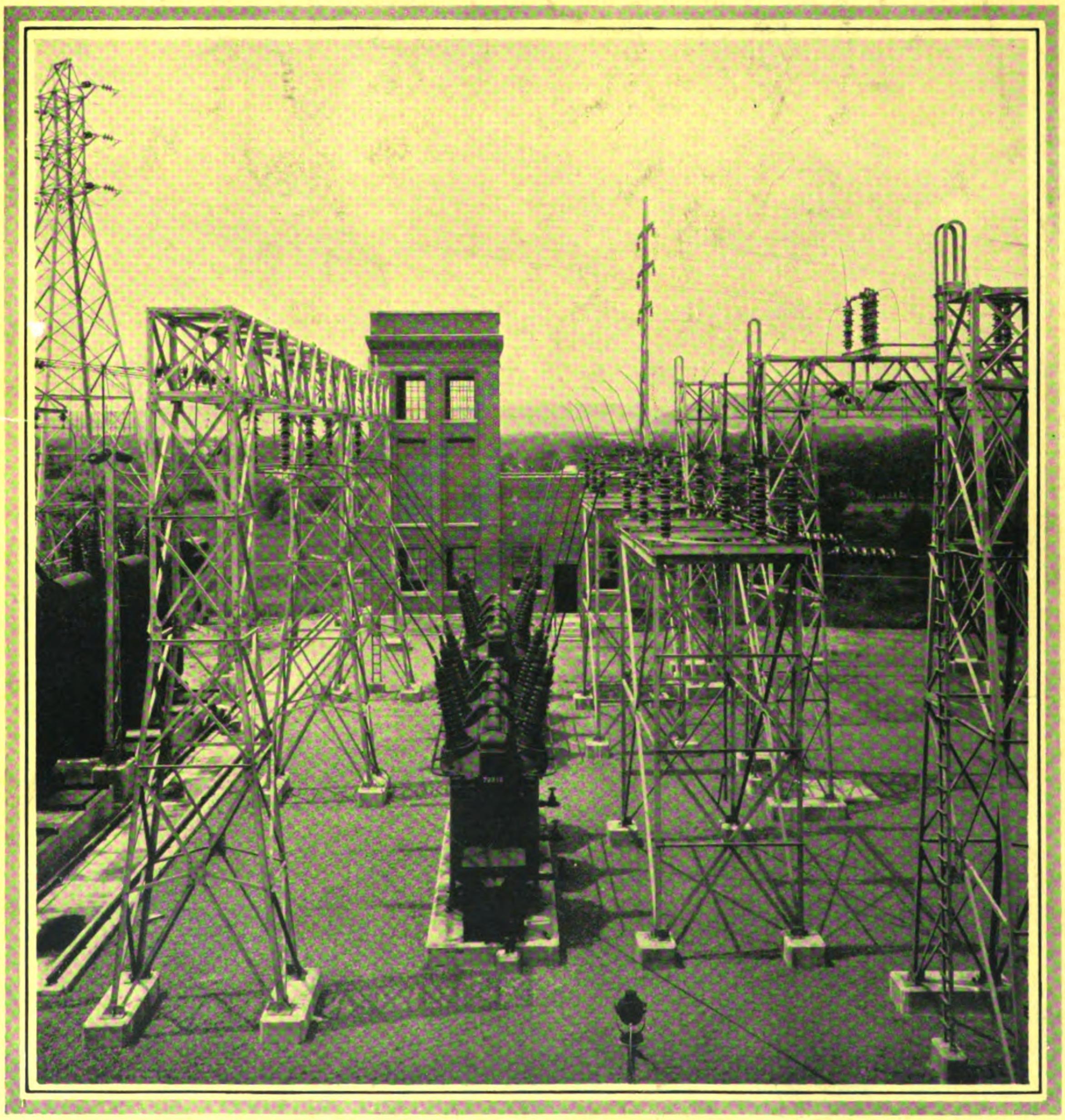
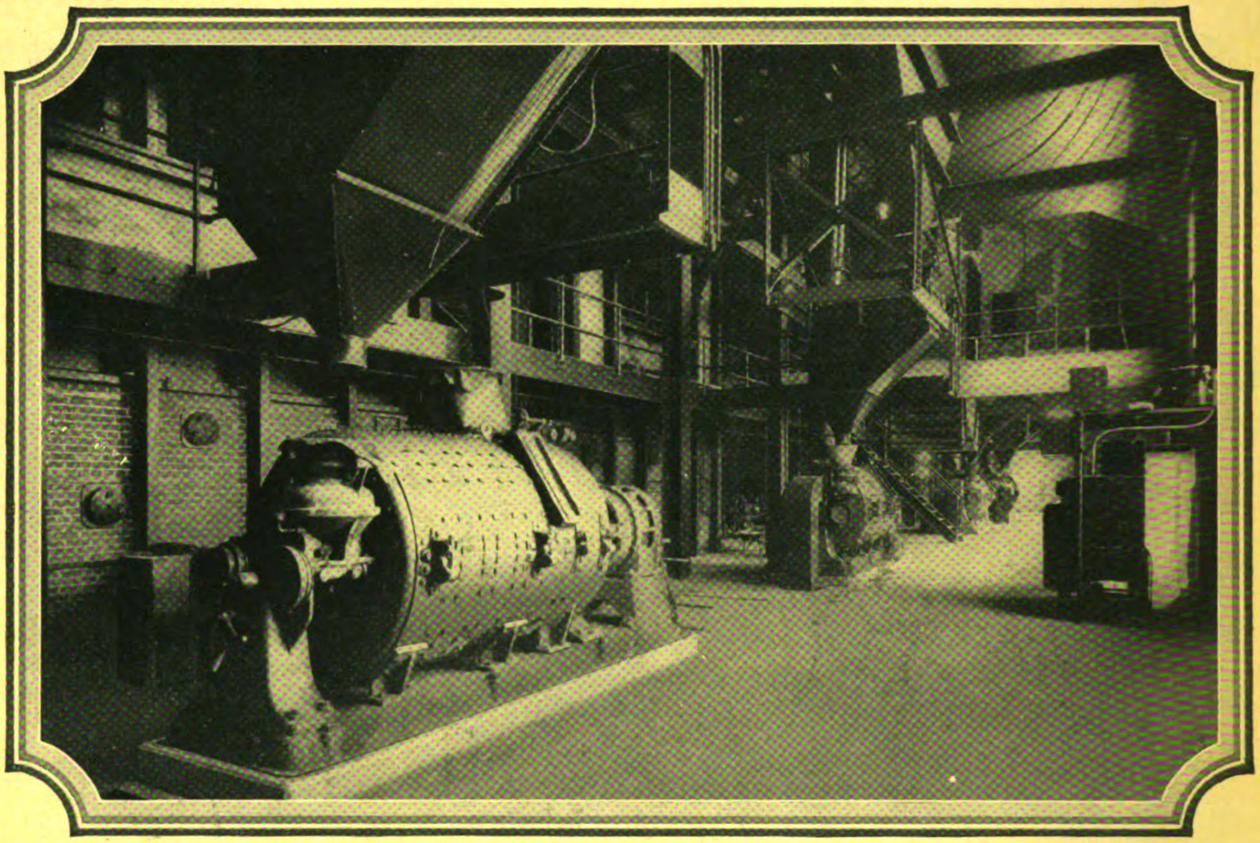
VOL. 31, No. 11 NOVEMBER, 1928



THE TOWERS AND THE WATCHMEN

Transmission lines and switch yard of a new outdoor substation. The building in the rear of the 2600-amp. 110,000-volt circuit breakers was erected for servicing the station equipment. (*See p. 579*)

In This Issue: ***Street Railways*** ***Carboloy Cutting Tools*** ***Outdoor Substation***
Crystal Structure ***Kenotron Testing Set*** ***High-voltage D-c. Machines***
Driven Grounds ***Low-voltage A-c. Networks*** ***Decline in Luminosity***



Aero Unit Pulverizers

THE four Aero unit pulverizers illustrated above, are firing four 1035 hp. Kidwell boilers in this up-to-date power plant. Normal operation is at 250 per cent rating, although the Aero pulverizers are capable of providing operation up to 333 per cent of the rated boiler capacity.

These Aero units have a capacity of pulverizing 10,000 lb. of coal per hour continuously and 12,000 lb. per hour for peak load operation. The pulverized fuel is fed directly to the furnace from the pulverizers, eliminating any storage of powdered coal. Control of combustion is easy and instantaneous over a wide range of operating loads.

Foster superheaters, economizers and water walls are installed in the boilers at this plant. The steam generating units are unusually compact and are designed for an operating efficiency of 85.6 per cent.

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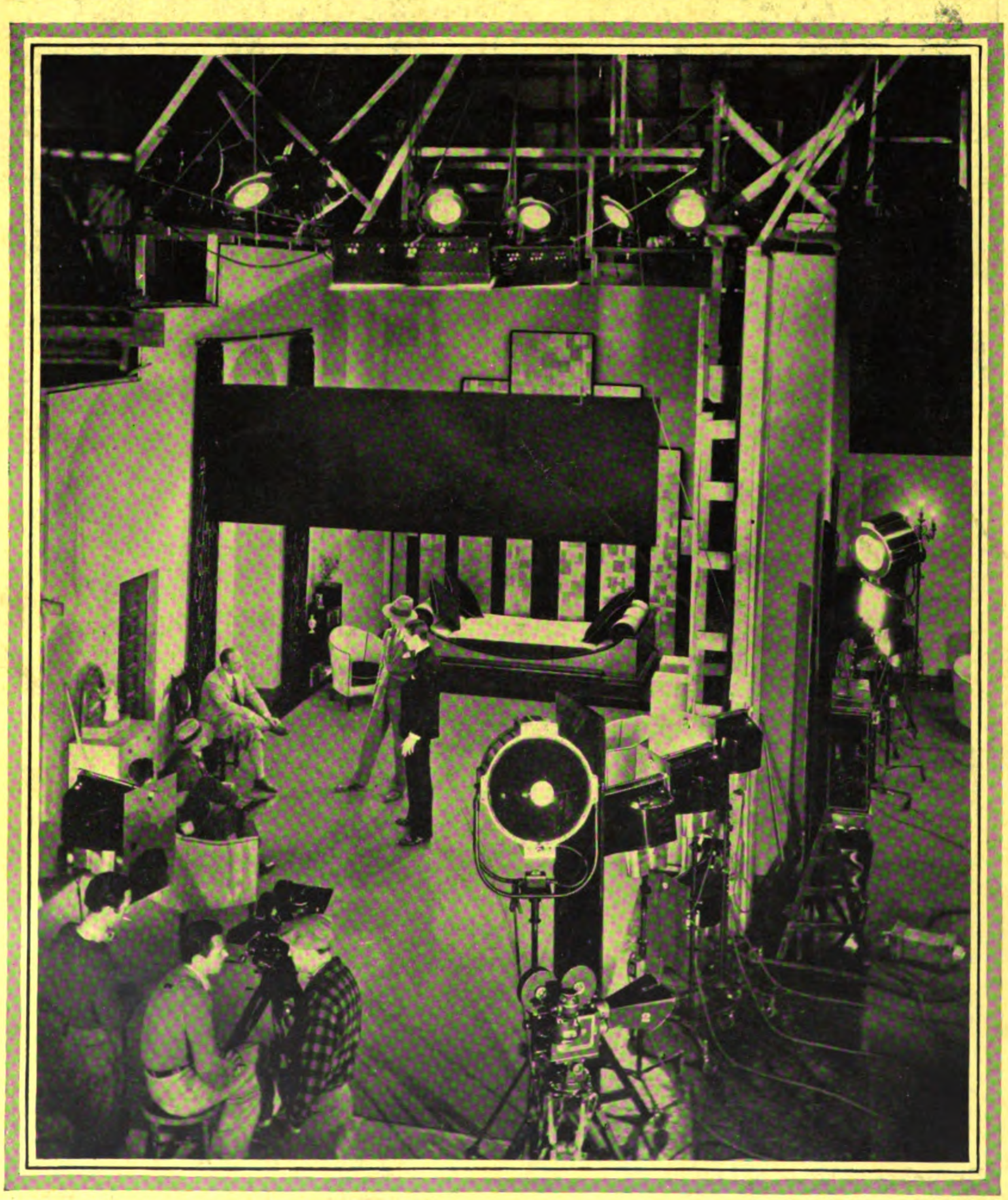

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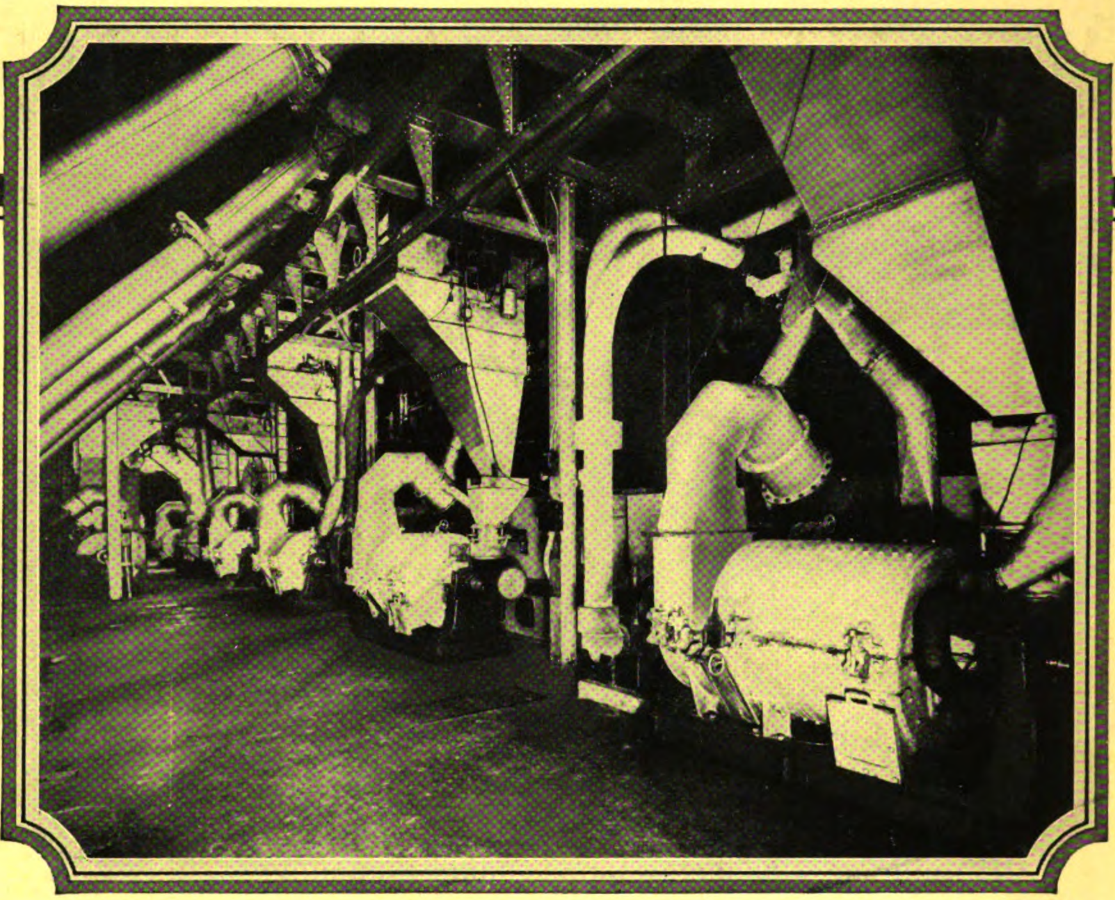
INCANDESCENT SUNSHINE IN THE MOVIE STUDIO

Exclusive use was made of the newly-perfected incandescent lighting units in the filming of "Our Dancing Daughters," a set of which is here shown. A description of this system of illumination appears on p. 648

In This Issue: ***Recording Instruments*** ***Resistor Furnaces*** ***Research and Progress***
Low-voltage A-c. Networks ***Current Transformers*** ***Flashing of Converters***
Oscillation Generators ***Car-lighting Equipment*** ***Motion Pictures and Mazda Lamps***



Pulverized Fuel Firing by the Aero Unit System

THE Aero unit system of pulverized fuel firing is a direct and efficient method of burning solid fuels. The coal is pulverized and mixed with primary air in the Aero machine. It is then delivered directly to the furnace, there being no storage of powdered coal.

Control of combustion with the Aero system is simple and practically instantaneous over a wide range of boiler ratings.

The Aero units shown above are firing seven 600-hp. Edge Moor boilers. Normal operation is at 250 per cent normal boiler rating. The Aero pulverizers, in this case, are designed for operation up to 300 per cent rating.

An investigation of this system of pulverized fuel firing is well merited. Its characteristics are reliability, low maintenance and instantaneous flexibility.

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